



COLLÈGE
DE FRANCE
—1530—



CIFAR
CANADIAN INSTITUTE
for ADVANCED RESEARCH

Supraconductivité à haute température dans les cuprates et les organiques: Où en est-on?

André-Marie Tremblay



Collège de France, 9, 16, 23 et 30 mars 2015
17h00 à 18h30



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Two pillars of Condensed Matter Physics

- Band theory
 - DFT
 - Fermi liquid Theory
 - Metals
 - Semiconductors: transistor
- BCS theory of superconductivity
 - Broken symmetry
 - Emergent phenomenon
 - Also in particle physics, astrophysics...



Breakdown of band theory

Half-filled band is metallic?



Half-filled band: Not always a metal

NiO, Boer and Verwey



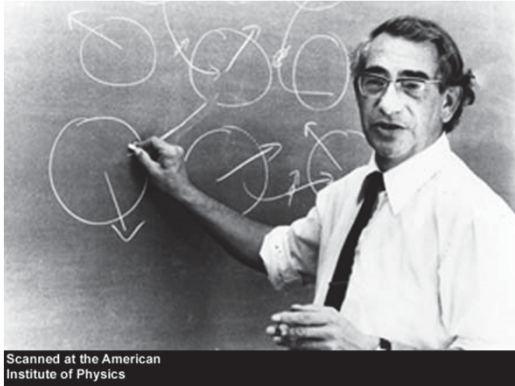
Peierls, 1937



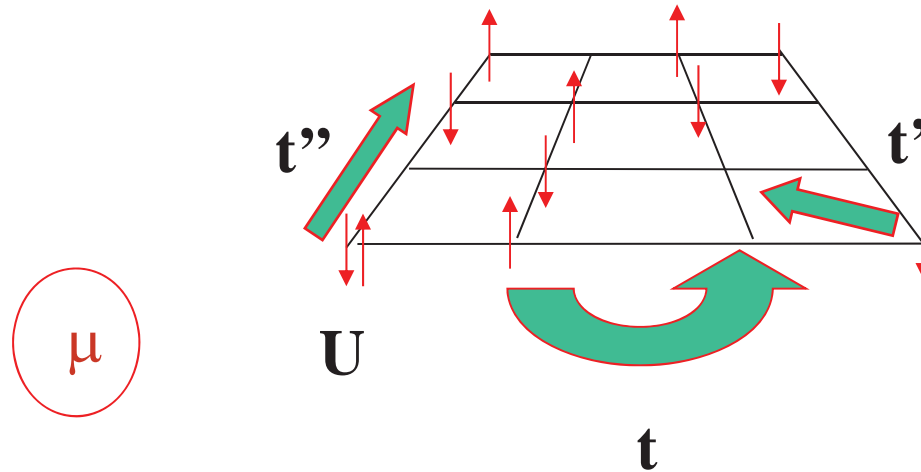
Mott, 1949



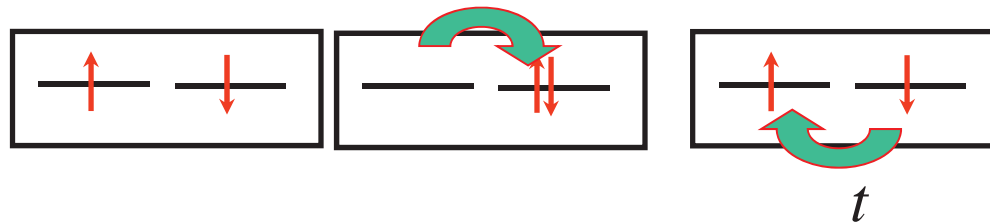
Hubbard model



1931-1980



$$H = - \sum_{\langle ij \rangle \sigma} t_{i,j} (c_{i\sigma}^\dagger c_{j\sigma} + c_{j\sigma}^\dagger c_{i\sigma}) + U \sum_i n_{i\uparrow} n_{i\downarrow}$$



Effective model, Heisenberg: $J = 4t^2 / U$

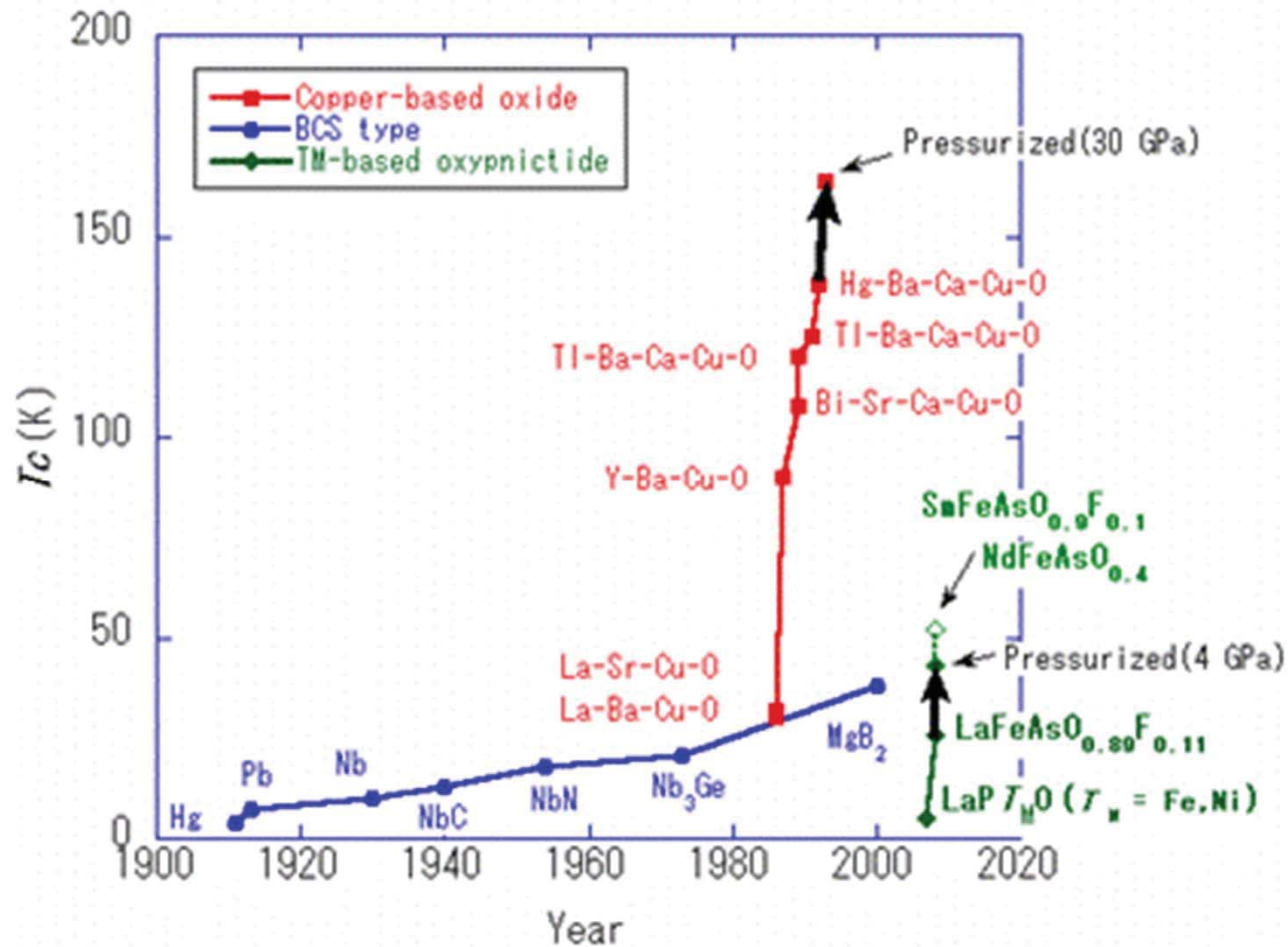


High temperature superconductors and layered organic superconductors

Failure of
BCS theory
Band structure
and more



New and old superconductors



H. Takahashi: JPSJ Online—News and Comments [June 10, 2008]



March meeting APS, 1987

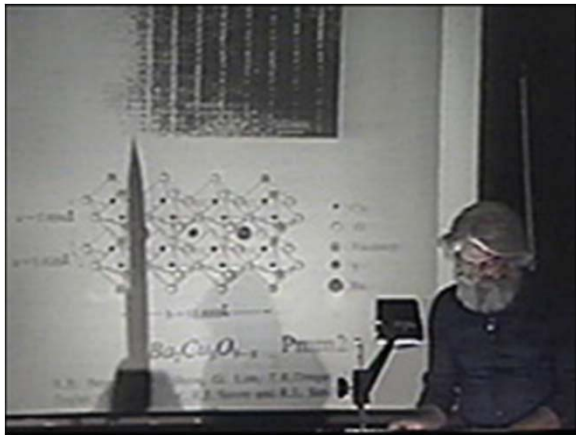
- New York Times headlines
"The Woodstock of Physics"

"They began lining up outside the New York Hilton Sutton Ballroom at 5:30PM for an evening session that would last until 3:00 AM"



15-18 Aug. 1969
500,000 participants







© A. Reymbaut

Atomic structure

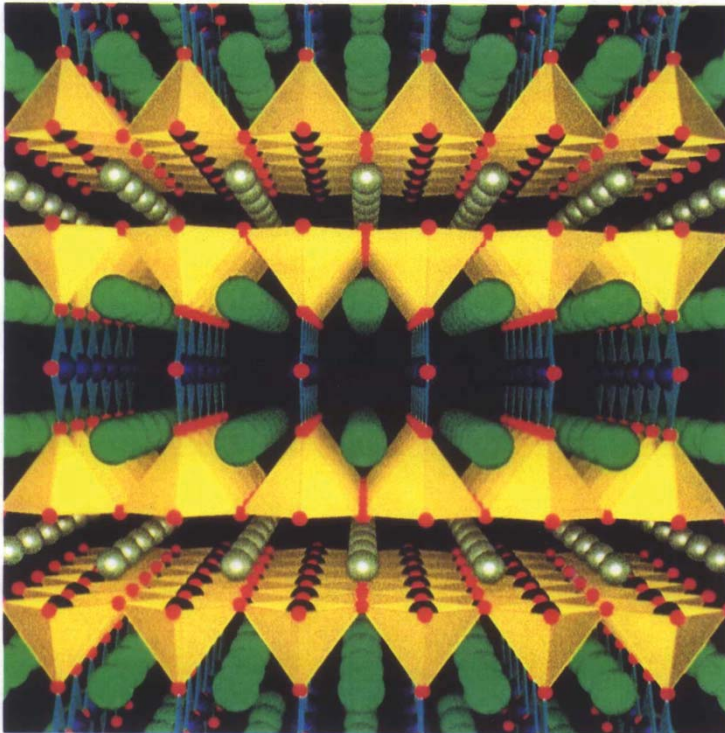
SCIENTIFIC AMERICAN

JUNE 1988
\$3.50

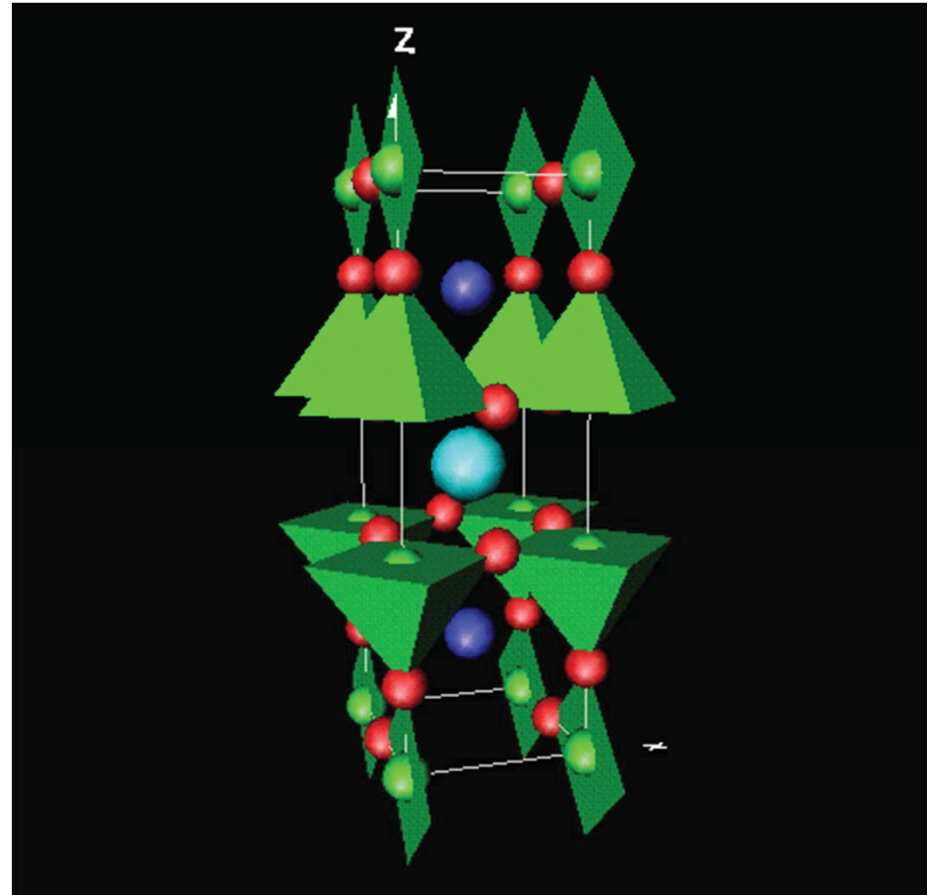
How nonsense is deleted from genetic messages.

R&D for economic growth: aggressive use of new technology.

Can particle physics test cosmology?

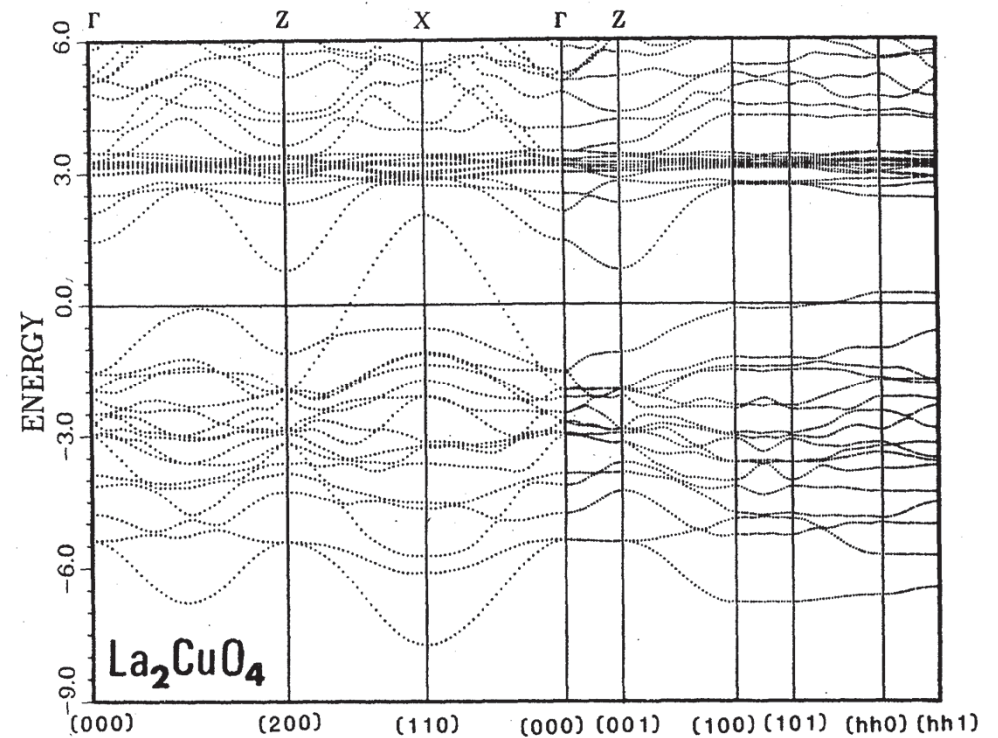


High-Temperature Superconductor belongs to a family of materials that exhibit exotic electronic properties.
 $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$ 92-37



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Band structure for high Tc

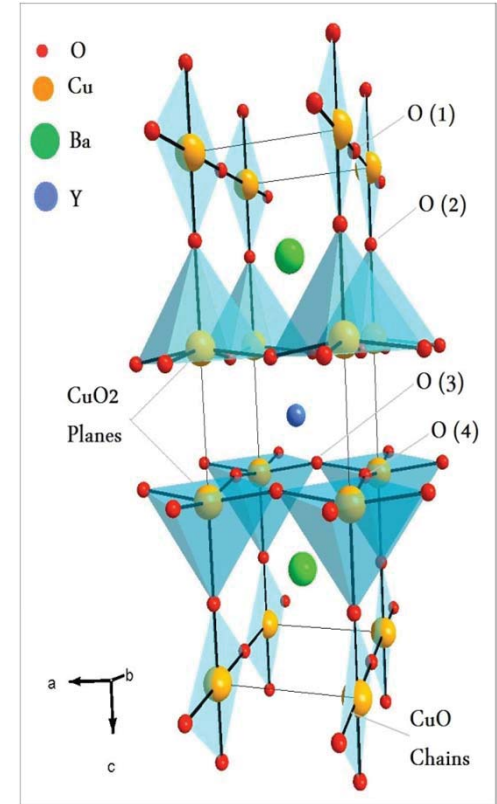
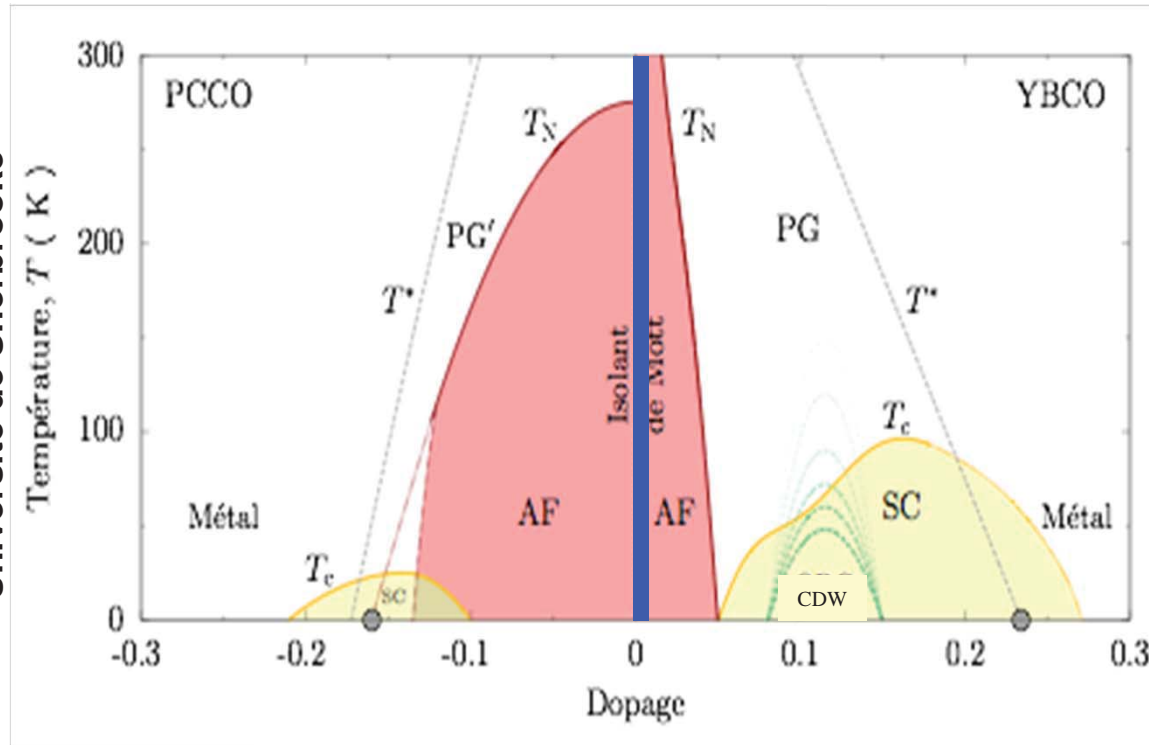


W. Pickett, Rev. Mod. Phys. 1989



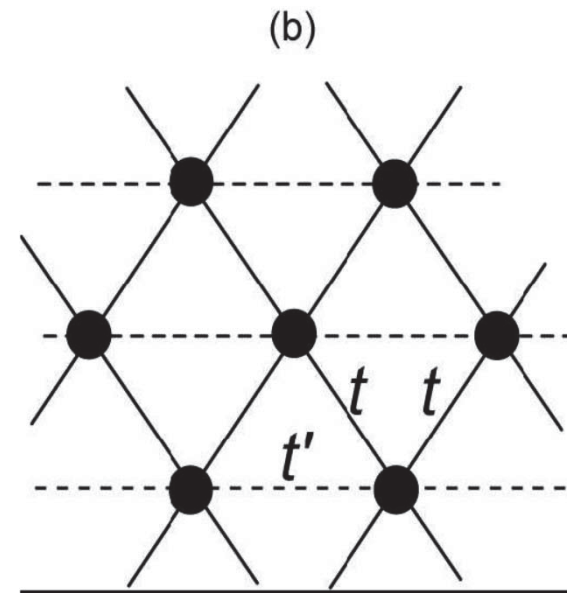
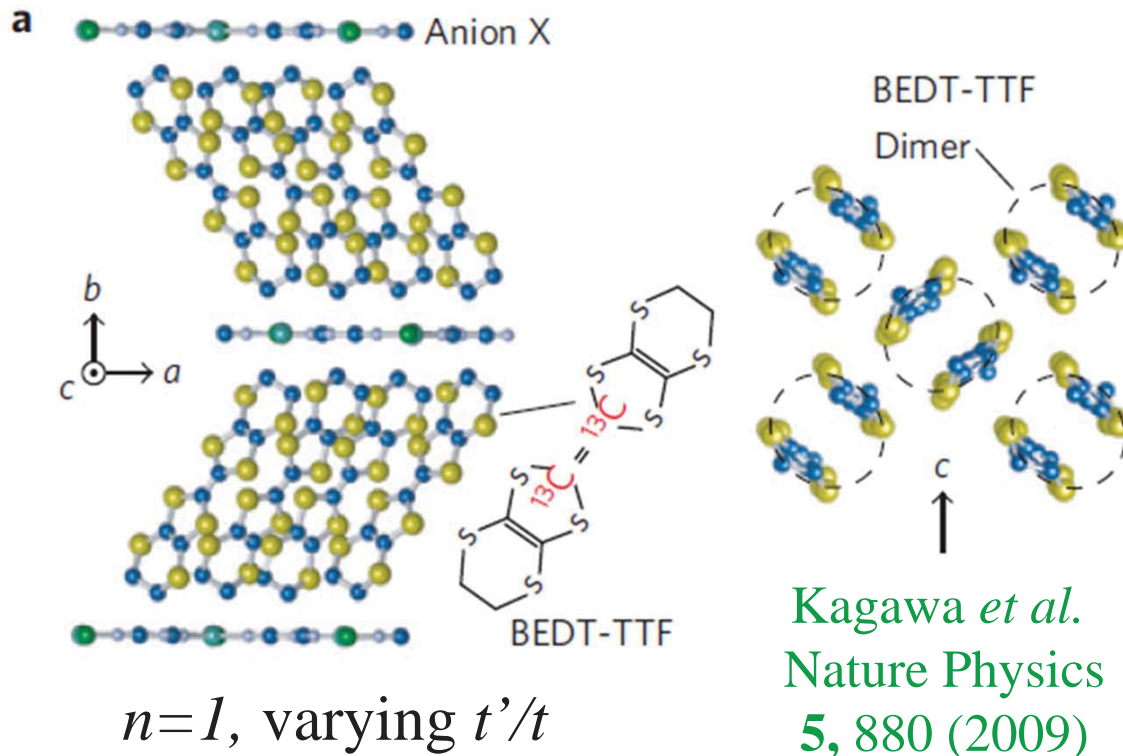
Our road map

Thèse de Francis Laliberté,
Université de Sherbrooke



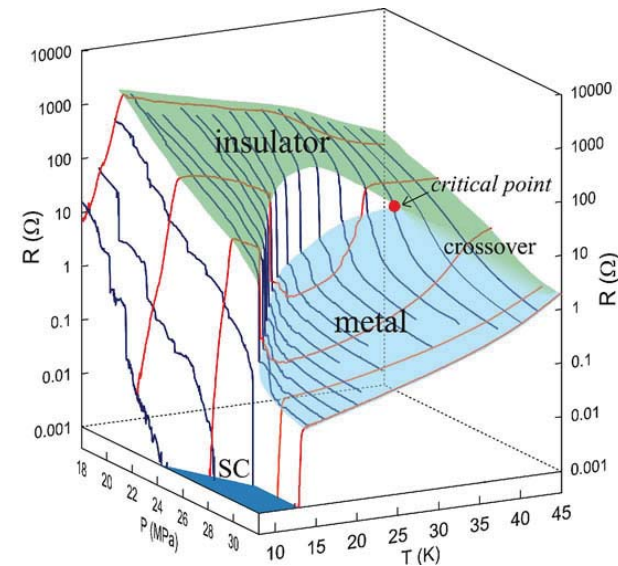
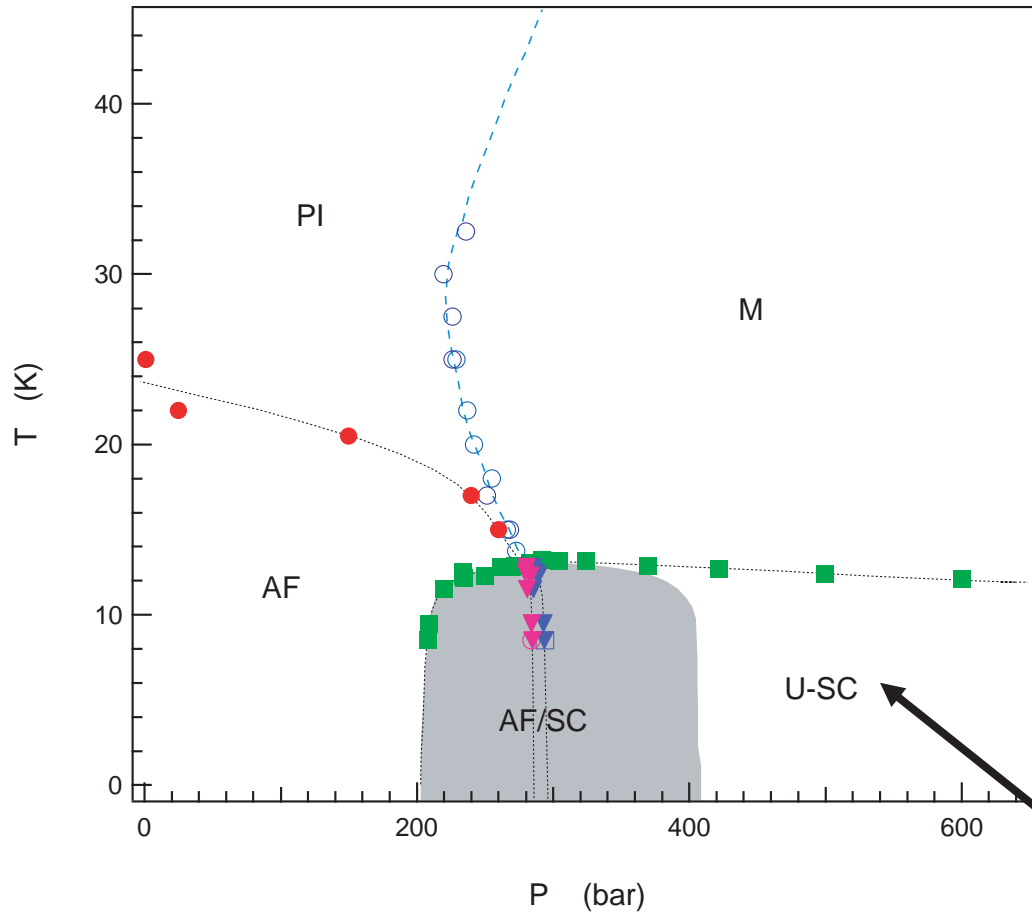
Hubbard on anisotropic triangular lattice

H. Kino + H. Fukuyama, J. Phys. Soc. Jpn **65** 2158 (1996),
 R.H. McKenzie, Comments Condens Mat Phys. **18**, 309 (1998)



$$H = \sum_{ij\sigma} (t_{ij} - \delta_{ij}\mu) c_{i\sigma}^\dagger c_{j\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

Phase diagram for organics



F. Kagawa, K. Miyagawa, + K. Kanoda
PRB **69** (2004) + Nature **436** (2005)

B_g for C_{2h} and B_{2g} for D_{2h}

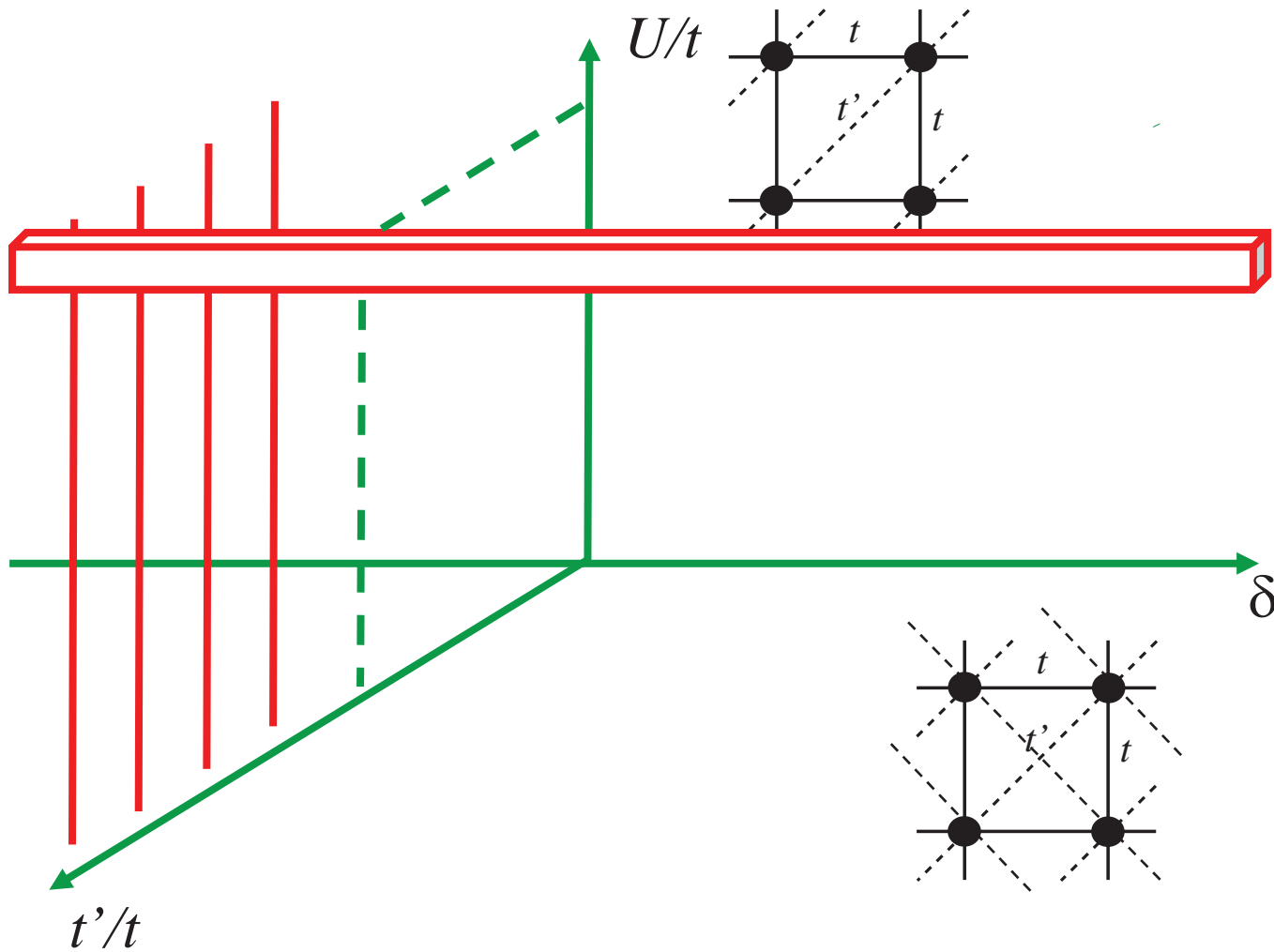
Powell, McKenzie cond-mat/0607078

Phase diagram ($X=Cu[N(CN)_2]Cl$)

S. Lefebvre et al. PRL **85**, 5420 (2000), P. Limelette, et al. PRL **91** (2003)

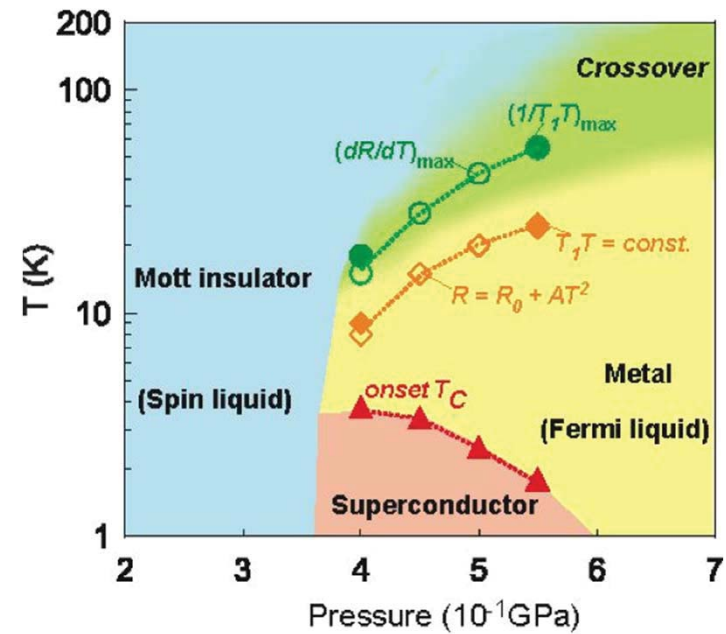
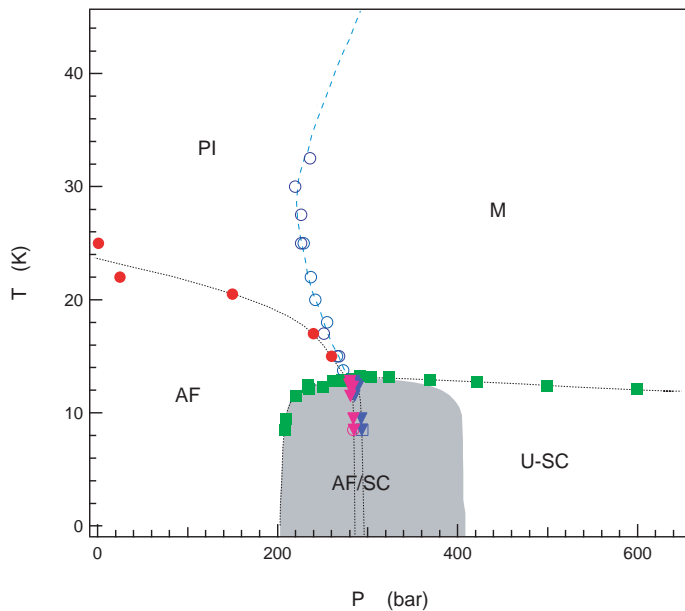
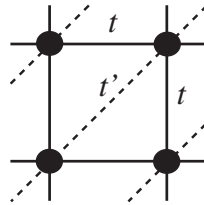


Perspective



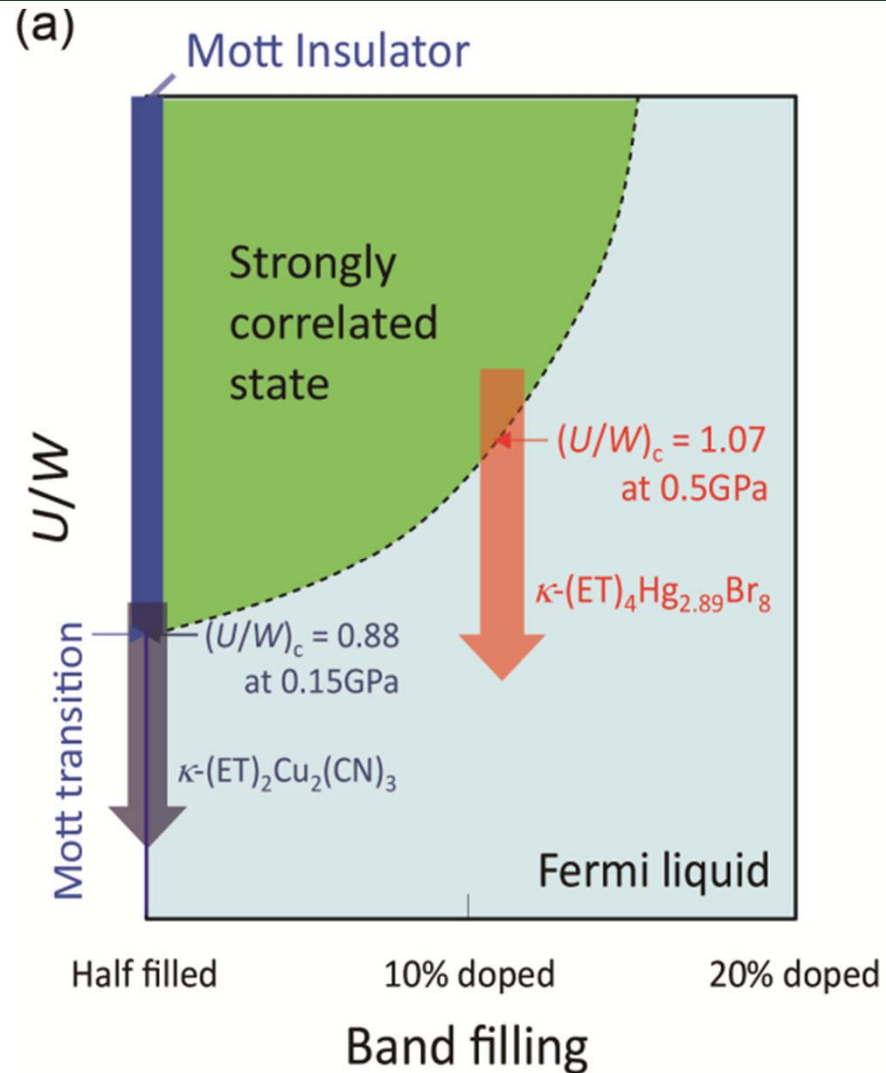
Phase diagram BEDT

$$t' = 0.6t$$



Y. Kurisaki, et al.
 Phys. Rev. Lett. **95**, 177001(2005)
 Y. Shimizu, et al. Phys. Rev. Lett. **91**, (2003)

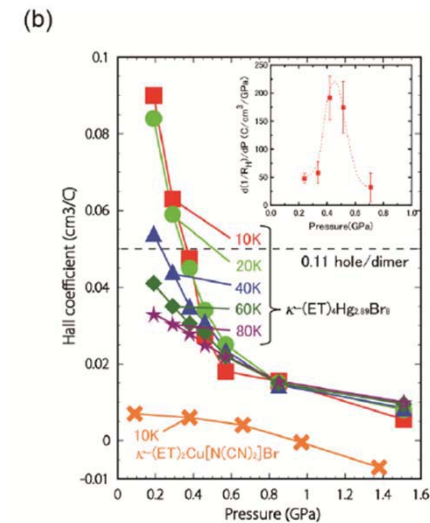
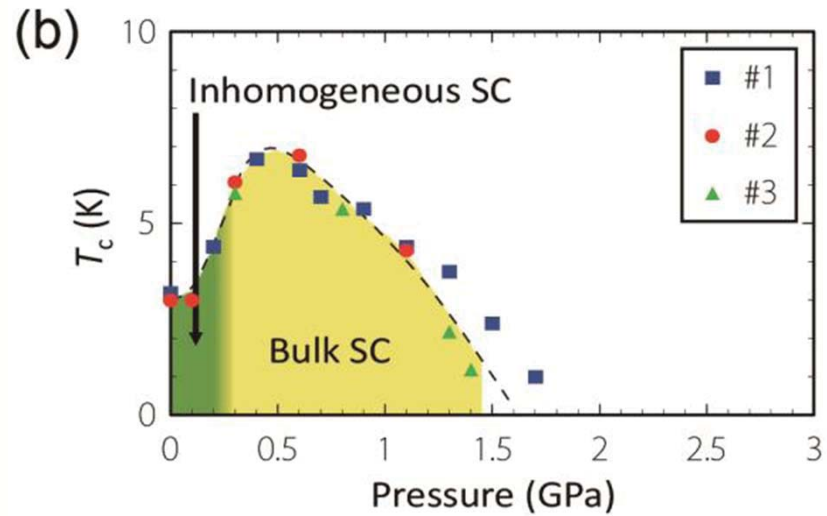
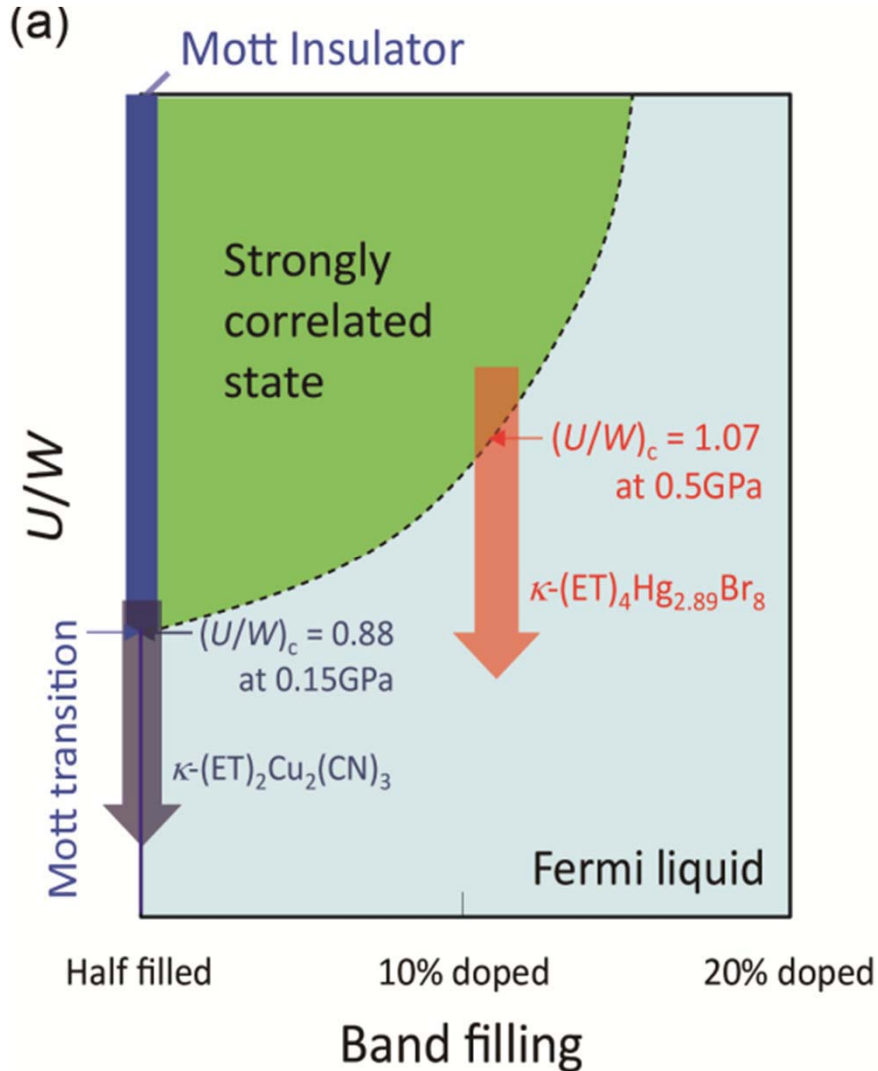
Doped organic



H. Oike, K. Miyagawa, H. Taniguchi, K. Kanoda PRL **114**, 067002 (2015)



Doped BEDT

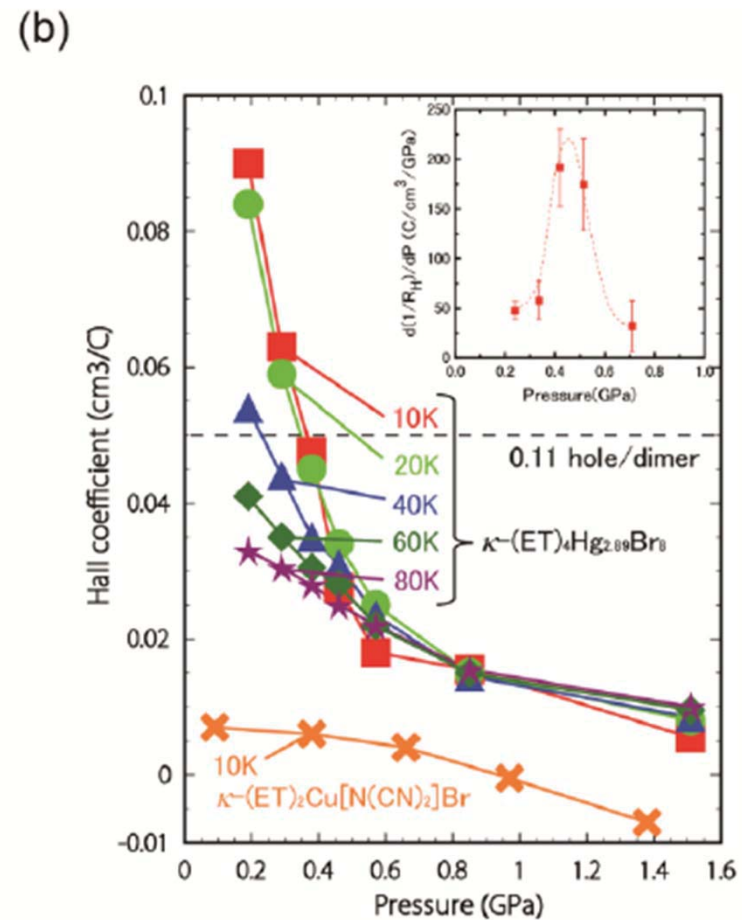
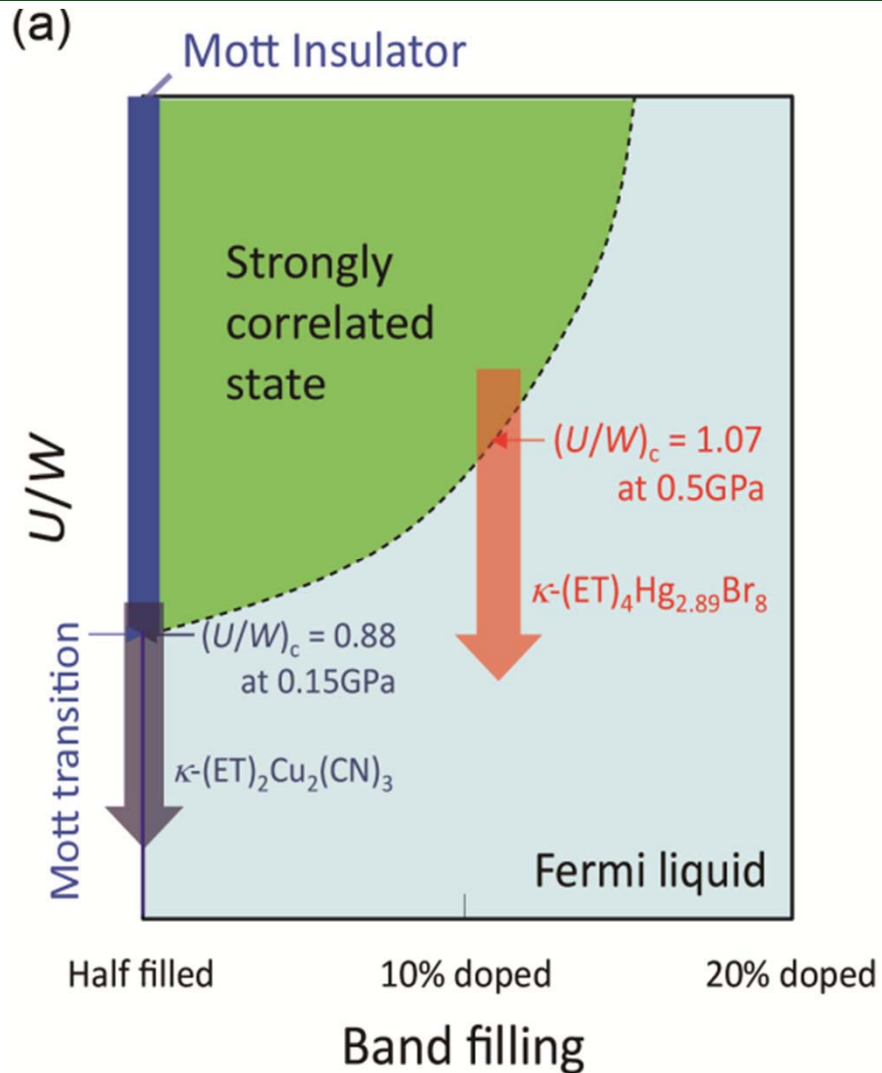


H. Oike, K. Miyagawa, H. Taniguchi, K. Kanoda PRL **114**, 06/002 (2015)



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Crossover to doped Mott insulator



H. Oike, K. Miyagawa, H. Taniguchi, K. Kanoda PRL **114**, 067002 (2015)



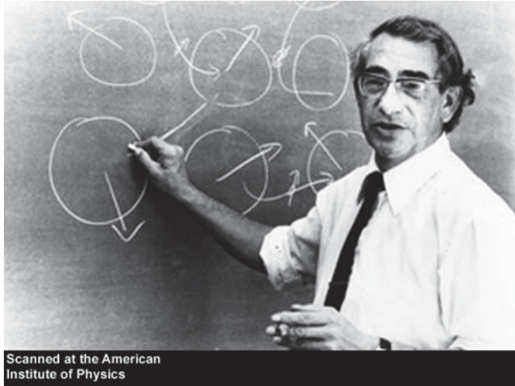
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2. The model

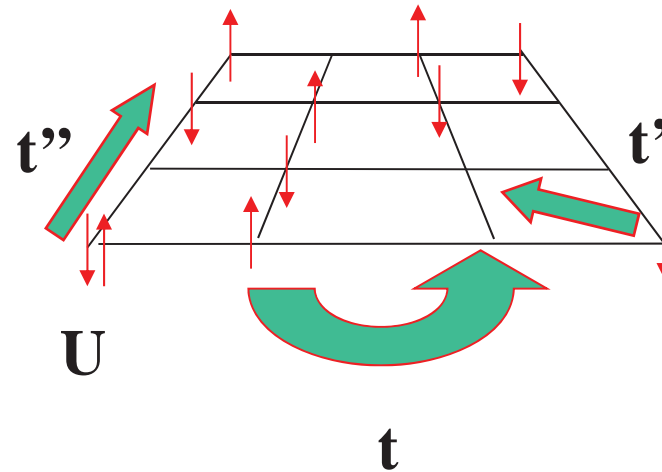
$$H = - \sum_{\langle ij \rangle \sigma} t_{i,j} (c_{i\sigma}^\dagger c_{j\sigma} + c_{j\sigma}^\dagger c_{i\sigma}) + U \sum_i n_{i\uparrow} n_{i\downarrow}$$



Hubbard model



1931-1980



$$H = - \sum_{\langle ij \rangle \sigma} t_{i,j} (c_{i\sigma}^\dagger c_{j\sigma} + c_{j\sigma}^\dagger c_{i\sigma}) + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

Attn: Charge transfer insulator



P.W. Anderson



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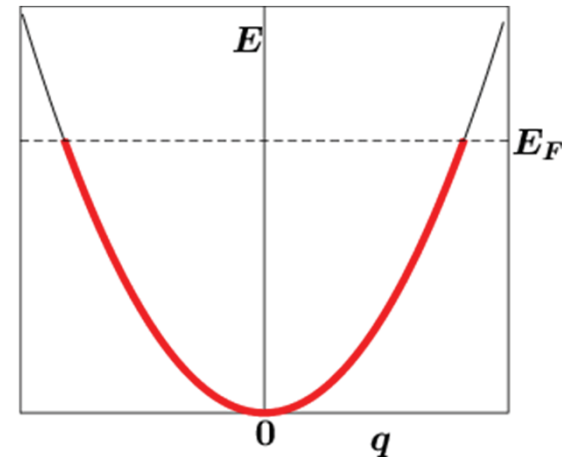
$$U = 0$$

$$H = -\sum_{\langle ij \rangle \sigma} t_{i,j} (c_{i\sigma}^\dagger c_{j\sigma} + c_{j\sigma}^\dagger c_{i\sigma})$$

$$c_{i\sigma} = \frac{1}{\sqrt{N}} \sum_{\mathbf{k}} e^{i\mathbf{k} \cdot \mathbf{r}_i} c_{\mathbf{k}\sigma}$$

$$H = \sum_{\mathbf{k}, \sigma} \varepsilon_{\mathbf{k}} c_{\mathbf{k}\sigma}^\dagger c_{\mathbf{k}\sigma}$$

$$|\Psi\rangle = \prod_{\mathbf{k}, \sigma} c_{\mathbf{k}\sigma}^\dagger |0\rangle$$



$$t_{ij} = 0$$

$$H =$$

$$U \sum_i n_{i\uparrow} n_{i\downarrow}$$

⋮

U



U



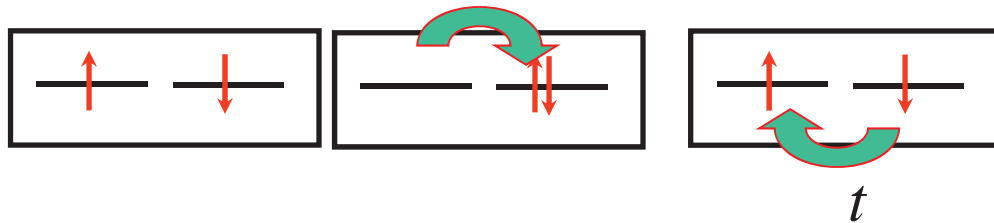
2^N

$$|\Psi\rangle = \prod_i c_{i\uparrow}^\dagger \prod_j c_{j\downarrow}^\dagger |0\rangle$$



Interesting in the general case

$$H = -\sum_{\langle ij \rangle \sigma} t_{i,j} (c_{i\sigma}^\dagger c_{j\sigma} + c_{j\sigma}^\dagger c_{i\sigma}) + U \sum_i n_{i\uparrow} n_{i\downarrow}$$



Effective model, Heisenberg: $J = 4t^2 / U$



Outline

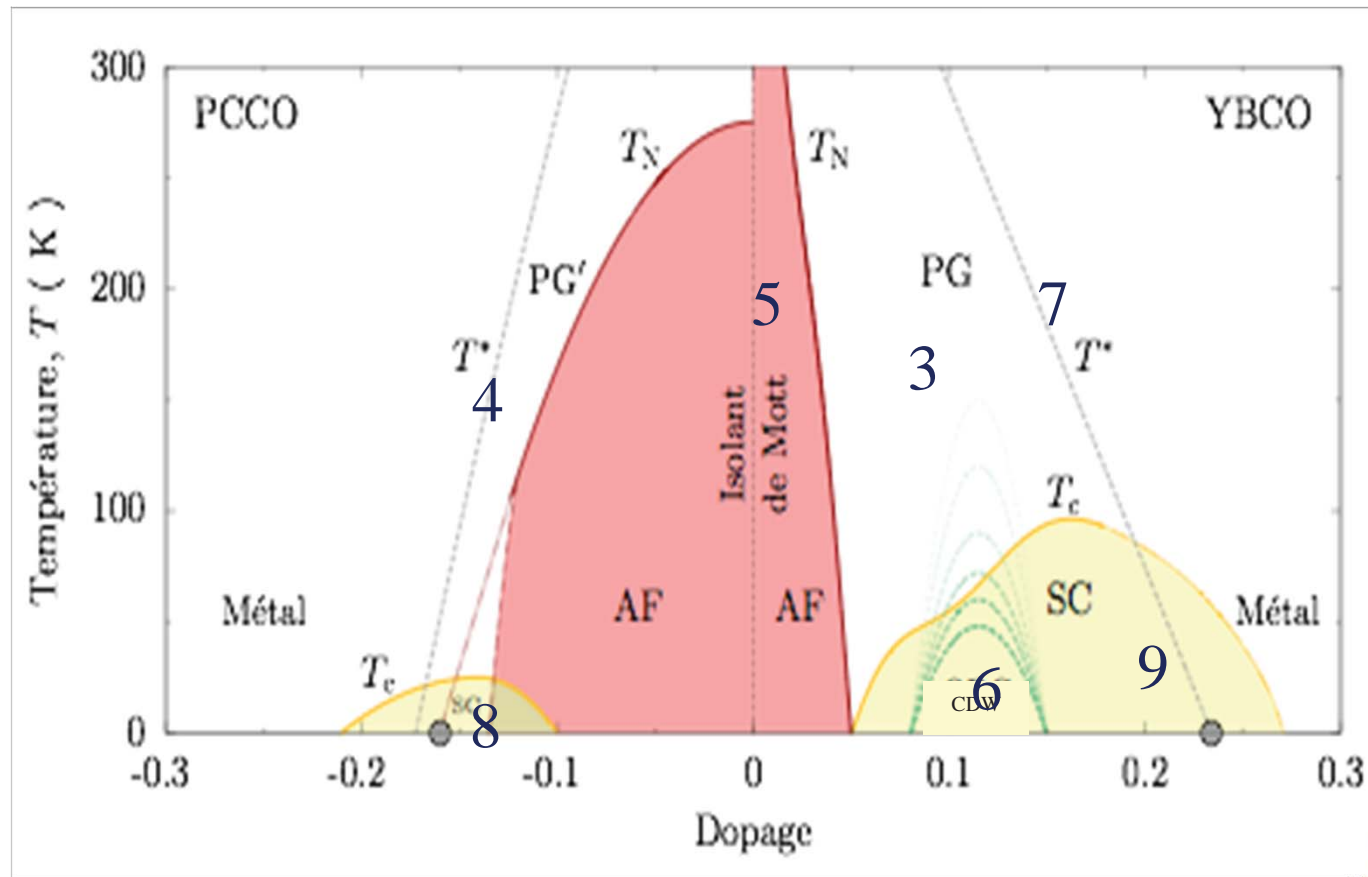
- Lecture 1: overview
 - What is the problem
 - Possible approaches and answers for organics
- Lecture 2 : h-doped
 - Strongly correlated superconductivity
 - Normal phase (pseudogap)
- Lecture 3: e-doped cuprates
 - Spin wave exchange (TPSC)
 - AFM quantum critical point
- Lecture 4
 - More on cluster generalizations of DMFT



Outline

For references, September 2013 Julich summer school
Strongly Correlated Superconductivity

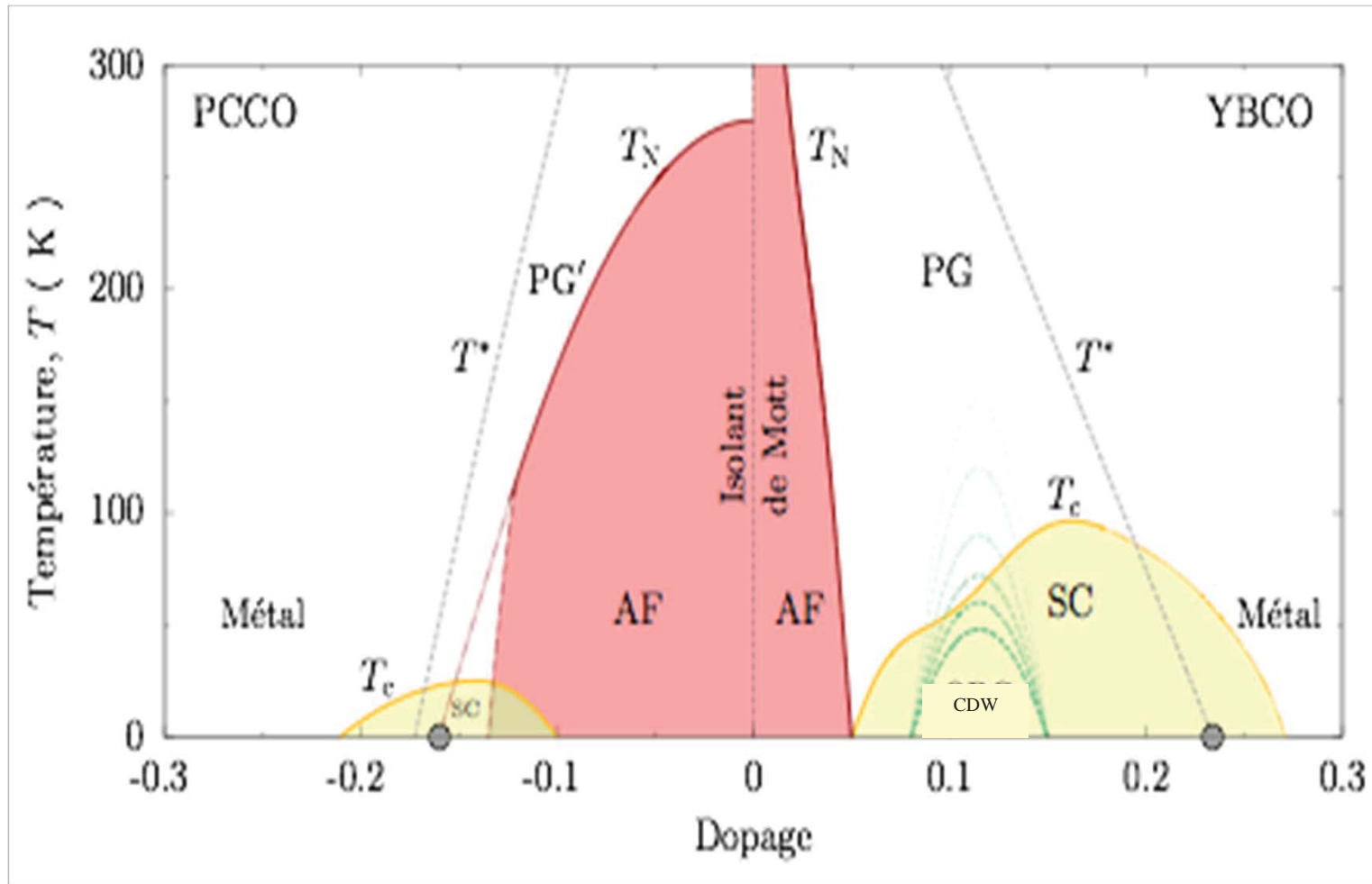
<http://www.cond-mat.de/events/correl13/manuscripts/tremblay.pdf>



3. A normal, normal state?

Our road map

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Université de Sherbrooke



**h-doped are strongly correlated:
evidence from the normal state**



Mott-Ioffe-Regel limit

$$\sigma = \frac{ne^2\tau}{m}$$

$$k_F\ell = \frac{2\pi}{\lambda_F}\ell \sim 2\pi$$

$$\sigma_{MIR} = \frac{e^2}{\hbar d}$$



Mott-Ioffe-Regel limit

$$\sigma = \frac{ne^2\tau}{m}$$

$$n = \frac{1}{2\pi d} k_F^2$$

$$\sigma = \left(\frac{1}{2\pi d} k_F^2 \right) \frac{e^2\tau}{m}$$

$$\ell = \left(\frac{\hbar k_F}{m} \right) \tau$$

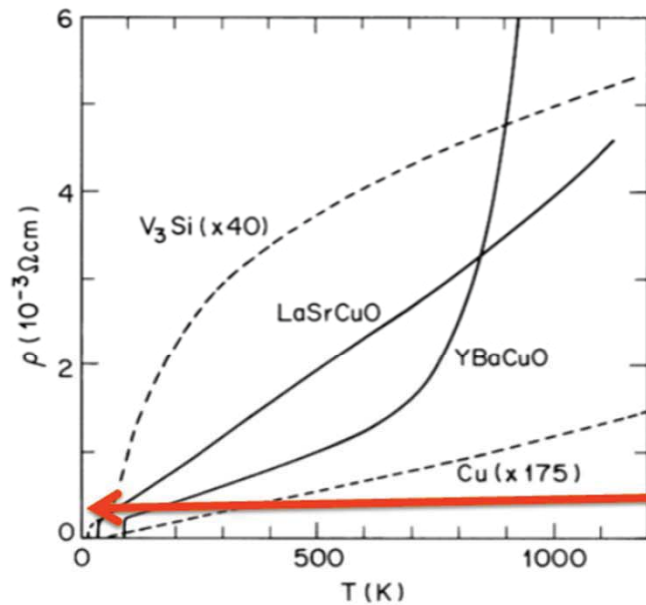
$$\sigma = \frac{1}{2\pi d} k_F e^2 \left(\frac{\ell}{\hbar} \right)$$

$$k_F \ell = \frac{2\pi}{\lambda_F} \ell \sim 2\pi$$

$$\sigma_{MIR} = \frac{e^2}{\hbar d}$$

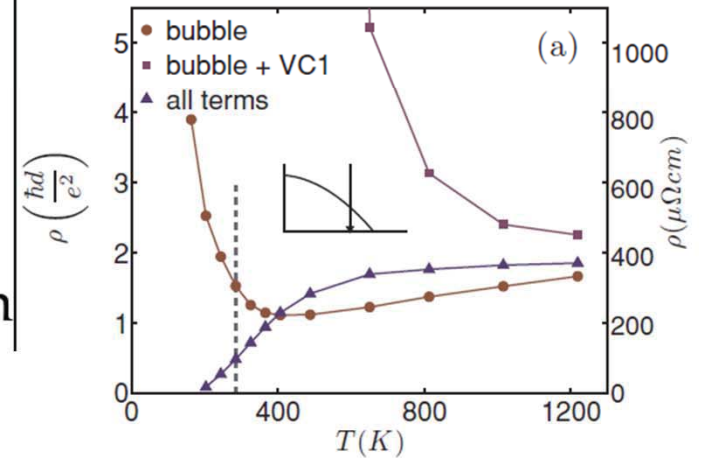


Hole-doped cuprates and MIR limit



Gurvitch & Fiory
PRL 59, 1337
(1987)

MIR limit
Mean-free path
~ Fermi wavelength



LSCO 17%, YBCO optimal

Dominic Bergeron & AMST
PRB 2011
TPSC

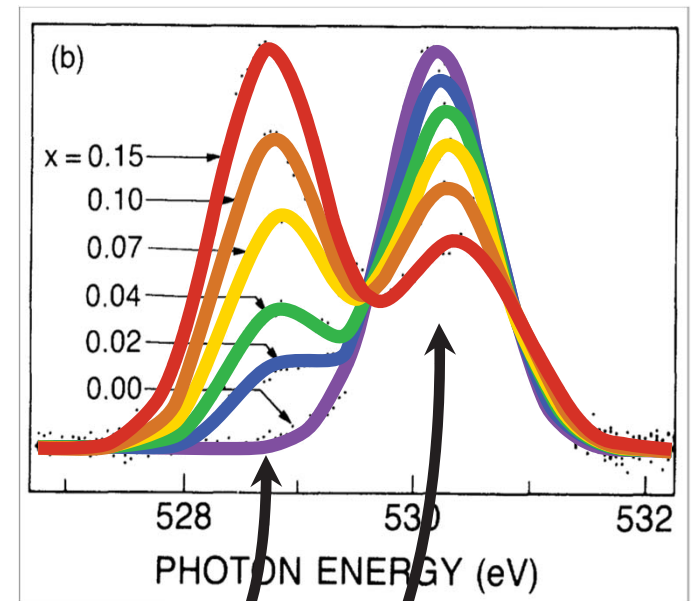
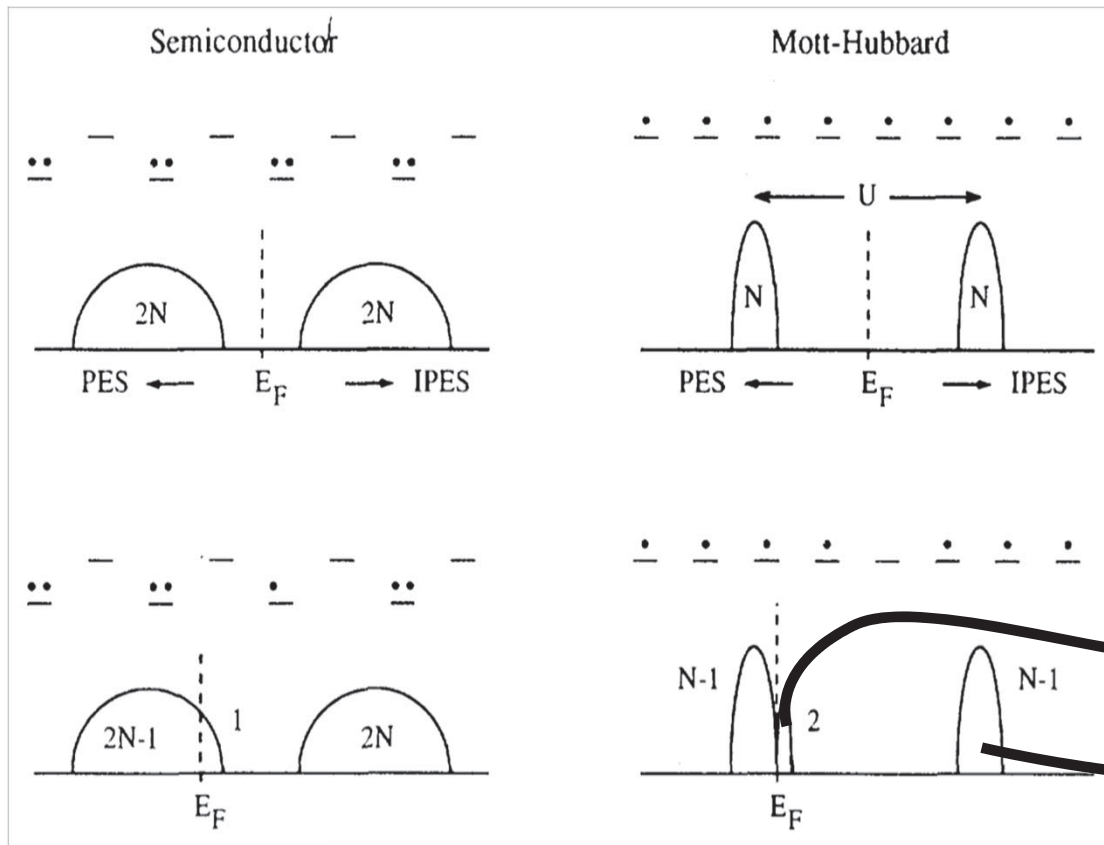
PHYSICAL REVIEW B **84**, 085128 (2011)

Optical and dc conductivity of the two-dimensional Hubbard model in the pseudogap regime and across the antiferromagnetic quantum critical point including vertex corrections



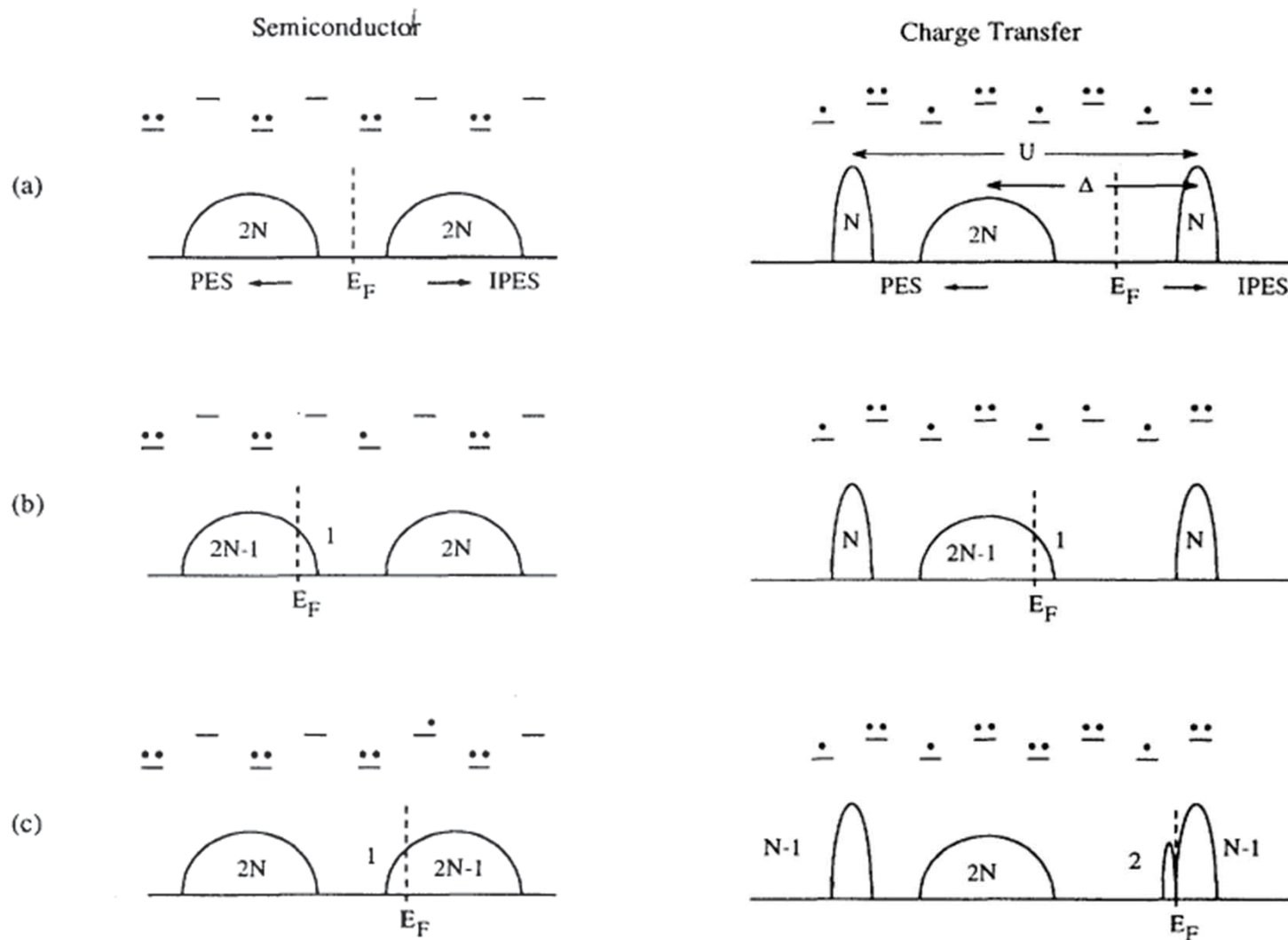
Experiment, X-Ray absorption

Meinders *et al.* PRB **48**, 3916 (1993)



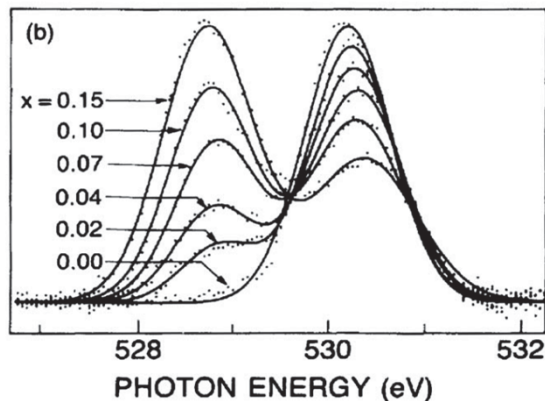
Chen *et al.* PRL **66**, 104 (1991)

Not obvious: Charge transfer insulator

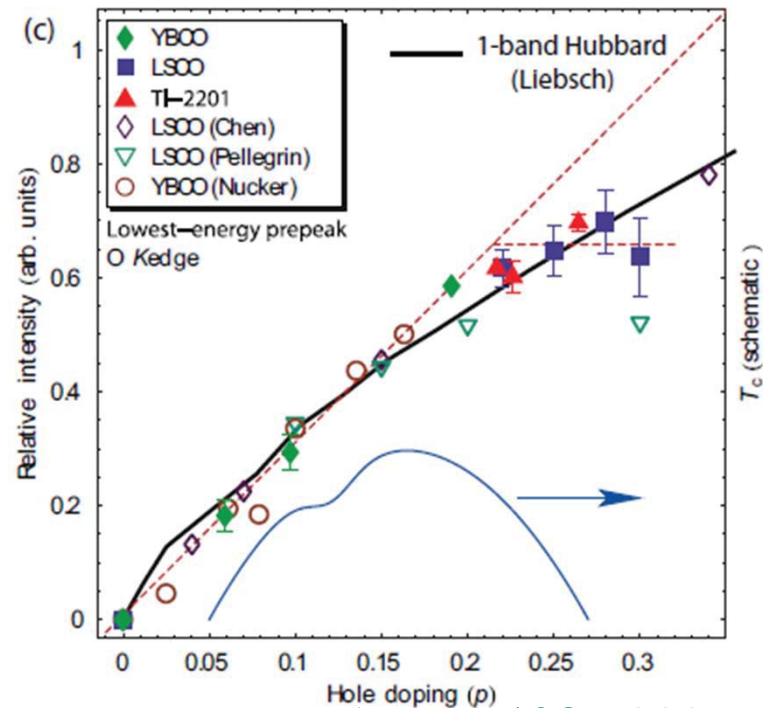


Meinders *et al.* PRB 48, 3916 (1993)

Experiment: X-Ray absorption



Chen et al. PRL **66**, 104 (1991)



Peets et al. PRL **103**, (2009),

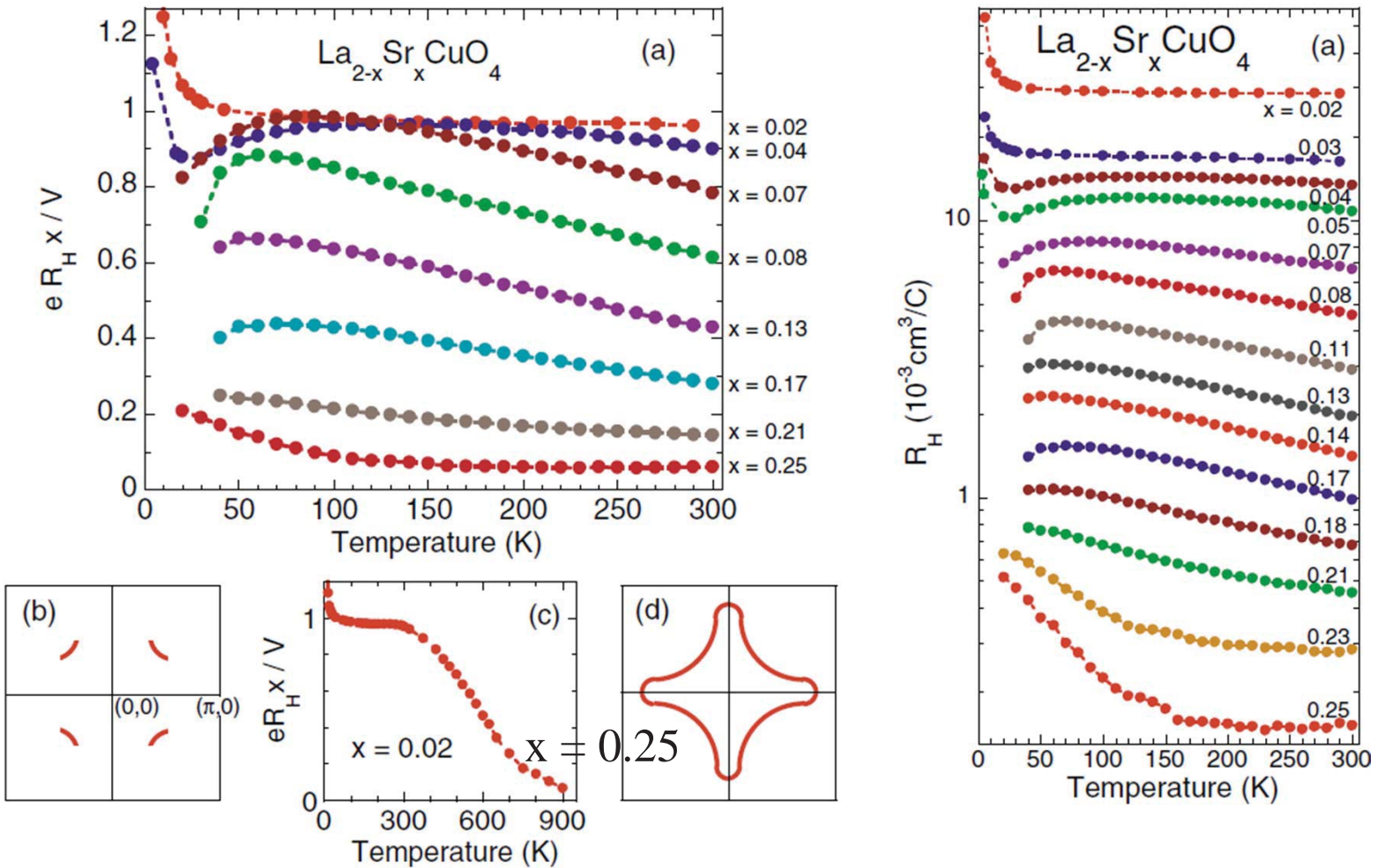
Phillips, Jarrell PRL, vol. **105**, 199701 (2010)

Number of low energy states above $\omega = 0$ scales as $2x +$
Not as $1+x$ as in Fermi liquid

Meinders et al. PRB **48**, 3916 (1993)

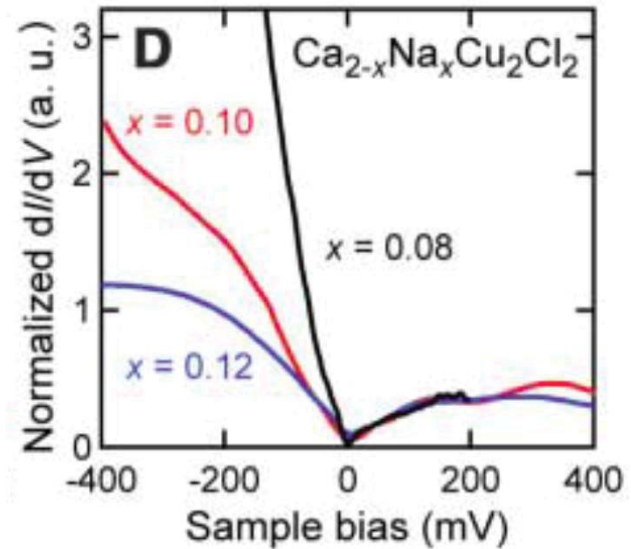


Hall coefficient



Ando et al. PRL **92**, 197001 (2004)

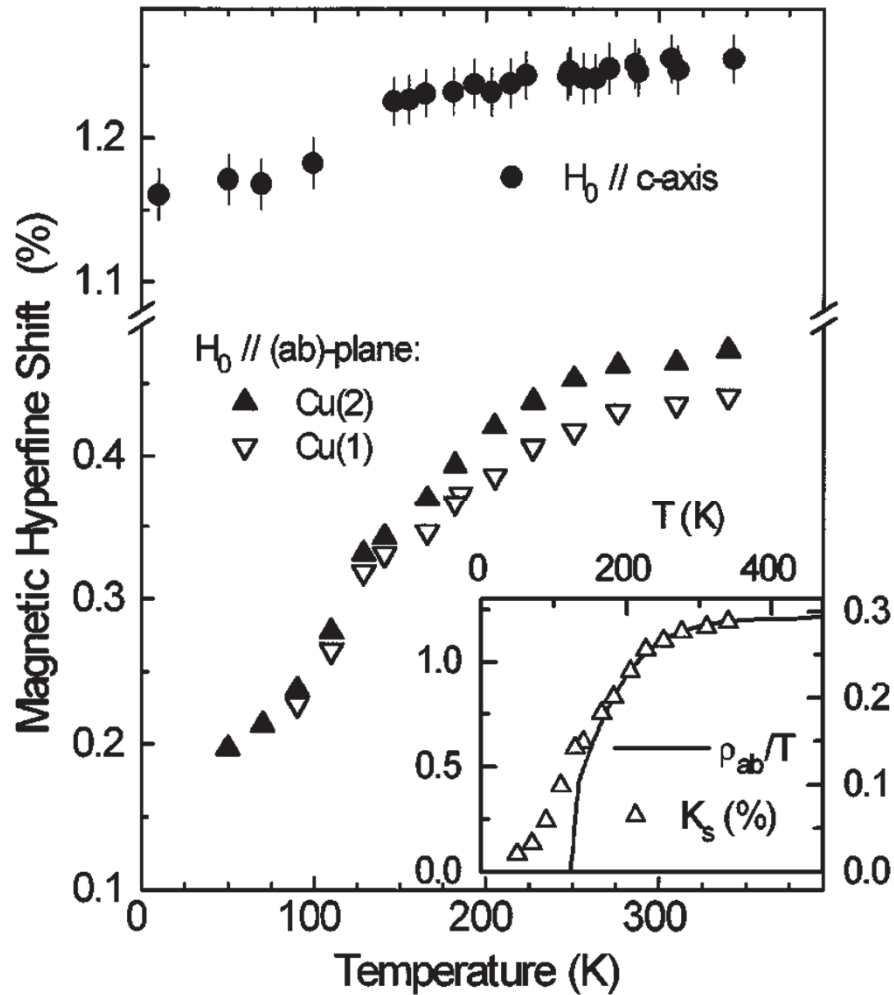
Density of states (STM)



Khosaka et al. *Science* **315**, 1380 (2007);



Spin susceptibility (Knight shift): Pseudogap



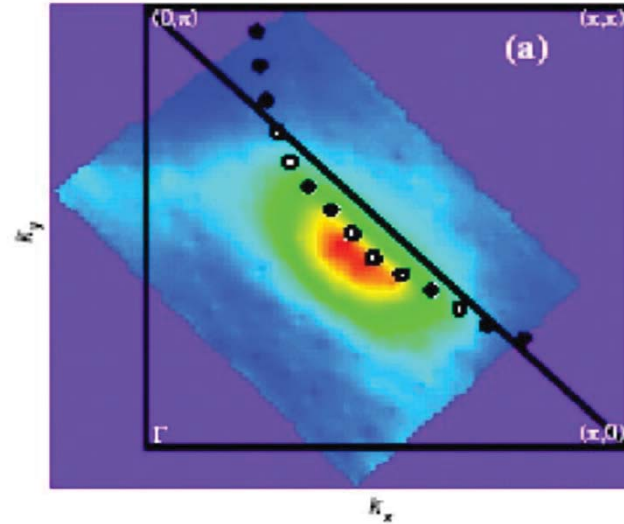
Underdoped Hg1223

Julien et al. PRL **76**, 4238 (1996)



ARPES: (Pseudogap)

Hole-doped, 10%



F. Ronning et al. Jan. 2002, $\text{Ca}_{2-x}\text{Na}_x\text{CuO}_2\text{Cl}_2$

Ronning *et al.* (PRB
2003)

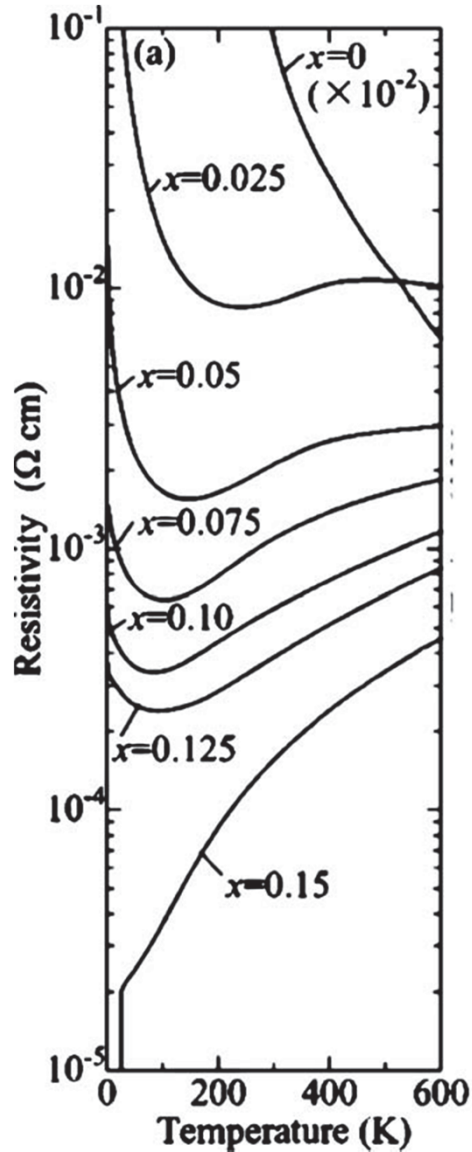


4. e-doped cuprates

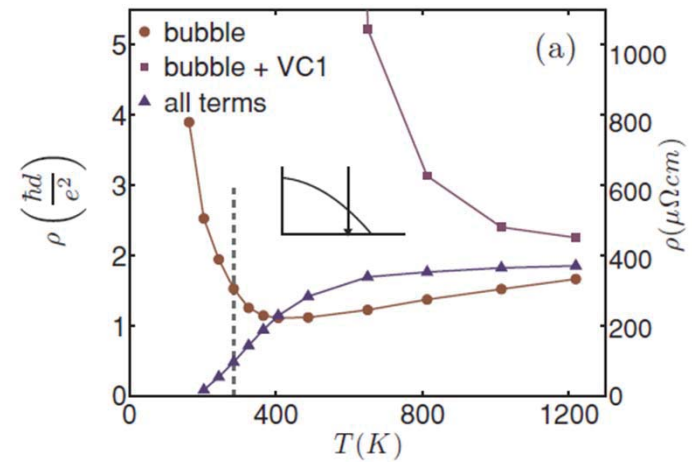
Less strongly coupled: evidence from
the normal state



Electron-doped and MIR limit



NCCO



Dominic Bergeron et al. TPSC
PRB **84**, 085128 (2011)

Onose et al. 2004



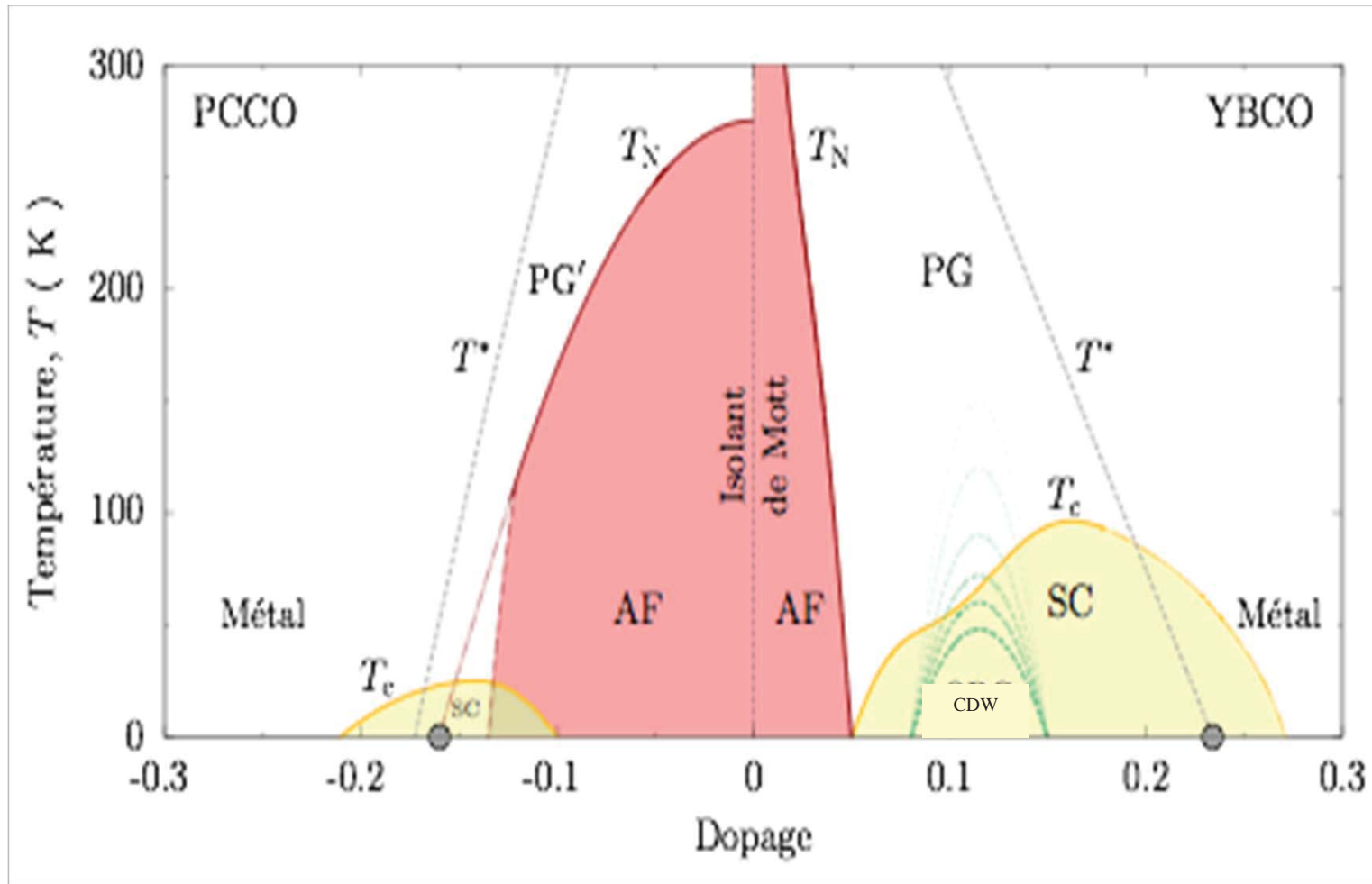
5. Weakly and strongly correlated antiferromagnets

What is a phase?



Our road map

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Université de Sherbrooke



Antiferromagnetic phase: emergent properties

- Some broken symmetries
 - Time reversal symmetry
 - Translation by one lattice spacing
 - Unbroken Time-reversal times translation by lattice vector \mathbf{a}
- Spin waves
- Single-particle gap



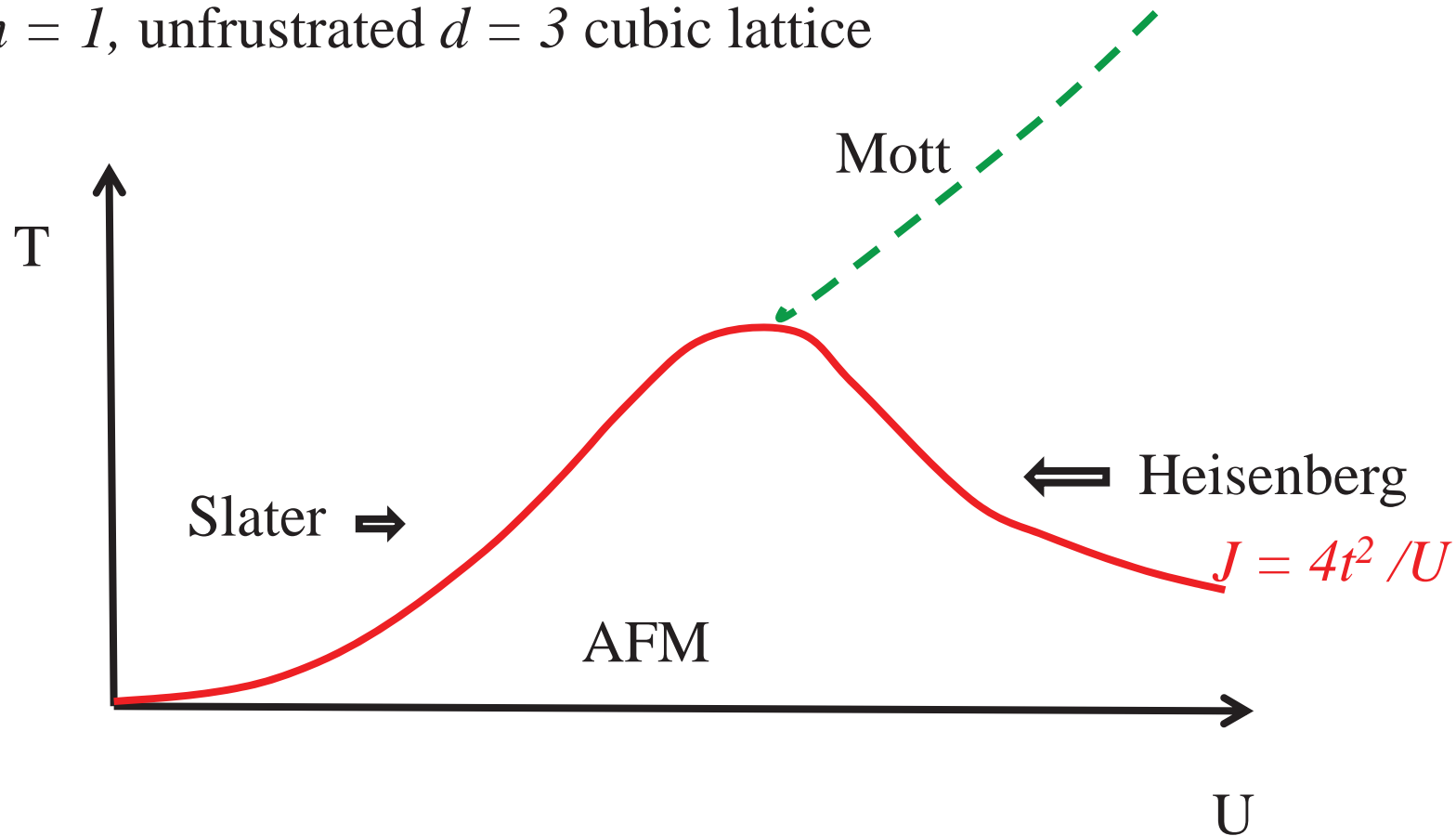
Differences between weakly and strongly correlated

- Different in ordered phase (finite frequency)
 - Ordered moment
 - Landau damping
 - Spin waves all the way or not to J
- Different, even more, in the normal state:
 - metallic in $d = 3$ if weakly correlated
 - Insulating if strongly correlated
 - Pressure dependence of T_N

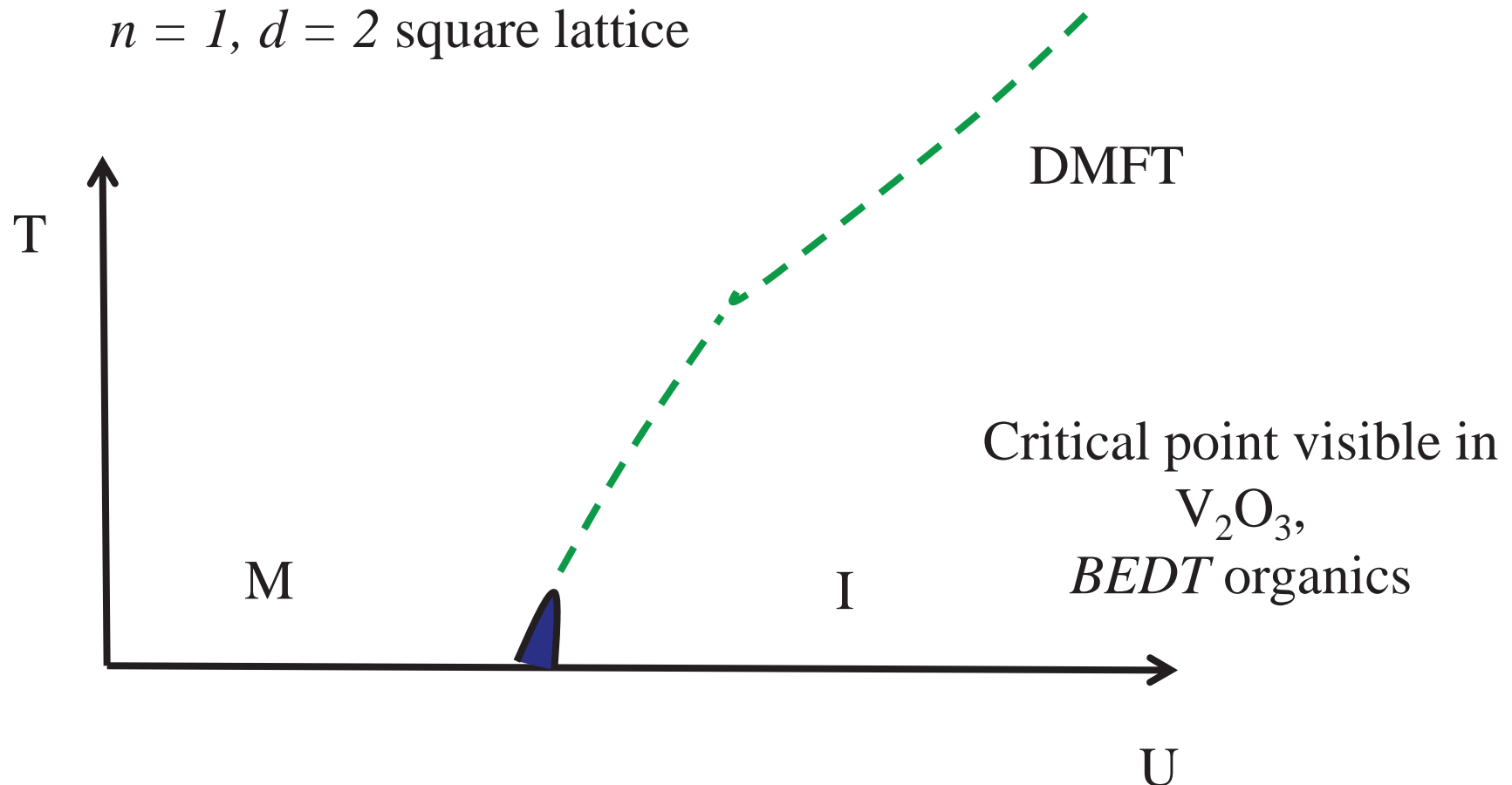


Local moment and Mott transition

$n = 1$, unfrustrated $d = 3$ cubic lattice



Local moment and Mott transition



Understanding finite temperature phase from a *mean-field theory* down to $T = 0$



Strong vs weak correlations

Contrasting methods

Ordered state

- Mean-field (Hartree-Fock) for AFM

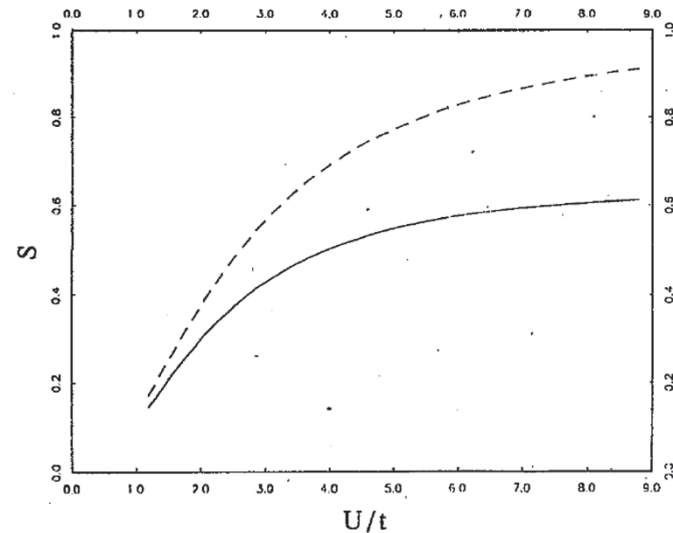
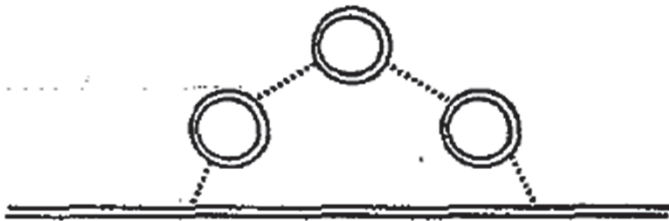


FIG. 7. The solid line represents the sublattice magnetization including the fluctuation effects. The dashed line is the mean-field result.

Schrieffer, Wen, Zhang, PRB 1989



More methods for ordered states, $n=1$

- Numerically, stochastic series expansion,
- High-temperature series expansion,
- Quantum Monte Carlo
- World-line
- Worm algorithms
- Variational methods
- Ground state of $S=1/2$ in $d=2$ is AFM, not spin liquid



In paramagnetic state



Theory difficult even at weak to intermediate correlation!

- RPA (OK with conservation laws)

- Mermin-Wagner

- Pauli



- Moryia (Conjugate variables HS $\phi^4 = \langle \phi^2 \rangle \phi^2$)

- Adjustable parameters: c and U_{eff}

- Pauli



- FLEX

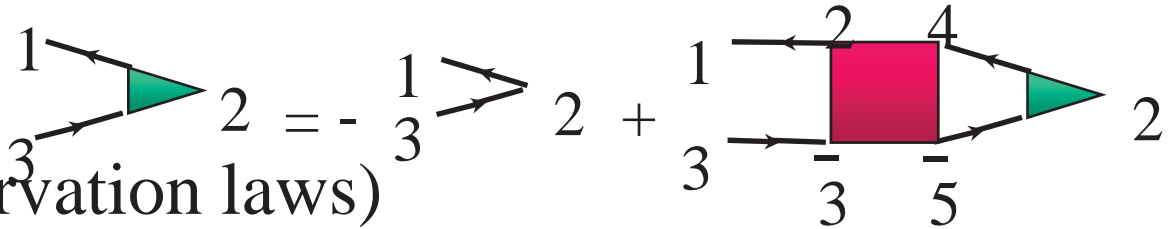
- No pseudogap

- Pauli



Σ

=



- Renormalization Group

- 2 loops

Zanchi Schultz, (2000)

Rohe and Metzner (2004)

Katanin and Kampf (2004)



Two-Particle Self-Consistent (idea)

- General philosophy
 - Drop diagrams
 - Impose constraints and sum rules
 - Conservation laws
 - Pauli principle ($\langle n_{\sigma}^2 \rangle = \langle n_{\sigma} \rangle$)
 - Local moment and local density sum-rules
- Get for free:
 - Mermin-Wagner theorem
 - Kanamori-Brückner screening
 - Consistency between one- and two-particle $\Sigma G = U \langle n_{\sigma} n_{-\sigma} \rangle$

Vilk, AMT J. Phys. I France, **7**, 1309 (1997); Allen et al. in *Theoretical methods for strongly correlated electrons* also cond-mat/0110130

(Mahan, third edition)



Doped Mott insulator : strong correlations

Normal state

At strong coupling

- Gutzwiller
- Variational approaches
- Slave particles (Review: Lee Nagaosa RMP)
- Extremely Correlated Fermi liquids (Shastry)



YRZ

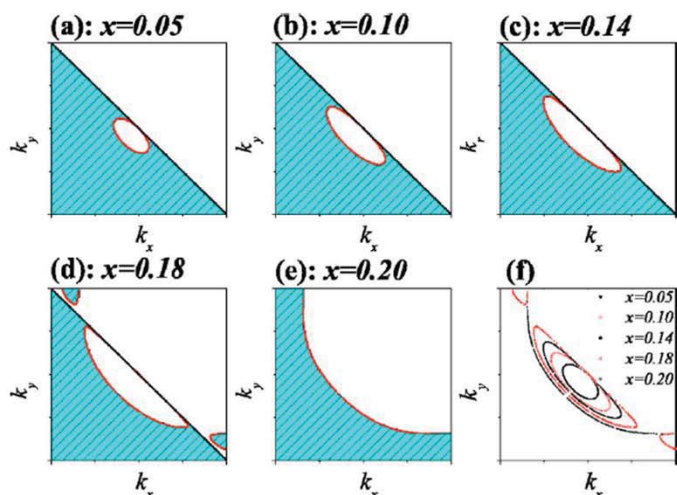
$$G^{RVB}(k, \omega) = \frac{g_t}{\omega - \xi(k) - \Delta_R^2 / [\omega + \xi_0(k)]} + G_{inc},$$

where $k = (k_x, k_y)$,

$$\xi_0(k) = -2t(x)(\cos k_x + \cos k_y),$$

$$\Delta_R(k) = \Delta_0(x)(\cos k_x - \cos k_y),$$

$$\xi(k) = \xi_0(k) - 4t'(x)\cos k_x \cos k_y - 2t''(x)(\cos 2k_x + \cos 2k_y) - \mu_p.$$



K.-Y. Yang, T.M. Rice, and F.-C. Zhang, Phys. Rev. B 73, 174501 (2006)

See numerous papers of Carbotte and Nicol and detailed discussions in

K. Le Hur and T.M. Rice, Annals of Physics 324, 1452 (2009)



Method

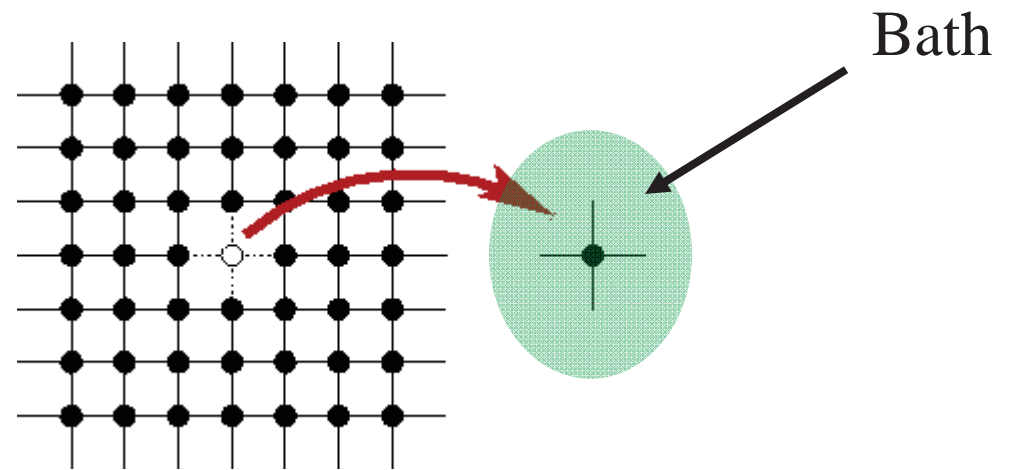
“The effect of concept-driven revolution is to explain old things in new ways. The effect of tool-driven revolution is to discover new things that have to be explained.”

Freeman Dyson *Imagined Worlds*



Mott transition and Dynamical Mean-Field Theory. The beginnings in $d = \text{infinity}$

- Compute scattering rate (self-energy) of impurity problem.
- Use that self-energy (ω dependent) for lattice.
- Project lattice on single-site and adjust bath so that single-site DOS obtained both ways be equal.



W. Metzner and D. Vollhardt, PRL (1989)

A. Georges and G. Kotliar, PRB (1992)

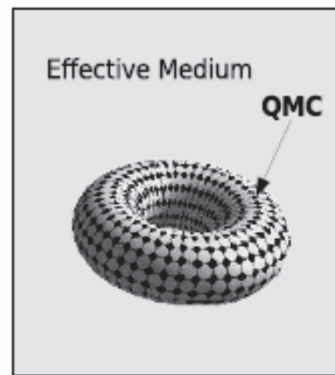
M. Jarrell PRB (1992)

A. Georges et al. RMP (1996)

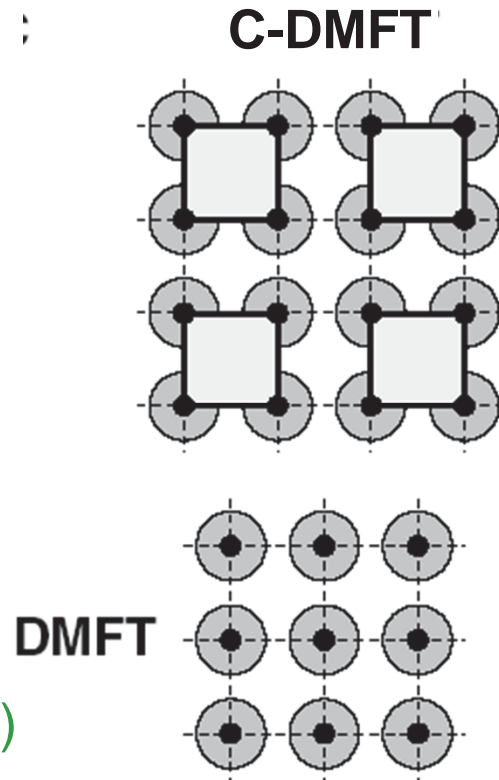
DMFT, ($d = 3$)



2d Hubbard: Quantum cluster method



DCA



Hettler ...Jarrell...Krishnamurty PRB **58** (1998)

Kotliar et al. PRL **87** (2001)

M. Potthoff *et al.* PRL **91**, 206402 (2003).

REVIEWS

Maier, Jarrell et al., RMP. (2005)

Kotliar *et al.* RMP (2006)

AMST *et al.* LTP (2006)



+ and -

- Long range order:
 - Allow symmetry breaking in the bath (mean-field)
- Included:
 - Short-range dynamical and spatial correlations
- Missing:
 - Long wavelength p-h and p-p fluctuations



Details on method in Lecture 4



Many active groups

- Paris: A. Georges, M. Ferrero, O. Parcollet
- Rutgers: K. Haule, G. Kotliar,
- Bâton Rouge: M. Jarrell
- Columbia: A. Millis
- Michigan: E. Gull
- Oakridge: Th. Maier, S.Okamoto
- Tokyo: M. Imada, Motome, Sakai
- Julich: A. Liebsch
- Graz: M. Aichhorn
- Hamburg: Potthoff
- LPS: M. Civelli
- ESRF: L. de Medici
- Trieste: M. Capone
- Vienna: Held
- Royal Holloway: G. Sordi
- Sherbrooke: D. Sénéchal, B. Kyung, P. Sémon, A.-M.S. Tremblay





Bio break

A.-M.S. Tremblay

“Strongly correlated superconductivity”

Chapt. 10 : *Emergent Phenomena in
Correlated Matter Modeling and
Simulation, Vol. 3*, E. Pavarini, E. Koch,
and U. Schollwöck (eds.)

Verlag des Forschungszentrum Jülich,
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