Three-terminal quantum-dot thermoelectrics

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Outline

Introduction

- Quantum dots and Coulomb blockade
- Quantum dots and Thermoelectrics
- Three-terminal thermoelectrics

Coulomb-coupled conductors

- Coulomb-blockade regime
- Chaotic cavities
- Resonant tunneling

Harvesting bosons

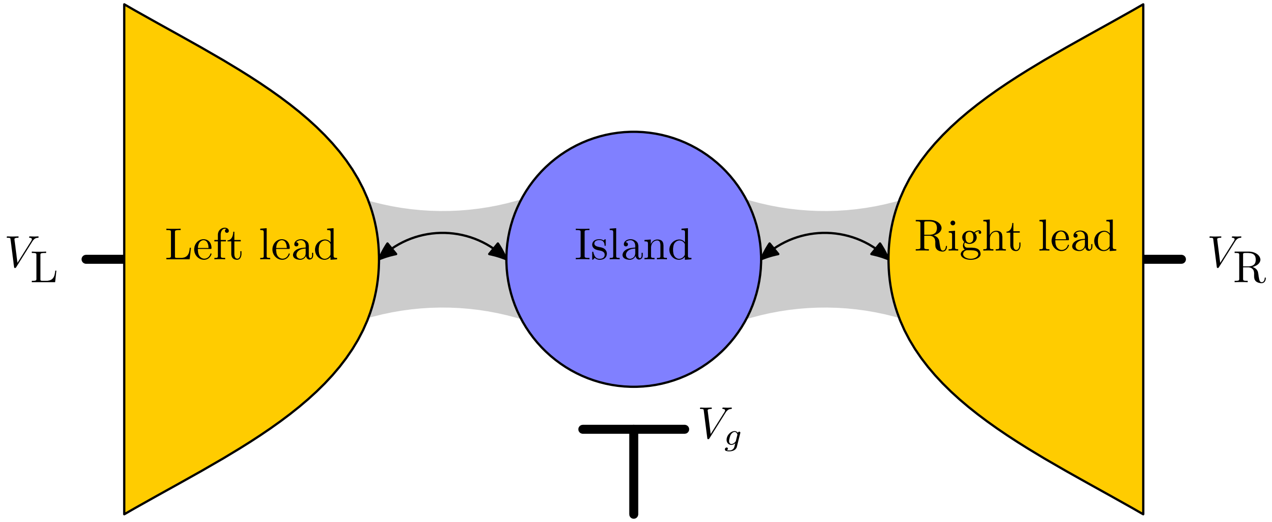
- Phonons
- Magnons
- Microwave photons

Summary

Introduction

Quantum dots

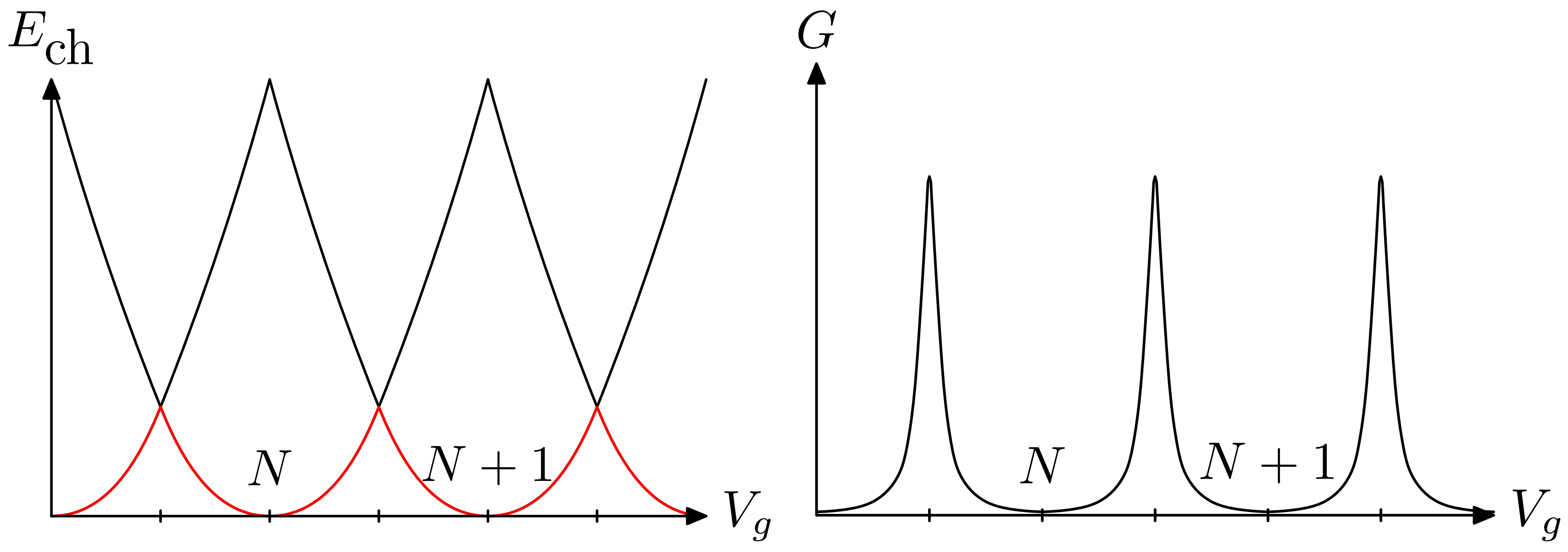
Confine electrons in all three spatial directions, quasi 0-dimensional
“Artificial atom”



Characteristic energy scales

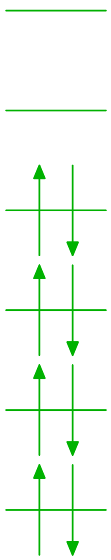
- Charging energy $E_\text{ch} = E_C(N - C_g V_g/e)^2$, $E_C = \frac{e^2}{2C}$
- Level quantization $\Delta\varepsilon$
- Tunnel couplings $\Gamma = 2\pi|t|^2\rho$

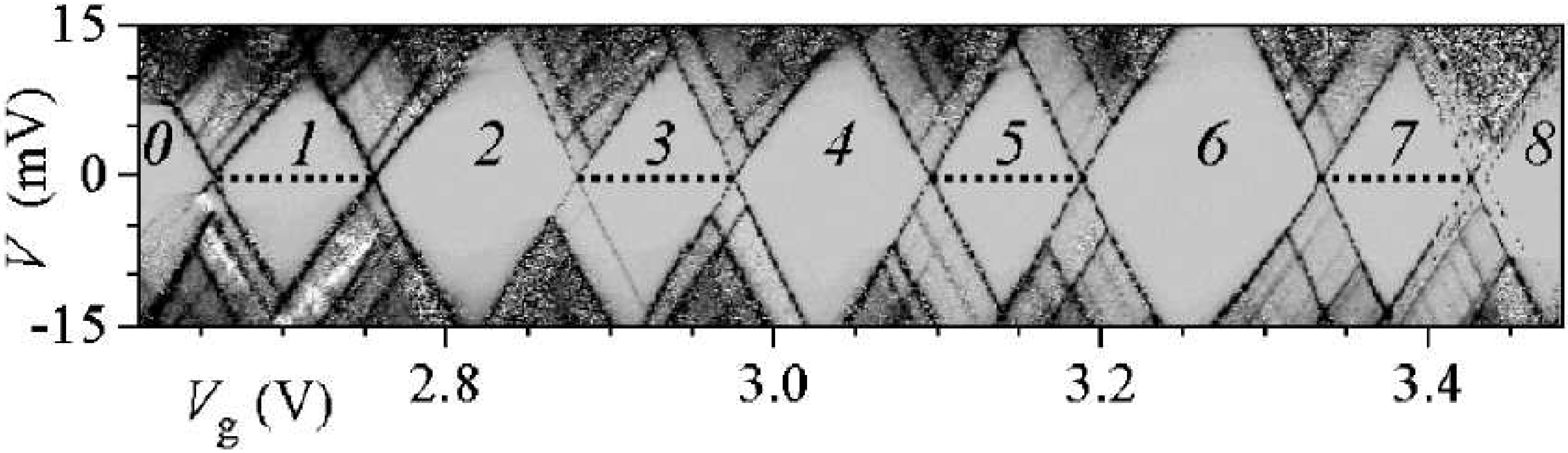
Coulomb blockade



- Charging energy $E_{\text{ch}} = \frac{e^2}{2C}(N - C_g V_g/e)^2$
- Charge fixed away from degeneracy points
- Transport only at degeneracy points
- Coulomb oscillations

Energy quantization and level scheme

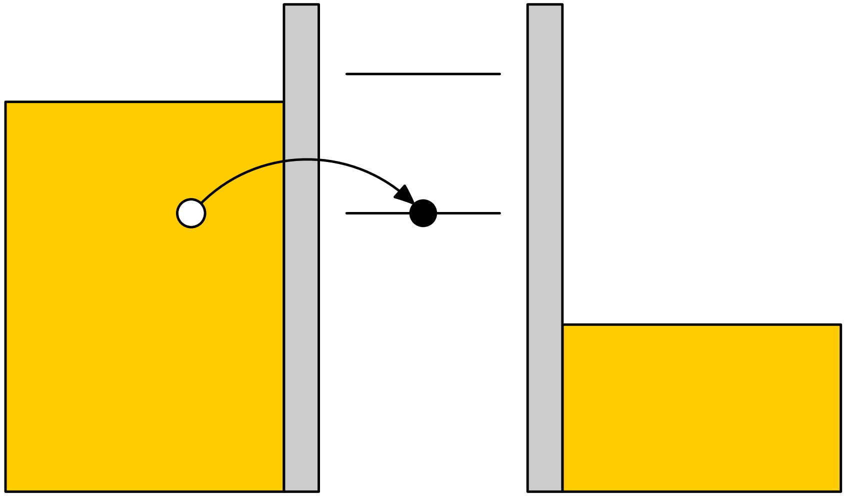




Cobden and Nygård, PRL 2002

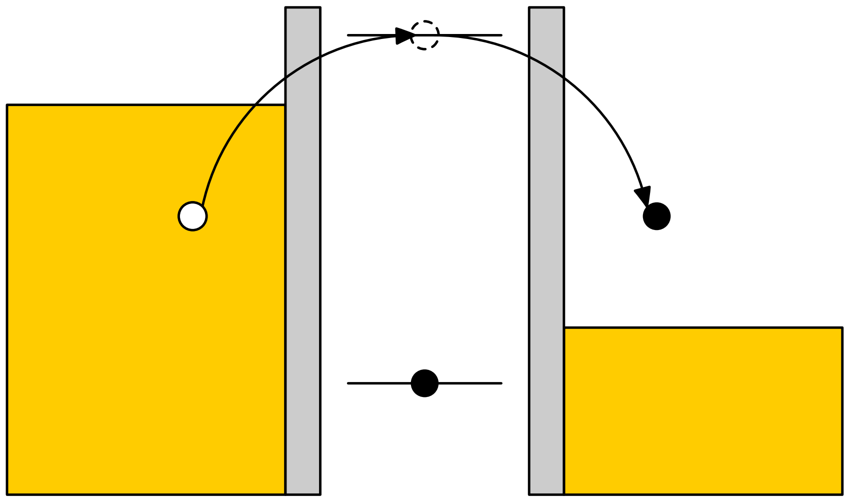
- Finite level spacing in small quantum dots $E_N = E_{\text{ch}} + \sum_{i=1}^{N} \varepsilon_i$
- Addition energy $\Delta = 2E_C + \Delta\varepsilon$
- $\Delta\varepsilon \ll k_{\text{B}}T, eV$: many levels involved → metallic island
- $\Delta\varepsilon \gg k_{\text{B}}T, eV$: only single level → quantum dot

Transport regimes



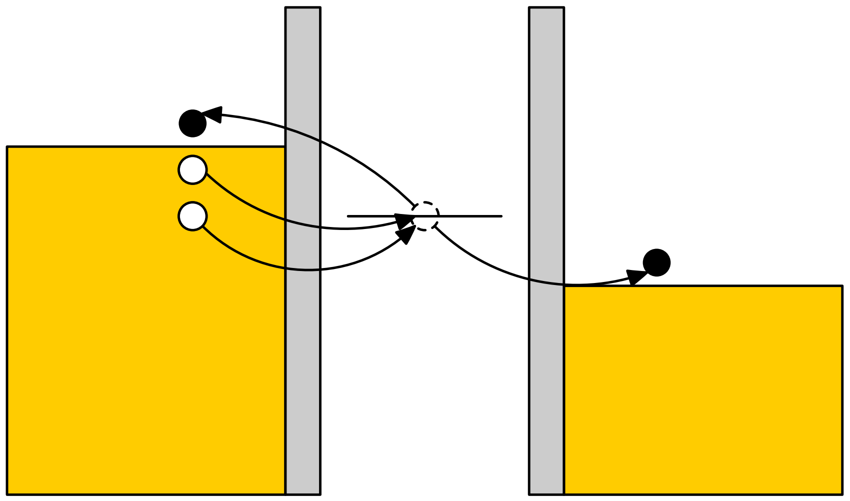
- Sequential tunneling
  - Tunneling of single electron
  - First order in Γ
  - Real occupation of the dot
  - Energy conservation
  - Dominant on resonance $k_{\mathrm{B}}T \gg \Gamma, |\varepsilon|$ or $eV \gg \Gamma, |\varepsilon|$

Transport regimes



- Cotunneling
  - Tunneling of two electrons
  - Second order in Γ
  - Virtual occupation of the dot
  - Energy conservation only for total process
  - Dominant off resonance $|\varepsilon| \gg k_\mathrm{B}T, eV, \Gamma$

Transport regimes

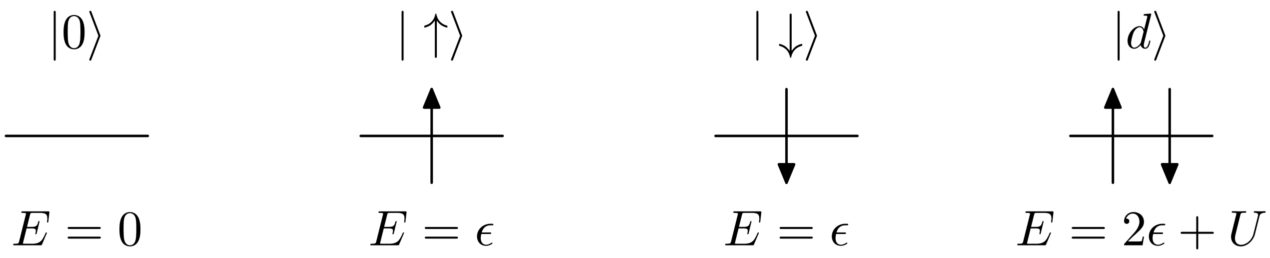


- Resonant tunneling
  - Many electrons tunnel
  - Nonperturbative in Γ
  - Complicated many-body effects (Kondo)
  - Dominant for strong coupling $\Gamma \gg k_\mathrm{B}T, eV$

Anderson model

Single-level quantum dot

$$H = \sum_{\sigma} \varepsilon n_{\sigma} + U n_{\uparrow} n_{\downarrow} + \sum_{r\mathbf{k}\sigma} \varepsilon_{\mathbf{k}} a^{\dagger}_{r\mathbf{k}\sigma} a_{r\mathbf{k}\sigma} + \sum_{r\mathbf{k}\sigma} t_r a^{\dagger}_{r\mathbf{k}\sigma} c_{\sigma} + \text{H.c.}$$



$\|0\rangle$	$\|\uparrow\rangle$	$\|\downarrow\rangle$	$\|d\rangle$
$E = 0$	$E = \epsilon$	$E = \epsilon$	$E = 2\epsilon + U$

- Level position ε (tunable)
- Coulomb energy U
- Tunnel coupling $\Gamma_r = 2\pi |t_r|^2 \rho$ (often tunable)
- Temperature T (tunable)
- Voltage V (tunable)

Master equation approach

- Probability P_χ to find quantum dot in state $\chi \in \{0, \uparrow, \downarrow, d\}$
- Occupation probabilities obey master equation

$$\dot{P}_\chi = \sum_{\chi'} W_{\chi\chi'} P_{\chi'}$$

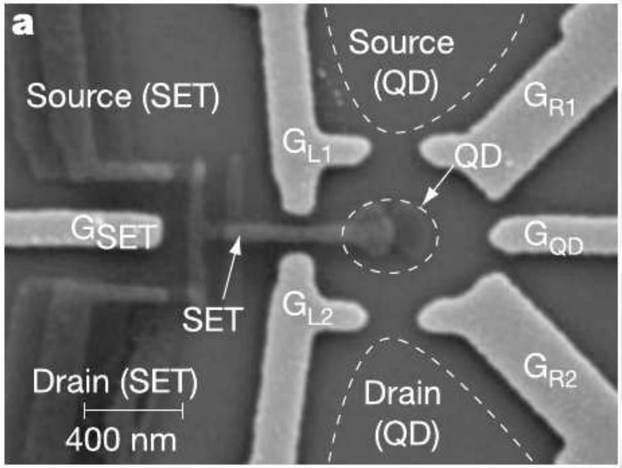
- Transition rates $W_{\chi\chi'}$ from Fermi's golden rule

$$W_{\uparrow 0} = \sum_r \Gamma_r f_r(\varepsilon)$$

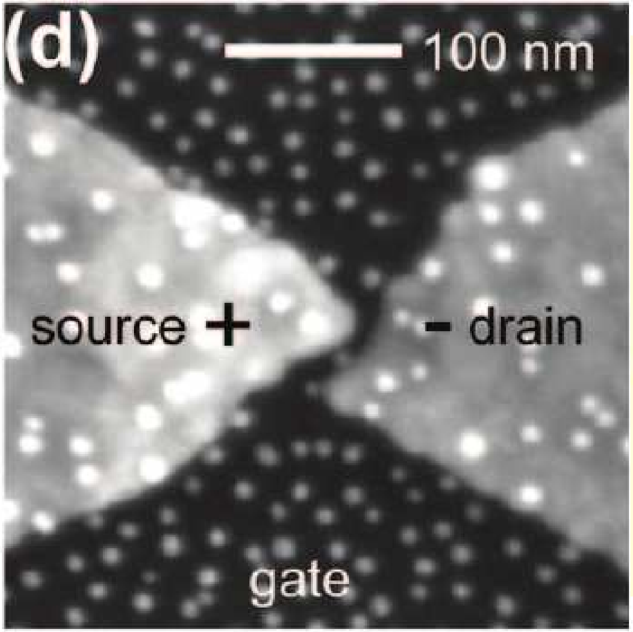
$$W_{\uparrow d} = \sum_r \Gamma_r [1 - f_r(\varepsilon + U)]$$

- Current $I = \sum_{\chi\chi'} W^I_{\chi\chi'} P_{\chi'}$

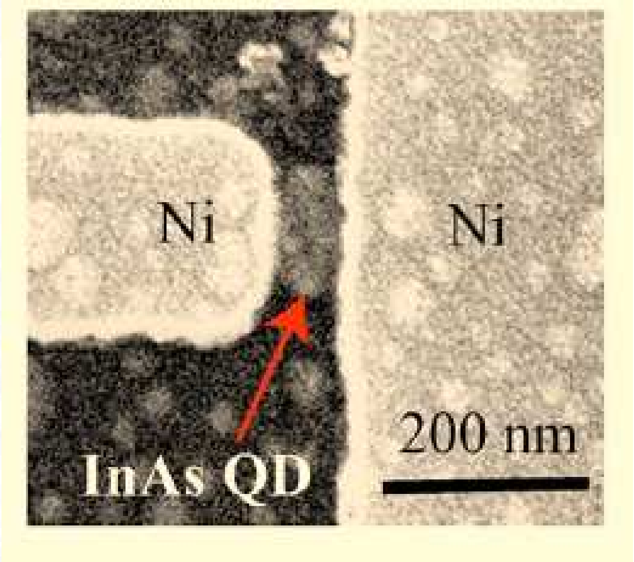
Experimental realizations



Lu et al., Nature 2003



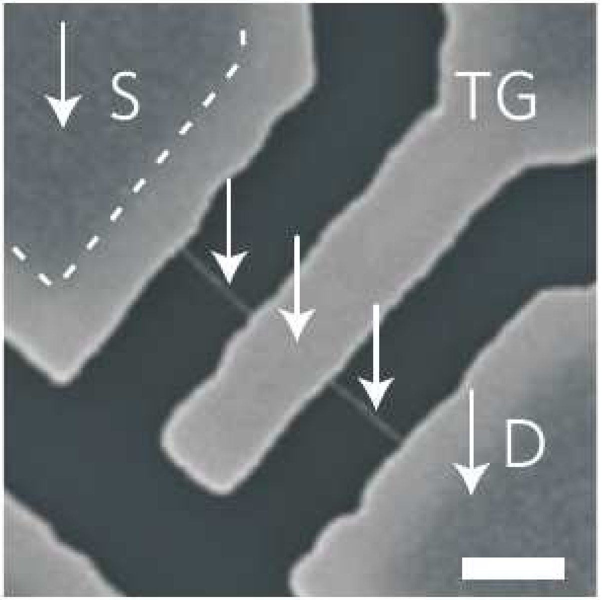
Kuemmeth et al., Nano Letters 2008



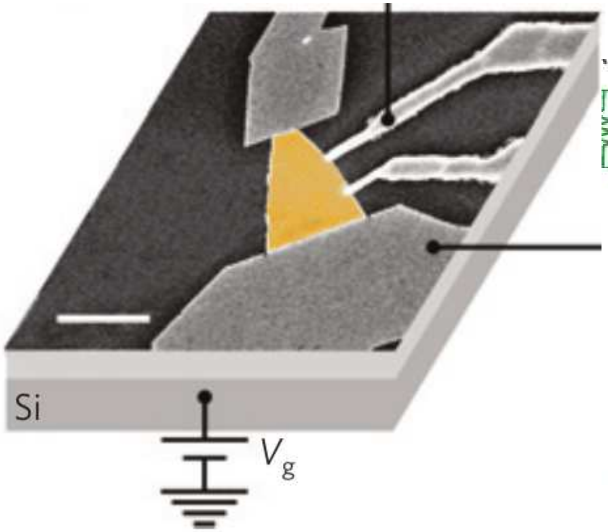
Hamaya et al., APL 2007

- 2-dimensional electron gas
- Metallic nanoparticles
- Self-assembled quantum dots

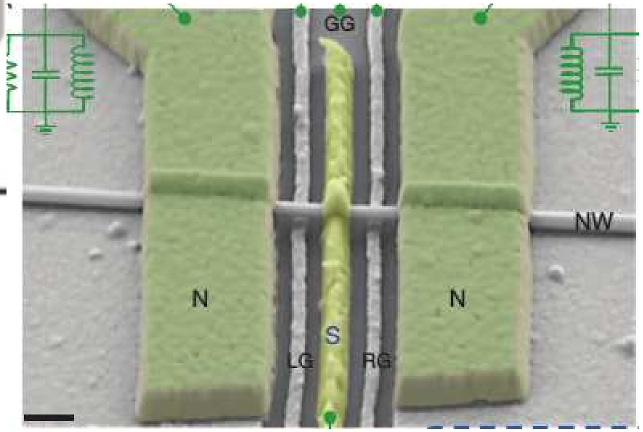
Experimental realizations



Leturcq, Nat. Phys. 2009



Dirks et al., Nat. Phys. 2011



Das et al., Nat. Comm. 2012

- Carbon nanotubes
- Graphene
- Nanowires

Thermoelectrics

- 1823 Seebeck effect: Heat → current
- 1834 Peltier effect: Current → Cooling

Advantages of thermoelectrics:

- No moving parts
- Scalable to the nanoscale
- Heat is ubiquitous

Disadvantage

- Low efficiency, small power

Mesoscopic physics and thermoelectrics

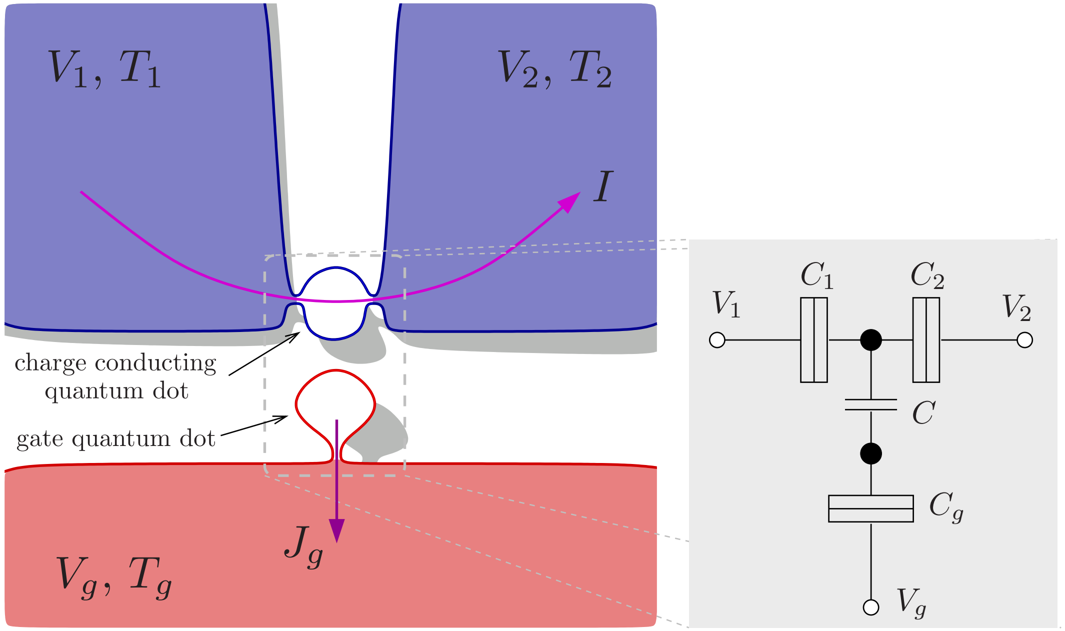
Fundamental research

- Theoretical analysis of thermopower of quantum dots, quantum point contacts Proetto PRB 1991, Beenakker and Staring PRB 1992, Nakpathomkun et al. PRB 2010
- Experiments on quantum dots Staring et al. EPL 1993, Dzurak PRB 1997, Godijn et al. PRL 1999, Scheibner et al. PRL 2005, PRB 2007, Svensson et al. NJP 2012, 2013
- Experiments on quantum point contacts Molenkamp et al. PRL 1990

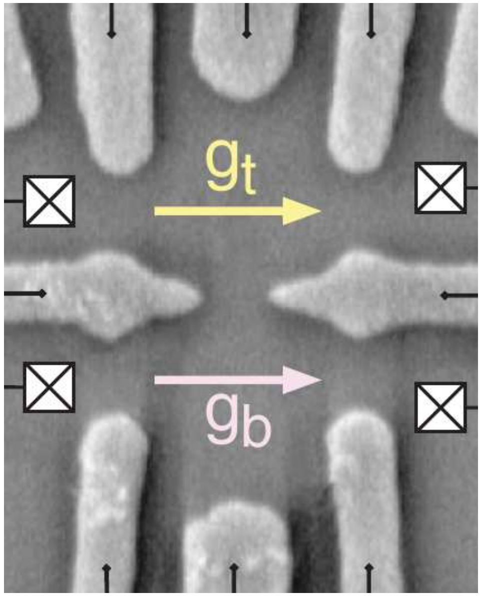
Can mesoscopic systems be useful for thermoelectric applications?

- Quantum wires and wells for thermoelectrics Hicks and Dresselhaus PRB 1993
- Sharp spectral features increase thermoelectric performance Mahan and Sofo PNAS 1996
- Electronic refrigerator Edwards et al. APL 1993, PRB 1995, Prance et al. PRL 2009

Three-terminal thermoelectrics



Sánchez, Büttiker, PRB 2011



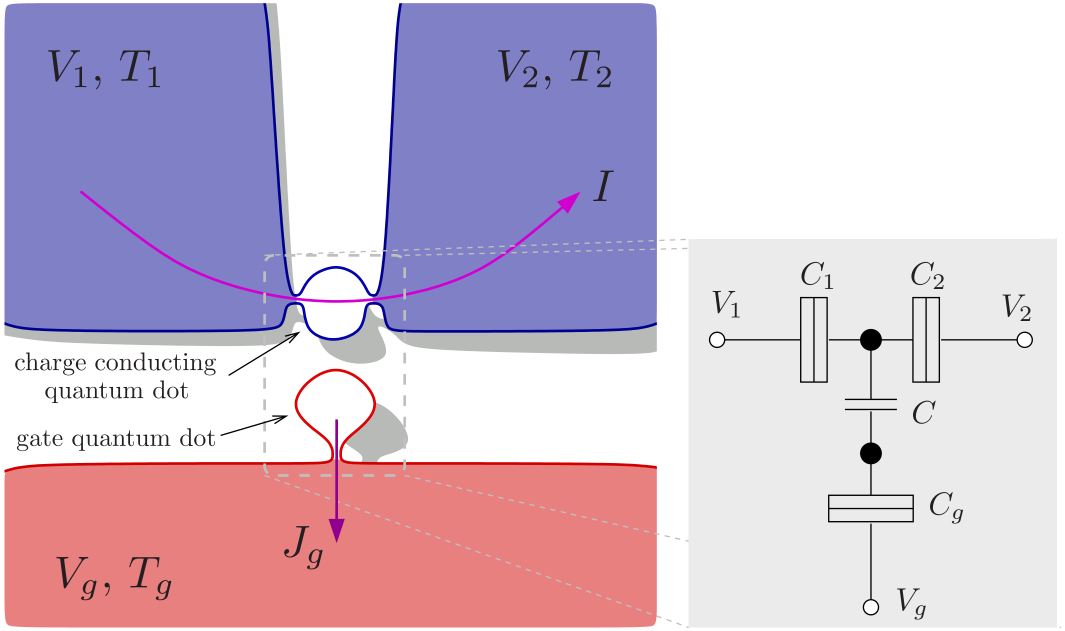
McClure et al., PRL 2007

- Connection to Coulomb-drag setups
- Crossed heat and charge currents
- Separation of heat source and rectifier
- Energy harvesting

Coulomb-coupled conductors

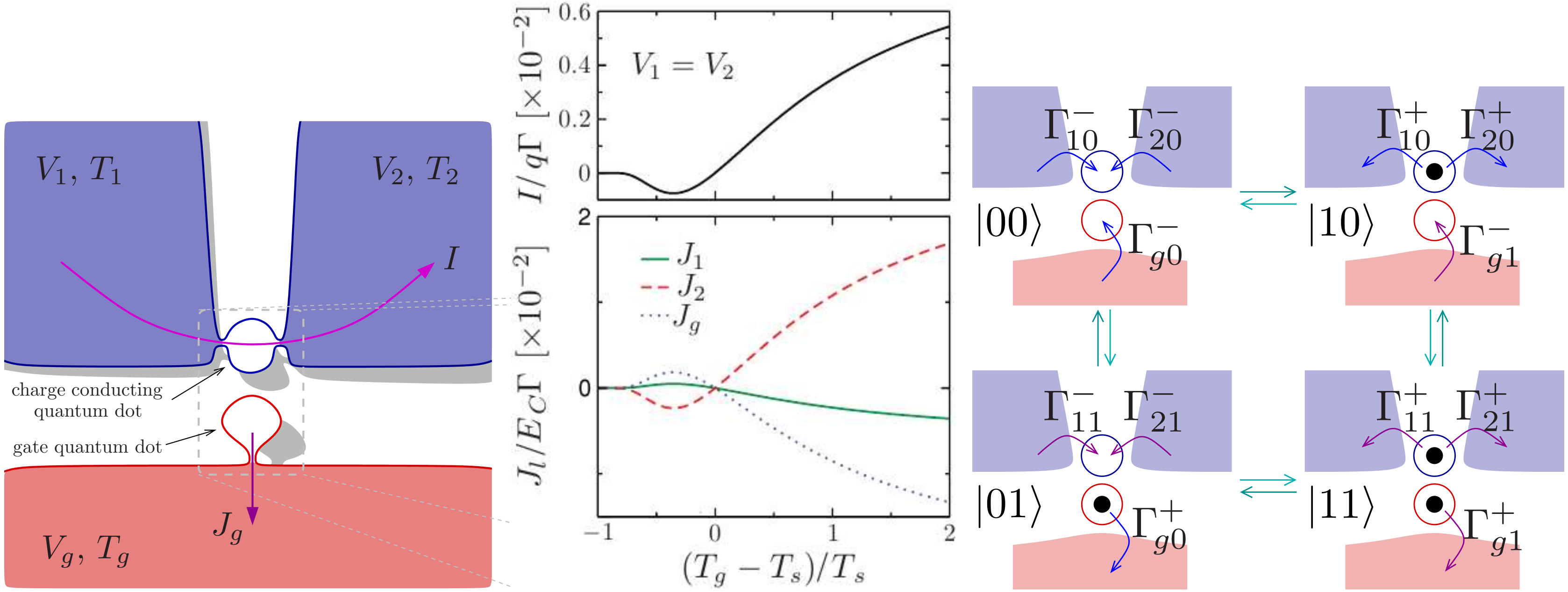
Coulomb-blockade regime

Sánchez, Büttiker, PRB 2011



- Coulomb-coupled quantum dots
- Exchange energy but no particles
- Conductor dot: two cold reservoirs
- Gate dot: single hot reservoir

Coulomb-blockade regime



- Drive current by temperature bias
- Energy-dependent, asymmetric tunnel barriers
- Optimal heat to charge current conversion
- One energy quantum of the bath transfers one charge quantum

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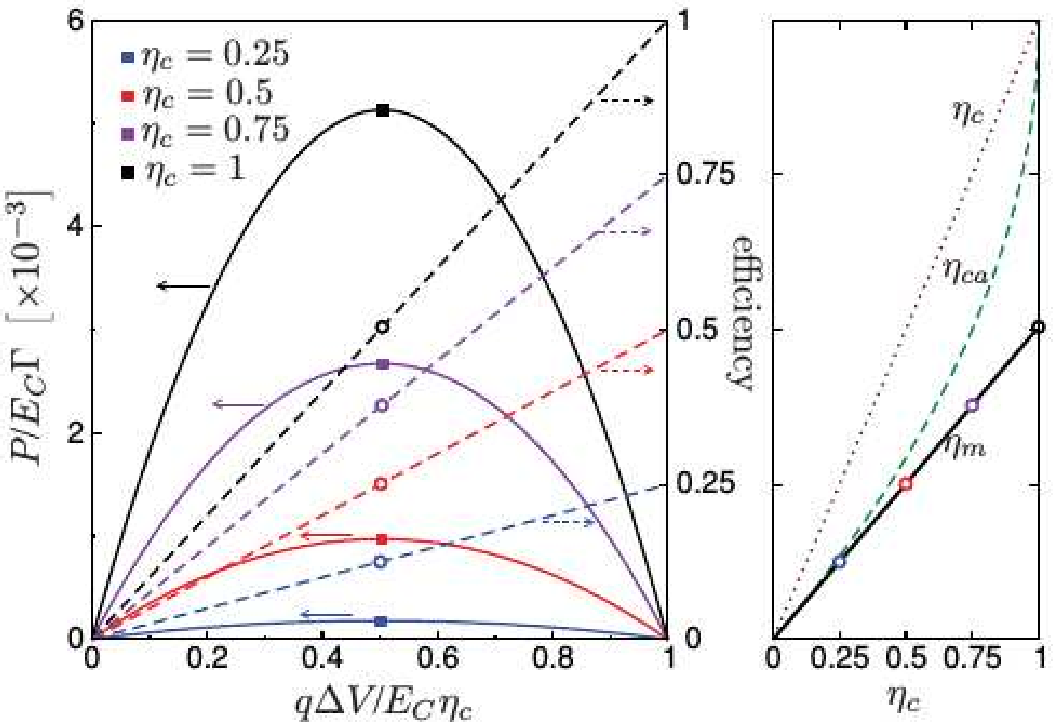
Coulomb-blockade regime



- Power $P = IV$
- Efficiency $\eta = P/J_g$
- Device reaches Carnot efficiency η_C at stopping voltage
- Efficiency at maximum power $\eta_{\text{maxP}} = \eta_C/2$

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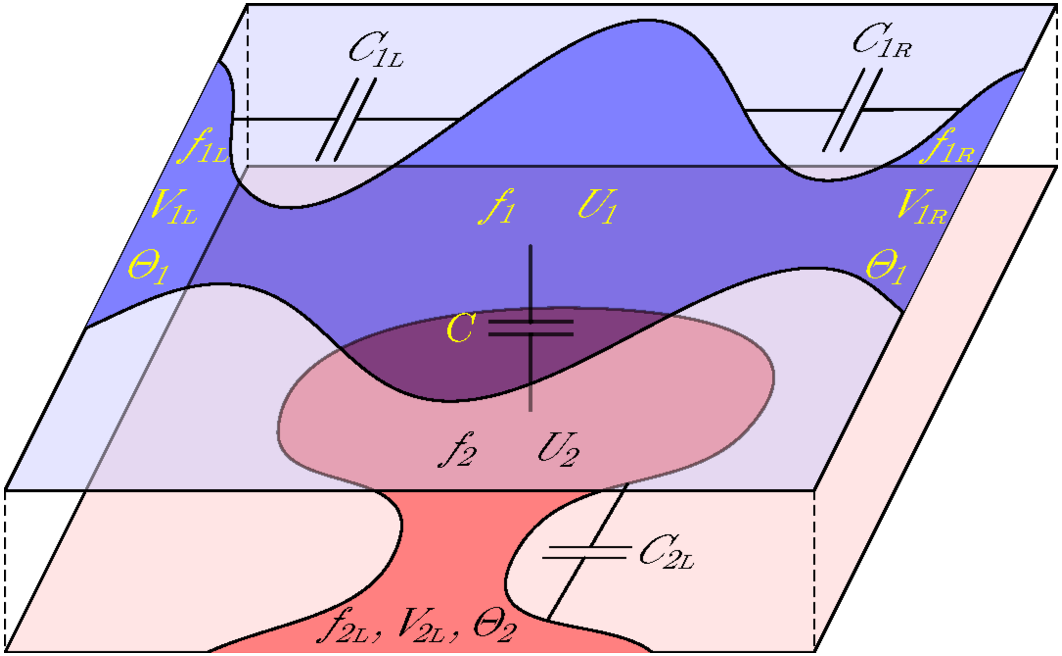
Chaotic cavities

BS, Sánchez, Jordan and Büttiker, PRB 2012



- Capacitively coupled chaotic cavities
- Open quantum dots: Large number N of transport channels
- How do current and power vary with N?
- Asymmetric, energy-dependent transmissions $T_r = T_r^0 - eT_r'\delta U$

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Chaotic cavities

- Current $I = \frac{\Lambda}{\tau_{RC}} k_{\mathrm{B}}(\Theta_1 - \Theta_2)$
- Asymmetry parameter $\Lambda = \frac{G_{\mathrm{L}} G'_{\mathrm{R}} - G_{\mathrm{R}} G'_{\mathrm{L}}}{(G_{\mathrm{L}} + G_{\mathrm{R}})^2}$ where $G_r = \frac{e^2}{h} T_r^0$ and $G'_r = \frac{e^3}{h} T'_r$
- RC time τ_{RC} determined by effective conductance and capacitance of double cavity
- Current independent of channel number, $I \sim 0.1\,\mathrm{nA}$
- Power scales as $1/N$, similar to Coulomb-blockade for a few open channels $P \sim 1\,\mathrm{fW}$
- Efficiency scales as $1/N^2$, few percent of η_C

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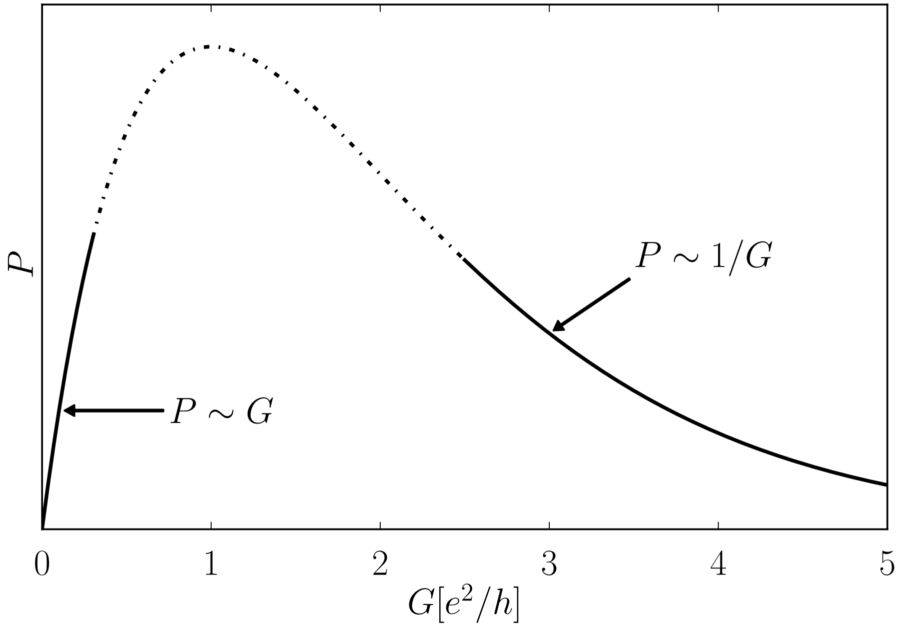
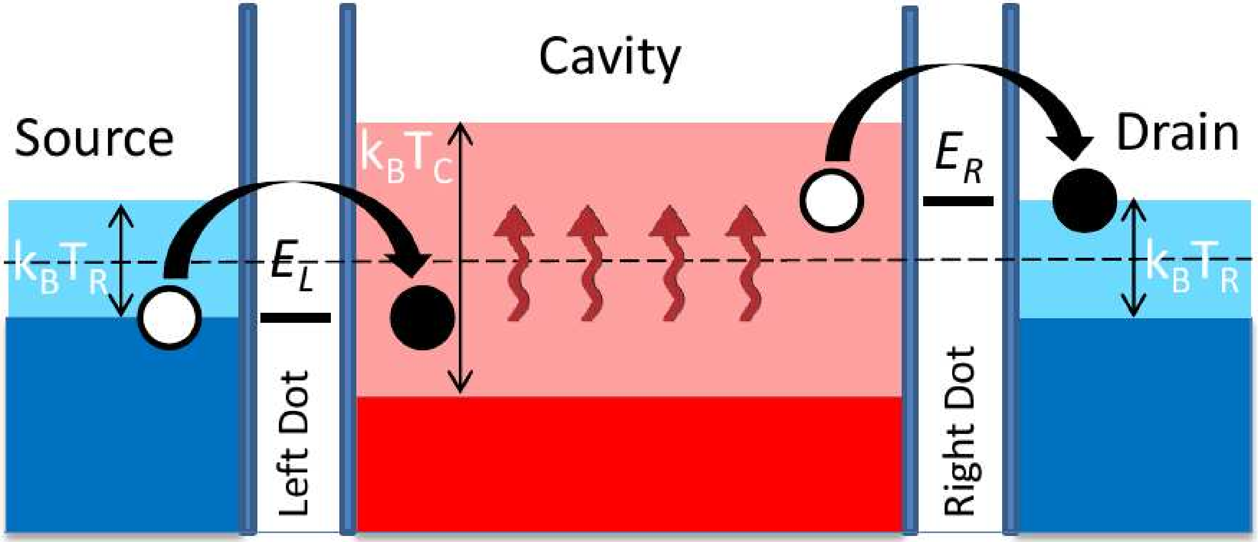
Power versus conductance



- Coulomb blockade regime: Power grows linear with conductance
- Open contacts: Power drops as inverse conductance
- Maximal power should be achieved for single channel
- $\Rightarrow$ Consider resonant tunneling through quantum dots

Resonant tunneling

Jordan, BS, Sánchez and Büttiker, PRB 2013



- Central cavity in thermal equilibrium with hot reservoir
- Cavity connected to two cold electronic reservoirs via quantum dots
- Quantum dots host single resonant level with width γ and energy $E_{\mathrm{L,R}}$

Resonant tunneling

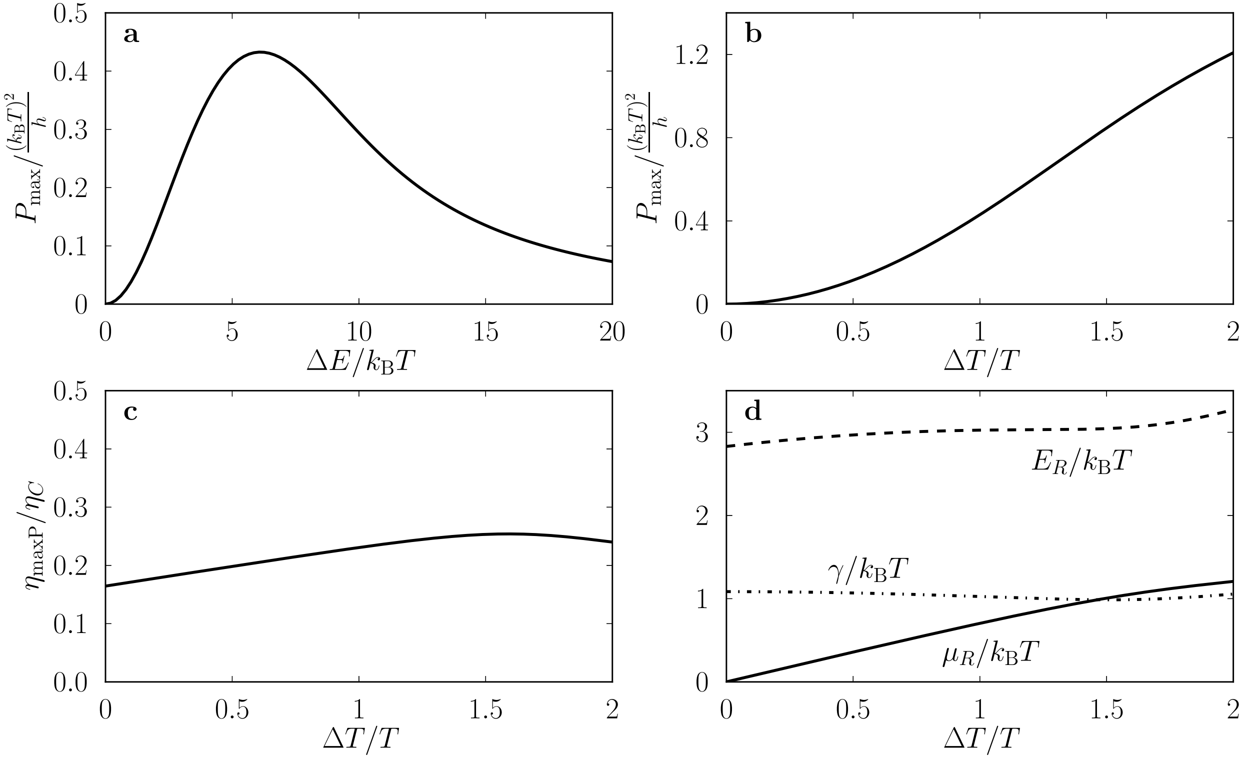
Scattering matrix approach

- Charge current $I_j = \frac{2e}{h} \int dE \, T_j(E)[f_j(E) - f_\mathsf{C}(E)]$
- Energy current $J_j = \frac{2}{h} \int dE \, E T_j(E)[f_j(E) - f_\mathsf{C}(E)]$
- Transmission $T_j(E) = \frac{\gamma^2}{(E-E_j)^2+\gamma^2}$
- Heat current J from hot reservoir
- Conservation of charge and energy

$$0 = I_\mathsf{L} + I_\mathsf{R}$$

$$0 = J + J_\mathsf{L} + J_\mathsf{R}$$

Resonant tunneling



- Numerically optimize ΔE, γ and V for maximal power
- Optimal values: $\Delta E \approx 6k_{\mathrm{B}}T$, $\gamma \approx k_{\mathrm{B}}T$
- Maximal power $P_{\mathrm{max}} \sim 0.4(k_{\mathrm{B}}\Delta T)^2/h$, about $0.1\,\mathrm{pW}$ at $\Delta T = 1\,\mathrm{K}$
- Efficiency at maximum power $\eta_{\mathrm{maxP}} \sim 0.2\eta_{\mathrm{C}}$

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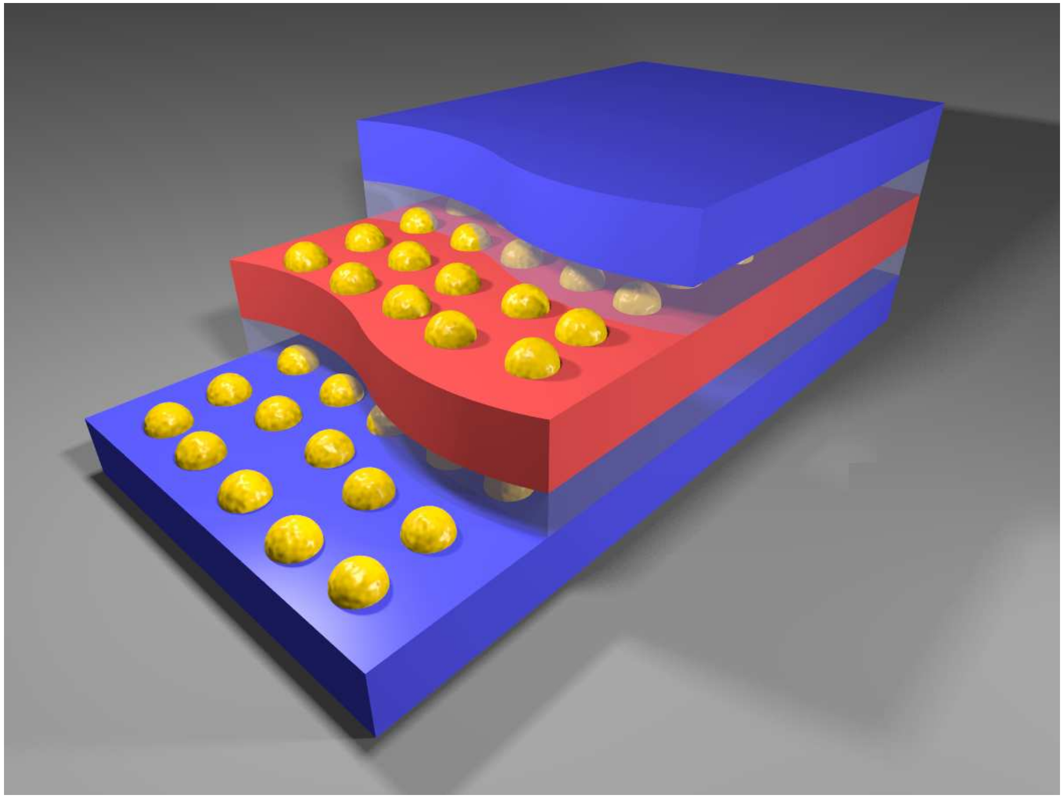
Resonant tunneling

Scaling

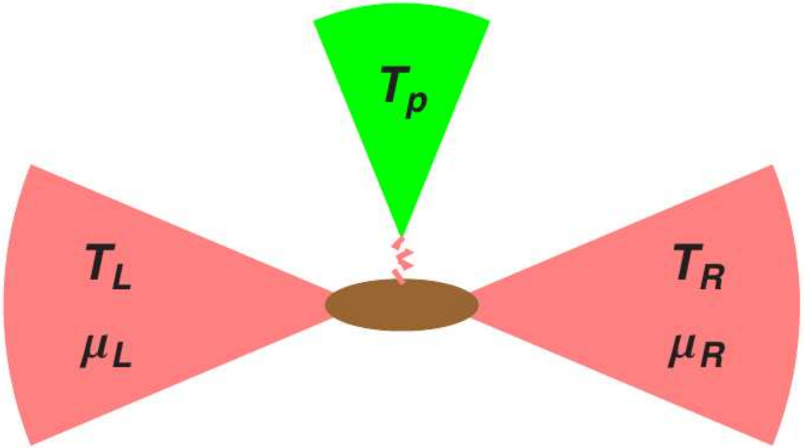


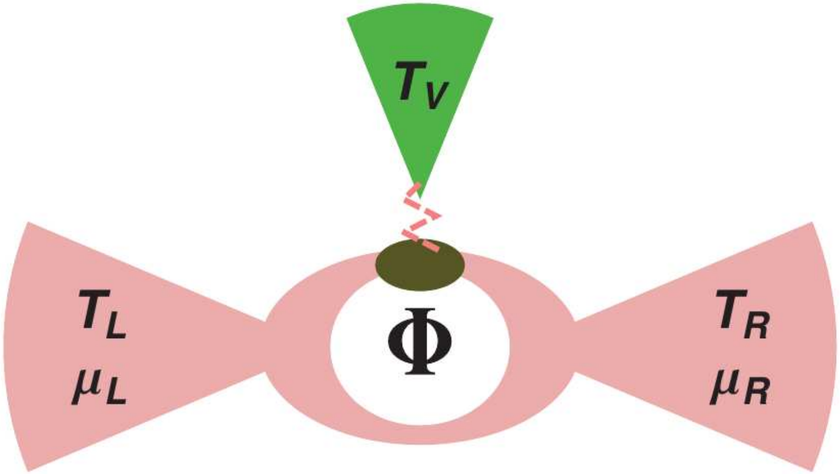
- Swiss cheese sandwich with self-assembled quantum dots
- Dot positions do not have to match
- Dot size of $100\,\mathrm{nm}^2$ yields $10\,\mathrm{W/cm}^2$ at $\Delta T = 10\,\mathrm{K}$
- Robust with respect to fluctuations of level positions

Harvesting bosons

Phonons

Entin-Wohlman et al. PRB 2010, PRB 2012





- Quantum dot coupled to electronic reservoirs and phonon bath
- Linear-response thermoelectrics
- Left-right and particle-hole symmetry broken: $\Gamma_L(E) \neq \Gamma_R(E)$
- Flux-dependence of response coefficients in Aharonov-Bohm geometry

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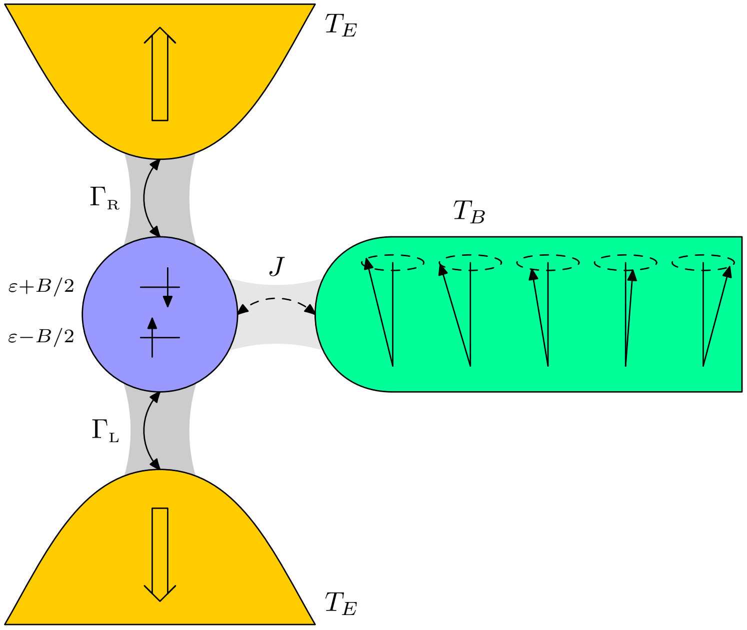
Magnons

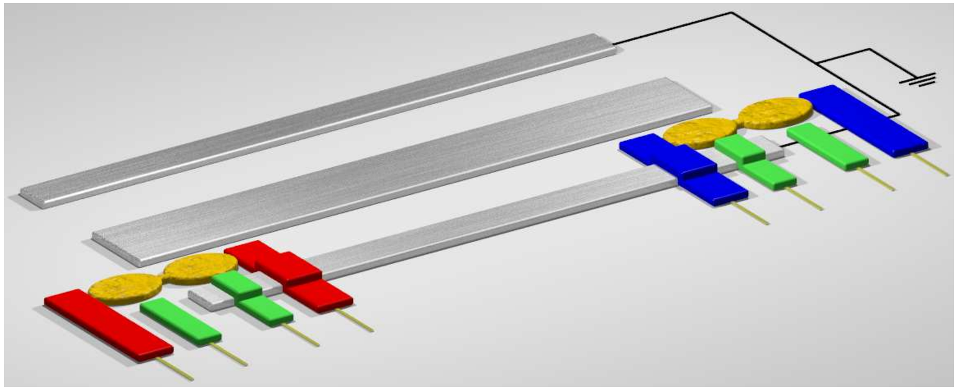
BS and Büttiker, EPL 2012

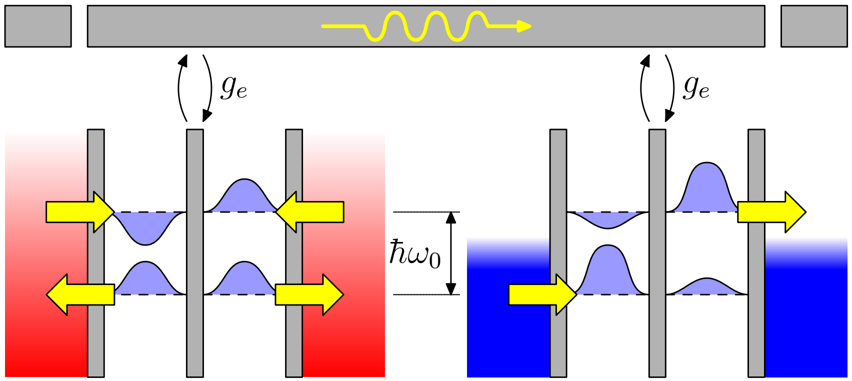


- Quantum dot coupled to ferromagnetic electrodes and ferromagnetic insulator
- Bridge between energy harvesting and spin caloritronics
- Drive pure spin current or spin-polarized charge current by magnons

Microwave photons

Bergenfeldt, Samuelsson, BS, Flindt and Büttiker, arXiv 2013





- Double quantum dots connected via superconducting cavity
- Combines circuit QED and thermoelectrics
- Separate hot and cold part by macroscopic distance
- Reduce leakage heat currents

Summary

Summary

- Three-terminal quantum-dot thermoelectrics
- Coulomb-coupled conductors
  - Coulomb blockade: High efficiency, small power
  - Open dots: Small efficiency, small power
  - Resonant tunneling: Large power, good efficiency
- Boson-driven heat engines
  - Phonons (hard to control)
  - Magnons: spintronics
  - Microwave photons: circuit QED