Turbulence in planetary cores

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Why care about turbulence in planetary cores?

- If you care about transport properties (κ, χ, ν, η, *etc*), then you should care about turbulence because the *effective* properties are likely to be orders of magnitude away from those measured in the lab or *ab initio*.
- In particular, energy dissipation, thermal and chemical mixing strongly depend upon turbulence.

(1/2)

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Why care about turbulence in planetary cores?

- Turbulence involves a large range of spatial and temporal scales. Numerical simulations can't cover that range. Neither do observations, but phenomena we observe at the large scales do depend upon the unseen scales.

(2/2)

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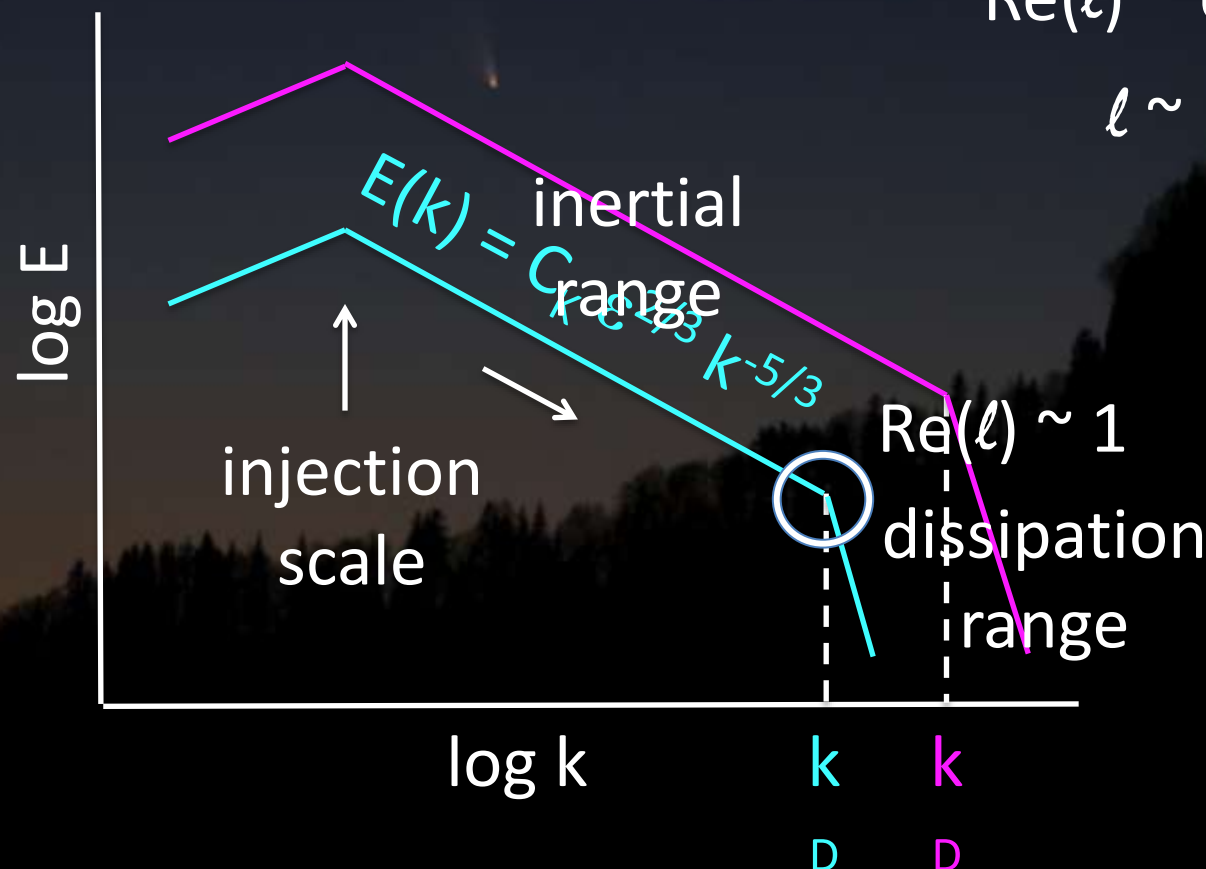
What do we know about turbulence?

- Most of what we know relates to hydrodynamic turbulence governed by the non-linear $(\mathbf{u}.\nabla)\mathbf{u}$ term in the Navier-Stokes equation.
- The role of turbulence appears very dull !
  It transfers energy from some ‘large scale’ at which it is injected to ‘small scales’ at which it can be dissipated by viscosity.

Kolmogorov's universal turbulence

$\mathrm{Re}(\ell) \sim U(\ell)\, \ell/\nu$

$\ell \sim 1/k$



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What turbulence in planetary cores?

- At what length- and time-scales is the turbulence regime dominated by rotation, by the magnetic field?
- Can we infer the evolution of flow velocity and small-scale magnetic field in these different regimes?
- How much energy is dissipated?
- What is the balance between viscous and ohmic dissipation?
- Which waves can propagate?

→ Introducing "τ-ℓ regime diagrams"

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“Turbulence in the core”, H-C. Nataf and N. Schaeffer, in *Treatise on Geophysics*, volume 9 “the Core” 2nd edition, Ed P. Olson, Elsevier, to appear in May 2015.

τ-ℓ regime diagrams assumptions

- Turbulence involves a wide range of scales
- Time-scales τ and length-scales ℓ are related by various physical processes
- Regime changes occur when $\tau(\ell)$ lines intersect
- Dimensionless numbers can we written as time-scale ratios
- The scale ℓ at which a dimensionless number is ~1 is more important than the value of that number at the integral scale
- Turbulent dynamics is controlled by the shortest time-scale process

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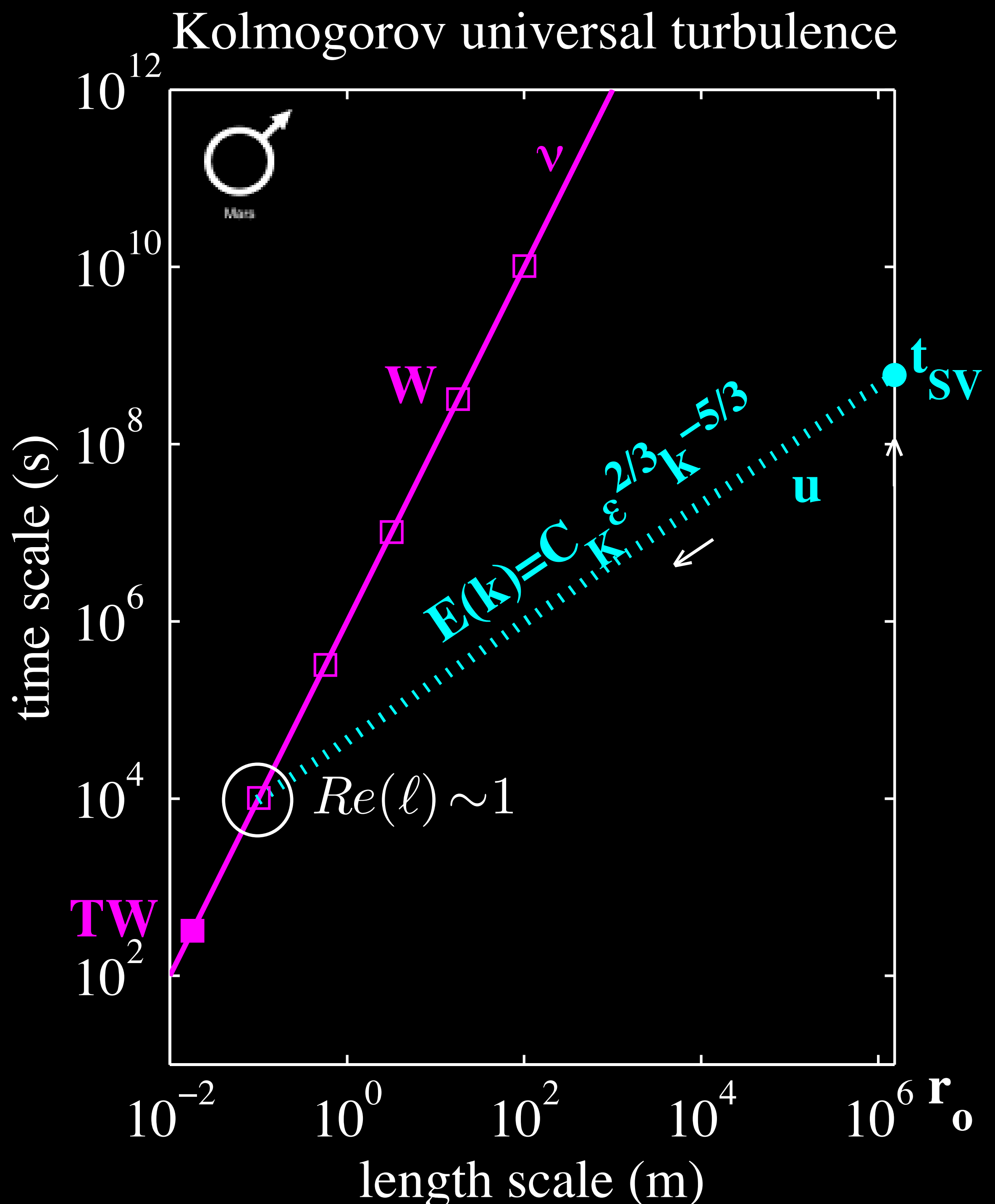
Illustration with the τ-ℓ diagram of classical Kolmogorov hydrodynamic turbulence, tuned to the size of Mars' core.

$\nu = 10^{-6}\ m^2/s$
t_{SV} = 20 years
r_o = 1500 km
$M_{oc} = 10^{23}$ kg



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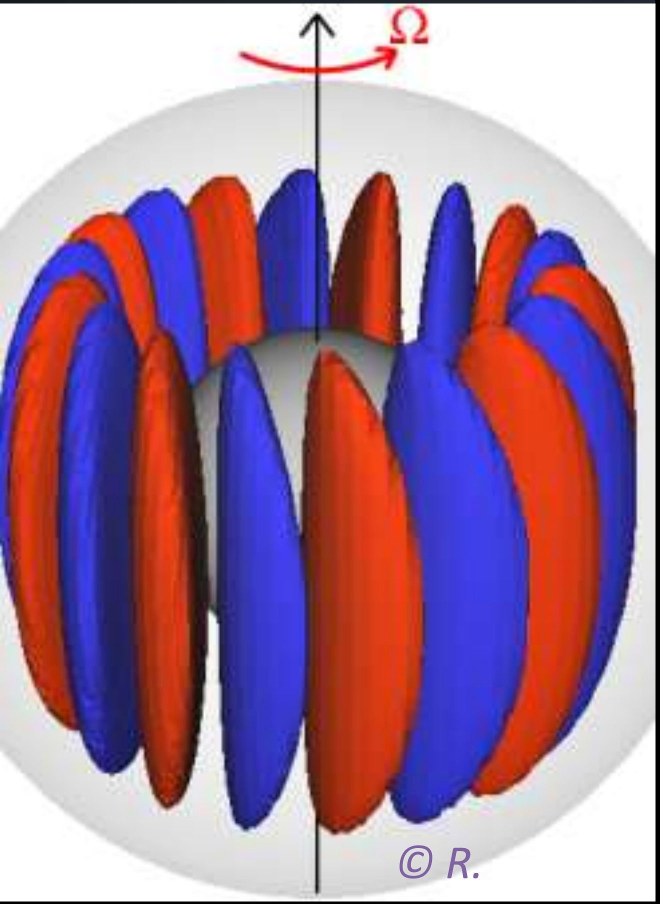
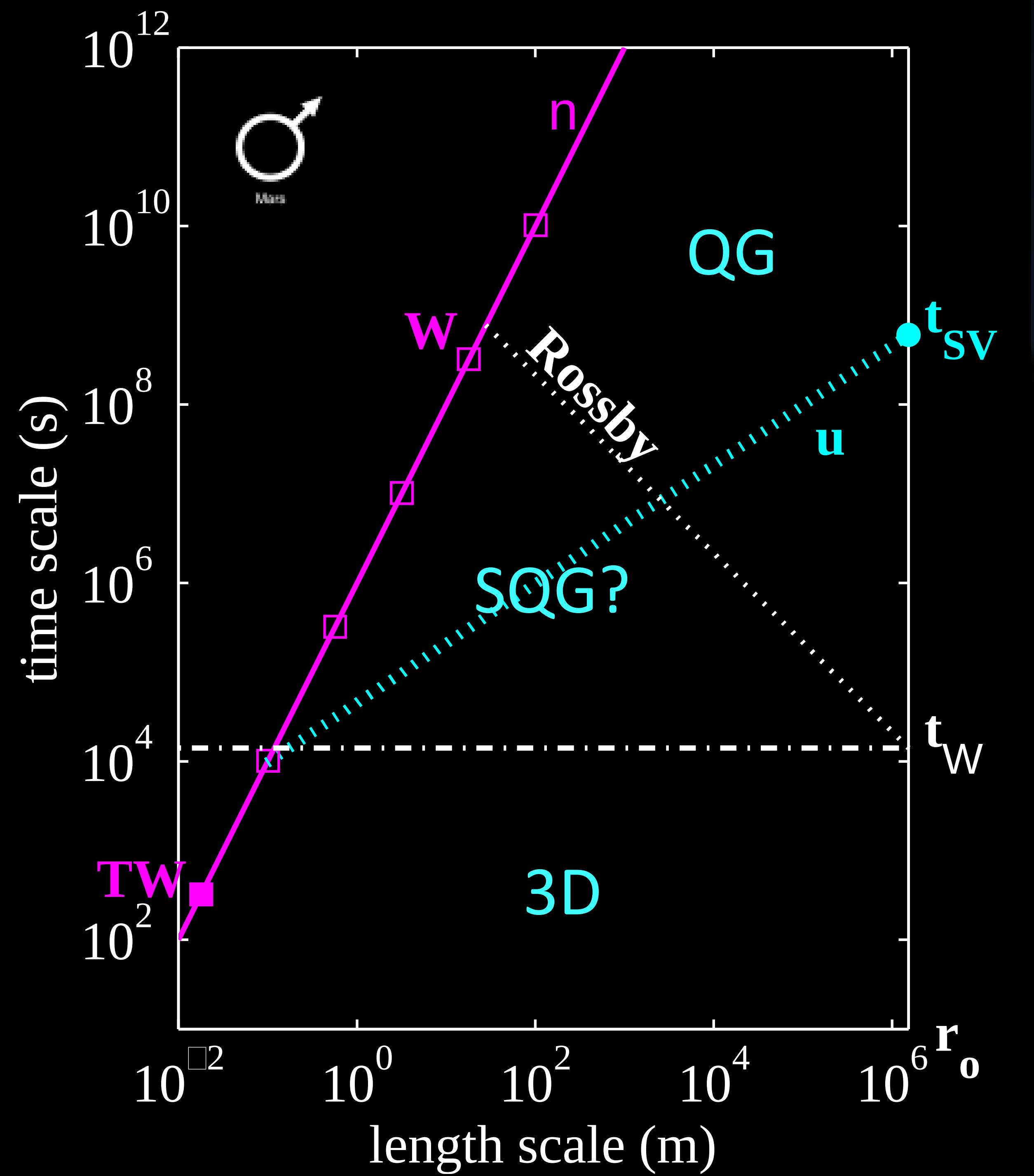
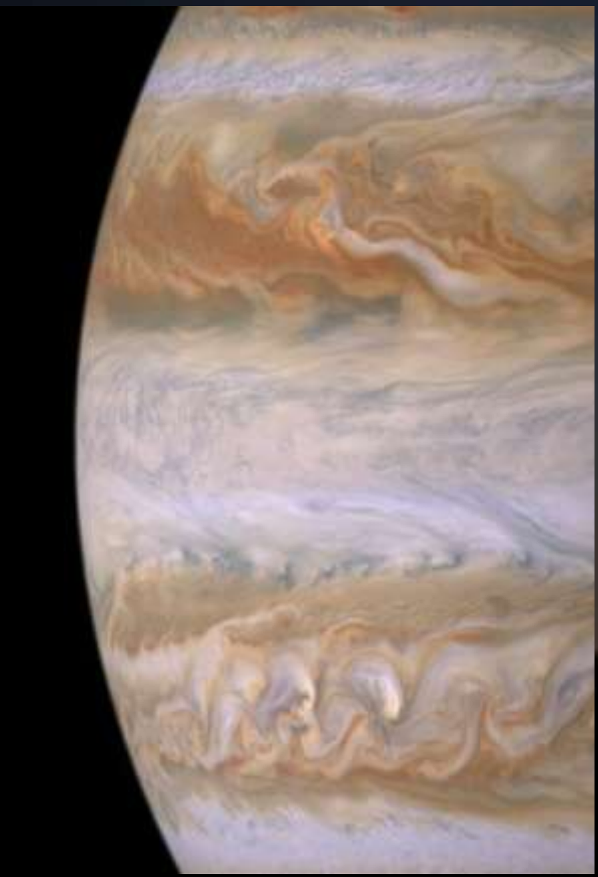
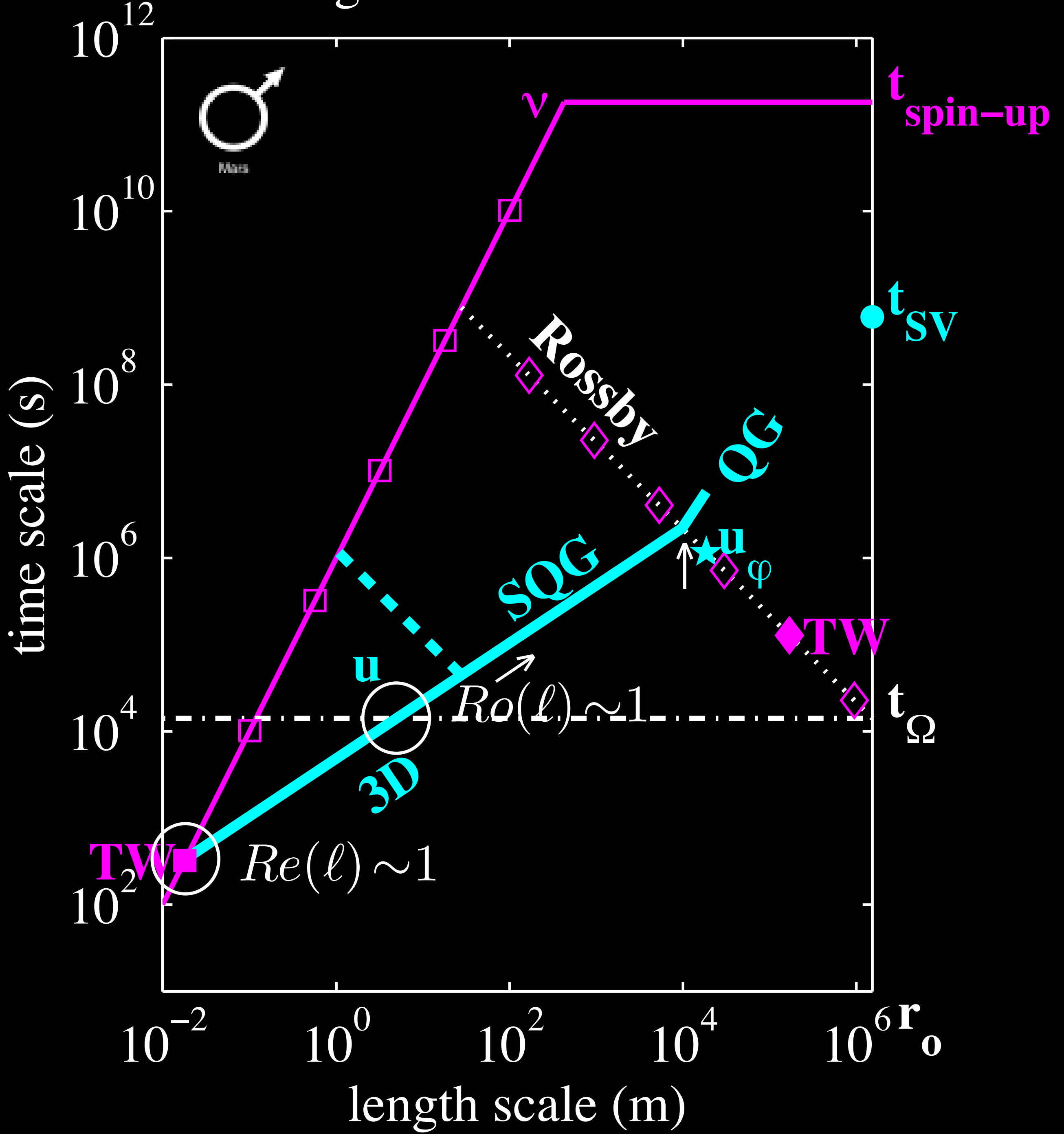
Let’s add rotation and sphericity.







non–magnetic turbulence in Mars



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Let’s add the magnetic field.

- While eddies are the building bricks of hydrodynamic turbulence, it is believed that Alfvén waves are those of magnetohydrodynamic turbulence, governed by the non-linear $(\mathbf{B}.\nabla)\mathbf{b}$ term in the Navier-Stokes equation and $(\mathbf{B}.\nabla)\mathbf{u}$ term in the induction equation.

Tobias et al, 2013

$$V_A = \frac{B}{\sqrt{\rho \mu_0}}$$

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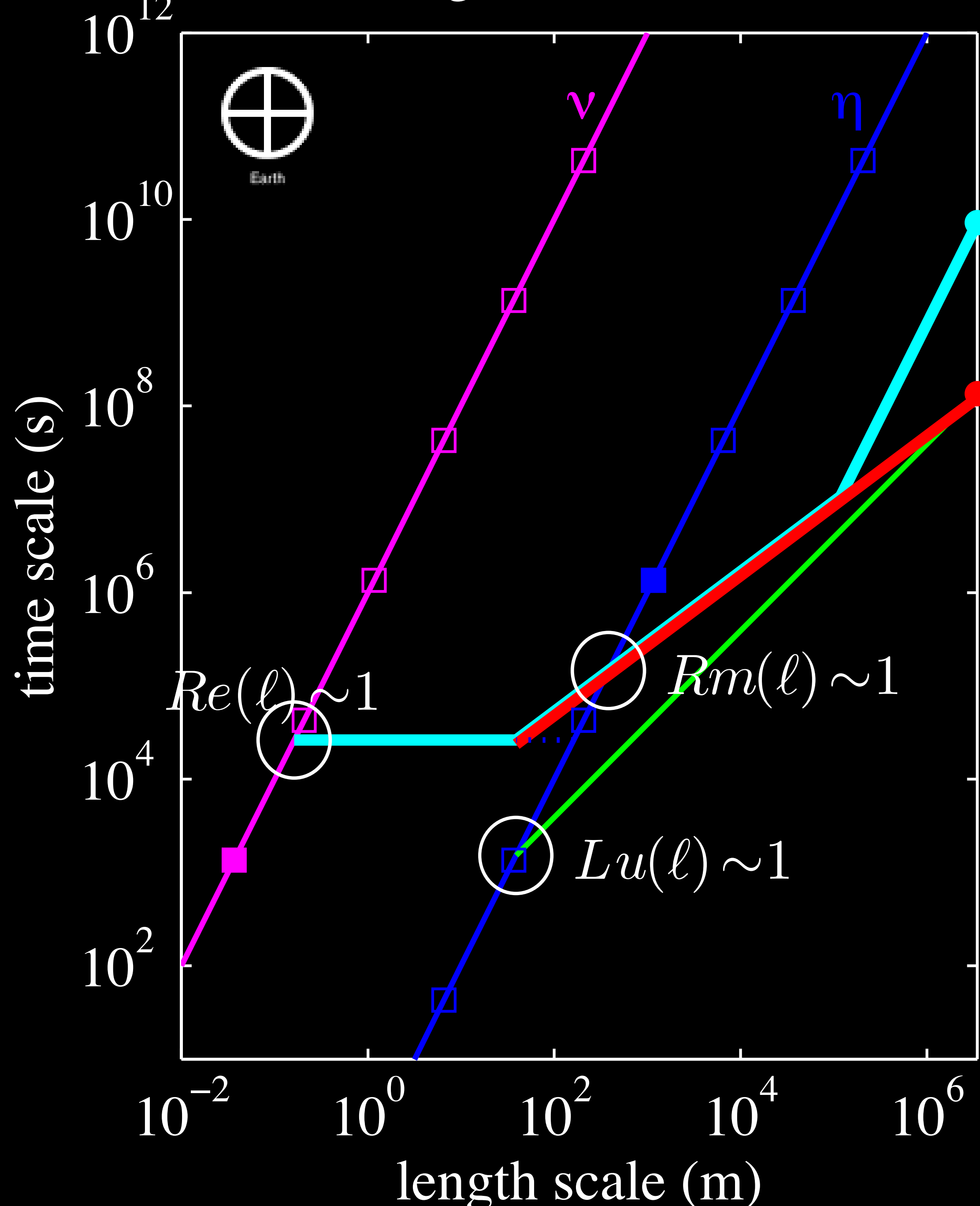
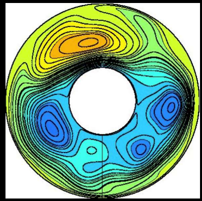
non–rotating Earth core turbulence





$\nu = 10^{-6}\ m^2/s$

t_{SV} = 300 years

r_o = 3480 km

$M_{oc} = 1.835\ 10^{24}$ kg

$t_{Alfvén}$ = 4.3 years

$\eta = 1.\ m^2/s$

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Let's have rotation *and* the magnetic field, as in the Earth.

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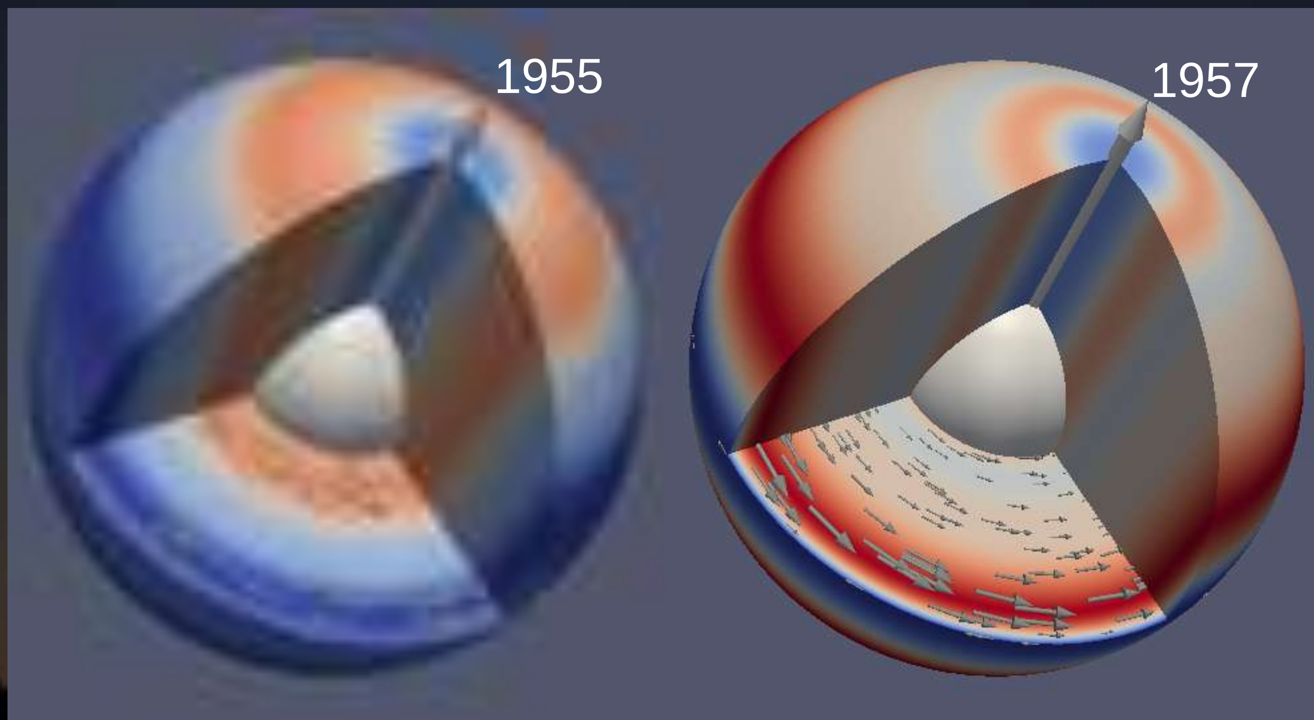
Torsional oscillations in the Earth's core

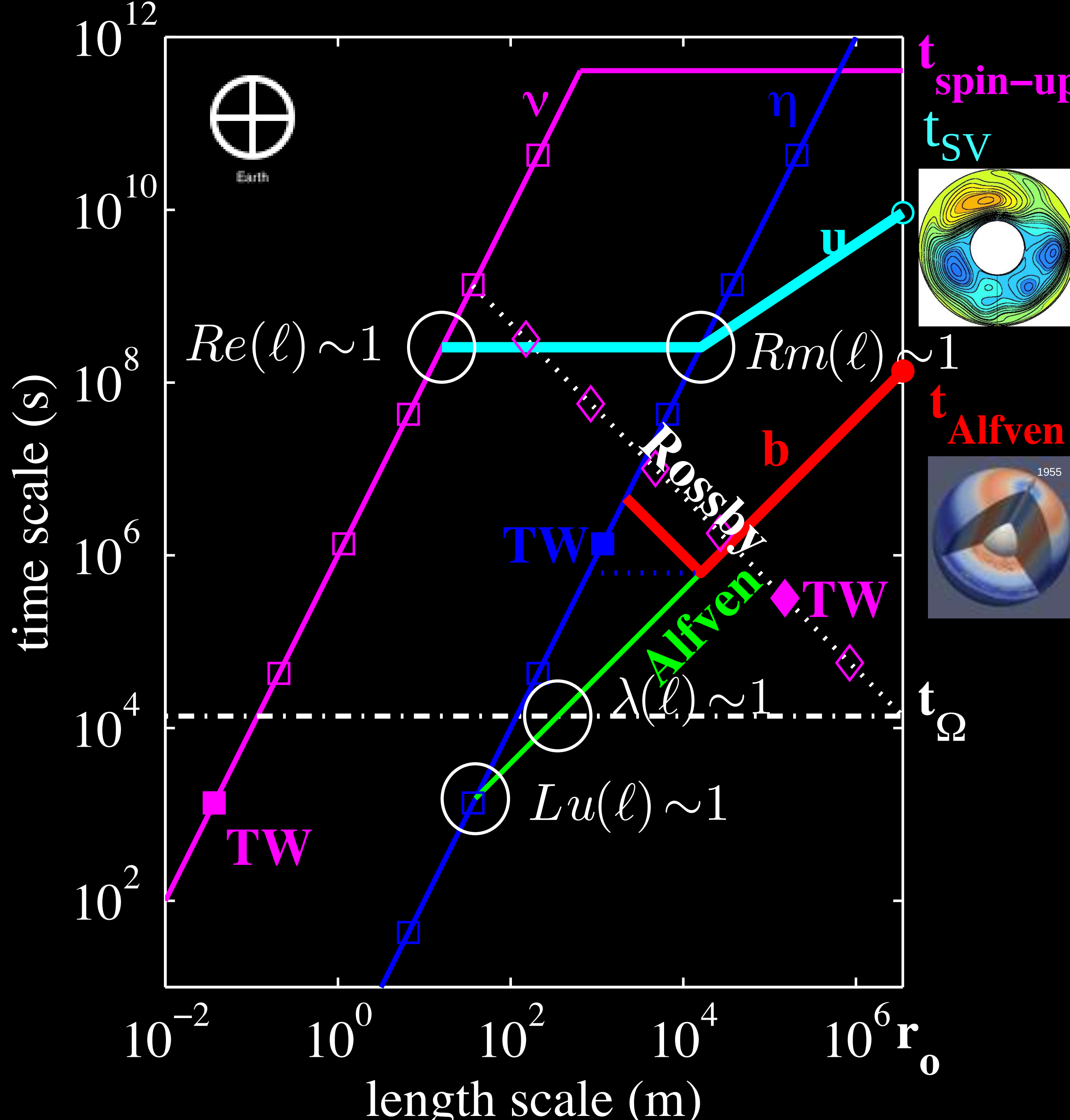


Gillet et al, Nature, 2010

$t_{Alfvén}$ = 4.3 years

$$V_A = \frac{B}{\sqrt{\rho \mu_0}}$$

Earth core turbulence



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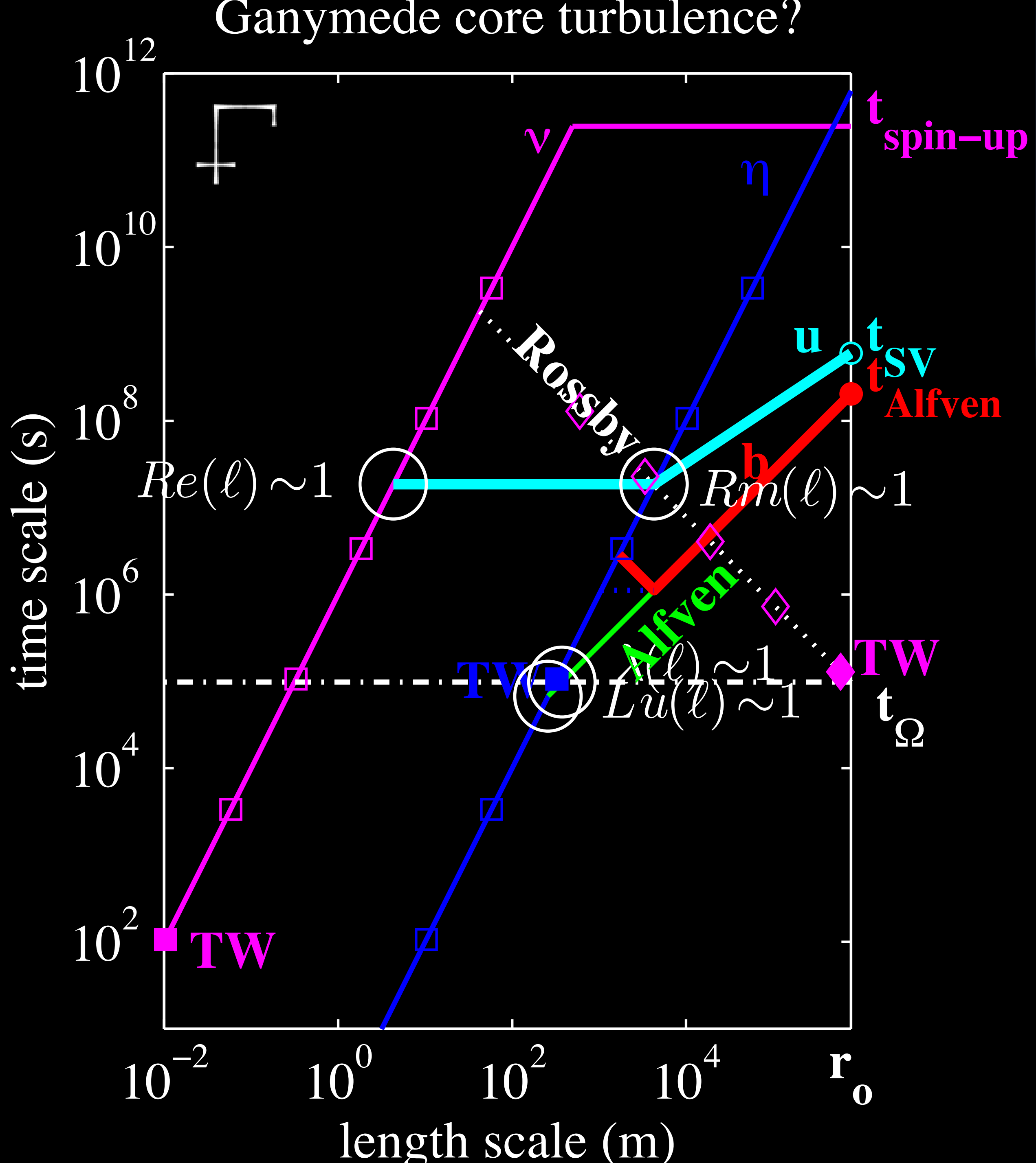

$\nu = 10^{-6}\ m^2/s$

t_{SV} = 20 years

r_o = 790 km

$M_{oc} = 10^{22}$ kg

$t_{Alfvén}$ = 6 years

$\eta = 1.\ m^2/s$

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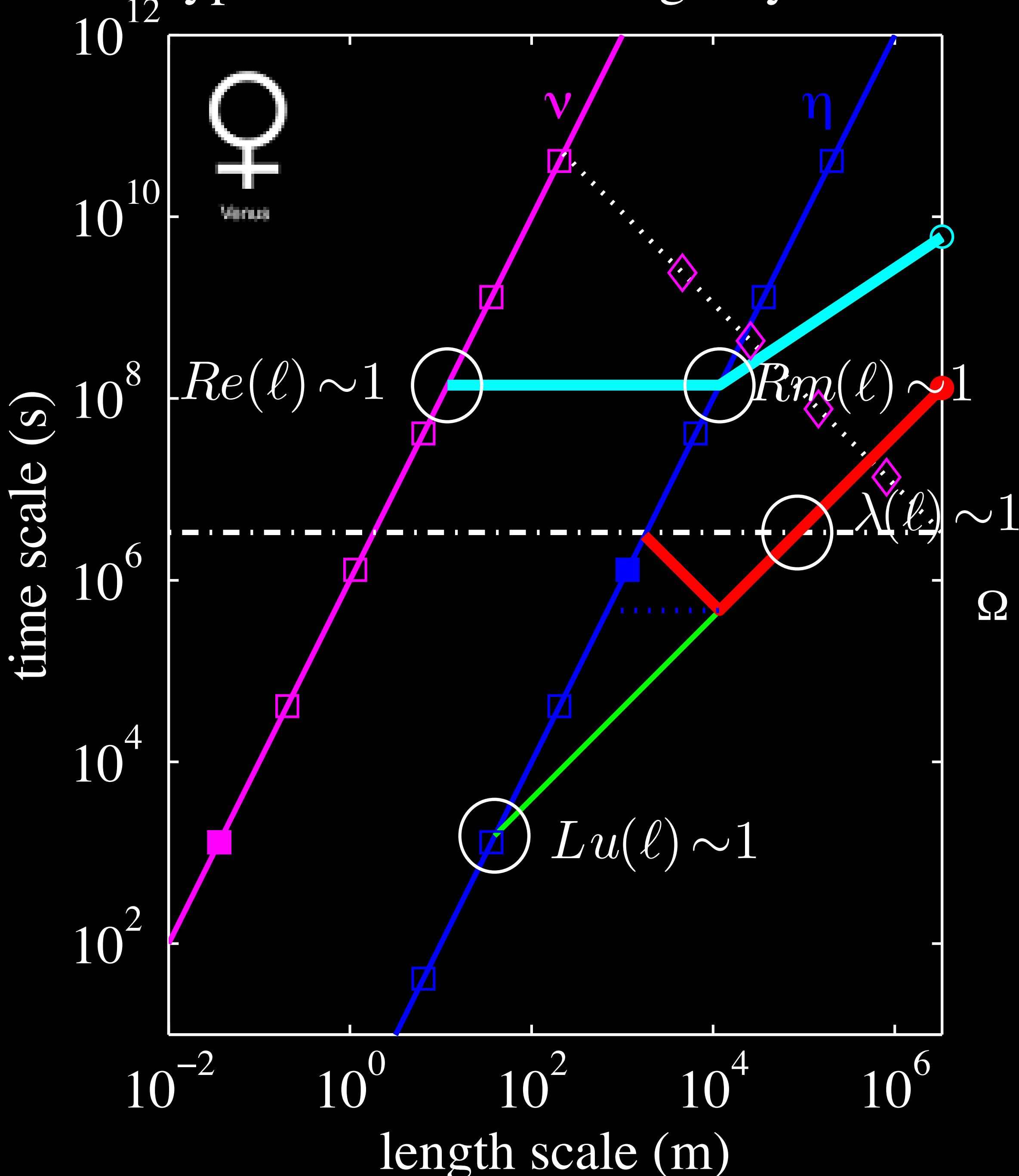
Could Venus have the same dynamo as the Earth?

Probably not...

a hypothetical Venusian geodynamo ???



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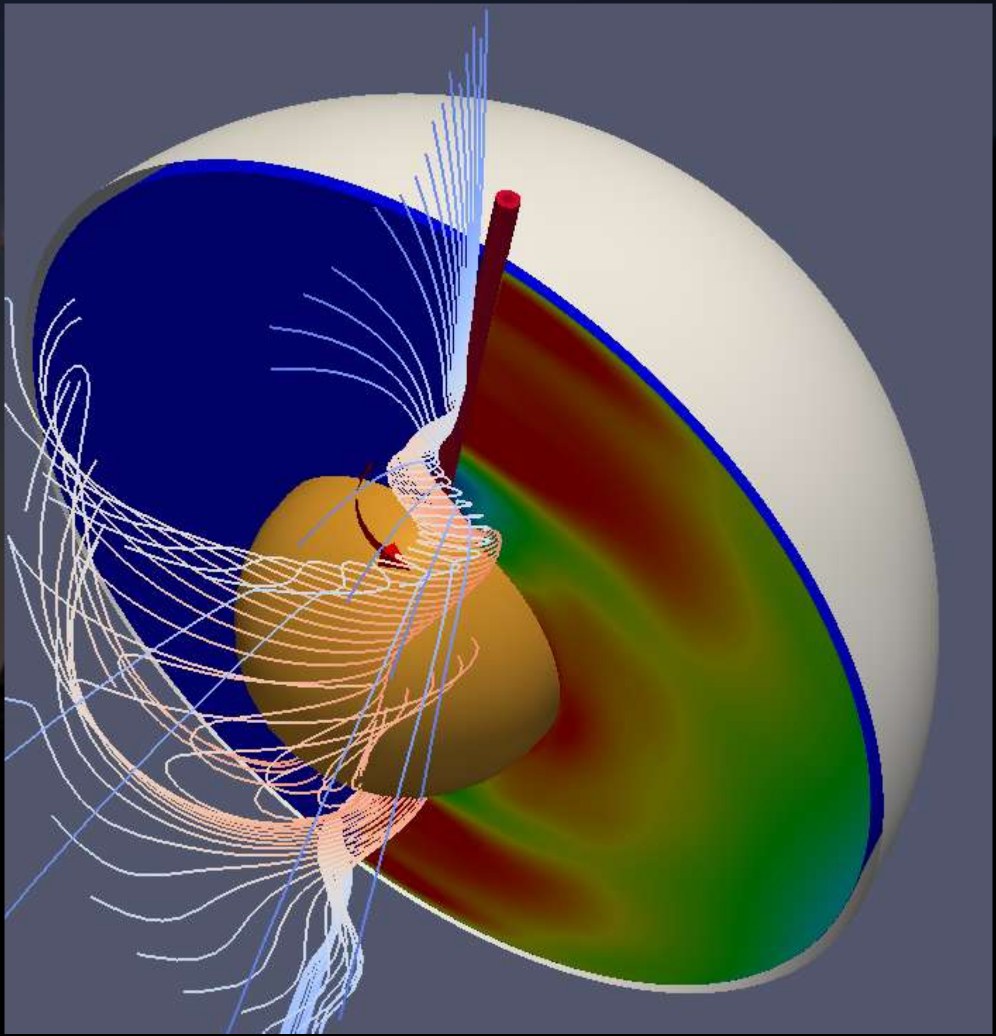
But do we know enough about turbulence in the magnetostrophic regime?

Turbulence *reduces* magnetic diffusivity in a liquid sodium experiment,

S. Cabanes, N. Schaeffer and H-C. Nataf,

Phys. Rev. Lett. **113**, 184501 – Published 28 October 2014.

The DTS experiment



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Conclusions

- I hope you like our τ-ℓ regime diagrams!
- Zonal jets in Mars' core?
- Rotation limits the dissipation of the dynamo.
- No magnetic field on Venus because it spins too slowly?
- We need to know a lot more about turbulence in planetary core conditions…
- We need to know a lot more about the internal structure of planets…

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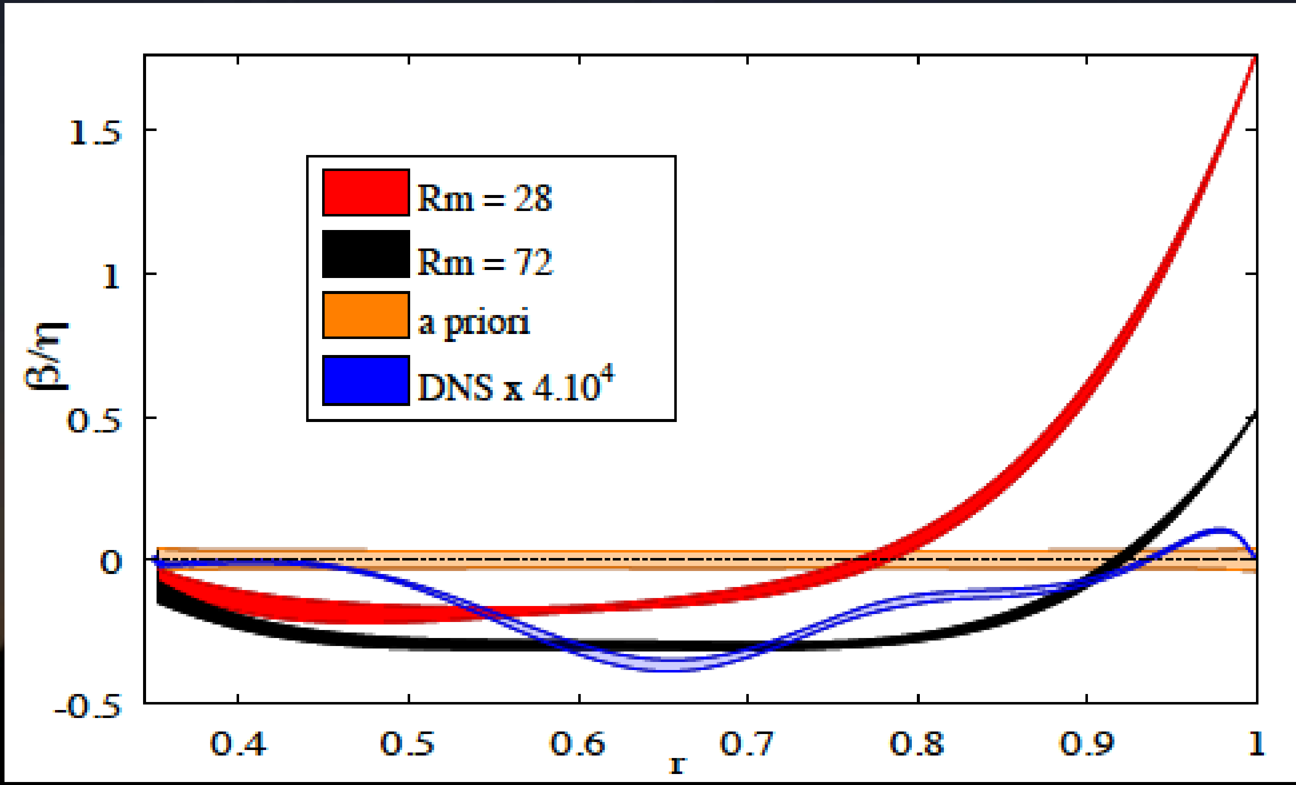
Negative β effect in the DTS liquid sodium experiment



Cabanes et al, PRL, 2014

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Alfvén waves (1942)

$$\rho \frac{\partial \mathbf{u}}{\partial t} = -\nabla p + \left(\frac{\mathbf{B}}{\mu} \cdot \nabla\right) \mathbf{b} + \rho \nu \nabla^2 \mathbf{u},$$

linearized Navier-Stokes eq^n

$$\frac{\partial \mathbf{b}}{\partial t} = (\mathbf{B} \cdot \nabla)\, \mathbf{u} + \frac{1}{\mu\sigma} \nabla^2 \mathbf{b}$$

linearized induction eq^n

Introducing Elsasser variables: $\mathbf{u}^{\pm} = \mathbf{u} \pm \mathbf{b}/\sqrt{\rho\mu}$ yields:

$$\frac{\partial \mathbf{u}^+}{\partial t} = -\nabla \frac{p}{\rho} + \left(\frac{\mathbf{B}}{\sqrt{\rho\mu}} \cdot \nabla\right) \mathbf{u}^+ + \nu \nabla^2 \mathbf{u} + \frac{1}{\mu\sigma} \nabla^2 \frac{\mathbf{b}}{\sqrt{\rho\mu}},$$

$$\frac{\partial \mathbf{u}^-}{\partial t} = -\nabla \frac{p}{\rho} - \left(\frac{\mathbf{B}}{\sqrt{\rho\mu}} \cdot \nabla\right) \mathbf{u}^- + \nu \nabla^2 \mathbf{u} - \frac{1}{\mu\sigma} \nabla^2 \frac{\mathbf{b}}{\sqrt{\rho\mu}}$$

from Alboussière et al, 2011

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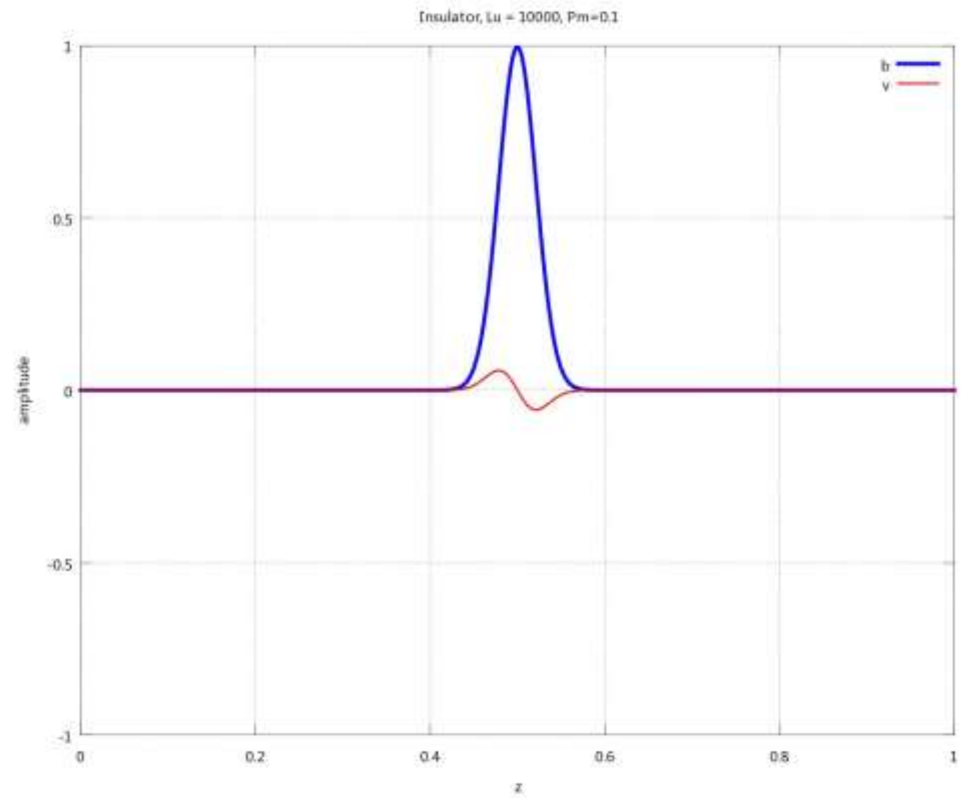
Properties of ideal Alfvén waves

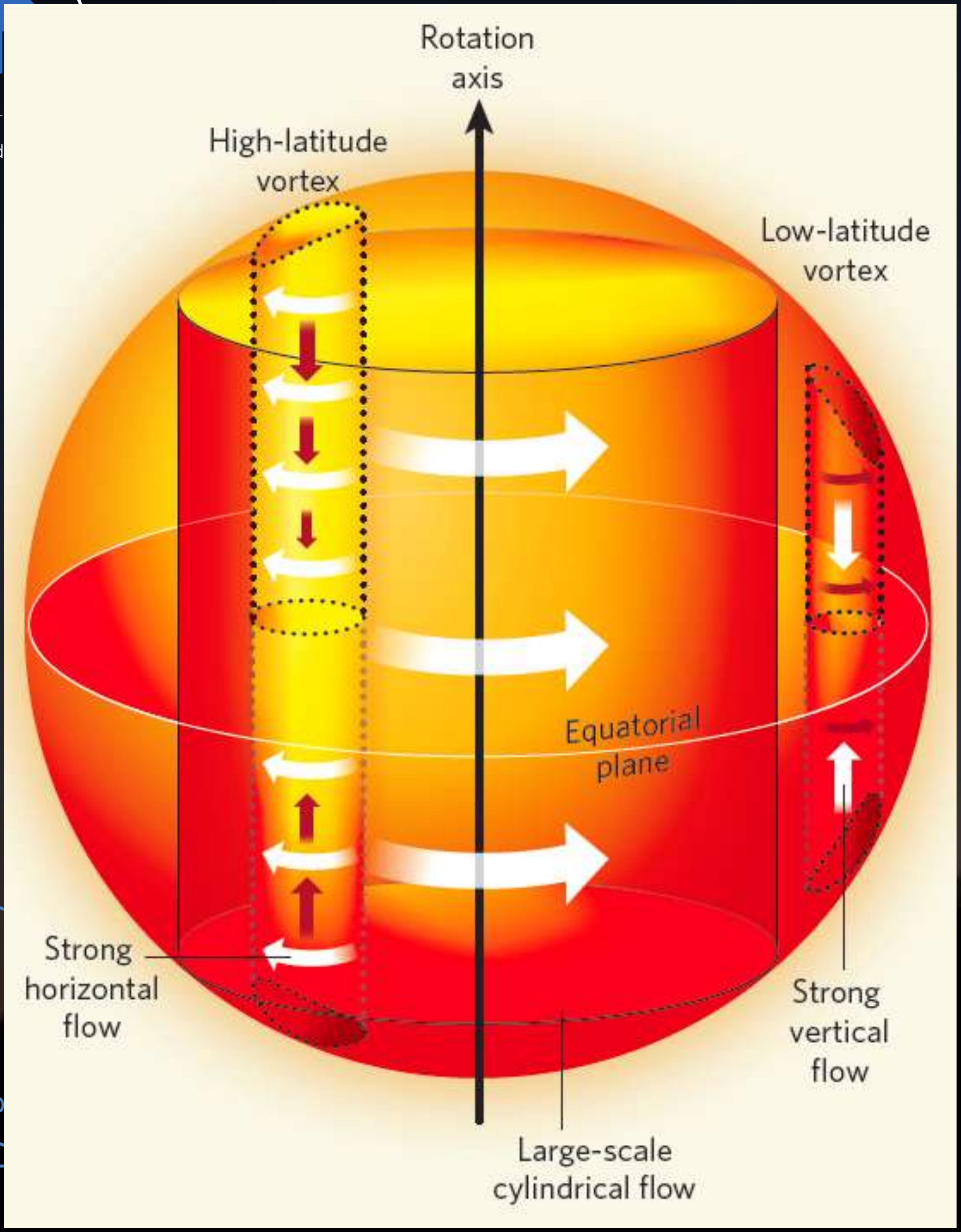
$$\frac{\partial \mathbf{u}^{\pm}}{\partial t} = \pm \left(\frac{\mathbf{B}}{\sqrt{\rho\mu}} \cdot \nabla \right) \mathbf{u}^{\pm}$$

- Transverse
- Non-dispersive
- Alfvén velocity
- Energy equipartition

With vanishing diffusivities



courtesy Ph. Cardin

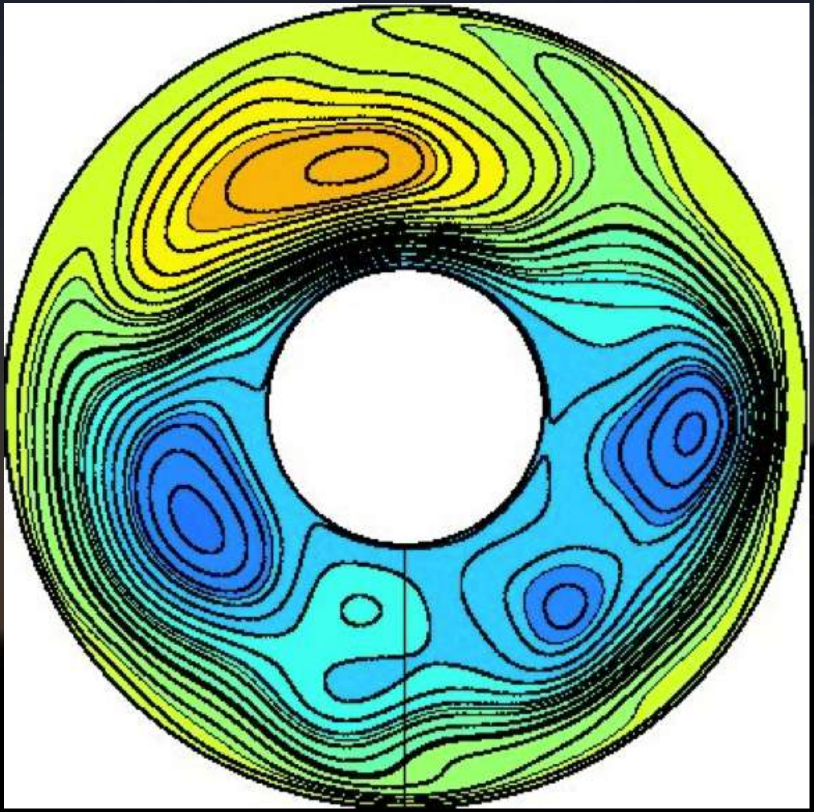


Jackson, news & views, Nature, 2010

- At “short” time scales, the Earth’s rapid rotation imposes its law: quasi-geostrophic flow.

Jault, 2008
Gillet, Schaeffer & Jault, 2012
Schaeffer & Pais, 2011

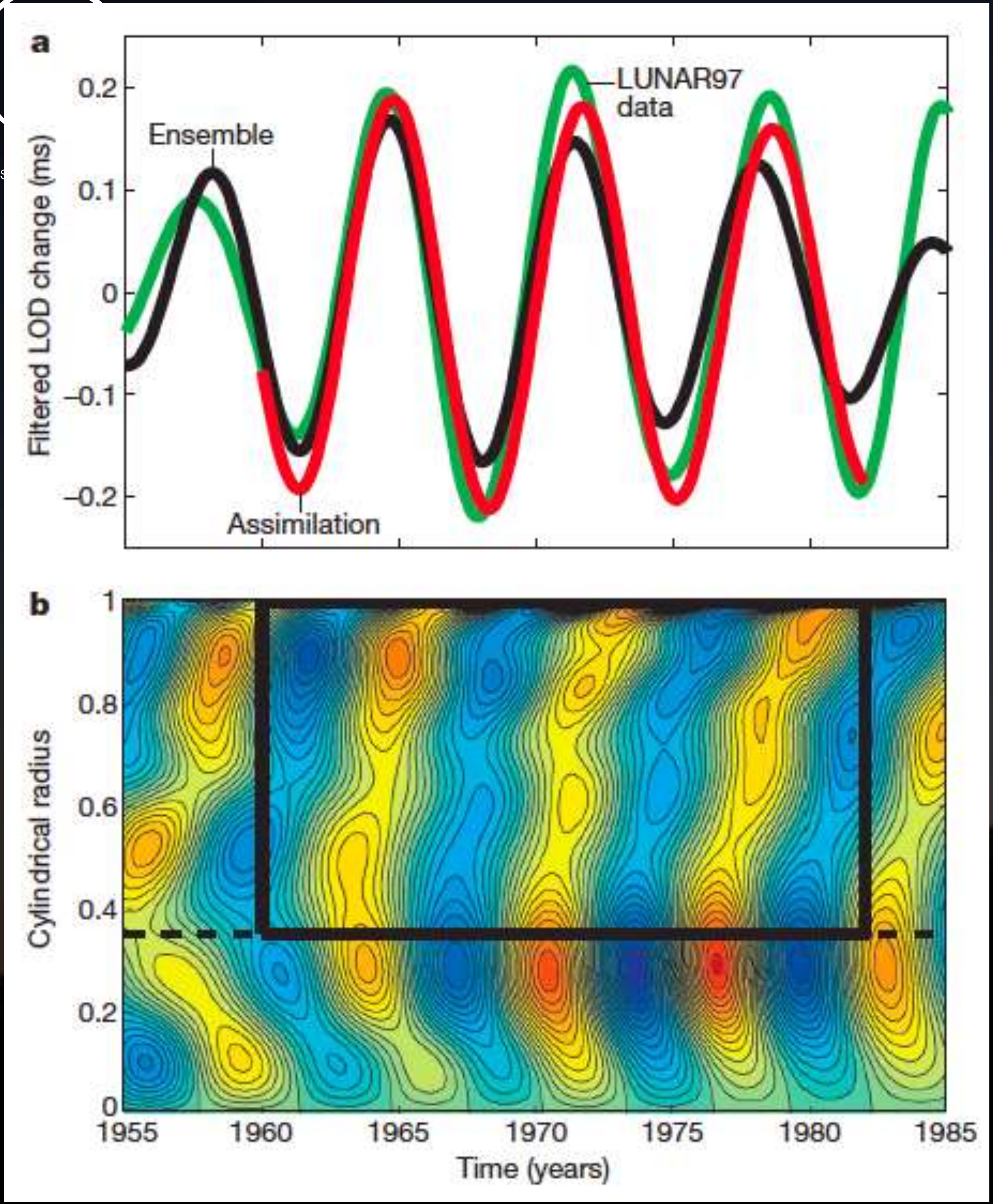
Discovering the flow inside the core: a large non-axial anti-cyclone



Equatorial map of the stream function, from data of year 2000

$t_{SV} \approx 300$ years

Pais & Jault, 2008



- and Alfvén waves that jerk the Earth !

(LOD = Length of Day)

±0.4km/year

$$V_A = \frac{B}{\sqrt{r\, m_0}}$$

$t_{Alfvén} \approx 4.3$ years

Gillet et al, Nature, 2010

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1. Kolmogorov-Obukhov-Corrsin versus Bolgiano scaling:

Kolmogorov and Bolgiano scaling in thermal convection: the case of Rayleigh-Taylor turbulence, G. Boffetta, F. De Lillo, A. Mazzino, S. Musacchi, 2011.

Turbulent convection at very high Rayleigh numbers, J. J. Niemela, L. Skrbek, K. R. Sreenivasan & R. J. Donnelly, *Nature*, 2000.

NB: report Nu α $Ra^{0.309}$ for $10^6 < Ra < 10^{17}$.

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2. Time-scale for QG columns:

$$t = \frac{L_z}{\Omega \ell}$$

On the evolution of eddies in a rapidly rotating system, P.A. Davidson, P.J. Staplehurst & S.B. Dalziel, *JFM*, 2006.

3. 'Why anisotropic turbulence is neither weak nor two-dimensional'

Critical balance in magnetohydrodynamic, rotating and stratified turbulence: towards a universal scaling conjecture, S.V. Nazarenko & A.A. Schekochihin, *JFM*, 2011.

4. Rhines scale for zonal jets:

$$\frac{u_j}{\ell} \approx \beta = \frac{2\Omega\cos\phi}{r_o}$$

Motion in the Interiors and Atmospheres of Jupiter and Saturn: Scale Analysis, Anelastic Equations, Barotropic Stability Criterion, A.P. Ingersoll & D. Pollard, *Icarus*, 1982.

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5. MHD turbulence by collisions of Alfvén waves:

Tobias S., Boldyrev & Cattaneo, in *10 Chapters on Turbulence*, Eds Davidson, 2012.

if equipartition continues down to the dissipation scale, then:

$$B_0 \nabla u = \eta \nabla^2 b$$

implies that this happens when:

$$\frac{t_h(\ell)}{t_{Alfvén}(\ell)} \gg 1$$

and at that point, one also has $Rm_\ell \gg N_\ell \gg 1$

6. Almost no fluctuations in our DTS experiment with global rotation

Schmitt D., T. Alboussière, D. Brito, P. Cardin, N. Gagnière, D. Jault, and H-C. Nataf, Rotating spherical Couette flow in a dipolar magnetic field: experimental study of magneto-inertial waves, *JFM*, 604, 175-197, 2008.

Nataf H-C. and N. Gagnière, On the peculiar nature of turbulence in planetary dynamos, *C.R. Physique*, 9, 702-710, 2008.