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Summary

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# **Retrodictions of late Paleogene Mantle Flow**

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L. Colli, S. Ghelichkhan, J. Oeser, M. Mohr, B. Schuberth, L. Vynnytska, A. Horbach, A. Friedrich, R. Pail

(LRZ, SAMPLE-SPP, TOPO-AFRICA)

Collège de France December 1st, 2016

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#### OUTLINE

- Introduction (forward models, dynamic topography)
- Theory (equations, twin experiments on convergence, boundary conditions)
- Simple Geodynamic Earth Models (model initialisation and uniqueness)
- Retrodictions (sensitivity to model parameters and tomographic input model)
- Conclusions (retrodictions are an powerful tool to learn about past Earth dynamics)



- Achievements
  - many of them
  - high resolution
  - comp. efficient
  - scenario simulations
- Frontiers
  - forward vs. inverse
  - link to observation
  - explicit histories



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(Zurich group)



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(Boulder group)



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- Achievements
  - many of them
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  - explicit histories



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paleo shorelines

5 Myr:



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#### 10 Myr:





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#### 20 Myr:





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#### 30 Myr:





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45 Myr:





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#### 60 Myr:





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#### 70 Myr:



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#### 80 Myr:



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#### 90 Myr:



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# Africas Elevation History

• **Topo Africa**: French sister program of the German **DFG SAMPLE SPP** to study the topographic evolution of Africa.



Burke & Gunnell. (2008)

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# Gravity and Dynamic Topography



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Colli et al. (2016)

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#### Response Kernels of Dynamic Earth Models



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#### Admittance vs. Wavelength



Colli et al. (2016)

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# The inverse approach to geodynamic flow modeling

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#### Initial state estimates from which one may start a model

a) Run convection models for a long time (with surface velocities given by known plate motion histories), i.e. longer than one mantle overturn,  $\approx 150$  Ma.

b) run convection backward in time (e.g., Moucha et al., Steinberger et al.)

 $\Rightarrow$  c) pose fluid dynamic inverse problem based on history matching

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#### The Adjoint equations of mantle convection

Mathematical procedures (chain rule, partial integration, adjoint operators) lead to the adjoint equations:

$$\partial_{t}\Psi + \mathbf{v} \cdot \nabla\Psi + k\nabla^{2}\Psi + \alpha\rho_{0}\mathbf{g} \cdot \phi = \partial_{T}\hat{\chi}(T)$$
$$\nabla \cdot \eta \left(\nabla\phi + (\nabla\phi)^{T}\right) + \Psi\nabla T = 0$$
$$\nabla \cdot \phi = 0$$

The scalar field  $\Psi$  is the so called *adjoint temperature*. It has to satisfy the above equations and provides sensitivity information about the initial state.



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The Adjoint equations of mantle convection

We solve the *adjoint* equations in global mantle flow models

- terminal condition on temperature
- adjoint diffusion operator stable vs. time-reversal
- iterative procedure: computationally expensive, but is beginning to become feasible in 3D

#### $\Rightarrow$ optimise for suitable flow histories (backwards in time)

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The (variational) adjoint approach to data assimilation

By iteratively adjusting the initial condition, one corrects the model trajectory over the whole time window, providing an *optimal* fit to the observational (here terminal condition) constraints.

Dashed line corresponds to initial (unconstrained) guess of the model trajectory.



(From **Fournier et al. 2012** with an application to dynamo models. Similiar approaches are used in meteorology, oceanography, glacier dynamics, hydrology.)

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#### The compressible adjoint equations of mantle convection

Mathematical procedures (chain rule, partial integration, adjoint operators) lead to the *compressible* adjoint equations:

$$\partial_t \Psi + \mathbf{v} \cdot \nabla \Psi - (\gamma - 1) \Psi \nabla \cdot \mathbf{v} + k \nabla^2 \Psi + \alpha \rho_0 \mathbf{g} \cdot \phi = \partial_T \hat{\chi}(T)$$
  
$$\nabla \cdot \eta \left( \nabla \phi + (\nabla \phi)^T \right) + \Psi \nabla T = 0$$
  
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The scalar field  $\Psi$  is the so called adjoint temperature. It has to satisfy the above equations and provides sensitivity information about the initial state. (from Ghelichkhan & Bunge, 2016)

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# inverse mantle convection models

# Twin Experiments (convergence)



#### Twin Experiments

Evolve Reference Twin from *initial* to *final* state.

Time period corresponding to 50 Myrs ( $\approx 1/2$  overturn).



Initial and final state for reference twin (red=hot, blue=cold). (from Ghelichkhan & Bunge, 2016)

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#### Twin Experiments

Iterate the adjoint many times from *initial* to *final* state.



Initial and final state reconstructions for increasing (top to bottom: 0,2,6) iterations (red=hot, blue=cold). Note that the initial state error is nearly eliminated through the inversion after 6 iterations. (from Ghelichkhan & Bunge, 2016)

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#### Twin Experiments

Convergence per iteration for *initial* and *final* state compared to reference twin.



Initial and final state residual. (from Ghelichkhan & Bunge, 2016)
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### regularisation of the inversion through knowledge of the surface velocity field

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**Twin Experiments** 

#### Twin Experiments: surface velocity

Evolve reference convection model (Twin) from *initial* to *final* state.

- Pick a time period corresponding to 50 Myrs ( $\approx 1/2$  overturn).
- See, if one recovers the intial state.



(a-d) temperatures (red=hot, blue=cold) at initial (a), intermediate (b,c) and final (d) state, (from Vynnytska & Bunge, 2014) 



#### Twin Experiments: surface velocity

First guess for *initial* condition.

Take a simple 1-D profile as the *first guess* for the unknown temperature of the initial state.



First guess model temperature initial condition (from Vynnytska & Bunge, 2014)



#### Twin Experiments: surface velocity

First guess for *initial* condition.

Take a simple 1-D profile as the *first guess* for the unknown temperature of the initial state.

This unrealistic *first guess* state is equivalent to assuming there is *no convection*.



First guess model temperature initial condition (from Vynnytska & Bunge, 2014)

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**Recovered** initial state temperature after **20** forward and adjoint iterations.

*Left* with assimilated history of model surface velocities.

*Right* with unconstrained (free slip) model surface.



first guess model temperature initial condition (from Vynnytska & Bunge, 2014)

The inversion is *unsuccessful* with unconstrained (free slip) model surface.

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True model temperature initial condition (from Vynnytska & Bunge, 2014)



Best guess model temperature initial condition with assimilated (left) and unconstrained surface velocities (right) (from Vynnytska & Bunge, 2014)

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Best guess model temperature initial condition with assimilated (left) and unconstrained surface velocities (right) (from Vynnytska & Bunge, 2014)

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**Initial State** RMS error as a function of adjoint iteration



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**Initial State** RMS error as a function of adjoint iteration

(note divergence for model with *free-slip* surface)



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**Initial State** RMS error as a function of adjoint iteration

(note divergence for model with *free-slip* surface) (we understand this result as a consequence of Serrin's *uniqueness* theorem)



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**Final State** RMS error as a function of adjoint iteration.



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**Final State** RMS error as a function of adjoint iteration.

(note there is convergence, i.e. the cost function is reduced, for either model)



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horizontal motion of Earth's surface is reconstructed for past  $\approx 200$  million years

provides boundary condition for velocity

one effectively exploits Serrin's theorem





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## compressibility effects

(These should be included, if one wants to apply geodynamic adjoint models to

seismically inferred Earth structure.)

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Initial and final state reconstructions for consistent (left), mixed (middle) and inconsistent (right) model (red=hot, blue=cold). Reference Twin (right most figure) (from Ghelichkhan & Bunge, 2016)

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Initial and final state reconstructions for consistent (left), mixed (middle) and inconsistent (right) model (red=hot, blue=cold). Reference Twin (right most figure) (from Ghelichkhan & Bunge, 2016)

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Evolve Reference Twin (right column) from *initial* to *final* state.

Time period corresponding to 50 Myrs ( $\approx 1/2$ overturn).



Initial and final state reconstructions for consistent (left), mixed (middle) and inconsistent (right) model (red=hot, blue=cold). Reference Twin (right most figure) (from Ghelichkhan & Bunge, 2016)

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Initial and final state reconstructions for consistent (left), mixed (middle) and inconsistent (right) model (red=hot, blue=cold). Reference Twin (right most figure) (from Ghelichkhan & Bunge, 2016)

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(from Ghelichkhan & Bunge, 2016)

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# Application of simple Geodynamic Earth models (GEMS) to seismic structure

(Uniqueness, and effect of model initialisation)

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- Schuberth et al., 2009a,b, Davies et al., 2012
- Schaber et al., 2009
- Goal:
  - compare geodynamic with seismic models by going through the convection process and mapping geodynamic to elastic variation



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#### Simple Geodynamic Earth Models

**Final State** 3 viscosity layers Isochemical **80** million grid points

initialized from present-day structure for unknown heterogeneity 40 million years ago



(from Horbach et al., 2014)

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Simple Geodynamic Earth Models

Initial State Corrections shown after 1 (left), 2 (middle) and 7 (right) iterations

only small corrections are needed after 7 iterations (right most column)



(from Horbach et al., 2014)

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#### Simple Geodynamic Earth Models

Optimal **Initial State 40 million years ago** 3 viscosity layers Isochemical **80** million grid points

initialized from **present-day** structure for unknown heterogeneity 40 million years ago



(from Horbach et al., 2014)


### Simple Geodynamic Earth Models





(from Horbach et al., 2014)

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### Simple Geodynamic Earth Models

*Optimal* Initial State computed from **four** different First Guesses present-day (a), rotated (b), blank (c), backward advection (d)





Residual at final state vs. Iteration for four ' initial guess' models: a) tomo b) rotated tomo c) blank mantle d) backw. advection.

Backward advection (cyan curve on the right) is a poor

initialisation for upper mantle structure



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## Retrodictions

Global High Resolution (pprox 670 million grid points) Geodynamic Earth Models

(sensitivity to tomographic input model and viscosity profile)

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#### Model Overview

- ( $\approx$  670 million grid points), grid point distance  $\approx 10 km$
- per adjoint iteration  $\approx 1$  million Core Hours
- six adjoint iterations per model
- four different models with Earthlike convective vigor
- pyrolite composition assumed for the sake of simplicity
- sensitivity to tomographic input structure and lower mantle viscosity

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### Tomographic input model



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(from Colli et al., 2017)

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### Viscosity profiles and dynamic topography kernels



(from Colli et al., 2017)



Conversion from Elastic to Geodynamic heterogeneity



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(from Colli et al., 2017)

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### Misfit Reduction



(from Colli et al., 2017)

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### Hemispheric Retrodictions (Atlantic Realm)





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### Histograms



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Model AM2

# 40 Ma









Upper Mantle Temperature



Upper Mantle Flow

Velocities



Dynamic Topography

(Imax 20)

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Model AM2

# 35 Ma











Whole Mantle Model Thermal Field Whole Mantle Model Flow Velocities Upper Mantle Temperature Upper Mantle Flow Velocities Dynamic Topography

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Model AM2

# 30 Ma











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Model AM2

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Dynamic Topography

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Model AM2

# 20 Ma











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Model AM2

# 15 Ma











Whole Mantle Model Thermal Field Whole Mantle Model Flow Velocities

Upper Mantle Temperature Upper Mantle Flow Velocities Dynamic Topography

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Model AM2

# 10 Ma











Whole Mantle Model Thermal Field Whole Mantle Model Flow Velocities Upper Mantle Temperature Upper Mantle Flow Velocities

Dynamic Topography

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Model AM2

# 5 Ma











Whole Mantle Model Thermal Field Whole Mantle Model Flow Velocities Upper Mantle Temperature Upper Mantle Flow Velocities

Dynamic Topography

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Model AM2

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Whole Mantle Model

Thermal Field

Whole Mantle Model

Flow Velocities

Upper Mantle

Temperature

Upper Mantle Flow

Velocities

**Dynamic Topography** 

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### 40 Ma



Thermal Field

Upper Mantle Velocity

Dynamic Topography (Imax 40)

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## 35 Ma



Thermal Field

Upper Mantle Velocity

Dynamic Topography (Imax 40)

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## 30 Ma



Thermal Field

Upper Mantle Velocity

Dynamic Topography (Imax 40)

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### 25 Ma



Thermal Field

Upper Mantle Velocity

Dynamic Topography (Imax 40)

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Thermal Field

Upper Mantle Velocity

Dynamic Topography (Imax 40)

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Thermal Field

Upper Mantle Velocity

Dynamic Topography (Imax 40)

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Thermal Field

Upper Mantle Velocity

Dynamic Topography (Imax 40)

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Thermal Field

Upper Mantle Velocity

Dynamic Topography (Imax 40)

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Thermal Field

Upper Mantle Velocity

Dynamic Topography (Imax 40)

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### Computational aspects

- $10^9 10^{10}$  free parameters (resolution dependent)
- iterative conjugate gradient scheme
- $\approx$  10 iterations needed to reach convergence
- 1 forward and 1 adjoint simulation per iteration
- need to store u and T at each time-step in the forward simulation

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- forward and adjoint equations are similar  $\rightarrow$  same numerical code can be used
- This is expensive.

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#### Conclusion

Growing model complexity makes it attractive to *test* geodynamic simulations by retrodictions

Uniqueness properties make plate motions the *input* rather than the output of a retrodiction

Compressible adjoint equations allow us to apply retrodictions to seismically derived mantle structure

Retrodictions open exiting possibilities for collaborative work *across* the Earth Sciences

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