Particules Élémentaires, Gravitation et Cosmologie

Année 2004-2005

Interactions fortes et chromodynamique quantique I: Aspects perturbatifs

Cours VII: 12 avril 2005

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2. Jet hadronization as a branching process

   2.1 From one- to many-parton inclusive x-sections

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   2.6 Quark vs. gluon jets

1. Summary of lecture no. 6

- Polarized DIS, connection w/ axial currents and their matrix elements, the «spin» crisis.
- Elements of Regge-theory & its naive connection to small-x in DIS: $(s/s_0)^{\alpha(0)}$ <--> $x^{-\alpha(0)}$
- Pushing DGLAP to smaller and smaller x

1. Double scaling limit: valid for

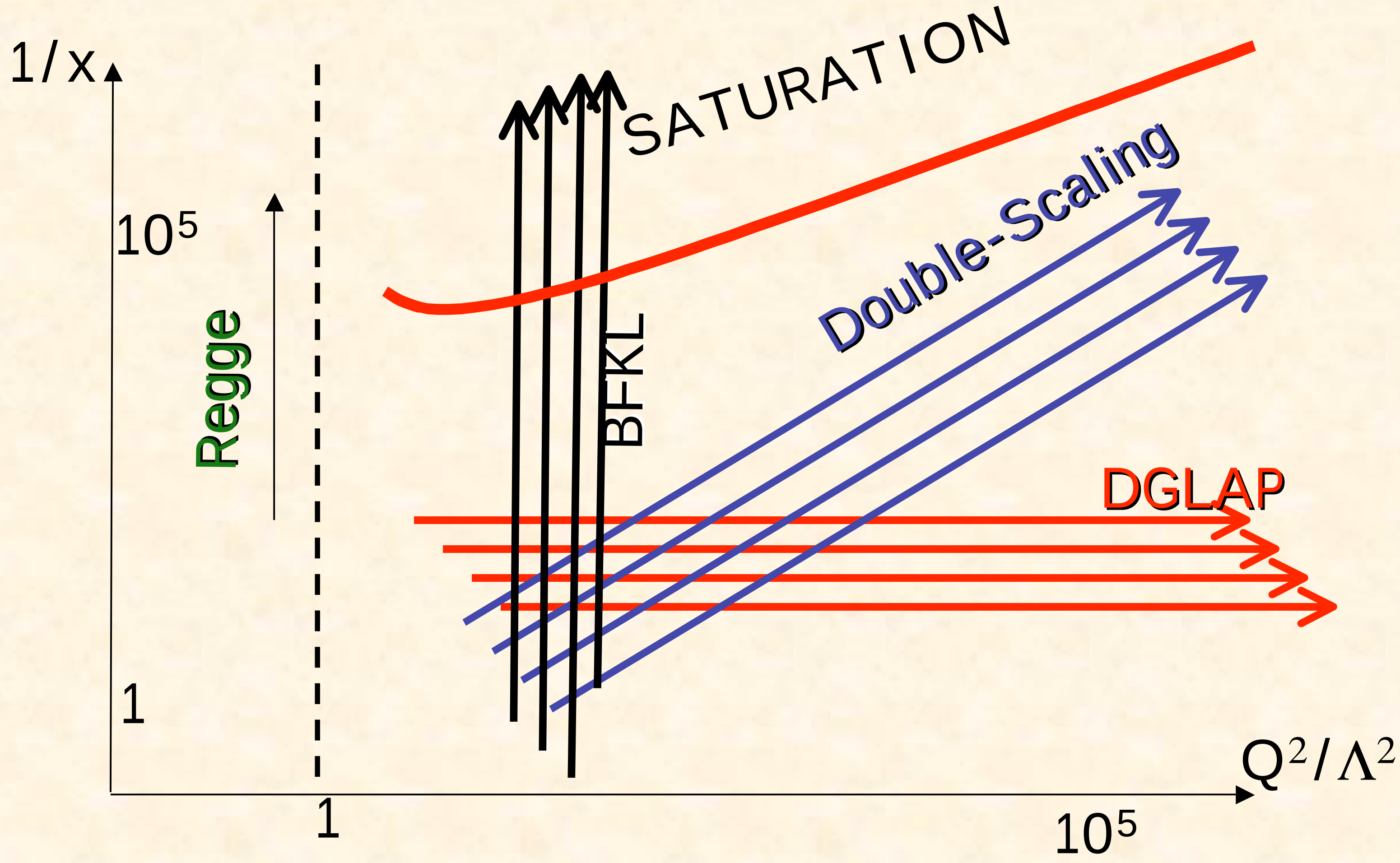
$$\alpha(Q^2)\log(1/x) \sim \log(s/Q^2)/\log(Q^2/\Lambda^2) < O(1)$$

and in apparent agreement with data

2. BFKL(mainly GS's seminar): trying to resum all leading log(1/x) => violating Froissart bound and not in good agreement with the data

Something must intervene to stop an IR catastrophe!

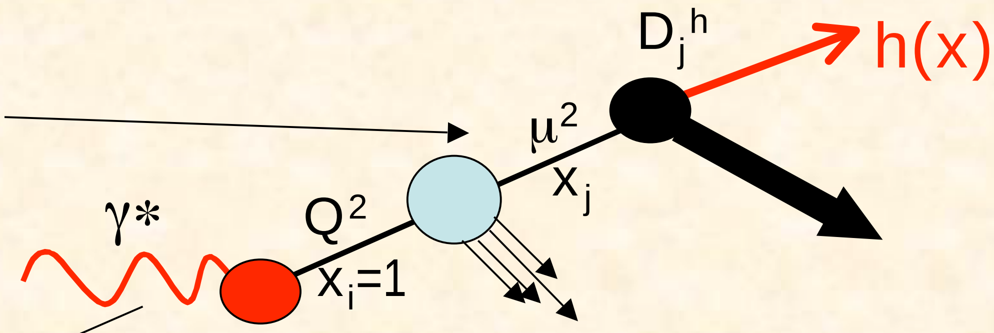
Saturation when $\alpha(Q^2)\, xg(x, Q^2) \sim Q^2/\Lambda^2$?

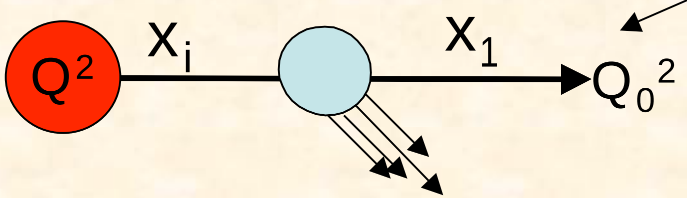


What is the situation if instead we look at fragmentation, i.e. at jet evolution and hadronization? At finite x there was a close similarity with DIS. But what happens as x->0?

2.1 From 1 to n-parton incl. x-section: Jet calculus

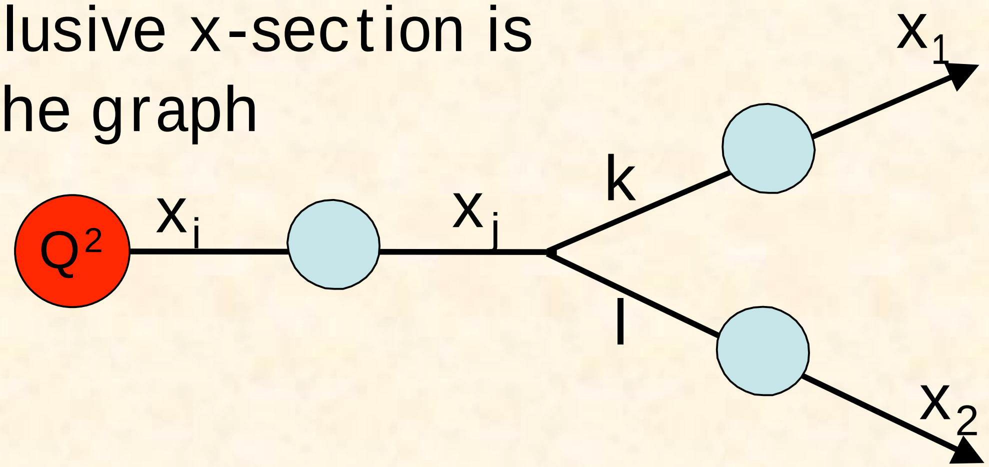
Let us go back to the one-particle incl. x-section (Lect. 4) but at parton level (with virtuality Q_0^2)





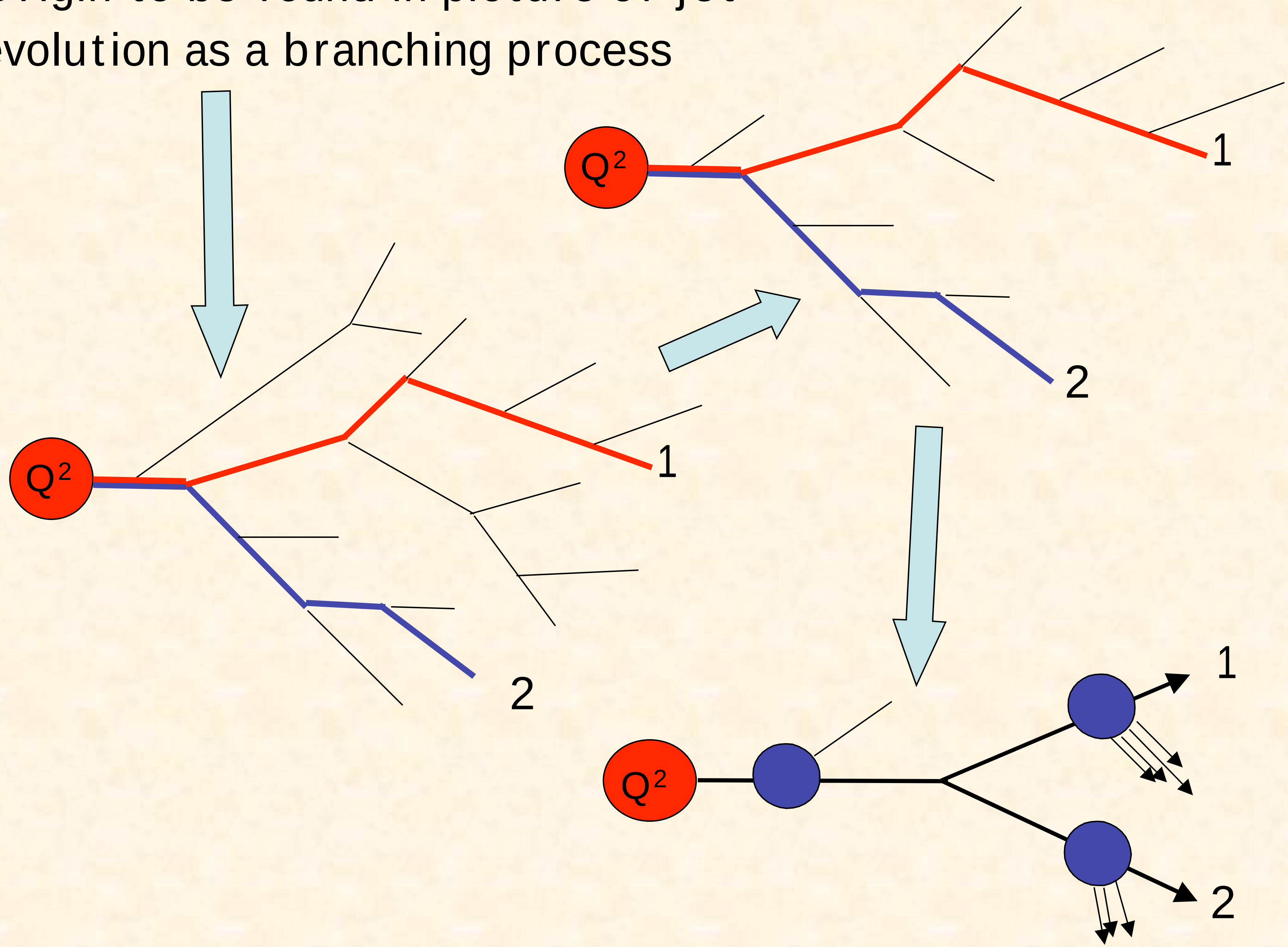
$$\sigma_H^i(x_i, Q^2) E_i^1(x_1/x_i; Q^2, Q_0^2)$$

The two-parton inclusive x-section is given by the graph

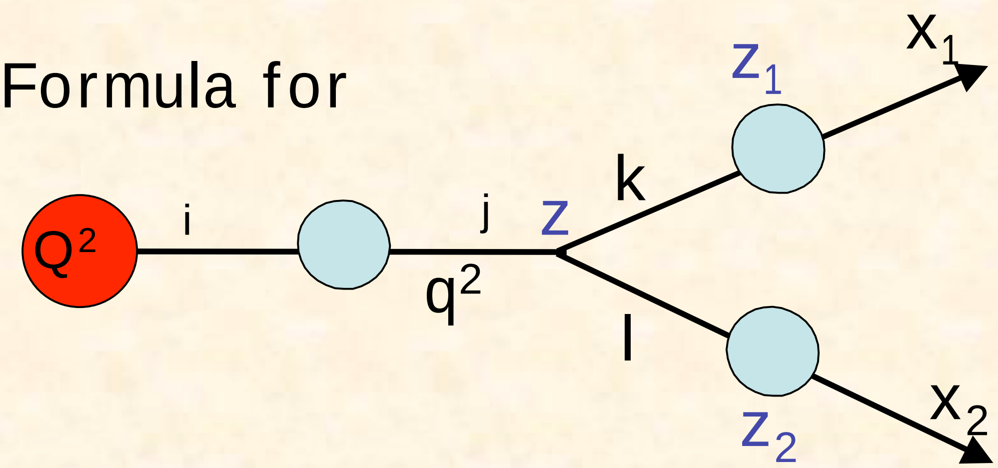


Why? Which is the corresponding mathematical expression?

Origin to be found in picture of jet evolution as a branching process



Math. Formula for



$$\sigma_H^i(x_i,Q^2)\int\frac{dq^2}{q^2}\frac{\alpha(q^2)}{2\pi}\int dx_j dz dz_1 dz_2 E_i^j(x_j/x_i;Q^2,q^2)\hat{P}_j^{kl}(z)$$

$$E_k^1(z_1;q^2,Q_0^2)E_l^2(z_2;q^2,Q_0^2)\delta(x_1-x_j z z_1)\delta(x_2-x_j(1-z)z_2)$$

Here P_j^{kl} is our P_R of lect. 4, i.e. w/out virtual contr. ~ $\delta(1-z)$

Simplifies a lot by going over to (n_1,n_2) moment

(moment conservation at the vertex)

If we keep q^2 fixed gives also the dependence on θ_{12}

One finds $\langle q^2\rangle \sim \alpha(Q^2)\,Q^2$

2.2 Sudakov form factor

What is the price to pay for having no real emission, i.e. for the original parton i to keep all its x_i? (NB: still have to keep the final parton at a finite virtuality Q_0^2 or else we get 0!)

Let $\Delta(Q^2, Q_0^2)$ be the probability that no emission takes place down to an off-shellness Q_0^2 in a jet produced at the scale Q^2. As it is intuitive, Δ (the Sudakov form factor) satisfies an evolution equation containing just the virtual part P_v of the GLAP kernel. In formulae:

$$Q^2 \frac{\partial \Delta(Q^2, Q_0^2)}{\partial Q^2} = \int \frac{dz}{z} \frac{\alpha}{2\pi} P_v(z) \Delta(Q^2, Q_0^2) = -\int dz P_r(z) \frac{\alpha}{2\pi} \Delta(Q^2, Q_0^2)$$

whose solution is:

$$\Delta(Q^2, Q_0^2) = exp\left(- \int_{Q_0^2}^{Q^2} \frac{dq^2}{q^2} \int_{\varepsilon}^{1-\varepsilon} dz \frac{\alpha(?)}{2\pi} P_r(z) \right)$$

$$\Delta(Q^2, Q_0^2) = exp\left(-\int_{Q_0^2}^{Q^2} \frac{dq^2}{q^2} \int_{\varepsilon}^{1-\varepsilon} dz \frac{\alpha(?)}{2\pi} P_r(z) \right)$$

This is clearly a very small number as the gap between Q^2 and Q_0^2 becomes large..

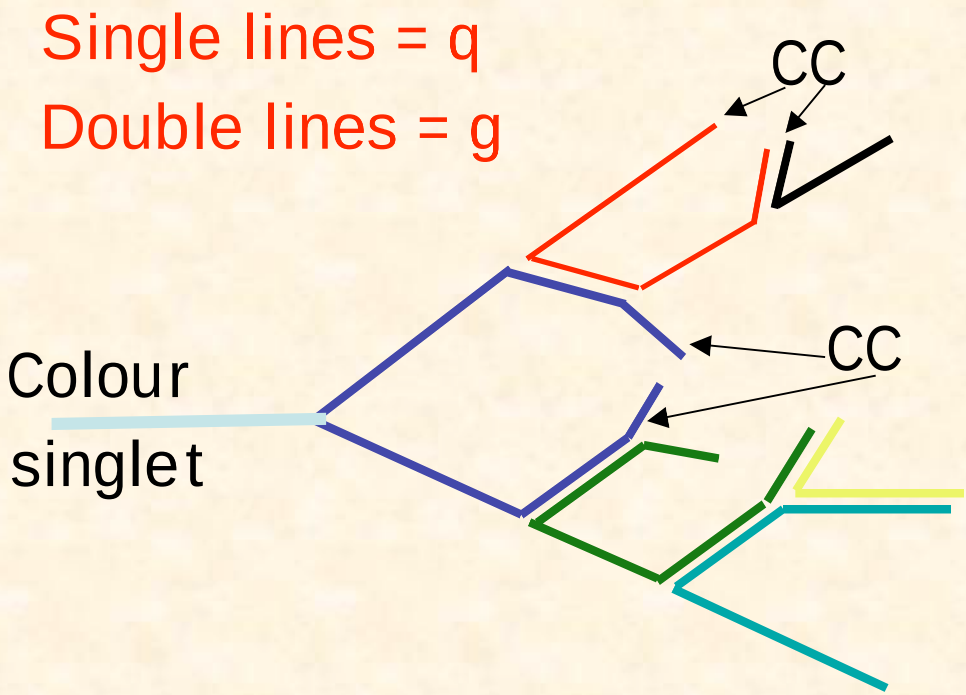
In order to be more quantitative we need to specify both ε and the argument of α. Naive guess: $\varepsilon = Q_0^2/q^2$, $\alpha(?) = \alpha(q^2)$ giving a power-like suppression

A more careful analysis suggests that $\alpha(?) = \alpha(z(1-z)\, q^2)$ while angular ordering (see 2.4) will give $\varepsilon^2 = Q_0^2/q^2$ .The former gives more suppression (bigger α) while the latter does the opposite (smaller phase space). One finds a suppression that is stronger than any power, but not as strong as an exponential:

$$\Delta(Q^2, Q_0^2) \sim \exp(-c \log Q^2 \operatorname{loglog} Q^2)$$

2.3 Colour structure and preconfinement

We can try to follow not only the flow of energy in the branching process but also that of colour. It is basically as indicated in the following picture (really valid for large-N)



It leads to the concept of colour-connected (CC) partons, those that share a colour line. The overall colour of two colour connected partons is reduced (for two quarks it's 0) and thus CC partons are good precursors of the final hadrons. The question is: what is the mass distribution of CC systems?

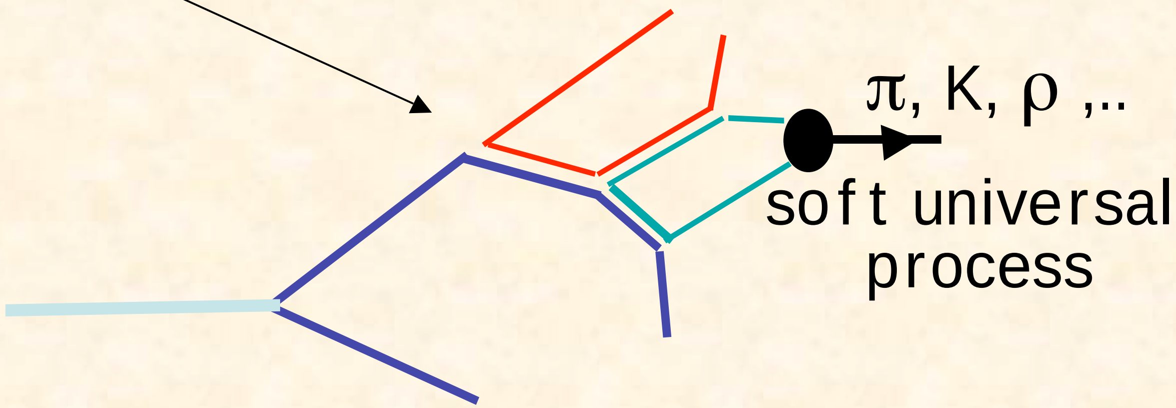
Previously we have seen that the distribution of the inv. mass^2 of two arbitrary partons is broad ($\langle M^2 \rangle \sim \langle q^2 \rangle \sim \alpha(Q^2)\, Q^2$). Something dramatically different happens for M_{CC}^2. The reason is related to our Sudakov F.F.!

The Sudakov FF was the price to pay for keeping the qq* pair colour connected (parton emission breaks the connection) while sending its inv. mass higher and higher. Thus the Q^2-dependence of $\Delta(Q^2, Q_0^2)$ should give the M_{CC}^2 distribution. Since Δ falls faster than any power as $Q^2 \gg Q_0^2$ it will enforce $\langle M_{CC}^2 \rangle \sim Q_0^2$.

We thus arrive at the following interesting conclusion:
The perturbative branching automatically organizes the final partons (at off-shellness Q_0^2) into CC clusters of mass just a few times larger than Q_0^2 and independent of Q^2.

One can now postulate that these clusters «hadronize», i.e. produce known light hadrons through a non-perturbative universal process that does not involve much reshuffling of momentum. => «jet- shape variables» computed at parton level should reflect those observed experimentally at hadronic level

Example

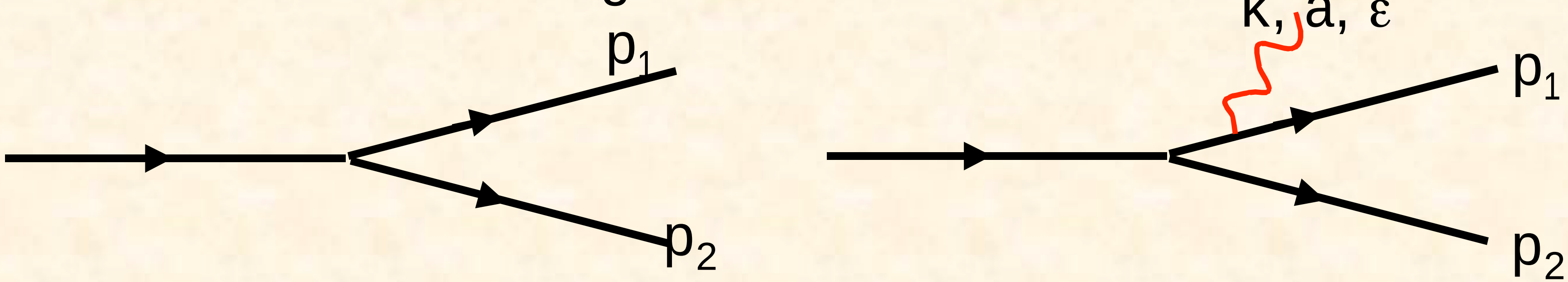


In order to make this recipe as effective as possible actual value of Q_0^2 should be optimized: sufficiently large in order to trust pQCD, sufficiently low to have to make minimal guesses on the final hadronization pattern (few GeV looks reasonable)

2.4 Interference and angular ordering

I will be rather brief since this has been discussed at length in some of the seminars.

Consider an elementary step in the branching process and the one with one more soft gluon emitted



Amplitude for latter process is that of former times a factor (NB: summing amplitudes => interference effects!)

$$g\left(T_1^a \frac{p_1 \cdot \varepsilon}{p_1 \cdot k} + T_2^a \frac{p_2 \cdot \varepsilon}{p_2 \cdot k}\right)$$

This shows:

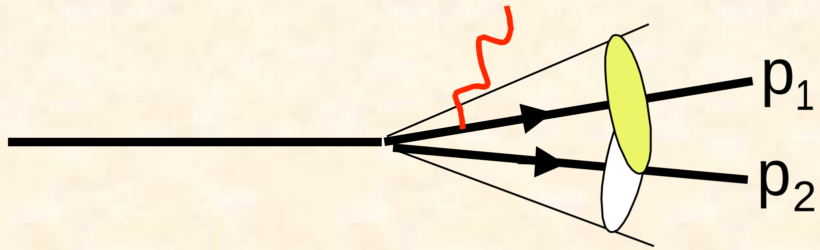
IR singularity when k --> 0

When p_1 parallel to p_2 what counts is $(T_1^a + T_2^a)$

In practice, $p_1 // p_2$ means that the angle between the two vectors is smaller than at least one of those they form with k

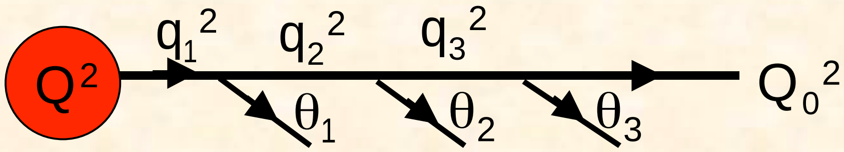
In this case, since the sum of the two generators tends to cancel (they correspond to the colour of their common parent) the process is suppressed in this kinematical region

==> IR enhancements are only present for gluons emitted inside two overlapping cones obtained by rotating p_1 around p_2 and viceversa. This is the angular ordering due to quantum interference.



Luckily this interference effect does not destroy the probabilistic character of the branching process: it just reduces the phase space available for each emission

How does it restrict available phase space?

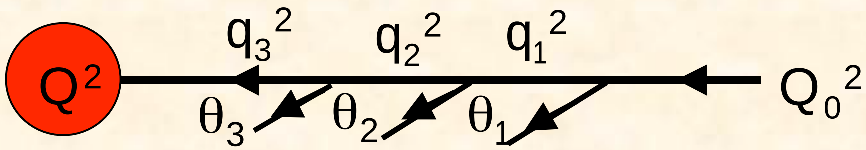


$q_i^2 \sim x_i^2 z(1-z)\ \theta_i^2$

$\theta_1 > \theta_2 > \theta_3 \Rightarrow q_1^2/x_1^2 > q_2^2/x_2^2 > q_3^2/x_3^2$ ..

$q_2^2 < x_2^2\ /x_1^2\ q_1^2 = z^2\ q_1^2$ (stronger than simple q^2-ordering)

Note difference wrt DIS:



Since x-ordering now opposite of q^2-ordering the latter already implies angular ordering

2.5 Small-x behaviour and multiplicities

Angular ordering means basically

$q_1^2/x_1^2 > q_2^2/x_2^2 > q_3^2/x_3^2$ ...i.e. $q_2^2 < q_1^2\, x_2^2\, /x_1^2 = z^2\, q_1^2$

This implies a modification of the DGLAP equation into

$$Q^2 \frac{\partial D^i(x,Q^2)}{\partial Q^2} = \sum_j \int_x^1 \frac{dz}{z} \frac{\alpha(?)}{2\pi} P_j^i(z) D^j(x/z, z^2 Q^2)$$

Similar to the equation we encountered for small-x in DIS (DSL) except for the rescaling of the argument by z^2. After inserting also the argument of α, we finally obtain the small-x distribution and total multiplicity.

Instead of

$$xF(x,Q^2) = exp\left(2\tilde{c}\sqrt{log(1/x)log\frac{logQ^2/\Lambda^2}{logQ_0^2/\Lambda^2}}\right)$$

we find

$$xD(x,Q^2) = e^{c\sqrt{logQ^2/Q_0^2}}exp\left(-c'\frac{(log1/x-\frac{1}{4}logQ^2/Q_0^2)^2}{(logQ^2/Q_0^2)^{3/2}}\right)$$

$$c = \sqrt{\frac{6}{\pi\beta_0}}$$

The total multiplicity in the jet is dominated by the gaussian peak:

$$\langle n\rangle \sim e^{c\sqrt{logQ^2/Q_0^2}}$$

Both predictions, including the falloff of D above $x \sim (Q_0^2/Q)^{1/2}$, are well verified in the data

In order to connect the partonic prediction to real data we can proceed in two ways:

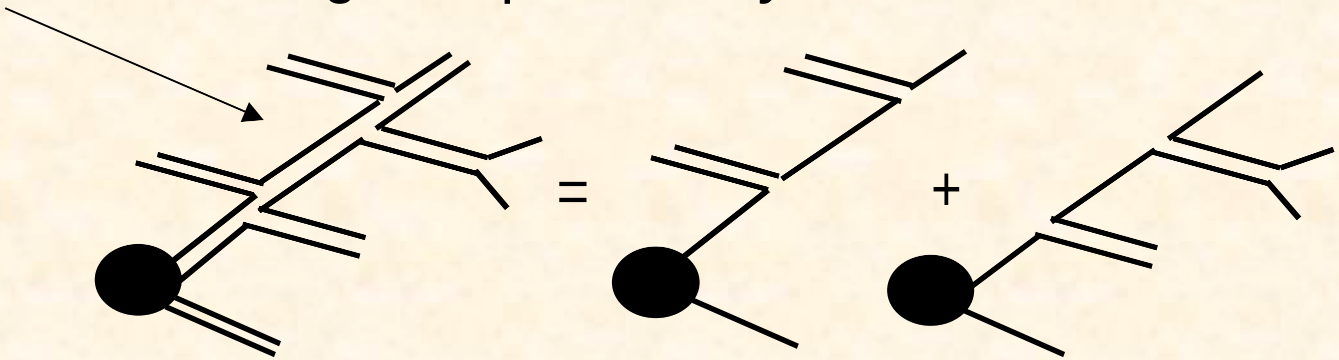
1. Assemble the final partons in CC pairs and convert those into hadrons via some phenomenological recipe. Although the overall normalization is lost we can still check the shape of the x-distribution or the growth of multipicity
2. Keep Q_0 somewhat large (compared to Λ) and then interpret each off-shell final parton as a «mini-jet». The above result then tells us how the number of these minijets depends on our « resolution » scale Q_0.

- Since $n(Q, Q_0) = n(Q, Q_1)\, n(Q_1, Q_0) = ..$ The process is self-similar, like a fractal (jets inside jets, inside jets..)
- Self-similarity eventually ends .. when we reach the QCD confinement scale Λ...

2.6 Quark vs. Gluon jets

Because of their larger colour charge gluons radiate more than quarks, about twice as much ($C_A/C_F = 2N^2/(N^2-1) \Rightarrow 9/4$)

At large N we can understand this as due to the two colour lines of the gluon radiating independently:



Gluon jets are also softer, e.g. the fastest hadron emerging from a gluon jet should have a smaller $\langle x \rangle$ than the fastest hadron coming out of a quark jet. Also, the leading hadron in a quark jet should carry some memory of the quark flavour, while, of course, a gluon jet is more « democratic »...

Finally, gluon jets have a broader opening angle than quark jets. This can be seen from the leading order formula:

$$\frac{\sigma_{2\,jets}}{\sigma_T} = 1 - 4C_{F,A}\frac{\alpha(Q^2)}{\pi}\log\varepsilon\log\delta_{q,g}$$

for quark and gluon jets, respectively. Thus, if we want the same fraction of the total x-section with the same fraction of energy (1- ε) in the two jets, we need to take

$$\delta_g = (\delta_q)^{C_F/C_A} = (\delta_q)^{4/9} > \delta_q$$

All these features are confirmed by MC simulations and can also be seen in the data.

General conclusion of last two lectures: our understanding of small-x physics is better for jet hadronization than for DIS!

Some bibliography on pQCD

1. T. Muta, Foundations of QCD, World Scientific Pub. Co. (1987);
2. Perturbative QCD, ed. A. H. Mueller, World Scientific Pub. Co. (1989)
3. Yu. L. Dokshitzer, V.A. Khoze, A. H. Mueller and S.I. Troyan, Basis of Perturbative QCD, Editions Frontiers (1991)
4. R. K. Ellis, W. J. Stirling and B.R. Webber, QCD and Collider Physics, Cambridge University Press, 1996

Next week:

Last Lecture: Symmetries and Anomalies

Last seminar: Twistors and Gauge theory