College de France, May 14, 2013

SUPERFLUIDTY IN ULTRACOLD ATOMIC GASES

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PLAN OF THE LECTURES

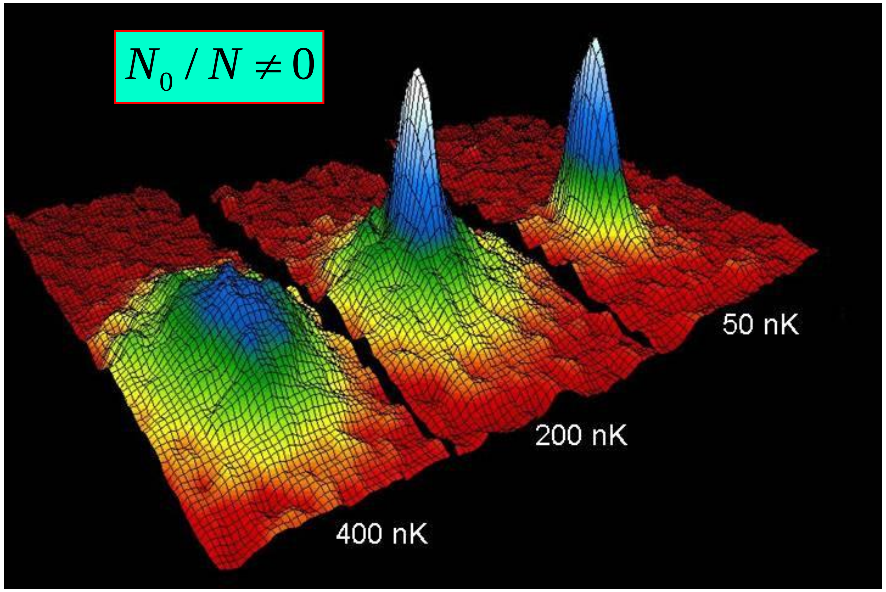
Lecture 1. **Superfluidity** in ultra cold atomic gases:
examples and open questions (May 14)

Lecture 2. A tale of two sounds (**first** and **second sound**) (May 21)

Lecture 3. **Spin-orbit** coupled Bose-Einstein condensed gases:
quantum phases and **anisotropic dynamics** (May 28)

Lecture 4. **Superstripes** and **supercurrents** in spin-orbit coupled
Bose-Einstein condensates (June 4)

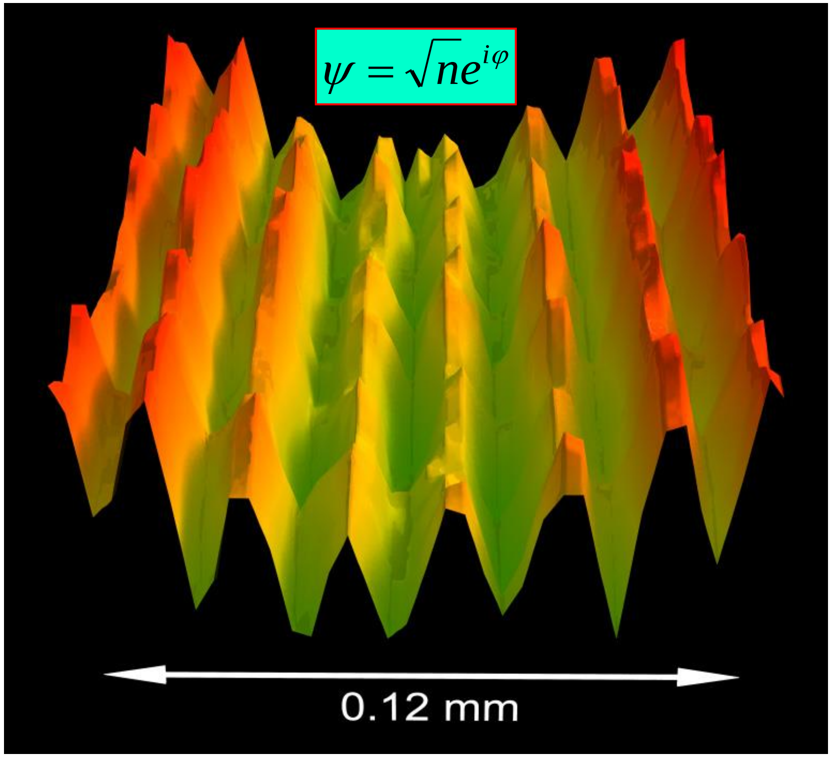
Bose-Einstein condensation:
first experiments



1995
(Jila+Mit)

(Macroscopic
occupation
of sp state)

1996 Mit
(coherence +
wave nature)



What was **new** with BEC in trapped atomic gases ?

- Bose-Einstein condensation in both **momentum** and **coordinate** space

- **Diluteness** (Gross-Pitaevskii eq. for order parameter)

New important knobs are now available. They permit to increase the effects of correlations

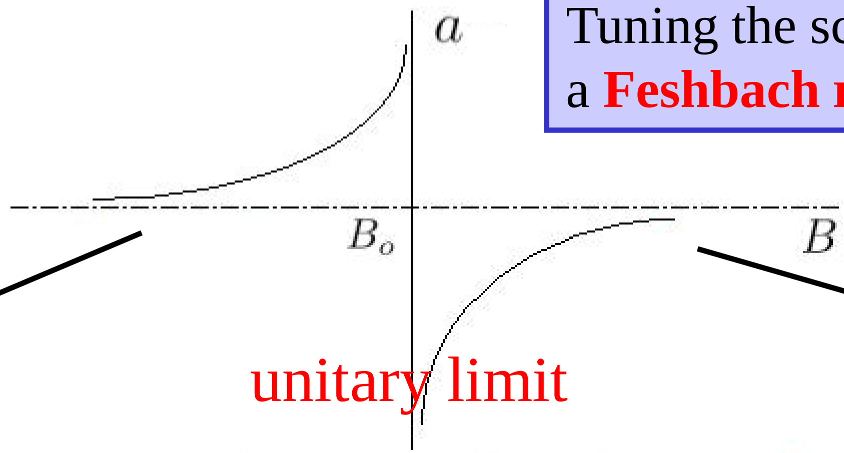
- **Tuning** of **scattering length** (BEC-BCS crossover in Fermi gases)

- **Tuning** the **external conditions**, optical lattices and the superfluid-Mott insulator transition, 1D and 2D configurations, adding disorder …

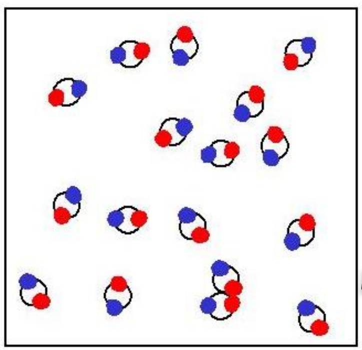
Fermi Superfluidity: the BEC-BCS Crossover

(Eagles, Leggett, Nozieres, Schmitt.Rink, Randeria)

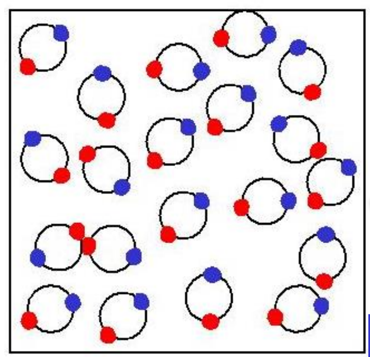
Tuning the scattering length through a **Feshbach resonance**



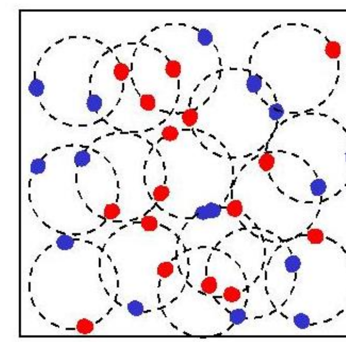
BEC regime
(molecules)



unitary limit



BCS regime
(Cooper pairs)



Dilute Bose gas
(size of molecules much smaller than interparticle distance

At unitarity scattering lenght is much larger than interparticle distance: strongly interacting superfluid

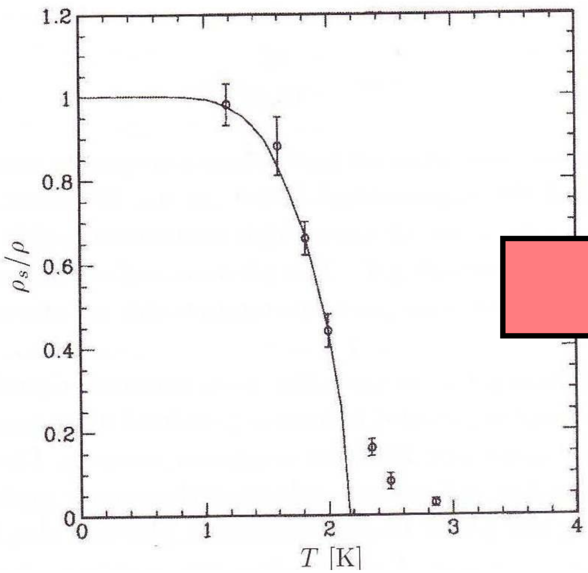
Important (and old) **questions**

- Connections between BEC and superfluidity
- Can the condensate and superfluid densities be different ?

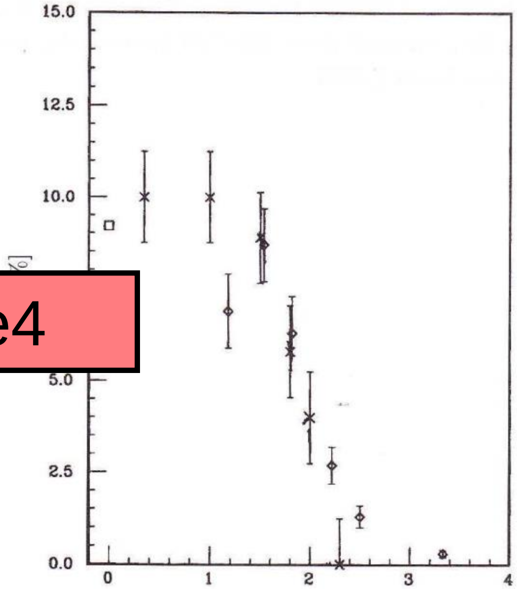
Some answers

- **Gross-Pitaevskii** equation for the 3D BEC order parameter predicts important **superfluid** features (quantized vortices, irrotational hydrodynamic flow …)
- In **dilute 3D BECs** condensate and superfluid densities are practically equivalent (quantum depletion is small). Crucial differences emerge in low D (**BKT superfluidity**) and in strongly interacting superfluids (**liquid Helium, unitary Fermi gas**). Measurement of superfluid density in **strongly interacting Fermi gas is now available**

Superfluid He4

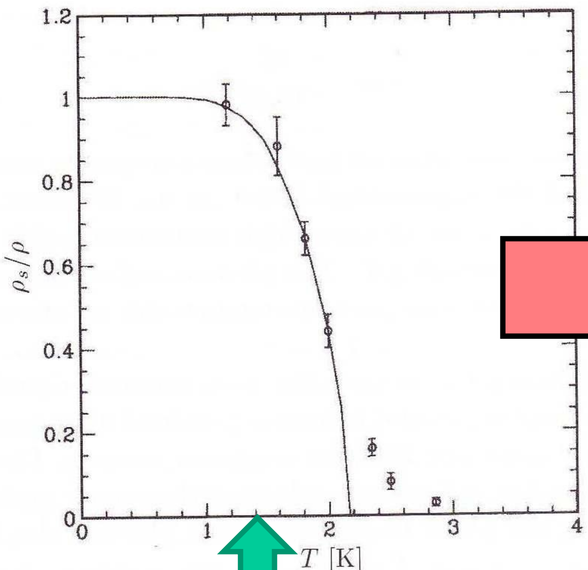


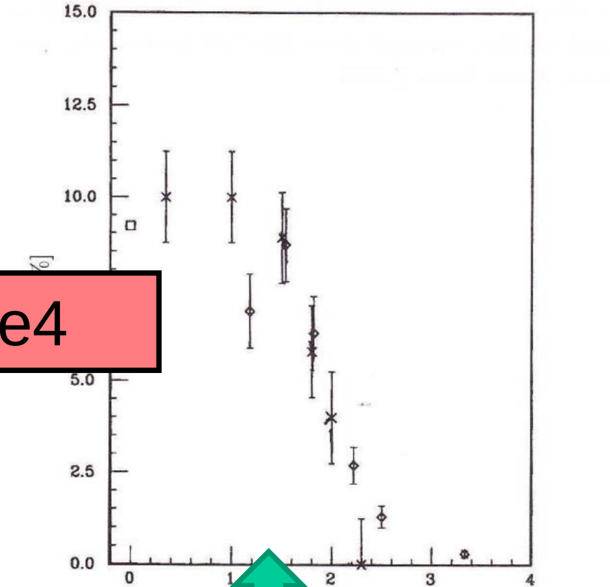
Superfluid fraction



Condensate fraction

Superfluid He4

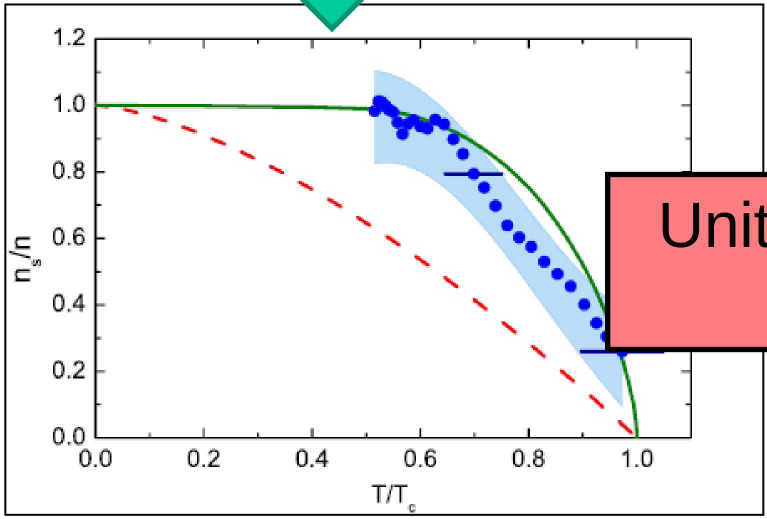


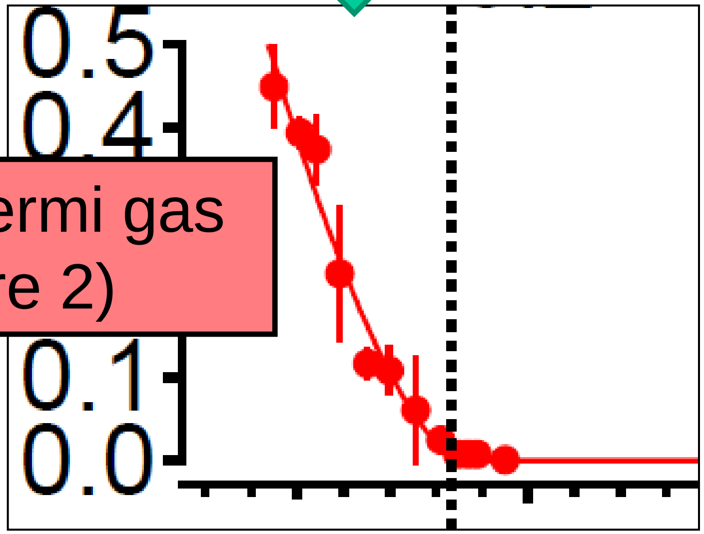


Superfluid fraction

Condensate fraction

Unitary Fermi gas (lecture 2)





Sidorenkov et al., IBK-TN, 2013

Ku et al., MIT, 2012

Superfluid features in atomic gases (selection)

- **Quantized vortices** and **solitons**

- Absence of **viscosity**
  (critical velocity and supercurrents)

- Search for **supersolidity**

- **Lambda transition** and specific heat

- Hydrodynamic behavior (irrotationality, collective oscillations, first and **second sound**)

Quantized Vortices and **solitons exhibit unique topological features**

They are characterized by a peculiar behavior of the phase of the order parameter

$$\Psi(\vec{r}) = e^{i\vartheta} \, | \Psi(\vec{r}) |$$

quantized vortex

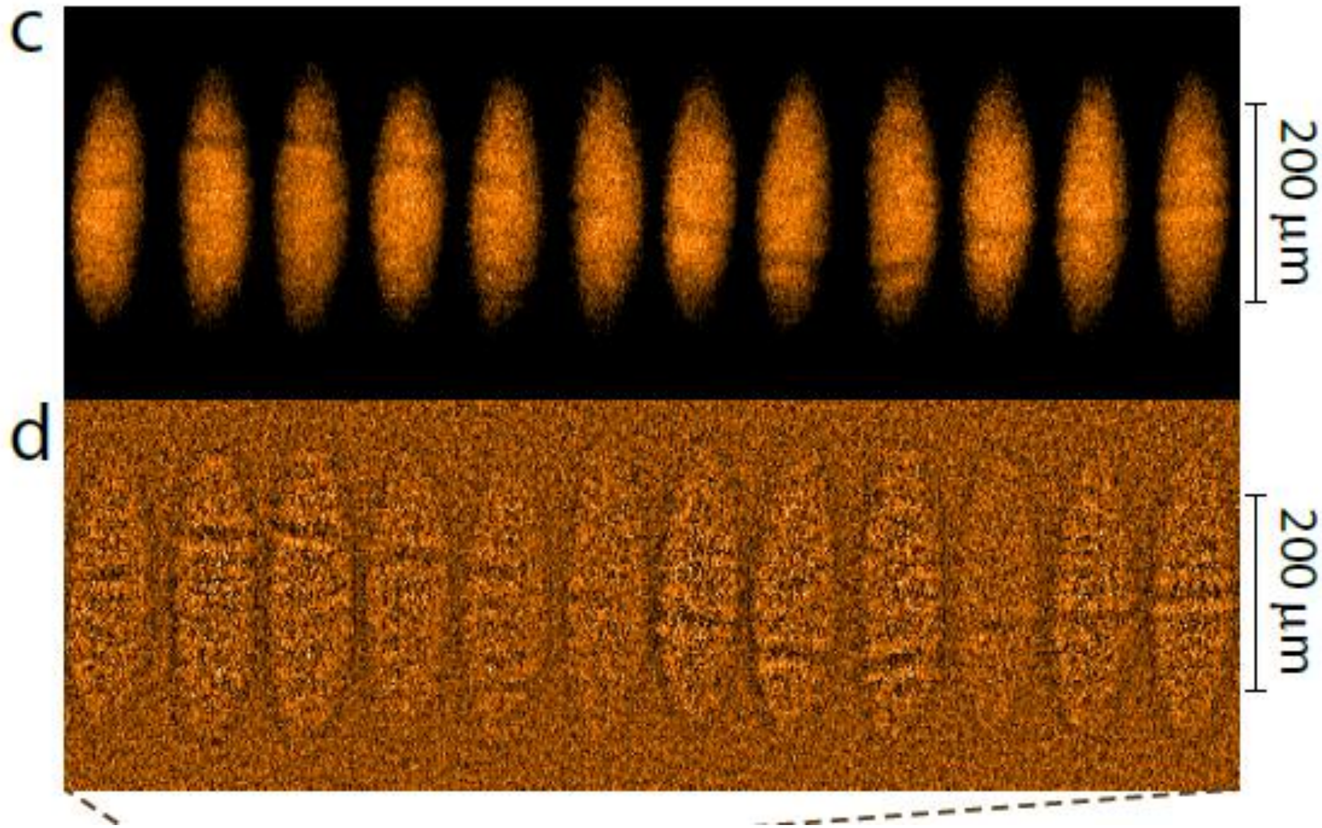
$$\Psi(z) = \mathrm{sgn}(z) \, | \Psi(z) |$$

dark soliton

Due to the singularity of the phase at r=0 (vortex) and z=0 (dark soliton) the density of the superfluid is suppressed in the vicinity of the core. Gross-Pitaevskii theory predicts that the density exactly vanishes on the core. What happens in more correlated fluids (for example in the unitary Fermi gas ?)

Recent **Mit experiment** on the oscillation of a soliton of superfluid Fermi gas in a harmonic trap raises dramatically the question of the structure of its **core** and of its **effective mass**

Yefsah et al., arXiv:1302.4736

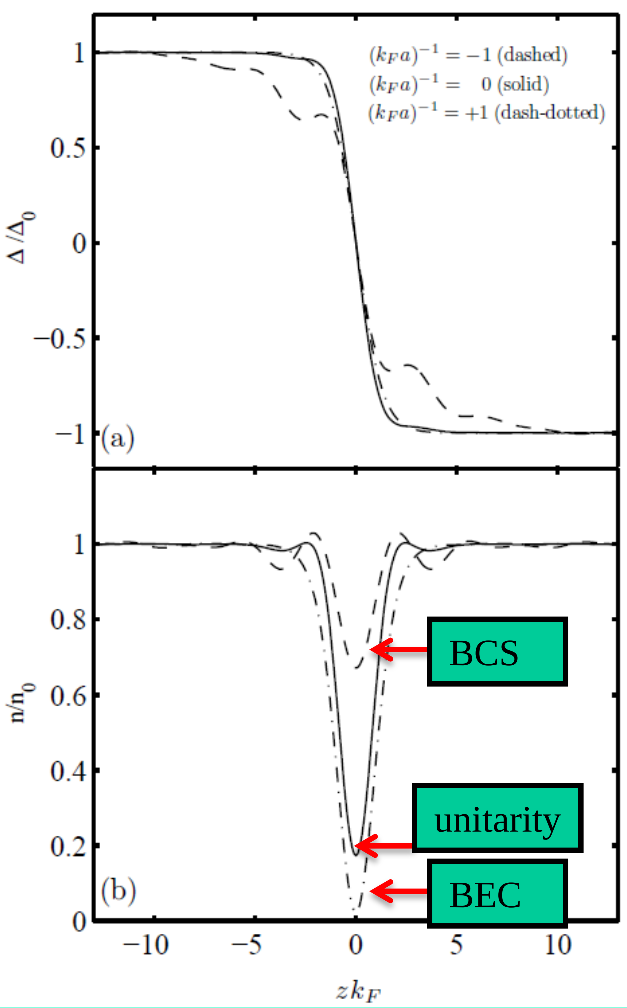


Bogoliubov de Gennes theory predicts that at unitarity the **core of the dark soliton** is **partially filled**, differently from what happens in a dilute Bose gas

Useful quantity characterizing the soliton core is the so called **deficit of atoms**

$$N_S = \int_{-\infty}^{+\infty} dx (n_1(x) - n_1(+\infty))$$

This quantity is crucial for the calculation of the **effective mass** of the soliton.



Antezza et al., PRA 2007

Effective mass of a dark soliton (oscillation in harmonic trap)

The effective mass of a dark soliton is a crucial ingredient describing the oscillation of the soliton in a harmonic trap.

$$\frac{M^*}{M} = \left(\frac{T_S}{T_Z}\right)^2 = 1 + \frac{\hbar n_1(+\infty)}{m_B N_S}\frac{d\Delta\varphi}{dv}$$

Its value is the result of two different effects (Scott et al. PRL 2011):

- a **dynamic effect** accounted for by the derivative of the phase difference (left to right) with respect to the velocity of the soliton.

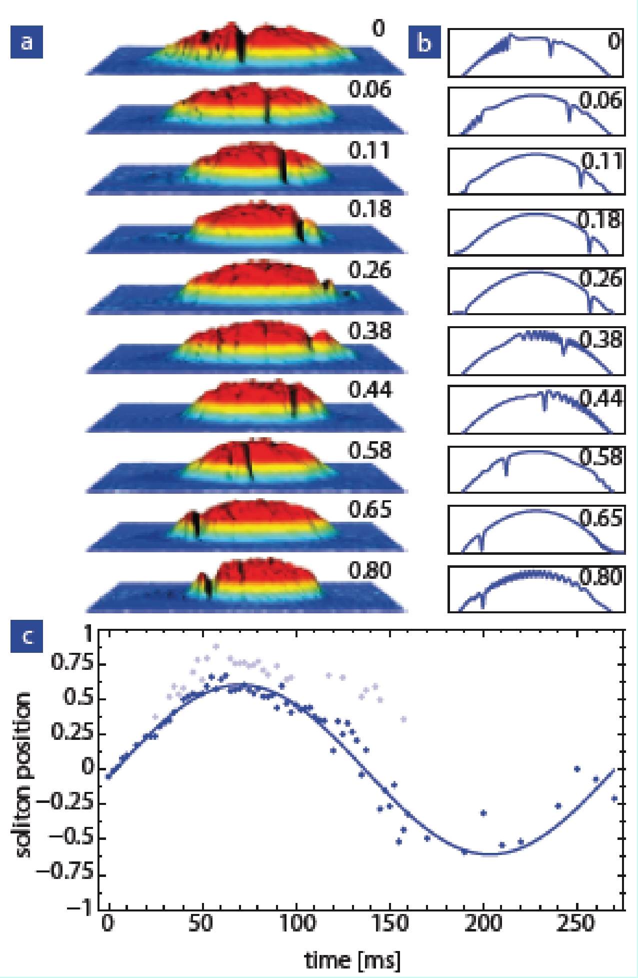
- An **equilibrium** ingredient given by deficit of particles in the soliton region

$$N_S = \int_{-\infty}^{+\infty} dx\left(n_1(x) - n_1(+\infty)\right)$$

In a **dilute Bose-Einstein condensed gas** both the gradient of the phase and the deficit of particles can be calculated analytically starting from GP equations.

The result is (Busch and Anglin, 2001, Konotop and Pitaevskii, 2004

$$\boxed{\left(\frac{T_S}{T_Z}\right)^2 = 2}$$



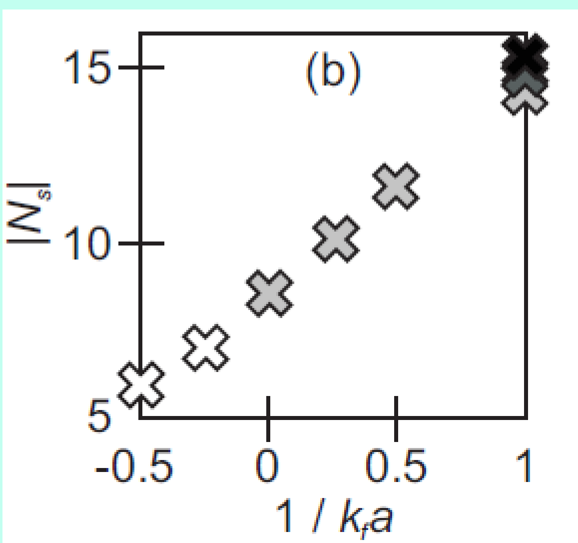
Frequency of oscillation is decreased
By factor $\sqrt{2}$
Experiments in 1D harmonic traps
confirm the prediction of theory
with reasonably good accuracy
Becker et al. (2008), Weller et al. (2008)

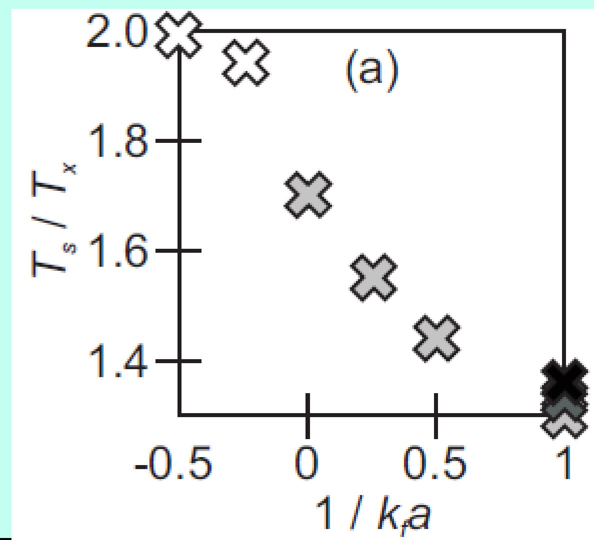
Solitons in an interacting Fermi gas

In Scott et al. (PRL, 2011) we calculated the velocity gradient $d(\Delta\varphi)/dv$ of the phase difference and the deficit N_S of the soliton density along the **BEC-BCS** crossover, by solving the BdG equations.

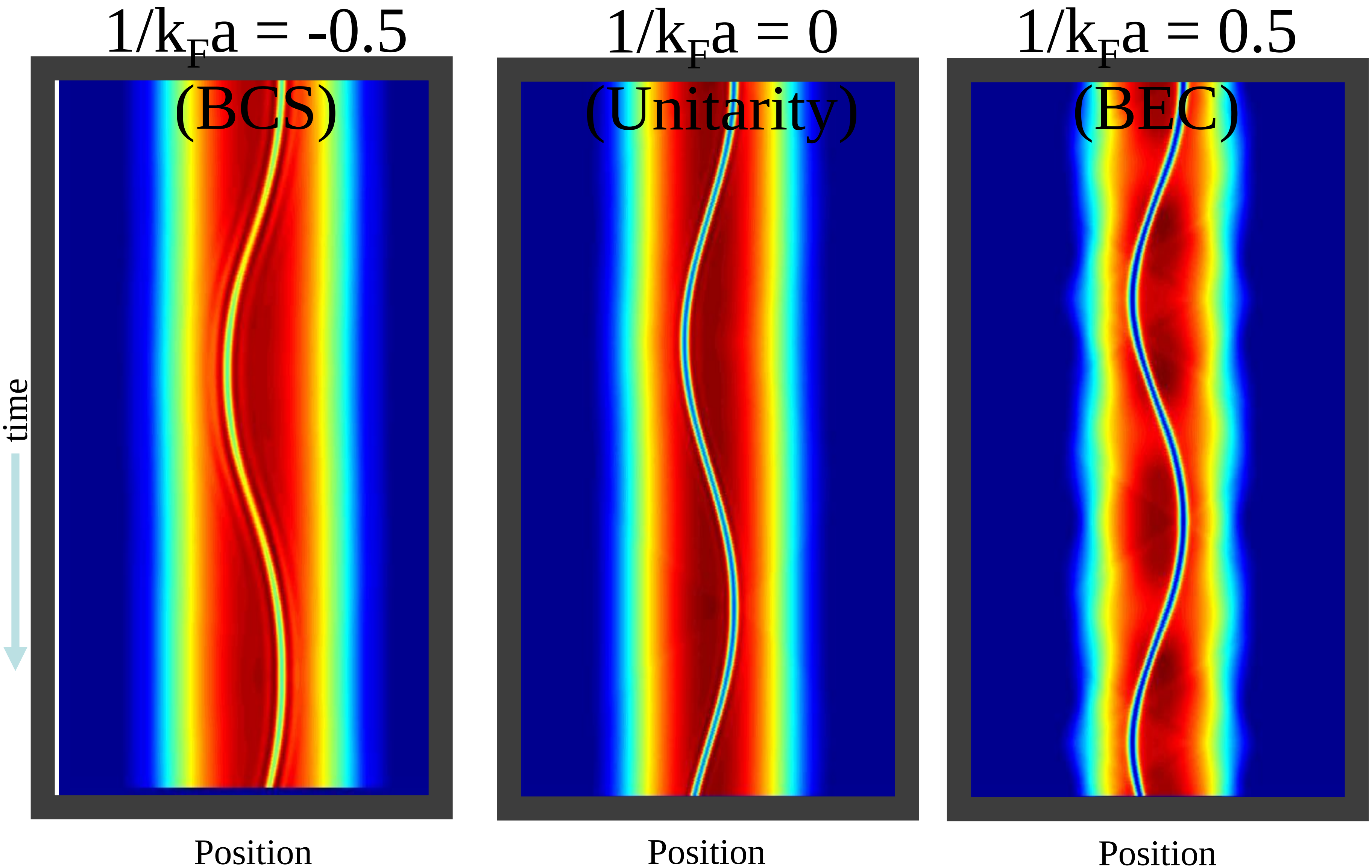
We find:

- The velocity gradient is practically constant along the crossover
- The defict N_S decreases (in modulus) as one moves from BEC to unitarity. The oscillation time accordingly increases



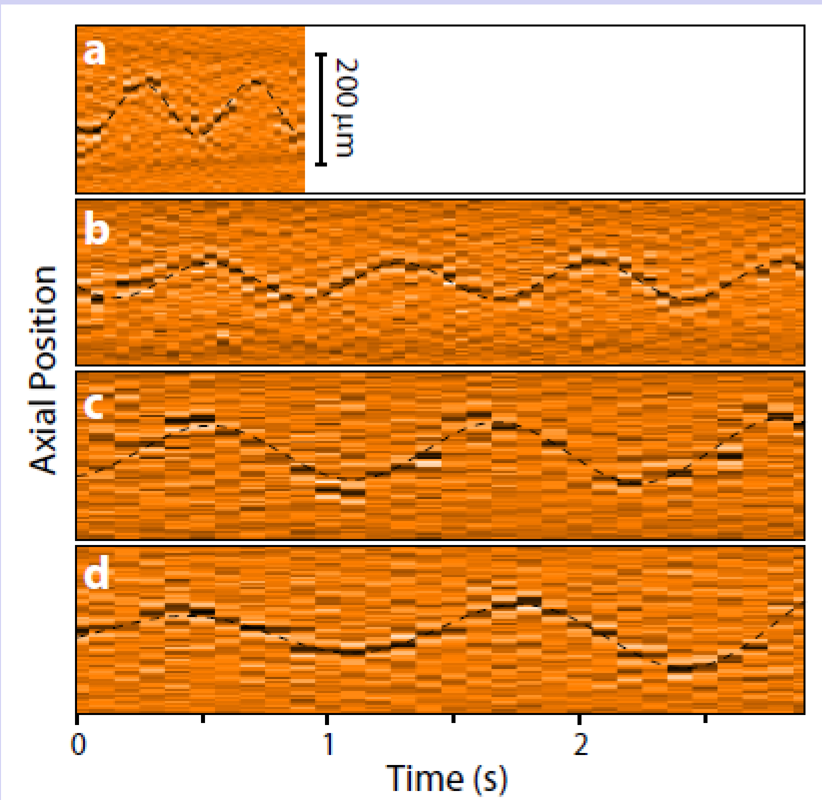


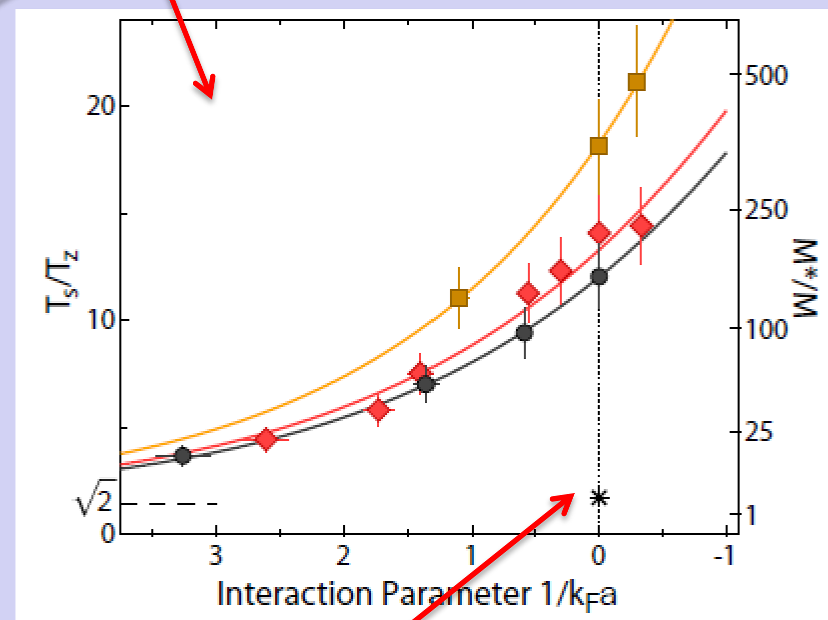
Soliton oscillations in harmonic trap
(solution of time dependent Bogoliubov equations, Scott et al. 2011)



Recent measurements of the **soliton oscillation** in harmonic trap reveals much stronger increase of effective mass when one moves from BEC to unitarity. Huge discrepancy with predictions of mean field theory (BdG eqs)

Mit exp:
Yefsah et al., arXiv:1302.4736





BdG prediction
Scott et al., PRL 2011

Heavy Solitons in a Fermionic Superfluid

arXiv:1302.4736

Tarik Yefsah, Ariel T. Sommer, Mark J.H. Ku, Lawrence W. Cheuk, Wenjie Ji, Waseem S. Bakr, and Martin W. Zwierlein
MIT-Harvard Center for Ultracold Atoms, Research Laboratory of Electronics, and Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA

We attribute the large relative effective mass M^*/M in the strongly interacting regime to the filling of the soliton with uncondensed fermion pairs resulting from strong quantum fluctuations. These can also reside inside the soliton [14–19], in addition to fermionic Andreev bound states. A substantial filling of the soliton will reduce the number $|N_s|$ of atoms missing inside the soliton, therefore considerably weaken the restoring harmonic force from the trap and strongly increase M^*/M. Mean-field theory for the BEC-BCS crossover heavily underestimates the role of quantum fluctuations already on the BEC side,

- Can we calculate quantum fluctuations inside soliton **beyond mean field theory** ?
- Is the question relevant also for the **effective mass** of a **quantized vortex** ?
- Can we envisage an **experiment** to measure the effective mass of a quantized vortex ?

Landau's critical velocity and supercurrents

Two different critical velocities :

- **Impurity moving with velocity v**
  Critical velocity fixed by Landau's criterion
  (energetic instability, energy of
  excitation spectrum becomes negative)

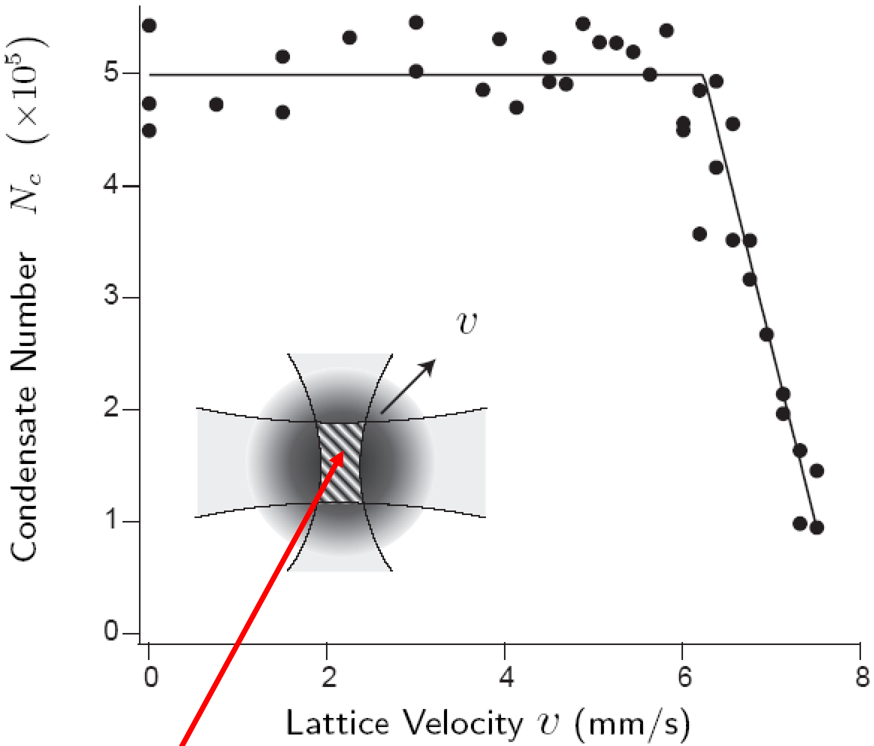
$$v_c = \min_p \frac{\varepsilon(p)}{p} \neq 0$$

- **Critical current**.
  Superfluid can support a metastable supercurrent up to
  a critical velocity

In uniform superfluids the two criteria are equivalent
(consequence of Galilean invariance)

In non uniform superfluids (eg. optical lattice) the two criteria are different and current can exhibit **dynamic instability** (appearence of imaginary component in excitation spectrum)

Example of Landau's critical velocity: uniform Fermi superfluid at unitarity (energetic instability)

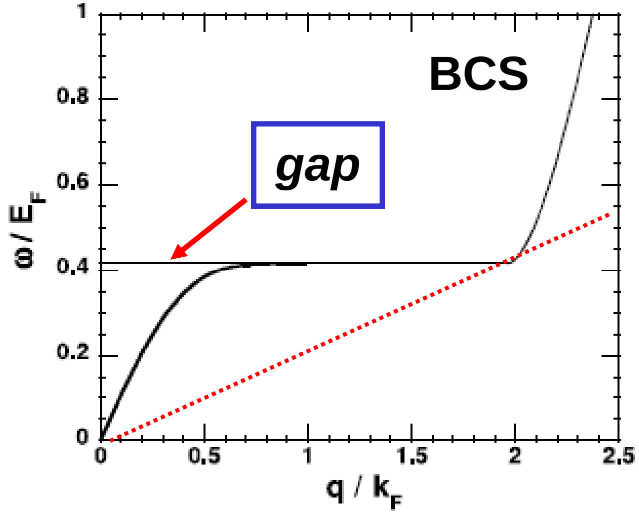


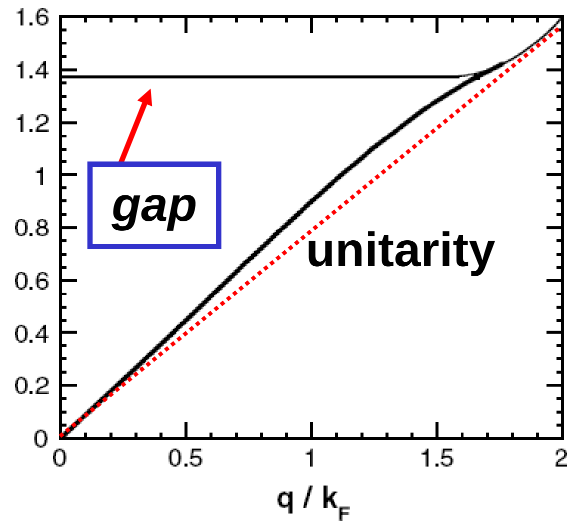
$$v_c = \min_p \frac{\varepsilon(p)}{p}$$

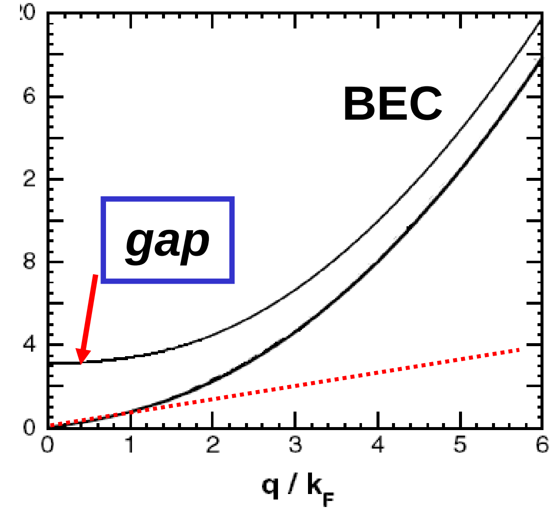
Above critical velocity dissipative effect produced by moving optical lattice is observed

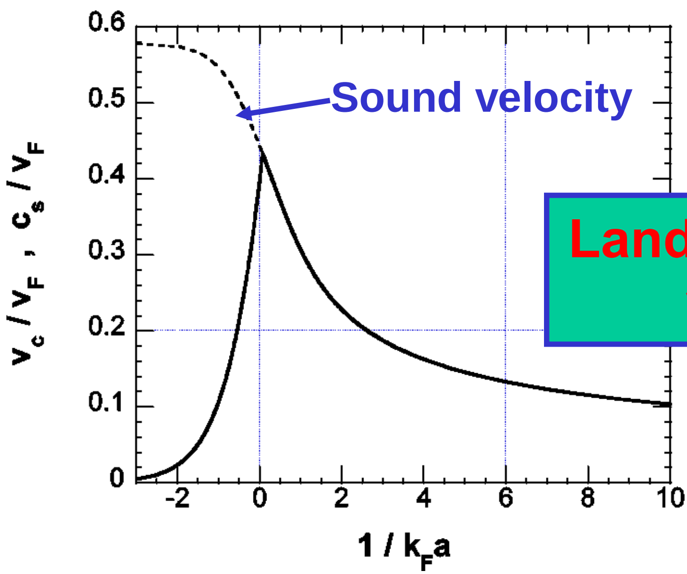
(Mit, Miller et al, 2007)

Dispersion law along BCS-BEC crossover
(Cobescot, M. Kagan, Stringari, 2006)

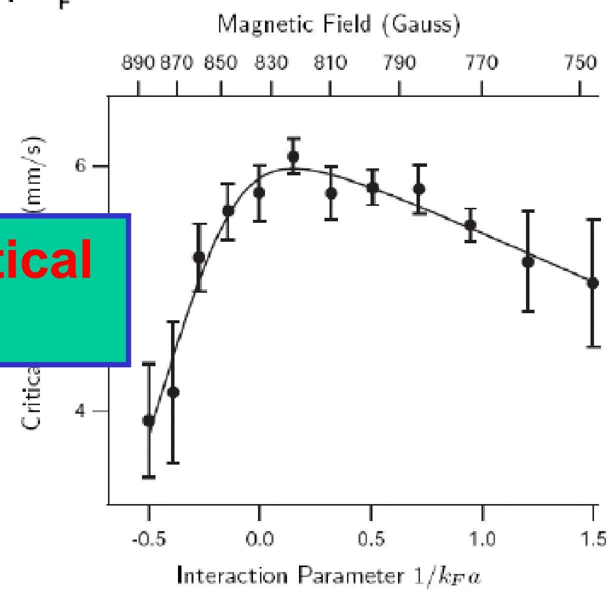






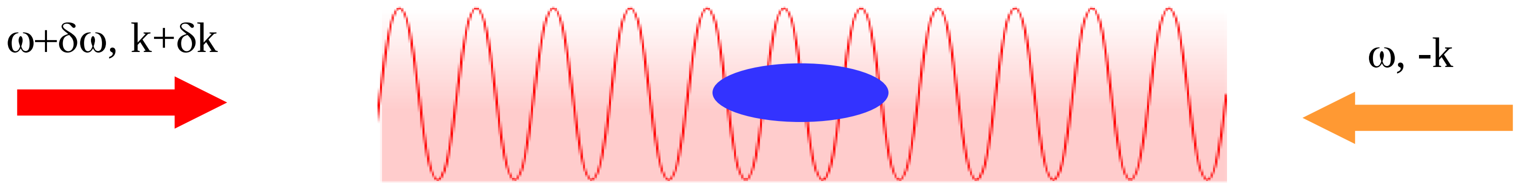


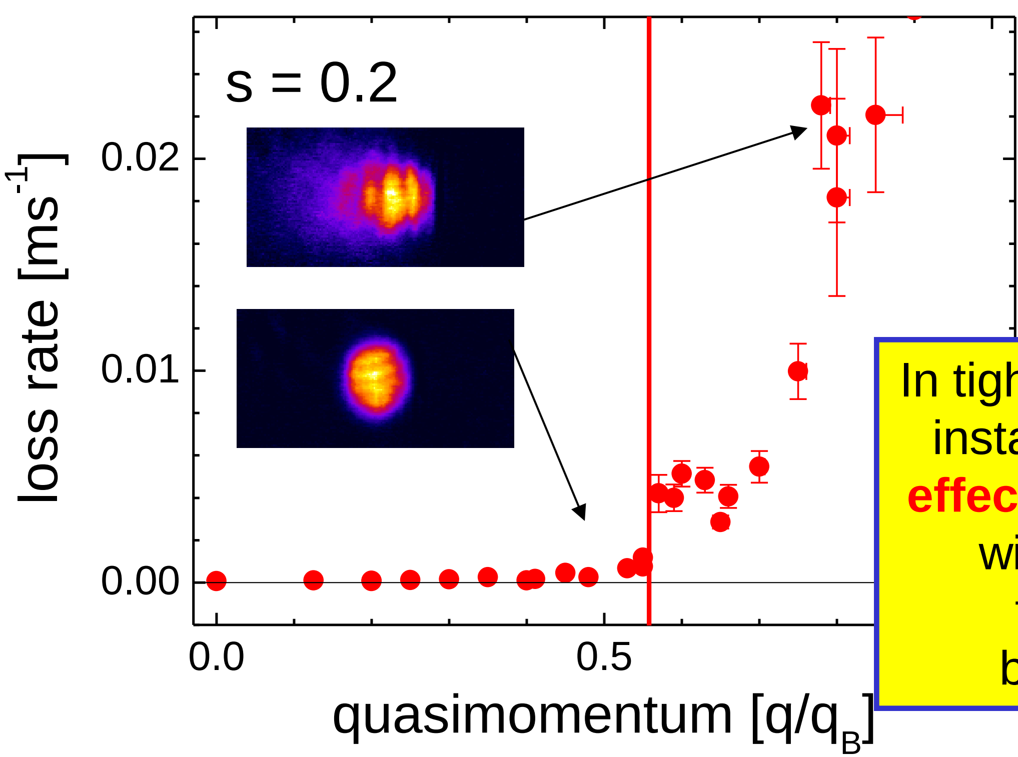
Landau's critical velocity



Landau's critical velocity is highest near unitarity !!

Example of dynamic instability : BEC in moving periodic potential





(Fallani et al., 2004)

In tight binding limit dynamic instability starts when the **effective mass** associated with the presence of the optical lattice becomes **negative**

RECENT WORK ON PERSISTENT CURRENTS

Measured critical velocity in 2D superfluids (ENS)
Persistent current in ring geometry with weak links (Nist)
Critical velocities in spinor mixtures (Cambridge)
…..

Question addressed in Lecture 4

Are Landau's criterion for crititical velocity and stability criterion for persistent current always equivalent in uniform superfluids ?

Recent possibility of generating **uniform** superfluids with Spin-Orbit Hamiltonians **breaking Galilean invariance**

Non trivial consequence on the stability of the supercurrent

Search for **supersolidity** in ultracold atomic gases

Supersolidity is characterized by co-existence of two sponatenoeusly broken continuous symmetries:

- **Gauge symmetry** yielding BEC and superfluidity

- **Translational invariance** yielding crystalline structure

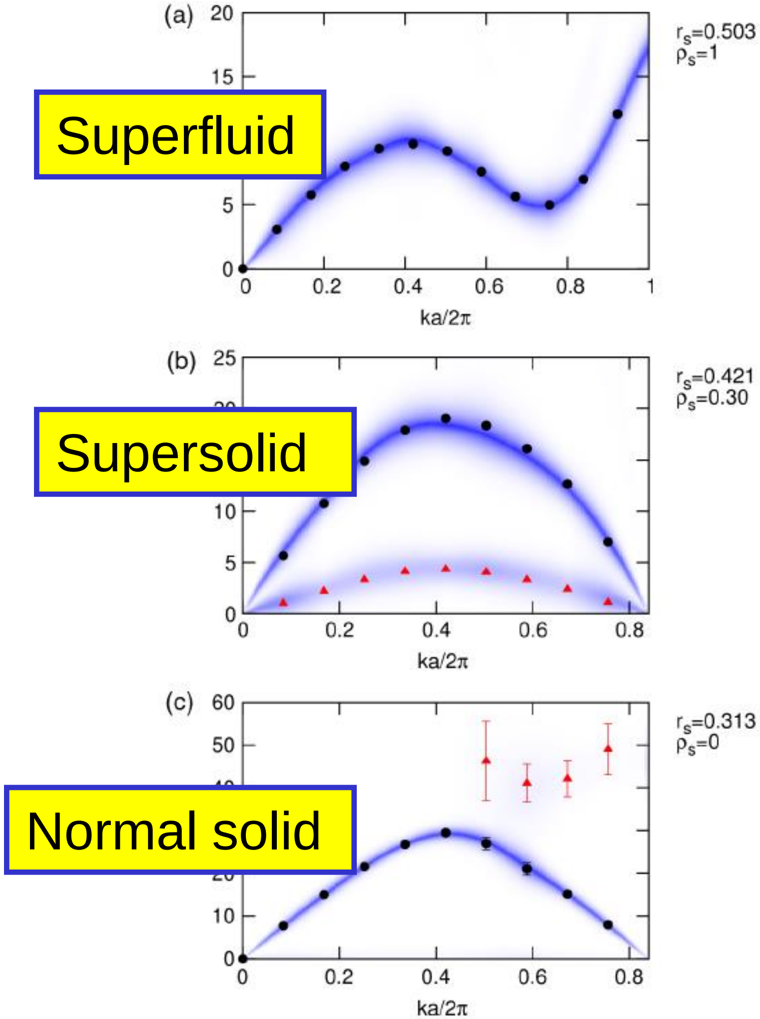
- First attempts to observe supersolidity in **solid helium** (Kim and Chan, Nature 2004) by observing quenching of moment of inertia

- **No conclusive proof** of supersolidity still available (Balibar, Nature 2012).

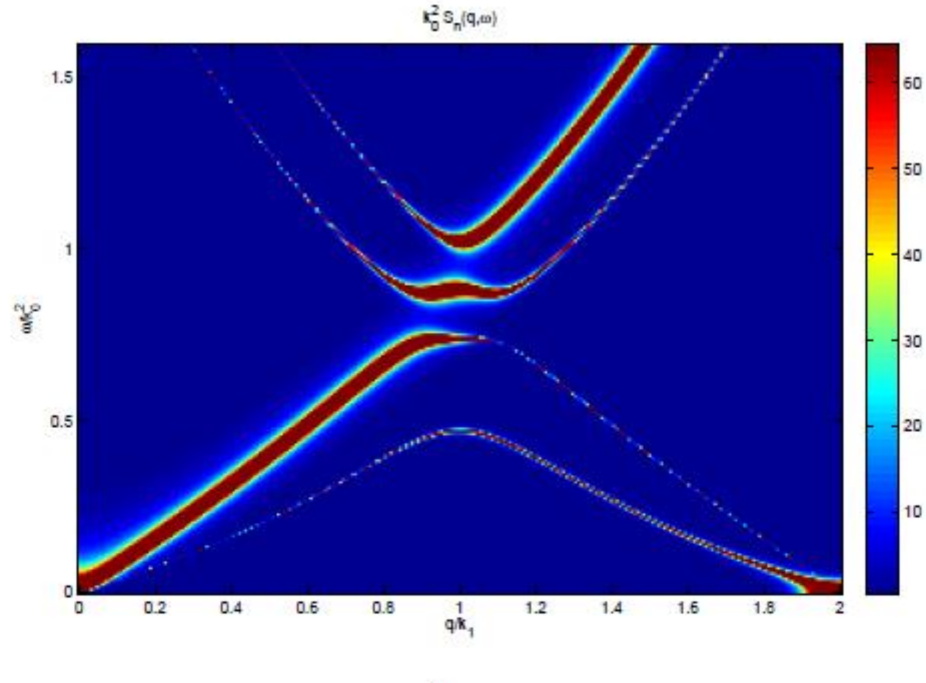
Recent theoretical proposals to realize supersolidity in ultracold atomic gases:

- **Rydberg atoms** with dipolar potentials softened at short distance

- Superstripe phase in **spin-orbit coupled BEC's** (Lecture 4)



Excitation spectrum of a Bose gas with soft core repulsive potential
Saccani et al, PRL 2012



Double gapless band in the superstripe phase of a spin-orbit coupled BEC
Yun Li et al. PRL 2013

Thermodynamics of a strongly interacting Fermi superfluid gas:

The lambda transition

Thermodynamics and Universality of the Unitary Fermi gas (1/a=0)

Absence of interaction parameter implies that thermodynamics should obey universal law (Ho, 2004)

$$\frac{P}{k_B T}\lambda_T^3 = f_p(\mu / k_B T)$$

$$n\lambda_T^3 = f_p^{'}(\mu / k_B T)$$

where $\lambda_T = \sqrt{2\pi\hbar^2 / mk_B T}$ is thermal wave-length and $f_p(x)$ is **dimensionless, universal function**.

All thermodynamic functions (entropy, compressibilities, specific heats etc.) can be expressed in terms of $f_p(x)$
exact behavior known only at large negative x (classical regime) and at large positive x (phonon regime). Calculation of $f_p(x)$ requires non trivial many-body approaches at finite T.

Universal function $f_p(x)$ and thermodynamic functions are now **available experimentally**.

Thermodynamics of interacting Fermi gas

Recent major contributions: ENS (Nascimbene et al., Nature 2010) and MIT (Ku et al., Science 2012)

MIT experiment has provided first direct evidence of lambda transition in specific heat. **Pressure** is measured by integrating radial density profile and using LDA result

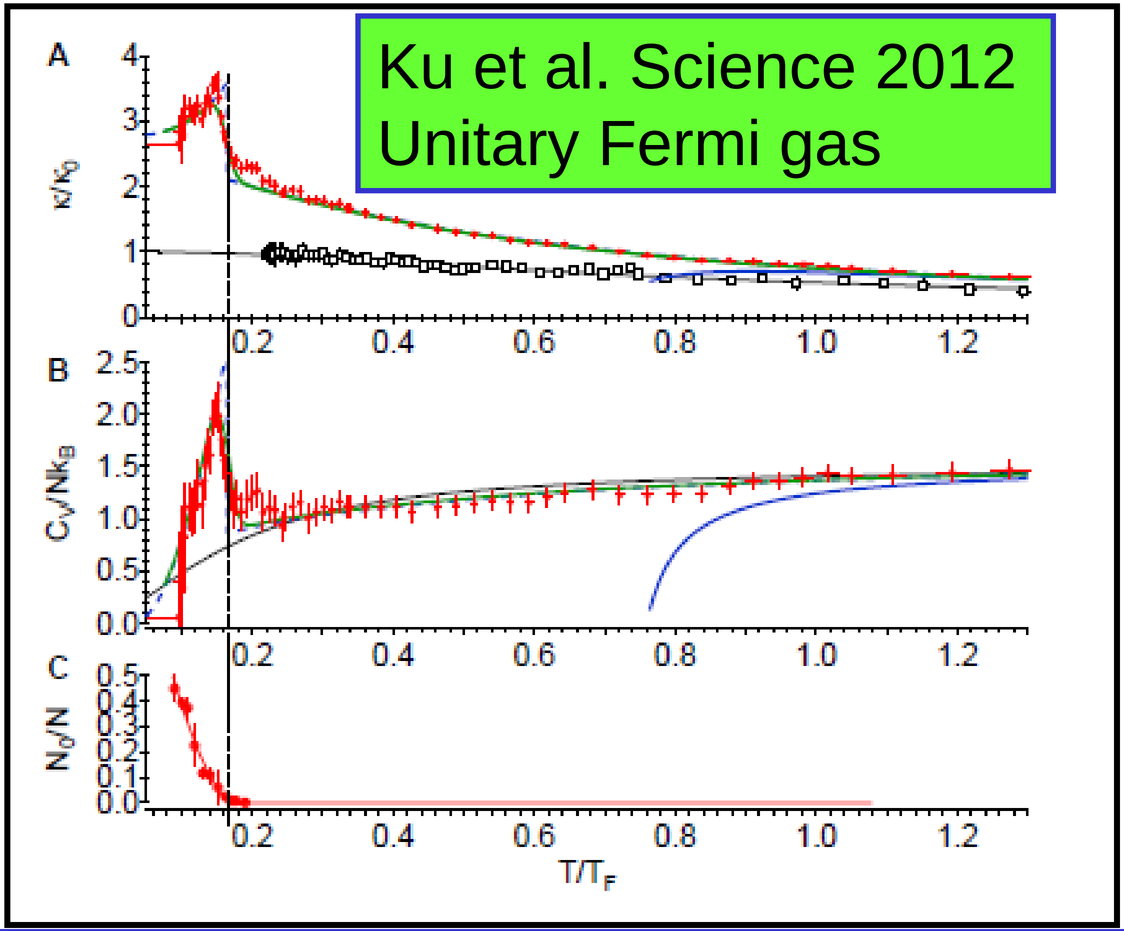
$$n_1(z) = \int n(\vec{r})dxdy = \frac{2\pi}{m\omega_\perp^2}P(x=y=0)$$

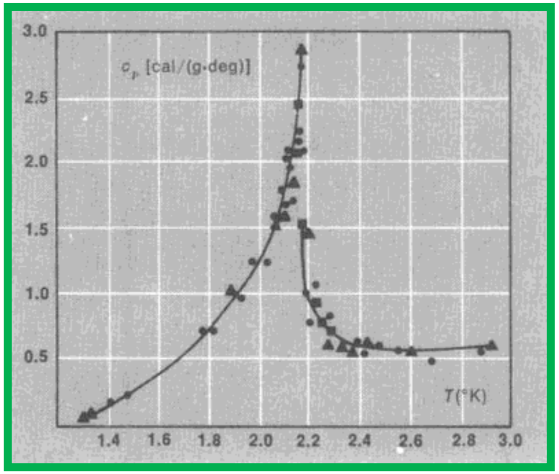
holding in harmonic traps

$$V_{ext} = (1/2m)[\omega_\perp^2(x^2+y^2)+\omega_z^2 z^2$$

In MIT exp measurement of T was replaced by measurement of **compressibility**

$$\kappa = -\frac{1}{n^2}\left(\frac{dn}{dV_{ext}}\right)_T$$





Superfluid He4

Experimental determination of critical temperature

$$T_C / T_F = 0.167(13)$$

(determined by peak in specific heat and onset of BEC)
in agreement with many-body predictions (Burowski et al. 2006; Haussmann et al. (2007); Goulko and Wingate 2010)

Universal function $\boxed{f_p(\mu / k_B T)}$ gives access to all thermodynamic quantitities, except superfluid density

Question: how to **measure** the **superfluid density** ?

(not available from equlibrium thermodynamics, needed **transport** phenomena)

Determination of **superfluid density** in strongly interacting Fermi gases through measurement of **second sound**

(Innsbruck-Trento collaboration Nature, this week !)

Next Lecture