COLLÈGE DE FRANCE 1530

Chaire de Physique de la Matière Condensée

Des oxydes supraconducteurs aux atomes froids

- la matière à fortes corrélations quantiques -

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Cycle 2009-2010
Cours 2 - 12 mai 2010

Cours 2: Effet Kondo (suite),
réalisations expérimentales

Séminaire :

COLLÈGE DE FRANCE 1530

Serge FLORENS
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Effets Kondo exotiques dans les nanostructures

Cet exposé fera le point sur les réalisations plus inhabituelles de l'effet Kondo, dans lesquelles le moment magnétique localisé n'est pas forcément exactement compensé. Une présentation générale des concepts théoriques (groupe de renormalisation, sous et sur-écrantage, transitions de phases quantiques d'impureté) sera donnée, ainsi qu'une revue des systèmes expérimentaux (points quantiques semiconducteurs et avancées récentes en électronique moléculaire). À travers des exemples choisis, on essaiera de présenter un état de l'art du sujet, et de mettre en évidence quelques questions encore ouvertes.

Informations pratiques :

- Les documents de cours (diapos + audio) et seminaires (diapos) sont mis en ligne sur le site de la chaire:

http://www.college-de-france.fr/default/EN/all/phy_mat/index.htm

- Ce sont les seules « notes de cours »…

- Si vous souhaitez recevoir par email chaque semaine les informations liées au cours et au séminaire, merci de m'adresser un courriel:

(antoine.georges@polytechnique.edu)

0. Reminders and clarifications on previous lecture:

Anderson « impurity » model:

$$H = H_{\rm c} + H_{\rm at} + H_{\rm hyb}$$

$$H_{\rm c} = \sum_{\mathbf{k}\sigma} \varepsilon_{\mathbf{k}}\, c^{\dagger}_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma}$$

Conduction electron host (``bath'', environment)

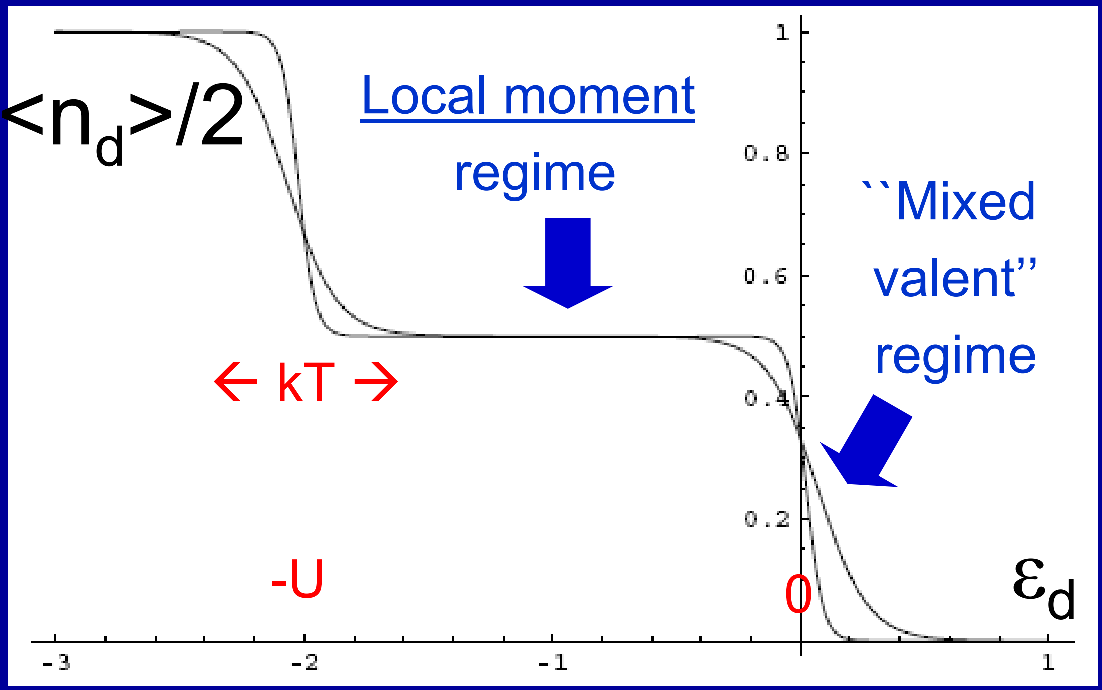
$$H_{\rm at} = \varepsilon_d \sum_{\sigma} d^{\dagger}_{\sigma} d_{\sigma} + U\, n^d_{\uparrow} n^d_{\downarrow}$$

Single-level ``atom''

$$H_{\rm hyb} = \sum_{\mathbf{k}\sigma} V_{\mathbf{k}} (c^{\dagger}_{\mathbf{k}\sigma} d_{\sigma} + d^{\dagger}_{\sigma} c_{\mathbf{k}\sigma})$$

Transfers electrons between bath and atom – Hybridization, tunneling

"Coulomb staircase", different regimes:



Plot of $n_d/2$ vs. ε_d for $U = 2$ at $\beta = 30$ and $\beta = 10$.

Note: middle of staircase has particle-hole symmetry, for: $\varepsilon_d = -U/2$

(There, states |2> and |0> have same energy = 0)

Symmetry ω→ -ω e.g. in spectral function of d-level

Effective hamiltonian in local moment regime: the Kondo model

$$H_{\text{eff}} = J_K\,\vec{S}_d \cdot \vec{S}_c + \text{pot.scattering}$$

$$\vec{S}_c = \sum_{\mathbf{k}\mathbf{k}'\alpha\alpha'} \frac{1}{2} c^\dagger_{\mathbf{k}'\alpha'} \vec{\sigma}_{\alpha\alpha'} c_{\mathbf{k}\alpha}$$

$$J_K = 2V^2 \left[\frac{1}{\varepsilon_d + U} - \frac{1}{\varepsilon_d}\right]$$

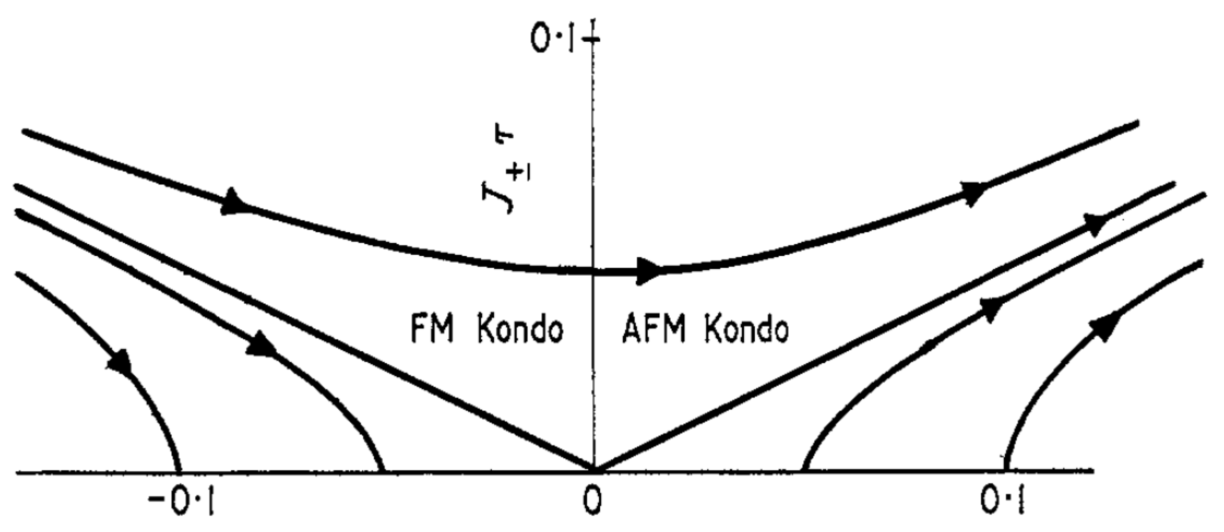
$$V_{\text{pot}} = -\frac{V^2}{2} \left[\frac{1}{\varepsilon_d + U} + \frac{1}{\varepsilon_d}\right]$$

→ In symmetric case: $J_K = 8V^2/U \ , \ V_{\text{pot}} = 0$

AF coupling flows to strong-coupling under renormalization. i.e. integrating out $\varepsilon_{\mathbf{k}} \in [D - \delta D, D]$

$$\frac{dJ_z}{dl} = J_\perp^2$$

$$\frac{dJ_\perp}{dl} = J_z J_\perp$$



Kondo scale: $$T_K \simeq \Lambda e^{-1/J_K \rho_c} \simeq \Lambda e^{-\frac{\pi |\varepsilon_d (\varepsilon_d + U)|}{2\Gamma U}}$$

(1st order expression)

(Λ: cutoff, e.g. bandwidth)

The strong-coupling fixed point:
singlet formation and local Fermi liquid

- Singlet ground-state formed between impurity spin and conduction electrons

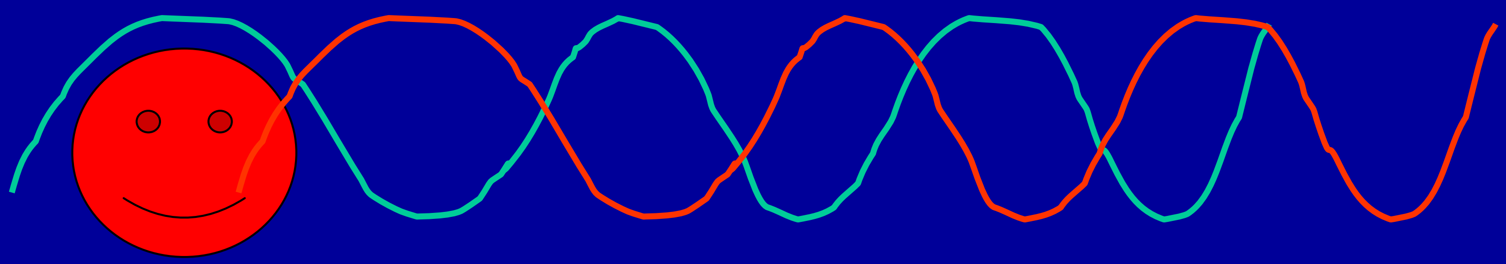
- cf. ground-state wavefunction with 1 conduction electron site:

$$|\Psi_0\rangle = \sqrt{1-\eta^2}\,|\mathcal{S}\rangle + \eta\,|\mathcal{D}\rangle$$
$$|\mathcal{S}\rangle \equiv \frac{1}{\sqrt{2}}\left[|\uparrow,\downarrow\rangle - |\downarrow,\uparrow\rangle\right]$$
$$|\mathcal{D}\rangle \equiv \frac{1}{\sqrt{2}}\left[|\uparrow\downarrow,0\rangle + |0,\uparrow\downarrow\rangle\right]$$

with $\eta \sim \frac{V}{U} \ll 1.$

- Seen from the conduction electron viewpoint:

N sites → N-1 sites (impurity site inaccessible) → **$\pi/2$ "phase shift"**



The conduction electrons viewpoint:

$$G_{\mathbf{k}\mathbf{k}'}(i\omega_n) = \frac{1}{i\omega_n - \varepsilon_{\mathbf{k}}}\delta_{\mathbf{k}\mathbf{k}'} + \frac{V^*_{\mathbf{k}}}{i\omega_n - \varepsilon_{\mathbf{k}}} G_d(i\omega_n) \frac{V_{\mathbf{k}'}}{i\omega_n - \varepsilon_{\mathbf{k}'}}$$

→ $$T_{\mathbf{k}\mathbf{k}'}(i\omega_n) = V^*_{\mathbf{k}} G_d(i\omega_n) V_{\mathbf{k}'}$$ Scattering T-matrix

Total scattering cross-section ~ Im T ~ $V^2 A_d(\omega)$ - \`optical' theorem

→ Need to understand spectral function of impurity orbital

Define phase-shift by:

$$T_{\mathbf{k}\mathbf{k}'}(\omega + i0^+) = -\,|T_{\mathbf{k}\mathbf{k}'}|\, e^{i\delta_{\mathbf{k}\mathbf{k}'}(\omega)}$$

Note: at particle-hole symmetry: T is purely imaginary → $\delta = \pi/2$

d-level spectral function, wide bandwidth limit, Fermi-liquid considerations:

$$A_d(\omega) = \frac{1}{\pi} \frac{\Gamma - \Sigma''(\omega)}{[\omega - \varepsilon_d - \Sigma'(\omega)]^2 + [\Gamma - \Sigma''(\omega)]^2}$$

Hence, at low-frequency:

$$\Sigma'(\omega) = \Sigma(0) + \left(1 - \frac{1}{Z}\right)\omega + \cdots$$

$$\Sigma''(\omega) = -A\,\omega^2 + \cdots$$

$$A_d(\omega \simeq 0) = \frac{Z}{\pi} \frac{\widetilde{\Gamma}}{(\omega - \widetilde{\varepsilon}_d)^2 + \widetilde{\Gamma}^2}$$

Resonance with renormalized level position and width, overall spectral weight Z:

$$\widetilde{\varepsilon}_d = Z\,[\varepsilon_d + \Sigma(0)] \quad , \quad \widetilde{\Gamma} = Z\,\Gamma$$

In particular, in particle-hole symmetric case (LM regime) $\varepsilon_d = -\frac{U}{2}$

$$A_d(\omega \simeq 0) = \frac{Z}{\pi} \frac{Z\Gamma}{\omega^2 + (Z\Gamma)^2}$$

$$A_d(\omega = 0) = \frac{1}{\pi\Gamma}$$

Width, Weight ~ Z

Height unchanged !

The T-matrix, at T=0 and ω=0 (wide bandwidth):

$$G_d(i0^+) = \frac{1}{-(\varepsilon_d + \Sigma'(0)) + i\Gamma}$$ → $$\tan\delta = \frac{\Gamma}{\varepsilon_d + \Sigma'(0)}$$

phase shift at T=ω=0

Hence :

$$|G_d|^2 = \frac{1}{\Gamma^2}\frac{1}{1+(\varepsilon_d+\Sigma_0)^2/\Gamma^2} = \frac{1}{\Gamma^2}\sin^2\delta$$

So that, finally:

$$G_d(i0^+) = -\frac{1}{\Gamma}\,\sin\delta\, e^{i\delta}$$

$$A_d(\omega=0) = \frac{\sin^2\delta}{\pi\Gamma}$$

$A_d(0)$ pinned at its U=0 value in symmetric case !

Im T takes maximal value

→ Unitary limit scattering

$$G_c = G_{c0} + [G_{c0}]^2 V^2 G_d$$

Local d.o.s of conduction electrons at ω=0:

$$A_c(\omega=0, T=0) \equiv -\frac{1}{\pi}\mathrm{Im}\sum_{\mathbf{kk}'} G_c^{\mathbf{kk}'} = \rho_c\,(1-\sin^2\delta)$$

d.o.s vanishes in symmetric case → Kondo screening `hole'

Friedel's sum-rule

(valid at T=ω=0, wide bandwidth)

Exact relation between

the phase shift and the occupancy of the atomic orbital !

$$\delta = \frac{\pi\, n_d}{2}$$

Why is this remarkable ?

- Phase-shift is a low-energy property (Ad(0))
- Occupancy integrates over all energies (integral of Ad overω<0)

Non-perturbative proof : see later – or see bibliography

(in the context of the AIM: Langreth, Phys Rev 150 (1966) 516

Impurity contribution to resistivity :

$$\sigma_{\mathrm{imp}}(T) = \frac{ne^2}{m}\int_{-\infty}^{+\infty} d\omega\, \tau_{\mathrm{tr}}(\omega,T)\left(-\frac{\partial f}{\partial\omega}\right)$$

Kubo formula for c-electrons

$$\tau_{\mathrm{tr}}^{-1}(\omega,T) = 2c_{\mathrm{imp}}\mathrm{Im}T^{\mathrm{adv}} = c_{\mathrm{imp}}\frac{2}{\rho_c}\Gamma A_d(\omega,T)$$

`Optical' theorem

$$\frac{R_u}{R_{\mathrm{imp}}(T)} = \int_{-\infty}^{+\infty} d\omega\left(-\frac{\partial f}{\partial\omega}\right)\frac{1}{\pi\Gamma A_d(\omega,T)}$$

Unitary limit resistivity :

$$R_u = c_{\mathrm{imp}}\,\frac{2m}{ne^2\pi\rho_c}$$

T=0 :

$$R_{\mathrm{imp}}(T=0) = R_u\sin^2\delta = R_u\sin^2\left(\frac{\pi n_d}{2}\right) = R_u\sin^2\left(\frac{\pi n_d}{2(2l+1)}\right)$$

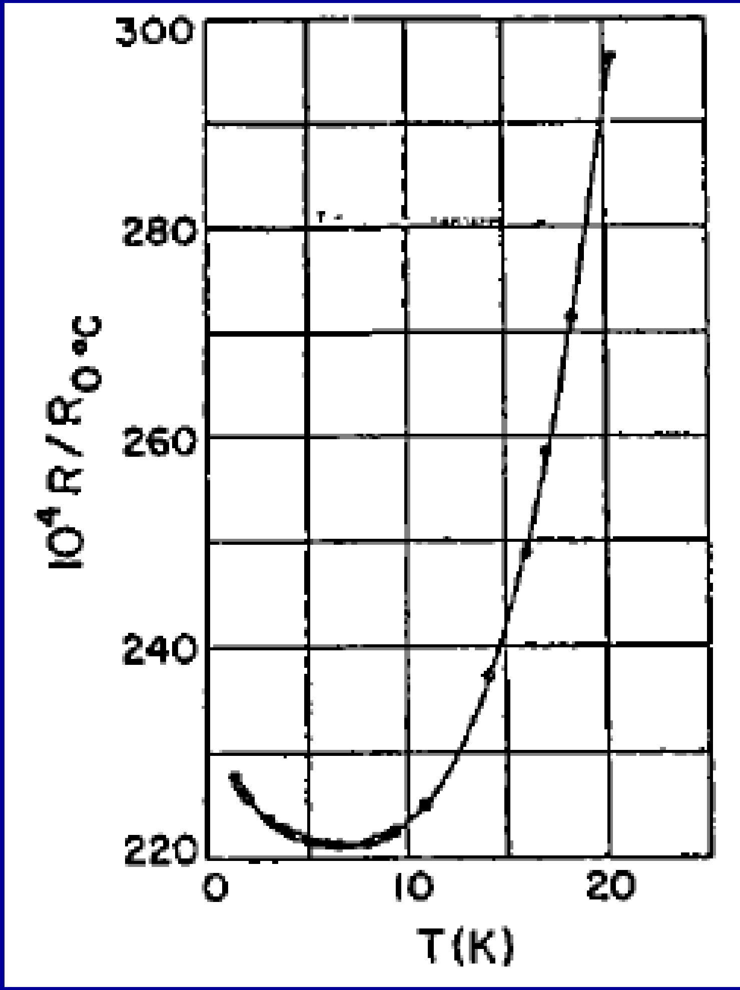
Finite-T, Kondo regime:

$$A_d(\omega,T) \to \frac{1}{\pi\Gamma}\,a\left[\frac{\omega}{T_K},\frac{T}{T_K}\right]$$

$$\frac{R_u}{R_{\mathrm{imp}}(T)} = \int_{-\infty}^{+\infty} dx\,\frac{1}{4\cosh^2\left(\frac{x}{2}\frac{T_K}{T}\right)}\,\frac{1}{a(x,\frac{T}{T_K})}$$

Experiments on dilute magnetic impurities in metals

1. Historical context: magnetic impurities in metals



De Haas, de Boer
and van den Berg,
Physica 1 (1934) 1115

``The resistivity of the gold wires measured (not very pure) has a minimum.''

An experiment contemporary to Kondo's paper and demonstrating that the effect comes from Fe-moments :

PHYSICAL REVIEW VOLUME 135, NUMBER 4A 17 AUGUST 1964

Resistivity of Mo-Nb and Mo-Re Alloys Containing 1% Fe

M. P. SARACHIK, E. CORENZWIT, AND L. D. LONGINOTTI
Bell Telephone Laboratories, Murray Hill, New Jersey
(Received 19 March 1964)

The resistivity of a series of Mo-Nb and Mo-Re alloys, with and without 1% Fe, has been measured at room temperature, and between 1.5 and 77°K. Large effects are observed near the alloy composition where the iron acquires a localized magnetic moment. These effects appear both as an excess temperature-independent scattering and in the form of large anomalies at low temperatures. Interpreted in the light of current theories of localized moments, the resistivity results confirm the existence of virtual bound states near the Fermi level. In addition, the anomalous behavior of the resistivity at low temperatures has been directly related to the existence of a localized magnetic moment.

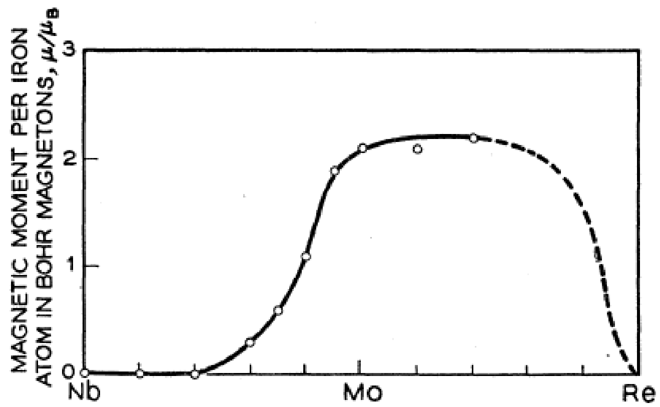


FIG. 1. Magnetic moment of an iron atom dissolved in various Mo-Nb and Mo-Re alloys as a function of alloy composition, according to Clogston *et al.*

Note added in proof. A recent theory by J. Kondo [Progr. Theoret. Phys. (Kyoto) (to be published)] predicts that a minimum exists whenever there is a negative $s-d$ exchange integral. This theory gives the observed linear dependence on concentration, and apparently gives the correct temperature dependence. I would like to thank Dr. Kondo for sending a preprint of his work prior to publication.

Kondo's

Resistance minimum:

$$\rho(T) = aT^5 + c_{\rm imp}\rho_0 - c_{\rm imp}\rho_1 \ln\frac{T}{D}$$

$$\Rightarrow T_{\rm min} \sim c_{\rm imp}^{1/5}$$

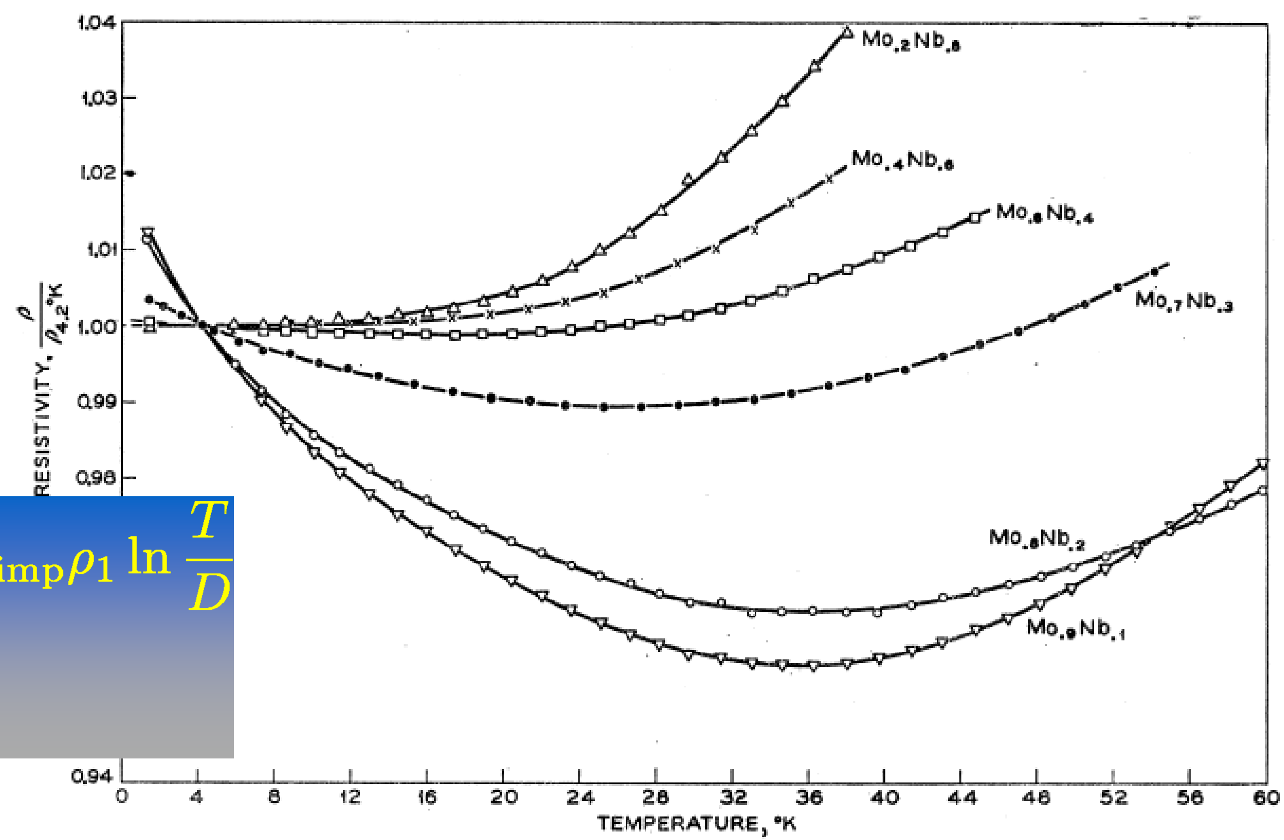
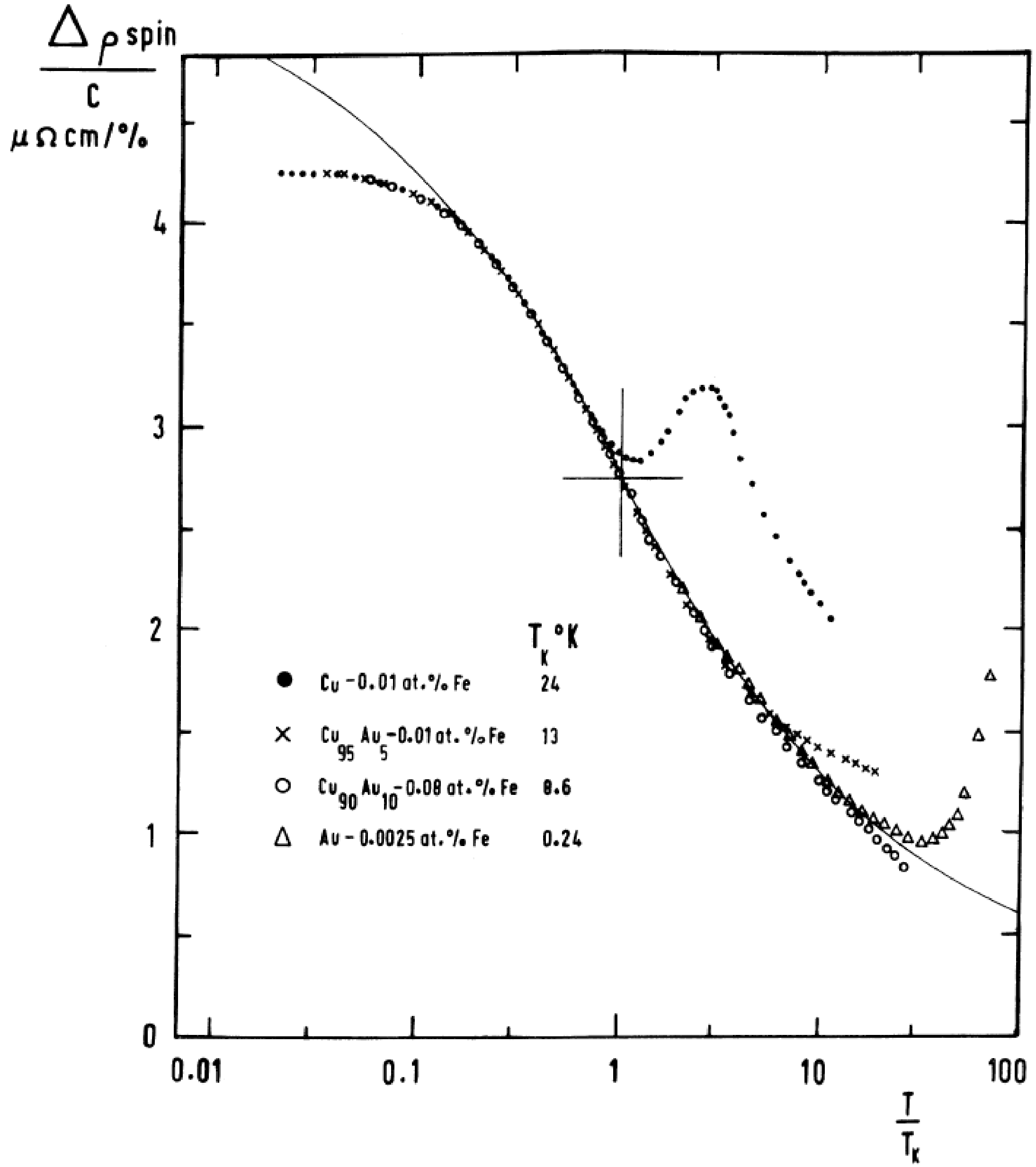


FIG. 3. Resistivity vs temperature for various Mo-Nb alloys containing 1% Fe. Resistivities are normalized at 4.2°K.

Scaling with T/T_K ?



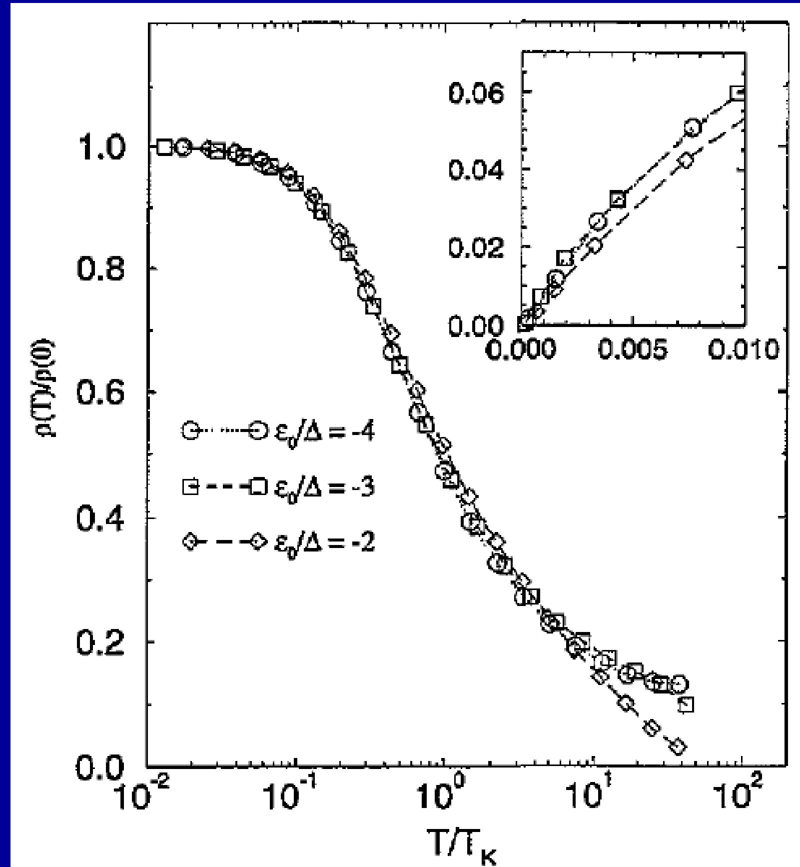
Experiments: Loram et al.

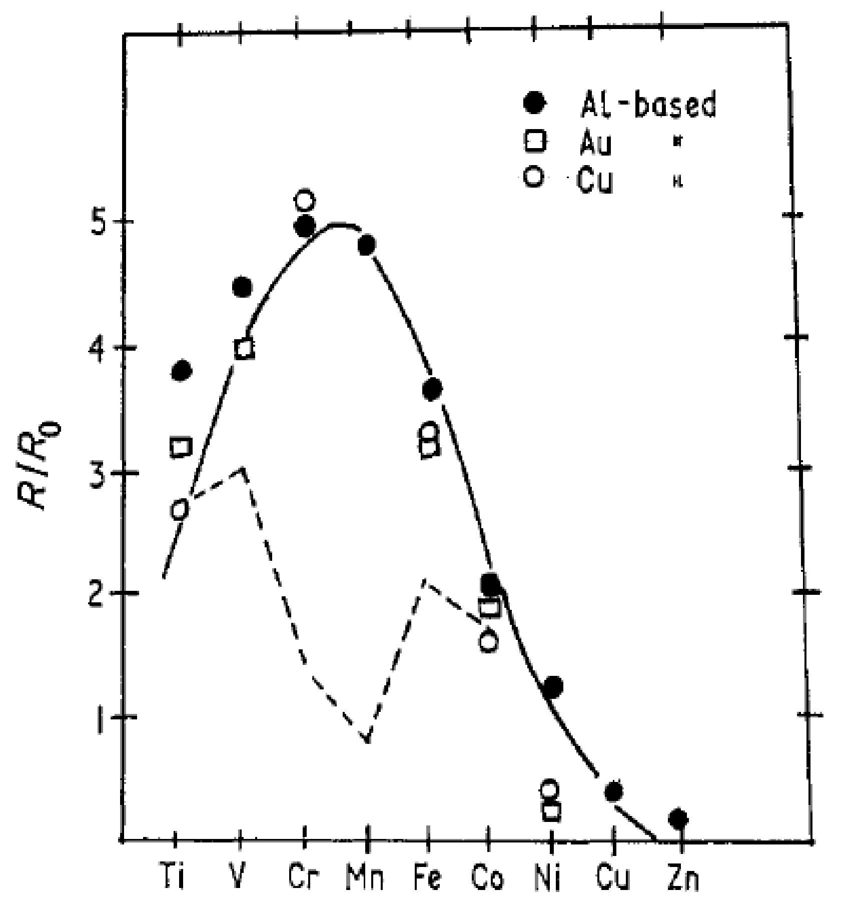
Phys Rev B 2 (1970) 857

NRG theoretical calculation:

Costi and Hewson

J. Phys. Cond. Mat. 6 (1994) 2519





$$R(T=0) = R_u \sin^2 \frac{\pi n_d}{2(2l+1)} \quad ??$$

Details of deviations
(asymmetry, etc…)
claimed to be
reasonably accounted for
by additional effects
e.g. crystal fields, etc…

Impurity resistivities normalized to the R_0 values extrapolated to $T = 0$ for Au-, Cu- and Al-based alloys (after Grüner and Zawadowski 1972).

Single-orbital Anderson model ignores many realistic aspects - especially: orbital degeneracy

$$H = \sum_{k,\sigma} \epsilon_k a_{k\sigma}^{+} a_{k\sigma} + \sum_{\sigma,m} \epsilon_{\mathrm{d}} n_{m\sigma} + \sum_{k,m,\sigma} (V_{km} a_{k\sigma}^{+} a_{m\sigma} + \mathrm{cc})$$
$$+ \tfrac{1}{2} U \sum_{m,m'} n_{m\sigma} n_{m'-\sigma} + \tfrac{1}{2}(U-J) \sum_{m\neq m',\sigma} n_{m\sigma} n_{m'\sigma}$$
$$- \tfrac{1}{2} J \sum_{m\neq m',\sigma} a_{m\sigma}^{+} a_{m-\sigma} a_{m'-\sigma}^{+} a_{m'\sigma} + \tfrac{1}{2} J \sum_{m,\sigma} n_{m\sigma} n_{m-\sigma} + (\text{crystal field})$$

J: Hund's coupling

Rotational invariance in spin and orbital space:

Cf. Caroli, Caroli and Fredkin Phys Rev 178 (1969) 599

Dworin and Narath Phys Rev Lett 25 (1970) 1287

2. The study of scattering by magnetic impurities in metals has been rejuvenated recently by experimental studies on **mesoscopic wires**

- Quantronics group, SPEC, CEA-Saclay
(F.Pierre, A.Anthore, B.Huard, H.Pothier, D.Esteve, et al.)
- Grenoble group (L.Saminadayar, C.Bäuerle et al.)

→ See also seminar by S.Florens…

1997: a much discussed article …

VOLUME 78, NUMBER 17 PHYSICAL REVIEW LETTERS 28 APRIL 1997

Intrinsic Decoherence in Mesoscopic Systems

P. Mohanty, E. M. Q. Jariwala, and R. A. Webb
Center for Superconductivity Research, Department of Physics, University of Maryland, College Park, Maryland 20742
(Received 17 December 1996)

We present measurements of the phase coherence time τ_ϕ in six quasi-1D Au wires and clearly show that τ_ϕ is temperature independent at low temperatures. We suggest that zero-point fluctuations of the phase coherent electrons are responsible for the observe[d] functional form for the temperature dependence and present th[e] value of τ_ϕ. This explains the observed temperature depend[…] and 2D systems reported to date. [S0031-9007(97)03022-6]

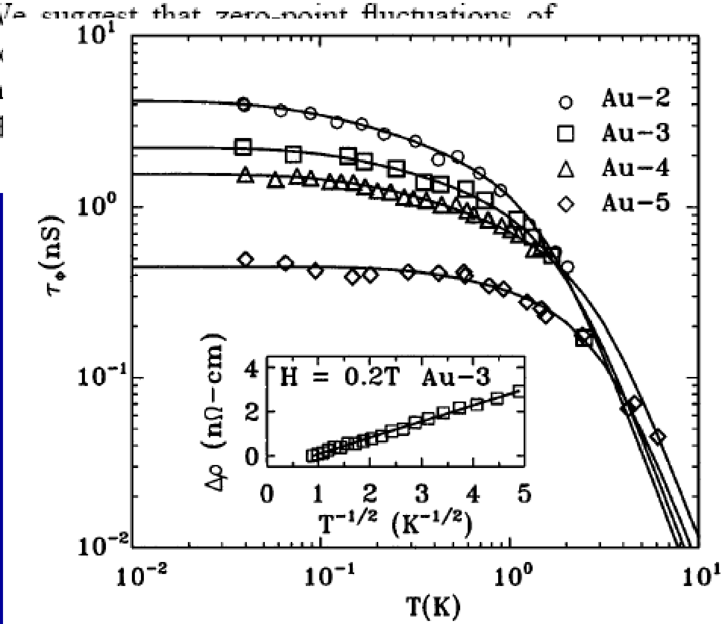


FIG. 2. Temperature dependence of τ_ϕ for four Au wires. Solid lines are fits to Eq. (1) with phonons. The inset is the EE contribution to $\Delta\rho$ with the theoretical prediction.

Measure of the phase-coherence (dephasing) time from weak-localization Magnetoresistance experiments

From 2002 → : growing evidence that these effects are largely due to magnetic impurities.

- Detailed magnetoresistance measurements and comparison to theory of dephasing in Kondo systems:

F.Pierre et al PRB 68 (2003)

B. Huard et al. PRL 95 (2005) 036802

Lin et al. J. Phys. Cond Matt 14 (2002) R501 –exp. Review-

Goppert et al. PRB 66 (2002) 155328

Zarand et al. PRL **93**, 107204 (2004)

Micklitz et al. PRL 96 (2006) 226601

- `Relax' experiments probing electron distribution function

→ Energy dependent kernel for inelastic scattering

H.Pothier et al. PRL 79 (1997) 3450

A. Anthore et al. PRL 90 (2003) 076806

Kaminsky and Glazman PRL 86 (2001) 2400

+ A number of papers by the Grenoble group, e.g.:

Anomalous Temperature Dependence of the Dephasing Time in Mesoscopic Kondo Wires

Félicien Schopfer, Christopher Bäuerle, Wilfried Rabaud, and Laurent Saminadayar

Phys. Rev. Lett. **90**, 056801 (2003) –

Scaling of the Low-Temperature Dephasing Rate in Kondo Systems

Mallet et al. Physical Review Letters, vol. 97, 226804 (2006)

+ many others

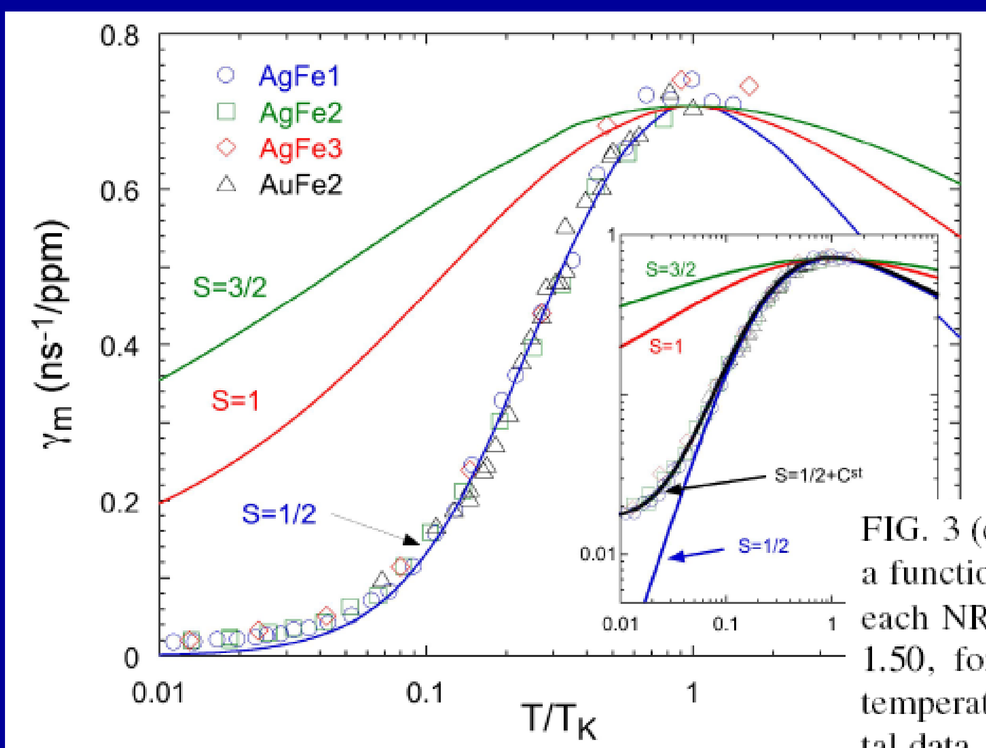
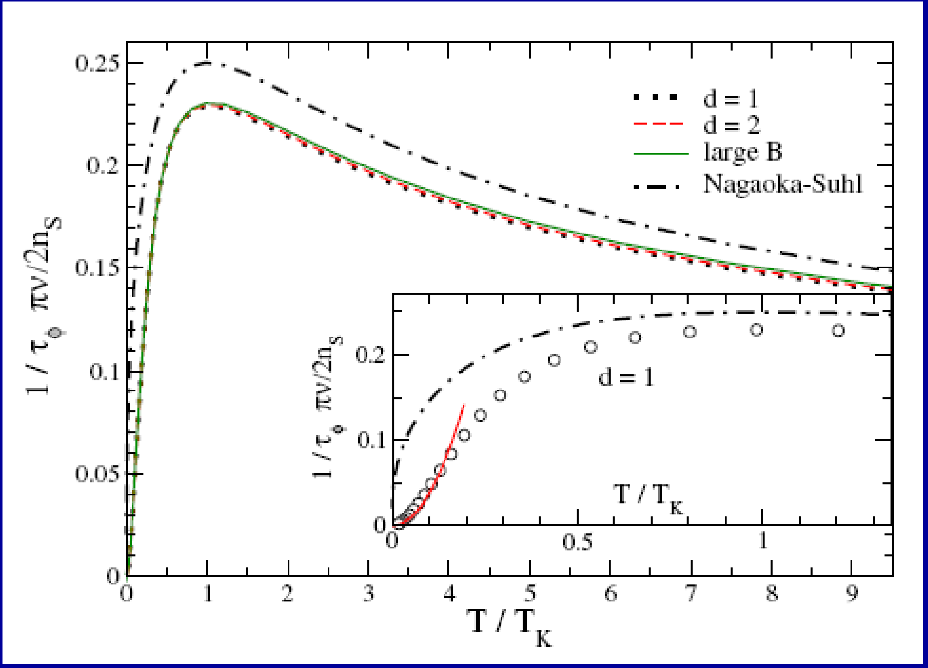


FIG. 3 (color online). Dephasing rate per magnetic impurity as a function of (T/T_K^γ). The NRG results, keeping 1900 states for each NRG iteration and using the discretization parameter $\Lambda = 1.50$, for $S = 1/2$, $S = 1$, and $S = 3/2$ have been scaled in temperature such that the maxima coincide with the experimental data. Inset: Same data on a log-log scale. The thick solid line corresponds to $S = 1/2$ with a constant background added.

Dephasing rate in Kondo systems:

$$\frac{1}{\tau_\varphi(\epsilon, T)} = \frac{2n_S}{\pi\nu}[\pi\nu \, \mathrm{Im}[T^A(\epsilon)] - |\pi\nu T^R(\epsilon)|^2].$$



Zarand et al.

PRL **93**, 107204 (2004)

Micklitz et al.

PRL 96 (2006) 226601

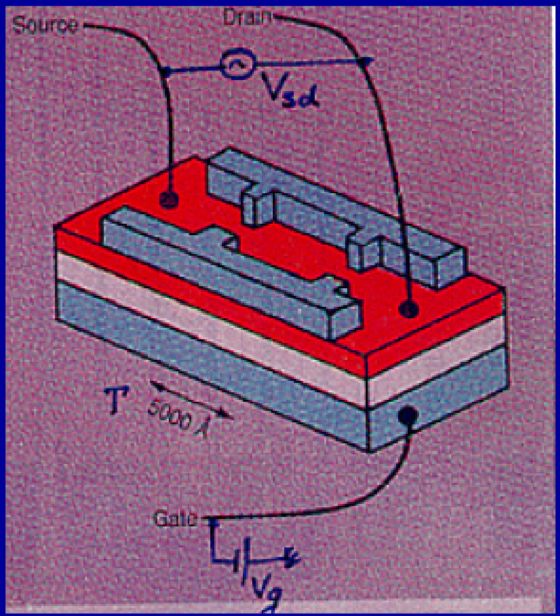
FIG. 3 (color online). Universal dephasing rate calculated via NRG. Accidentally, there is almost no dependence on the dimension d. The dotted-dashed line shows the result of a Nagaoka-Suhl resummation [26] $1/\tau_\varphi = (n_S/2\pi\nu)[(\pi^2 3/4)/(\pi^2 3/4 + \ln^2 T/T_K)]$, using our definition of T_K [25]. While the Fermi liquid behavior is recovered for $T \lesssim 0.1 T_K$, the calculated prefactor of the T^2 behavior (solid line) is not exactly reproduced due to numerical problems [23].

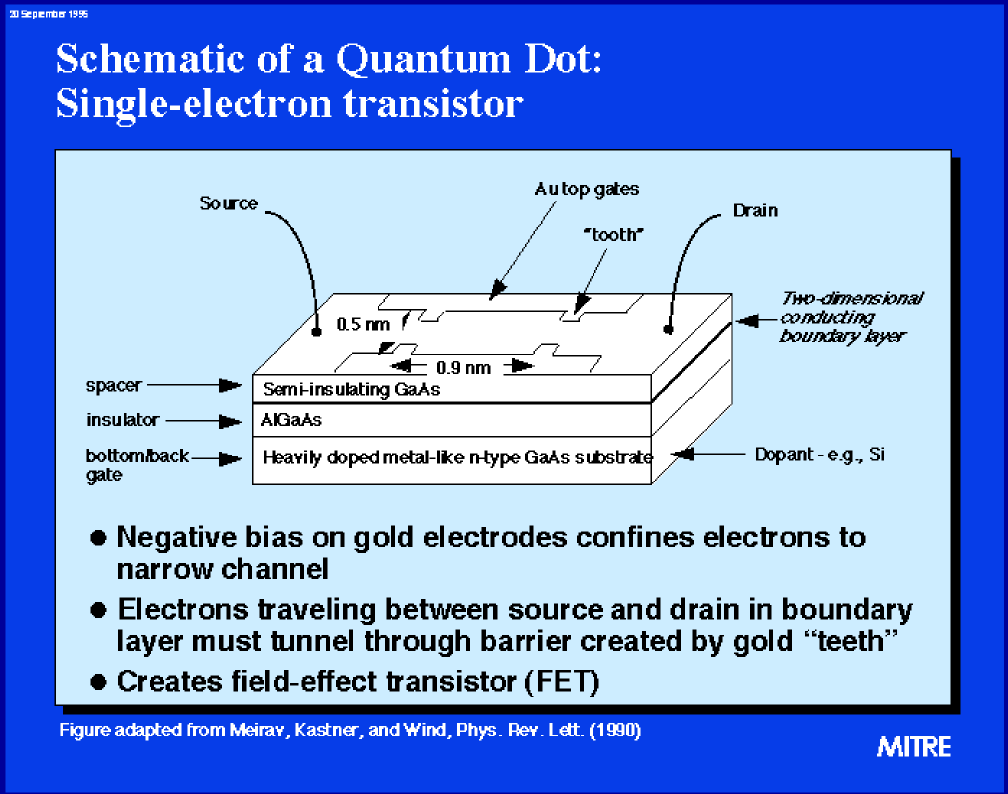
$$\gamma_{sf}(T) = \frac{c}{\pi\hbar\nu_F} \frac{\pi^2 S(S+1)}{\pi^2 S(S+1) + \ln(T/T_K)^2}$$

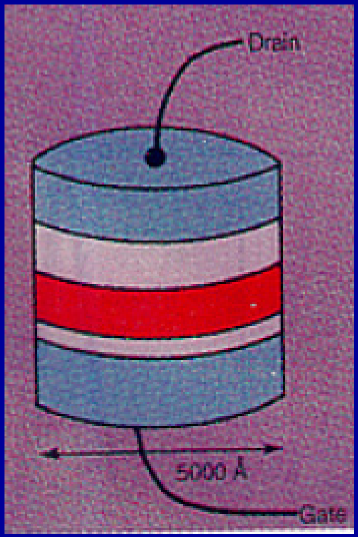
3. A quantum dot as an `artificial' magnetic impurity:

when the Kondo effect suppresses the Coulomb blockade !

Quantum dots :







Cf. e.g. M.Kastner
Physics Today, 1993

Extremely simplified model: a slight modification of the Anderson single-impurity model (w/ 2 baths)

$$H = H_{\rm dot}[d_\sigma, d_\sigma^\dagger] + \sum_{p=L,R} \sum_\sigma \left[V_p \, d_\sigma^\dagger a_{p\sigma} + h.c + E_p \, a_{p\sigma}^\dagger a_{p\sigma} \right]$$

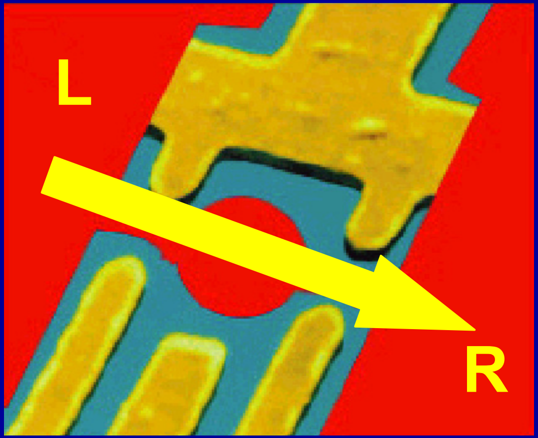
Hybridization to the leads ⮥ Leads ⮥

$$H_{\rm dot} = \varepsilon_d \sum_\sigma n_{d\sigma} + U n_{d\uparrow} n_{d\downarrow}$$

Coulomb blockade on the dot ⮥

Valid for widely separated energy levels on the dot, considering a single level (NOT correct for a metallic island, OK for 2DEG dots).

Conductance through dot :



Left junction, Kubo formula:

$$G_L \sim \frac{e^2}{h}\frac{1}{\omega}\langle j_{Ld} \cdot j_{Ld}\rangle|_{\omega\to 0}$$

$$j_{Ld} \sim -i\, eV_{\mathbf{k}L}\left(c^\dagger_{\mathbf{k}\sigma L} d_\sigma - d^\dagger_\sigma c_{\mathbf{k}\sigma L}\right)$$

→ $$G_L = \frac{8e^2}{h}\Gamma_L \int_{-\infty}^{+\infty} d\omega \left[-\frac{\partial f}{\partial \omega}\right] \pi A_d(\omega, T)$$

Adding the 2 junctions in series:

$$G = \left[\frac{1}{G_L} + \frac{1}{G_R}\right]^{-1} = \frac{8e^2}{h}\frac{\Gamma_L \Gamma_R}{(\Gamma_L + \Gamma_R)^2}\int_{-\infty}^{+\infty} d\omega \left[-\frac{\partial f}{\partial \omega}\right] \pi \Gamma A_d(\omega, T)$$

Conductance (cont'd)…

Specialize to L-R symmetric device, for simplicity:

$$G_{L=R}(T) = \frac{2e^2}{h} \int_{-\infty}^{+\infty} d\omega \left[-\frac{\partial f}{\partial \omega} \right] \pi \Gamma A_d(\omega, T)$$

Notes:

1- Compare to formula for resistivity ! G ~ R quite remarkable !

$$\frac{R_u}{R_{\mathrm{imp}}(T)} = \int_{-\infty}^{+\infty} d\omega \left(-\frac{\partial f}{\partial \omega} \right) \frac{1}{\pi \Gamma A_d(\omega, T)}$$

2- → ~ A Landauer formula generalized to tunneling into an interacting system $\Gamma A_d(\omega,T)$ plays the role of transparency of barrier

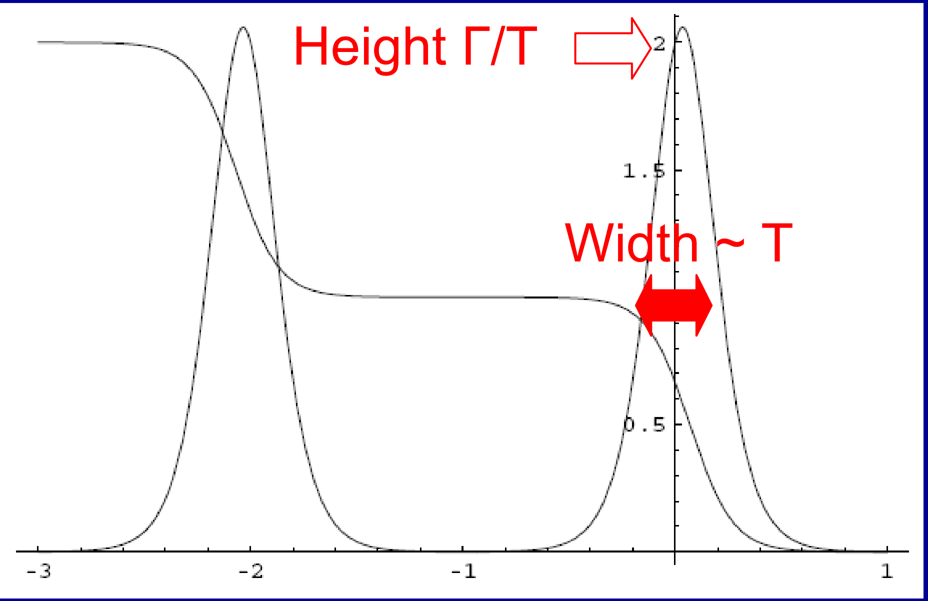
3- Generalization to out of equilibrium, e.g. I(V) for finite voltage Is an outstanding problem. General formula based on Keldysh has been Derived (Meir and Wingreen, PRL 68 (1992) 2512) but concrete calculations Difficult ! Numerous recxent works (Saleur et al., Andrei et al.) – an active field

High-temperature regime T>>Γ : Coulomb blockade

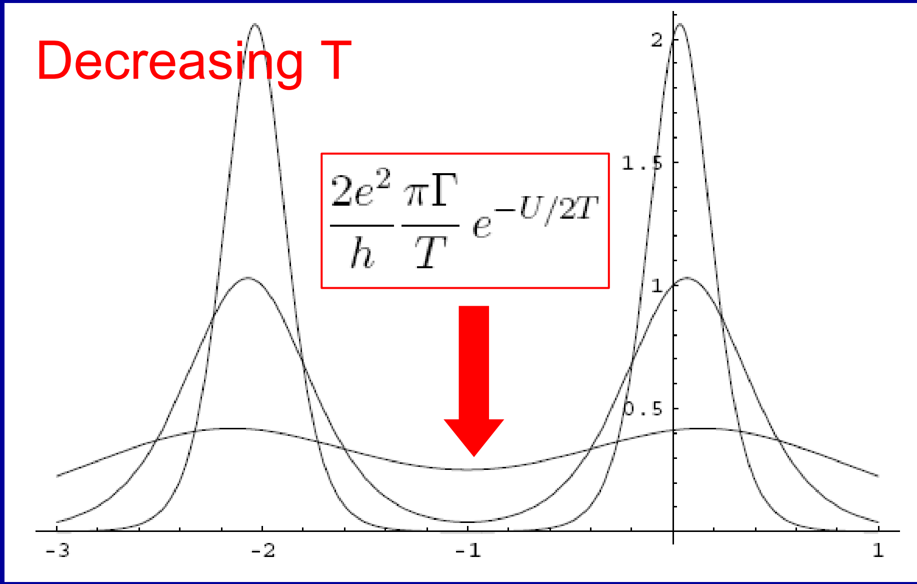
Use isolated atom form of spectral function $\frac{n_d}{2}\,\delta(\omega-\varepsilon_d-U)+(1-\frac{n_d}{2})\,\delta(\omega-\varepsilon_d)$

$$G(T\gg\Gamma)\simeq\frac{2e^2}{h}\frac{\pi\Gamma}{T}\left[(1-\frac{n_d}{2})\frac{1}{4\cosh^2\frac{\varepsilon_d}{2T}}+\frac{n_d}{2}\frac{1}{4\cosh^2\frac{\varepsilon_d+U}{2T}}\right]$$

$$\frac{n_d}{2}=\frac{1}{Z}\left(1\times e^{-\beta\varepsilon_d}+1\times e^{-\beta(2\varepsilon_d+U)}\right)=n_d(T,\varepsilon_d)$$



Plot of n_d and G vs. ε_d for $U=2$ at $\beta=10$.



Plot of G vs. ε_d for $U=2$ at $\beta=2,5,10$.

Low-T regime: the Kondo effect suppresses Coulomb blockade !

Using the value of $A_d(0)$ above, and the Friedel sum-rule:

$$G(T=0)_{L=R} = \frac{2e^2}{h} \sin^2 \frac{\pi n_d}{2}$$

Maximum conductance $2e^2/h$ restored at midpoint of Coulomb staircase !

Scaling form vs. T/T_K :

$$\frac{G(T)}{G(T=0)} = \int_{-\infty}^{+\infty} dx \, \frac{a(x, T/T_K)}{4\cosh^2 \frac{T_K}{T} x}$$

Theoretical prediction of suppression of Coulomb blockade:

- L.Glazman and M.Raikh, JETP Lett. 47 (1988) 452
- T.K.Ng and P.A.Lee, Phys Rev Lett 61 (1988) 1768
- Cf. also: J. Appelbaum, PRL 17 (1966) 91

Wave-function interpretation:

Virtual transitions create admixture of components with 0 or 2 electrons on the dot in the wave-function.

→ Restoration of charge fluctuations

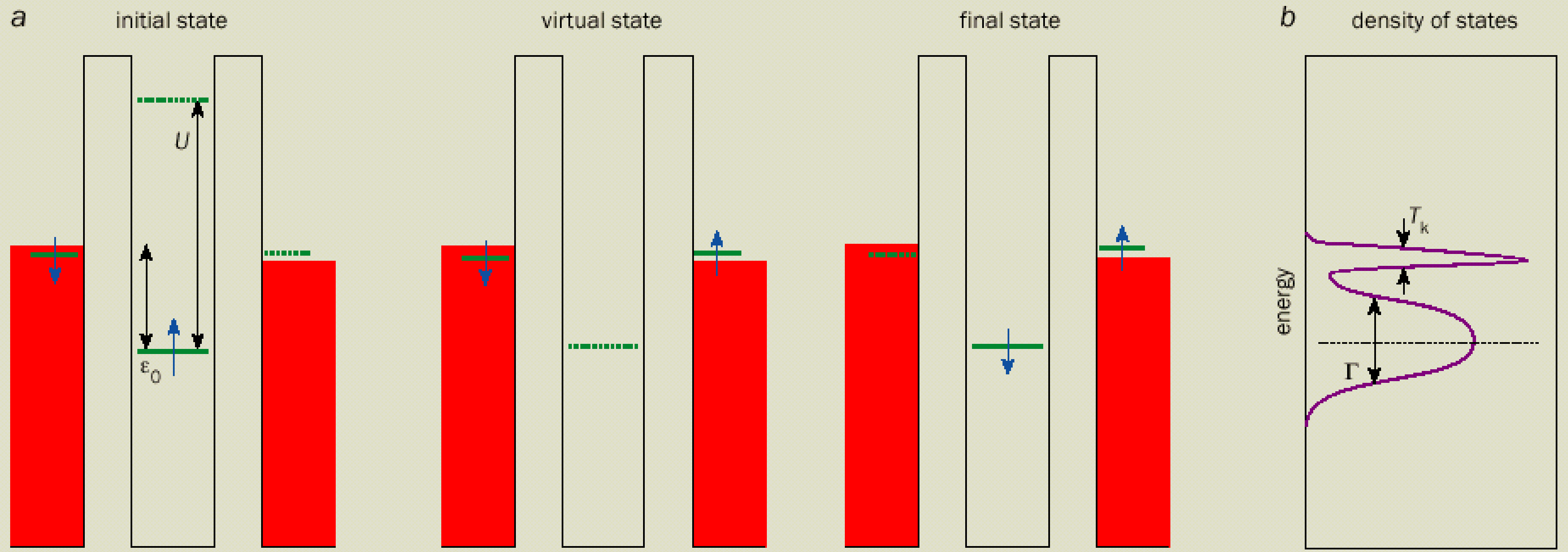
$$|\Psi_0\rangle = \sqrt{1-\eta^2}\,|\mathcal{S}\rangle + \eta\,|\mathcal{D}\rangle$$

$$|\mathcal{S}\rangle \equiv \frac{1}{\sqrt{2}}\left[|\uparrow,\downarrow\rangle - |\downarrow,\uparrow\rangle\right]$$

$$|\mathcal{D}\rangle \equiv \frac{1}{\sqrt{2}}\left[|\uparrow\downarrow,0\rangle + |0,\uparrow\downarrow\rangle\right]$$

with $\eta \sim \frac{V}{U} \ll 1.$

2 Spin flips



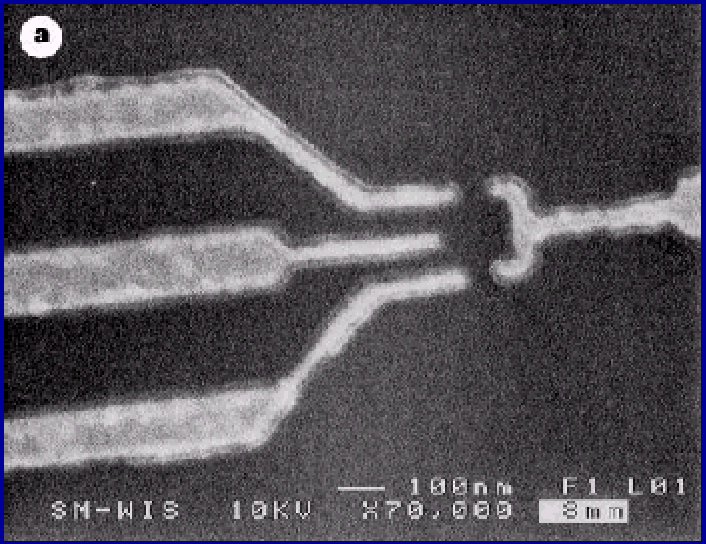
(*a*) The Anderson model of a magnetic impurity assumes that it has just one electron level with energy ε_0 below the Fermi energy of the metal (red). This level is occupied by one spin-up electron (blue). Adding another electron is prohibited by the Coulomb energy, *U*, while it would cost at least $|\varepsilon_0|$ to remove the electron. Being a quantum particle, the spin-up electron may tunnel out of the impurity site to briefly occupy a classically forbidden "virtual state" outside the impurity, and then be replaced by an electron from the metal. This can effectively "flip" the spin of the impurity. (*b*) Many such events combine to produce the Kondo effect, which leads to the appearance of an extra resonance at the Fermi energy. Since transport properties, such as conductance, are determined by electrons with energies close to the Fermi level, the extra resonance can dramatically change the conductance.

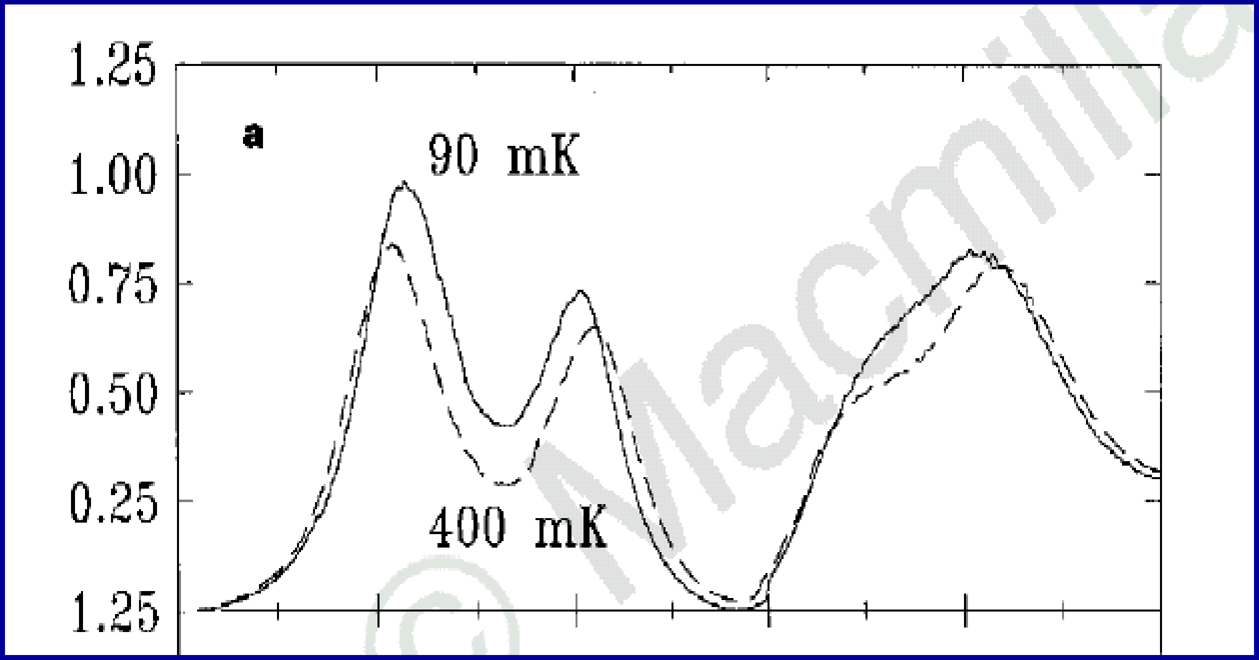
Kondo effect in a single-electron transistor

D. Goldhaber-Gordon*†, Hadas Shtrikman†, D. Mahalu†, David Abusch-Magder*, U. Meirav† & M. A. Kastner*

NATURE | VOL 391 | 8 JANUARY 1998

See also: D. G-G et al. PRL 81 (1998) 5225





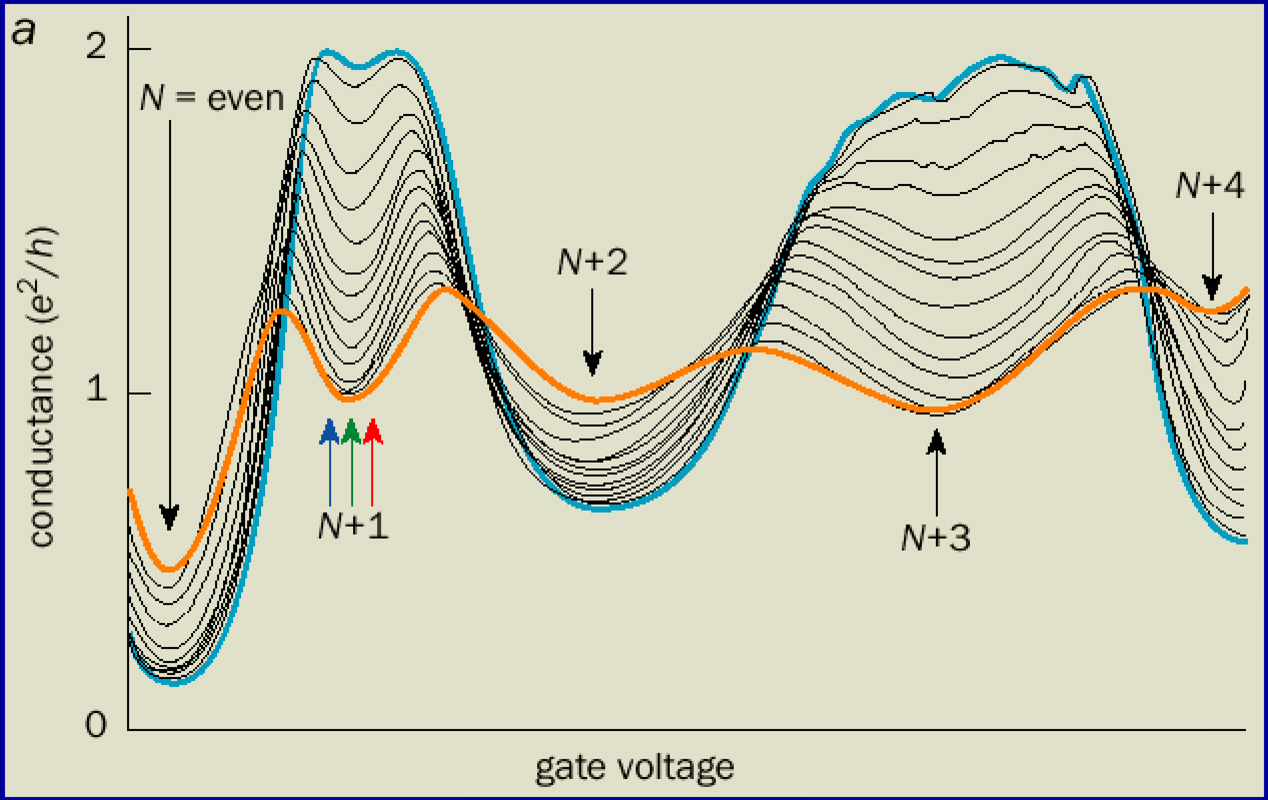
Orders of magnitude:

U ~ 1.9 meV

Γ ~ 0.3 meV

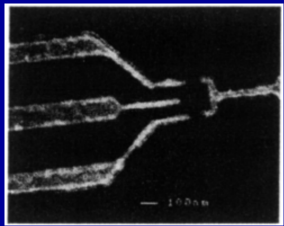
Range of T_K:

40mK → 2,5 K



(*a*) The conductance (*y*-axis) as a function of the gate voltage, which changes the number of electrons, *N*, confined in a quantum dot. When an even number of electrons is trapped, the conductance decreases as the temperature is lowered from 1 K (orange) to 25 mK (light blue). This behaviour illustrates that there is no Kondo effect when *N* is even. The opposite temperature dependence is observed for an odd number of electrons, i.e. when there is a Kondo effect.

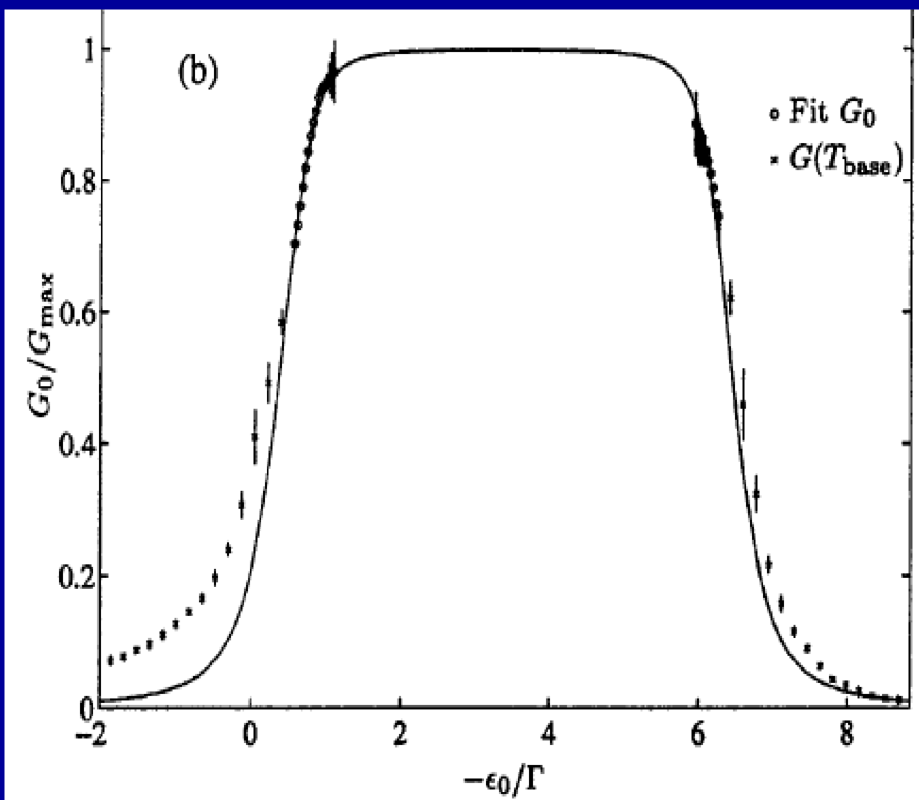
W G van der Wiel *et al*. 2000 The Kondo effect in the unitary limit *Science* **289** 2105–2108



Dependence on gate voltage :

Goldhaber-Gordon et al.

Phys Rev Lett 81 (1998) 5225



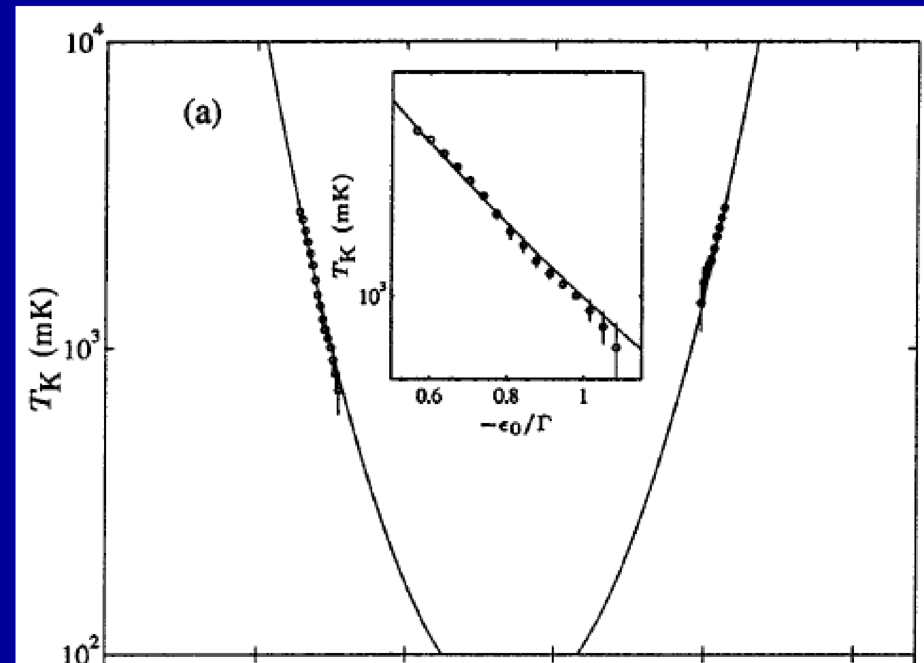


FIG. 5. (a) Fit values of T_K for data such as those in Fig. 3 for a range of values of ϵ_0 [22]. The dependence of T_K on ϵ_0 is well described by Eq. (1) (solid line). Inset: Expanded view of the left side of the figure, showing the quality of the fit. (b) Values of G_0 extracted from data such as those in Fig. 3 at a range of ϵ_0. Solid line: $G_0(\epsilon_0)$ predicted by Wingreen and Meir [4]. $G_{max} = 0.49e^2/h$ for the left peak, and $0.37e^2/h$ for the right peak.

Scaling of G(T)/G(0) vs. T/T_K

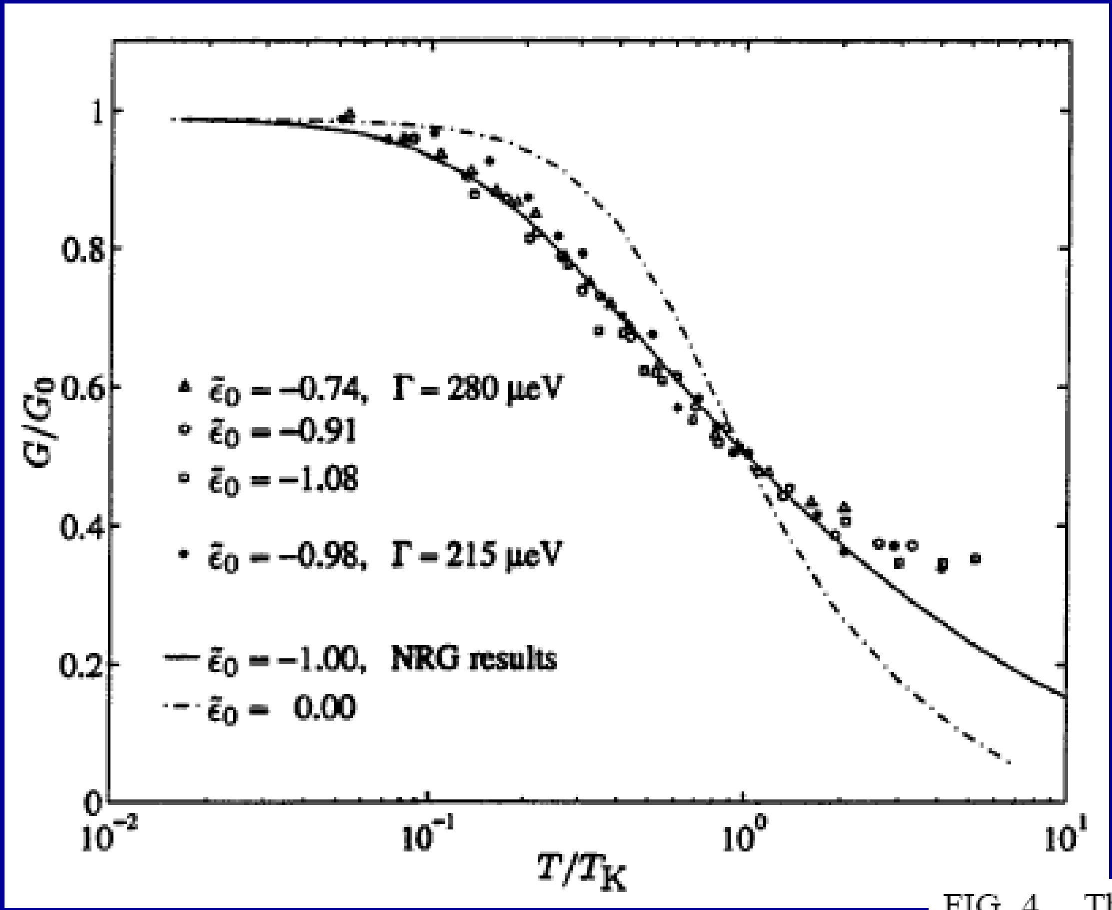


FIG. 4. The normalized conductance $\tilde{G} \equiv G/G_0$ is a universal function of $\tilde{T} \equiv T/T_K$, independent of both $\tilde{\epsilon}_0$ and Γ, in the Kondo regime, but depends on $\tilde{\epsilon}_0$ in the mixed-valence regime. Scaled conductance data for $\tilde{\epsilon}_0 \approx -1$ are compared with NRG calculations [13] for Kondo (solid line) and mixed-valence (dashed line) regimes. The stronger temperature dependence in the mixed-valence regime is qualitatively similar to the behavior for $\tilde{\epsilon}_0 = -0.48$ in Fig. 3(b).

$2e^2/h$ (t=1), yet noisy, conductors …

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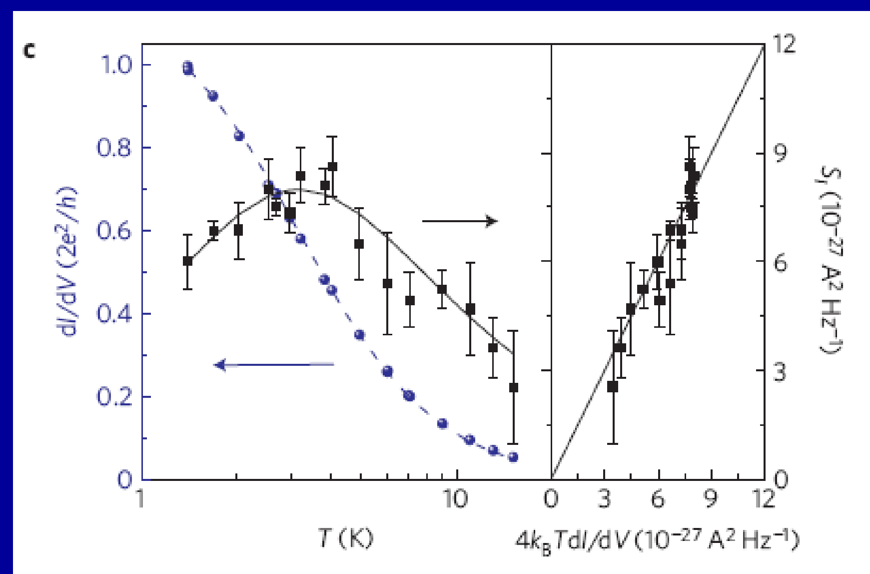
nature physics

Noisy Kondo impurities

T. Delattre[1,2], C. Feuillet-Palma[1,2], L. G. Herrmann[1,2], P. Morfin[1,2], J.-M. Berroir[1,2], G. Fève[1,2], B. Plaçais[1,2], D. C. Glattli[1,2,3], M.-S. Choi[4], C. Mora[1,2] and T. Kontos[1,2]*

In this letter, we show that, in contrast to the prediction of the non-interacting theory, a conductor in the Kondo regime can be noisy even though its conductance is very close to $2e^2/h$. We

SU(2) vs. SU(4) Kondo effect,…



The Kondo effect has implications for a variety of different physical systems…
which I didn't have time to deal with here…

- e.g. out of equilibrium transport/response
- Tunneling into magnetic impurities, magnetic atom

Adsorbed on surfaces etc…

Controlling the Kondo Effect of an Adsorbed Magnetic Ion Through Its Chemical Bonding

Aidi Zhao, Qunxiang Li, Lan Chen, Hongjun Xiang, Weihua Wang, Shuan Pan, Bing Wang, Xudong Xiao, Jinlong Yang,* J. G. Hou,* Qingshi Zhu

2 SEPTEMBER 2005 VOL 309 SCIENCE

We report that the Kondo effect exerted by a magnetic ion depends on its chemical environment. A cobalt phthalocyanine molecule adsorbed on an Au(111) surface exhibited no Kondo effect. Cutting away eight hydrogen atoms from the molecule with voltage pulses from a scanning tunneling microscope tip allowed the four orbitals of this molecule to chemically bond to the gold substrate. The localized spin was recovered in this artificial molecular structure, and a clear Kondo resonance was observed near the Fermi surface. We attribute the high Kondo temperature (more than 200 kelvin) to the small on-site Coulomb repulsion and the large half-width of the hybridized d-level.

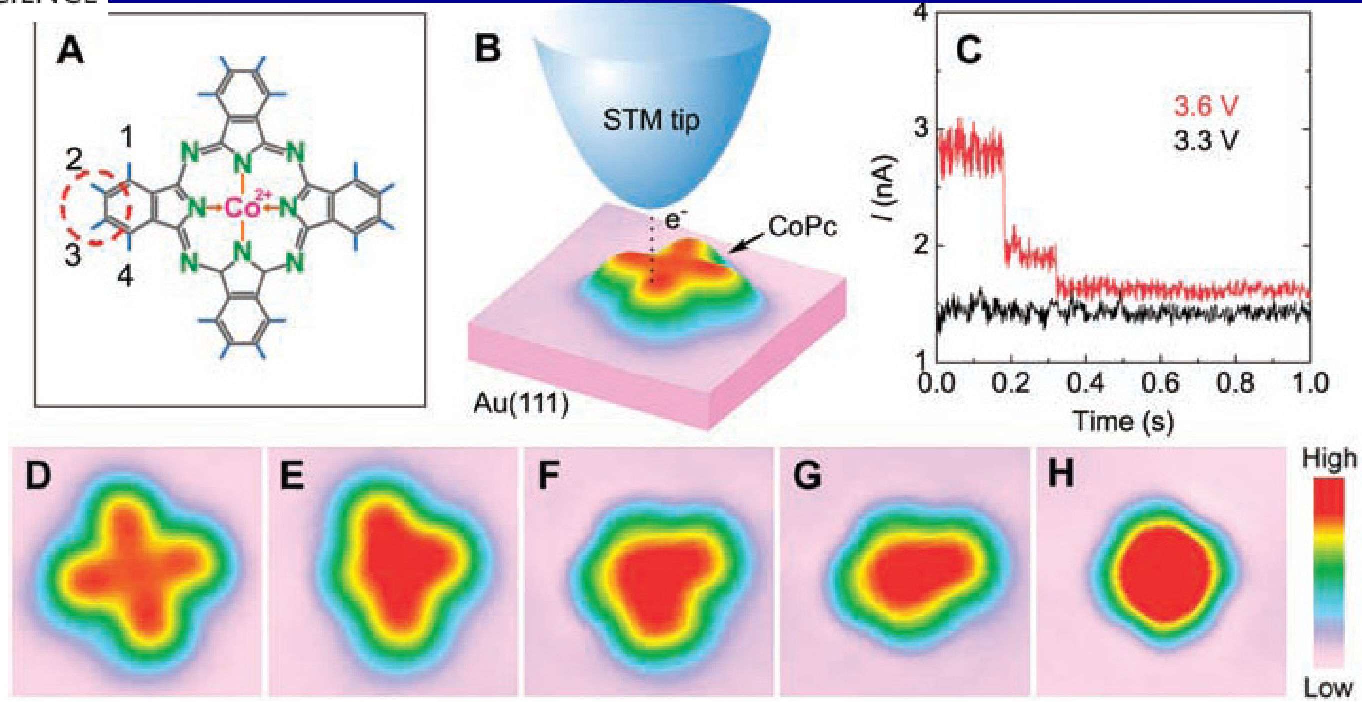


Fig. 1. STM tip–induced dehydrogenation of a single CoPc molecule. (**A**) Structural formula of the CoPc. Hydrogen atoms 2 and 3 of one lobe were dissociated in our experiments. (**B**) Diagram of the dehydrogenation induced by the STM current. (**C**) Current versus time during two different voltage pulses on the brink of one lobe. Black and red lines correspond to 3.3 V and 3.6 V, respectively. (**D** to **H**) STM images of a single CoPc molecule during each step of the dehydrogenation process, from (D) an intact CoPc to (H) d-CoPc. Image area, 25 Å by 25 Å. The color scale represents apparent heights, ranging from 0 Å (low) to 2.7 Å (high).

Kondo effect: ***in lieu of a conclusion…***

``No Hamiltonian so incredibly simple has ever previously done such violence to the literature and to national science budgets''

Attributed to Harry Suhl by P.W. Anderson
in his 1978 Nobel lecture
[Rev Mod Phys 50 (1978) 191 p. 195]

[Although the Ising model is surely a serious competitor…]

Friedel's sum-rule: non-perturbative proof (sloppy about contours…)

Friedel sum-rule

Wide bandwidth case. Non-perturbative proof, *valid for* $T = 0$.
Note: sloppy about contours and prescription for G.F,
We consider the Green's function $G_d^{-1}(\omega) = \omega - \varepsilon_d + i\Gamma - \Sigma(\omega)$

$$-\frac{\partial}{\partial\omega} \ln G_d(\omega) = \left[1 - \frac{\partial\Sigma}{\partial\omega}\right] G_d(\omega) \tag{1.59}$$

and use:

$$-\frac{1}{\pi}\int_C d\omega G_d(\omega) = \frac{n_d}{2} \tag{1.60}$$

Hence, integrating the above relation:

$$\frac{\pi n_d}{2} = \ln G_d(\omega = 0) - \ln G_d(\omega \to -\infty) + \int_C d\omega \frac{\partial\Sigma}{\partial\omega} G_d(\omega) \tag{1.61}$$

The last term will be shown to vanish, so that we get (remember Im$G_r < 0$, definition of phase shift, and using a branch-cut of the ln on the negative real axis):

$$\frac{\pi n_d}{2} = (\delta - \pi) - (-\pi) \tag{1.62}$$

so that the occupancy of the d-orbital and the phase-shift are related by

$$n_d = \frac{2}{\pi}\delta \tag{1.63}$$

A manifestation of *Friedel's sum-rule* in this context. This derivation for the AIM is due to Langreth.

To prove that the last term vanishes, we integrate it by part:

$$\int_C d\omega \, \frac{\partial G_d}{\partial \omega} \, \Sigma(\omega) \tag{1.64}$$

and observe that the self-energy is obtained from the Luttinger-Ward functional as:

$$\Sigma(\omega) = \frac{\delta \Phi[G]}{\delta G(\omega)} \tag{1.65}$$

so that the above reads:

$$\int_C d\omega \, \frac{\partial G}{\partial \omega} \, \frac{\delta \Phi[G]}{\delta G(\omega)} = \delta \Phi \tag{1.66}$$

This is the change of the LW functional when all frequencies are shifted.

Guesswork on the structure of the d-level spectral function (particle-hole symmetric case):

High Temperature $T >> T_K$: no Kondo effect

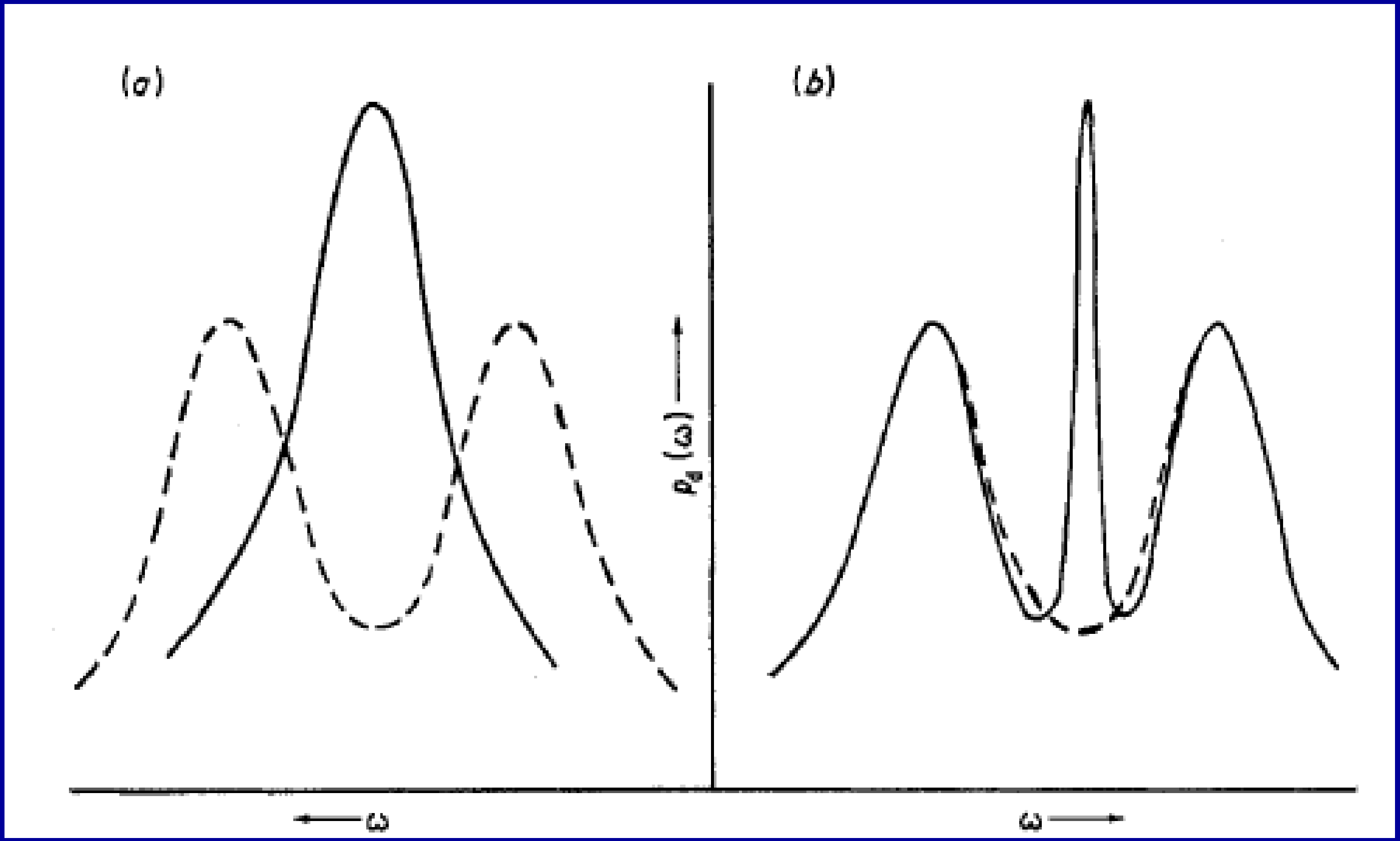
→ spectral function is made of 2 atomic-like transitions broadened by Γ

What happens at low – T ?

Constraint: Friedel sum-rule imposes at T=0 (in the wide bandwidth limit):

$$G_d(i0^+) = -\frac{1}{\Gamma}\,\sin\delta\, e^{i\delta}$$

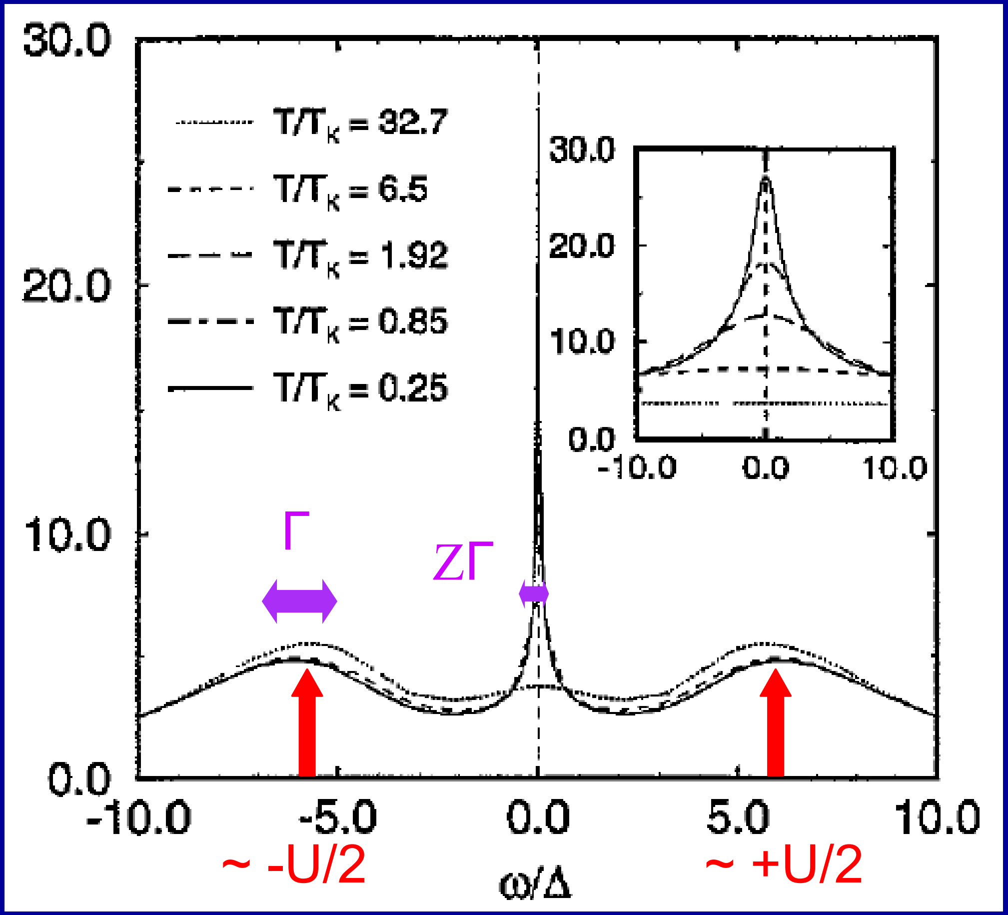
$$A_d(\omega = 0) = \frac{\sin^2\delta}{\pi\Gamma}$$


(a)
(b)
$\rho_d(\omega)$ ⟶
⟵ ω
ω ⟶

Numerical Renormalization Group (NRG) calculation

T.Costi and A.Hewson, J. Phys Cond Mat 6 (1994) 2519

Cf. seminar by Olivier Parcollet



$$Z \sim T_K/\Gamma \sim \exp -\frac{8\Gamma}{\pi U}$$