College de France, May 21, 2013

SUPERFLUIDTY IN ULTRACOLD ATOMIC GASES (A TALE OF TWO SOUNDS)

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PLAN OF THE LECTURES

Lecture 1. **Superfluidity** in ultra cold atomic gases:
examples and open questions (May 14)

Lecture 2. A tale of two sounds (**first** and **second sound**) (May 21)

Lecture 3. **Spin-orbit** coupled Bose-Einstein condensed gases:
quantum phases and **anisotropic dynamics** (May 28)

Lecture 4. **Superstripes** and **supercurrents** in spin-orbit coupled
Bose-Einstein condensates (June 4)

Major question: How to **measure** the **superfluid density** in an ultracold gas ?
(not available from equlibrium thermodynamics, needed **transport** phenomena)

Determination of **superfluid density** in strongly interacting Fermi gases (measurement of **second sound**)

(collaboration with the Innsbruck team, Nature, 15 May, online)

Dynamic theory for superfluids at finite temperature: Landau's Two-fluid HD equations

(hold in deep collisional regime $\omega\tau << 1$)

Irrotationality of superfluid flow

$$\frac{\partial}{\partial t}\rho + \vec{\nabla}(\vec{j}) = 0$$

$$\frac{\partial}{\partial t}s + \vec{\nabla}(s\vec{v}_N) = 0$$

$$m\frac{\partial}{\partial t}\vec{v}_S + \nabla(\mu(n) + V_{ext}) = 0$$

$$\frac{\partial}{\partial t}\vec{j} + \vec{\nabla}P + n\vec{\nabla}V_{ext} = 0$$

$$\rho = mn = \rho_S + \rho_N$$

$$\vec{j} = \rho_S\vec{v}_S + \rho_N\vec{v}_N$$

s is entropy density
P is local pressure

Ingredients:
- equation of state
- superfluid density

$$\frac{\partial}{\partial t}\rho + \vec{\nabla}(\vec{j}) = 0$$

$$\cancel{\frac{\partial}{\partial t}s + \vec{\nabla}(s\vec{v}_N) = 0}$$

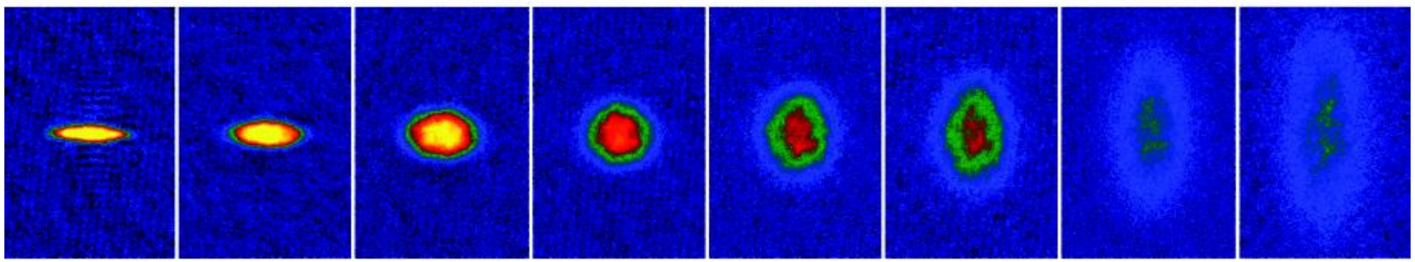
$$\left.\begin{aligned} & m\frac{\partial}{\partial t}\vec{v}_S + \nabla(\mu(n) + V_{ext}) = 0 \\ & \frac{\partial}{\partial t}\vec{j} + \vec{\nabla}P + n\vec{\nabla}V_{ext} = 0 \end{aligned}\right\}$$ *equivalent at T=0*

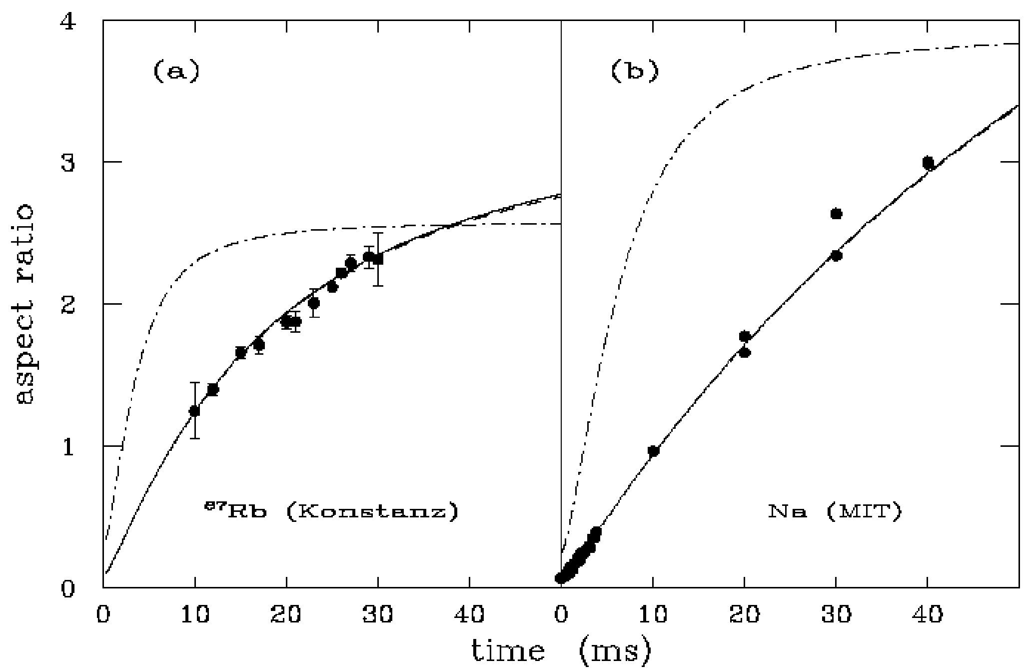
At T=0: $\rho = \rho_S \; ; \vec{j} = \rho\vec{v}_S$
eqs. reduce to
T=0 irrotational
superfluid HD equations

*At T=0 irrotational hydrodynamics follows from superfluidity (role of the phase of the order parameter). Quite successful to describe the macroscopic dynamic behavior of trapped atomic gases (Bose and Fermi) (**expansion**, **collective oscillations**)*

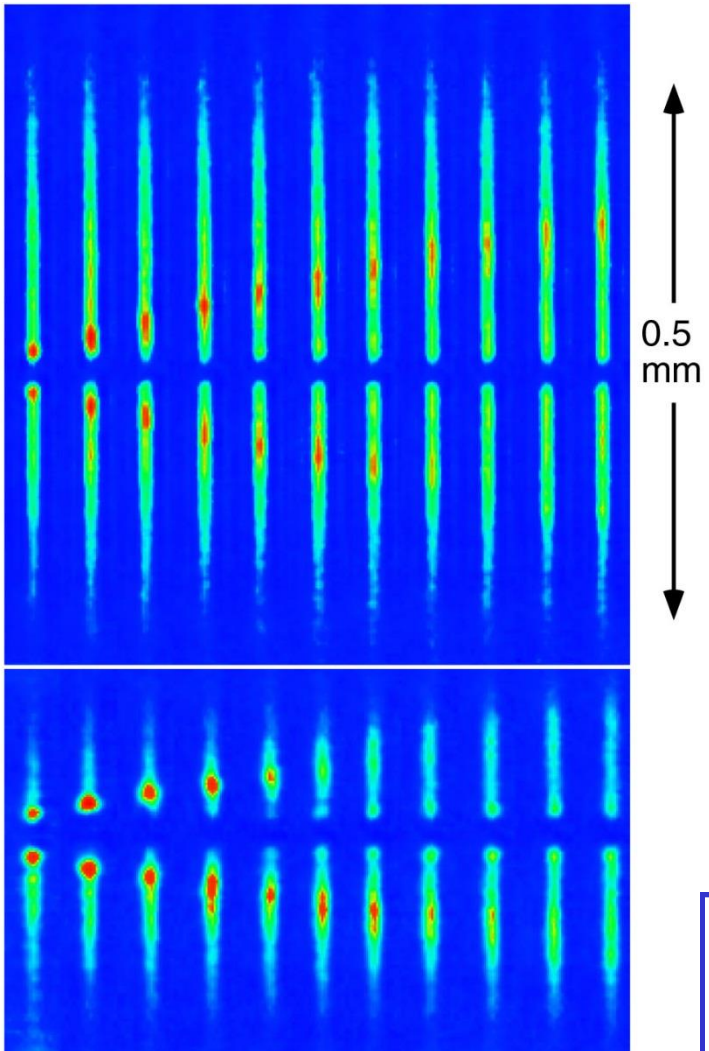
Hydrodynamics predicts anisotropic expansion of the superfluid

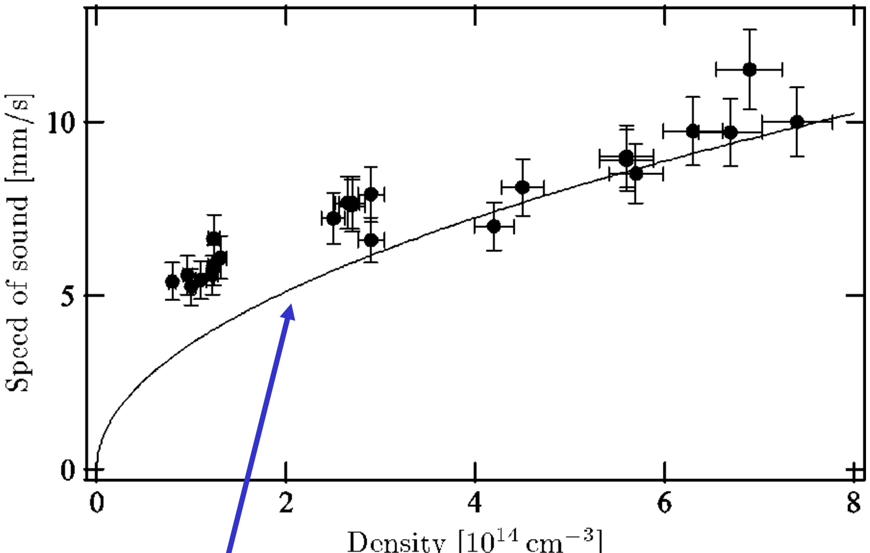
(Kagan, Surkov, Shlyapnikov 1996; Castin, Dum 1996,





T=0 Bogoliubov sound
(wave packet propagating in a dilute BEC, Mit 97)





sound velocity as a function of central density

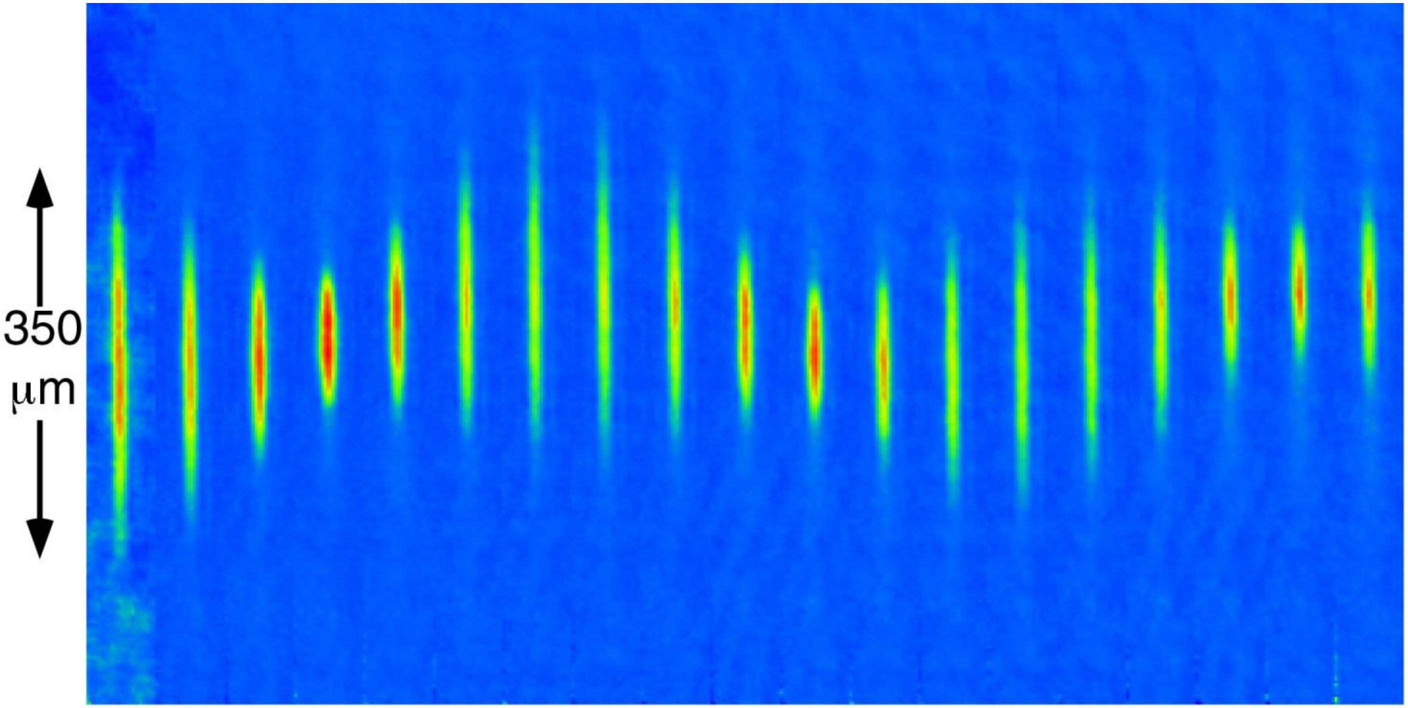
$$c = \sqrt{gn/2m}$$

factor 2 accounts for harmonic radial trapping (Zaremba, 98)

T=0 Collective oscillations in dilute BEC (axial compression mode) : checking validity of **hydrodynamic theory** of superfluids in **trapped gases**

Exp (Mit, 1997) $\omega = 1.57\omega_z$

HD Theory (S.S. 1996): $\omega = \sqrt{5/2}\,\omega_z = 1.58\omega_z$

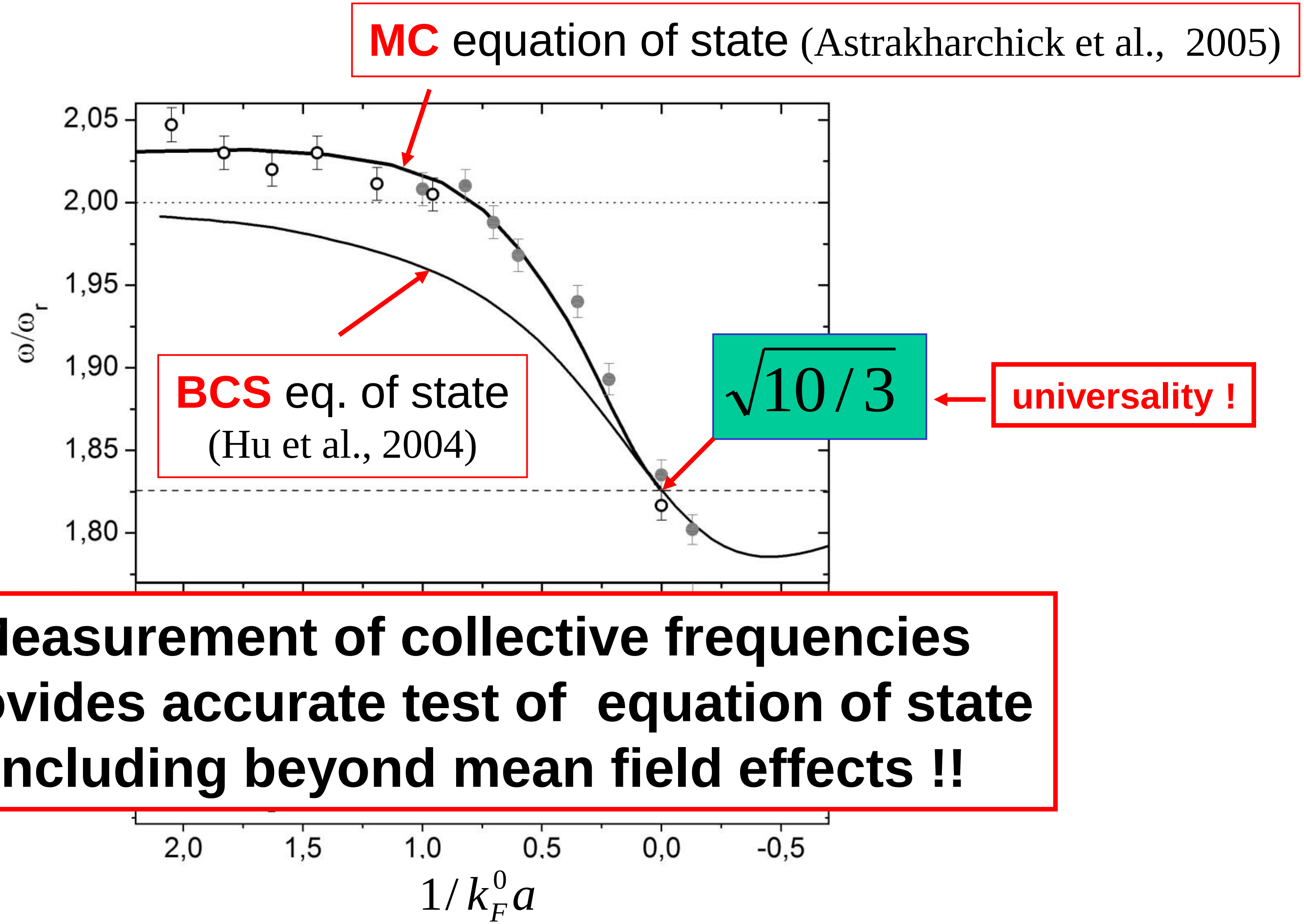


5 milliseconds per frame

T=0 breathing mode in elongated Fermi superfluids

Exp: Altmeyer et al. (Innsbruck 2007)

Theory: T=0 Hydrodynamics with Monte Carlo eq. of state



Measurement of collective frequencies provides accurate test of equation of state Including beyond mean field effects !!

SOLVING THE HYDRODYNAMIC EQUATIONS OF SUPERFLUIDS

AT FINITE TEMPERATURE

In **uniform matter** Landau equations gives rise to two solutions below the critical temperature:

First sound: superfluid and normal fluids move **in phase**

Second sound: superfluid and normal fluids move **in opposite phase**.

If condition $\boxed{\frac{c_2^2}{c_1^2}\frac{C_P - C_V}{C_V} << 1}$ is satisfied (small compressibility and/or small expansion coefficient) well satisfied by unitary Fermi gas)

second sound reduces to Isobaric oscillation (**constant pressure**)

$$c_2^2 = \frac{1}{m}\frac{n_s T s^2}{n_n C_P}$$

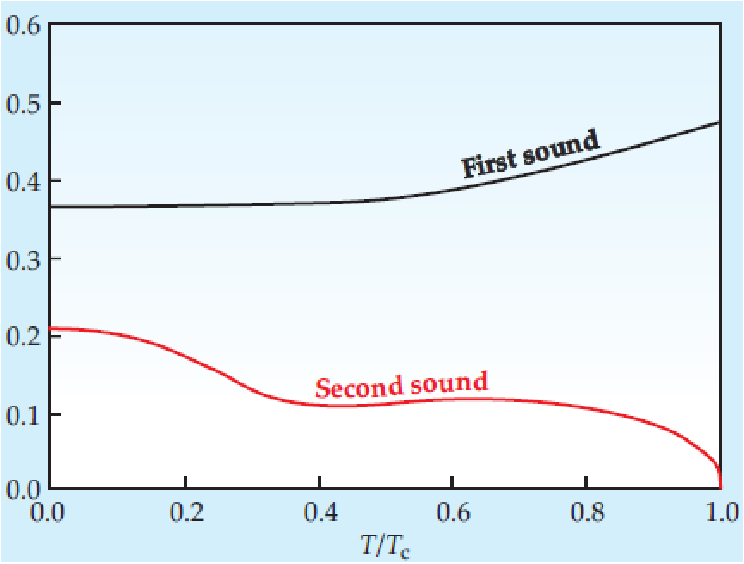
entropy

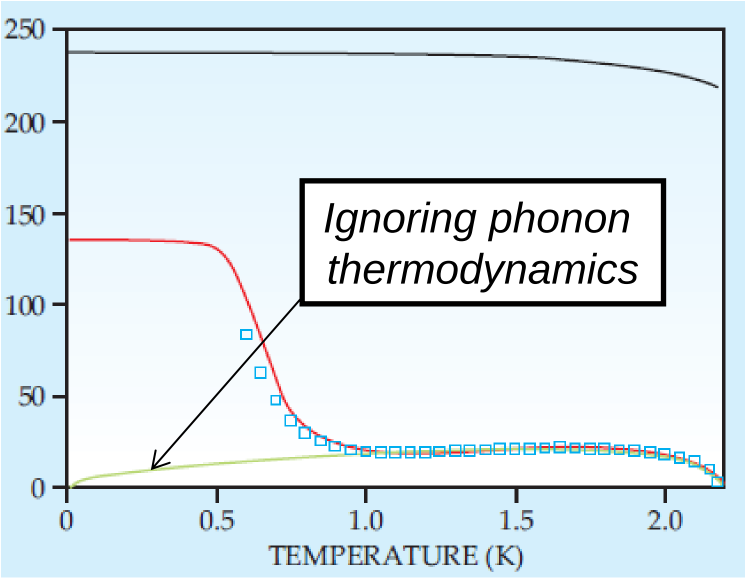
Specific heat

In this regime second sound velocity is fixed by superfluid density

First and second sound velocities in **uniform matter**

$$c_1^2 = \tfrac{1}{m}\left(\frac{\partial P}{\partial n}\right)_S \qquad c_2^2 = \tfrac{1}{m}\frac{n_s T s^2}{n_n C_P}$$

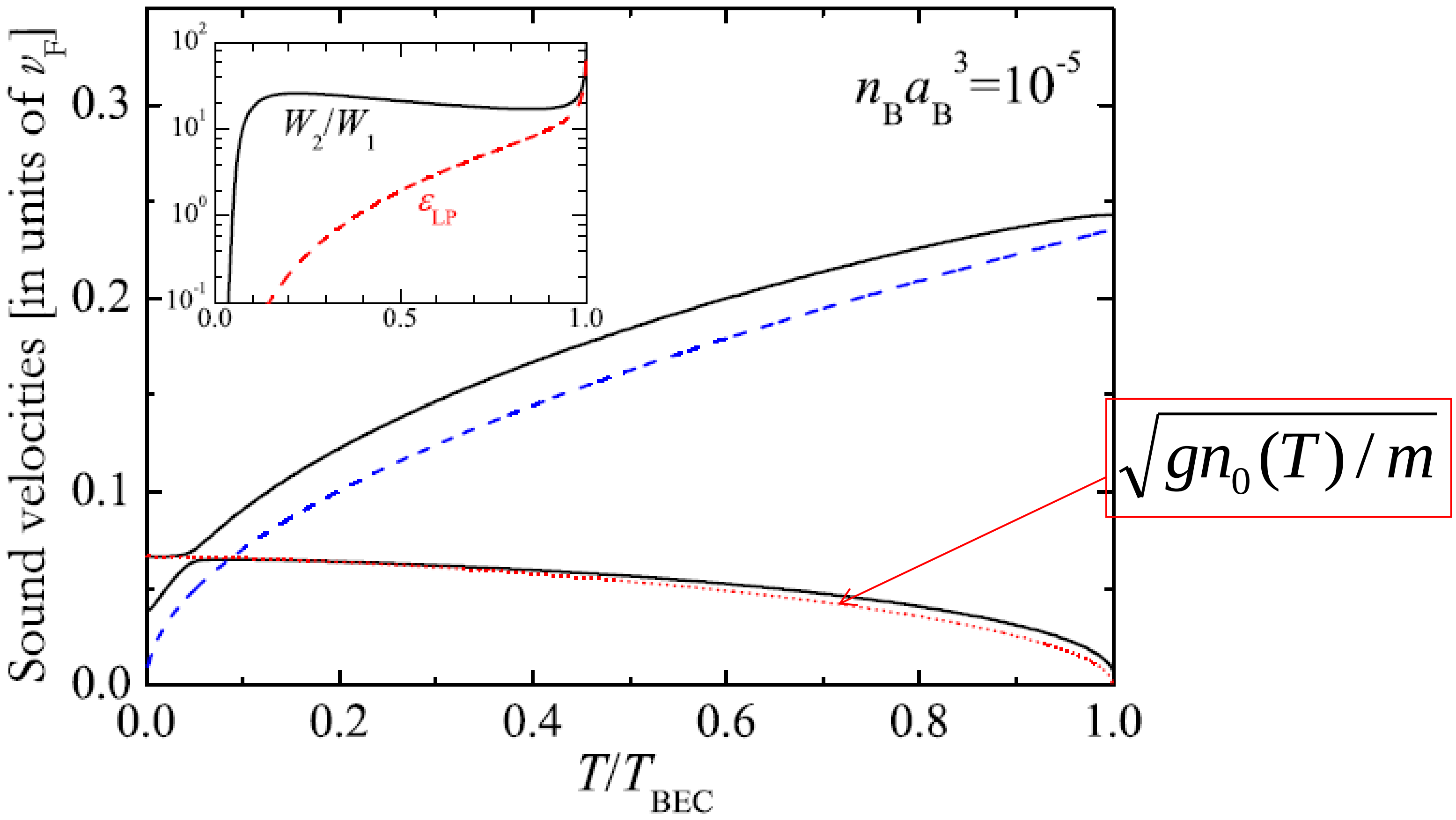




Unitary Fermi gas
H. Hu et al. 2009

Liquid He
(experiment, Peshkov 1946)

In dilute Bose gases the superfluid density practiclly concides with BEC density and second sound reduces to the oscillation of the **condensate** with the thermal part remaining at rest



What happens in the presence of **harmonic confinement** ?

$$V_{ext} = (1/2m)[\omega_\perp^2(x^2+y^2)+\omega_z^2 z^2$$

Can we derive solutions of HD equations at **finite temperature** ?

- Various theoretical (either numerical and analytical) studies at finite T available for **isotropic 3D harmonic** trapping (Castin, Levin, Griffin, Hu, Taylor, Trento team ….)

- **Exp.** results available in **elongated** geometries $\omega_z << \omega_\perp$ (both discretized solutions and sound propagation)

First sound scaling solutions available at unitarity for any T

- For **isotropic harmonic** trapping an exact **scaling** solution (breathing oscillation) of the Schrodinger equation can be proven at **unitarity**. The frequency of the oscillation is twice the harmonic frequency (Castin, 2004).

- In a recent paper (Hou et al. 2013) we have proven that at **unitarity** Landau's two fluid HD equations admit exact **scaling** solutions for **arbitrary deformed** harmonic trapping

$$\vec{v}_S = \vec{v}_N = \beta(t)\vec{r}_\perp + \delta(t)\vec{z}$$

$$n(\vec{r},t) = e^{2\alpha(t)+\gamma(t)} n(\vec{r}\,')$$

$$s(\vec{r},t) = e^{2\alpha(t)+\gamma(t)} s(\vec{r}\,')$$

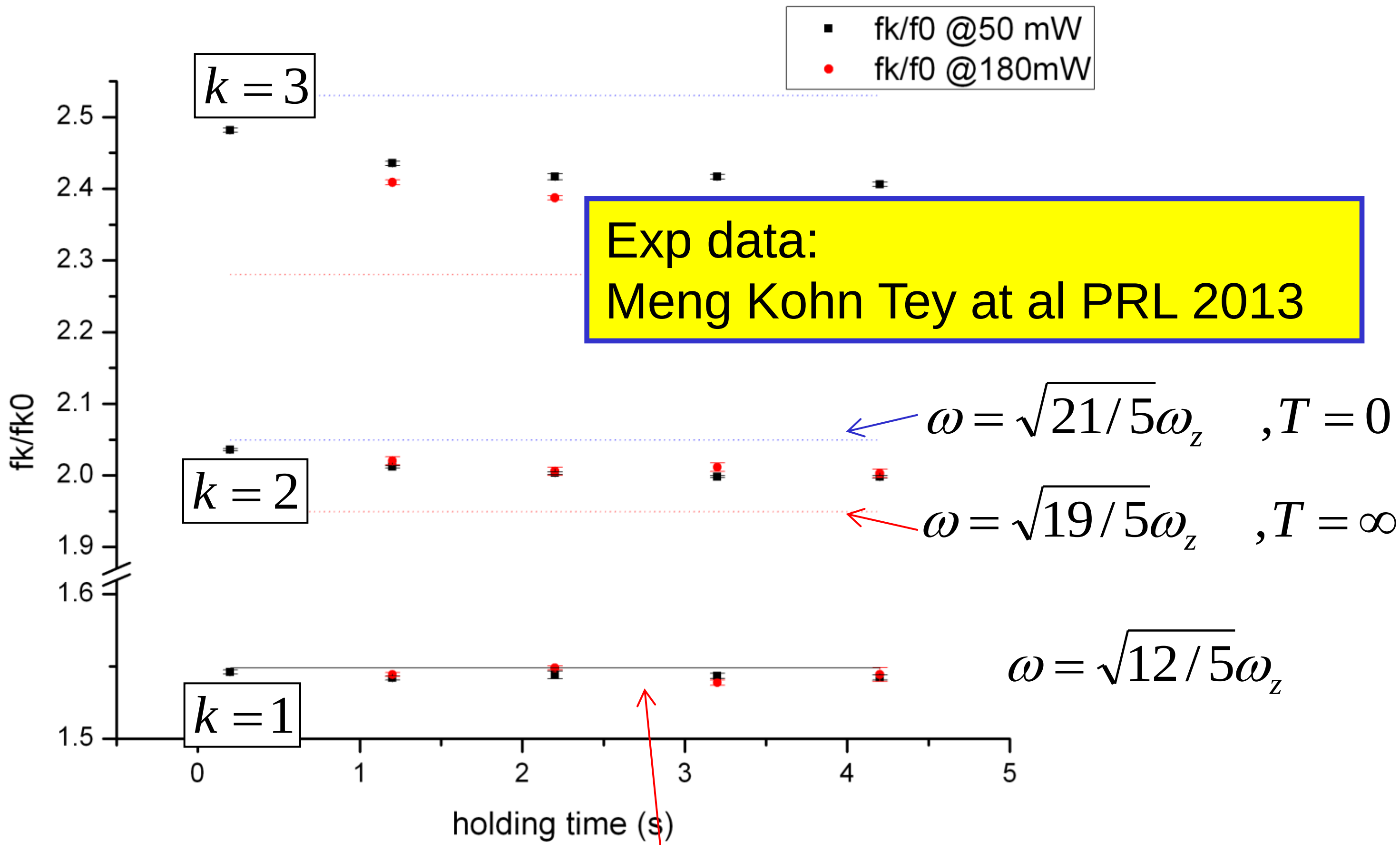
$$T(t) = e^{(2\alpha(t)+\gamma(t))2/3} T$$

$$\vec{r}\,' = (e^{\alpha(t)}x, e^{\alpha(t)}y, e^{\gamma(t)}z)$$

$$\boxed{\lambda = \omega_z / \omega_\perp} \qquad \omega^2 = \left(\frac{5}{3} + \frac{4}{3}\lambda^2 \pm \frac{1}{3}\sqrt{16\lambda^4 - 32\lambda^2 + 25}\right)\omega_\perp^2$$

Frequency **does not depend** on **Temperature**

- Compared to Castin's theorem our result holds only in hydrodynamic regime, but applies to more relevant experimental situations (deformed harmonic traps)



Exp data:
Meng Kohn Tey at al PRL 2013

Temperature increases with holding time (heating effect)

Frequency of **scaling axial mode (k=1) does not vary with T**

From 3D to 1D at finite temperature

In the presence of tight radial trapping $\omega_z << \omega_\perp$
(still LDA in radial direction)
3D Hydrodynamic equations can be reduced to **1D form** (Bertaina et al. 2011)

- New equation of state ($P_1(n_1) \neq P(n)$)
- Easier experimental conditions
- Easier realization of HD condition $\omega_z \tau << 1$
- New role of viscosity and thermal conductivity ensuring 1D form of equations $\omega << \omega_\perp^2 \tau$
- Easier theoretical calculation

1D hydrodynamic condition

$$\omega << \omega_\perp^2 \tau$$

Implies that both normal velocity field and temperature variations do **NOT depend on radial variable**.

Follows from the condition $\eta >> m n_{1D} \omega$

(viscous penetration depth larger than radial size)

Independence of superfluid velocity on radial variables follows from equation $m\partial_t \vec{v}_S + \nabla \delta\mu(n) = 0$ ⇨ T=0 1D Hydrodynamics

In a **tube with hard walls**, independence of normal velocity on radial coordinates implies vanishing of normal velocity field (only superfluid can move: **fourth sound** in superfluid helium).

With radial **harmonic** trapping also the **normal** part can **move** .

1D Hydrodynamic equation for **first** sound at unitarity:

$$\boxed{m(\omega^2-\omega_z^2)v_z-\frac{7}{5}m\omega_z^2 z\partial_z v_z+\frac{7}{5}\frac{P_1}{n_1}\partial_z^2 v_z=0}$$

$$n_1=\int n\,dxdy \qquad P_1=\int dxdyP\propto n_1^{7/5}f(T/n_1^{2/5})$$

Solutions are discretrized because of axial trapping

at T=0 ($P_1\propto n_1^{7/5}$) $\quad v_1=z^k+....\quad$ at large T ($P_1=Tn_1$)

$$\boxed{\omega^2=\frac{1}{5}(k+1)(k+5)\omega_z^2} \qquad \boxed{\omega^2=\frac{1}{5}(7k+5)\omega_z^2}$$

Lowest frequency solutions:

- **Sloshing** (k=0, $\omega=\omega_z, v_z=const$)
- **Scaling Axial breathing** (k=1, $\omega=\sqrt{12/5}\,\omega_z$, $v_z=z$)

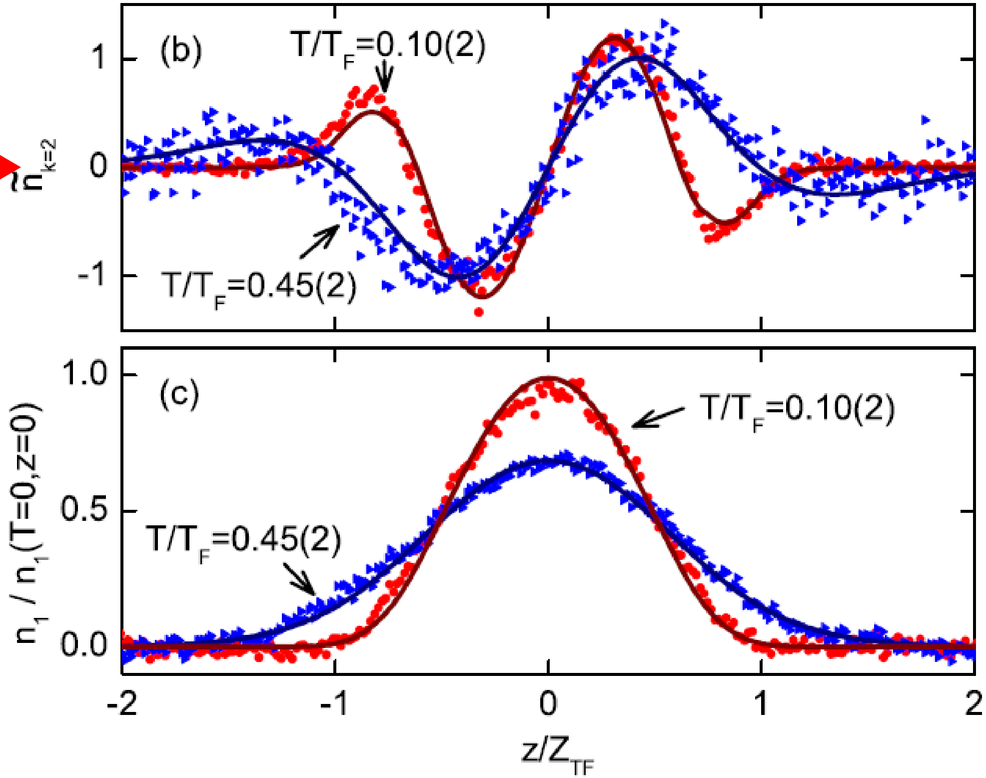
  are temperature independent

Higher nodal modes (k=2) exhibit Temperature dependence (test of EoS and of 1D hydrodynamic approximation)

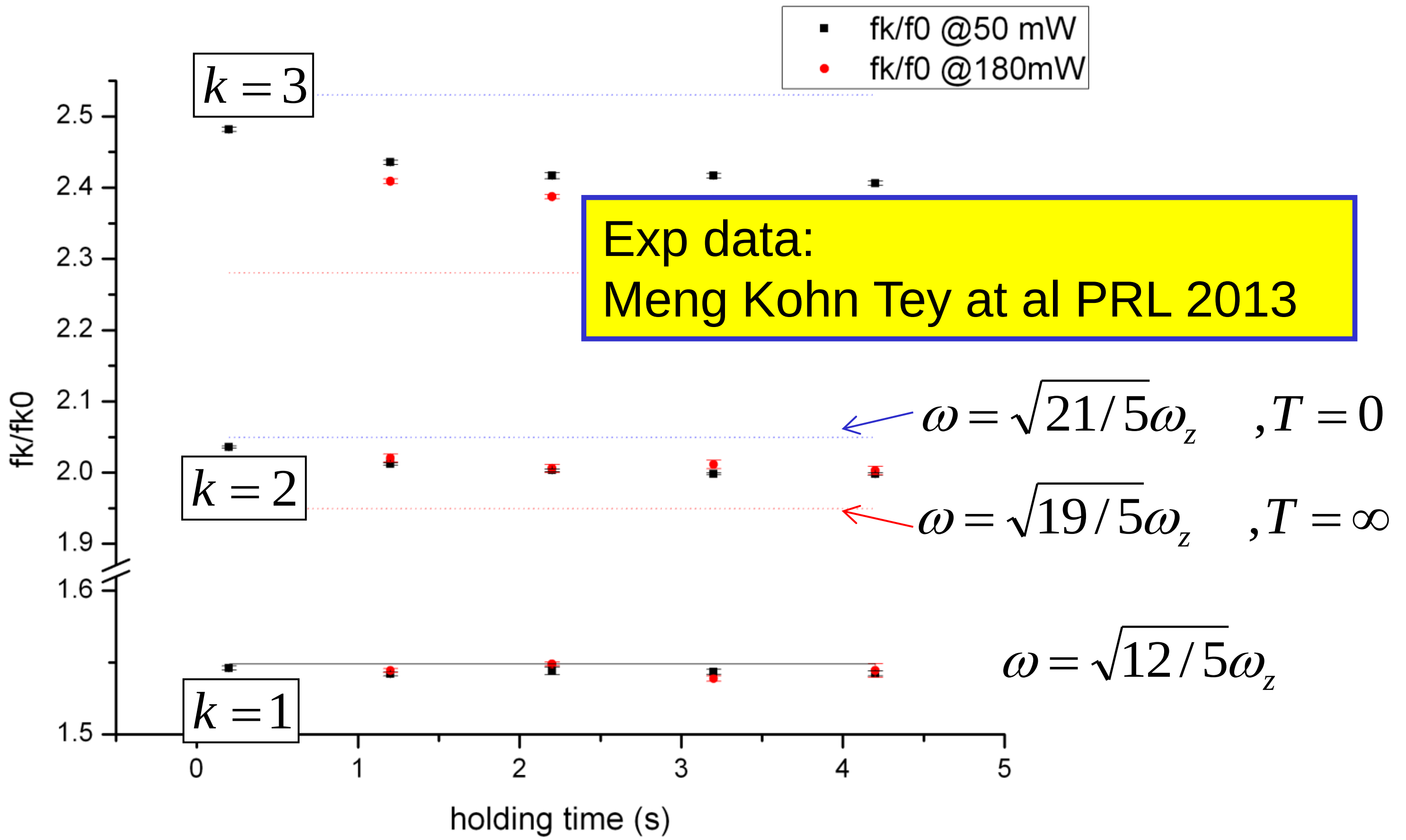
Higher nodal modes can be excited by proper density modulation of laser perturbation

Density modulations of k=2 mode at different T

Equilibrium density profiles at different T



Exp data and theory predictions: Meng Kohn Tey at al PRL 2013



Temperature increases with holding time (heating effect)

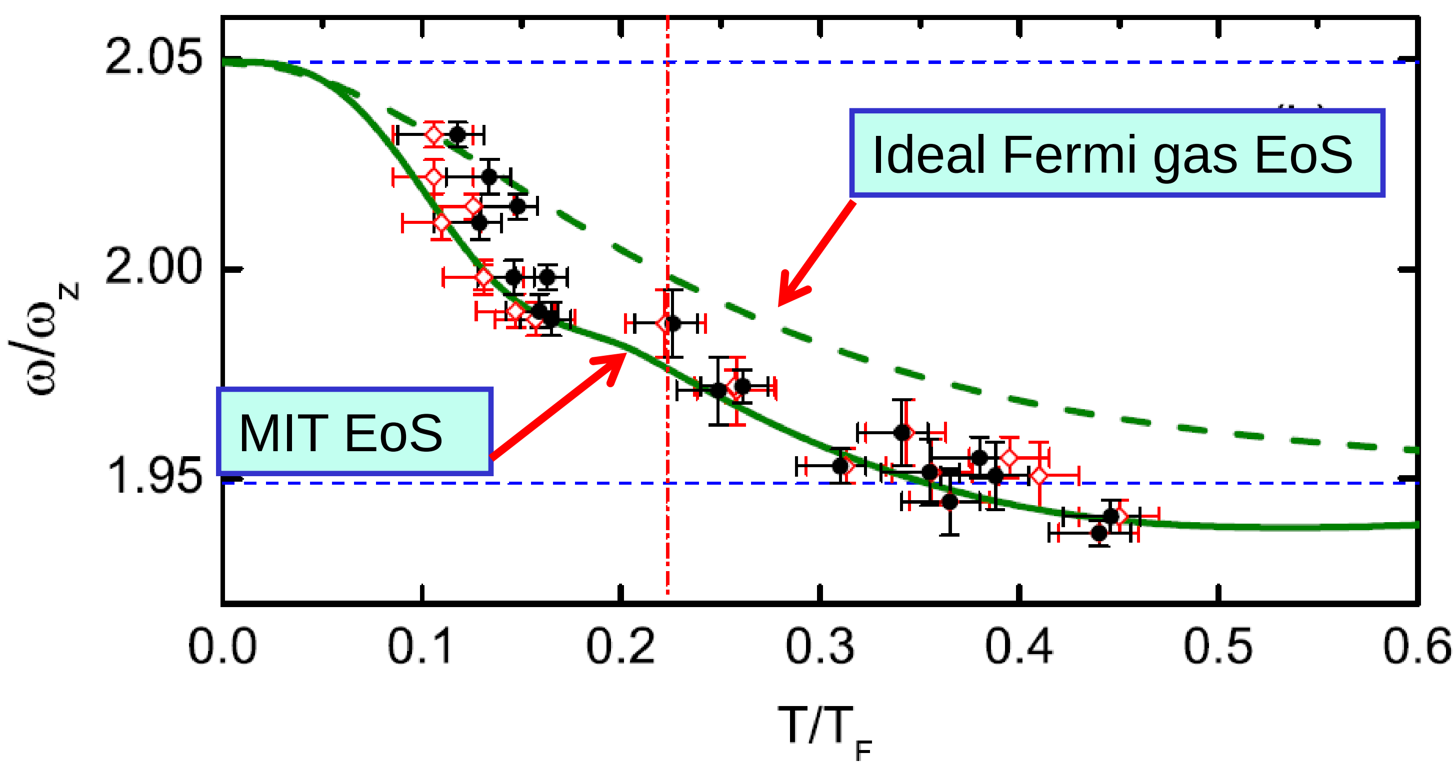
Frequency of scaling axial mode (**k=1**) remains **constant**
Frequency of higher nodal modes (**k=2,3**) **decreases**

Measured temperature dependence of k=2 mode

Theory predictions obtained
solving 1D HD eqs.
with MIT equation of state

$$m(\omega^2-\omega_z^2)v_z-\frac{7}{5}m\omega_z^2 z\partial_z v_z+\frac{7}{5}\frac{P_1}{n_1}\partial_z^2 v_z=0$$

Meng Kohn Tey at al PRL 2013 (IBK-MIT-Trento collaboration)



Measurement of **second sound** and determination of the **superfluid density** in a strongly **interacting Fermi gas**

(Innsbruck- Trento collaboration)

First measurements of second sound carried out at **Utrecht** (2009) in a dilute 1D like Bose gas

- density wave of the condensate, thermal cloud practically remains at rest).
- Sound velocity fixed by temperature dependence of condensate fraction

Phys. Rev. A **80**, 043605 (2009)

Sound propagation in a Bose-Einstein condensate at finite temperatures

R. Meppelink, S. B. Koller, and P. van der Straten[1]

[1]*Atom Optics and Ultrafast Dynamics, Utrecht University, P.O. Box 80,000, 3508 TA Utrecht, The Netherlands*

(Dated: September 18, 2009)

We study the propagation of a density wave in a magnetically trapped Bose-Einstein condensate at finite temperatures. The thermal cloud is in the hydrodynamic regime and the system is therefore described by the two-fluid model. A phase-contrast imaging technique is used to image the cloud of atoms and allows us to observe small density excitations. The propagation of the density wave in the condensate is used to determine the speed of sound as a function of the temperature. We find the speed of sound to be in good agreement with calculations based on the Landau two-fluid model.

More interesting conditions are expected to occur in the **interacting Fermi** gas at unitarity:

- Large **space overlap** between superfluid and normal densities
- **Superfluid** density **different** from pair **condensate** density

In a recent paper we have provided a combined exp + theory investigation of the **propagation of second sound** and of **superfluid density** in a strongly interacting **Fermi** gas

Second sound and the superfluid fraction in a resonantly interacting Fermi gas

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Yan-Hua Hou[1], Lev Pitaevskii[1,2], and Sandro Stringari[1]

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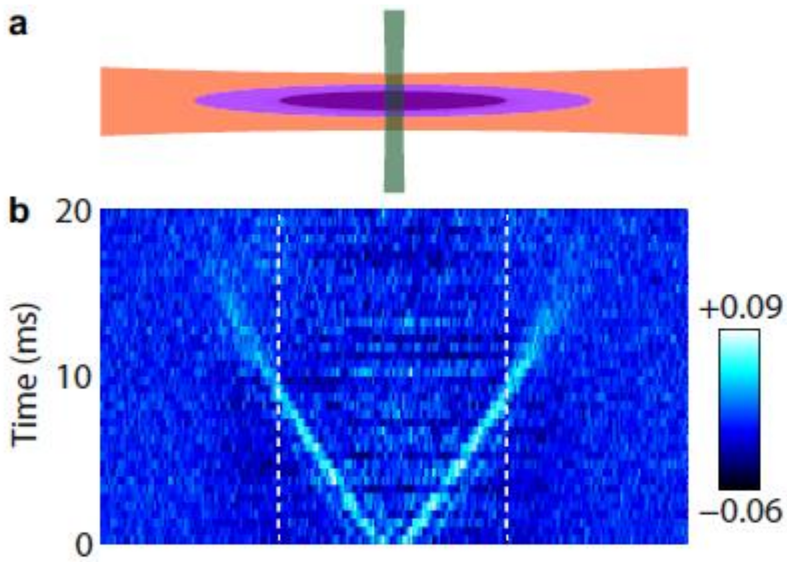
[2] *Kapitza Institute for Physical Problems RAS, Kosygina 2, 119334 Moscow, Russia*

(Dated: February 13, 2013)

arXiv: 1302.2871
Nature, 15 May online

Both **first** and **second** sound have been investigated

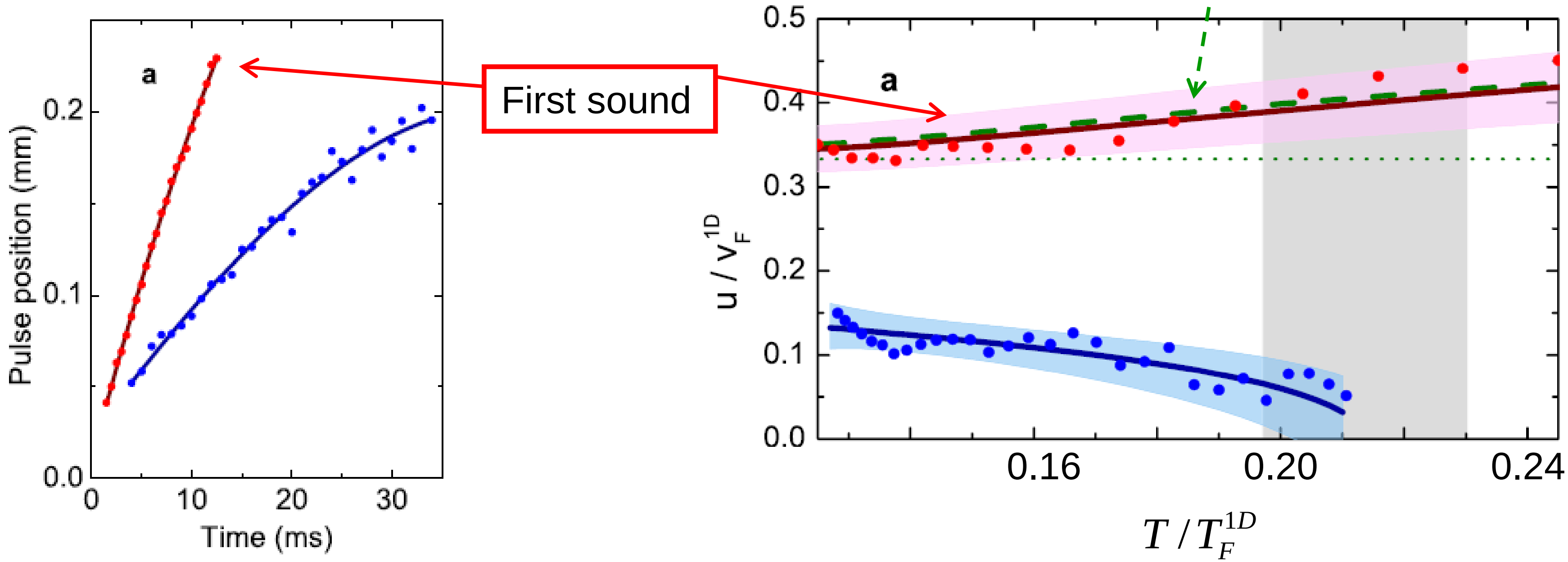
To excite **first sound** one suddenly turns on a repulsive (green) laser beam in the center of the trap [similar tecnhnique used at Mit (1998) and Utrecht (2009) to generate Bogoliubov sound in dilute BEC and at Duke (2011) to excite sound in a Fermi gas along the BEC-BCS crossover at T=0]



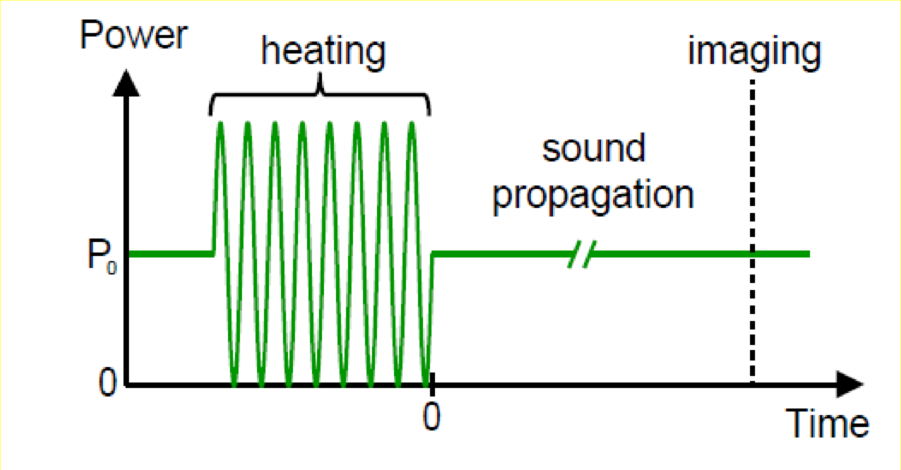
Velocity of **first sound** of radially trapped unitary Fermi gas given by adiabatic law (excellent approximation due to small thermal expansion) also below critical temperature

$$\boxed{mc^2 = \frac{7}{5}\frac{P_1}{n_1}}$$

By measuring velocity of the signal at different times (different pulse positions) one extracts behavior as a function of T/T_F^{1D}. T is fixed, but T_F decreases as the perturbation moves to the periphery (lower density)



To excite **second sound** one keeps the repulsive (green) laser power constant with the exception of a short time modulation producing local heating in the center of the trap

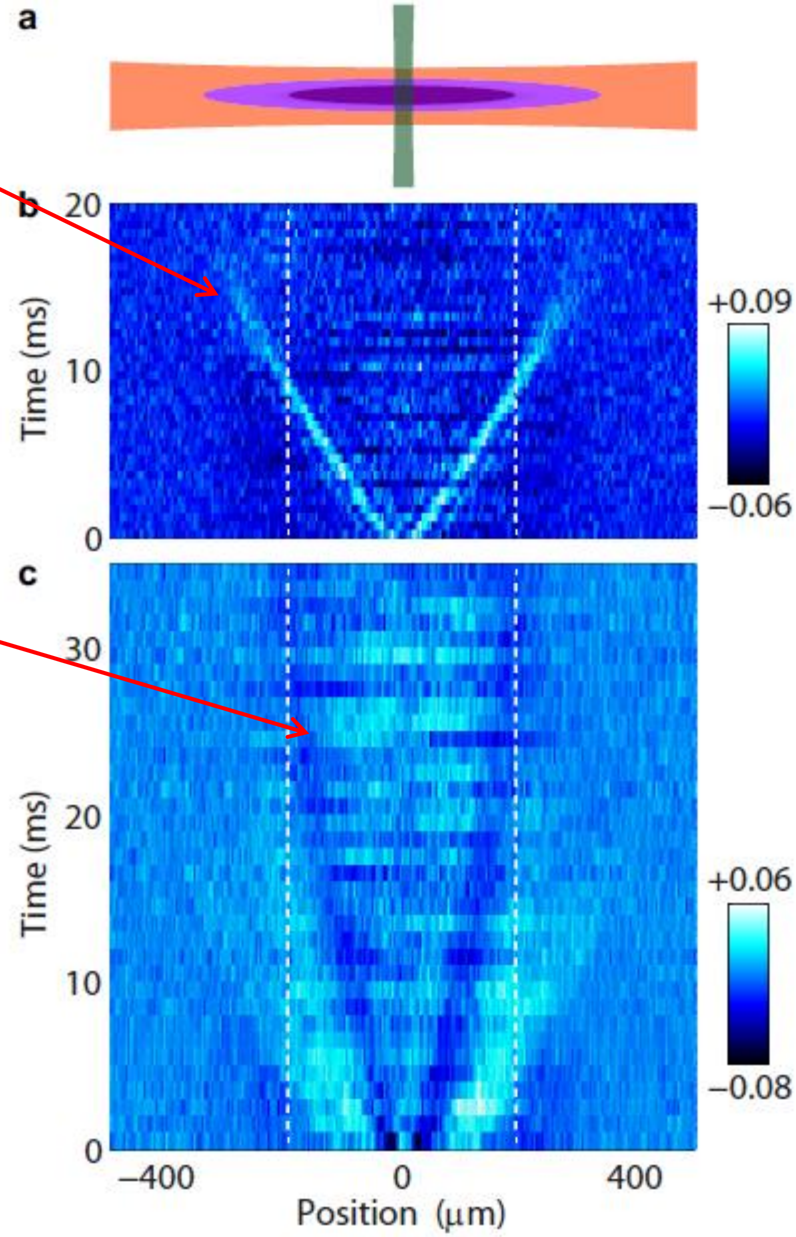


The average laser power is kept constant to limit the excitation of pressure waves (first sound)

First sound
propagates also beyond the boundary between the superfluid and the normal parts

Second sound
propagates only within the region of co-existence of the super and normal fluids.

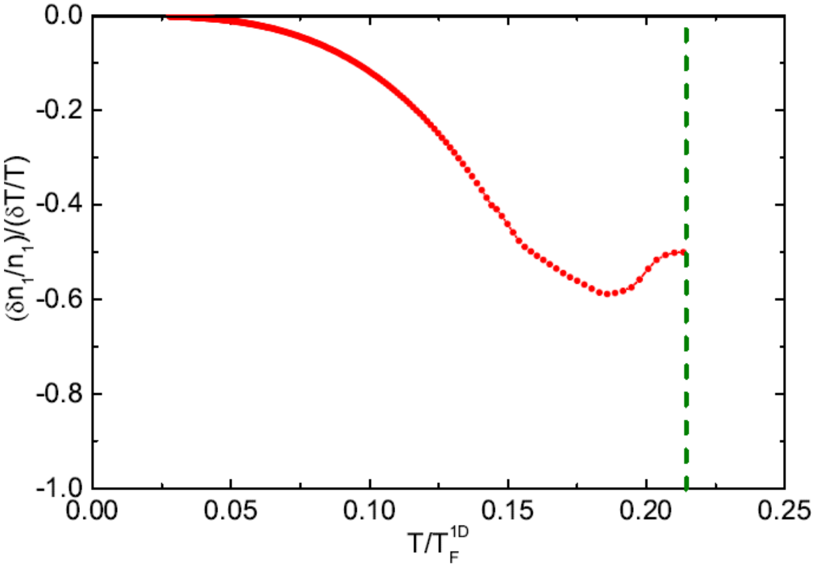
Second sound is basically an isobaric wave, but signal is visbile because of small, but **finite thermal expansion**.



Second sound:
relative density and temperature variations are fixed by thermal expansion (consequence of isobaric nature of second sound) and are not negligeable (except at very small temperatures)

In Innsbruck experiment second sound is **excited** via a **thermal** perturbation and **detected** imaging the propagation of the **density** signal.

$$\frac{\delta n_1 / n_1}{\delta T / T} = -T\alpha_1$$

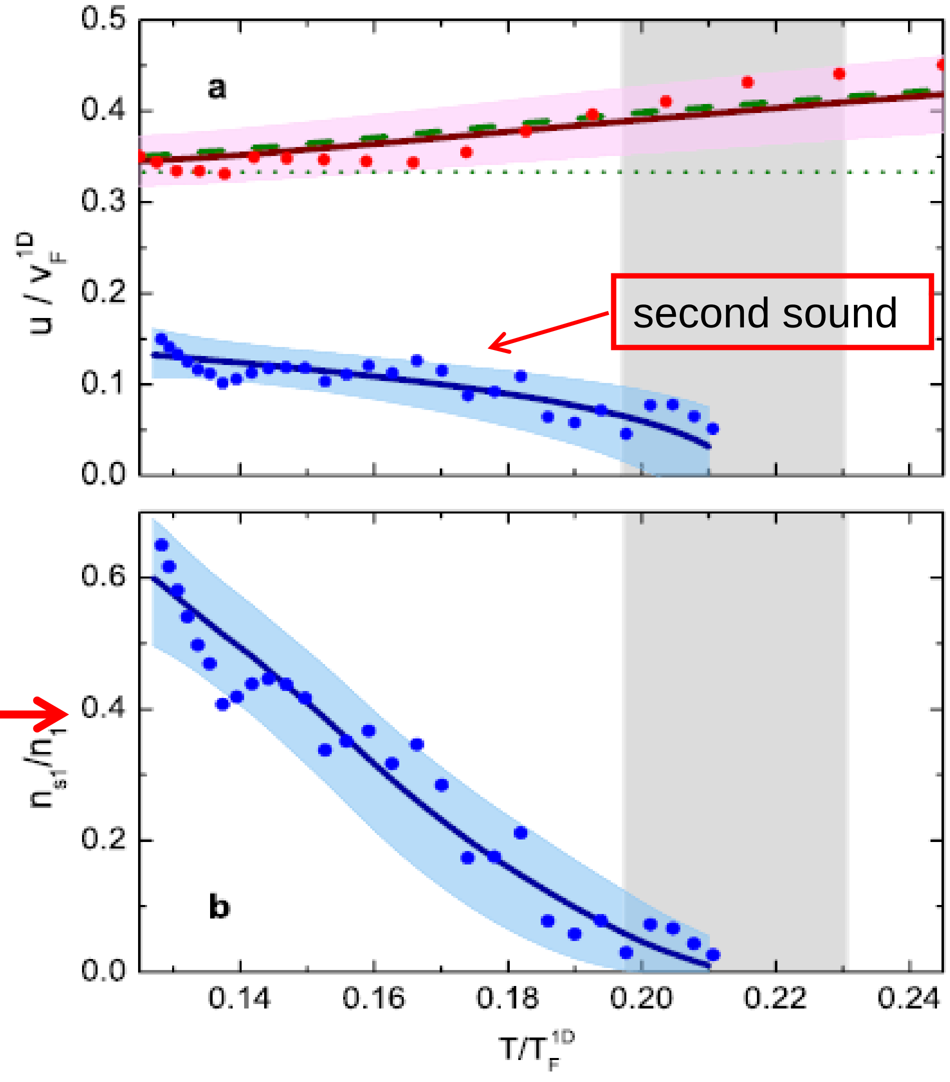


Relative density vs T fluctuations during the propagation of second sound (thermodynamics from MIT)

From measurement of 1D second sound velocity and relationship with 1D superfluid density

$$mc_2^2 = \frac{n_{s1} T s_1}{n_{n1} C_{P1}}$$

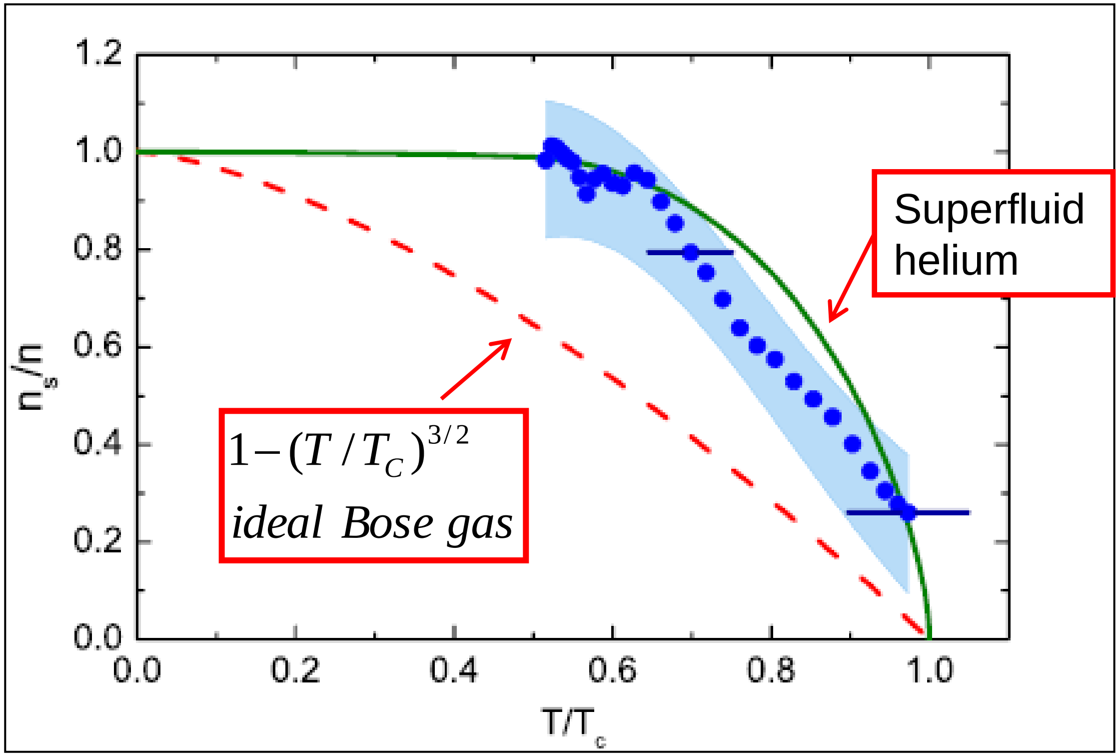
one extracts 1D superfluid density



From integral definition

$$n_{s1} = \int n_s dxdy$$

one can reconstruct 3D superfluid fraction

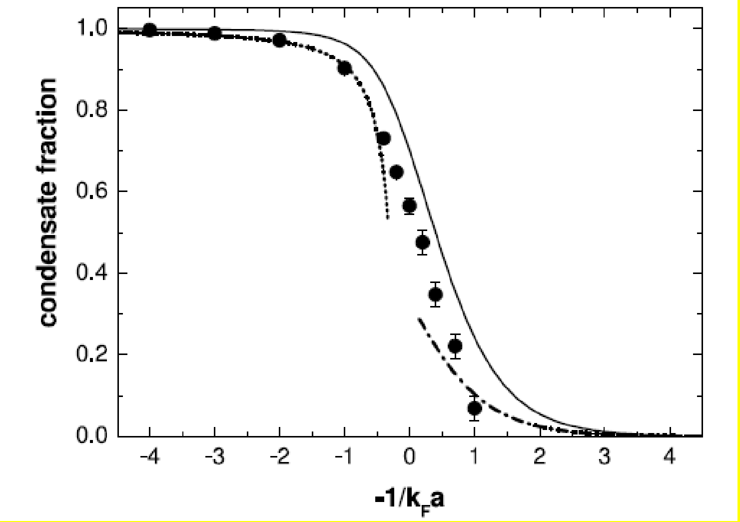


Some comments:

- Superfluid fraction of **unitary Fermi gas** behaves similarly to **superfluid helium** (strongly interacting superfluid)

- Very **different** behavior compared to dilute **BEC gas.** New benchmark for **many-body calculations**

- Superfluid density **differs** significantly from condensate **fraction of pairs** (about 0.5 at T=0, Astrakharchik et al 2005)

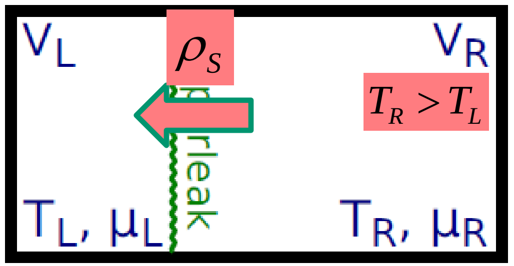


- Condensation **fraction of pairs** measurable by **fast ramping** of scattering length to BEC side (**bimoda**l distribution) (Jila 2004, Mit (2004, 2012))

Other questions concerning superfluid density

- Behavior in **2D** (BKT transition). Superfluid density has a **jump** at the transition. Possible strategies to measure ρ_S
  i) **second sound** in 2D dilute Bose gas
  ii) measurement of **moment of inertia**
  iii) **transverse** response function

- Control of superfluid flow via **superleak** (only superfluid can flow)
  Thermomechanical effect. 'Cooling by heating a superfluid'
  (Papoular et al, PRL 2012)



By heating the rhs ($T_R > T_L$) one predicts a flow of the superfluid from right to left with consequent **increase of quantum degeneracy in the left hand side**.

Change in density fixed by behavior of chemical potential at fixed density

$$\boxed{\delta n = -n^2 k_T \partial_T \mu|_n \, \delta T}$$