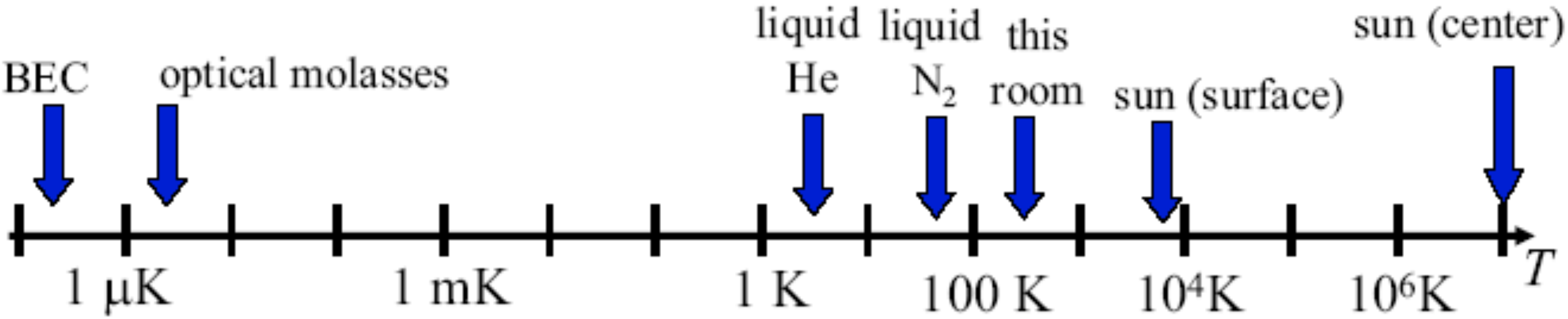
Chaire de Physique de la Matière Condensée

III.3 Transport mésoscopique et effets thermoélectriques dans les gaz atomiques ultra-froids

Antoine Georges

Cycle 2014-2015
1er juin 2015 – III.3

Ultra-Cold Atomic Gases



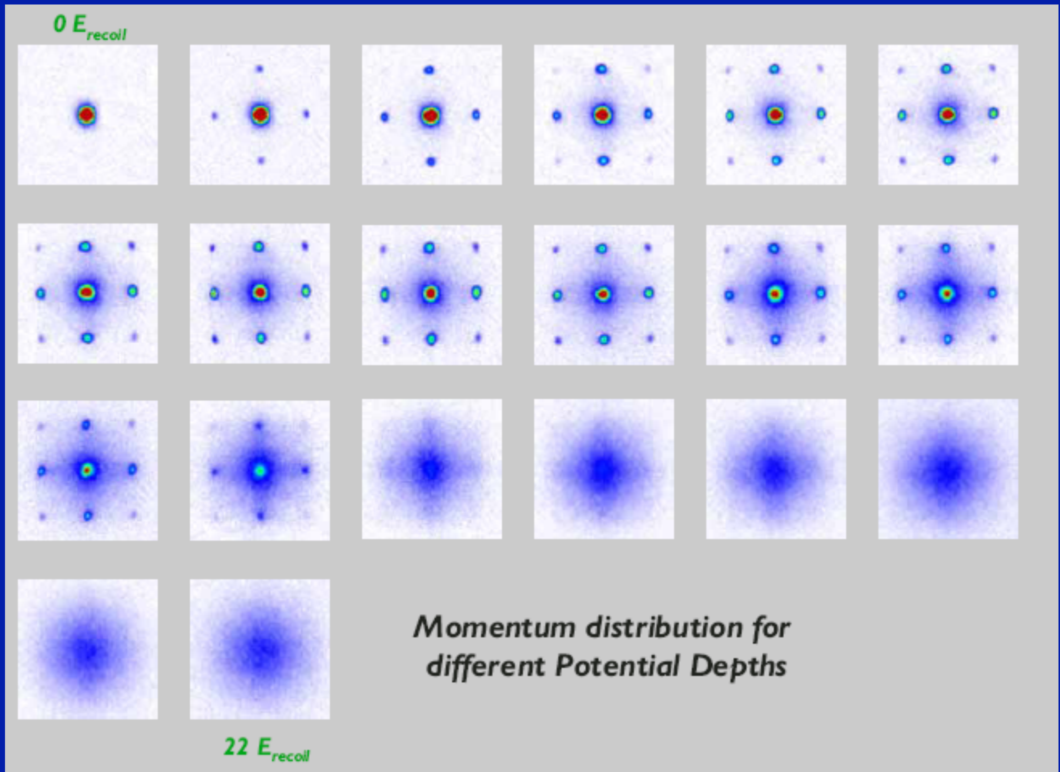
BEC

Nobel 2001
E. Cornell , W. Ketterle , C. Wieman

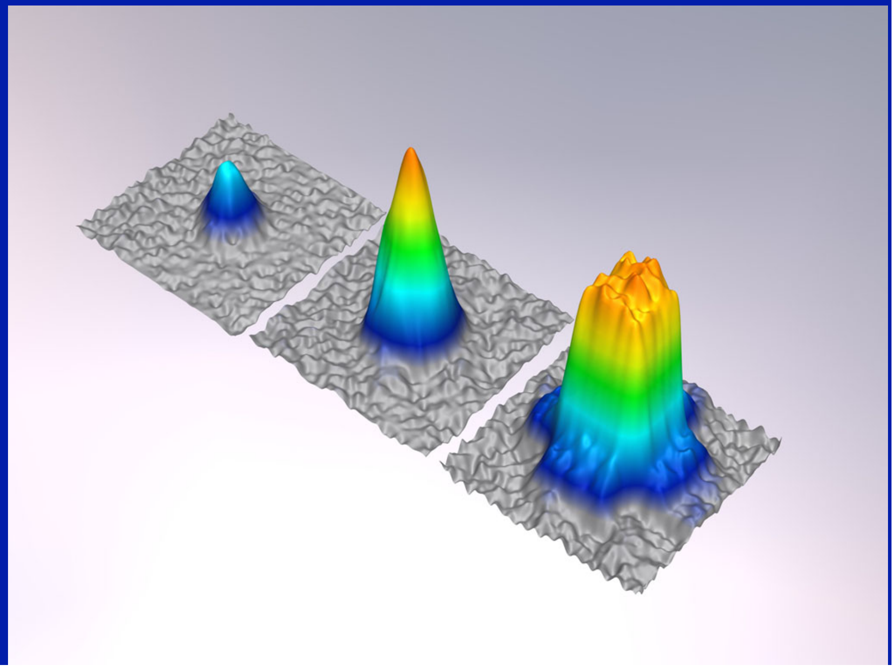
COOLING

Nobel 1997
S. Chu, C. Cohen-Tannoudji, W. Phillips

Crossing Borders: from Quantum Optics to Condensed Matter Physics



Imaging Fermi Surfaces
Michael Köhl et al.
PRL 94, 080403 (2005)
[ETHZ]

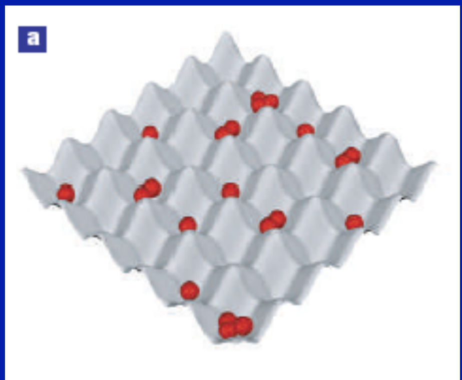


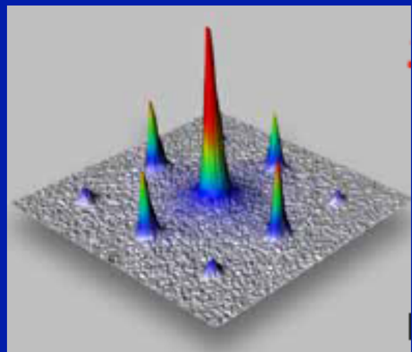
Superfluid to Mott Insulator Transition
Greiner, Bloch, Esslinger et al.
Nature 2002 [Garching MPI]

Quantum phase transition from a superfluid to a Mott insulator in a gas of ultracold atoms

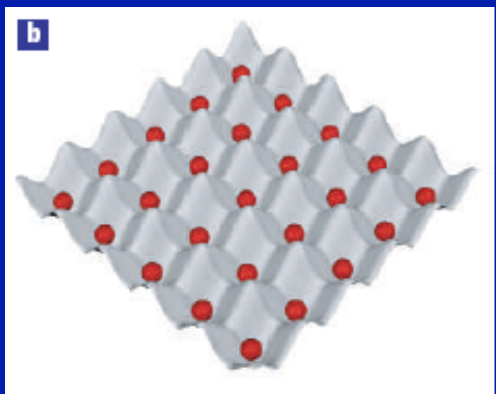
NATURE | VOL 415 | 3 JANUARY 2002

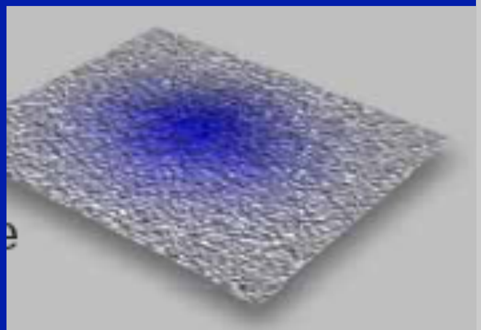
Markus Greiner*, Olaf Mandel*, Tilman Esslinger†, Theodor W. Hänsch* & Immanuel Bloch*

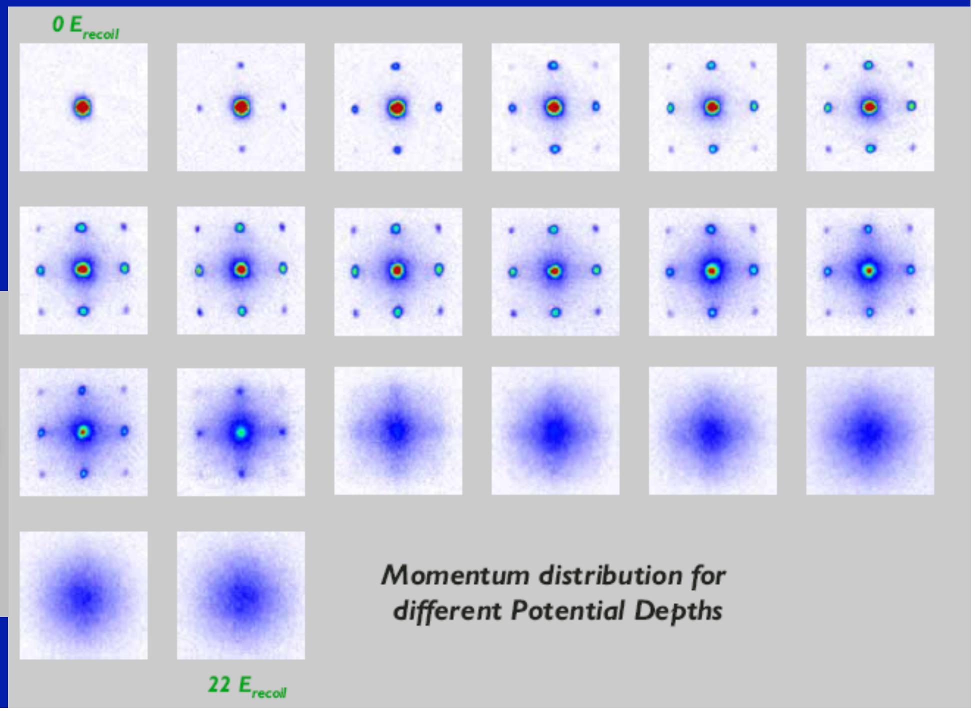




Phase coherence between wells
in superfluid phase >interference pattern

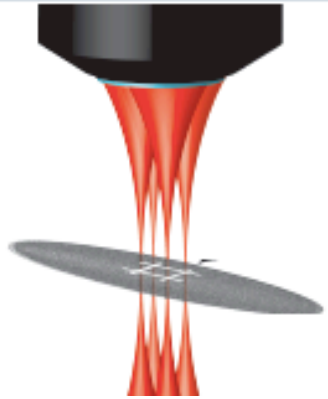






Cold Atoms and Condensed Matter Physics: comparing characteristic scales

	Cold Fermionic atoms	Electrons in a solid
Density	10^{12} cm^{-3}	10^{22} cm^{-3} (Metals)
Mass	6 (Li), 40 (K)	5.4 10^{-4}
Fermi Temperature	µK	10^4 K
Temperature	100 nK	10 mK
Charge	0	-1
Interactions	Contact, *tunable*	Coulomb, material dep.
Potential shaping	Laser light	growing, lithography

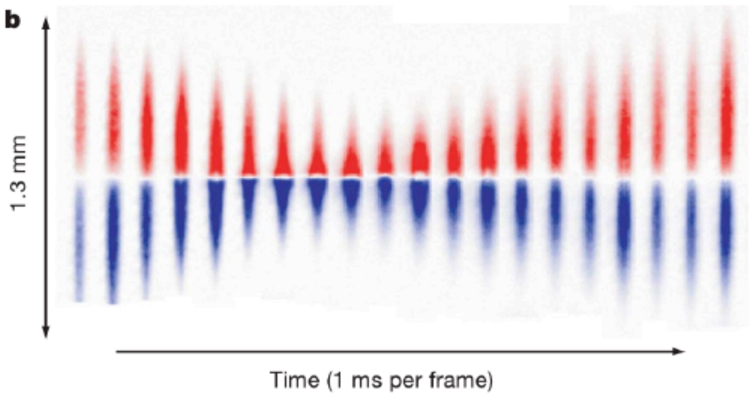


Slide: courtesy
J-P Brantut

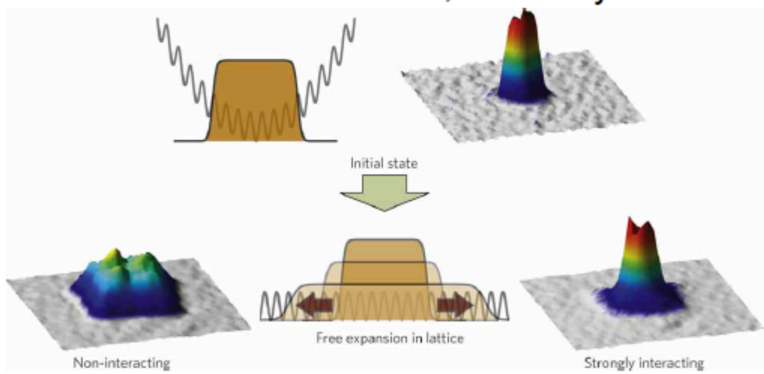
An emerging field: transport experiments with ultra-cold atomic gases

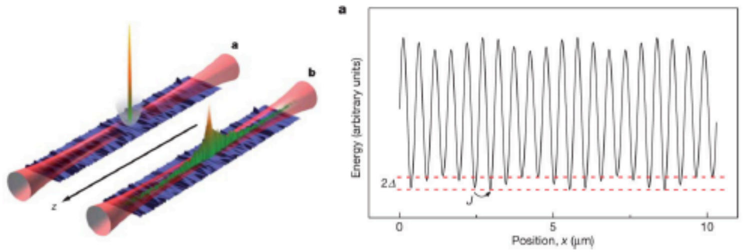
Introduction - Transport and cold atoms

Spin transport (MIT, 2011)
A. Sommer *et al.*, Nature



Interactions (LMU, 2012)
U. Schneider *et al.*, Nat. Phys.

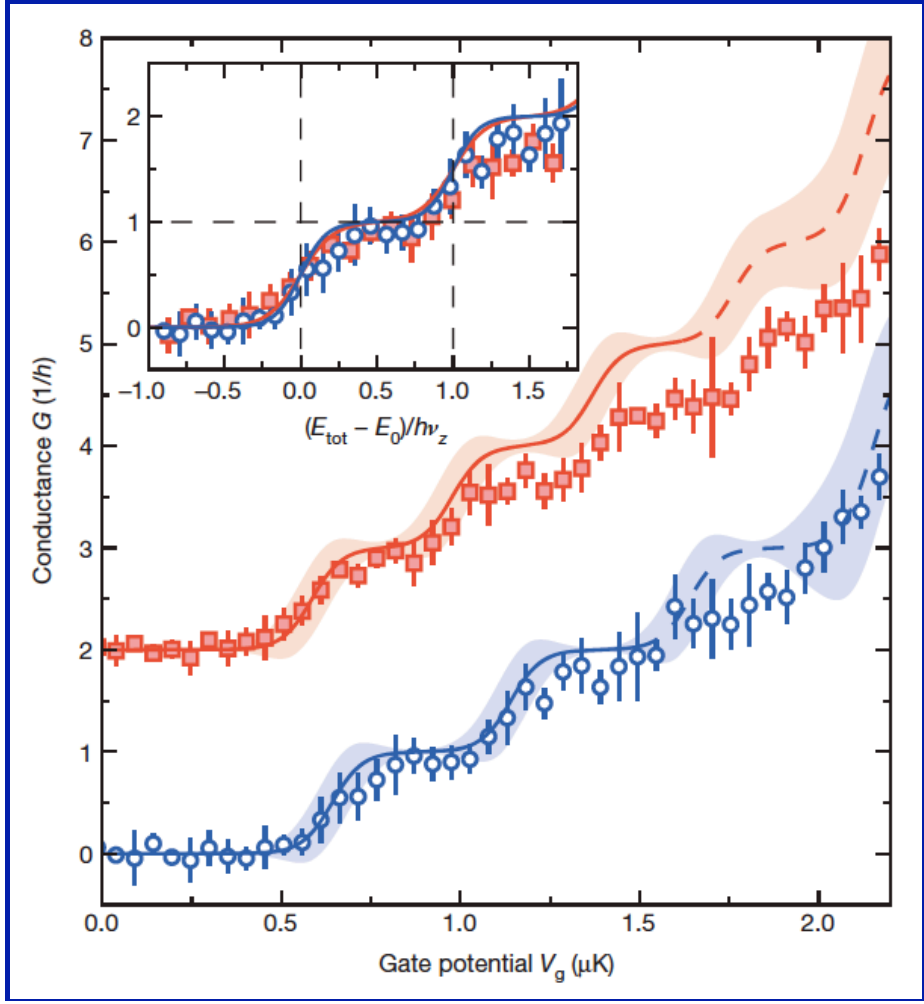


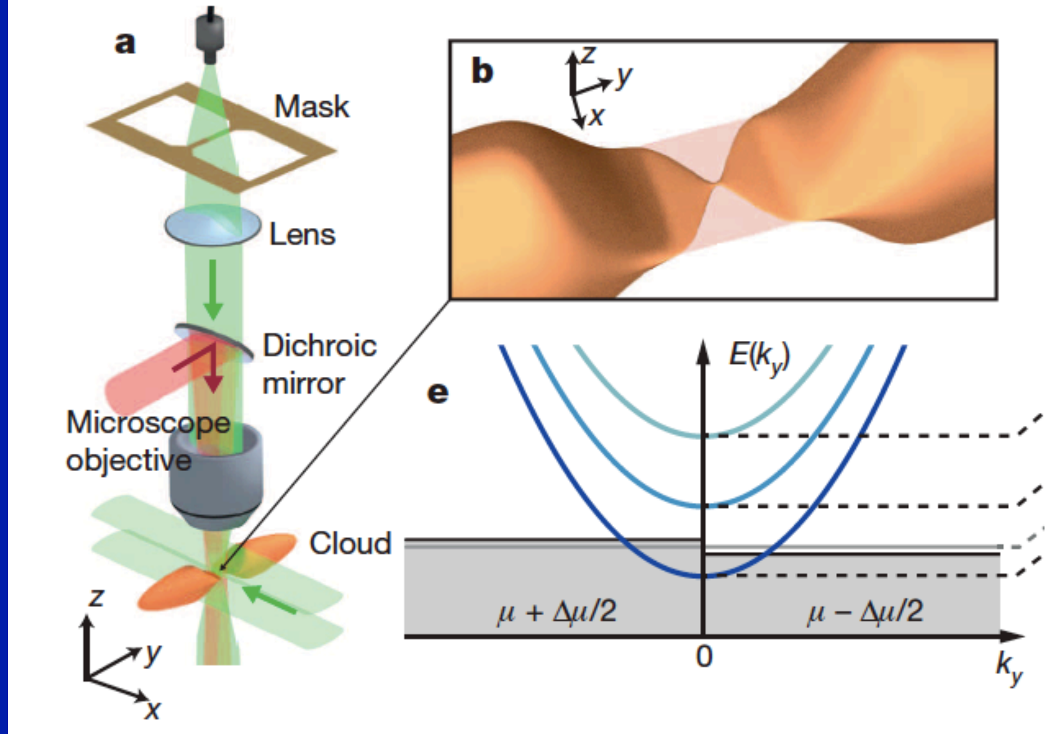


Disorder (Inst. d'optique - LENS, 2008)
J. Billy *et al.*-G. Roati *et al.*, Nature

Also:
H. Ott *et al*, Phys. Rev. Lett. 92, 160601 (2004)
S. Palzer *et al*, Phys. Rev. Lett. 103, 150601 (2009)
J. Catani *et al*, Phys. Rev. A 85, 023623 (2012)
K.K. Das *et al*, Phys. Rev. Lett. 103, 123007 (2009)
And many others ...

Conductance Quantization observed !





Imprinted potential realizing a quantum point-contact

Krinner et al.
Nature 517, 65 (2015)
Tilman Esslinger's group
@ETH-Zurich

Cold atoms in a warm atmosphere – Thanks to:

ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Experiment: (ETH Zürich)

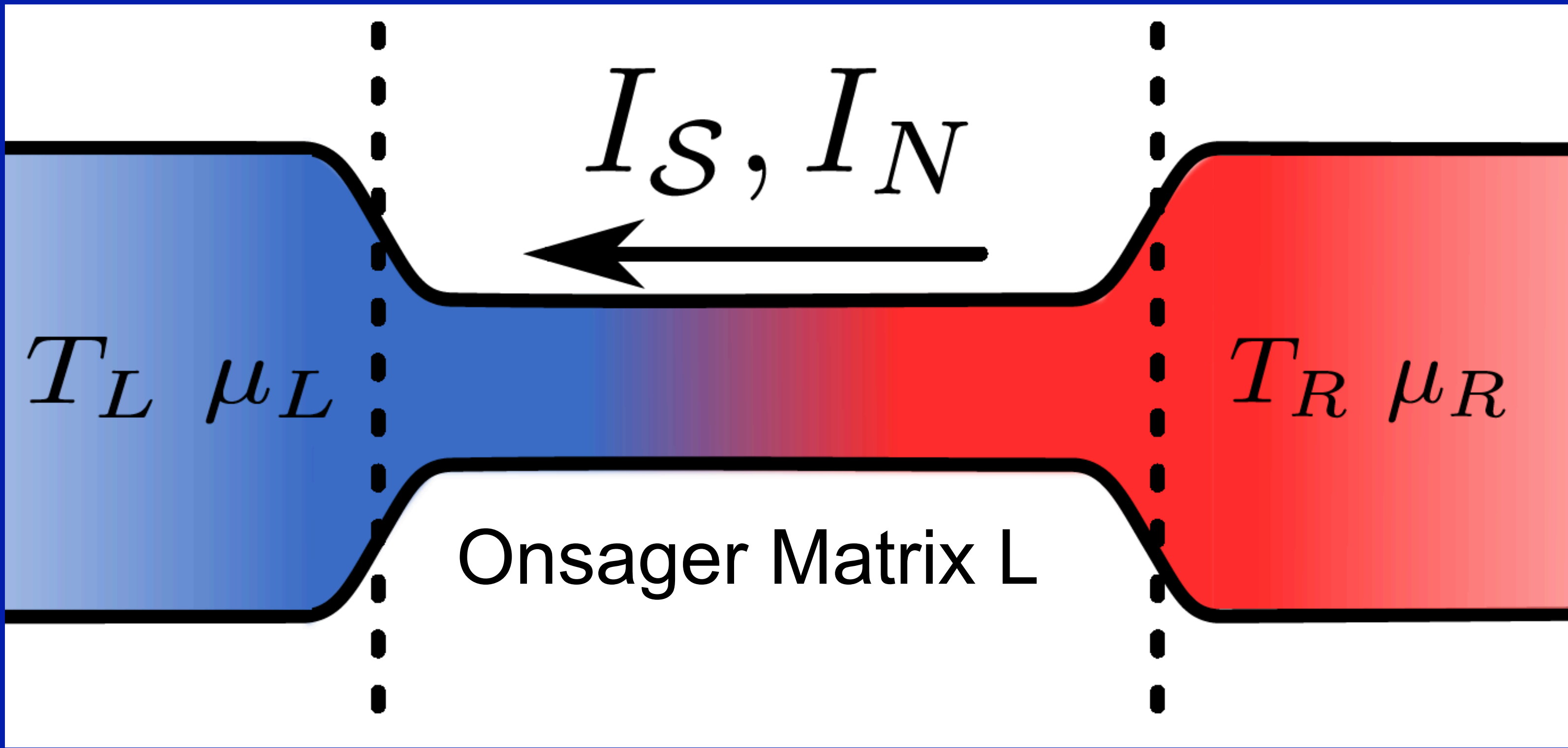
S. Krinner
J.P. Brantut,
J. Meineke
D. Stadler
T. Esslinger
D. Husmann
S. Haüsler
M. Lebrat

Theory … :

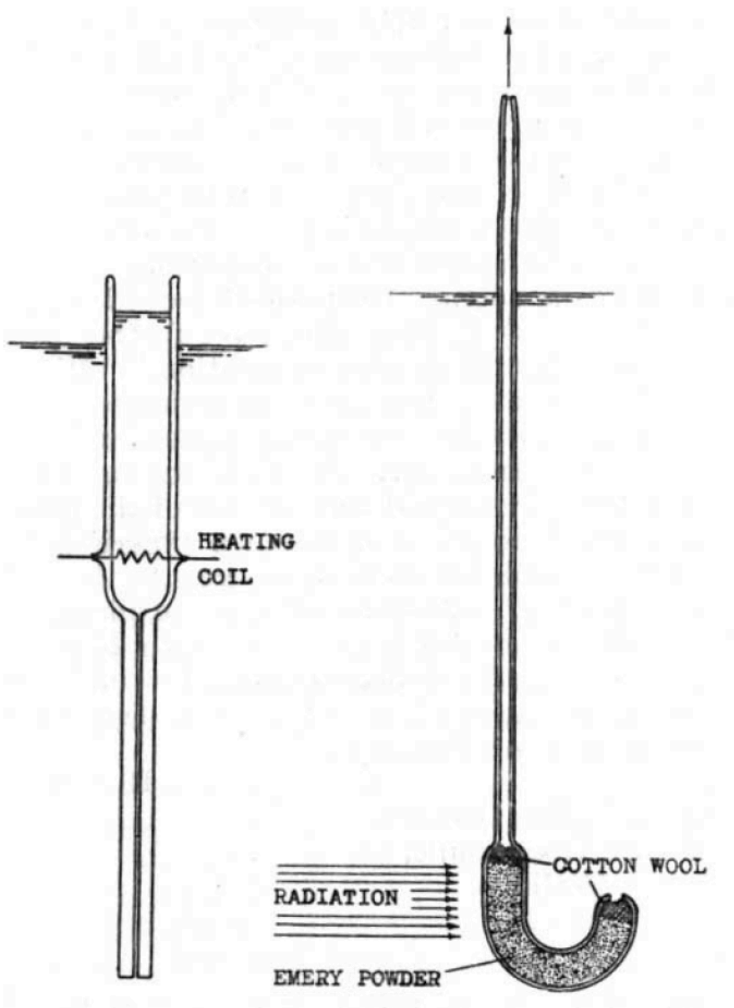
C. Grenier (Ecole Polytechnique)
C. Kollath (University of Bonn)
A. Georges (College de France)

Generic set-up in the following:

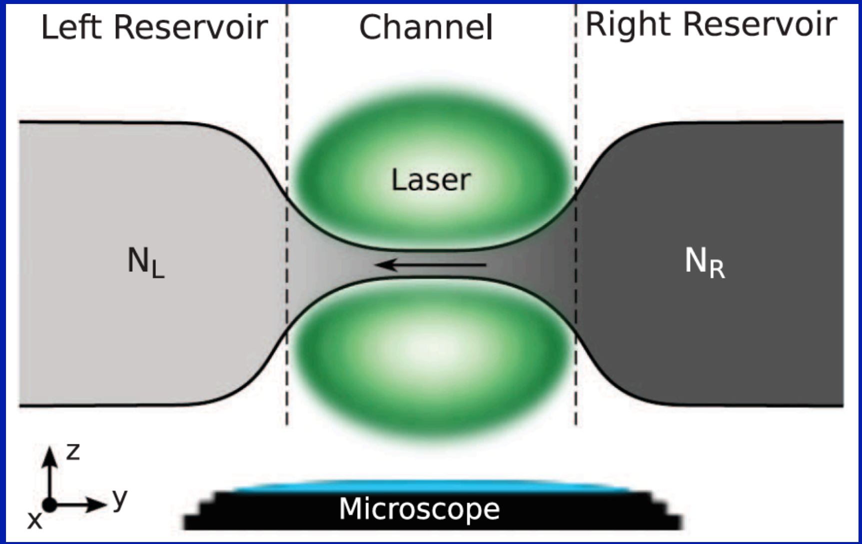
Two reservoirs and a constriction



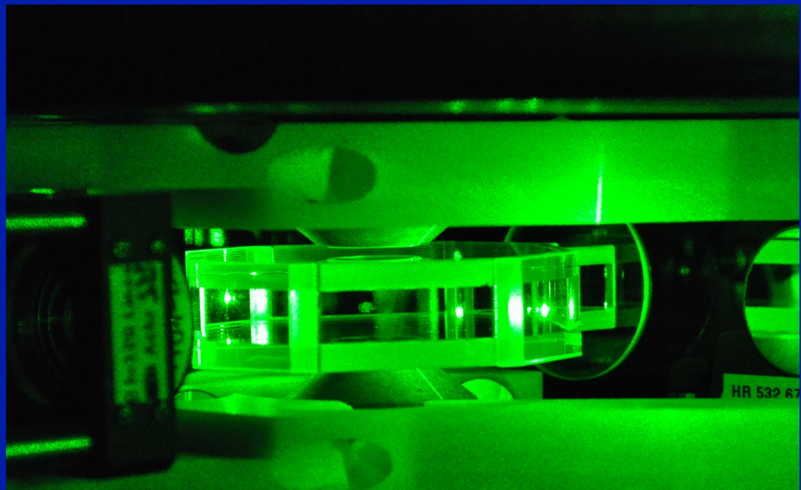
Constrictions and Quantum Fluids: Older and Newer Incarnations



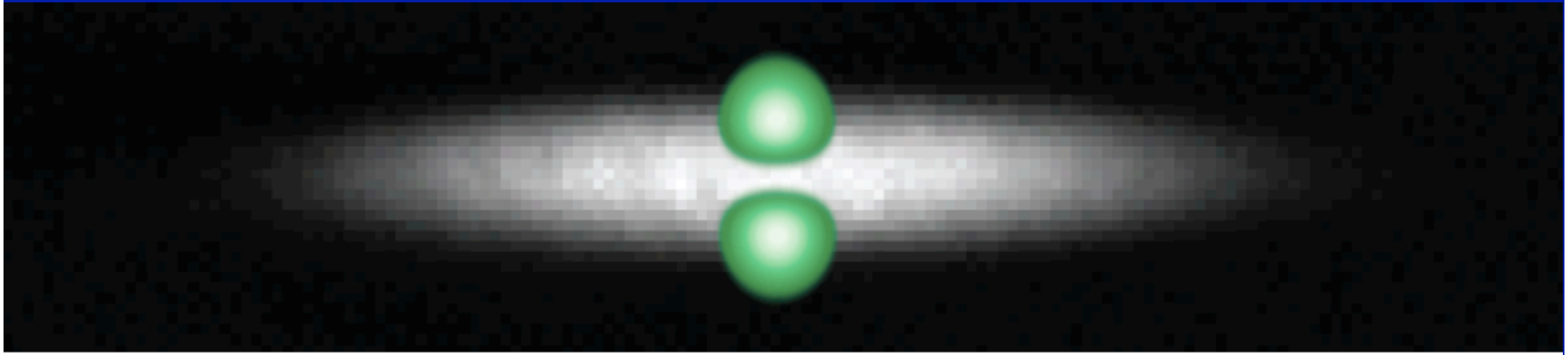
\`\`Superlink''
Allen and Jones,
Nature, 1938



Brantut et al.
Science, 337, 1069 (2012)



Picture:
Courtesy T.Esslinger et al. ETH

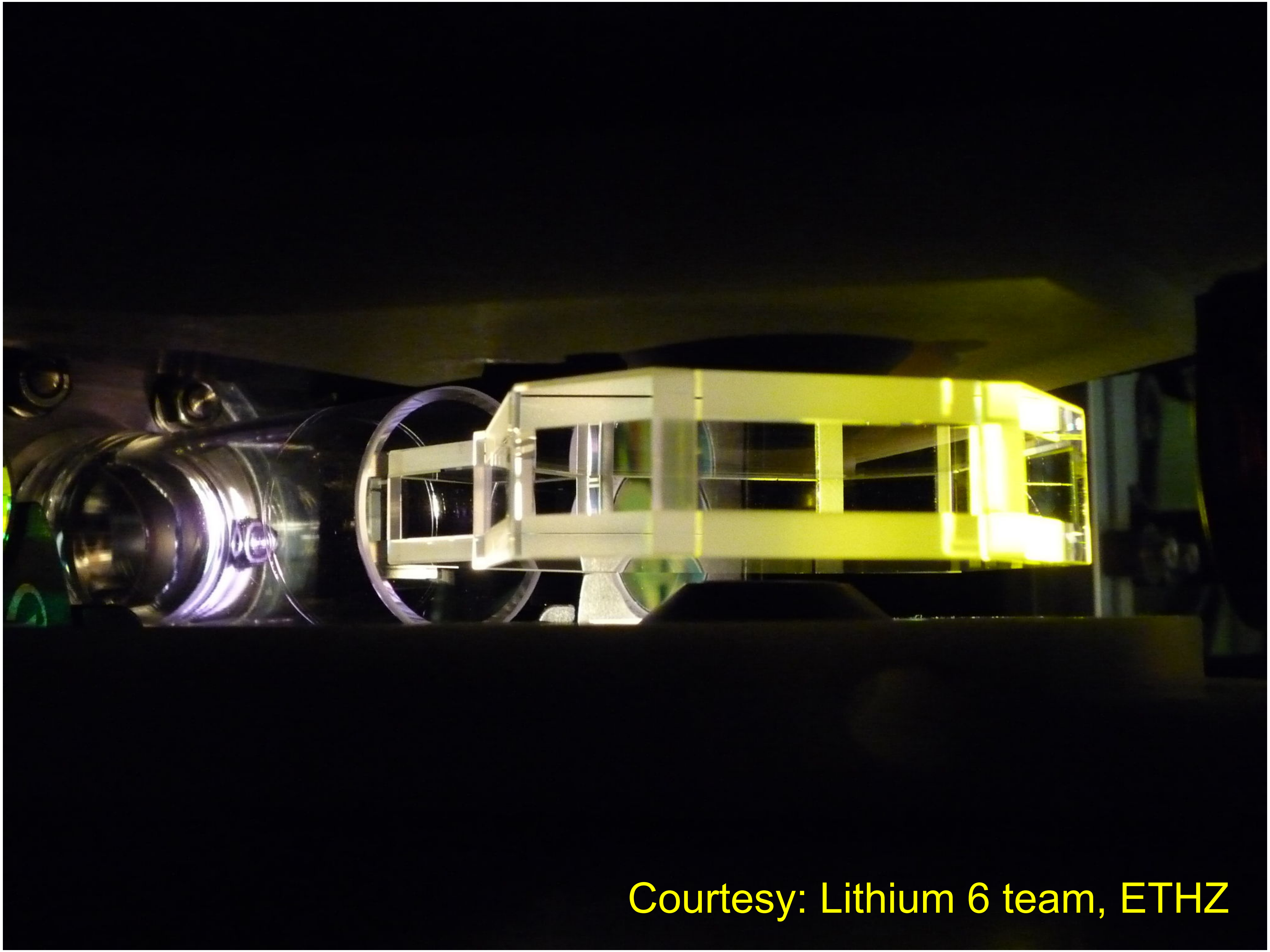


Repulsive TEM_{01} laser beam on the center of the cloud

Trap frequency up to 11 kHz

Creates a narrow multimode, *ballistic* channel

Courtesy: JP Brantut


Courtesy: Lithium 6 team, ETHZ

Onsager Coefficients describing transport in the constriction
in the linear response regime

Particle and entropy currents:

$$\boxed{\begin{aligned} I_N &= L_{11}\Delta\mu + L_{12}\Delta T \\ I_S &= L_{21}\Delta\mu + L_{22}\Delta T \end{aligned}}$$

N,S particle number and entropy
I_N,I_S: currents

$$\Delta\mu \equiv \mu_L - \mu_R$$
$$\Delta T \equiv T_L - T_R$$

Measuring the Conductance by transient `discharge' - from ballistic to diffusive -

Science, 337, 1069 (2012)

Conduction of Ultracold Fermions Through a Mesoscopic Channel

Jean-Philippe Brantut, Jakob Meineke, David Stadler, Sebastian Krinner, Tilman Esslinger*

In a mesoscopic conductor, electric resistance is detected even if the device is defect-free. We engineered and studied a cold-atom analog of a mesoscopic conductor. It consists of a narrow channel connecting two macroscopic reservoirs of fermions that can be switched from ballistic to diffusive. We induced a current through the channel and found ohmic conduction, even when the channel is ballistic. We measured in situ the density variations resulting from the presence of a current and observed that density remains uniform and constant inside the ballistic channel. In contrast, for the diffusive case with disorder, we observed a density gradient extending through the channel. Our approach opens the way toward quantum simulation of mesoscopic devices with quantum gases.

Dynamics of equilibration:

The thermodynamic properties of the reservoirs AND the transport in the constriction BOTH play a role

Consider the simplest case with an initial particle-number imbalance, no temperature imbalance, $L_{12}=0$, and linear-response applies (small deviations from equilibrium) :

Dynamics of the particle flow:

$$\frac{d}{dt}\Delta N = -I_N = -L_{11}\,\Delta\mu$$

Thermodynamics in the reservoirs: $\Delta N = \kappa\,\Delta\mu \;,\; \kappa \equiv \frac{\partial N}{\partial \mu}\Big|_T$

(κ ~ compressibility of gas in reservoir)

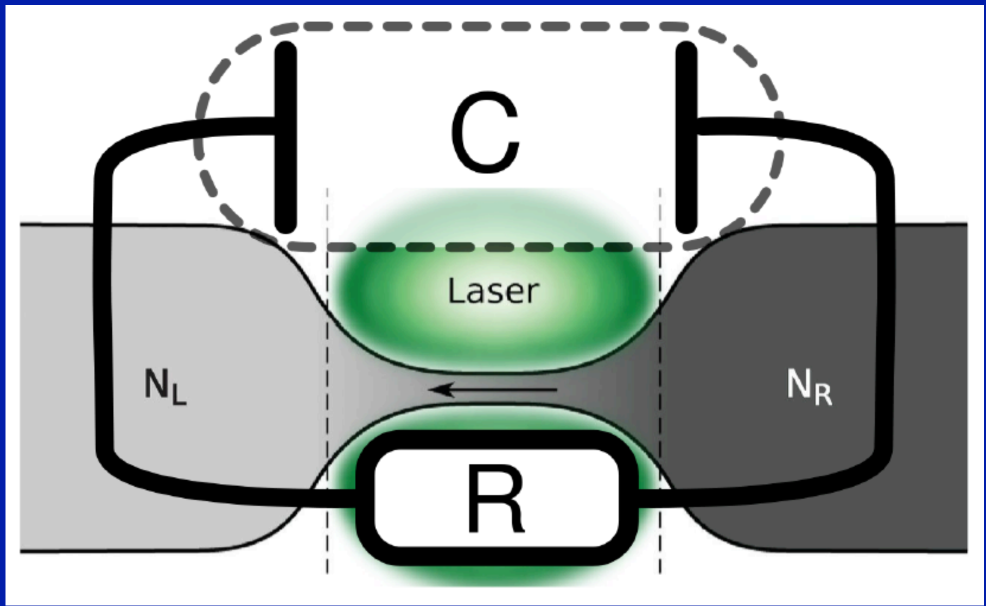
Combining:

$$\frac{d}{dt}\Delta\mu = -\frac{L_{11}}{\kappa}\,\Delta\mu$$

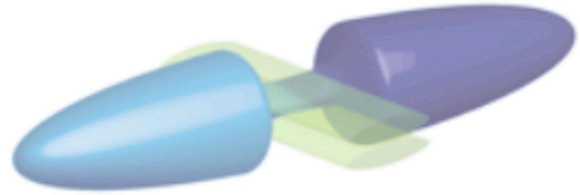
Same for ΔN

$$\boxed{\{\Delta N(t), \Delta\mu(t)\} = \{\Delta N_0, \Delta\mu_0\}\, e^{-t/\tau_\mu} \quad , \quad \tau_\mu = \frac{\kappa}{L_{11}}}$$

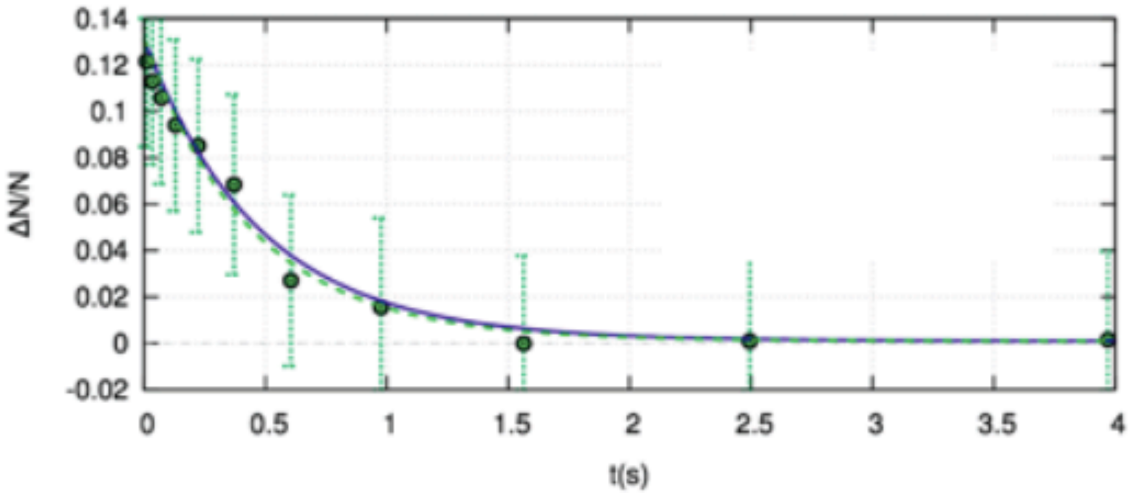
cf. discharge of a capacitor:

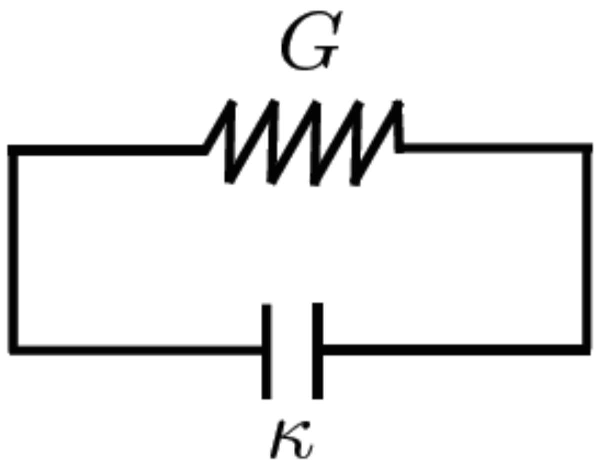


$$Q(t) = Q_0\, e^{-t/\tau} \quad , \quad \tau = RC = \frac{C}{G} = \frac{C}{e^2 L_{11}}$$



Atomic flow through the channel





Ballistic channel :

$\kappa/G = 481(30)\,\mathrm{ms}$ *Experimental fit*

$\kappa/G = 450(30)\,\mathrm{ms}$ *Landauer-Büttiker + ideal reservoirs*

Slide: JP Brantut

(Calculation: C.Grenier)

Thermal equilibration between reservoirs in the absence of thermoelectric effects (L_{12}=0)

$$\Delta T(t) = \Delta T_0 \, e^{-t/\tau_T} \quad , \quad \tau_T = \frac{C_\mu/T}{L_{22}} = \frac{C_\mu/T}{G_{th}/T}$$

With the heat capacity at constant chemical potential:

$$C_\mu = T \, \frac{\partial S}{\partial T}\Big|_\mu$$

In the presence of coupling between T and μ either via transport (L_{12}) or thermodynamics (dilatation coeff.), <u>the evolution of μ and T become coupled</u>

A note in passing: a novel (?) interpretation of Wiedemann-Franz law

For a free Fermi gas, as T→0 (cf. previous lectures):

$$\frac{G_{th}/T}{G} \rightarrow \frac{\pi^2}{3} \quad , \quad \frac{C_\mu/T}{\kappa} \rightarrow \frac{\pi^2}{3}$$

Hence, the particle and thermal equilibration times are the same as T→ 0 !

$$\boxed{\frac{\tau_\mu}{\tau_T} \rightarrow 1}$$

Probing Thermo`electric’ Effects in Ultra-Cold Gases

A Thermoelectric Heat Engine with Ultracold Atoms

Science, 342, 713 (2013)

See also: Cheng Chin et al.

Jean-Philippe Brantut,[1] Charles Grenier,[2] Jakob Meineke,[1]* David Stadler,[1] Sebastian Krinner,[1] Corinna Kollath,[3] Tilman Esslinger,[1]† Antoine Georges[2,4,5]

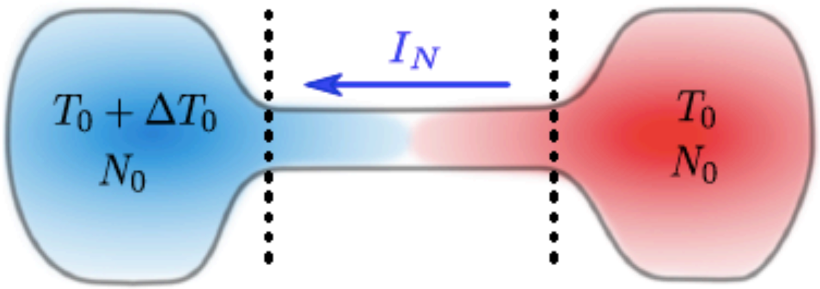
Thermoelectric effects, such as the generation of a particle current by a temperature gradient, have their origin in a reversible coupling between heat and particle flows. These effects are fundamental probes for materials and have applications to cooling and power generation. Here, we demonstrate thermoelectricity in a fermionic cold atoms channel in the ballistic and diffusive regimes, connected to two reservoirs. We show that the magnitude of the effect and the efficiency of energy conversion can be optimized by controlling the geometry or disorder strength. Our observations are in quantitative agreement with a theoretical model based on the Landauer-Büttiker formalism. Our device provides a controllable model system to explore mechanisms of energy conversion and realizes a cold atom–based heat engine.

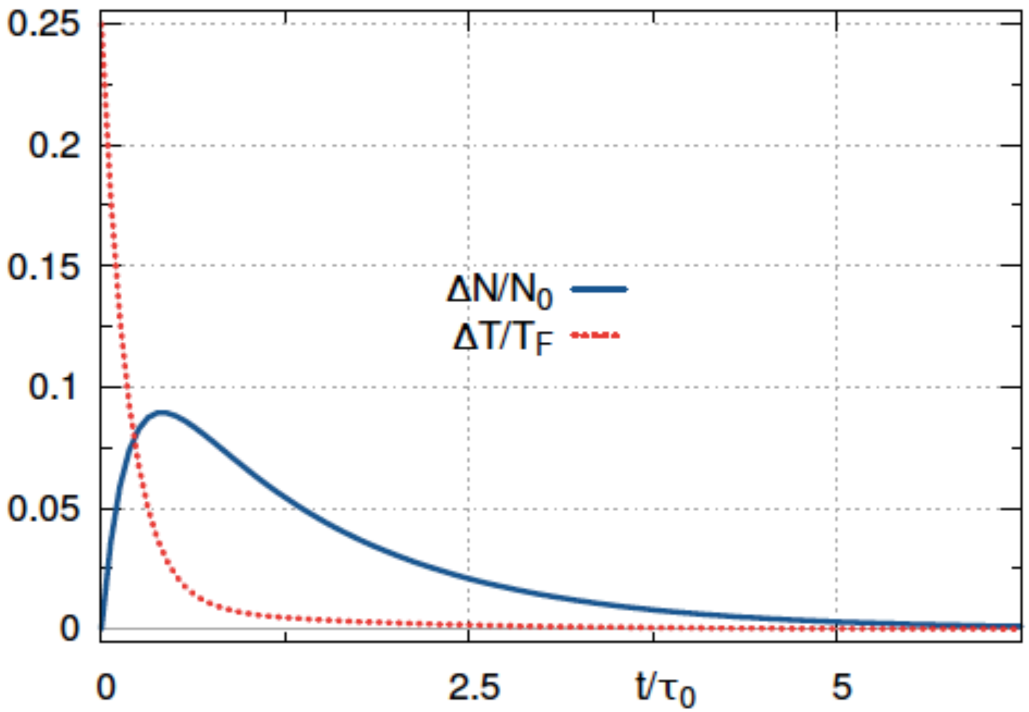
« Smoking-gun » for thermoelectric effects: the theoretical proposal arXiv:1209.3942

Two steps

i. Prepare reservoirs with equal particle number and different temperatures, with closed constriction

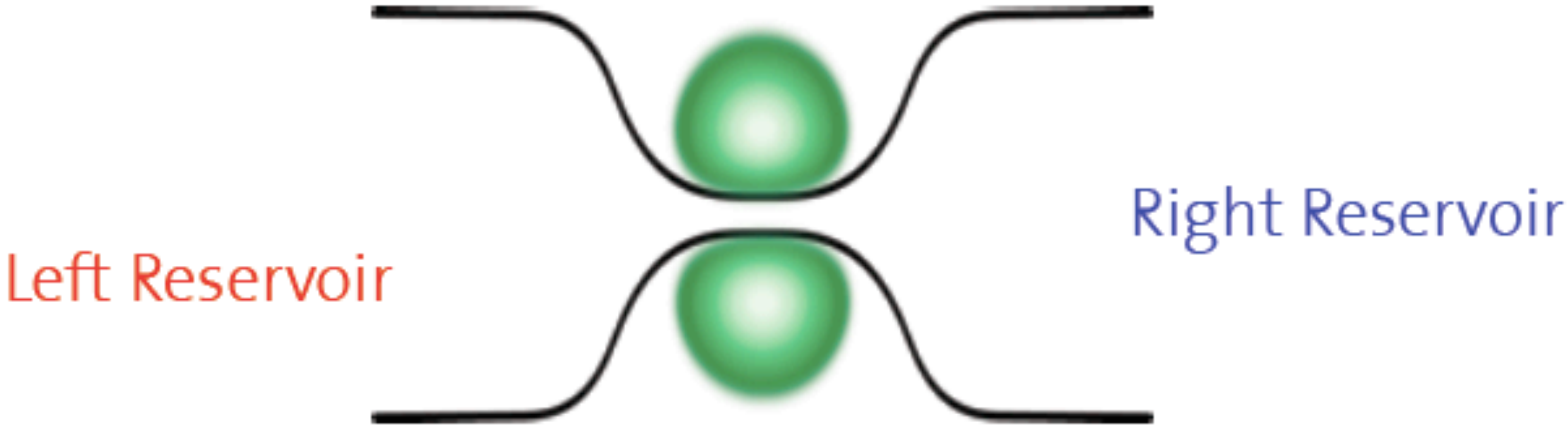
ii. Open the constriction and monitor particle number





During equilibration: particle first flow from hot to cold, then backflow from cold to hot !

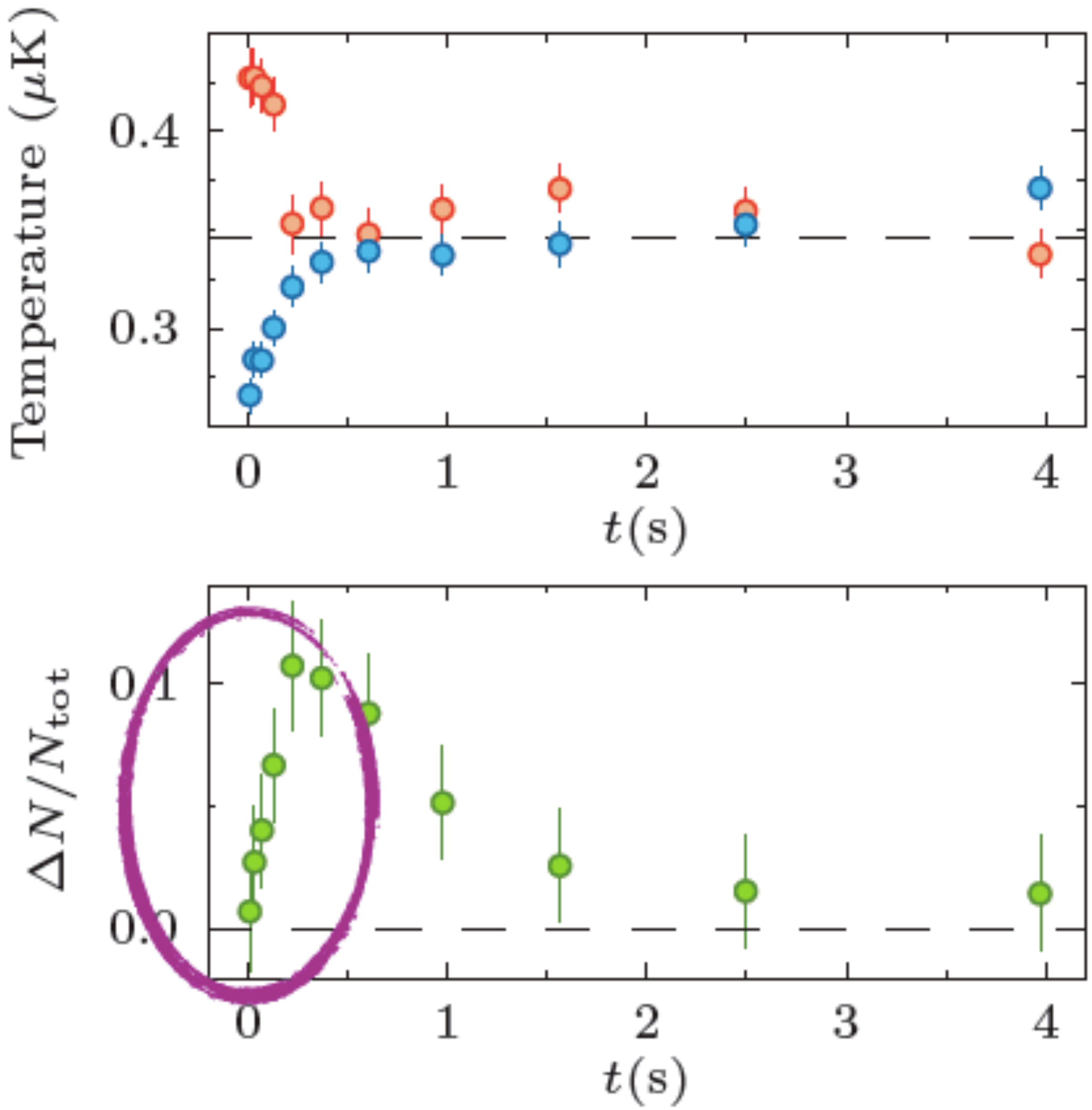
Initial flow against the bias !



$$T_{\mathrm{Left}} > T_{\mathrm{Right}}$$

$$\boxed{\mu_{\mathrm{Left}} < \mu_{\mathrm{Right}}}$$

negative potential bias

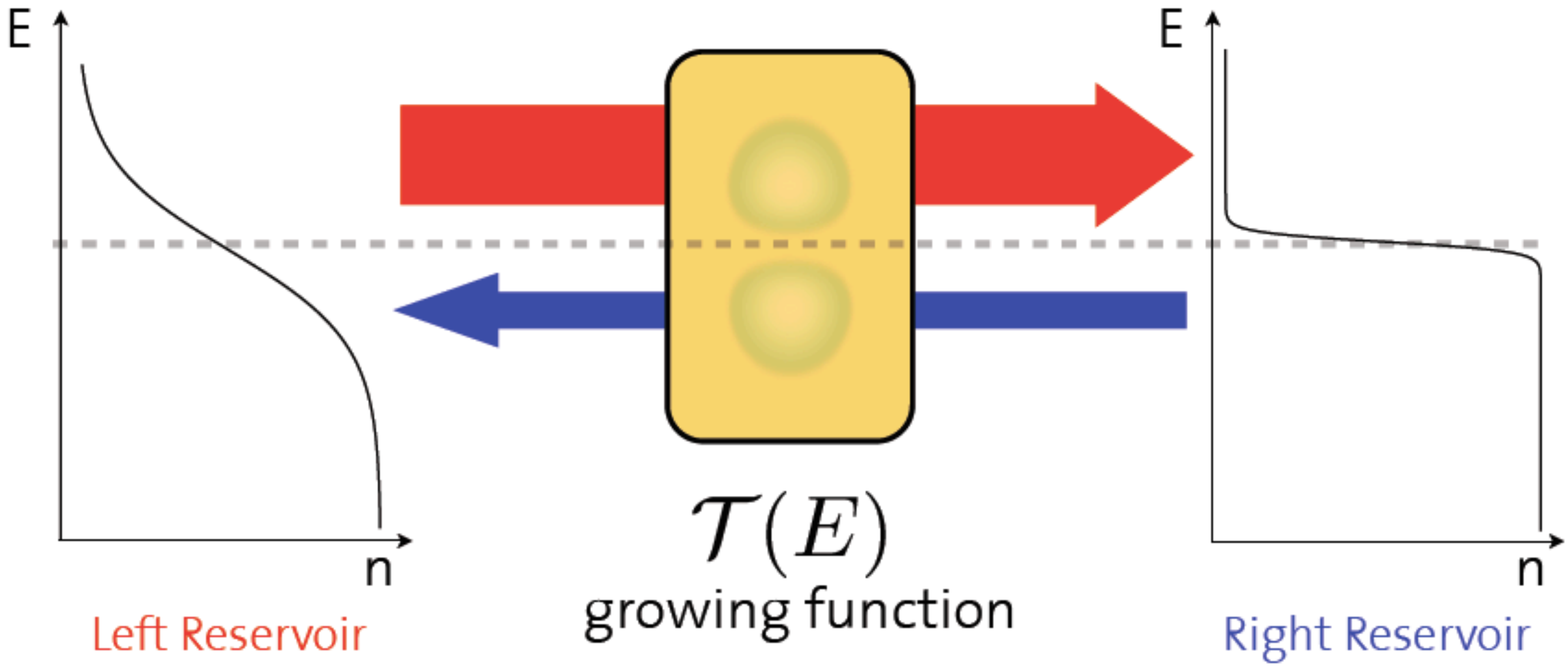


Entropy flow from hot to cold

Particle flow from hot to cold

...flows against the potential bias...

Simple physical picture: transmission increases with energy and more high-energy states are populated in hot reservoir



Thermoelectric transport through a cold atomic gas constriction - Quantitative theory -

Thermodynamics of the reservoirs: non-diagonal terms

$$\begin{pmatrix} \Delta N \\ \Delta S \end{pmatrix} = \underline{K} \begin{pmatrix} \Delta \mu \\ \Delta T \end{pmatrix} = \begin{pmatrix} \kappa & \kappa\alpha_r \\ \kappa\alpha_r & C_\mu/T \end{pmatrix} \begin{pmatrix} \Delta \mu \\ \Delta T \end{pmatrix}$$

$$K_{11} \equiv \kappa = \frac{\partial N}{\partial \mu}\Big|_T = -\frac{\partial^2 \Omega}{\partial \mu^2}$$

$$K_{12} = K_{21} \equiv \alpha_r \kappa = \frac{\partial N}{\partial T}\Big|_\mu = -\frac{\partial^2 \Omega}{\partial \mu \partial T} = \frac{\partial S}{\partial \mu}\Big|_T$$

$$K_{22} \equiv \frac{C_\mu}{T} = \frac{\partial S}{\partial T}\Big|_\mu = -\frac{\partial^2 \Omega}{\partial T^2}$$

Grand-potential: $\Omega \equiv -k_B T \ln Z_{gc}$, $S = -\frac{\partial \Omega}{\partial T}\Big|_\mu$, $N = -\frac{\partial \Omega}{\partial \mu}\Big|_T$

Expressions for a free Fermi gas:

For a free Fermi gas, these coefficients are easily calculated from:

$$N(\mu,T) = \int d\varepsilon D(\varepsilon)\, f\left(\frac{\varepsilon-\mu}{k_B T}\right)$$

$$S(\mu,T) = -k_B \int d\varepsilon D(\varepsilon)\, [f \ln f + (1-f)\ln(1-f)]$$

with $D(\varepsilon)$ the density of states. This leads to:

$$K_{11} = J_0 \ , \ \ K_{12} = K_{21} = k_B J_1 \ , \ \ K_{22} = k_B^2 J_2$$

where J_n are integrals with the *dimension of energy*:

$$J_n = \int d\varepsilon D(\varepsilon) \left(\frac{\varepsilon-\mu}{k_B T}\right)^n \left(-\frac{\partial f}{\partial \varepsilon}\right)$$

Note formal similarity with the expression of the Onsager coefficients for transport ! Transport function → DOS

Same (general) constraints apply (around equilibrium state):

$$K_{11} \geq 0\,,\ K_{22} \geq 0\,,\ \det K \geq 0$$

$$\kappa = \left.\frac{\partial N}{\partial \mu}\right|_T \quad = \quad \int_0^\infty d\varepsilon\, g_r(\varepsilon) \left(-\frac{\partial f}{\partial \varepsilon}\right)$$

$$T\alpha_r\kappa = \left.\frac{\partial N}{\partial T}\right|_\mu = \left.\frac{\partial S}{\partial \mu}\right|_T \quad = \quad \int_0^\infty d\varepsilon\, g_r(\varepsilon)\,(\varepsilon - \mu) \left(-\frac{\partial f}{\partial \varepsilon}\right)$$

$$\frac{C_N}{T} + \kappa\alpha_r^2 \quad = \quad \int_0^\infty d\varepsilon\, g_r(\varepsilon)\,(\varepsilon - \mu)^2 \left(-\frac{\partial f}{\partial \varepsilon}\right)$$

Note: $\alpha_r > 0$ for a Fermi gas with DOS growing with energy

$$\ell \equiv \frac{C_N/T}{\kappa} > 0$$

Thermodynamic analogue of Lorenz number

Transport coefficients of channel

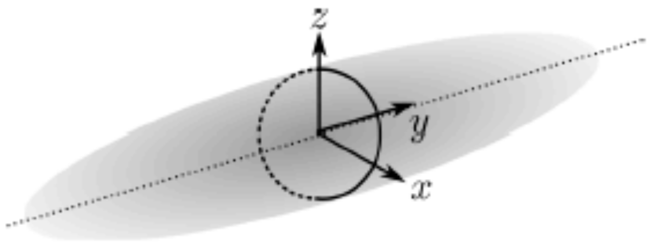
$$
\begin{aligned}
G &= \frac{1}{h}\int_0^\infty d\varepsilon\,\Phi(\varepsilon)\left(-\frac{\partial f}{\partial \varepsilon}\right) \\
T\alpha_{ch}G &= \frac{1}{h}\int_0^\infty d\varepsilon\,\Phi(\varepsilon)\,(\varepsilon-\mu)\left(-\frac{\partial f}{\partial \varepsilon}\right) \\
\frac{G_T}{T}+G\alpha_{ch}^2 &= \frac{1}{h}\int_0^\infty d\varepsilon\,\Phi(\varepsilon)\,(\varepsilon-\mu)^2\left(-\frac{\partial f}{\partial \varepsilon}\right)
\end{aligned}
$$

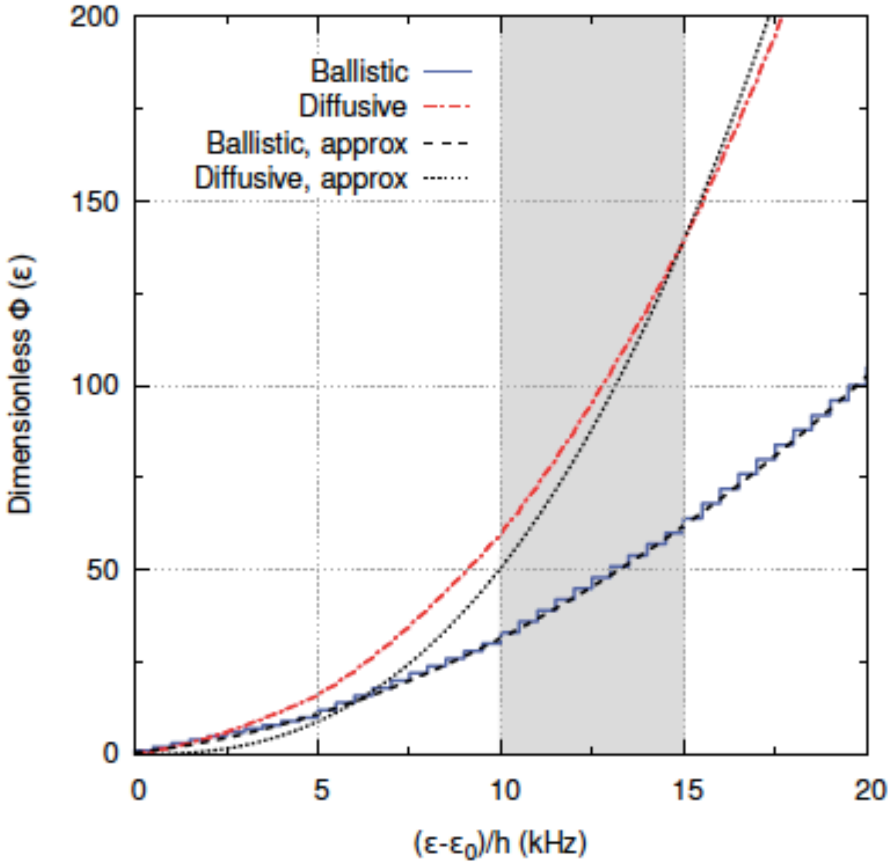
$$L \equiv \frac{G_T/T}{G}$$ Lorenz number

Transmission coefficient

$$\begin{aligned}\Phi(\varepsilon) &= \sum_{n_z=0}^{\infty}\sum_{n_x=0}^{\infty}\int_0^{\infty} dk_y \, \frac{\hbar k_y}{M}\mathcal{T}(k_y) \\ &\cdot\delta\left(\varepsilon - \hbar\omega_x(n_x+1/2) - \hbar\omega_z(n_z+1/2) - \frac{\hbar^2 k_y^2}{2M}\right) \\ &= \sum_{n_z=0}^{\infty}\sum_{n_x=0}^{\infty}\mathcal{T}(\varepsilon - \hbar\omega_x(n_x+1/2) - \hbar\omega_z(n_z+1/2)) \\ &\cdot\vartheta(\varepsilon - \hbar\omega_x(n_x+1/2) - \hbar\omega_z(n_z+1/2)) \,,\end{aligned}$$

$$\Phi(\varepsilon) = \sum_{n_x, n_z} \int dk_y \frac{\hbar k_y}{M} T(k) \delta\left(\varepsilon - \frac{\hbar^2 k_y^2}{2M} - n_z h\nu_z - n_x h\nu_x\right)$$





Here $\mathcal{T}(\varepsilon) \to \Phi(\varepsilon)$

Ballistic : $T(k) = 1,$

$$\Phi(\varepsilon) \simeq \frac{1}{2}\left(1 + \left\lfloor \frac{\varepsilon - \varepsilon_0}{h\nu_x} \right\rfloor\right)\left(1 + \left\lfloor \frac{\varepsilon - \varepsilon_0}{h\nu_z} \right\rfloor\right)$$

Diffusive : $T(k) \simeq \frac{l(k)}{\mathscr{L}},$

$$\Phi(\varepsilon) \simeq \frac{4}{15} \frac{\tau_s}{\mathscr{L}} \sqrt{\frac{2}{M}} \frac{(\varepsilon - \varepsilon_0)^{5/2}}{h\nu_x h\nu_z}$$

For $\nu_z = 5\ kHz,\ \nu_x = 0.5\ kHz$

Currents and Discharge

$$\begin{pmatrix} I_N \\ I_S \end{pmatrix} = -G \begin{pmatrix} 1 & \alpha_{\mathrm{ch}} \\ \alpha_{\mathrm{ch}} & L + \alpha_{\mathrm{ch}}^2 \end{pmatrix} \begin{pmatrix} \mu_{\mathrm{c}} - \mu_{\mathrm{h}} \\ T_{\mathrm{c}} - T_{\mathrm{h}} \end{pmatrix}$$

$$\tau_0 \frac{d}{dt} \begin{pmatrix} \Delta N \\ \Delta T \end{pmatrix} =$$

$$- \begin{pmatrix} 1 & -\kappa(\alpha_{\mathrm{r}} - \alpha_{\mathrm{ch}}) \\ -\dfrac{\alpha_{\mathrm{r}} - \alpha_{\mathrm{ch}}}{\ell\kappa} & \dfrac{L + (\alpha_{\mathrm{r}} - \alpha_{\mathrm{ch}})^2}{\ell} \end{pmatrix} \begin{pmatrix} \Delta N \\ \Delta T \end{pmatrix}$$

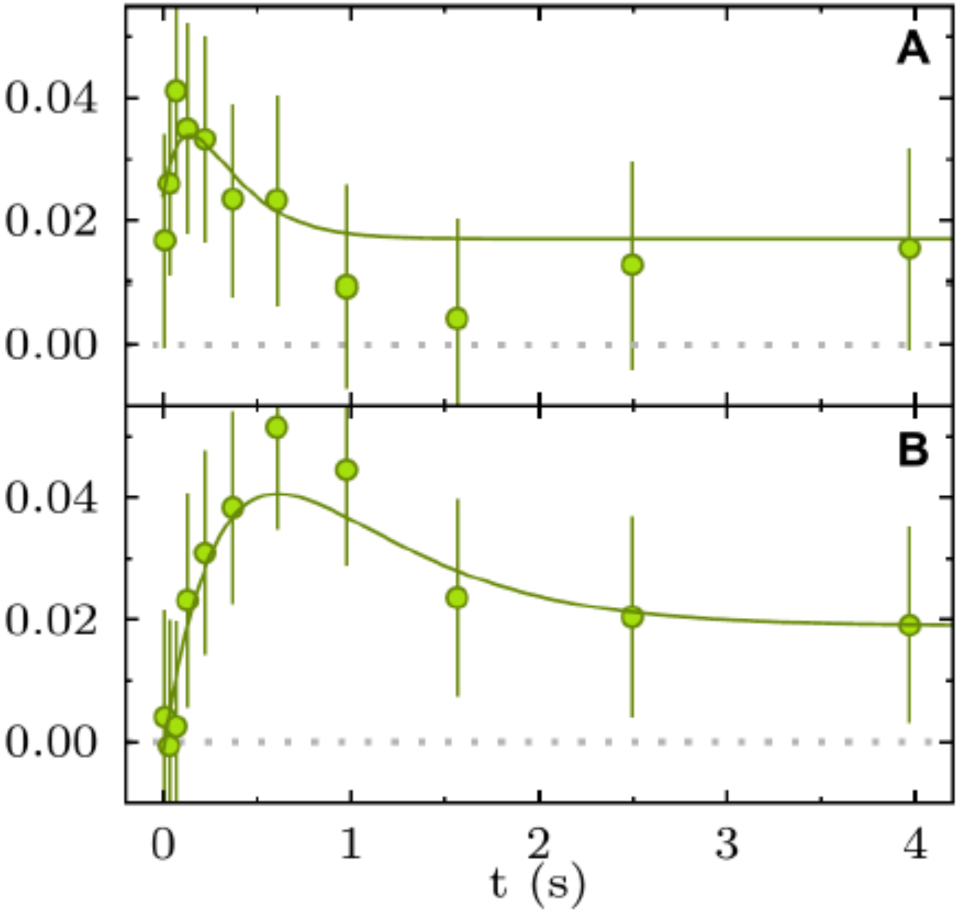
$$
\begin{aligned}
(N_c - N_h)(t) &= \left\{ \frac{1}{2}\left[e^{-t/\tau_-} + e^{-t/\tau_+}\right] + \left[1 + \frac{L+\alpha^2}{\ell}\right] \frac{e^{-t/\tau_-} - e^{-t/\tau_+}}{2(\lambda_+ - \lambda_-)} \right\} \Delta N_0 \\
&\quad + \frac{\alpha\kappa}{\lambda_+ - \lambda_-}\left[e^{-t/\tau_-} - e^{-t/\tau_+}\right] \Delta T_0 \qquad \text{(S14)} \\
(T_c - T_h)(t) &= \left\{ \frac{1}{2}\left[e^{-t/\tau_-} + e^{-t/\tau_+}\right] - \left[\frac{L+\alpha^2}{\ell} - 1\right] \frac{e^{-t/\tau_-} - e^{-t/\tau_+}}{2(\lambda_+ - \lambda_-)} \right\} \Delta T_0 \\
&\quad + \frac{\alpha}{\ell\kappa(\lambda_+ - \lambda_-)}\left[e^{-t/\tau_-} - e^{-t/\tau_+}\right] \Delta N_0 \qquad \text{(S15)}
\end{aligned}
$$

The initial temperature difference and particle imbalance are denoted by ΔT_0 and ΔN_0, respectively. The inverse time-scales $\tau_\pm^{-1} = \tau_0^{-1}\lambda_\pm$ are given by the eigenvalues of the transport matrix $\underline{\Lambda}$

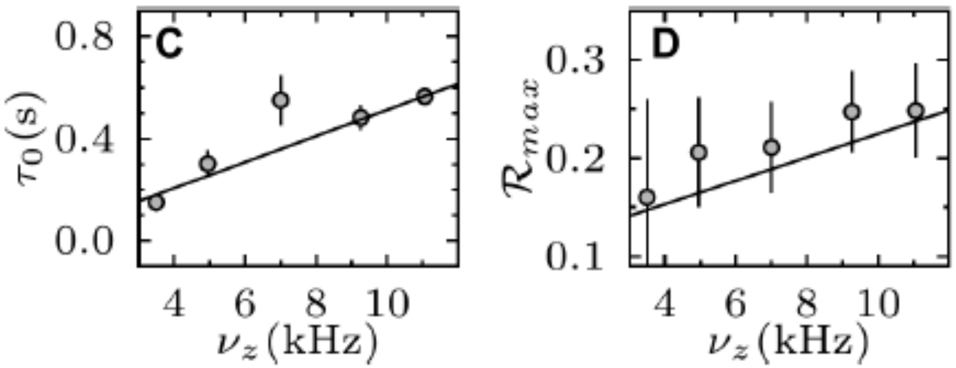
$$
\lambda_\pm = \frac{1}{2}\left(1 + \frac{L+\alpha^2}{\ell}\right) \pm \sqrt{\frac{\alpha^2}{\ell} + \left(\frac{1}{2} - \frac{L+\alpha^2}{2\ell}\right)^2}. \qquad \text{(S16)}
$$

All the effective transport coefficients are ratios that depend only on the variable $\frac{\mu}{k_B T}$.

Comparison data-theory in the ballistic case

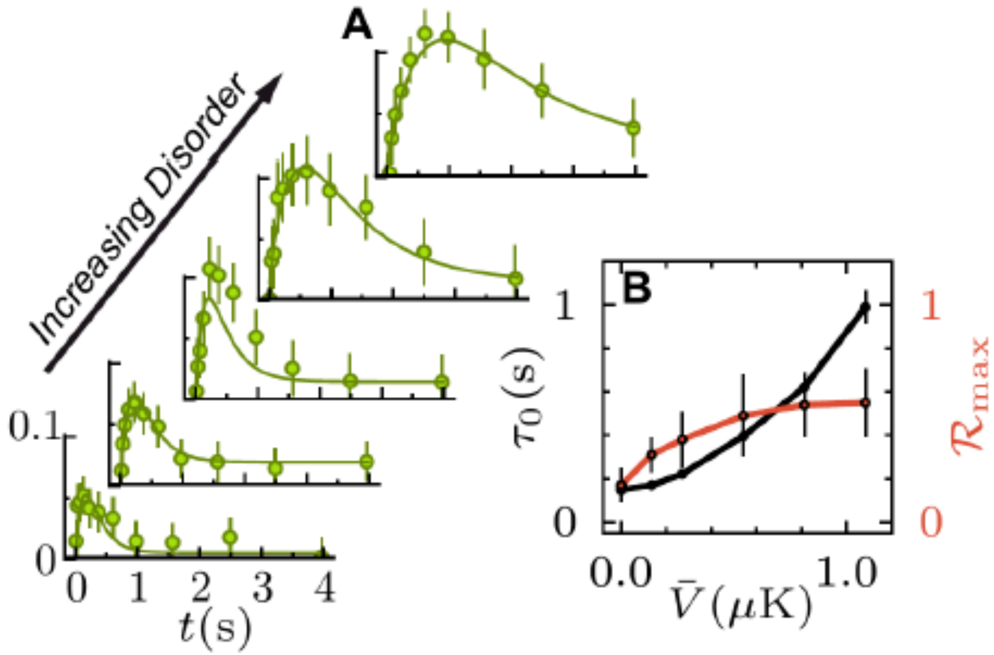


Particle imbalance vs. time for :
A 3.5 kHz and **B** 9.3 kHz



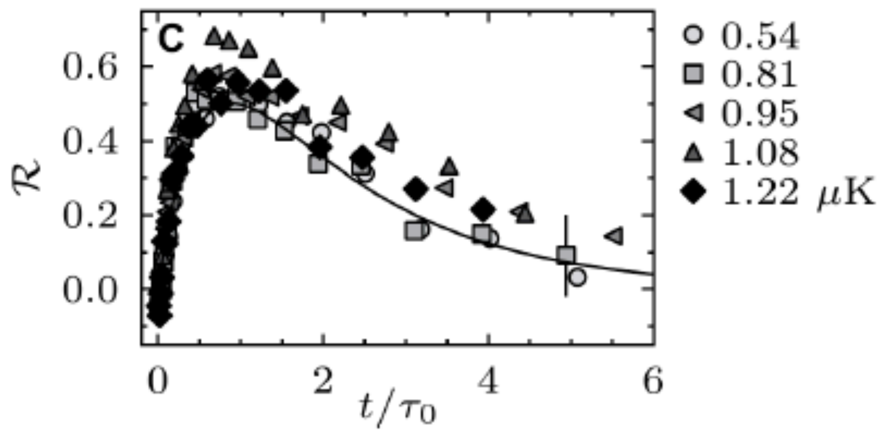
Characteristics vs confinement
(ab-initio predictions):

- τ_0: Good agreement with theory vs ν_z
- Response $\mathscr{R}_{max} = \dfrac{T_F \Delta N(max)}{N_{tot} \Delta T_0}$

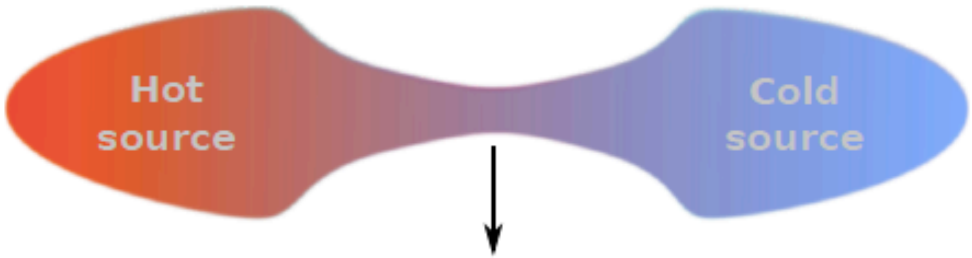


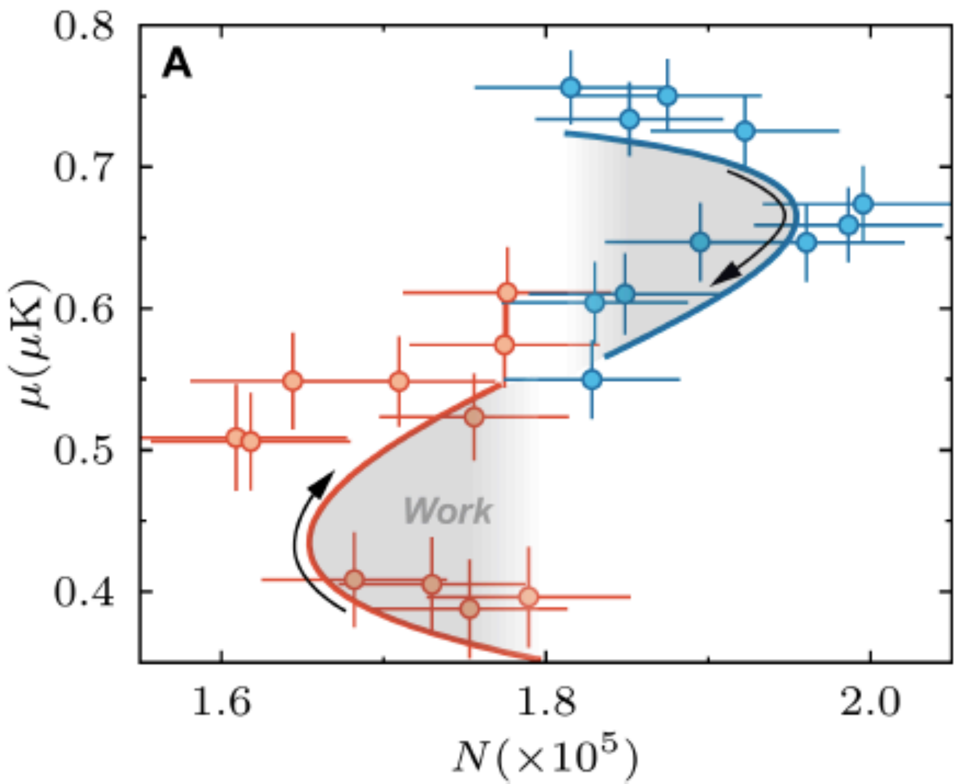
- Thermoelectric effect grows with disorder
- **At strong disorder**
  The effect saturates : Constant τ_S ✓
- Seebeck coefficient ≠ Conductivity

Rescaled evolution of particle imbalance :
Universal regime



The setup as a heat engine





- Reservoirs ≡ Hot and Cold sources
- Channel : converts heat into (chemical) work

- Evolution in the $\mu - N$ plane
- Access to thermodynamic evolution
  ⇒ Extraction of work

QUESTION : Efficiency of the process ?

Efficiency

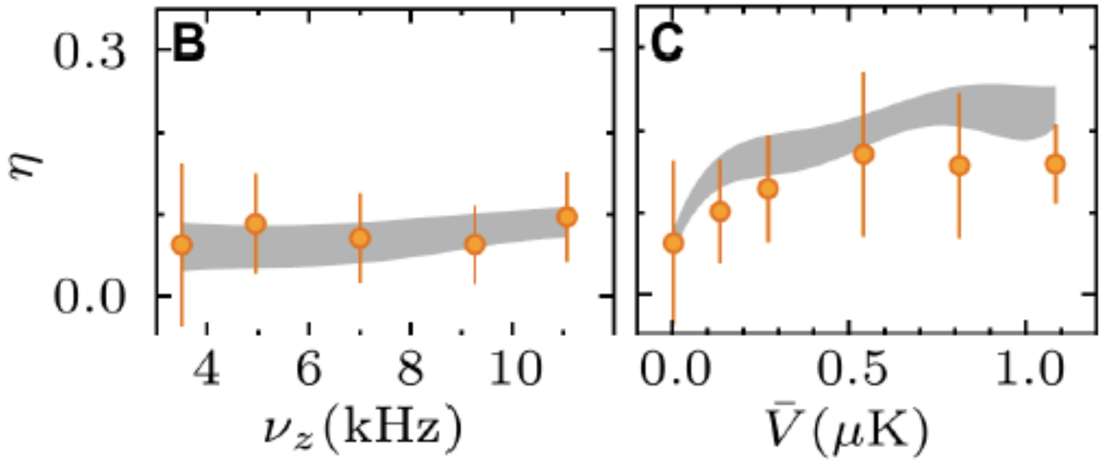
No DC regime $\Rightarrow$ compare work, not power

Expression for the efficiency : compare output chemical work to heat

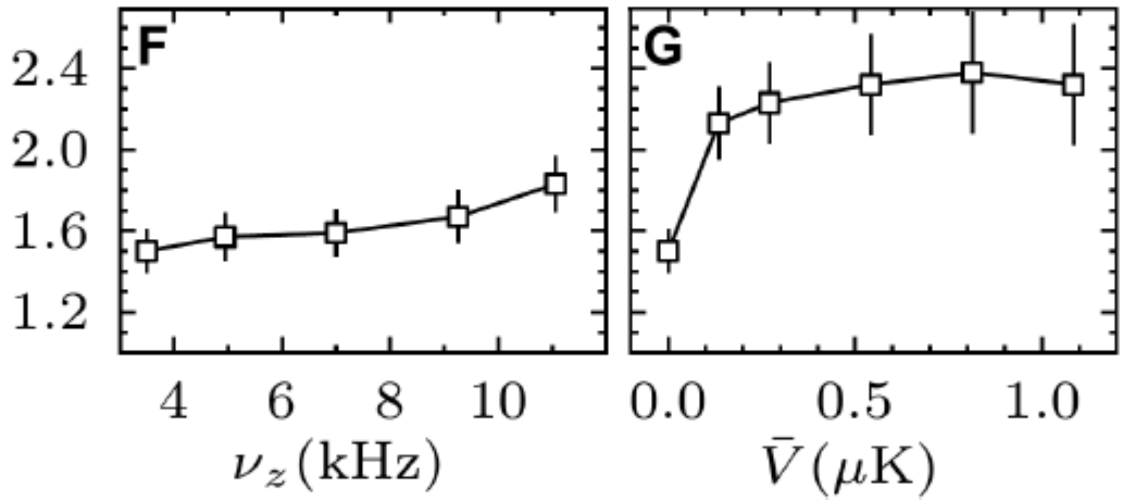
$$\eta \equiv \frac{Work}{Heat} = \frac{\int_{evolution} \Delta\mu \cdot d\Delta N}{\int_{evolution} \Delta T \cdot d\Delta S} = \frac{\int_0^\infty dt \Delta\mu \cdot I_N}{\int_0^\infty dt \Delta T \cdot I_S}$$

Solution to transport equations $\Rightarrow$ η **in terms of transport coefficients :** ℓ, L, α

$$\boxed{\eta = \frac{-\alpha\alpha_r}{\ell + L + \alpha^2 - \alpha\alpha_r}}$$



- η grows with confinement and speckle
- Slow dynamics $\Rightarrow$ most efficient



For the channel only :

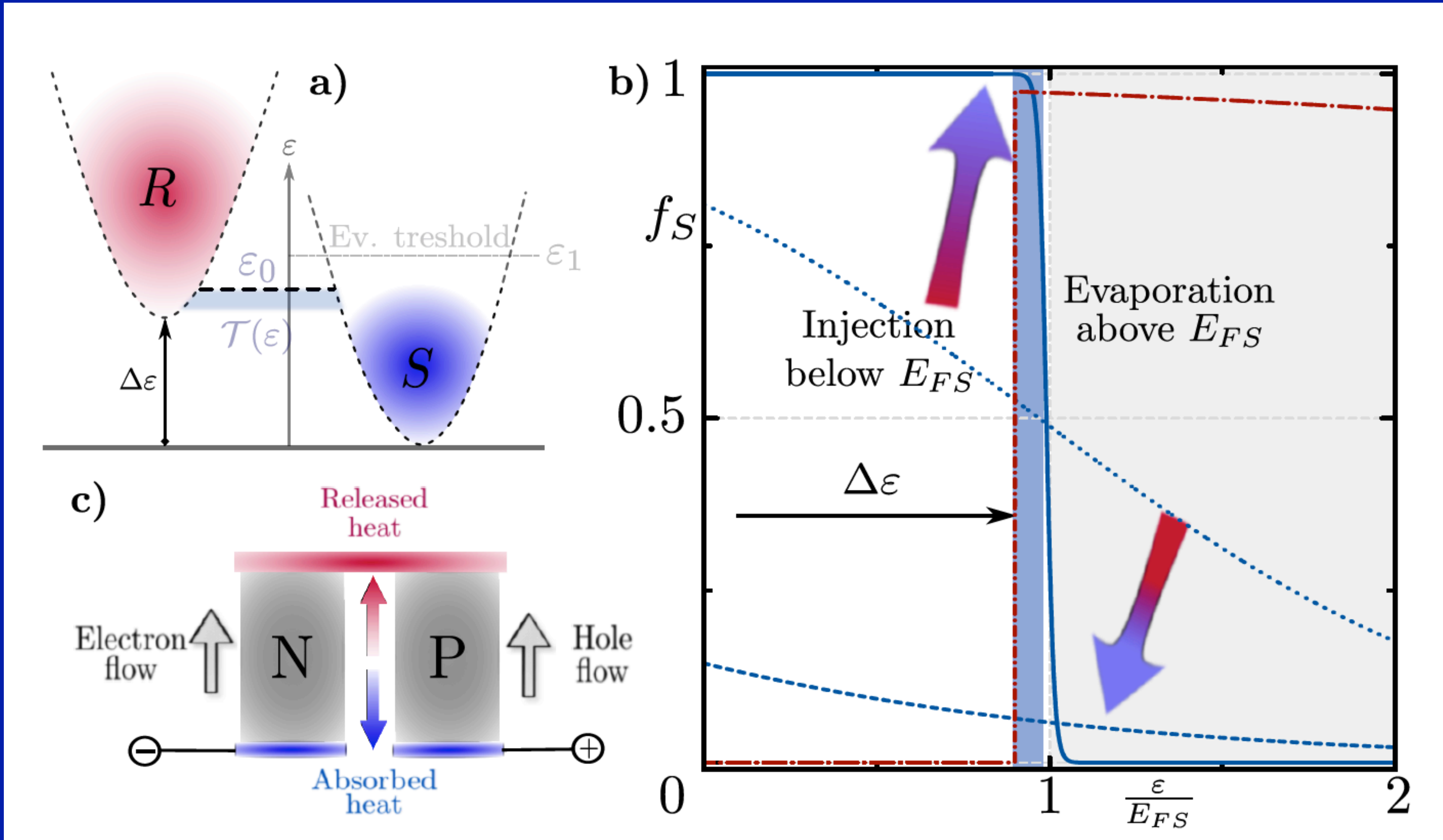
Thermoelectric figure of merit $ZT = \frac{\alpha_{ch}^2}{L}$

$ZT \rightarrow 2.4$: > than any material

Prospects: Peltier-cooling of atomic gases

(= Evaporating particles AND holes !)

C.Grenier, C.Kollath & AG – Phys Rev Lett 113, 200601 (2014)



$$
\begin{aligned}
I_N &= \frac{1}{h}\int d\varepsilon \mathcal{T}(\varepsilon)[f_R(\varepsilon) - f_S(\varepsilon)] \\
&= -\int d\varepsilon g_R(\varepsilon)\frac{df_R}{dt}(\varepsilon) = \int d\varepsilon g_S(\varepsilon)\frac{df_S}{dt}(\varepsilon).
\end{aligned}
$$

In this expression, $g_R(\varepsilon) = (\varepsilon - \Delta\varepsilon)^2/((h\nu)^3)\vartheta(\epsilon - \Delta\varepsilon)$ and $g_S(\varepsilon) = \varepsilon^2/((h\nu)^3)\vartheta(\varepsilon)$ are the density of states in the reservoir and in the system, with ϑ the Heaviside function. The coupled evolution of the two distribution functions is, thus, given by

$$
g_R(\varepsilon)\frac{df_R(\varepsilon)}{dt} = -\frac{\mathcal{T}(\varepsilon)}{h}[f_R - f_S](\varepsilon), \tag{1}
$$

$$
g_S(\varepsilon)\frac{df_S(\varepsilon)}{dt} = \frac{\mathcal{T}(\varepsilon)}{h}[f_R - f_S](\varepsilon) - \Gamma_{\text{ev}}(\varepsilon)g_S(\varepsilon)f_S(\varepsilon). \tag{2}
$$

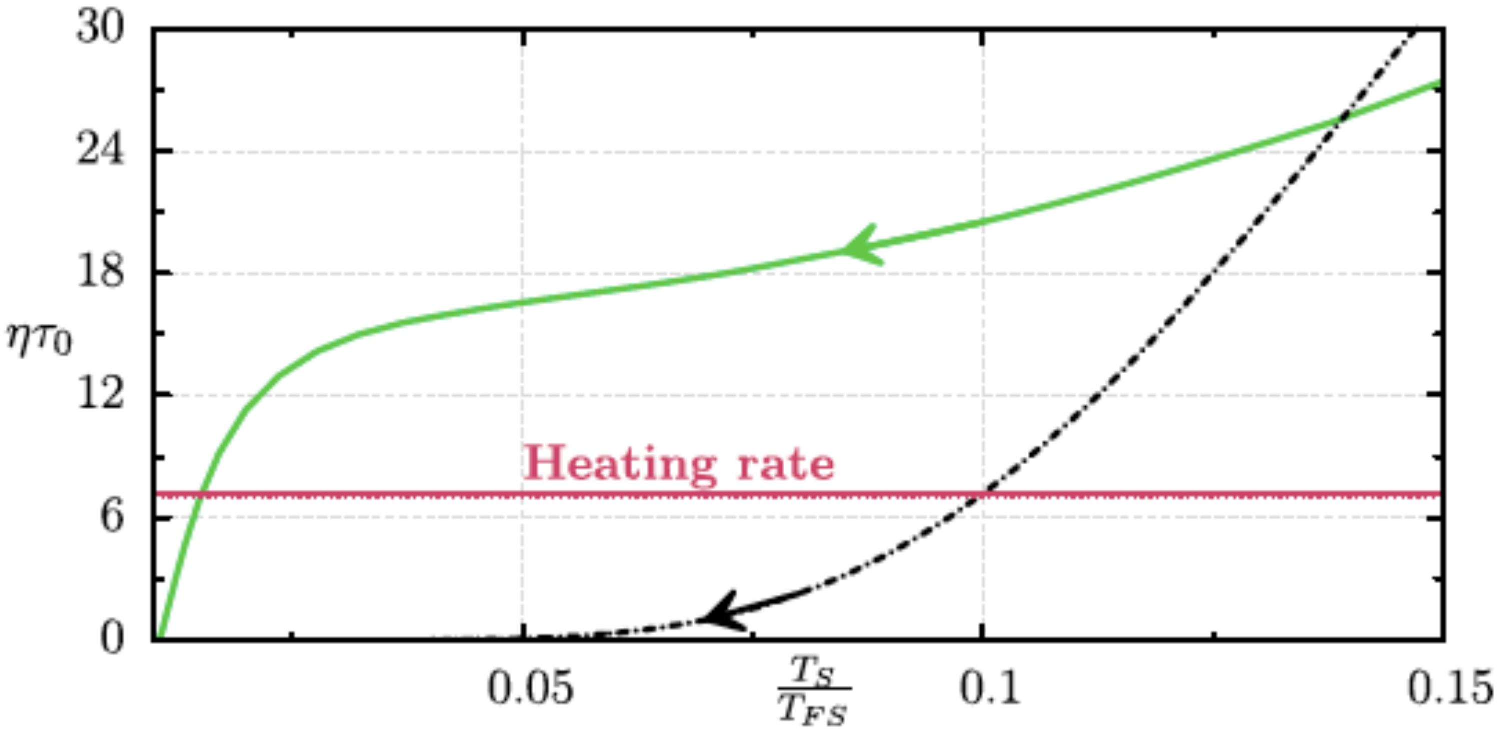


FIG. 3 (color online). Dimensionless cooling rate $\eta(t)\tau_0$ as a function of T_S/T_{FS}, for the same parameters as in Fig. 2. The black dashed curve is for evaporative cooling only. Arrows indicate the direction of the time evolution. The horizontal (red) line indicates a typical heating rate (see, e.g., [43]) limiting these cooling processes.

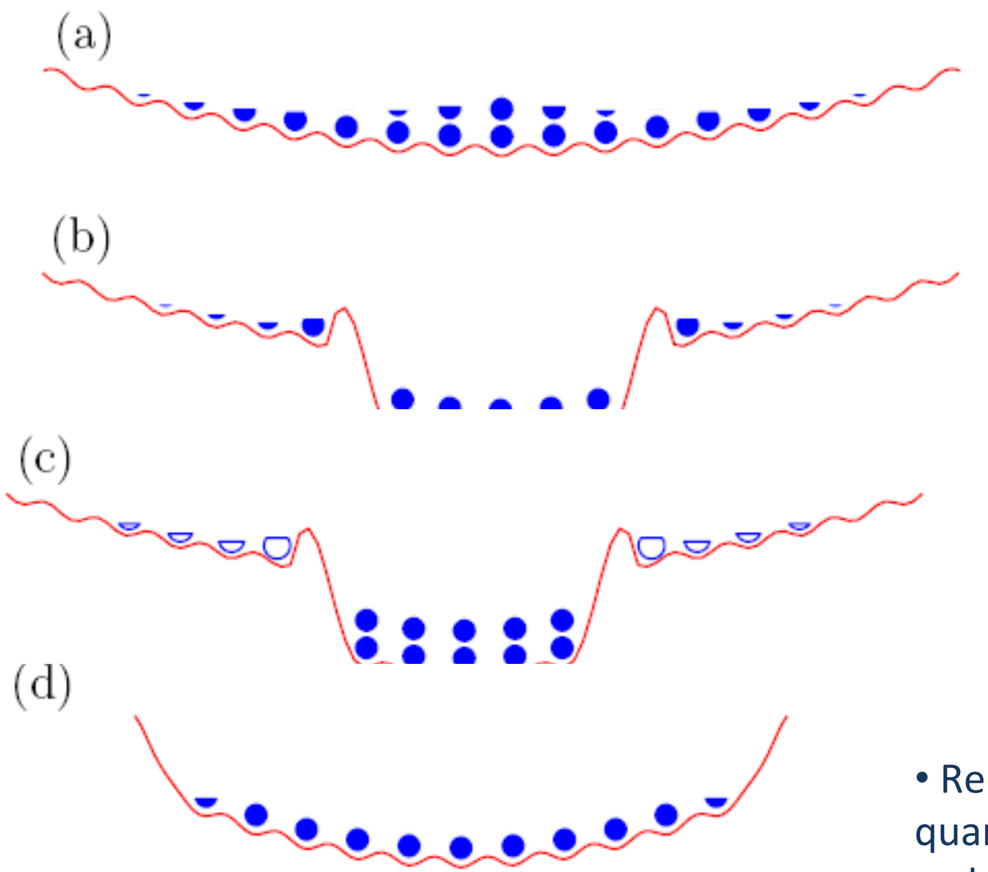
Other Cooling procedures

RAPID COMMUNICATIONS

PHYSICAL REVIEW A 79, 061601(R) (2009)

Cooling fermionic atoms in optical lattices by shaping the confinement

Jean-Sébastien Bernier,[1] Corinna Kollath,[1] Antoine Georges,[1] Lorenzo De Leo,[1] Fabrice Gerbier,[2] Christophe Salomon,[2] and Michael Köhl[3]



- Fermionic atoms pre-cooled in a dipole trap
- Apply a 3d optical lattice potential
- Done in the presence of weak U

- Create a dimple in the middle of the trap
- Raise potential barriers

- Remove the atoms from the storage region

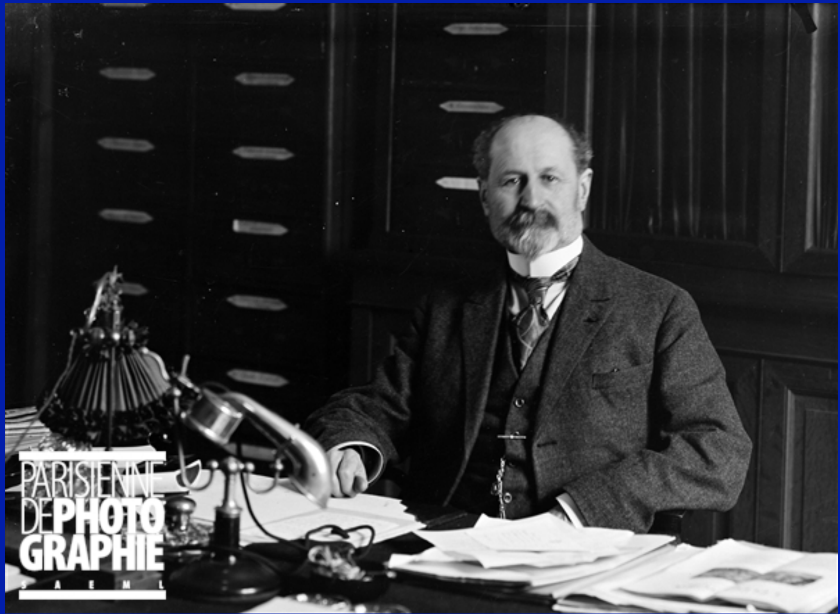
- Relax the core region to the desired quantum
  phase

MERCI DE VOTRE PARTICIPATION AU COURS !

… Une note personnelle…

Campus de l'Université
Joseph Fourier

Il y a 123 ans…



Paul Janet 1863-1937

RÉPUBLIQUE FRANÇAISE

ACADÉMIE DE GRENOBLE

ANNÉE SCOLAIRE 1892-1893

FACULTÉ Des Sciences DE GRENOBLE

COURS D'ÉLECTRICITÉ INDUSTRIELLE

M. JANET, DOCTEUR ÈS SCIENCES

CHARGÉ DU COURS

COURS PUBLIC. — Le JEUDI à 3 h. 1/2. — Etude approfondie des machines dynamo-électriques; caractéristiques; propriétés magnétiques et électriques des matériaux; projet d'une dynamo à courant continu de puissance donnée.

Transport de l'Energie mécanique; traction électrique; tramways et locomotives. Eclairage; lampes à incandescence; lampes à arc. Canalisation et distribution de l'énergie électrique; stations centrales; systèmes divers.

CONFÉRENCE PRATIQUE. — Le VENDREDI à 10 h. et demie. Mesures électriques industrielles.

Le Cours public s'ouvrira le JEUDI 12 JANVIER 1893

Grenoble, le 27 décembre 1892.

Le Doyen de la faculté des Sciences,

RAOULT.

Vu et approuvé :
Le Recteur de l'Académie,
G. BIZOS

Physical Interpretation of the Coefficients of the Thermodynamic Matrix :

K_{11} ~ Compressibility :

Pressure in grand-canonical ensemble, given extensivity of $\Omega = V\omega(\mu, T)$:

$$p(\mu, T) = -\frac{\partial \Omega}{\partial V} = -\frac{1}{V}\Omega(\mu, T)$$

With $n \equiv N/V$ the density, the equation of state will be given by:

$$p(n, T) = p_{gc}\left[\mu(n, T), T\right]$$

From which it follows that:

$$\frac{\partial p}{\partial n}|_T = \frac{\partial p}{\partial \mu}|_T \frac{\partial \mu}{\partial n}|_T = n\frac{\partial \mu}{\partial n}|_T$$

A variation of volume corresponds to (from $n = N/V$):

$$\frac{\delta V}{V} = -\frac{\delta n}{n}$$

The isothermal compressibility is usually defined as:

$$\kappa_T \equiv -\frac{1}{V}\frac{\partial V}{\partial p}|_T = \frac{1}{n}\frac{\partial n}{\partial p}|_T$$

So that:

$$\boxed{\kappa_T = \frac{1}{n^2}\frac{\partial n}{\partial \mu}|_T = \frac{1}{n^2 V}K_{11} \equiv \frac{1}{n^2 V}\kappa}$$

Must be positive (otherwise phase separation)

K_{12} ~ Thermal expansion coefficient at constant μ:

$$\alpha_\mu \equiv \frac{1}{V}\frac{\partial V}{\partial T}|_\mu = -\frac{1}{n}\frac{\partial n}{\partial T}|_\mu$$

$$\boxed{K_{12} \equiv \kappa\alpha_r = V\frac{\partial n}{\partial T}|_\mu = -N\alpha_\mu}$$

Importantly for the following, this coefficient can be positive or negative. Alternatively its sign can be related to the variation of μ as a function of temperature at constant density:

We note that a variation at constant density implies:

$$K_{11}\delta\mu + K_{12}\delta T = 0 \Rightarrow \frac{\partial \mu}{\partial T}|_n = -\frac{K_{12}}{K_{11}} = -\alpha_r$$

Hence:

$$\boxed{\frac{\partial n}{\partial T}|_\mu = -n^2\kappa_T\frac{\partial \mu}{\partial T}|_n \quad \left(= n^2\kappa_T\alpha_r = \frac{1}{V}K_{12}\right)}$$

μ decreases with T → $\alpha_r > 0$ → Δn , ΔT same sign at constant μ

We have seen that $K_{22} = C_\mu / T$

At constant density: “stopping condition”

$$K_{11}\delta\mu + K_{12}\delta T = 0 \Rightarrow \delta\mu = -\frac{K_{12}}{K_{11}}\delta T$$

$$\delta S = K_{21}\delta\mu + K_{22}\delta T = \left(K_{22} - \frac{K_{12}^2}{K_{11}}\right)\delta T = \frac{\mathrm{det}K}{K_{11}}\delta T$$

cf. analogy with thermal conductivity calculation

$$\boxed{\mathrm{det}K = \kappa\frac{C_N}{T} \geq 0}$$

Positivity of K_{22} and det K follows from the second principle of thermodynamics