String Cosmology I: General Considerations
For more details on String Cosmology see, for instance:

• G. Veneziano, Les Houches 1999, hep-th/0002094
• M. Gasperini & G. Veneziano, hep-th/0703055

For even more details see:

Outline

1. Puzzles in hot-big-bang cosmology
2. Conventional inflation: successes and new questions
3. String cosmology:
   • Short-distance motivations
   • Large-distance motivations
   • Useful plots, examples of solutions
   • The exit problem
4. Initial conditions, fine-tuning issues, open problems
Puzzles of Hot Big Bang cosmology

Immediately after the Big Bang, today’s observable Universe (“our patch” hereafter) was far too big for its age. For instance, a Planck-time ($t_P \sim 10^{-43}$ s) after the BB the size of our patch was $\sim 10^{28}$ times larger than the Planck length $l_p$ ($l_p \sim 10^{-33}$ cm = maximal correlation length @ $t = t_P$).

It consisted of $\sim 10^{84}$ causally disconnected regions. With the decelerating expansion of standard cosmology our patch has always been larger than the horizon:

Q₁: Why homogeneity on large-scales?

Also: spatial curvature redshifts like $a^{-2}$, i.e. slower than matter ($a^{-3}$) and radiation ($a^{-4}$).

Since today’s Universe appears to be $\sim$ spatially flat (from CMB data, see next lecture) it must have been (almost) exactly flat right after the BB. Q₂: Why?
CONVENTIONAL HOT-BIG BANG MODEL

Present horizon
Present distance from far-away cluster

now
here

H-1

太少 big for its age!

Big Bang
If we accept that the Universe had a beginning, \( t > 0 \), there appears to be only one way out: 
Make the primordial Universe much smaller! 
This is the way conventional inflation works. 
By introducing a phase of **accelerated** (typically exponential) expansion in early cosmology, it makes our initial patch (much) smaller than \( cH^{-1} \)

During inflation spatial curvature, as well as matter and radiation, are quickly red-shifted away. Instead, the potential energy that is responsible for inflation behaves like a (slowly rolling) cosmological constant and remains (nearly) constant. After a few Hubble times the Universe becomes essentially empty, cold, and spatially flat. This is why inflation needs a reheating mechanism giving rise to an “effective (big?) bang”
End of inflation, "effective bang"

Towards the Big Bang

STANDARD INFLATION

Present horizon

Present distance from far galaxy

$\mathcal{H}^{-1}$

Now

Time

Space
Inflationary models are usually agnostic about the real Big Bang. They assume that whatever preceded inflation was kind enough to generate, at least within our patch, suitable initial conditions for the onset of inflation (an inflaton displaced from the minimum of its flat potential & homogeneous over a few Hubble lengths).

The nice thing about inflation is that, once such conditions are satisfied, it washes out all details of the initial conditions, making robust predictions: (once a specific model of inflation is assumed)
The **counterpart** is that we cannot get information about pre-inflationary, short-distance physics (controversy over transplanckian problem?)

Also a number of questions arise:

What is the inflaton?
What produces its very flat potential?
How come it started far from the minimum of its potential?
How come it was homogeneous over several Hubble scales?

and also

Implementing conventional inflation in the SM of particle physics, or in string theory, turned out to be problematic (brane inflation?)
Is \( t > 0 \) a necessity or a myth? (is conventional inflation the only way out?)

- It is a necessity if we consider GR as an exact theory suffering no short-distance (large-curvature) corrections at both the classical and the quantum level.
- Simple dimensional arguments show that quantum effects become \( O(1) \) at the latest when the Planck scale is reached, but possibly earlier.
- Alternatively, CGR softens already at an energy (distance) scale well below (above) the Planck scale. This is what happens, for instance, in weakly-coupled string theory.
- New ways to solve the problems of standard cosmology (an older, rather than a smaller Universe)?
Here

Our horizon today

Present distance from far galaxy

end of inflationary phase: Bang!

PRE BIG BANG
Even an accelerated contraction can do the job!
What is important is that
\( \frac{da}{dt} \) and \( \frac{d^2a}{dt^2} \)
have the same sign!
Both are characterized by a bounce in curvature!
Here is a diagram showing the relationship between time and space in the context of the Big Bang and the expansion of the universe.

- **Our horizon today**
- **Present distance from far galaxy**

At the end of the collapsing phase, there was a **Bang!**

The diagram illustrates the current state of the universe with the present distance from a far galaxy shown as $|H|^{-1}$. The state before the Big Bang is labeled as **EKPYROTIC**.

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Remarks

• The **disadvantage** of these cosmologies is that they are sensitive to what happens at (or what replaces) the big bang
• Its **counterpart** is that the physics of the big bang becomes accessible to present cosmological observations
• This is not the only option: spacetime could be an **emerging** concept out of a state in which we cannot even talk about space and time
• Finally, this scenario is not logically **incompatible** with conventional inflation. A more modest aim would be to generate from a **generic** previous phase the initial conditions that are suitable to turning on inflation
• In this latter case details of that primordial phase will be presumably washed out
Motivating a pre-bang cosmology in string theory

A. Short-distance Motivations

Fundamental string-length scale $l_s$ appears to imply:

1. maximal $T$, density, curvature, ..
2. minimal BH size, compactification radius,
3. $\Delta x > \hbar/\Delta p + \alpha'\Delta p > l_s$

Q: Is the BB singularity avoided?
Elimination of singularities through $l_s \neq 0$?

- String theory «resolves» some singularities of GR
- Those associated with cosmology (big bang) are harder to deal with (SUSY breaking), but are also likely to be eliminated/reinterpreted (new d.o.f.) *
- If so, we may conceive new scenarios in which the big bang, rather than representing the beginning of time, is the result of a previous phase in which space-time curvature (~ Hubble parameter $H$) grew up to some maximal, finite value
- String effects would then force the Universe to «bounce» after going through a high-curvature «string phase»
- The Big Bang becomes a «Big Bounce»

*) AdS/CFT may help clarify this issue.
Motivating a pre-bang cosmology in string theory

B. Large distance motivations

1. **New fields**: $G_N$ is a field: $G_N \sim e^{\phi} \sim \alpha$. This ubiquitous scalar field, the dilaton $\phi$, controls the strength of all forces, modifies EE’s, provides novel symmetries, is essential for T-duality.

2. **Extra dimensions** (important for perturbations, see next lecture) provide other scalar fields, associated with their sizes and shapes (so-called moduli)
All these fields have vanishing perturbative mass, because of SUSY.

If they remain light (at NP level after SUSY breaking), they may induce «short-distance» modifications of gravity, threaten the equivalence principle and universality of free-fall, induce space-time variations of the above «constants», etc. (see next lecture)

A very active field of experimental and theoretical research.

But they may become sufficiently heavy to be completely harmless today...
However, if the Universe was originally in a very weak coupling regime, the dilaton and other moduli started as basically massless and could have participated in the cosmological evolution. They provide a crucial ingredient for arguing about what the pre-bang phase may have looked like: T-duality.

![Diagram showing weak and strong coupling](image-url)

An anti-friction term pushes the dilaton up.

\[ V(\phi) \]

\[ \phi = \phi_0 \]
Use of T-duality to describe pre-bang phase

The duality symmetries of the string-cosmology equations naturally allow for inflationary solutions.

Scale-factor duality, combined w/ T, acts as follows

\[ a(t) \rightarrow a^{-1}(-t), \phi(t) \rightarrow \phi(-t) - 2d \log a(-t) \]

\[ H(t) \equiv \dot{a}/a(t) \Rightarrow H(-t); \dot{H}(t) \Rightarrow -\dot{H}(-t) \]

Maps FRW type cosmologies into inflationary ones thanks to a growing effective \( G_N \sim e^\phi \)

Basically, the Friedman equation,

\[ H^2 = G_N \rho \]

becomes:

\[ H^2 \sim e^\phi \rho \]

and H can grow because of growth of \( G_N \sim e^\phi \)
Useful plots

1. Coupling and curvature vs. time

\[ \phi = \phi_0 \]

\[ O(1) \]

\[ O(1/N) \]

pert. in \( e^\phi \)

pert. in \((l_sH)^2\)

pert. in derivatives

t
Useful plots

2. Coupling and curvature (parametric)

\[ V(\phi) \]

\[ \phi_0 \]

\[ 1_s^2 H^2 \]
Useful plots

3. \( \phi \) and expansion rates (parametric).

Arrows correspond to growing time

In perturbation theory we cannot connect DDI and FRW!
We may hope that $\alpha'$ and/or loop corrections will allow for a smooth transition to the unstable trivial vacuum via FRW (from sing.?) and DDI (towards sing.?).
Basic assumption of PBB cosmology

Let’s make one assumption, suggested by our pictures: The Universe (or at least our patch) started its evolution from deeply inside the small coupling, small curvature regime, but otherwise in a generic state.

⇒ At least for a while, we can describe it in terms of the tree-level, low-energy effective action, the one thing we know well. The problem of initial conditions becomes more tractable than in standard inflation (no transplanckian problem, for instance) ... but we have to pay the price later!
Let us limit our attention to metric, dilaton and antisymmetric tensor fields. Then, in \( D=Dc=10=d+1 \),

\[
\Gamma_{eff} \sim \int d^{d+1}x \sqrt{|g|} e^{-\phi} \left( R + g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{12} (dB)^2 \right)
\]

Assuming, for the moment, all fields to be functions of time only (Bianchi I-type cosmologies), we can construct the general solution (w/ explicit \( O(d,d) \)-invariance).
For \( B=0 \) this reads (for \( t<0 \)):

\[
ds^2 = -dt^2 + \sum_i (-t)^{2\alpha_i} dx^i dx^i,
\]

\[
\phi = - \left( 1 - \sum_i \alpha_i \right) \log(-t)
\]

\[
\bar{\phi} = \phi - \frac{1}{2} \log (\det g_{ij}) = -\log(-t)
\]

NB: unlike for Kasner, isotropic solutions allowed; SFD manifest
At very early times these solutions are in the weak-curvature regime (the initial coupling, a modulus, can be made arb. small)

\[ ds^2 = -dt^2 + \sum_i (-t)^{2\alpha_i} dx^i dx^i, \]
\[ \phi = -(1 - \sum_i \alpha_i) \log(-t) \]
\[ 1 = \sum_i \alpha_i^2. \]

A particularly interesting symmetric case is given by:

\[ \alpha_i = -1/3 \ (i = 1, 2, 3); \quad \alpha_i = +1/3 \ (i=4, ..9) \]

=> Our 3-D space grows isotropically, the internal compact space shrinks isotropically; metric is Ricci-flat; 10-D dilaton is constant, effective 4-D coupling grows.

It will turn out to be “phenomenologically” favoured
Initial conditions and fine-tuning issues

1. General considerations, APT
2. Spherical symmetry
3. Planar symmetry
4. Almost realistic situations
5. Answering Fine-tuning allegations
6. BKL oscillations and chaos

Some points sketched below: more details available on transparencies (upon request)
Under the assumption of Asymptotic Past Triviality (APT) one can reduce the problem of getting an inflationary solution out of some generic initial conditions to a well-known problem in CGR: Gravitational Collapse.

Technically this is done by some field-redefinitions which amount to going to the Einstein frame

$$\Gamma^E_{\text{eff}} \sim \int d^{d+1}x \sqrt{|g^{(E)}|} \left( R - \frac{1}{d-1} \partial_\mu \phi \partial^\mu \phi - \frac{1}{12} e^{-\frac{4}{d-1} \phi} (dB)^2 \right)$$

$$g_{\mu \nu} = g^{(E)}_{\mu \nu} e^{\frac{2}{d-1} (\phi - \phi_0)}$$
For $B=0$ this reduces to the much-studied case of gravitational collapse of a minimally-coupled massless scalar field. The general case is difficult to analyze. However, given that the action, in both frames, is simply rescaled under either a constant shift of the dilaton or a rescaling of the coordinates, the collapse criteria are scale and dilaton-shift invariant (leaving us with two free parameters, integration constants)

When a horizon is formed the behaviour inside it is cosmological, typically ending at a big-crunch singularity. Depending on the behaviour of the dilaton, this can become a big bang from the point of view of the string metric. NB: GC is very generic in GR, and so is DDI! (see cartoons)

Of course, we can only trust the solutions until curvature or coupling become large, whichever comes first! But, depending on those two moduli, this can take a very very long time, answering fine-tuning allegations...
Initial chaotic sea of massless waves as perturbation of the trivial vacuum

Overdense regions
Our big bang (r=0)

Onset of collapse/inflation

Observable U today

Another collapse, big bang, Universe

Initial chaotic sea of massless waves

A. Buonanno, T. Damour & GV, 1999
In some cases much can be said about the collapse criteria
1. Spherical symmetry (Christodoulou, Choptuik, BDV,..)
2. Planar symmetry (FKV, BV,..). Analytic solutions
3. Collision of finite-front waves (KV, see lect. 4)
The exit problem

This is the main unsolved theoretical problem of PBB cosmology. Most naïve attempts don’t work: no go theorems. Most promising less naïve approaches include:
1. Exit through a SFD-invariant (non-local) potential
2. Late time attraction to a fixed point (with constant positive $H$ and $\frac{d\phi}{dt}$, thanks to $\alpha'$ corrections (GMV); completion of exit to FRW via loop corrections (BM, CC)
3. Exit at strong coupling and small curvature via production of D0-branes (MR)
4. Exit after going through a weak coupling string-hole phase at which temperature, density, curvature and entropy bounds are all saturated (GV)
5. Quantum tunnelling a la WdW...
Other big-bounce scenarios

1. Ekpyrotic/cyclic I
2. Ekpyrotic/cyclic II
3. Brane inflation
4. Ekpyrotic III?
The ekpyrotic Universe I
(Khouri, Ovrut, Steinhardt & Turok '01)

Large 5th dimension

xyz

Our brane  A 3rd brane moving in the bulk  Hidden brane

BB as result of impact of 3rd brane on ours
Before the BB

BB is just the collapse of the 5th dimension to zero size. This means having the BB at zero coupling, i.e. the opposite of what is assumed in PBB scenario.

⇒ PBB phase is a contracting phase (even in the string metric) & the BB is a scale-factor bounce.

In both cases it is a curvature bounce
Brane inflation
(Burgess, Quevedo, Tye, KKLT, ...)

Here the extra dimensions are just those orthogonal to the two parallel branes. A brane-antibrane system is not static, there is a non-trivial potential and the distance between the two branes plays the role of the inflaton. More sophisticated versions using fluxes (KKLT, KKMMT) have been proposed in order to stabilize some moduli.
Conclusions so far

1. On purely theoretical grounds these cosmologies are better motivated and, at least some of them, less contrived than conventional slow-roll inflation.

2. The real question however is whether they do as well as slow-roll inflation in explaining cosmological data; this is the issue we will try address in the final lecture.