College de France, May 28, 2013

SUPERFLUIDTY IN ULTRACOLD ATOMIC GASES (1D SPIN ORBIT COUPLED BEC'S)

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PLAN OF THE LECTURES

Lecture 1. **Brief summary of superfluidity** in ultracold gases.
Some open questions

Lecture 2. A tale of two sounds (**first and second sound**)

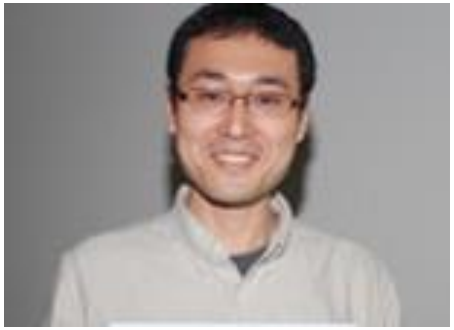
Lecture 3. Spin-orbit (SO) coupled Bose-Einstein condensed gas:
new quantum phases and **anisotropic superfluidity**

Lecture 4. **Superstripes and supercurrents** in SO coupled BECs

Yun Li

Giovanni Martone

Lev Pitaevskii



Tomoki Ozawa

- Recent **experimental realization** of syntethic gauge fields is providing new challenging many-body configurations in ultra-cold atomic gases

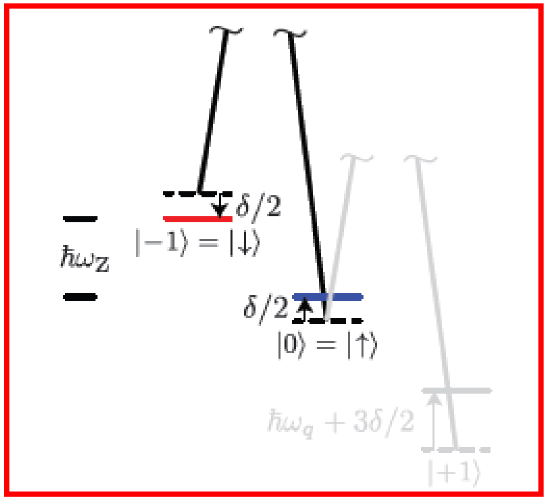
- **Conservation laws are modified** due to spin coupling

- **New quantum phases** (stripes, spin polarized, vortices) and new phase transitions in both Bose and Fermi gases

- **New dynamic** properties

- **Center of mass** oscillation in harmonic trap **does not obey Kohn's theorem**. Consequence of violation of Galilean invariance. (Experiments already available)

- Elementary excitations in uniform gases: **quenching of sound velocity**, appearence of **rotons**



Two detuned and **polarized** laser beams provide Raman transitions between two spin states giving rise to the Hamitonian

$$h_0^{lab} = \frac{\vec{p}^{\,2}}{2} + \frac{\Omega}{2}\sigma_x \cos(2k_0 x - \Delta\omega_L t) + \frac{\Omega}{2}\sigma_y \sin(2k_0 x - \Delta\omega_L t) - \frac{\omega_Z}{2}\sigma_z$$

The Hamiltonian is not translationally invariant. It is however invariant with respect to helicoidal translations of the form (**continuous** symmetry)

$$e^{id(p_x - k_0\sigma_z)}$$

consisting of a rigid **translation** plus **rotation** in **spin space**

The peculiar symmetry property of h_0^{lab}
suggests the introduction of the unitary transformation

$$\boxed{e^{i\Theta\sigma_z/2}}$$ with $\Theta = 2k_0 x - \Delta\omega_L t$

In the spin rotated frame h_0 takes the spin-orbit form

$$\boxed{\begin{aligned} h_0 &= e^{-i\Theta\sigma_z/2} h_0^{lab} e^{i\Theta\sigma_z/2} = \\ &\frac{1}{2}[\left(p_x - k_0\sigma_z\right)^2 + p_\perp^2] + \frac{1}{2}\Omega\sigma_x + \frac{1}{2}\delta\sigma_z \end{aligned}}$$

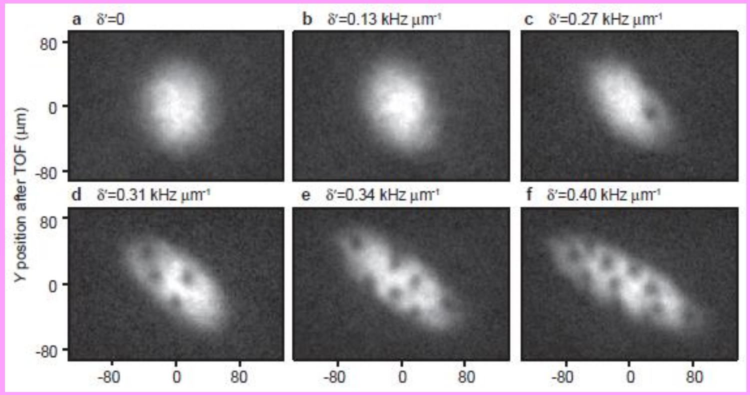
1D spin-orbit coupling

characterized by equal Rashba and Dresselhaus coupling
plus Raman and effective Zeeman field $\delta = \Delta\omega_L - \omega_Z$

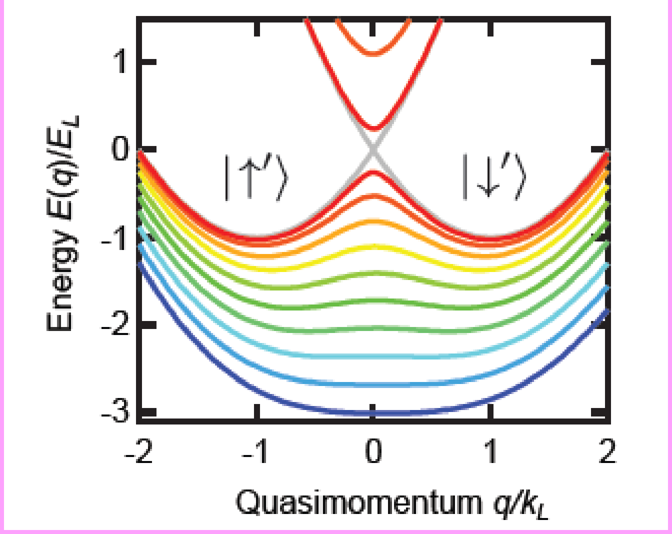
The two body interaction term is not affected by the transformation.

Different strategies (both pursued by Spielman team at Nist)

- Spatially dependent detuning ($\Delta\omega_L(z)$) in the strong Raman coupling (Ω) regime gives rise to **effective Lorentz force** and vortices. The method is not subject to limitations of rotating systems. Possible route to quantum Hall regime.
(Lin et al., Nature 2009)



- Working with vanishing effective Zeeman field and small Raman coupling gives rise to the appearence of **two minima** which can host a Bose-Einstein condensate.
(Lin et al., Nature 2011)
Strategy of lectures 3 and 4



Differently from h_0^{lab} the Hamiltonian in the spin rotatated frame is **translationally invariant** , so, unless translational invariance is spontaneously broken (stripe phase), the **density** of the ground state configuration is **uniform** !

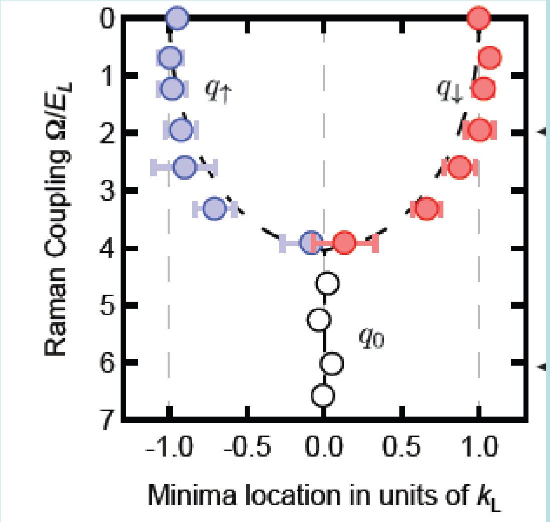
The full Hamitonian is given by $$H = \sum_i h_0(i) + \sum_{\alpha,\beta} \frac{1}{2} \int d\vec{r} g_{\alpha\beta} n_\alpha n_\beta$$

with $$h_0 = \frac{1}{2}[\left(p_x - k_0\sigma_z\right)^2 + p_\perp^2] + \frac{1}{2}\Omega\sigma_x + \frac{1}{2}\delta\sigma_z$$ $$[H, P_x] = 0$$

Interactions are treated in the mean field approximation.
We make the simplifying choice $$g_{\alpha\alpha} = g_{\beta\beta} \equiv g$$ $$\delta = 0$$

If $g_{\uparrow\uparrow} \neq g_{\downarrow\downarrow}$ one can choose an effective magnetic field to compensate the effect of asymmetry.

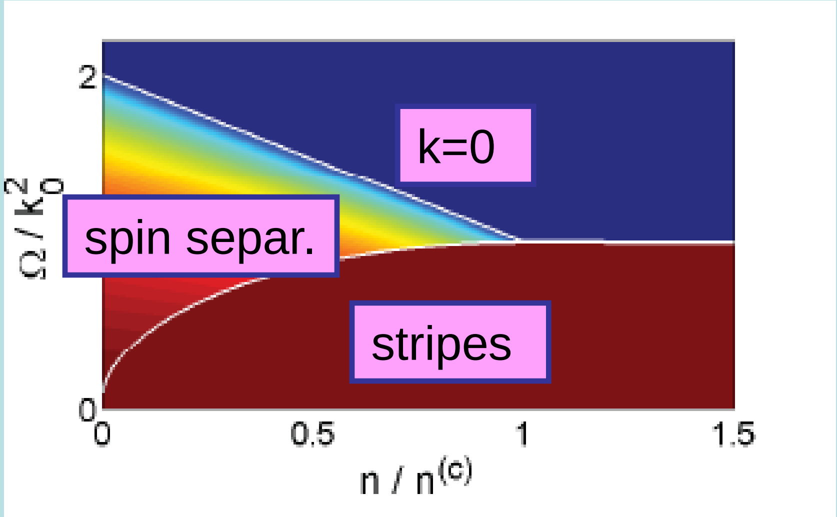
Experimental implementation of the SO hamiltonian with BECs by the Spielman team at NIST (Nature 2011)

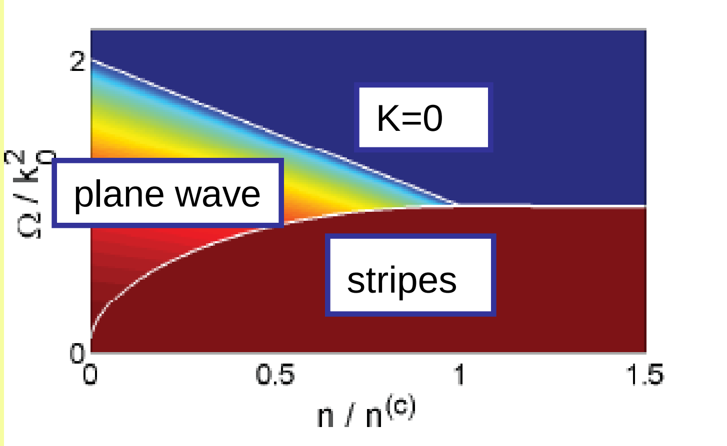


Theory of the new quantum phases:
Ho and Zhang (PRL 2011)
Many theoretical papers (……)
Recent Trento paper
(Yun Li, Pitaevskii, S. PRL 2012)

$$H = \sum_i h_0(i) + V_{\text{int}}$$

$$g_{\uparrow\uparrow} = g_{\downarrow\downarrow} > g_{\uparrow\downarrow}$$





Order parameter of the new phases

- **Stripe phase**

$$\Psi = \sqrt{\frac{N}{2V}}\left[\begin{pmatrix}\cos\theta \\ -\sin\theta\end{pmatrix} e^{ik_1x} + \begin{pmatrix}\sin\theta \\ -\cos\theta\end{pmatrix} e^{-ik_1x}\right] \Rightarrow \boxed{n(x) = n\left(1+\frac{\Omega}{2k_0^2}\cos 2k_1x\right)}$$

- **Plane wave** (or spin polarized) **phase.**

BEC occupies state

$$\Psi = \sqrt{\frac{N}{V}}\begin{pmatrix}\sin\theta \\ -\cos\theta\end{pmatrix} e^{-ik_1x} \quad or \quad +k_1$$

$$\cos 2\vartheta = k_1 / k_0$$

$$<\sigma_z> = \frac{k_1}{k_0}$$

with $k_1 = k_0\sqrt{1-\Omega^2/[2k_0^2 + n(g+g_{\uparrow\downarrow})/2]^2}$

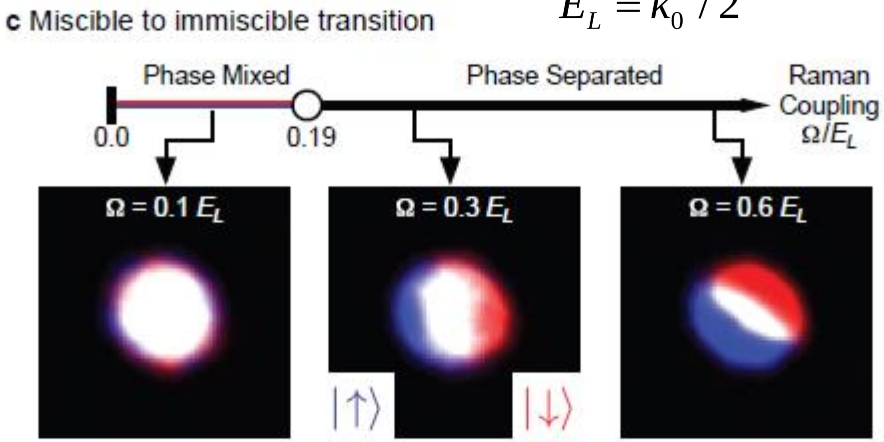
- **Zero momentum phase** ($k_1 = 0$)

$$\Psi = \sqrt{\frac{N}{2V}}\begin{pmatrix}1 \\ -1\end{pmatrix}$$

Nature of phase transitions

The transition between the **stripe** and the **plane wave** phase is of first order nature.
It has been observed at the predicted value of the Raman coupling
$$\Omega_{cr} = 2k_0^2 \sqrt{\frac{2\gamma}{1+2\gamma}}$$
(eq. holding in the small coupling limit $gn\,,\, g_{+-}n << k_0^2$
$\gamma = (g - g_{\uparrow\downarrow})/(g + g_{\uparrow\downarrow})$)

$E_L = k_0^2/2$



Lin et al., Nature 2011

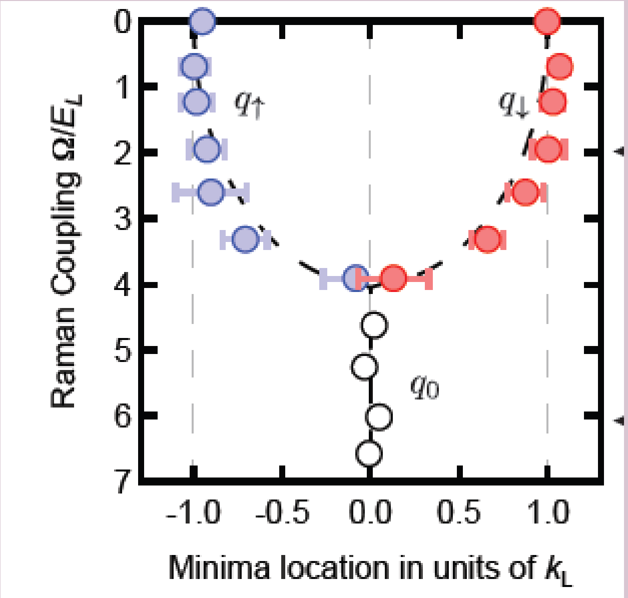
Density modulations are not visible in the **in situ profile** of the stripe phase (contrast and distance between stripes are too small)
Effects of stripes more easily revealed in the **excitation spectrum** (Lecture 4)

Nature of phase transitions

The transition between the **plane wave** and **the k=0** phase is second order.
It has been observed at the predicted value of the Raman coupling

$$\Omega = 2k_0^2 - \frac{1}{2}n(g - g_{\uparrow\downarrow}) \approx 2k_0^2$$

in small coupling limit

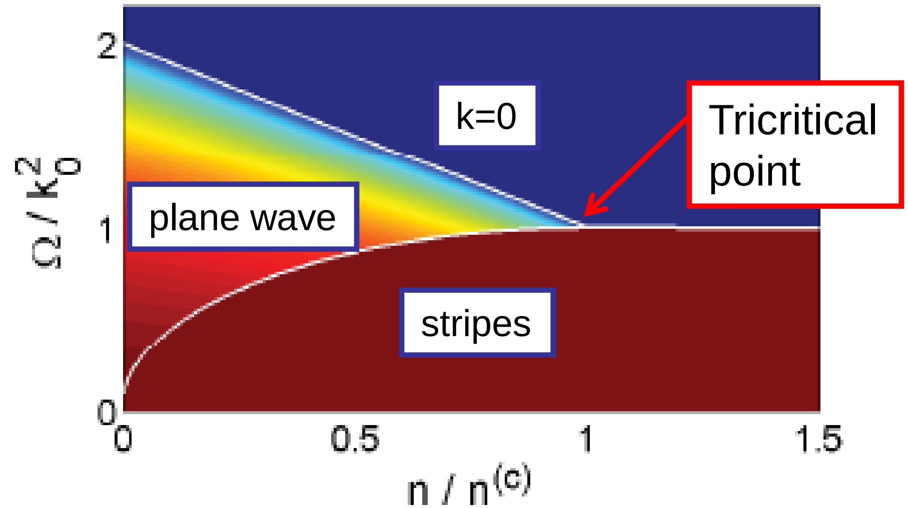


$E_L = k_0^2/2$

At the transition the **spin polarizability diverges**, with peculiar consequences on the collective oscillations and the propagation of sound.

Tricritical point

(Li et al. PRL 2012)



Above the critical density $$n_c = \frac{k_0^2}{2g}\frac{g+g_{\uparrow\downarrow}}{g-g_{\uparrow\downarrow}}$$

The plane wave phase disappears
and the transition takes place directly between the
stripe and the zero momentum phases

Value of critical density is very large in the case of Rb
because coupling constants $g_{\uparrow\uparrow}, g_{\downarrow\downarrow}, g_{\uparrow\downarrow}$ are practically equal

Consequences of spin-orbit coupling
on the **dynamics** of BECs

Center of mass oscillation in harmonic trap

Sound and rotons in uniform gases

This lecture only phases with **uniform** density
(plane wave and k=0 phases)

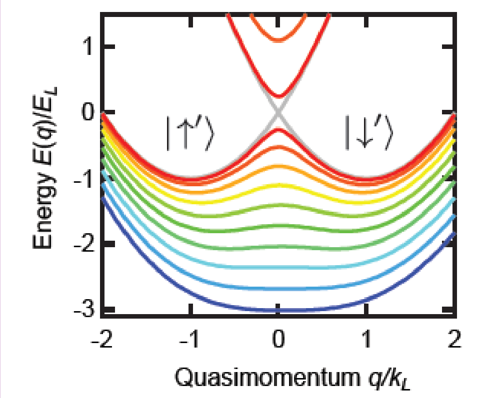
Next lecture: sound in **stripe** phase (supersolidity)

Semi-classical argument:

$$h_0 = \frac{1}{2}[(p_x - k_0\sigma_z)^2 + p_\perp^2] + \frac{1}{2}\Omega\sigma_x$$

The single-particle Hamiltonian gives rise to two branches with single-particle energy:

$$\varepsilon_\pm(\vec{p}) = \frac{p_x^2 + p_\perp^2 + k_0^2}{2} \pm \sqrt{k_0^2 p_x^2 + \frac{1}{4}\Omega^2}$$



Important spin-orbit effect on the **effective mass** of lower branch.
Second derivative calculated at the stationary point (local minima):

$$\frac{m}{m^*} = \frac{\partial^2 \varepsilon_-(\vec{p})}{\partial p_x^2} = \begin{cases} 1 - \dfrac{1}{4}\dfrac{\Omega^2}{k_0^4} & \textit{in plane wave phase} \\ 1 - \dfrac{2k_0^2}{\Omega} & \textit{in zero momentum phase} \end{cases}$$

vanishes at the second order phase transition $\Omega = 2k_0^2$

Profound effect on center of mass frequency and sound velocity

Center of mass oscillation

Oscillation in the presence of harmonic trapping:

$$V_{HO} = \frac{1}{2}\left(\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2\right)$$

(Yun Li, G. Martone and S.S, EPL 2012)

Coupling between center of mass ($X = \sum_i x_i$) and Spin ($\sigma = \sum_i \sigma_i$) degrees of freedom

Explicitly revealed by **commutation rule**

$$[H, X] = -i\left(P_x - k_0 \sigma_z\right)$$

- Reflects modification of **equation of continuity**
- Implies **new dynamic** behavior of center of mass coordinate

In the absence of spin-orbit coupling dipole operator X excites a single mode with frequency ω_x (Kohn's theorem).
Kohn's theorem is violated by spin-orbit coupling

Sum rule approach to the center of mass excitation

$$m_k(X) = \int d\omega\, S_X(\omega)\omega^k = \sum_n |<0|X|n>|(E_n - E_0)^k$$

Energy weigthed (k=1) sum rule (f-sum rule) :

$$m_1(X) = \frac{1}{2} < [X,[H,X]] > = \frac{N}{2}$$

- Comment: Despite the fact that $[H, X]$ is affected by spin-orbit the double commutator is not.

- **Universal** and **model independent f-sum rule.**

Inverse energy weigthed (k=-1) sum rule

(dipole polarizability sum rule) :

The $m_{-1}(X)$ sum rule can be calculated exactly in the presence of harmonic trapping using the (exact) commutation relation:

$$\boxed{[H, P_X] = i\omega_x^2 X} \qquad P_X = \sum_i p_{i,x}$$

(follows from translation invariance of two-body interaction)

One finds:

$$\boxed{m_{-1}(X) = \sum_n \frac{|<0|X|n>|^2}{E_n - E_0} = -\frac{i}{2\omega_x^2} <0|[X, P_X]|0> = \frac{N}{2\omega_x^2}}$$

- Neither **energy** weighted nor **inverse-energy** weighted sum rules are **affected** by **spin-orbit** coupling

Where does the spin orbit coupling enter dynamics ?

Cubic inverse energy weigthed (k=-3) sum rule

- Very sensitive to **low energy** part of excitation spectrum (low branch of excitation spectrum).
- Can be worked out using exact commutation rules

First step: use commutation relation $[H, P_X] = i\omega_x^2 X$

$$m_{-3}(X) = \sum_n \frac{|<0|X|n>|^2}{(E_n - E_0)^3} = \frac{1}{\omega_x^4}\sum_n \frac{|<0|P_X|n>|^2}{E_n - E_0} =$$
$$= \frac{1}{\omega_x^4}\sum_n \frac{|<0|P_X - k_0\sigma_z|n>|^2}{E_n - E_0} + \frac{k_0^2}{\omega_x^4}\sum_n \frac{|<0|\sigma_z|n>|^2}{E_n - E_0}$$

Second step: use commutation rule $[H, X] = -i\left(P_x - k_0\sigma_z\right)$ and identify **spin polarizability**

Cubic inverse energy weigthed (k=-3) sum rule

$$m_{-3}(X)=\frac{N}{2\omega_x^4}[1+k_0^2\chi(\sigma_z)]$$

With $\chi(\sigma_z)=2m_{-1}(\sigma_z)\equiv$ spin polarizability.

Comments on sum rules:

- Results for k=1, k=-1 and k=-3 sum rules **hold exactly** for spin-orbit Hamiltonian + 2-body int. + harmonic trapping

- Hold for both **Bose** and **Fermi** statistics.

- Are **not restricted** to **mean field** regime

- k=-3 sum rule emphasizes key role played by spin-orbit coupling through the **spin polarizability**

Frequency of lowest dipole mode

The ratio between $m_{-1}(X)$ and $m_{-3}(X)$ sum rules provides useful estimate for the frequency of the dipole oscillation:

$$\omega_D = \omega_x \sqrt{\frac{1}{1+k_0^2\chi(\sigma_z)}}$$

Comments:

- ω_D provides rigorous **upper bound** to lowest frequency excited by dipole operator X

- f-sum rule $m_1(X)$ useless to describe lowest frequency mode in the presence of spin-orbit coupling

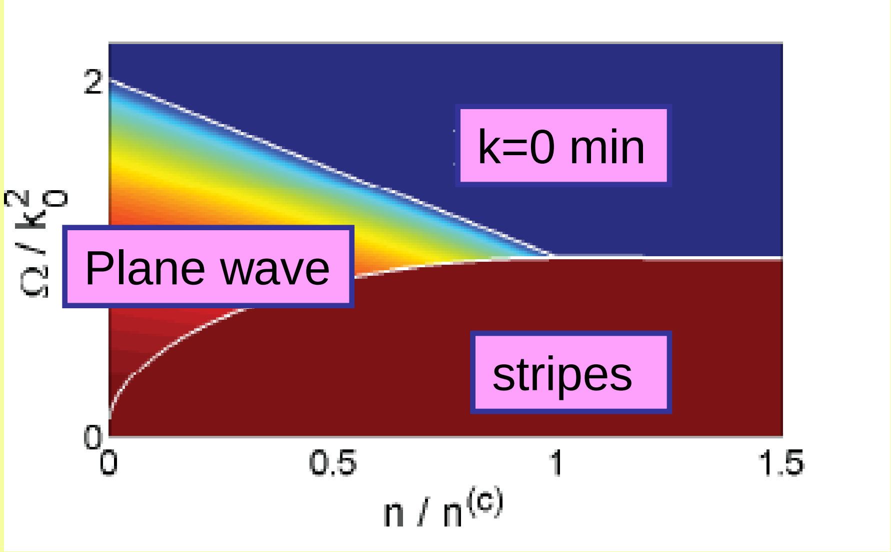
- result for ω_D is accurate for $\Omega >> \omega_x$. For smaller values of Raman coupling, lowest dipole mode is a pure spin oscillation with no coupling with center of mass oscillation

Behavior of spin polarizability

- Calculation of $\chi(\sigma_z)$ based on standard definition: evaluate spin polarization induced by external magnetic field

- results for polarizability depend on the phase considered

- Non trivial results for the behavior of $\chi(\sigma_z)$ at the transition between the quantum phases



Spin polarizability of spin-orbit coupled BEC

(weak coupling limit $gn\,,\,g_{+-}n << k_0^2$)

PLANE WAVE PHASE

$$\chi(\sigma_z) = \frac{\Omega^2}{k_0^2(4k_0^2 - \Omega^2)}$$

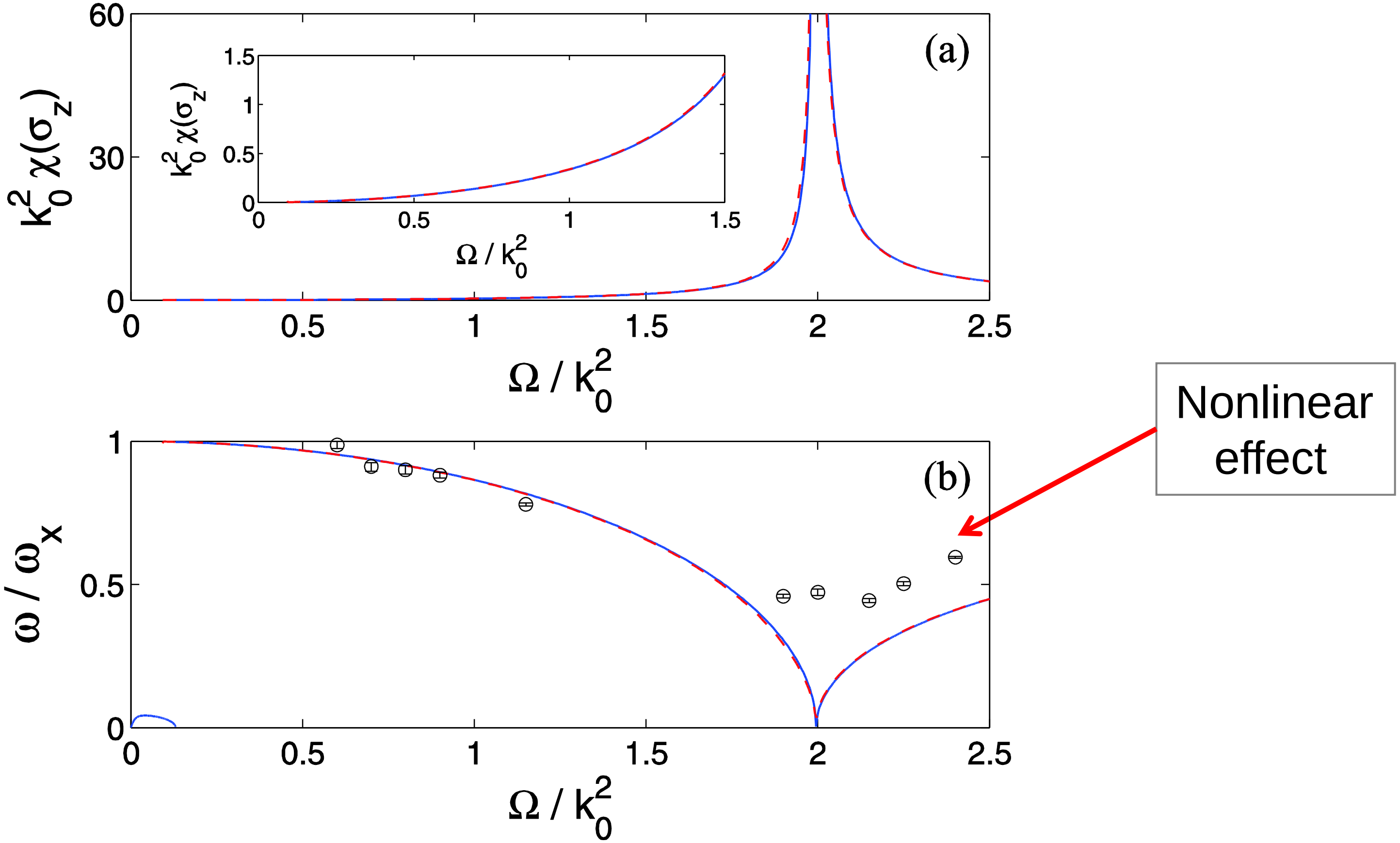
k=0 MOMENTUM PHASE

$$\chi(\sigma_z) = \frac{2}{\Omega - 2k_0^2}$$

- Spin polarizability **diverges** at the transition $\boxed{\Omega = 2k_0^2}$ between plane wave and k=0 phases (second order phase transition).

- Spin polarizabilities are **density independent** in the weak coupling limit (genuine effect of spin-orbit coupling)

Spin polarizability and dipole frequency
in weak coupling regime (exp: Zhang et al. PRL 2012)



- Divergent behavior of spin polarizability results in **strong quenching** of center of mass oscillation !

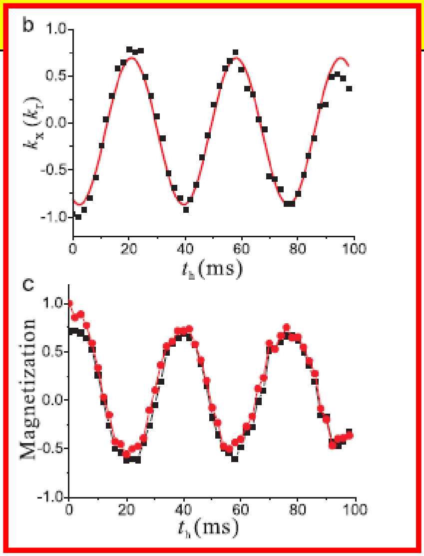
Effect of spin orbit coupling on amplitudes of oscillations

- During dipole oscillation, center of mass position X, momentum P_X and magnetization σ_Z oscillate in time

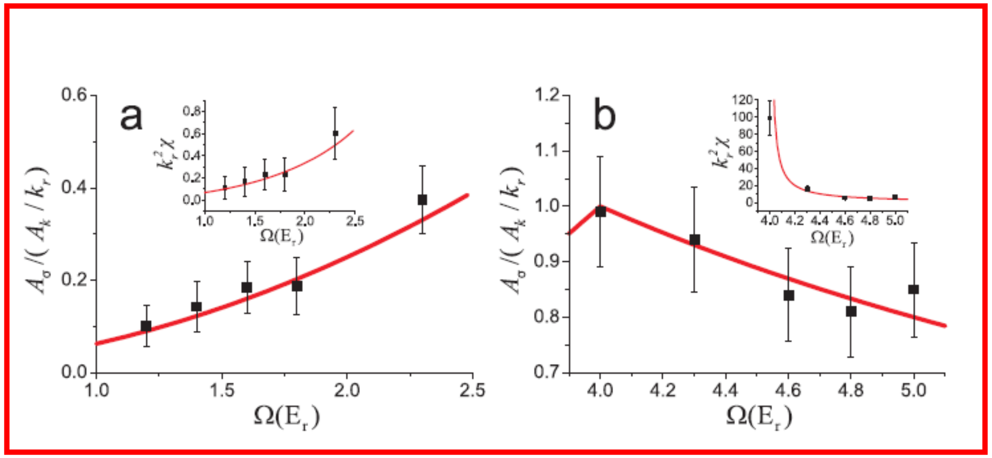
- Coupling between relative amplitudes determined by value of spin polarizability

$$A_\sigma = A_X k_0 \omega_x \chi(\sigma_z) \frac{1}{\sqrt{1+k_0^2\chi(\sigma_z)}}$$

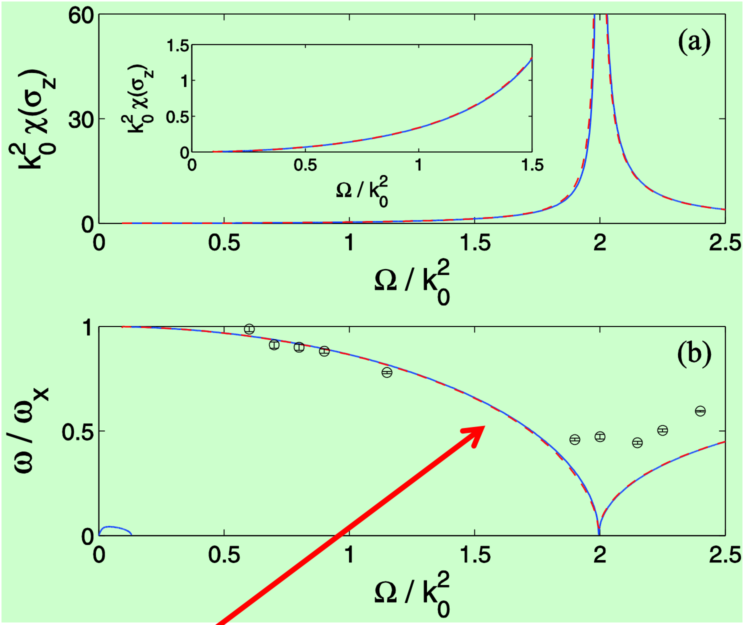
$$A_\sigma = \frac{A_P}{k_0} \frac{k_0^2\chi(\sigma_z)}{1+k_0^2\chi(\sigma_z)}$$



Experimental evidence of coupling used to measure spin polarizability (Zhang et al. PRL2012)



Open question raised by experiment on center of mass oscillation



- Why is the collective oscillation NOT observed in the Raman interval below the second order phase transition $\boxed{\Omega = 2k_0^2}$?

- Ocurrence of **dynamic instability** ! Lecture 4

Center of mass oscillation dramatically affected by spin-orbit coupling.
What about **Bogoliubov modes** in **uniform configurations ?**

- Bogoliubov modes can be obtained by solving linarized time-dependent Gross Pitaevskii equations

$$i\partial_t \Psi = \left[h_0 + \frac{1}{2}(g + g_{\uparrow\downarrow})\Psi^+\Psi + \frac{1}{2}(g - g_{\uparrow\downarrow})\Psi^+\sigma_z \Psi \sigma_z \right]\Psi$$

- A useful quantity: the dynamic response function: add a perturbation of the form $-\lambda e^{i(qx-\omega t)}$ and evaluate induced density fluctuations:

$$\delta\rho_q = \lambda \chi(q,\omega) e^{-i\omega t}$$

Result for density response function

(G. Martone et al. PRA 2012)

$$\chi(q,\omega) = \frac{-Nq^2[\omega^2 - 4k_1 q\omega + a(q)]}{\omega^4 - 4k_1 q\omega^3 + b_2(q) + k_1 q b_1(q)\omega + b_0(q)}$$

- Poles of response function give rise to two branches
- f-sum rule easily recovered for large ω $\quad \boxed{\chi(q,\omega) \to -q^2/\omega^2}$
- Response is not symmetric by exchange of q in – q in plane wave phase. Consequence of breaking of parity and time reversal inavariance of Hamiltonian.
- Values of functions $\boxed{a(q),\ b_0(q),\ b_1(q),\ b_2(q)}$ depend on whether one is in plane wave or single momentum phase

PLANE WAVE PHASE

BEC occupies state with

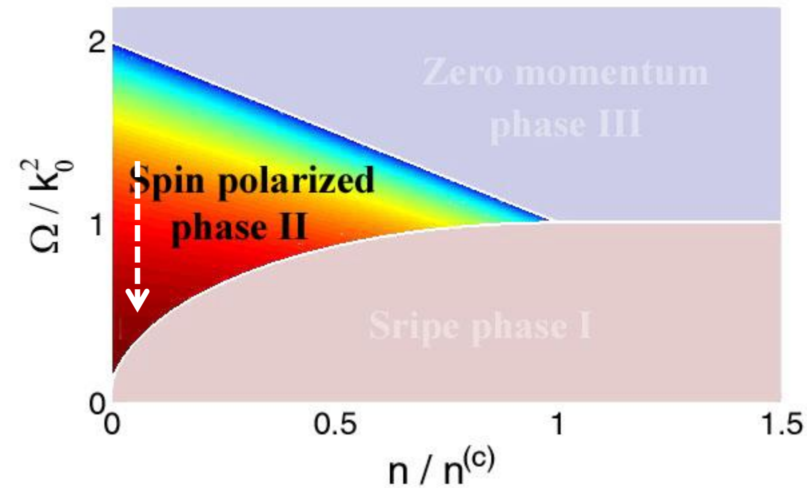
$$k_1 = k_0 \sqrt{1 - \frac{\Omega^2}{[2k_0^2 + n(g + g_{\uparrow\downarrow})/2]^2}}$$

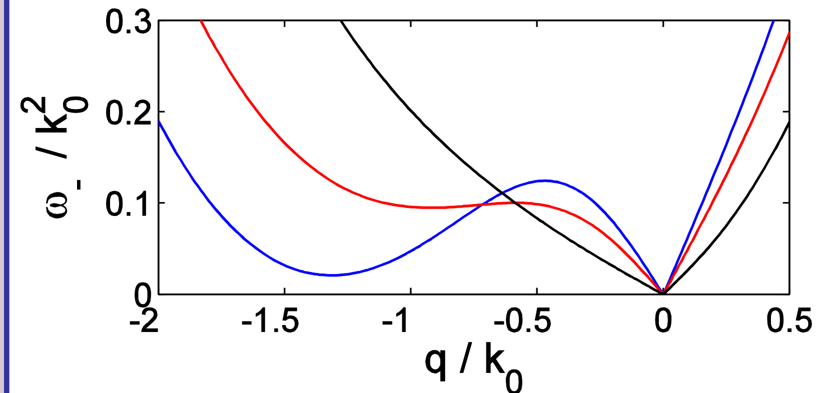
- Lower branch of excitation spectrum exhibits **maxon-roton structure** which becomes more pronounced as one lowers Raman coupling towards the transition to the stripe phase

- Lower branch contributes to **static structure factor**

- **Parity violation** in excitation pectrum:

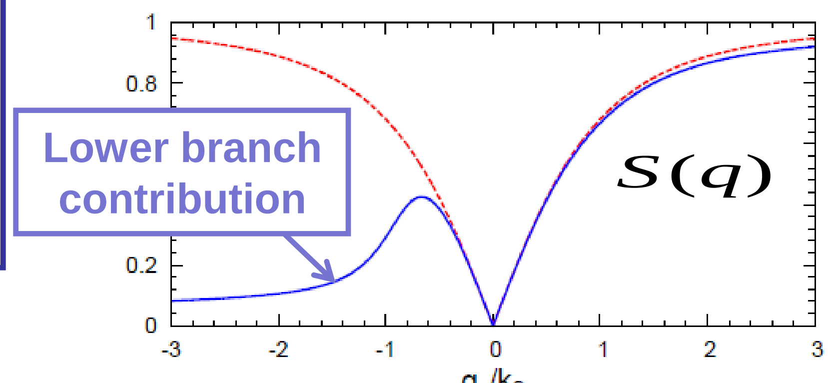
$$\omega(q) \neq \omega(-q)$$





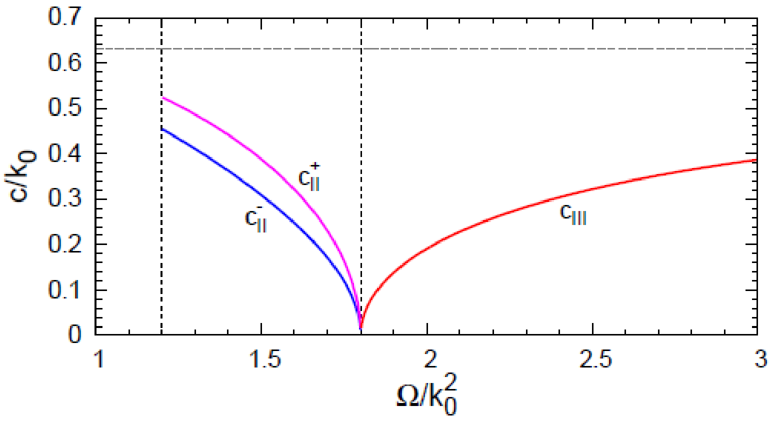
$G_1/k_0^2 = 0.5,\ G_2/k_0^2 = 0.12$

$\Omega/k_0^2 = 1.22,\ 1.33,\ 1.46$

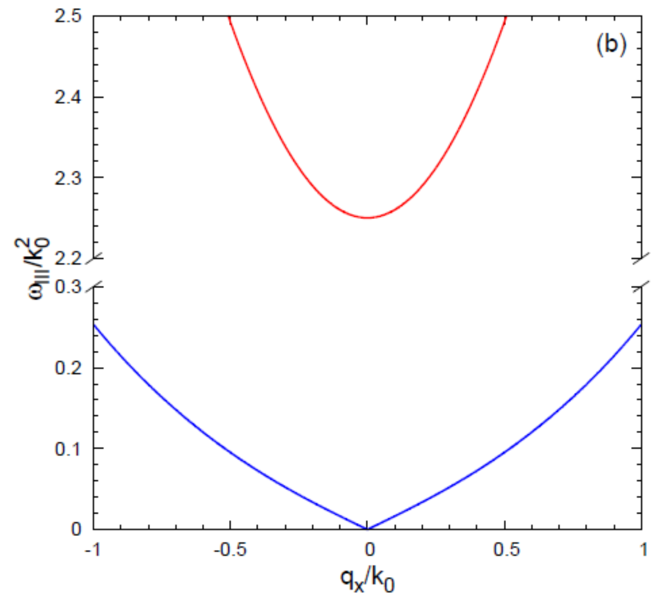


FROM PLANE WAVE TO ZERO MOMENTUM PHASE

- Sound velocity vanishes at the transition Raman frequency despite value of **compressibility** is **unchanged**. Consequence of **divergent** behavior of **spin polarizability** $$c^+c^- = \frac{n(\partial\mu/\partial n)}{1+k_0^2\chi_M}$$
- Sound wawes have mixed density and spin nature. Analogy with center of mass mode: $$\omega_D^2 = \frac{\omega_x^2}{1+k_0^2\chi_M}$$
- Sound velocity depends on sign of q with respect to $\boldsymbol{k_1}$
- In k=0 phase excitation spectrum is symmetric by exchange of q in to –q



If $g = g_{\uparrow\downarrow}$ and $gn << k_0^2$ sound velocity is fixed by effective mass $c^2 = gnm/m^*$



Conclusions and questions

Spin orbit coupling deeply affects the dynamic behavior of BEC gases

- **Violation of Kohn theorem** and strong quenching of dipole oscillation in harmonic trap
- Strong **suppression of sound velocity** near the transition between plane wave and zero momentum phase
- Emergence of **rotonic structure**. More and more pronounced as one approaches transition to the stripe phase

Questions addressed in Lecture 4

- **Instability of supercurrents** in the presence of spin-orbit coupling
- Excitation spectrum in stripe phase and manifestation of typical **supersolidity** effects

Other questions:
- Phase diagram and dynamics at **finite temperature**
- Dynamics of spin-orbit coupled **Fermi** superfluids
- Dynamics in **Rashba** configurations