

*“Enseigner la recherche en train de se faire”*



*Chaire de  
Physique de la Matière Condensée*

Seconde partie:  
Quelques questions liées au transport dans les  
matériaux à fortes corrélations électroniques

**Les mercredis dans l'amphithéâtre Maurice Halbwachs  
11, place Marcelin Berthelot 75005 Paris  
Cours à 14h30 - Séminaire à 15h45**

Antoine Georges

Cycle 2011-2012  
Partie II: 30/05, 06/06, 13/06/2012

# Séance du 6 juin 2012

- Séminaire : 15h45 -

*Nigel Hussey (University of Bristol)*

*High-temperature superconductivity and the Catch-22 conundrum*

# OUTLINE

- May, 30: Phenomenology, simple theory background. Mainly raise questions.
- June, 6: Answer some of these questions for a doped Mott insulator (simplest 1-site DMFT description, recent results)
- June, 13 (time permitting): some notions on thermoelectric properties

# In memoriam Bernard Coqblin



Directeur de Recherche au CNRS, LPS-Orsay  
Honorary Professor, Polish Academy of Sciences, Wroclaw  
Dr Honoris Causa, Univ. Federal do Rio Grande do Sul, Brasil

PHYSICAL REVIEW

VOLUME 185, NUMBER 2

10 SEPTEMBER 1969

## Exchange Interaction in Alloys with Cerium Impurities\*

B. COQBLIN† AND J. R. SCHRIEFFER

*Department of Physics, University of Pennsylvania, Philadelphia, Pennsylvania 19104*

(Received 4 March 1969)

# Ex: Ruthenates (remember: 3 FS sheets)

ab-plane:

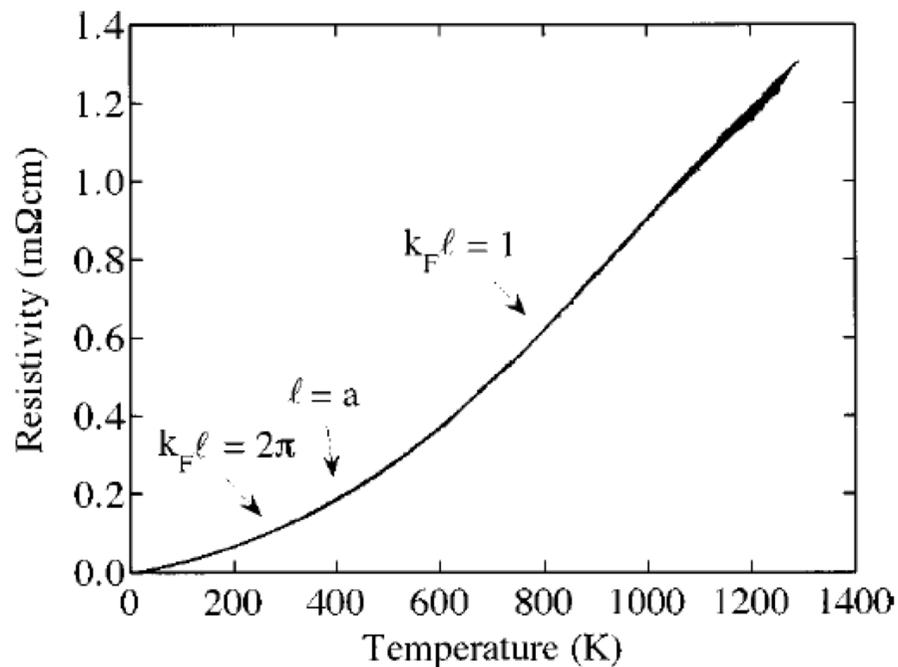


FIG. 1. The in-plane resistivity of Sr<sub>2</sub>RuO<sub>4</sub> from 4 to 1300 K. Three criteria for the Mott-Ioffe-Regel limit are marked on the graph, and there is no sign of resistivity saturation, so Sr<sub>2</sub>RuO<sub>4</sub> is a “bad metal” at high temperatures, even though it is known to be a very good metal at low temperatures.

- resistivity  
does cross IRM value

- Nothing dramatic is seen  
in  $\rho$  upon crossing IRM

Tyler, Maeno, McKenzie  
PRB 58 R10107 (1998)

# QUESTIONS :

- How low is  $T_{FL}$  and why ?
- What exactly happens to Landau quasiparticles at  $T_{FL}$  ?
- What are the current carrying entities for  $T_{FL} < T < T_{IRM}$  ?
- Is a Drude description applicable in this regime, despite the absence of Landau QPs ?
- Is there any signature of IRM in some physical observable (ARPES ? Optics ?)

# Why are these questions timely ?

- There is increasing evidence that there are indeed well-defined QPs in cuprates, in nodal regions
- These QPs may even be FL-like at low-enough  $T$ , certainly in overdoped (Hussey) and perhaps also in underdoped (Barisic)
- Quantum oscillations !
- Move away from the quest of infra-red stable NFL fixed points !
- Understand crossover scales, possibly momentum dependent, and physics (e.g. transport, and more) above  $T_{FL}$

# Some answers in a precise context: Doped Mott insulator with single-site DMFT

Recent results by: Xiaoyu Deng (EP), Jernej Mravlje (EP&CDF)  
Rok Zitko (Ljubljana) & AG



Building up on previous work by several authors, see e.g. G.Palsson et al. PRL 80, 475 (1998) and PhD Rutgers; Merino & McKenzie PRB 61, 7996 (2000), Limelette et al. PRL and Science 2003, Uhrig et al. recent papers, etc.

Final expression for conductivity, Kubo-bubble :

$$\begin{aligned} \text{Re } \sigma_{\mu\nu}(\vec{q} = \vec{0}, \omega) &= \\ &= \frac{2\pi e^2}{\hbar} \int d\omega' \frac{f(\omega') - f(\omega' + \omega)}{\omega} \int d\epsilon \Phi_{\mu\nu}(\epsilon) A(\epsilon, \omega') A(\epsilon, \omega' + \omega) \end{aligned}$$

Transport function contains information about **BARE** velocities:

$$\begin{aligned} \Phi_{\mu\nu}(\epsilon) &= \int_{\text{BZ}} \frac{d^d k}{(2\pi)^d} \frac{\partial \epsilon_{\vec{k}}}{\partial k_{\mu}} \frac{\partial \epsilon_{\vec{k}}}{\partial k_{\nu}} \delta(\epsilon - \epsilon_{\vec{k}}) , \\ \Phi(\epsilon) &= \frac{1}{d} \sum_{\mu} \Phi_{\mu\mu}(\epsilon) \end{aligned}$$

I hope I got factors of 2,  $\pi$ , e,  $\hbar$  etc... right !  
Dimensions are OK !

# Why now ?

- Could have been done 20 years ago... in principle (part of it has been explored, some key points were missed though)
- In practice:
- Need highly accurate impurity solvers down to low-T, with excellent resolution at low frequency (calculation of transport is exceedingly delicate)
- Need to handle real frequencies: a challenge to QMC methods

# Algorithms

- NRG (à la Wilson)
- CT-QMC (mostly HYB, also U-exp at hi-T)

E.Gull et al.

REVIEWS OF MODERN PHYSICS, VOLUME 83, APRIL–JUNE 2011

## Continuous-time Monte Carlo methods for quantum impurity models

Allows for analytic continuation using Pade approximants !!  
(M.Ferrero)

<http://ipht.cea.fr/triqs/>



# UNITS

Energy, Temperature, Frequency:

$\frac{1}{2}$  bandwidth  $D$  ( $=1$ ). Think of  $D = 1\text{eV} = 12000\text{ K}$

Note:  $\beta \equiv \frac{D}{k_B T}$

Resistivity

Ioffe-Regel-Mott value  $\sigma_M = \frac{e^2}{\hbar} \frac{\Phi(\epsilon_F)}{\epsilon_F}$ , ( $k_F l = 1$ )

Most calculations shown for  $U/D=4$  ( $>$  Mott MIT  $\sim 3$ )

## Transport function for quasi-2D free electrons :

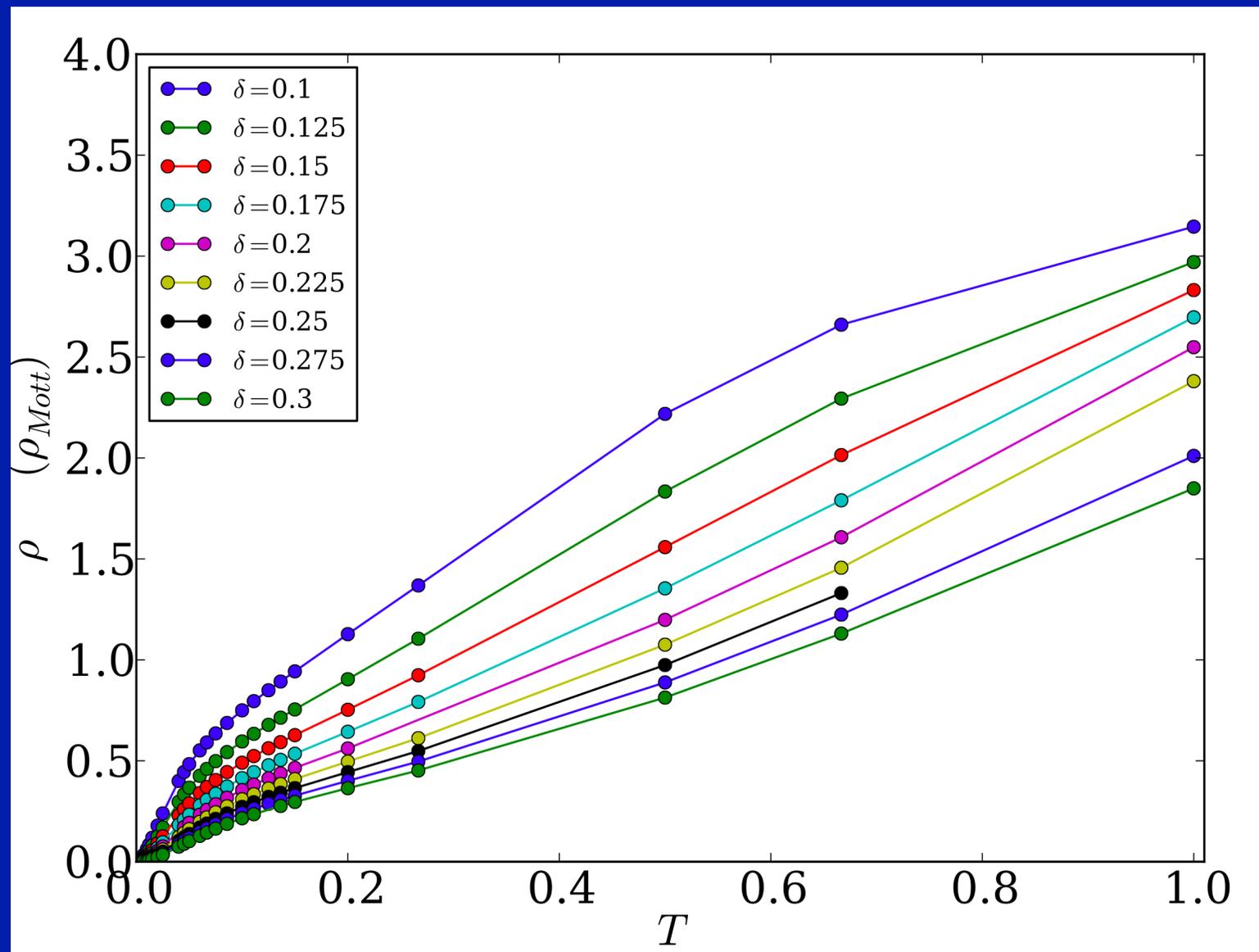
$$\Phi(\epsilon) = \frac{1}{2} \int_{-\pi/c_0}^{+\pi/c_0} \frac{dk_z}{2\pi} \int \frac{dk_x dk_y}{4\pi^2} \left( \frac{\hbar^2}{m} \right)^2 (k_x^2 + k_y^2) \delta \left[ \epsilon - \frac{\hbar^2}{2m} (k_x^2 + k_y^2) \right],$$

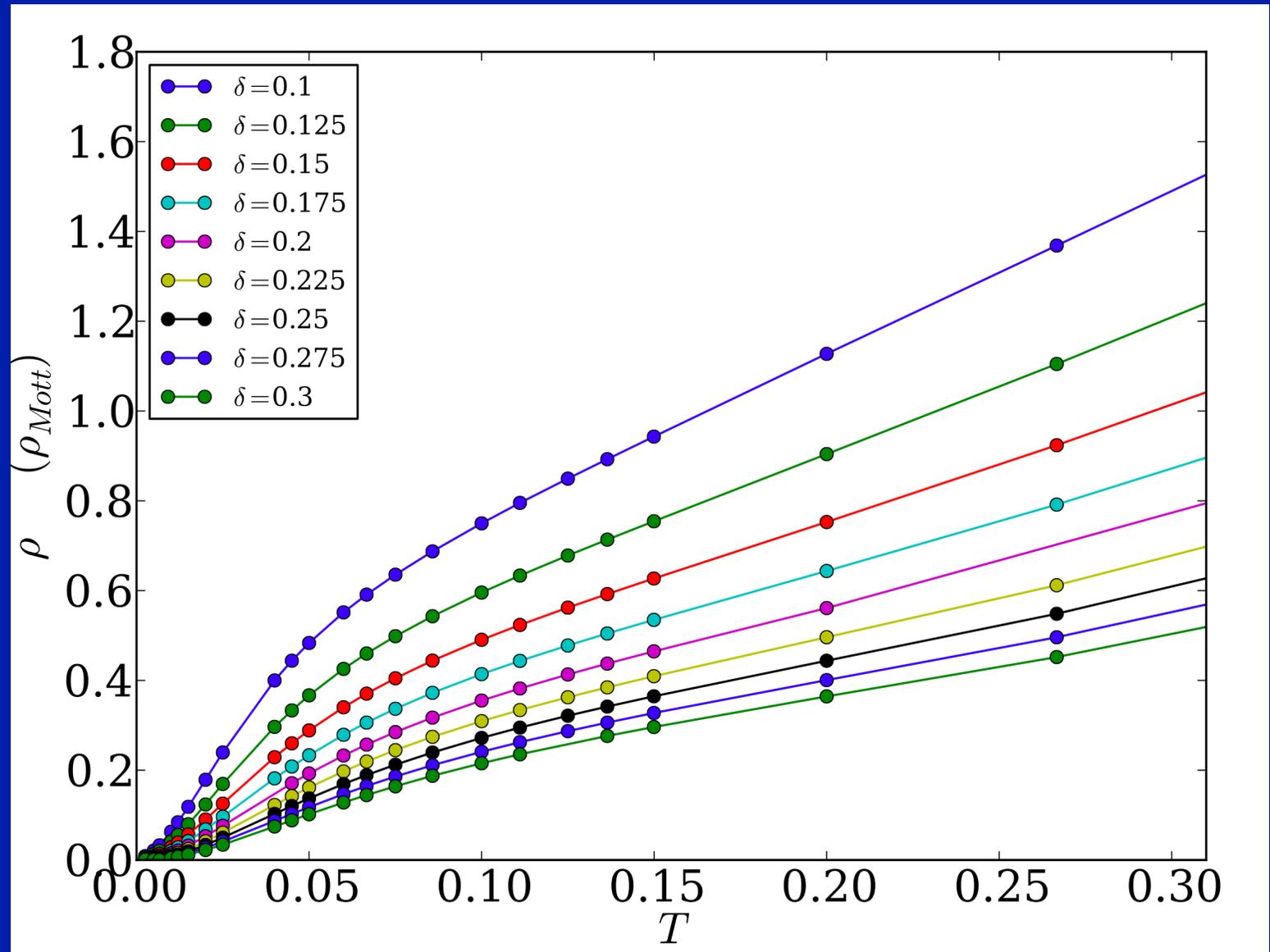
$$\Phi(\epsilon) = \frac{\epsilon}{2\pi c_0}$$

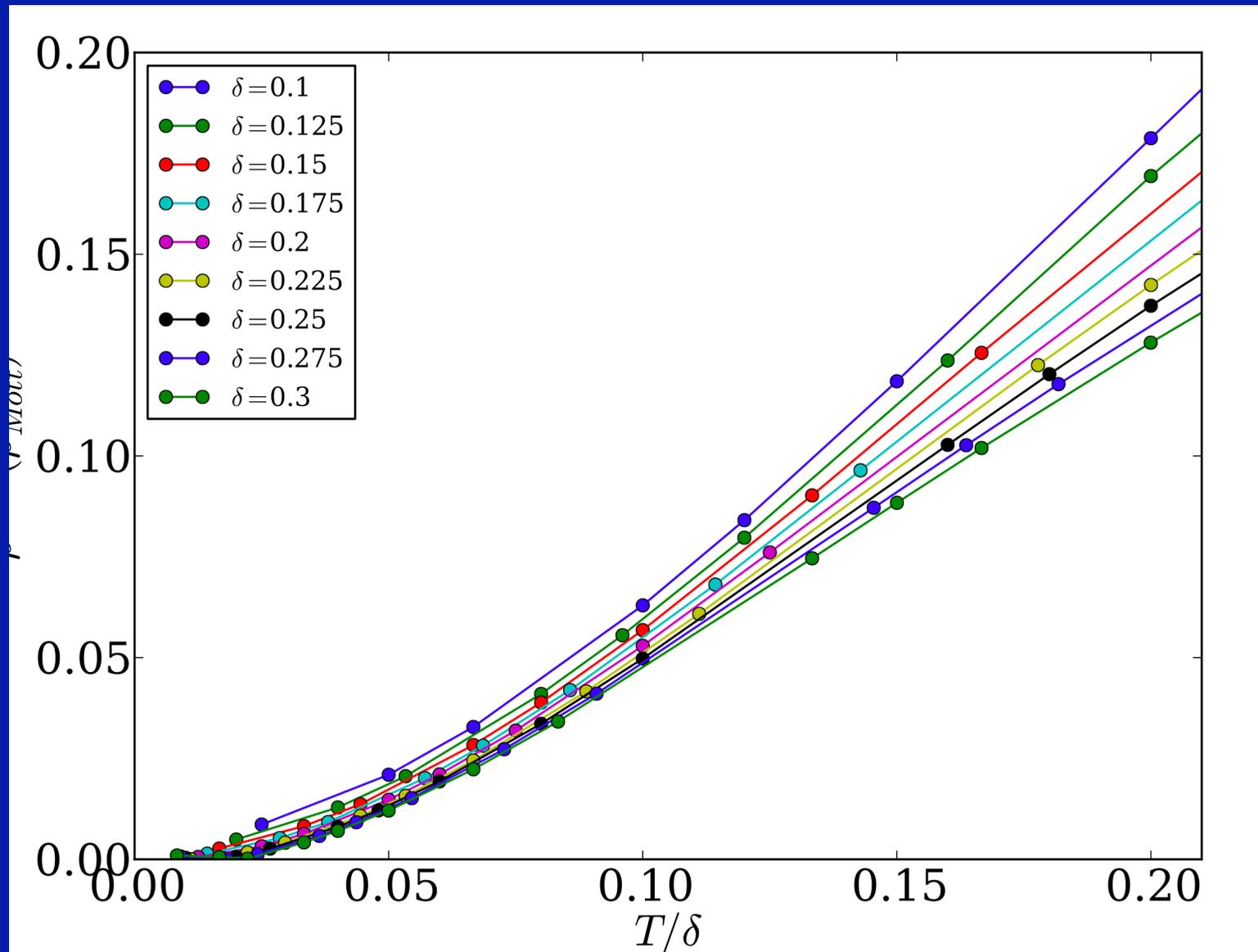
Hence, the IRM limit is naturally expressed in terms of  $\Phi(\epsilon_F)/\epsilon_F$

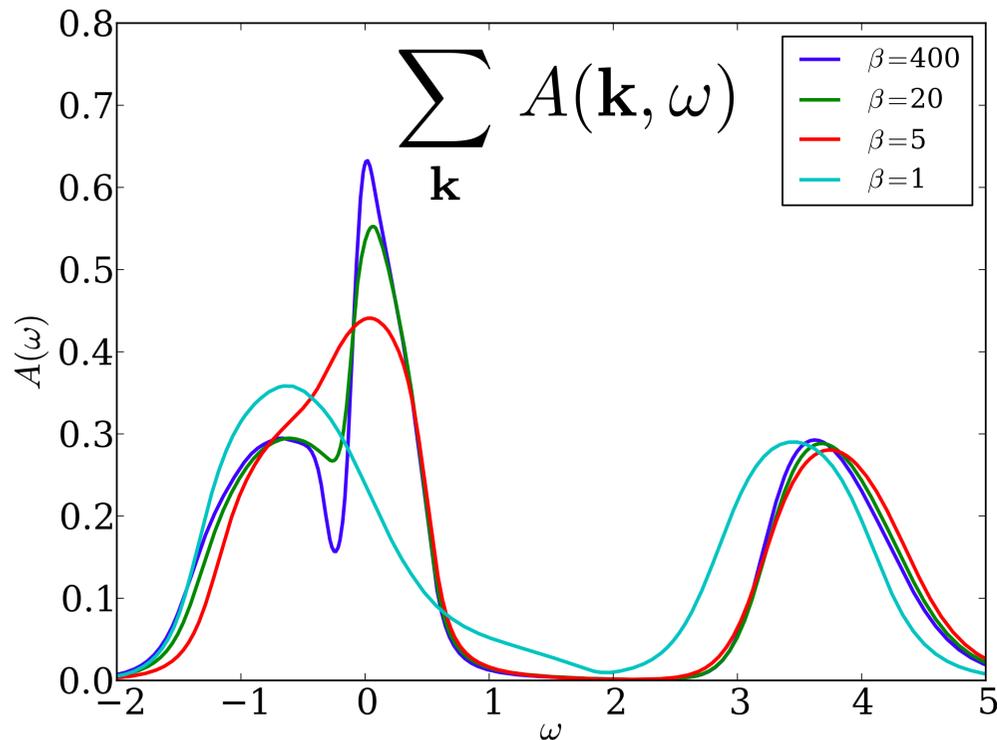
Drude, quasi-2D:

$$\sigma_{dc} = \frac{e^2}{\hbar} \frac{\Phi(\epsilon_F)}{\epsilon_F} (k_F l)$$







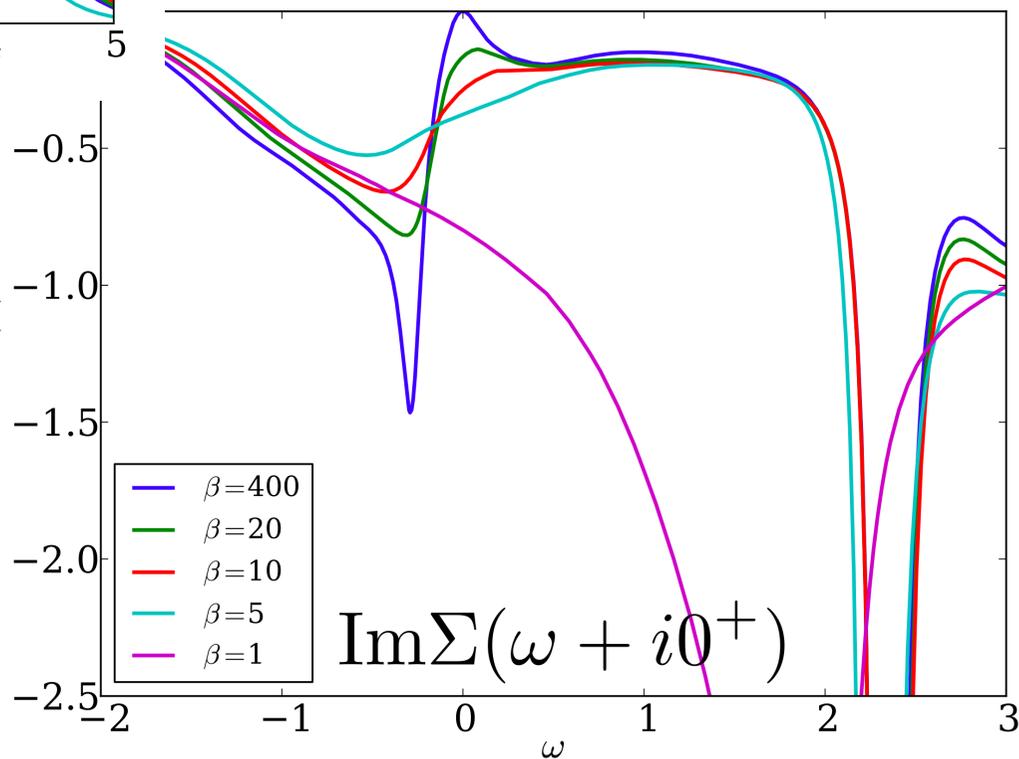


Doping: 20%

- Rich frequency dependence
- A lot of action in spectral properties as T is varied !

Total DOS

Self-energy  
"scattering rate"



# 1. The Fermi-Liquid regime

Local Fermi-liquid/Landau Theory description

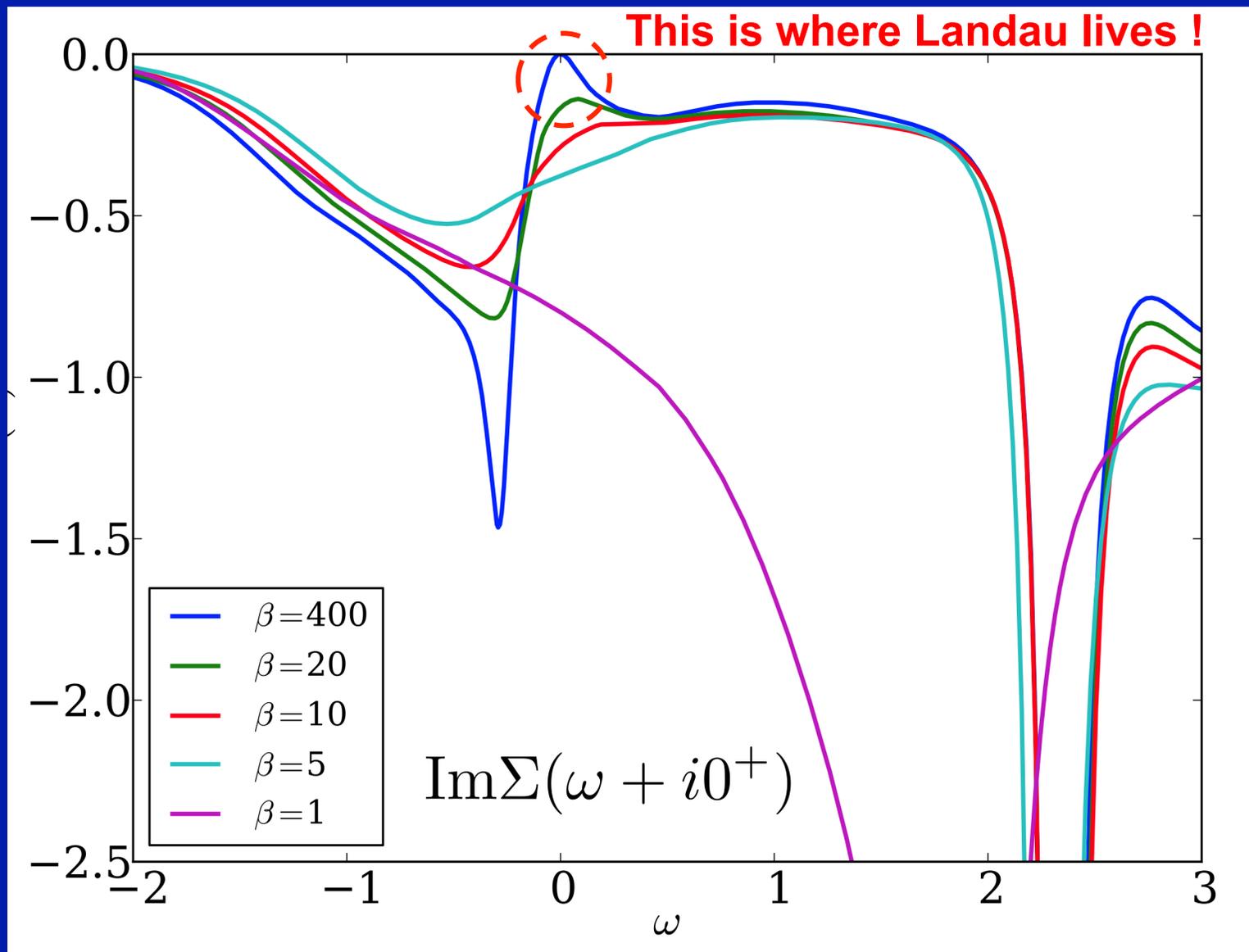
A self-consistent Kondo-like screening problem

DMFT (lattice) self-consistency  $\rightarrow$  intermediate coupling

$$\begin{aligned}\operatorname{Re}\Sigma(\omega + i0^+) &= \Sigma_0 + \left(1 - \frac{1}{Z}\right)\omega + \dots \\ -\operatorname{Im}\Sigma(\omega + i0^+) &= \frac{c}{D} [\omega^2 + (\pi T)^2]\end{aligned}$$

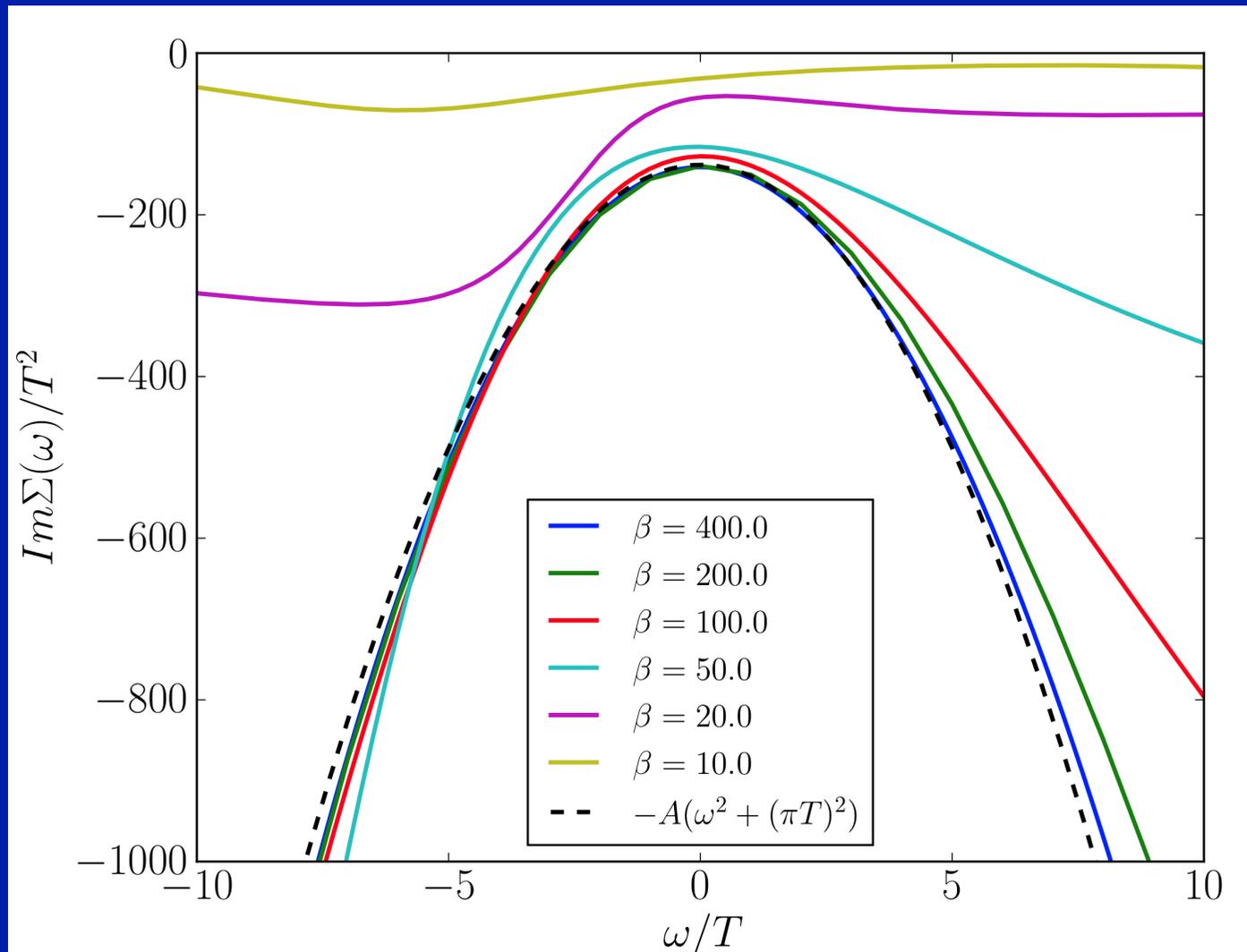
Luttinger theorem (large FS):

$$\mu - \Sigma_0(T = 0) = \mu_{U=0}(n) \equiv \epsilon_F$$



# Identifying the Fermi Liquid scale

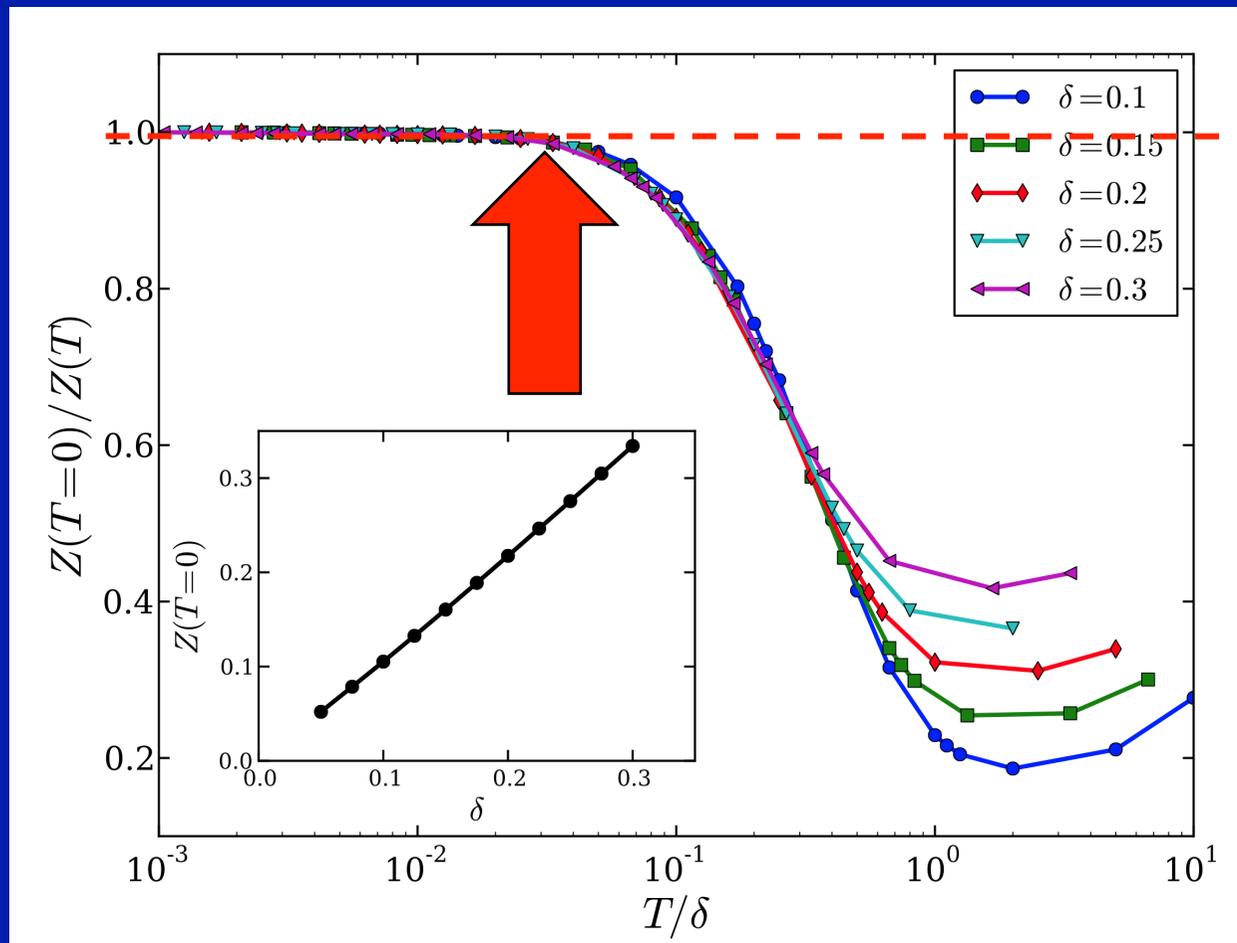
a. From  $\omega^2 + (\pi T)^2$  scaling (Pade)





c. From T-dep of “effective mass”  
 (useful: Matsubara...)

$$Z(0)/Z(T) = \text{const. for } T < T_{FL}$$



Brinkman-Rice  
 Behaviour  
 of

$$\frac{m^*}{m} = \frac{1}{Z} = \frac{1}{\delta}$$

# Fermi Liquid scale (U/D=4)

$$T_{\text{FL}}/D \simeq 0.05 \delta$$

\* A very low scale

(as compared to bare electronic scales) !

e.g.  $D=1\text{eV}$ ,  $\delta=10\%$   $\rightarrow$  60 K

\* Scales  $\sim$  doping

but much lower than 'Brinkman-Rice' scale  $\sim \delta D$   
(by 1/20)

# Resistivity in the FL regime: analytics

Low  $\omega, T$  scaling form of scattering rate:

$$-\text{Im}\Sigma/D = a \left[ \left( \frac{\omega}{\pi\delta} \right)^2 + \left( \frac{T}{\delta} \right)^2 \right] + \dots$$

$$a(U/D = 4) \simeq 5.5$$

→ On blackboard

$$\frac{\rho(T)}{\rho_M} = 1.22a \left( \frac{T}{\delta D} \right)^2 + \dots \simeq 0.017 \left( \frac{T}{T_{FL}} \right)^2$$

$$\rho(T_{FL}) \ll \rho_M$$

Note:  $Z \sim \delta$  drops out from  $A/\gamma^2 = \text{NON-UNIVERSAL constant}$

'Kadowaki Woods' 1986, TM Rice 1968

cf. N.Hussey JPSJ 74 (2005) 1107; B.Powell et al. Nature Physics 2009

# Sr<sub>1-x</sub>La<sub>x</sub>TiO<sub>3</sub>

Tokura et al. PRL 1993

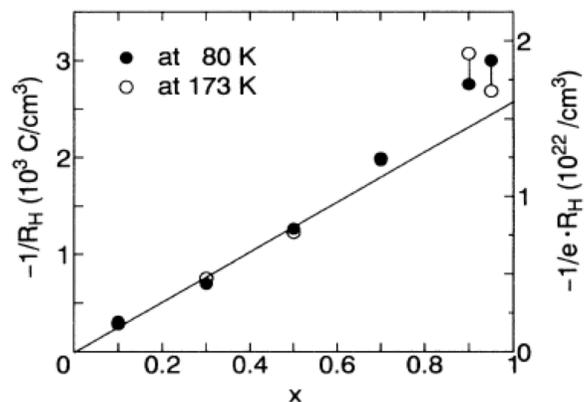
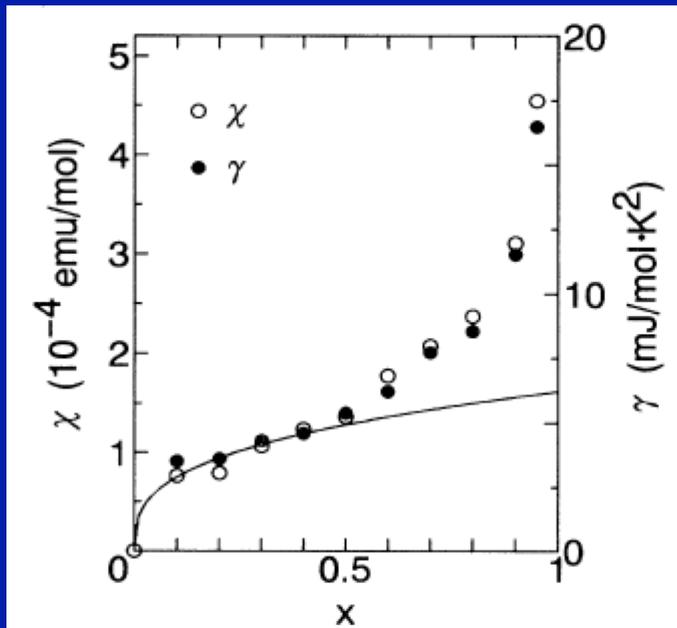
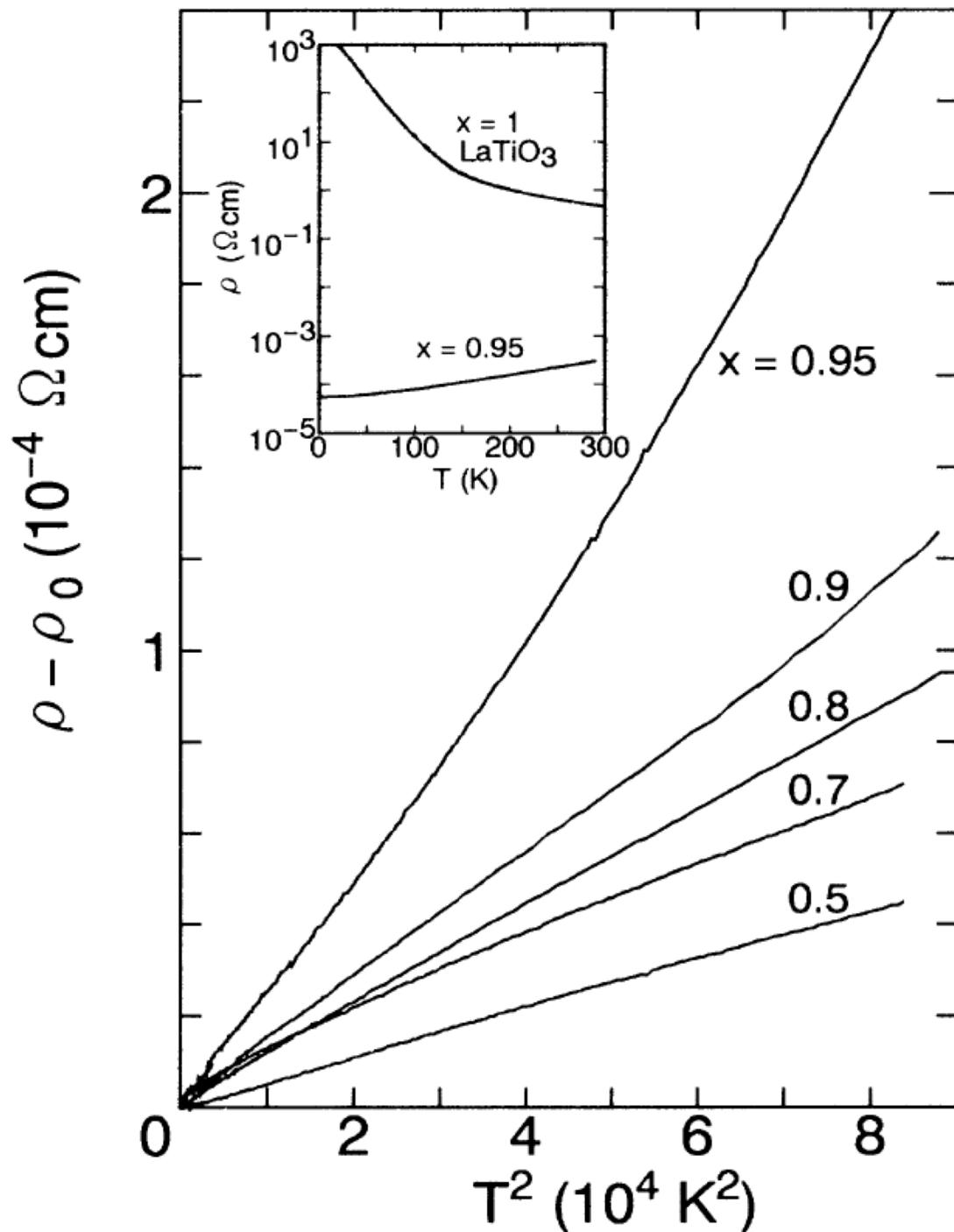
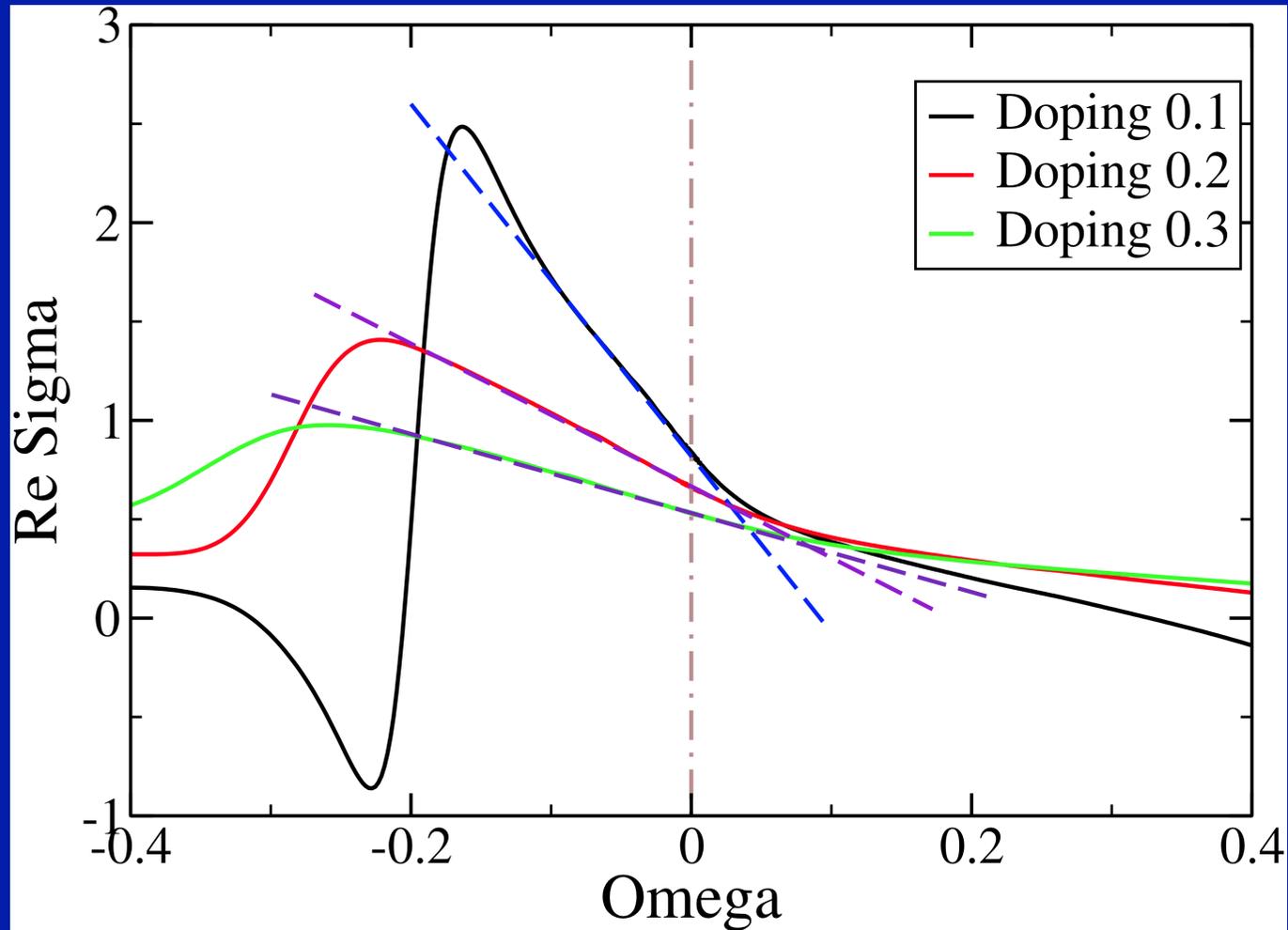


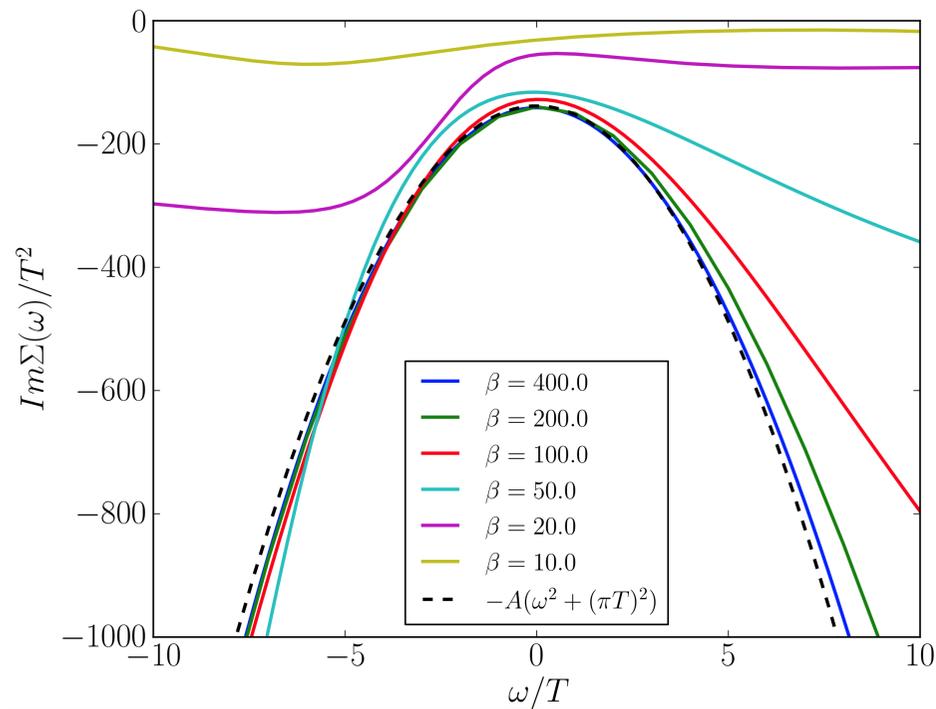
FIG. 2. The filling ( $x$ ) dependence of the inverse of Hall coefficient ( $R_H^{-1}$ ) in Sr<sub>1-x</sub>La<sub>x</sub>TiO<sub>3</sub>. Open and closed circles represent the values measured at 80 K and 173 K, respectively. A solid line indicates the calculated one based on the assumption that each substitution of a Sr<sup>2+</sup> site with La<sup>3+</sup> supplies the compound with one electron-type carrier per Ti site.



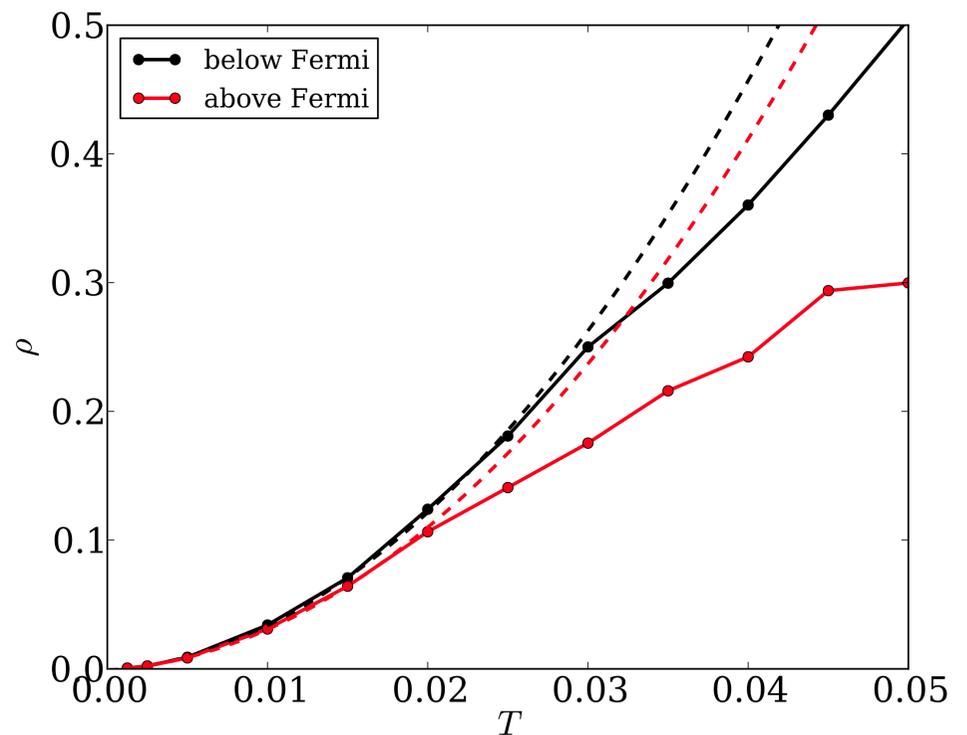
# Hole- and electron- like 'kinks' and the FL coherence scale



# Strong particle-hole asymmetry of the scattering rate in the FL regime



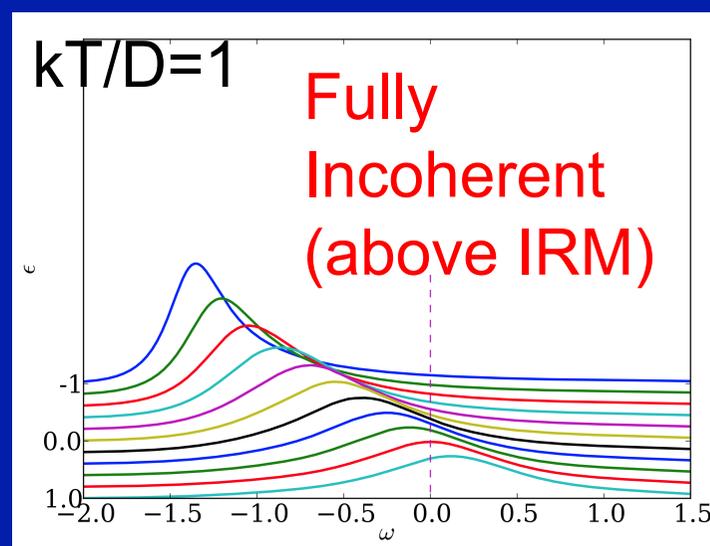
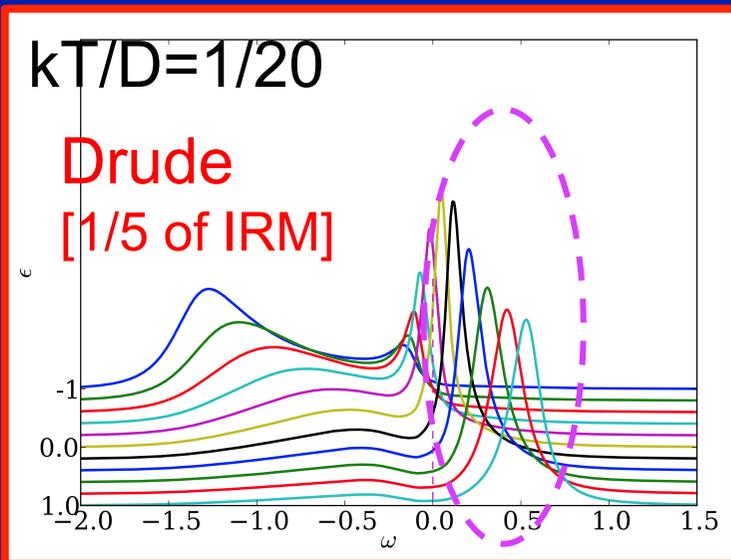
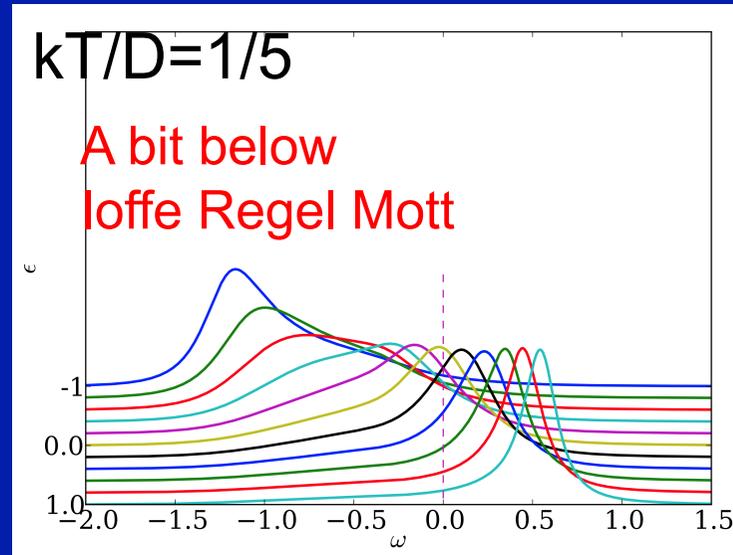
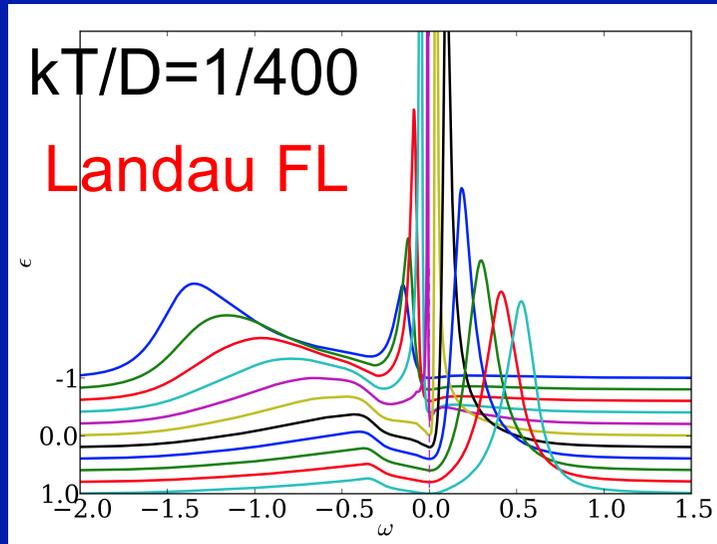
Positive  $\omega$  and negative  $\omega$  contributions to Kubo-Drude formula:



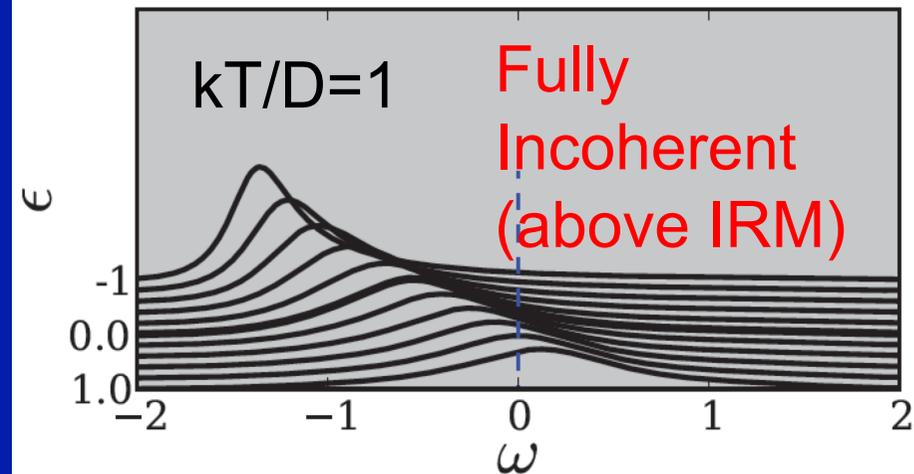
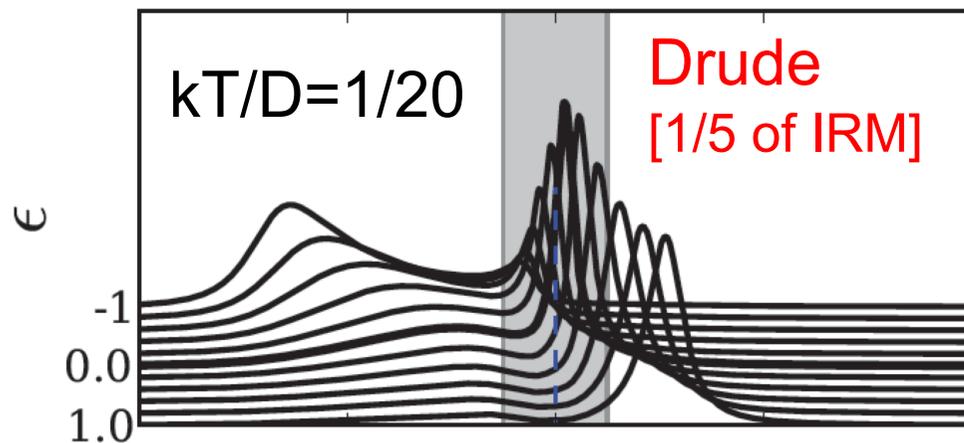
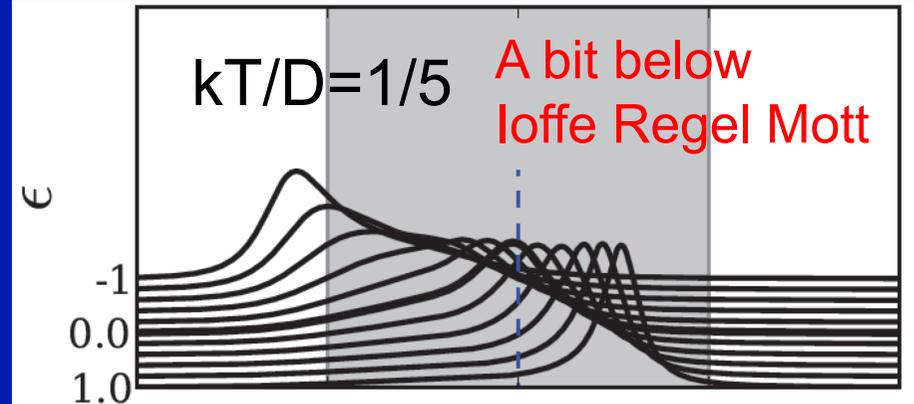
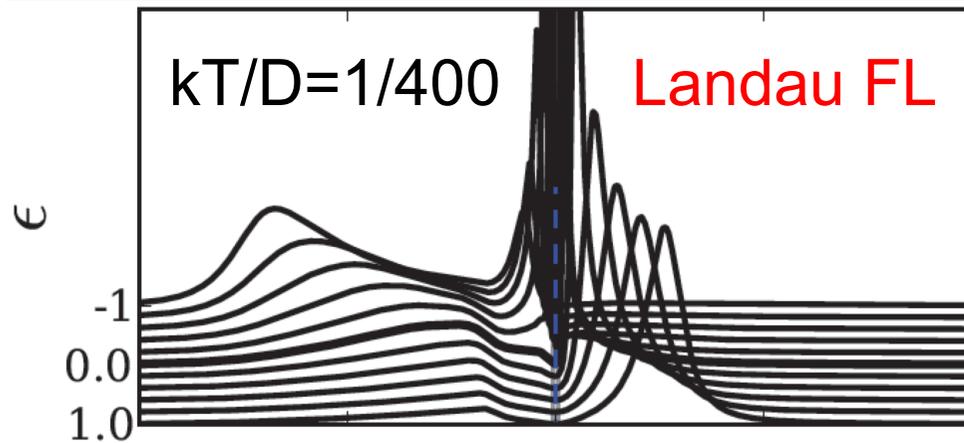
## 2. The 'Drude' regime for $T > T_{FL}$ ( $T < T_{IRM}$ )

Quasiparticles **SURVIVE** but they no longer obey Landau's theory

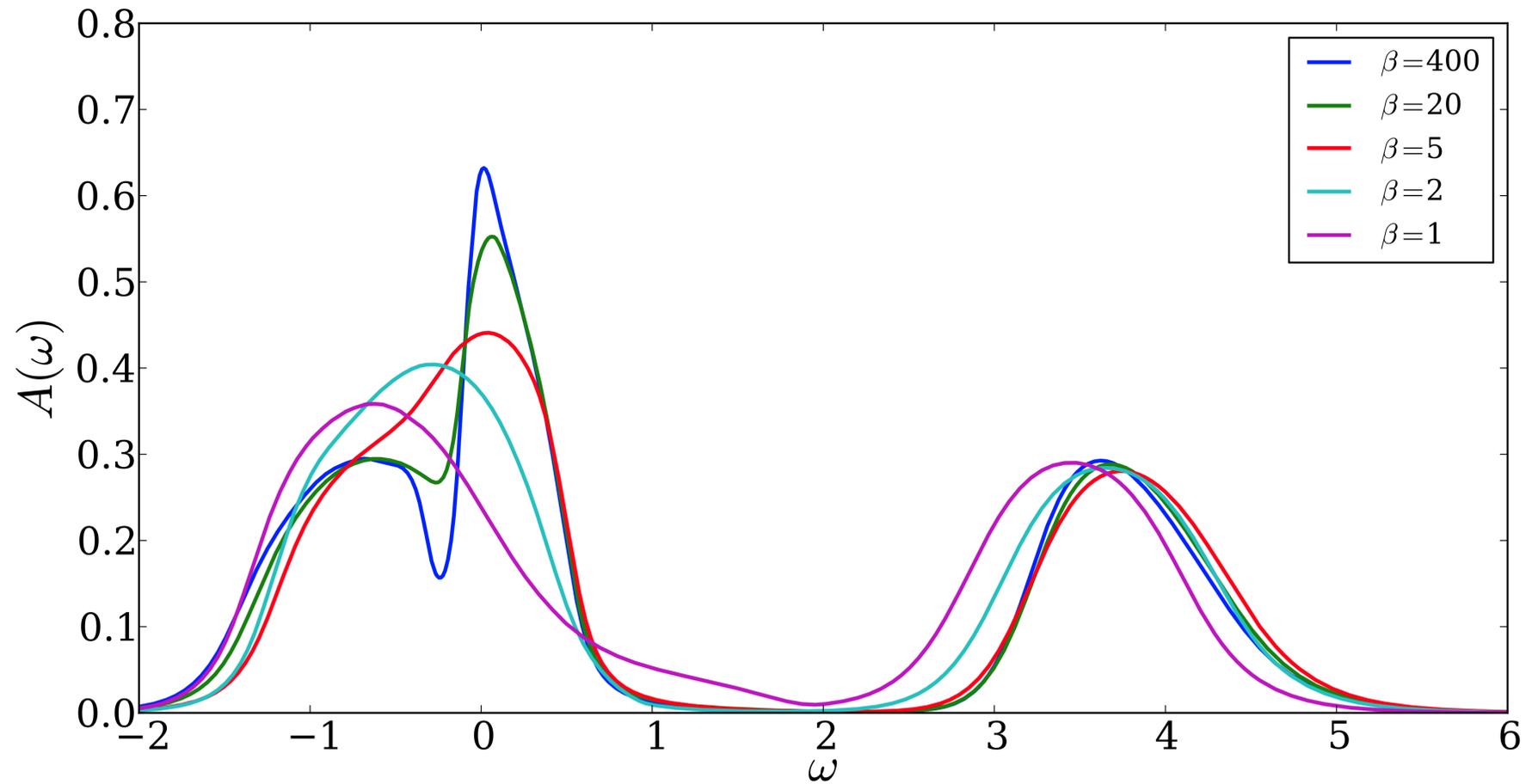
The 'dark side' of the Fermi surface



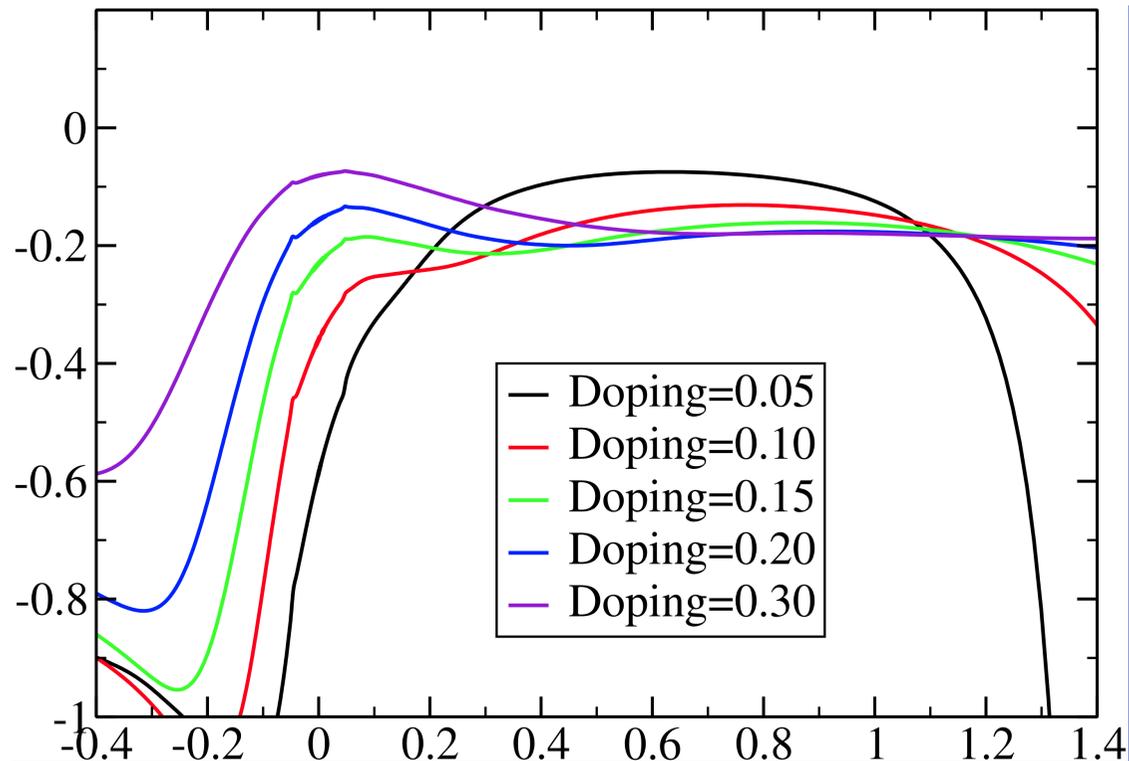
# Which excitations contribute to dc transport ?



# Total DOS

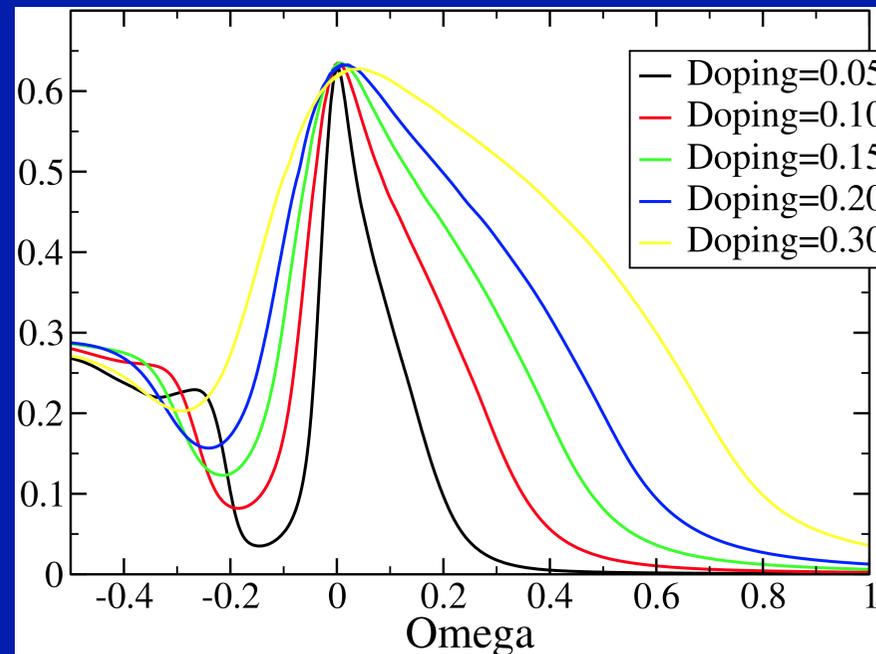
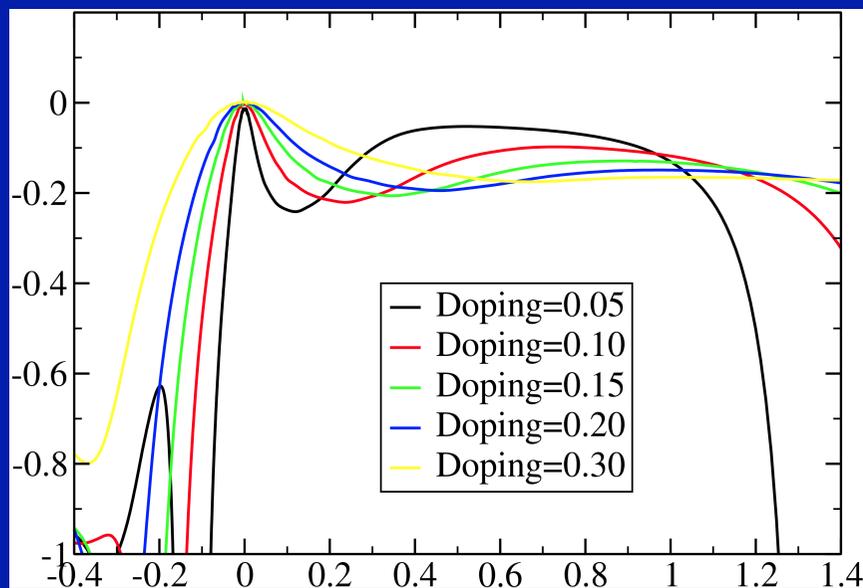


Clear 3-peak structure way above  $T_{FL}$

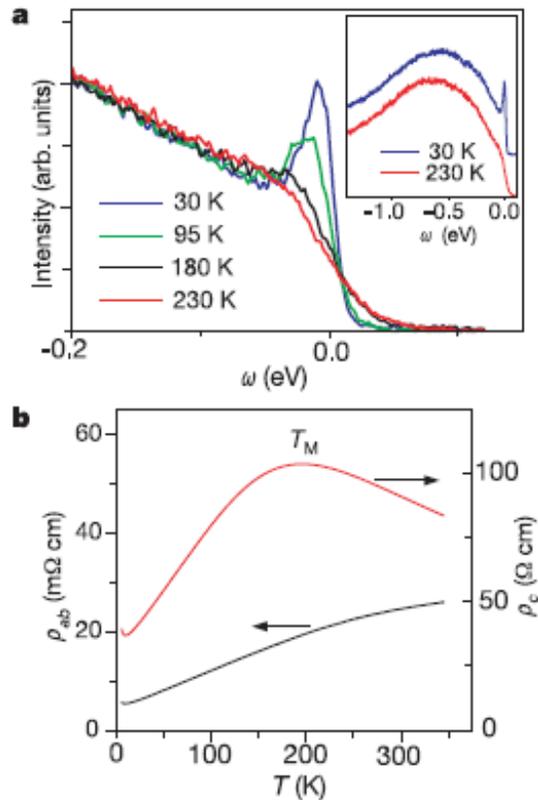


**'Drude' quasiparticles:**  
 Scattering rate  $\gg kT$   
 but  $\ll D$

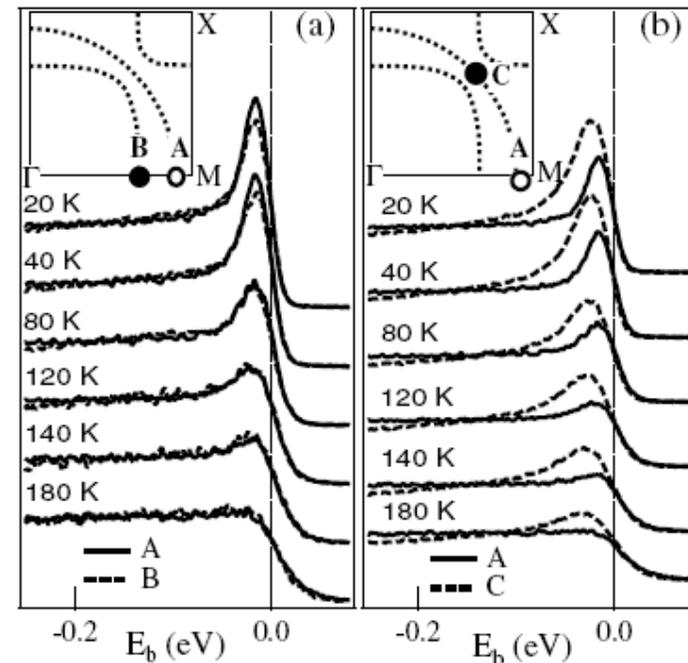
Weakly-temperature and  
 energy-dependent  
 ('plateau')  
 over some range



# Claims about destruction of quasiparticles as seen in ARPES: to be reconsidered



**Figure 2** Correlation between the ARPES and transport in  $(\text{Bi}_{0.5}\text{Pb}_{0.5})_2\text{Ba}_3\text{Co}_2\text{O}_y$ . **a**, The changes in energy distribution curves (vertical cross-sections of the ARPES data shown in Fig. 1) (for  $k = k_F$ ) with temperature. The inset shows the wide-range energy distribution curves. **b**, Transport data. The in-plane and the out-of-plane resistivities are measured on a sample from the same batch with a conventional four-probe technique.



**FIG. 2.** Temperature dependence of spectra at the three FSCPs: A, B, and C (see the insets). (a) Comparison of spectra between A and B. (b) Comparison of spectra between A and C. The insets show measurement locations in the Brillouin zone.

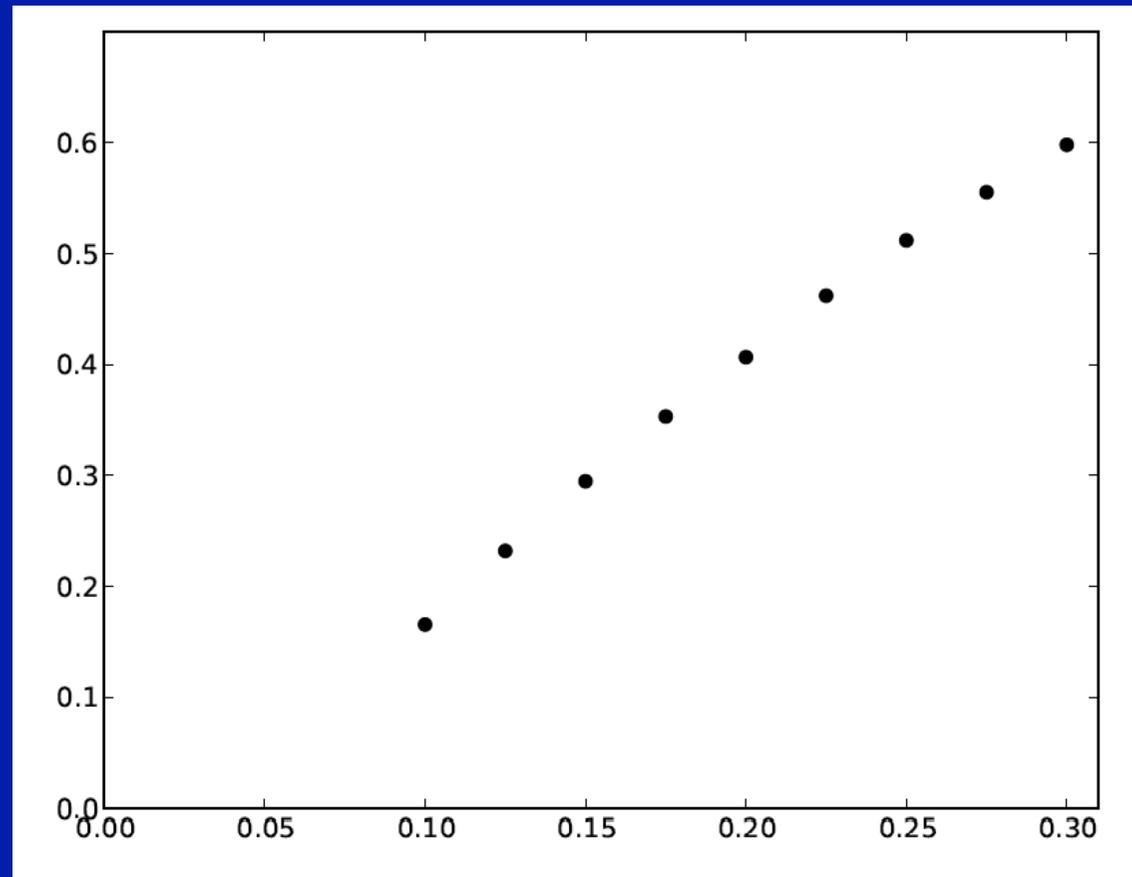
T.Valla et al. Nature 417 (2002) 628

Wang et al PRL 92 (2004) 137002

# When is the IRM 'limit' reached ?

The true meaning of the Brinkman-Rice scale  $\sim \delta D$

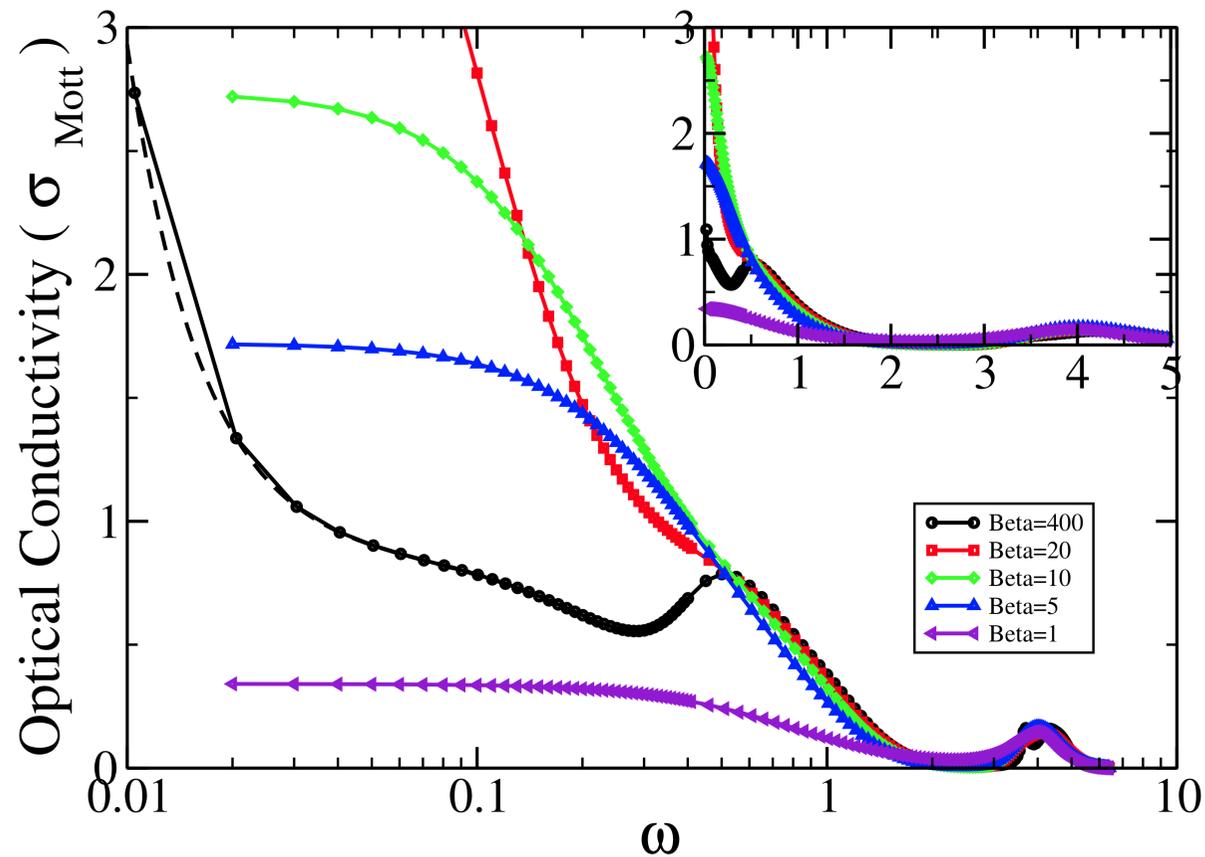
$T_{\text{IRM}}/D$



Doping

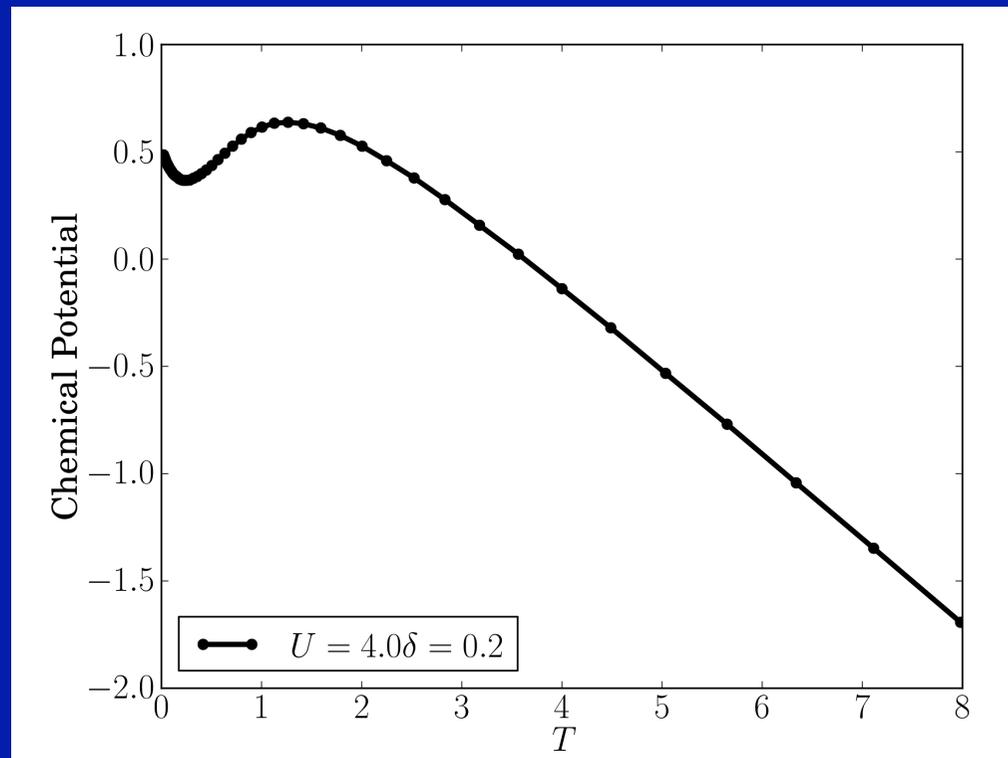
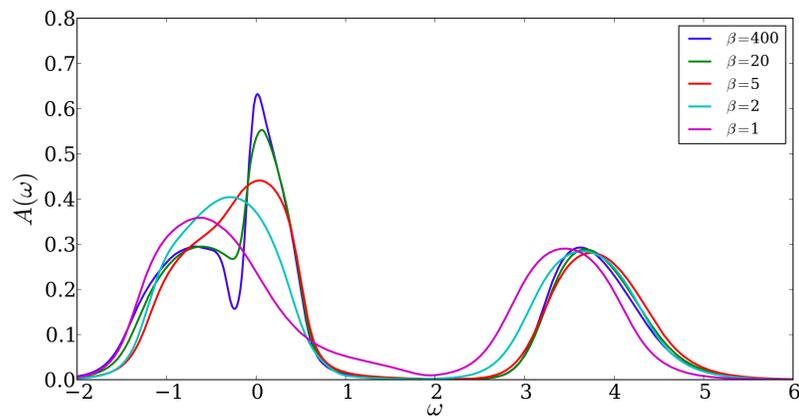
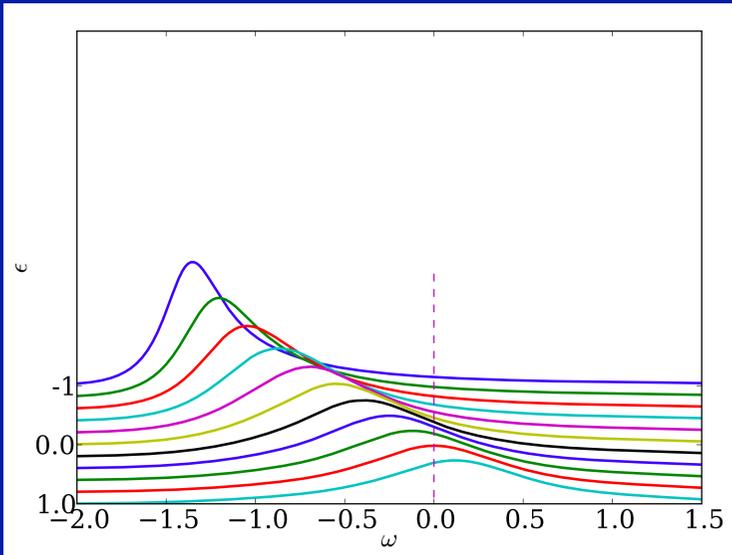
# Signatures of IRM in optics:

“IRM limit = changes in MIR range”



cf. N.Hussey, Takenaka, Takagi review in Phil Mag, 2004.

### 3. High temperatures: $T > T_{IRM}$ and beyond... Incoherent regime – Hubbard band physics ~ classical carriers in a rigid band



Chemical potential is linear in  $T$  at very hi- $T$

$$\tilde{\rho}(\omega, \epsilon) = \rho(\omega - \mu, \epsilon).$$

Hence the coefficients  $A_n$  from §3.4 become<sup>4</sup>:

$$A_n = \frac{\pi N}{4} \int d\omega \frac{(\beta\omega - \alpha)^n}{\cosh^2(\frac{\beta\omega - \alpha}{2})} \int d\epsilon \tilde{\rho}^2(\omega, \epsilon) \Phi(\epsilon).$$

We now expand the hyperbolic cosine in Taylor series around  $\beta = 0$ .

$$\frac{1}{\cosh^2(\frac{\beta\omega - \alpha}{2})} = \frac{1}{\cosh^2(\frac{\alpha}{2})} \left( 1 + \beta\omega \tanh(\frac{\alpha}{2}) + \frac{\omega^2 \beta^2}{4} \left[ 3 \tanh(\frac{\alpha}{2}) - 1 \right] \right).$$

Before we go any further we also define

$$\gamma_n = \int d\epsilon d\omega \omega^n \tilde{\rho}^2(\omega, \epsilon) \Phi(\epsilon) \quad \text{and let} \quad \tau = \tanh(\frac{\alpha}{2}) \quad \text{and} \quad \zeta = \frac{1}{4 \cosh^2(\frac{\alpha}{2})}.$$

T-linear above IRM  $\rho(T) \sim \frac{T}{\gamma_0 \zeta}$  From G.Palsson's PhD