



COLLÈGE
DE FRANCE
—1530—

Cuprates supraconducteurs : où en est-on ?

Pseudogap and Nodal/Antinodal Dichotomy:
Cluster-DMFT theoretical viewpoint
* Recent progresses *

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Collaborators:



Ecole Polytechnique:

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Saclay-IPhT: O.Parcollet

Columbia: E.Gull, A.J.Millis

Rutgers: **G.Kotliar**



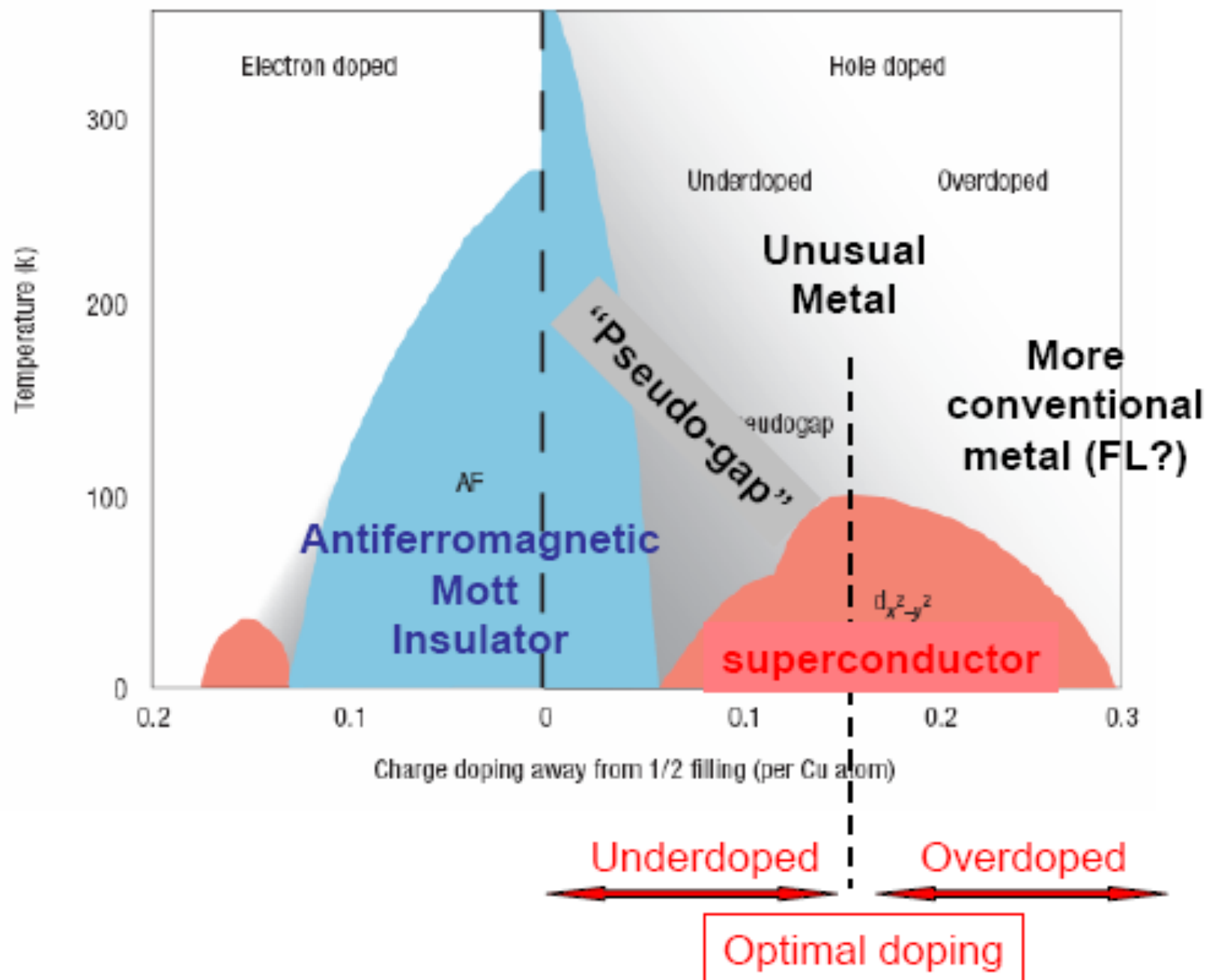
- Europhysics.Lett. 85 (2009) 57009

- Phys Rev B: 80, 064501 (2009); 82, 054502 (2010), 82 ?????

Acknowledgements:

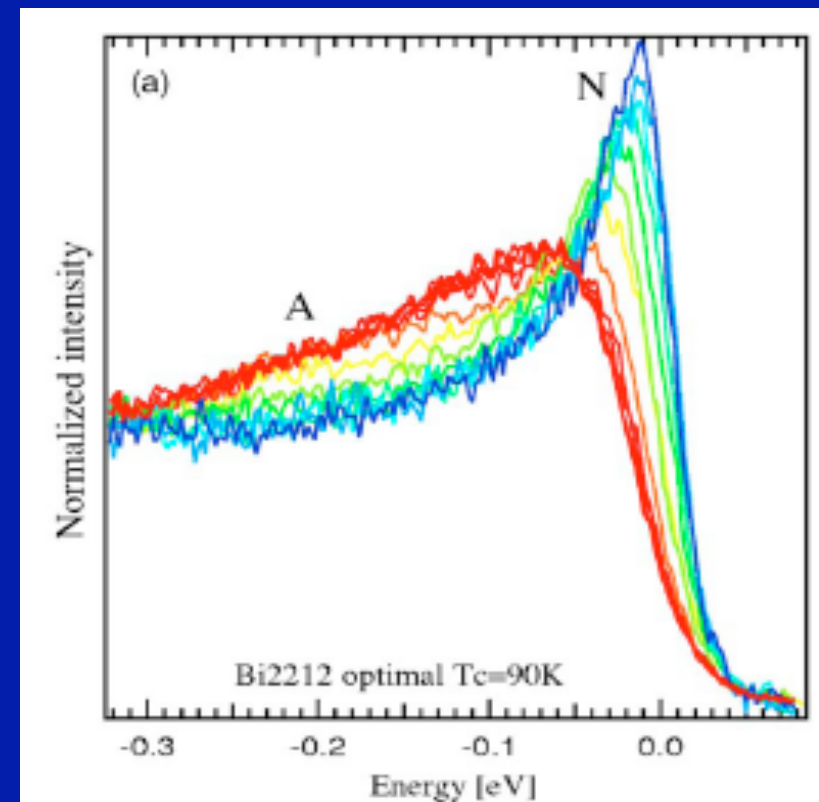
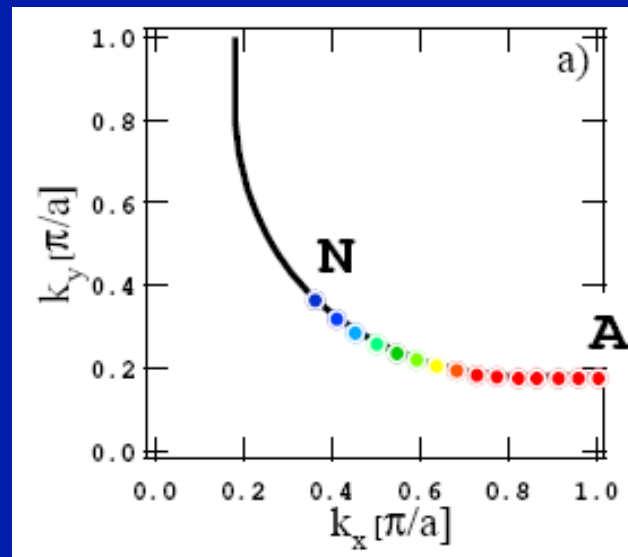
- M.Civelli, M.Capone, F.Lechermann, L.de Medici

- A.Sacuto, M.Le Tacon, S.Blanc, W.Guyard, M.Cazayous, Y.Gallais, D.Colson,
A.Forget



NORMAL state:

- “Nodal” regions display reasonably coherent quasiparticles
- In contrast, excitations in the “antinodal” regions e.g. $(0,\pi)$ are much more incoherent
AND they are (pseudo-) gapped below T^*

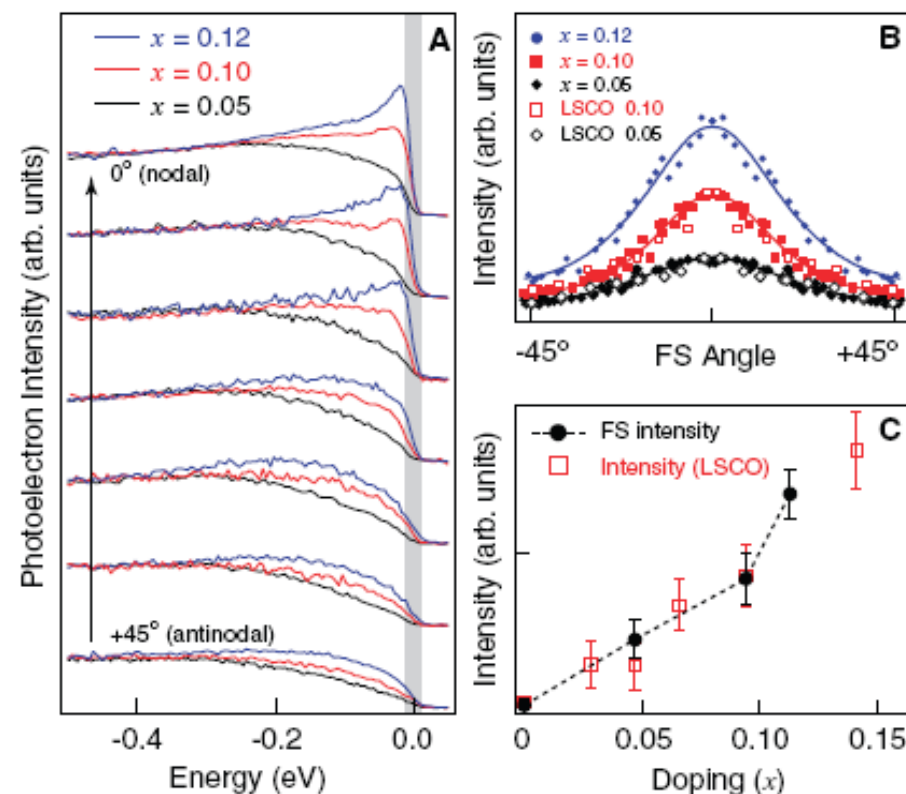
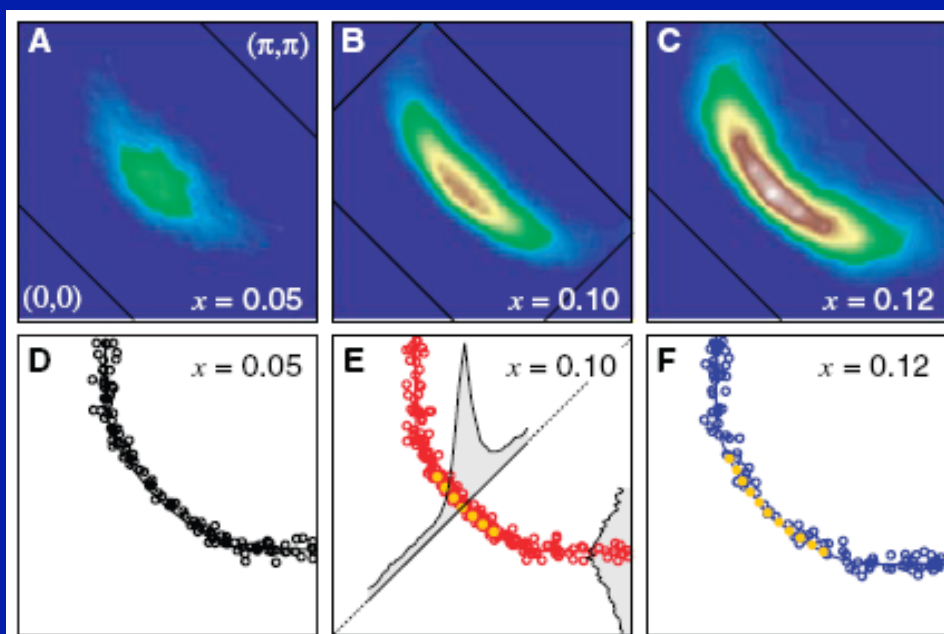


Kaminski et al., 2004 Bi2212
 $T_c=90K @ T=140K$

ARPES sees « Fermi arcs »



K. Shen et al. Science 2007



Physical origin of the pseudogap ?

* Fluctuations of the SC order parameter → most probably NOT

* Long-range ordering ?

cf. seminar by P. Bourges (orbital currents)

→ is it the cause of the PG or a resulting instability ? [chicken or egg ?]

• Signals formation of singlets due to strong antiferromagnetic superexchange [J] ?

cf. RVB ideas.

Crossover, possibly triggering secondary instability

* Is the PG phenomenon captured by the simplest model:
single-band Hubbard or t-J ?

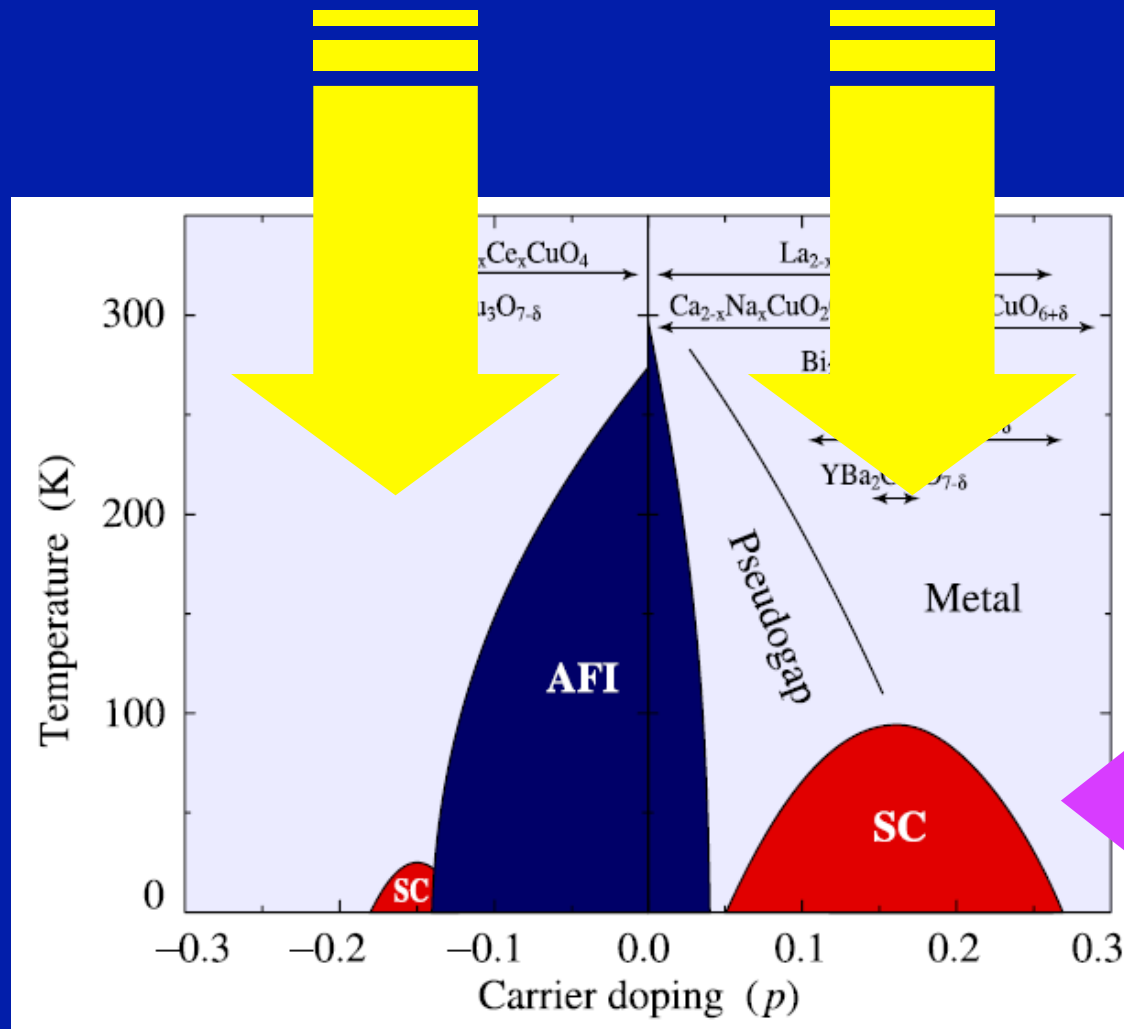
Why is this challenging for theory ?

- Approach to the Mott insulator: *quasiparticle coherence scale*
- Brinkman-Rice/Slave bosons/DMFT: *Uniform scale along Fermi surface (of order δt)*
- Need to take into account inter-site superexchange (J)

→ singlet formation/spatial correlations

Brinkman-Rice/DMFT leads e.g. to a large effective mass $\sim 1/\delta$
while in fact $\sim 1/J$ is expected for $J < \delta t$
(Indeed: spin entropy released at scale J)

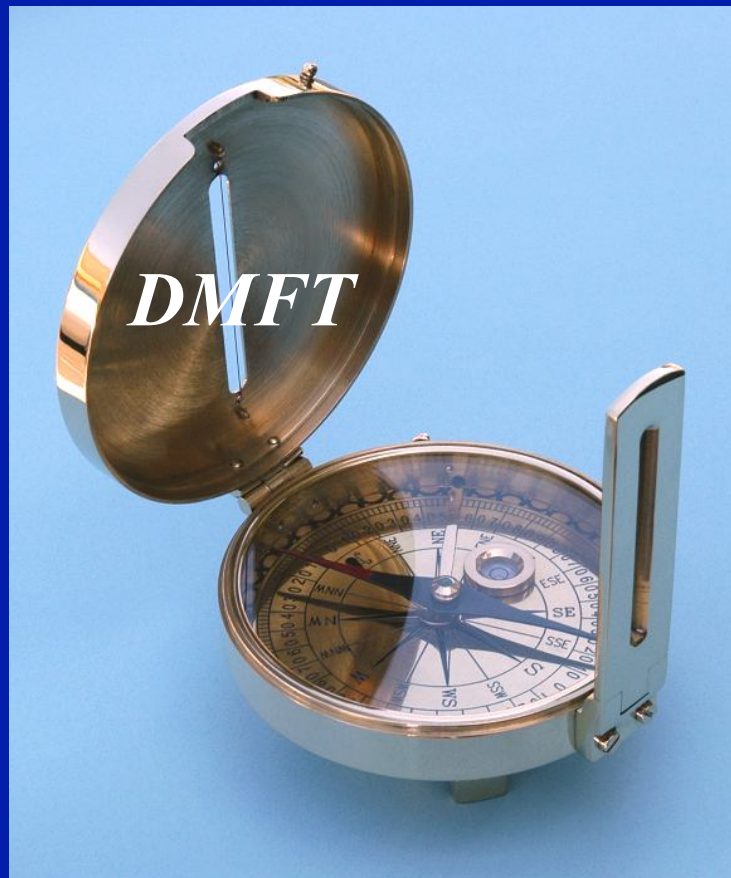
From high to low energy/temperature: DMFT and cluster extensions “top to bottom approach”



From high to
low
doping...



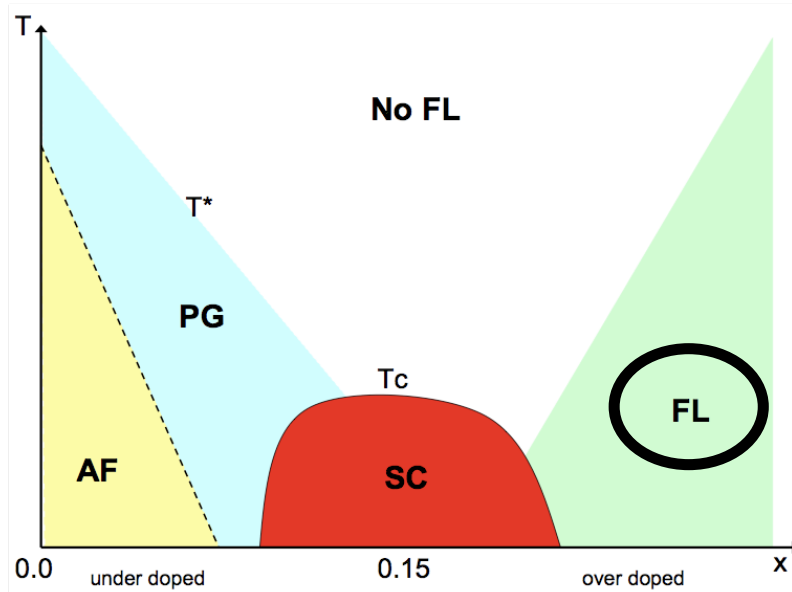
*Flowing down along RG trajectories...
...Need a « compass » to orient ourselves,*



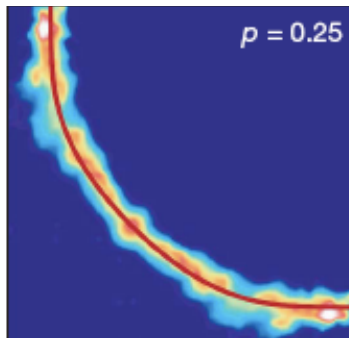
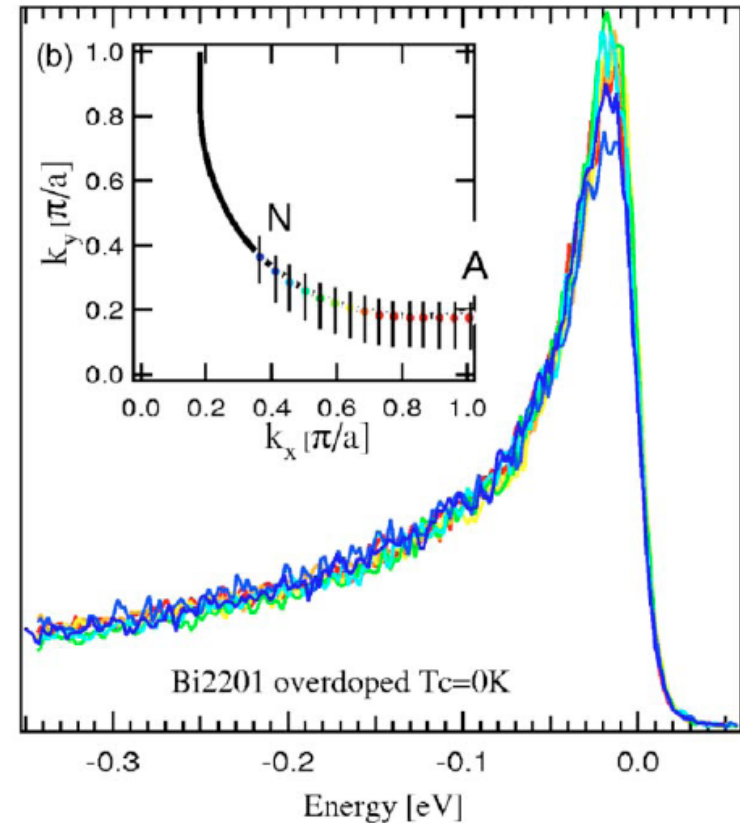
Starting from:

- High-temperature/*
- High-energy/*
- High-doping level ...*

At very high doping levels, to the right of the SC dome: physics ~ uniform in momentum space



*Kaminski et al.,
PRB (2005)*

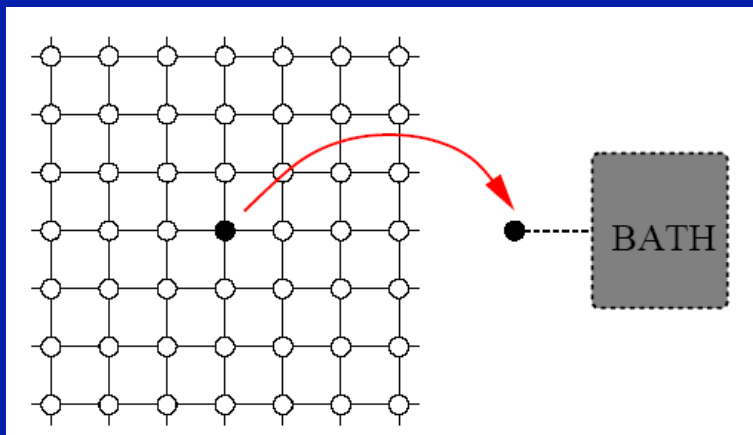


Platé et al., PRL (2005)
TI2201 OD

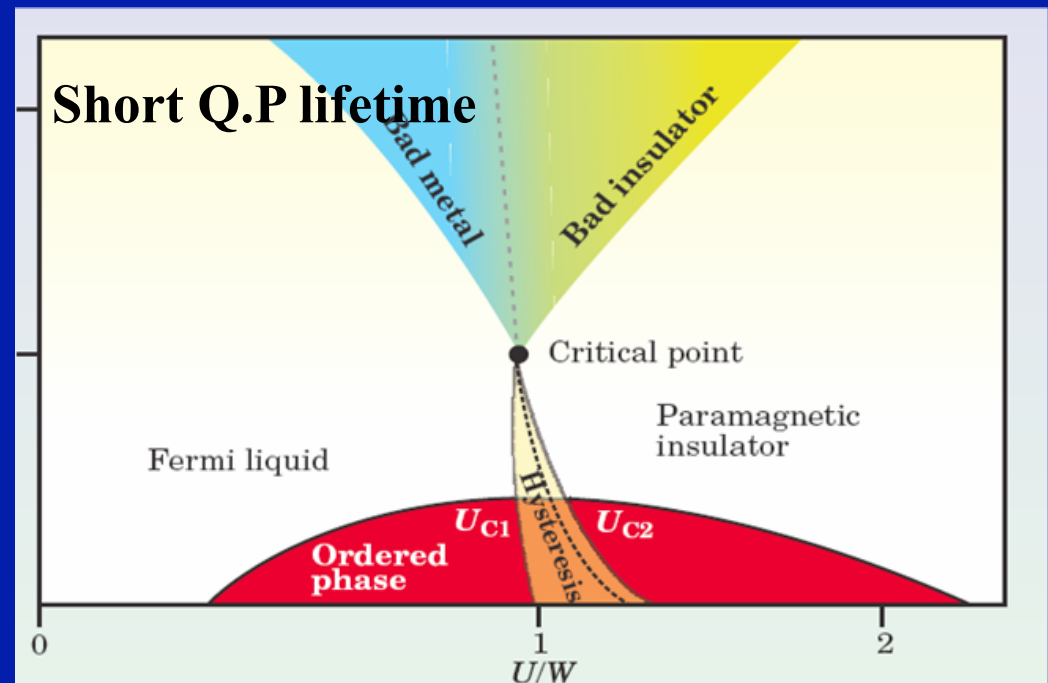
Single-site DMFT is a good approximation in this regime,
Properties are close to that of a Fermi-liquid at low-T, crossover to incoherent at higher T

Dynamical mean-field theory (single-site)

- Provides a simple theory of the proximity of the Mott transition
- Good at describing the **destruction of coherent quasiparticles** (small QP coherence scale, short lifetime near Mott transition)



T



Coherent excitations: k-space (wave)
Incoherent excitations: real space (particle)

2D Hubbard model (t, t', U)

$$H = \sum_{\mathbf{k}, \sigma=\uparrow, \downarrow} \varepsilon_{\mathbf{k}} c_{\sigma\mathbf{k}}^{\dagger} c_{\sigma\mathbf{k}} + U \sum_i n_{i\downarrow} n_{i\uparrow}$$

$$\varepsilon_{\mathbf{k}} = -2t(\cos(\mathbf{k}_x) + \cos(\mathbf{k}_y)) - 4t' \cos(\mathbf{k}_x) \cos(\mathbf{k}_y).$$

In the following:

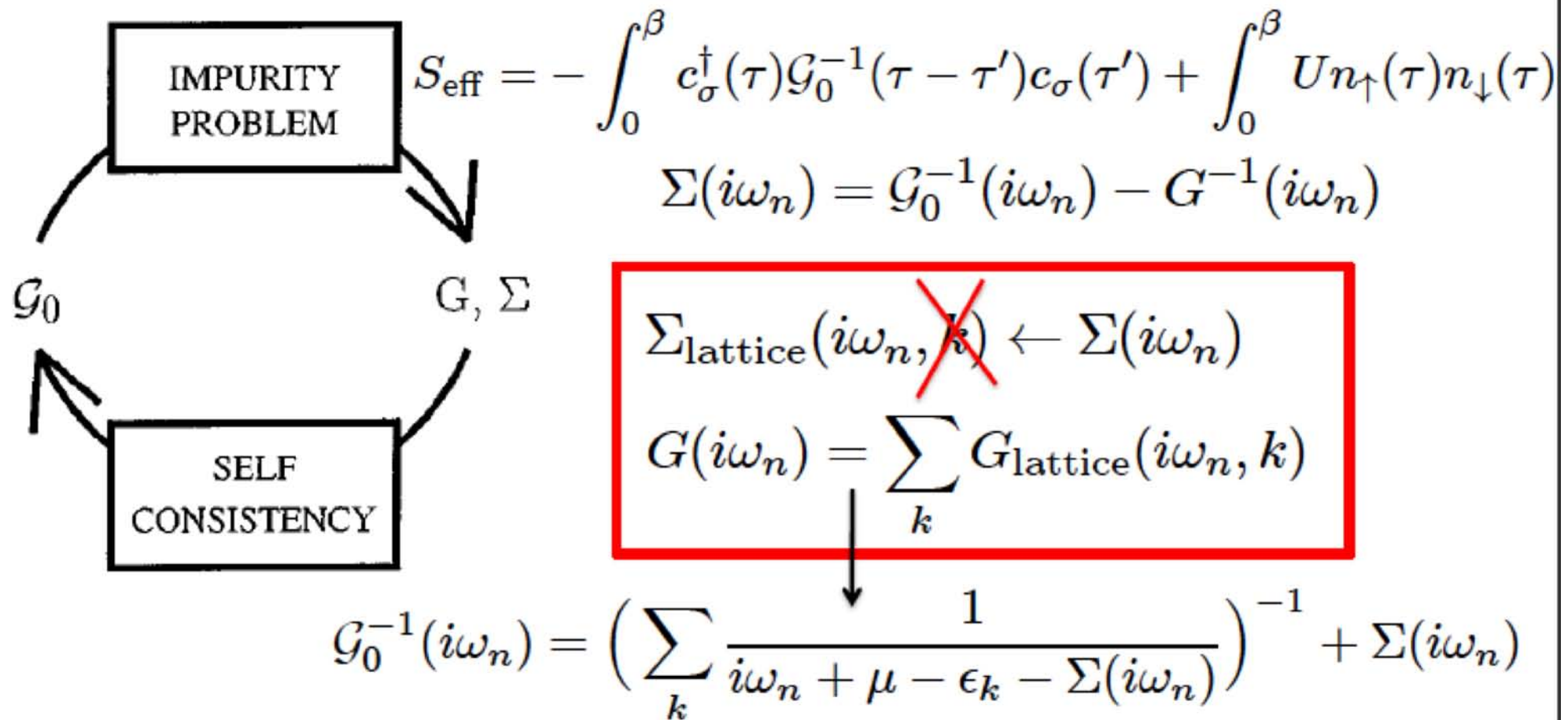
$$U/t = 10$$

$$t'/t = -0.3$$

Hole-doped

Unit of energy: $4t (= 1)$

DMFT equations



In DMFT the self-energy of the lattice is local: Z , m^* , coherence temperature, lifetimes are **constant along the Fermi surface**

Beyond DMFT: accounting for momentum dependence

“Cluster” extensions - come in various flavors, DCA, CDMFT etc...

For reviews see:

- ²⁷ T. Maier, M. Jarrell, T. Pruschke, and M. H. Hettler, *Rev. Mod. Phys.* **77**, 1027 (2005).
- ²⁸ G. Kotliar, S. Y. Savrasov, K. Haule, V. S. Oudovenko, O. Parcollet, and C. A. Marianett, *Rev. Mod. Phys.* **78**, 865 (2006).
- ²⁹ A. M. S. Tremblay, B. Kyung, and D. Senechal, *Low Temp. Phys.* **32**, 424 (2006).

Previous / Other work on (mostly small) cluster DMFT in 2D:

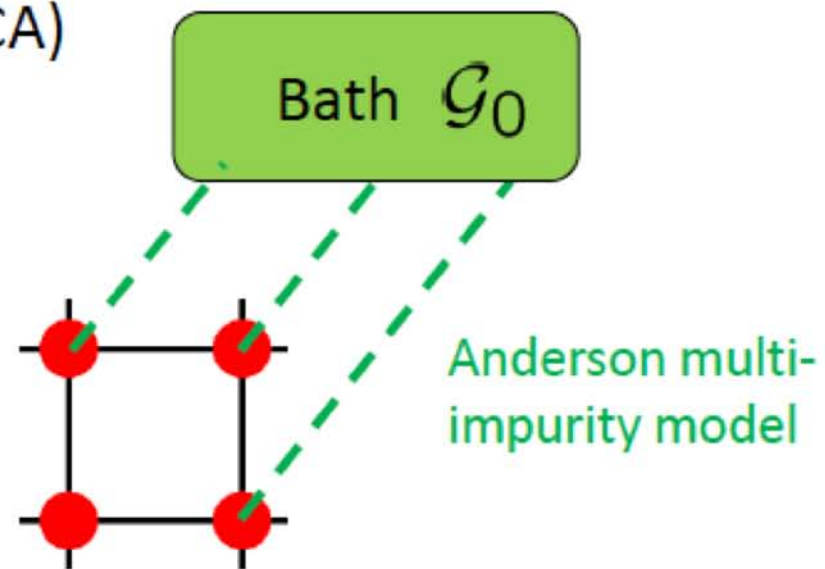
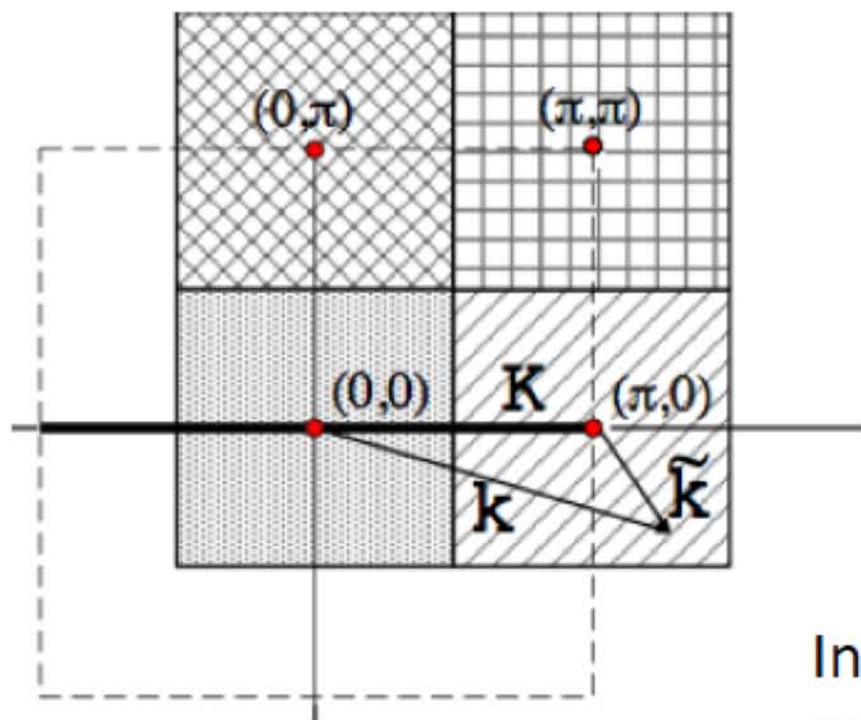
O. Parcollet, G. Biroli, and G. Kotliar, *Phys. Rev. Lett.* **92**, 226402 (2004), M. Civelli, M. Capone, S. S. Kancharla, O. Parcollet, and G. Kotliar, *Phys. Rev. Lett.* **95**, 106402 (2005), T. Maier, M. Jarrell, T.C. Schulthess, P.R.C. Kent, J.B. White, *Phys. Rev. Lett.* **95**, 237001 (2005), B. Kyung, S. S. Kancharla, D. Sénéchal, A.-M. S. Tremblay, M. Civelli, and G. Kotliar, *Phys. Rev. B* **73**, 165114 (2006), T. D. Stanescu and G. Kotliar, *Phys. Rev. B* **74**, 125110 (2006), A. Macridin, M. Jarrell, T. Maier, P. R. C. Kent, and E. D’Azevedo, *Phys. Rev. Lett.* **97**, 036401 (2006), Y. Z. Zhang and M. Imada, *Phys. Rev. B* **76**, 045108 (2007), M. Civelli, M. Capone, A. Georges, K. Haule, O. Parcollet, T. D. Stanescu, and G. Kotliar, *Phys. Rev. Lett.* **100**, 046402 (2008), E. Gull, P. Werner, X. Wang, M. Troyer, and A. J. Millis, *EPL* **84**, 37009 (2008), H. Park, K. Haule, and G. Kotliar, *Phys. Rev. Lett.* **101**, 186403 (2008), M. Ferrero, P. S. Cornaglia, L. D. Leo, O. Parcollet, G. Kotliar, and A. Georges, *EPL* **85**, 57009 (2009), M. Ferrero, P. S. Cornaglia, L. De Leo, O. Parcollet, G. Kotliar, and A. Georges, *Phys. Rev. B* **80**, 064501 (2009), M. Civelli, *Phys. Rev. B* **79**, 195113 (2009), A. Liebsch and N.-H. Tong, *Phys. Rev. B* **80**, 165126 (2009), N. Lin, E. Gull, and A. J. Millis, *Phys. Rev. B* **80**, 161105(R) (2009), S. Sakai, Y. Motome, and M. Imada, *Phys. Rev. Lett.* **102**, 056404 (2009), G. Sordi, K. Haule, and A. Tremblay, *Phys. Rev. Lett.* **104**, 226402 (2010), S. Sakai, Y. Motome, and M. Imada, arXiv:1004.2569, M. Ferrero, O. Parcollet, G. Kotliar, and A. Georges, arXiv:1001.5051.

Including a k-dependence

Dynamical cluster approximation (DCA)

M. H. Hettler et al., PRB (1998)

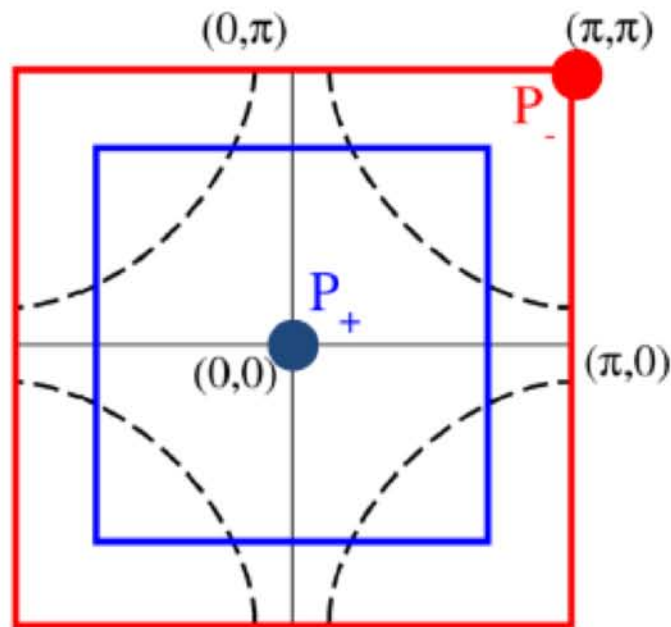
T. Maier et al., RMP (2005)



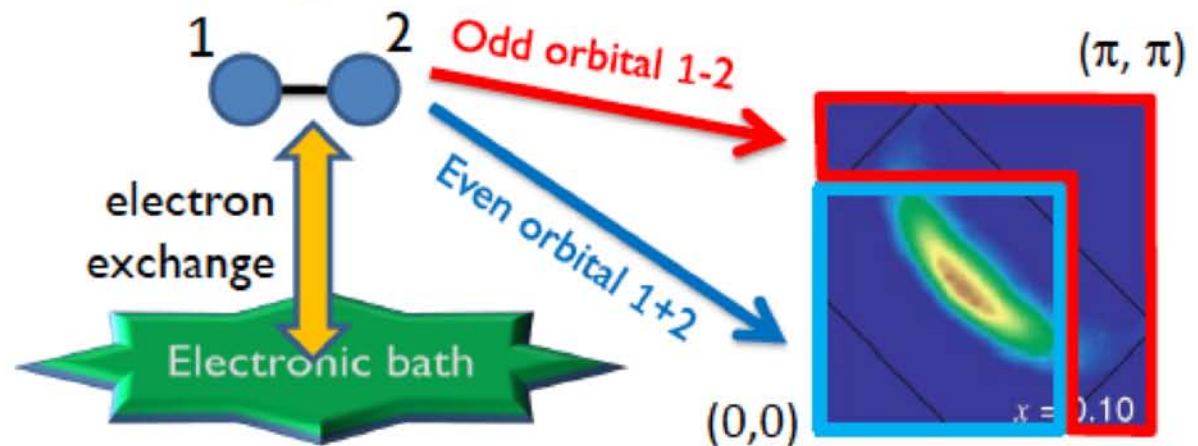
Cluster self-energies for every cluster momenta: $\Sigma(K)$

In DCA, the lattice self-energy is constant over the patches and equal to the cluster self-energies

Two-site DCA: A valence-bond DMFT

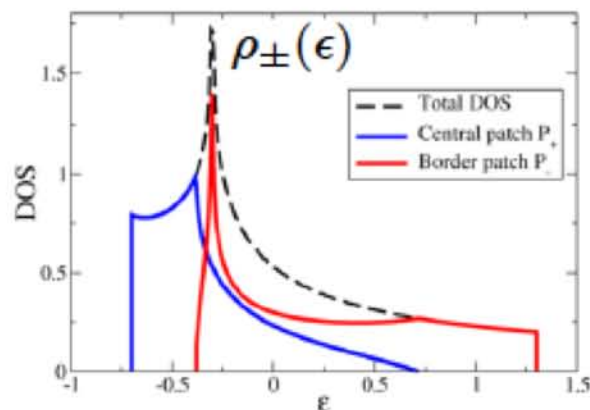


Two-site Anderson impurity model



$$\Sigma_{\text{lattice}} = \begin{cases} \Sigma_+ & \text{in the central patch} \\ \Sigma_- & \text{in the border patch} \end{cases}$$

One patch covers the nodal part of the BZ. The other covers the antinodal part



$$G_{\pm}(i\omega_n) = \int \frac{\rho_{\pm}(\epsilon)}{i\omega_n + \mu - \epsilon - \Sigma_{\pm}(i\omega_n)}$$

Solved by CTQMC + Padé approximants

Recent breakthroughs

entering a new age for such approaches...

Continuous-time quantum Monte Carlo (CT-QMC)

*Rubtsov 2005 Interaction expansion(CT-INT)

*Werner/AJM 2006 Hybridization expansion(CT-HYB)

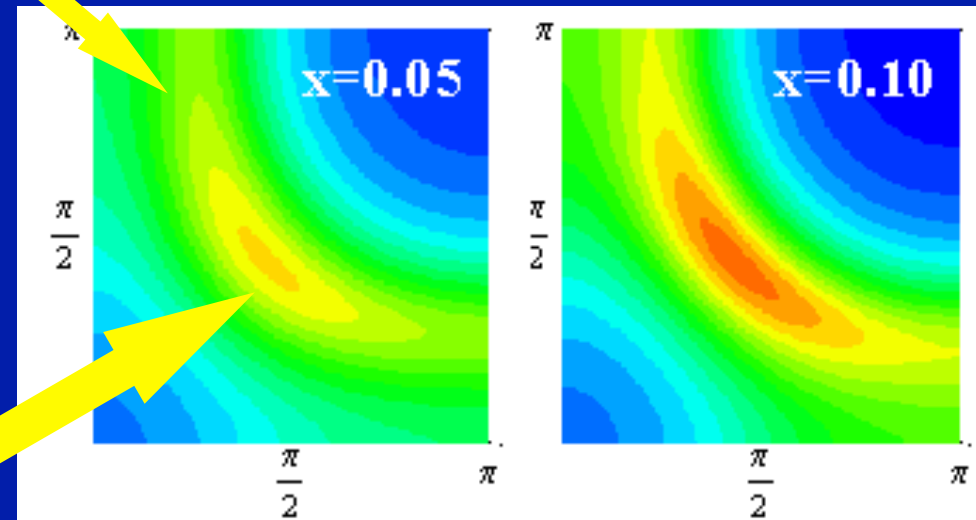
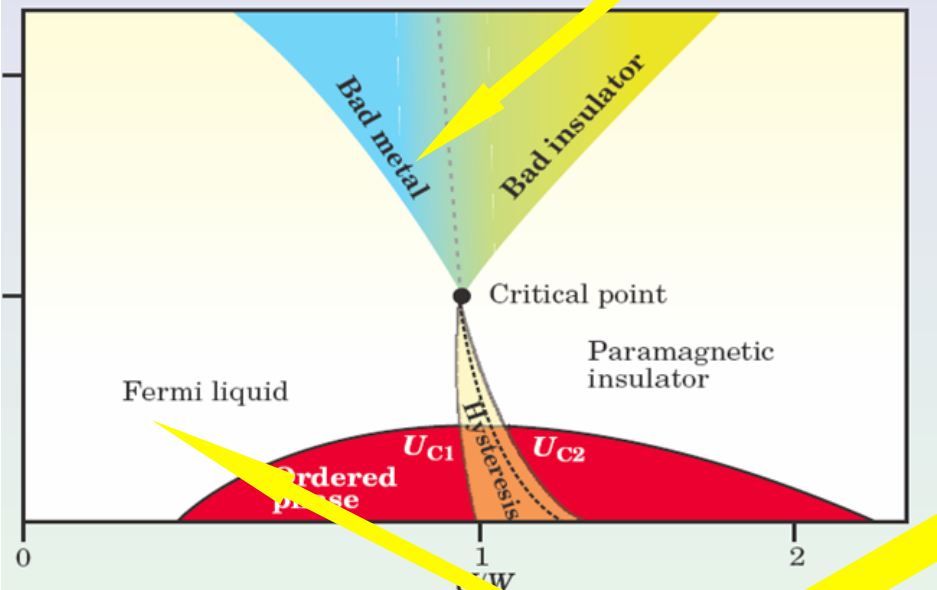
*Gull/Parcollet 2008 Auxiliary field (CT-AUX)

* Gull 2010 submatrix updates: convergence in cluster size reached for some parameter regime ! (full k-dependence resolved)

Early studies indicate that DCA/CDMFT schemes can capture pseudogap formation and nodal/antinode differentiation

PG: Huscroft et al., PRL 2001, N/AN: Civelli et al. PRL 2005

Antinodal Region



Nodal Region

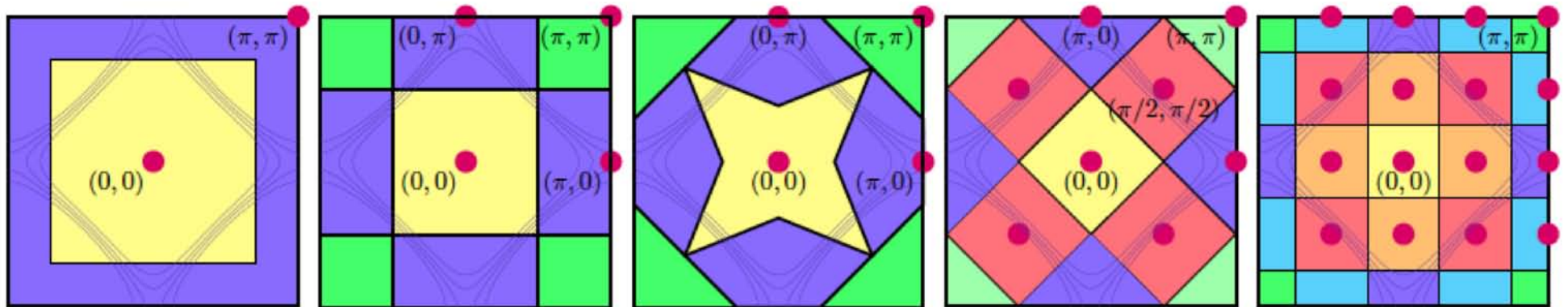
Region near $(\pi, 0)$ has higher scattering

Civelli et al., PRL 2005

How does this work ?

- Physical understanding as a momentum-selective Mott transition
 - Generic features
 - Systematic studies...
- from smallest to huge clusters !

Increasing k-space resolution...



[2,2]

[4,4]

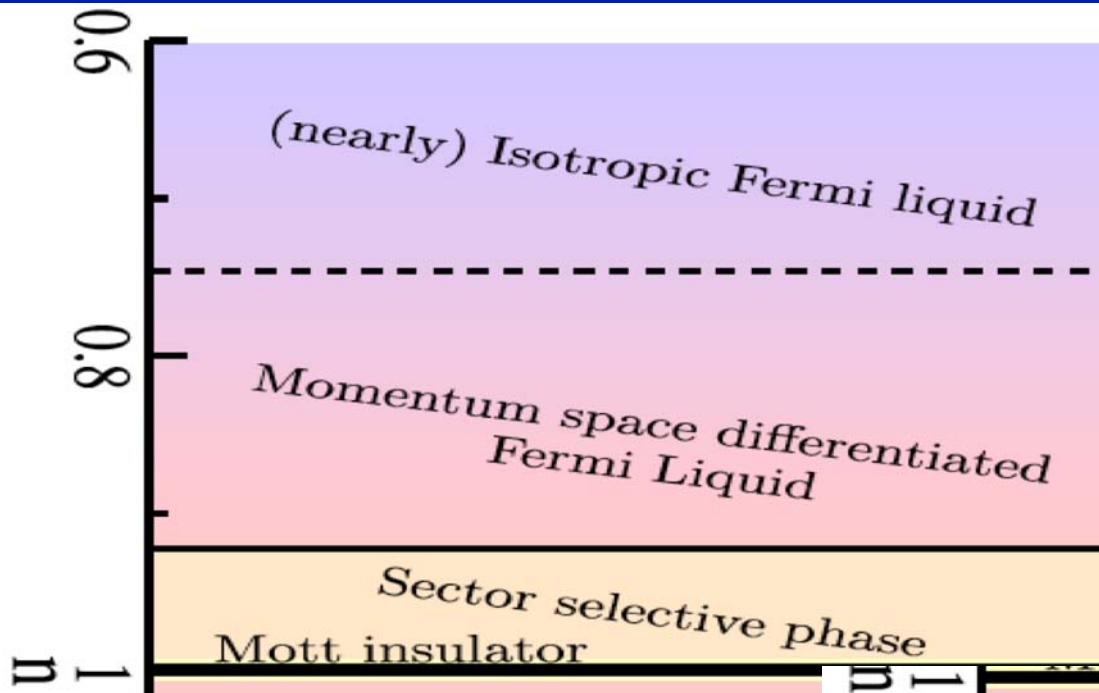
[4,4]*

[5,8]

[9,16]

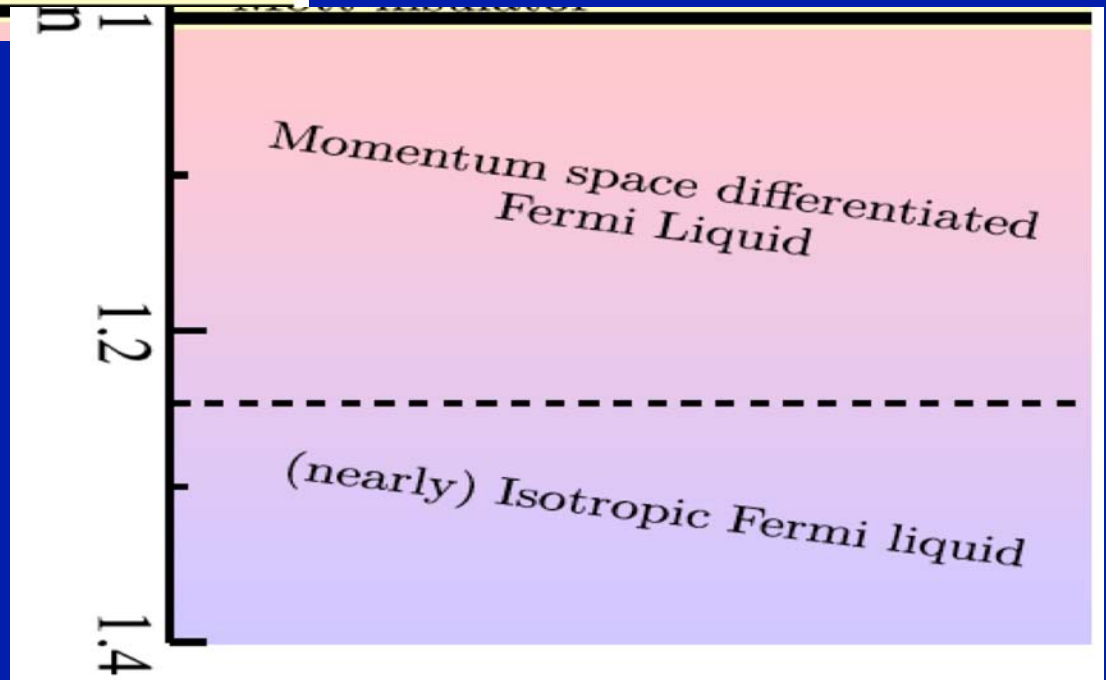
[Patches in $\frac{1}{4}$ BZ, Sites in real-space cluster]

Generic behavior
in different doping
regimes



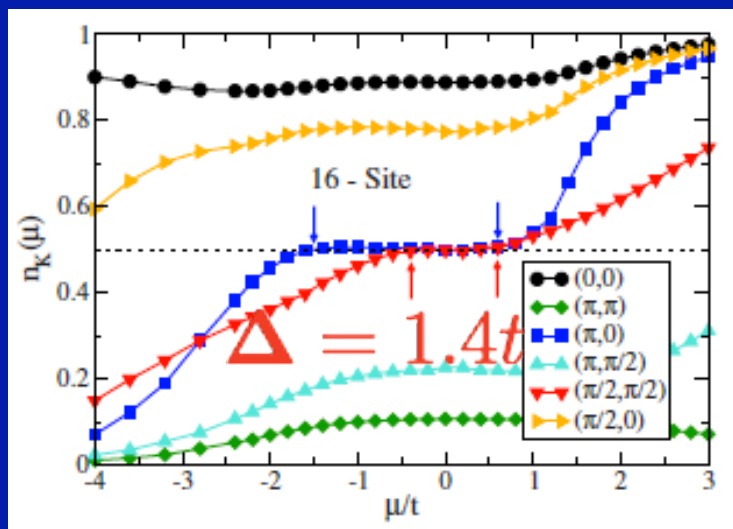
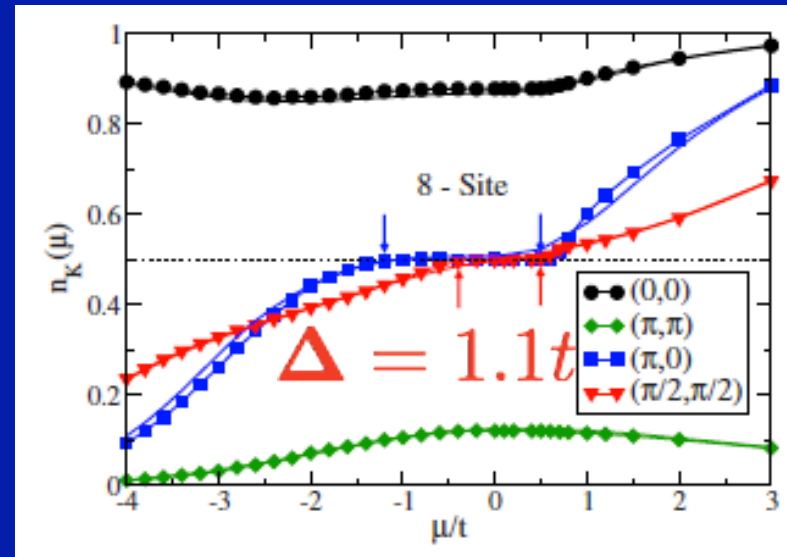
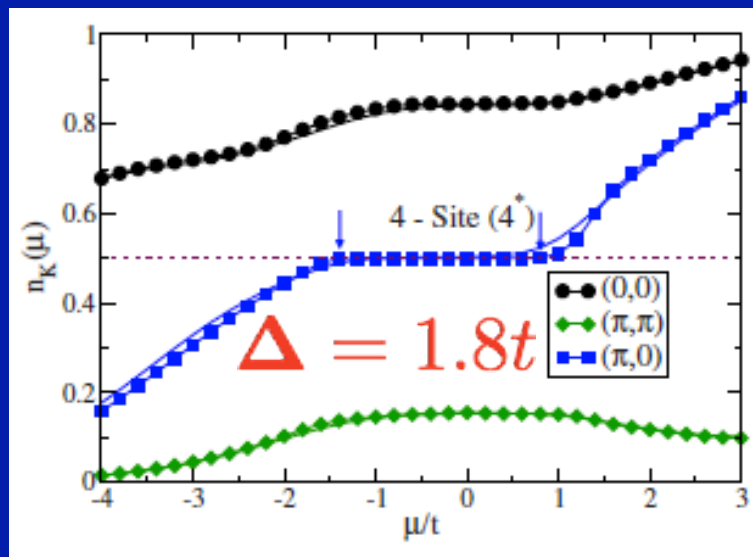
Hole doped

- Isotropic FL at hi-doping
- Momentum differentiation at intermediate doping
- Small HOLE-doping (only):
Momentum-sector selective Mott transition



Electron doped

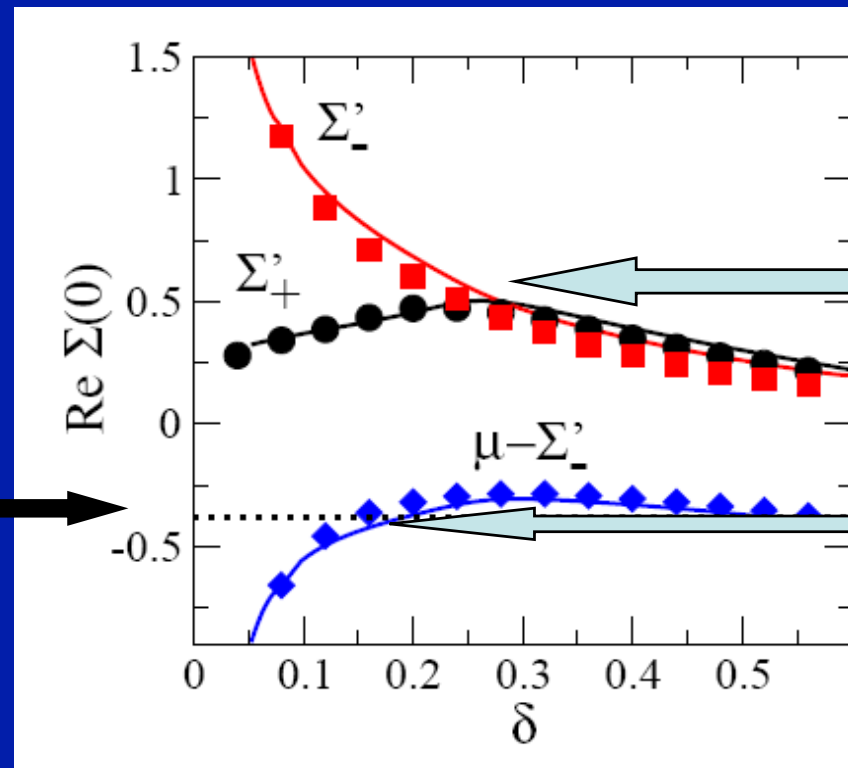
Momentum sector occupancy vs chemical potential: incompressibility of antinodes in sector-selective regime



Cluster size	n_{diff}^h	n_{diff}^e	Δ_g	n_{SST}^h	Δ_{SST}
2	0.66	1.27	1.4		
4	0.65	1.38	2.6		2.6
4^*	0.69	1.39	1.8	0.96	2.4
8	0.72	1.23	1.1	0.93	1.9
16	0.65	1.35	1.4	0.91	2.1

2-patch (VB-DMFT) theory adopts a somewhat special route to momentum selective transition:

Band edge of outer patch d.o.s

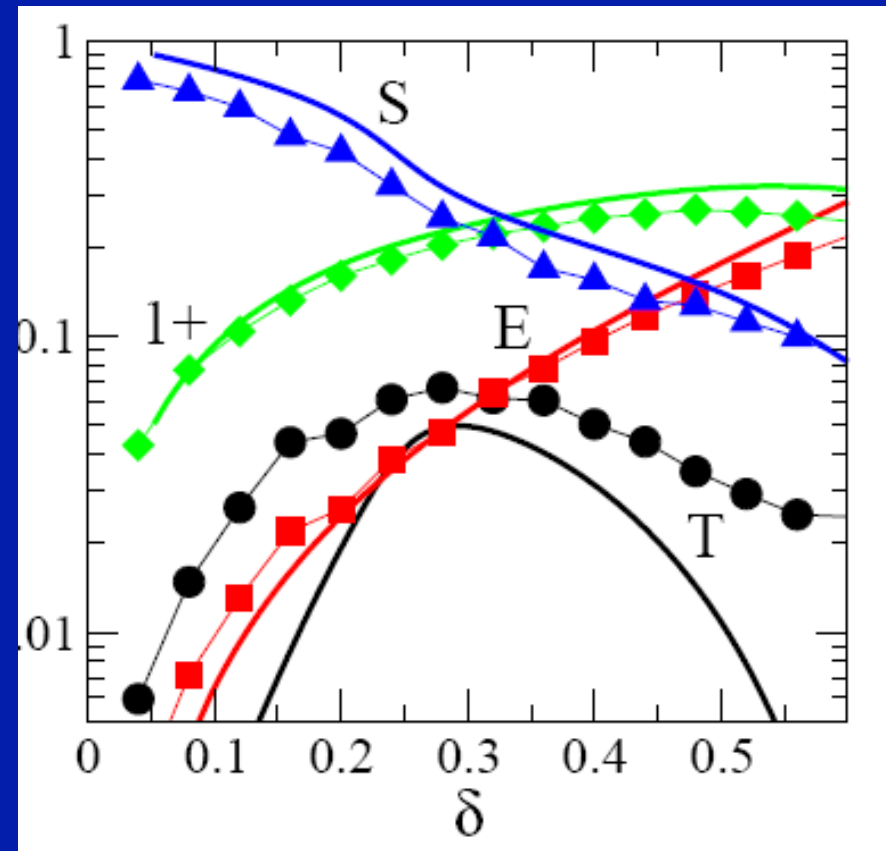


Onset of differentiation

Outer/odd
Orbital
gets (pseudo-)
gapped

Zero-frequency self-energies vs. doping

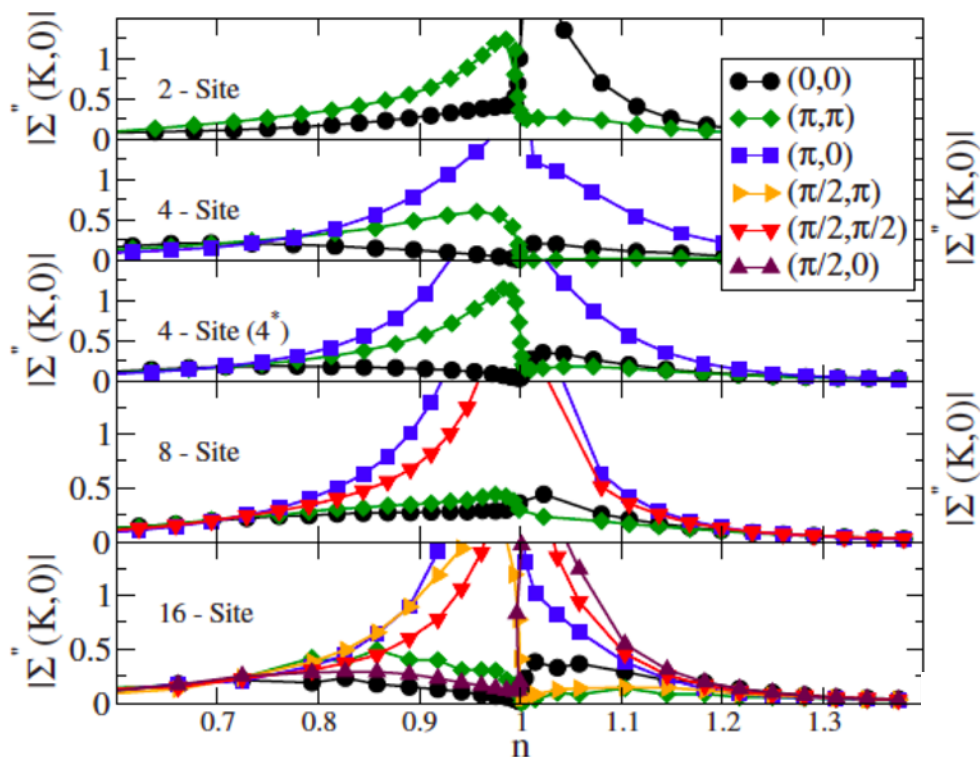
- Low doping MS regime is dominated by formation of singlets on bonds (evolves to ~ Brinkman-Rice at higher doping)



Statistical weights of the singlet (S) and 1-particle (1+) state vs. Doping in the CT-QMC calculation

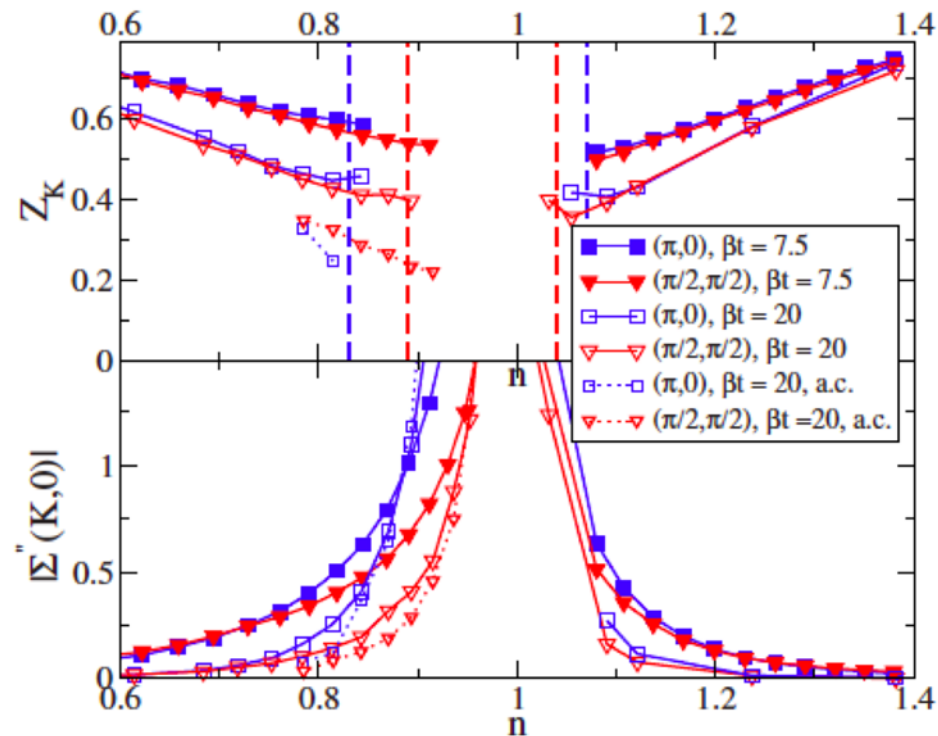
Momentum-differentiated regime at intermediate doping level

A metal with anisotropic scattering rate



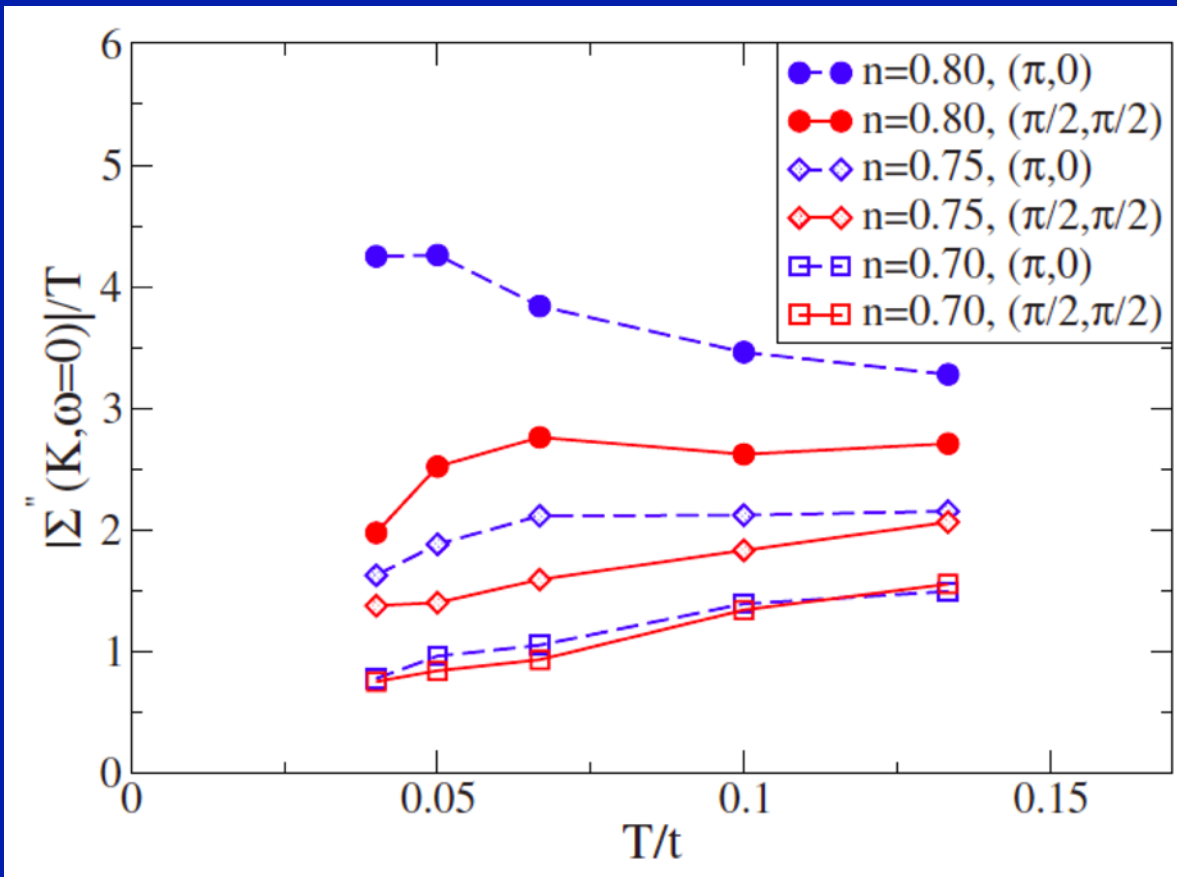
Antinodal scattering rate
Larger at small doping

No clear Brinkman-Rice
Behavior of QP weights Z



T-dependence of scattering rate

$$\Sigma''(0)/T$$



Nodal and Antinodal
Behave differently at
Small doping

Isotropic behavior at
High doping

Qualitatively consistent with transport experiments: cf. N.Hussey J.Phys.CondMat (2008)

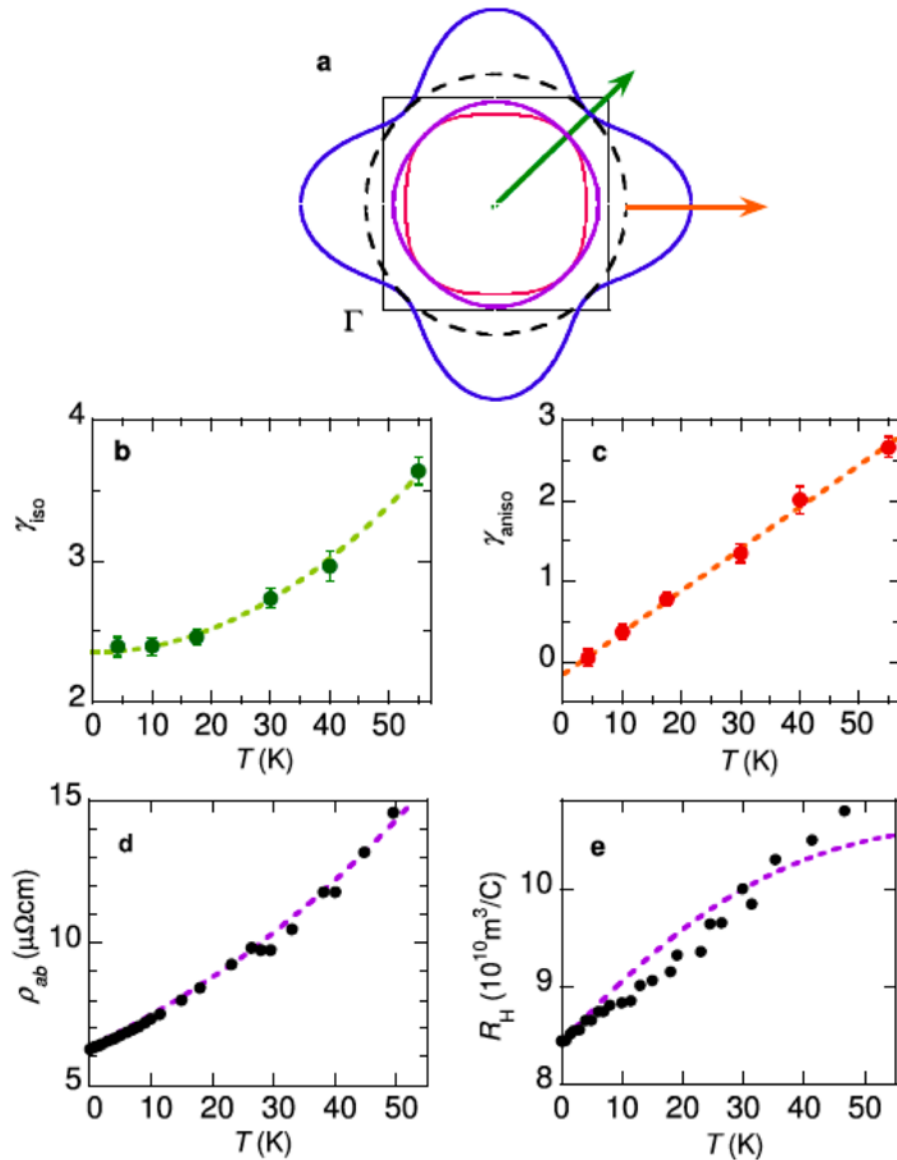
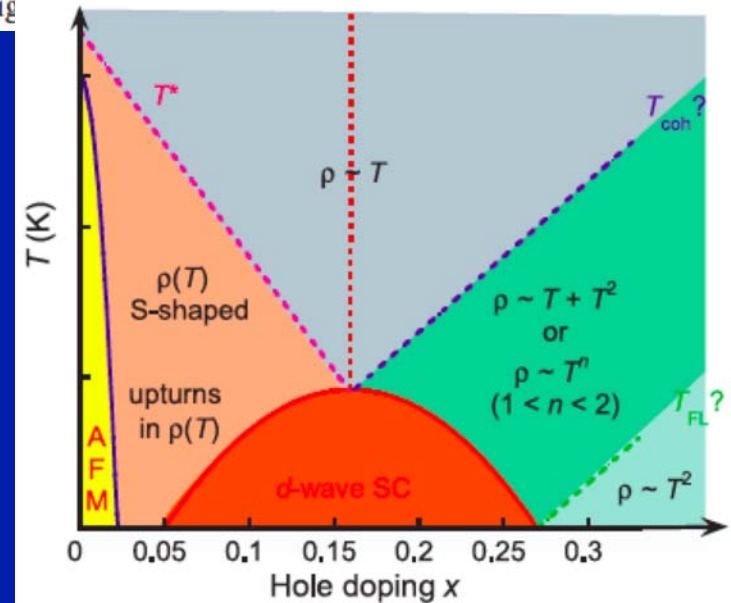
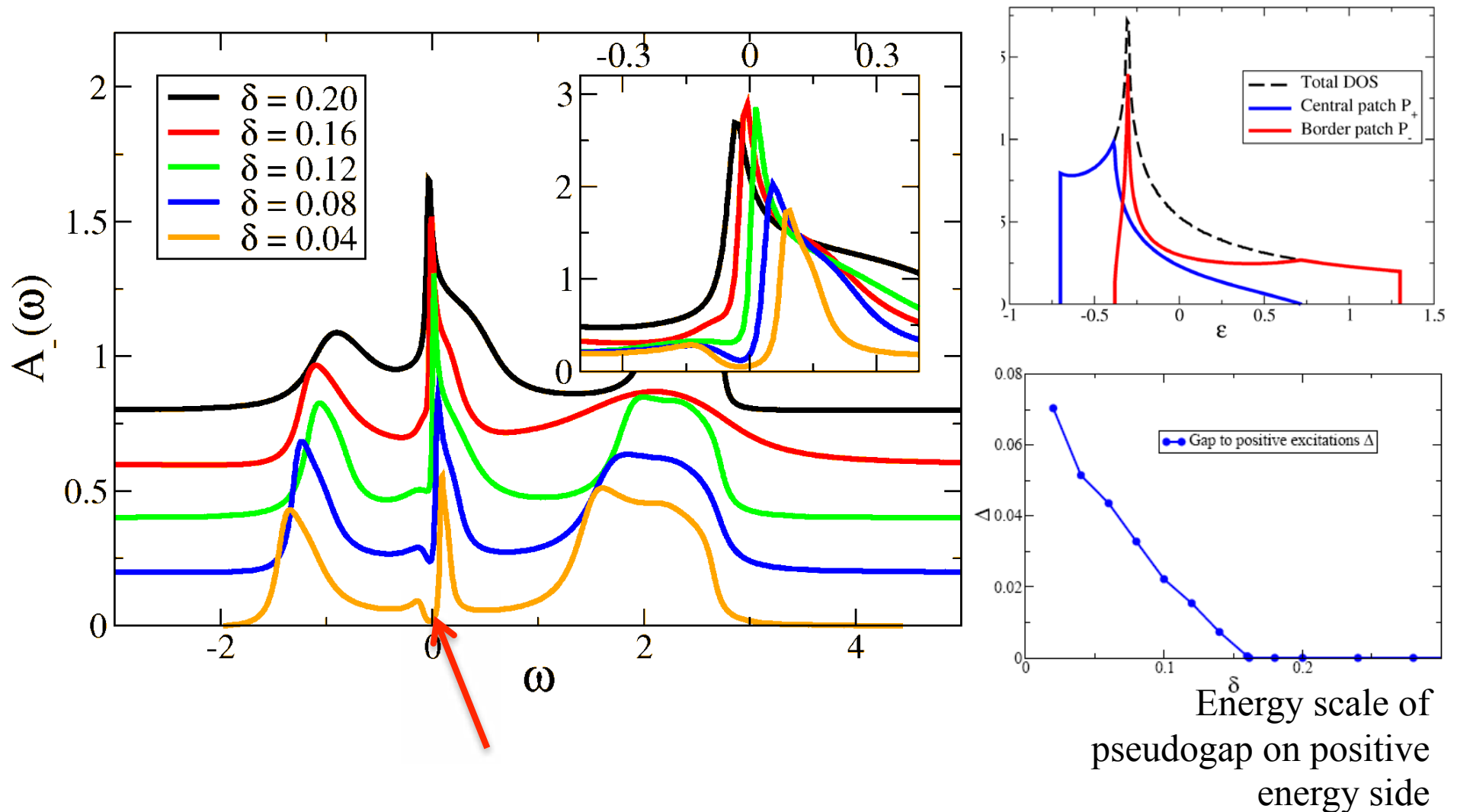


Figure 6. (a) Inner red curve: schematic 2D projection of the FS of overdoped TI2201. Adjacent purple curve: schematic representation of the d-wave superconducting gap. Outer blue curve: geometry of $(\omega_c \tau)^{-1}(\varphi)$. Dashed black line: isotropic part of $(\omega_c \tau)^{-1}(\varphi)$. (b) T dependence of γ_{iso} , i.e. the isotropic component of $(\omega_c \tau)^{-1}(\varphi)$ and sole contribution along the 'nodal' region indicated by the green arrow in (a). The dashed curve is a fit to $A + BT^2$. (c) T dependence of γ_{aniso} , i.e. the anisotropic component of $(\omega_c \tau)^{-1}(\varphi)$ and the additional contribution that is maximal along the 'anti-nodal' direction indicated by the orange arrow in (a). The dashed curve is a fit to $C + DT$. (d) Circles: $\rho_{ab}(T)$ data for overdoped TI2201 ($T_c = 15$ K) extracted from [49]. Dashed curve: simulation of $\rho_{ab}(T)$ from parameters extracted from the ADMR analysis. (e) Circles: $R_H(T)$ data for the same crystal [49]. Dashed curve: simulation of $R_H(T)$. Adapted with permission from *Nature Physics* 2 821, figure 2. Copyright



Pseudogap regime
(momentum sector-selective)

VB-DMFT/Antinode: not a sharp gap, a pseudogap!



At the antinode, a pseudogap appears below the transition.

Correlations have a strong effect (e.g. prominent Hubbard bands)

Pseudogap opening upon cooling:

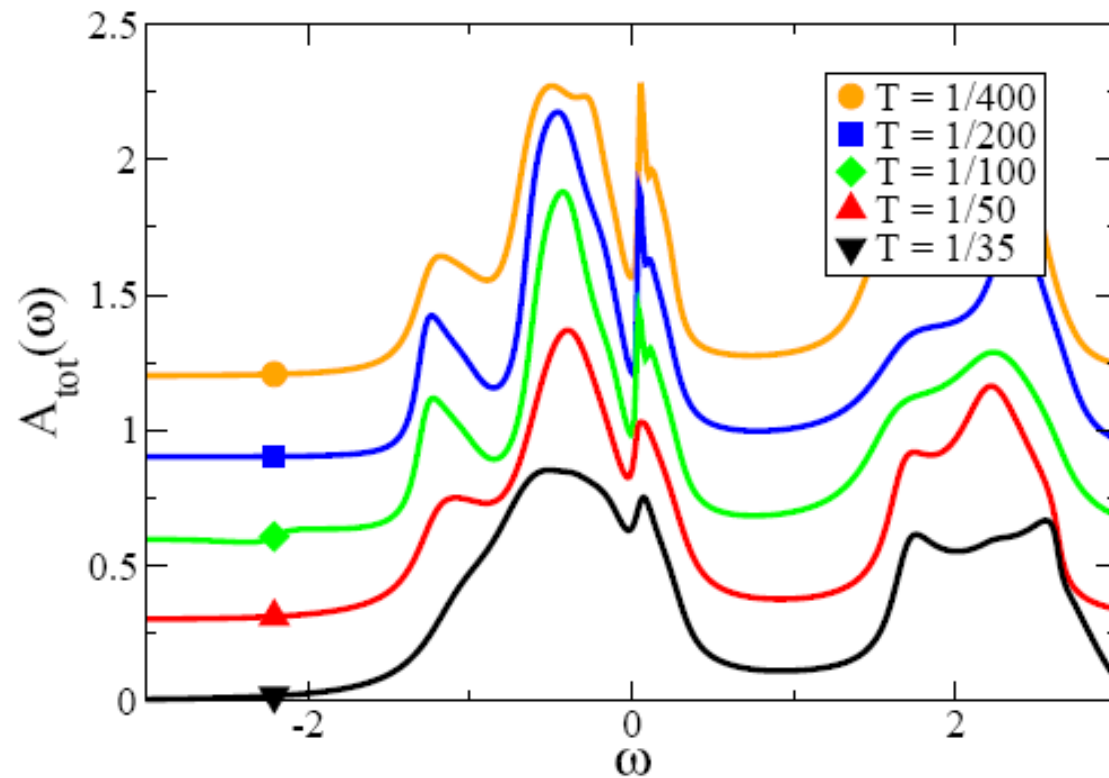
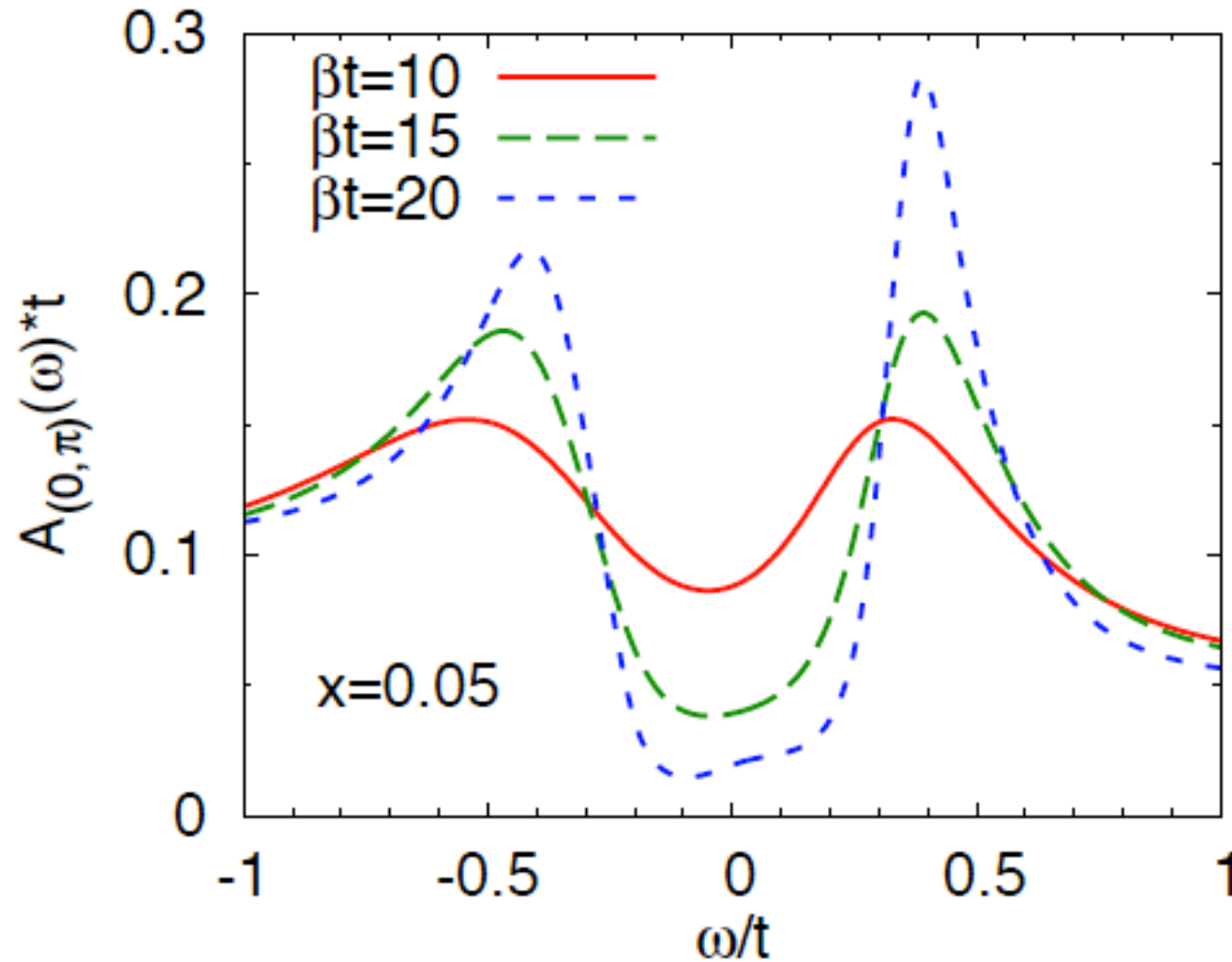


FIG. 10: (Color online) Total spectral function $A_{\text{tot}}(\omega)$ for various temperature at $\delta = 0.08$. A shift of 0.3 has been added between each curves for clarity.

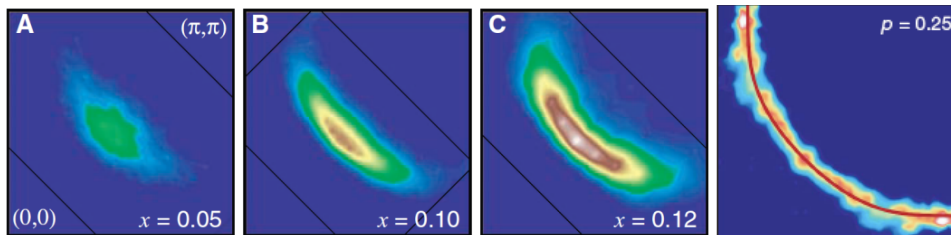
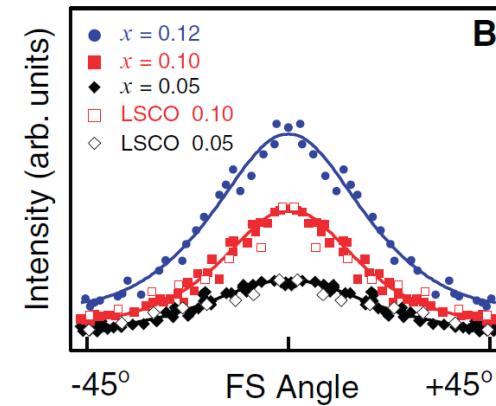
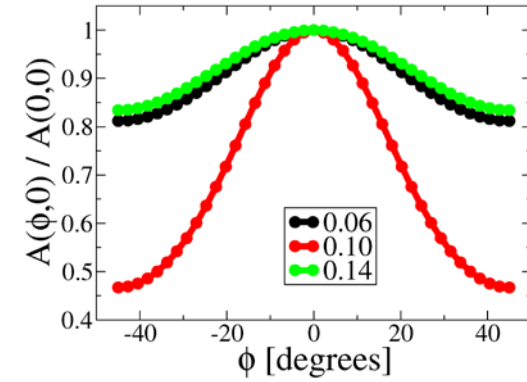
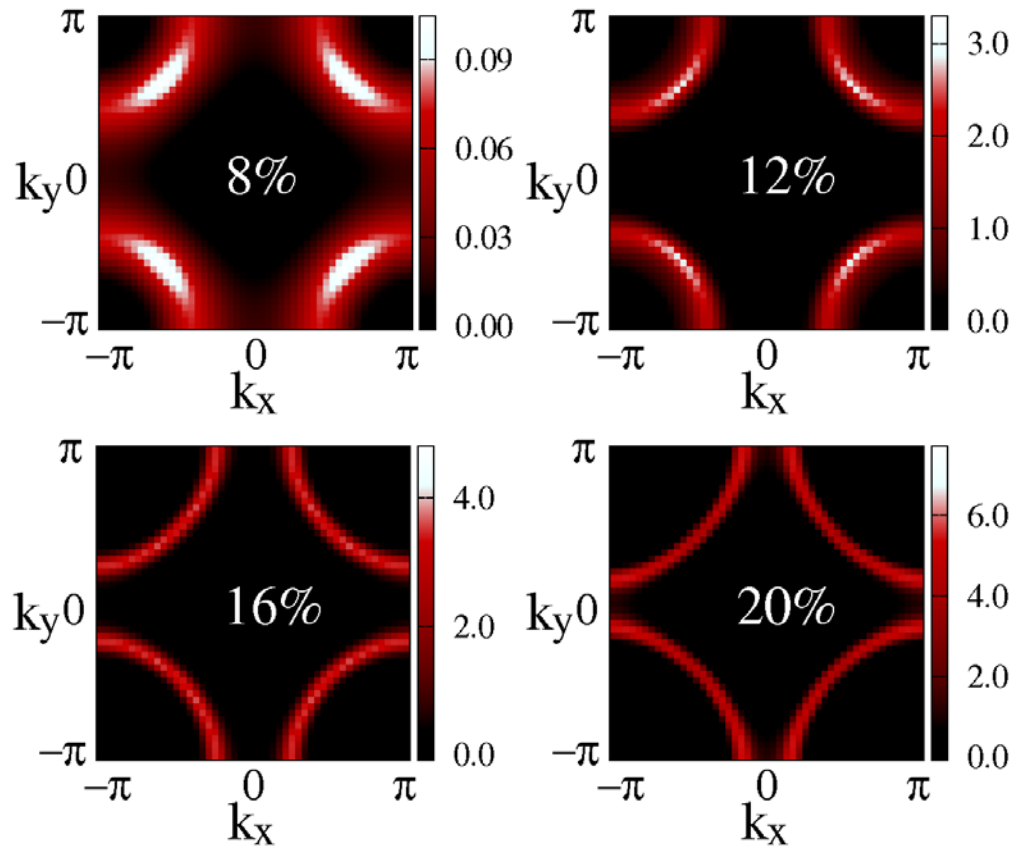
(Curves shifted upwards relative to one another, for clarity)

8-sites: pseudogap opening



Werner, Gull, Parcollet, Millis, Lin

Calculated ARPES intensity maps

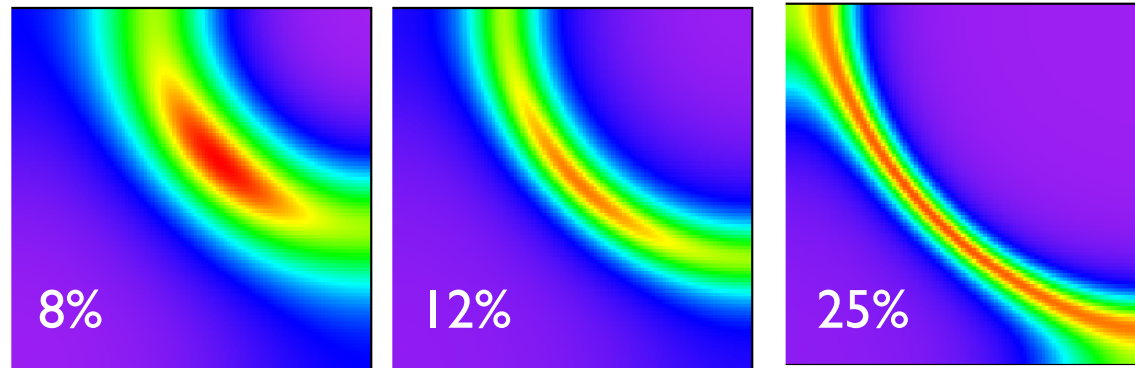


Shen et al., Science (2005)

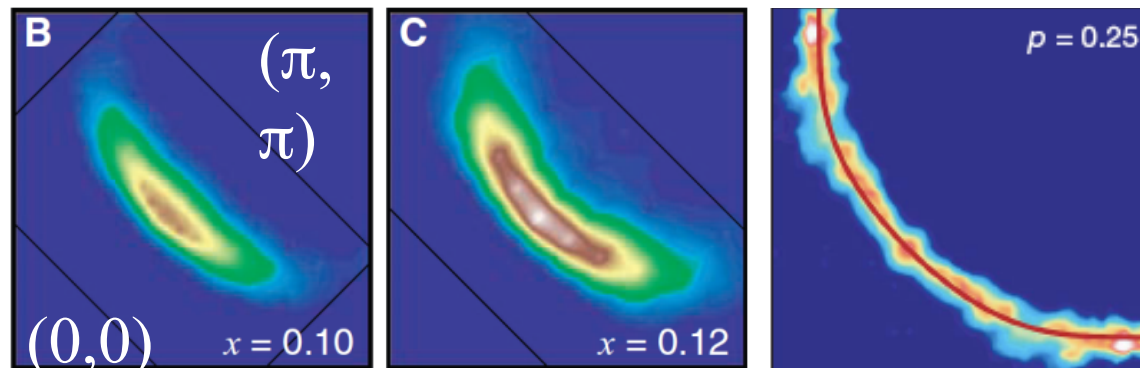
Maximum contrast
around 10%

With an (cumulant-) interpolation over the Brillouin zone...

Computed ARPES intensity maps



$A(k, \omega=0)$ contour map



*Shen et al., Science
(2005)*

*Platé et al., PRL
(2005)*

With an interpolation over the Brillouin zone...

C-axis optical conductivity

M.Ferrero et al. PRB (2010)

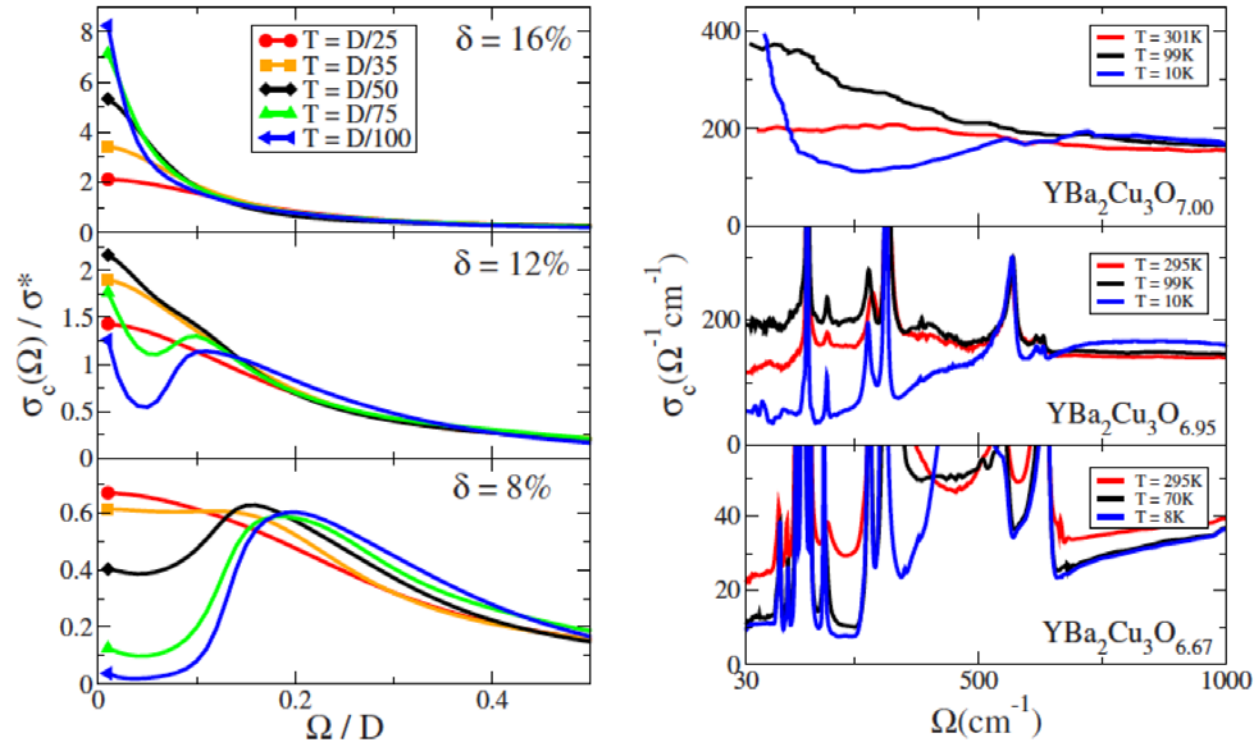
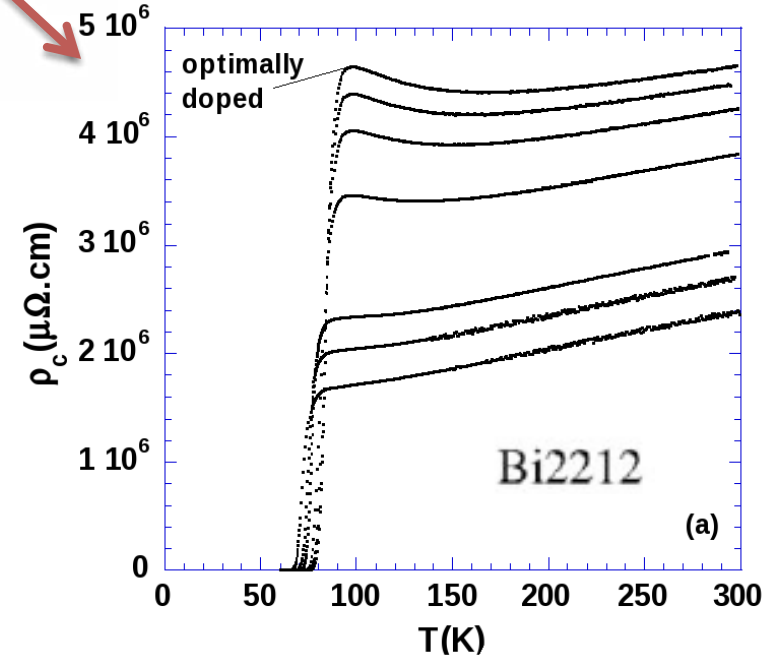
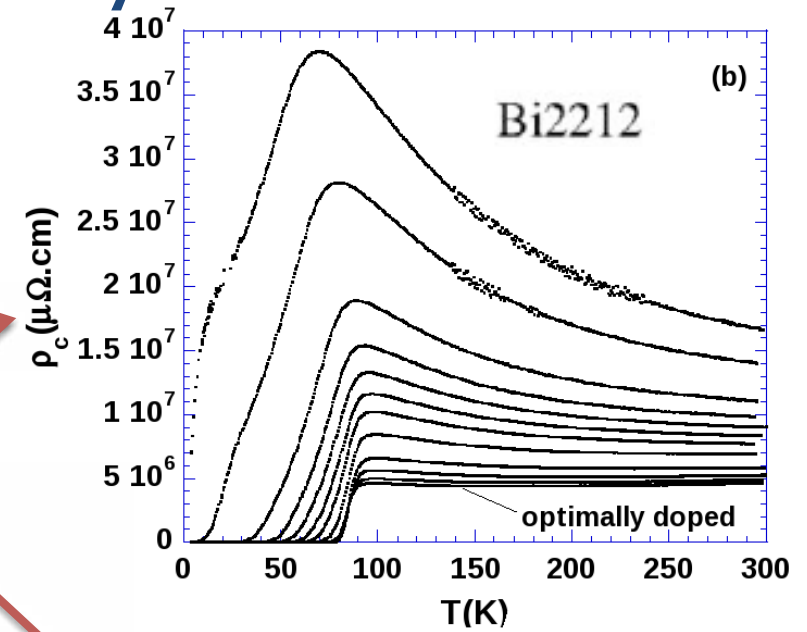
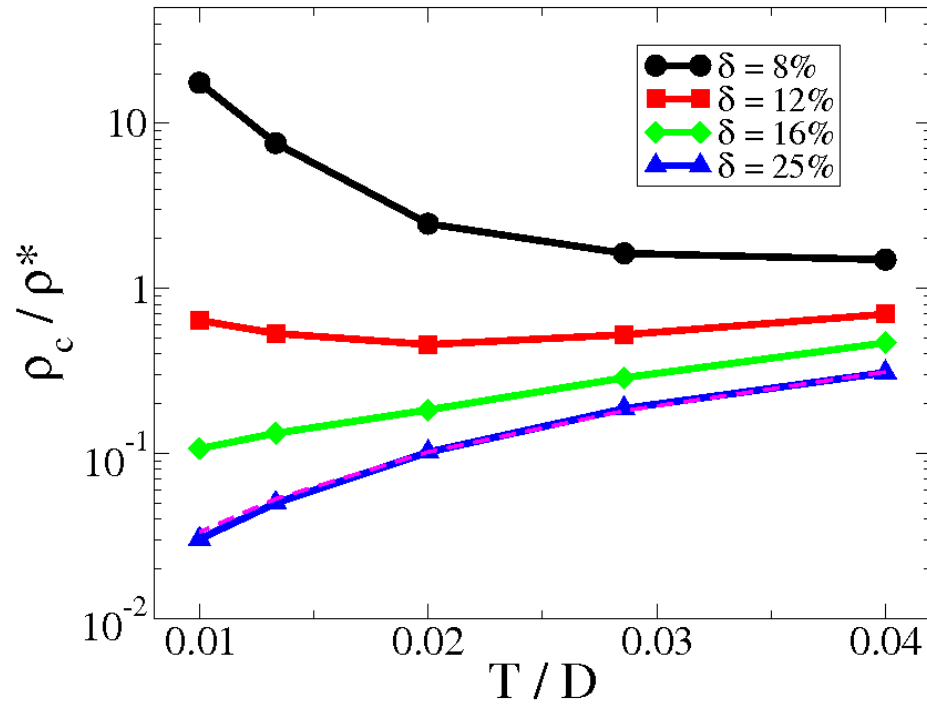


FIG. 2. (Color online) Left panel: the c -axis optical conductivity $\sigma_c(\Omega)$ calculated within VB-DMFT for three doping levels. σ_c is displayed in units of σ^* as defined in the text (σ^* is of order $50 \text{ } \Omega^{-1} \text{ cm}^{-1}$ for $\text{YBa}_2\text{Cu}_3\text{O}_y$). Frequency is normalized to the half-bandwidth $D \sim 1 \text{ eV} = 8000 \text{ cm}^{-1}$. Right panel: experimental data for the c -axis optical conductivity of $\text{YBa}_2\text{Cu}_3\text{O}_y$. The data for $\text{YBa}_2\text{Cu}_3\text{O}_{7.00}$ is taken from Ref. 8 where the phonon contribution was subtracted by fitting to five Lorentzian oscillators. The data for $\text{YBa}_2\text{Cu}_3\text{O}_{6.95}$ and $\text{YBa}_2\text{Cu}_3\text{O}_{6.67}$ are taken from Refs. 23 and 24.

$$\sigma_c(\Omega) = \frac{2e^2c}{\hbar ab} \int d\omega \frac{f(\omega) - f(\omega + \Omega)}{\Omega} \frac{1}{N} \sum_{\mathbf{k}} t_{\perp}^2(\mathbf{k}) A(\mathbf{k}, \omega) A(\mathbf{k}, \Omega + \omega), \quad (2)$$

C-axis resistivity



Overdoped: metallic
Underdoped: insulating
Optimally: minimum in resistivity

H. Raffy et al.

Take-home messages: cluster extensions of DMFT

- Proximity to Mott transition destroys antinodal quasiparticles
- Low-doping regime is dominated by the buildup of short-range singlet correlations (J), responsible for PG formation
- Minimal cluster-extension of DMFT accounts for nodal/antinodal differentiation
- PG appears as a momentum-sector selective Mott phase