Particules Élémentaires, Gravitation et Cosmologie

Année 2010-'11

Théorie des cordes: quelques applications

Cours IV: 11 février 2011

Résumé des cours 2009-'10: quatrième partie

Duality, branes and the 2^{nd} string revolution

Field and String-theoretic symmetries.	String version of KK: T-duality
D-strings and D-branes	M theory & unification

Symmetries of the effective action

The effective action of QST is invariant under GCT in spacetime (the principle underlying GR), under gauge transformations of the B field: B -> B + dΛ and under standard gauge transformations. These are symmetries that QST shares with QFT.

There are hints that these are only a tiny subset of a large symmetry underlying string theory and responsible for the huge degeneracy of its spectrum. Not much is known about the real extent of this symmetry; nonetheless some "stringy" symmetries are now well established.

Many are related to the compactification of the extra spatial dimensions in which strings like to "live".

In QFT gauge symmetries emerge from the KK mechanism. This phenomenon too takes a "new look" in QST.

Kaluza-Klein in QFT

Take one extra dimension of space (x^5) to be a circle of radius R and consider the 5-dimensional analog of the Einstein-Hilbert action with the metric parametrized as:

$$ds_5^2 = g_{\mu\nu} dx^\mu dx^\nu + e^{2\sigma}(dx^5 + A_\mu dx^\mu)^2$$

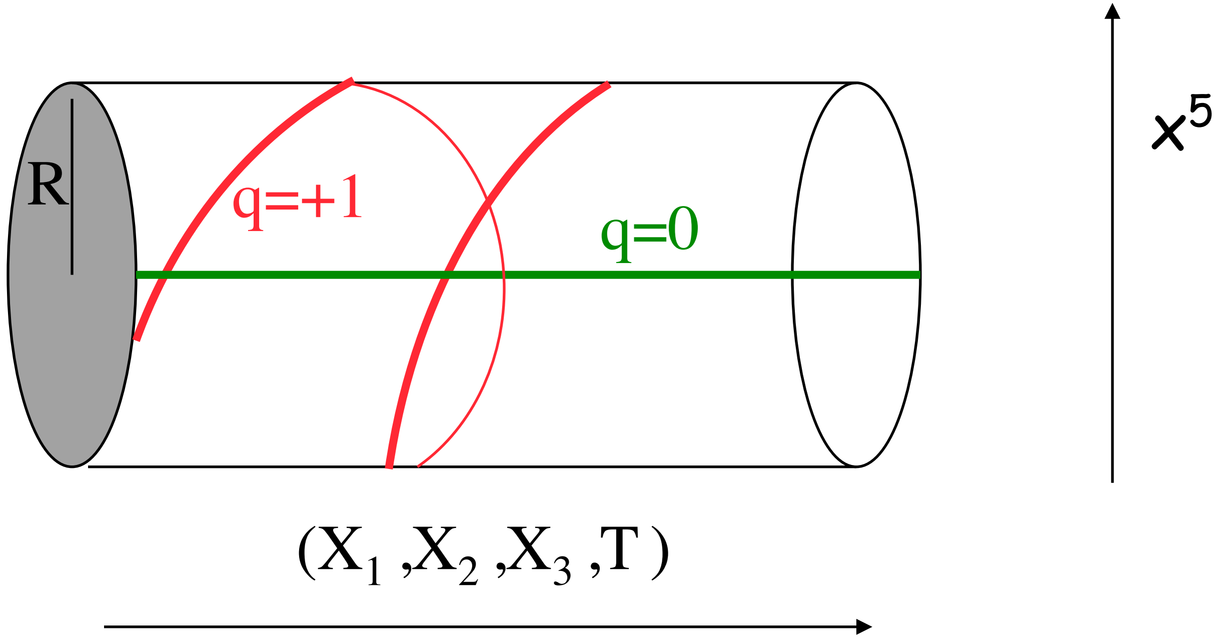
The $g_{\mu 5}$ component of the 5-dimensional metric is essentially A_μ, while g_{55} is associated with the physical radius of the circle (a "radion" field). If all the fields do not depend on x^5 one gets gravity + Maxwell's EM in 4-D:

$$S = \frac{1}{2\kappa_5^2} \int d^5x \sqrt{-g_5}\ R_5 \Rightarrow \frac{\pi R}{\kappa_5^2} \int d^4x \sqrt{-g_4}\ e^\sigma \left[R_4 - \frac{e^{2\sigma}}{4} F_{\mu\nu} F^{\mu\nu} \right]$$

$$l_P^2 \equiv 16\pi G = \frac{l_5^3}{2\pi\rho} \ ; \ \alpha_4 = \frac{l_P^2}{\rho^2} \ ; \ \rho \equiv e^\sigma R$$

Charged particles correspond to fields with non-trivial (periodic) dependence on x^5 and thus quantized p_5.

Typically they have masses O(h/R) (the scale of KK excitations) but there can be massless "zero modes".



$$x_5 \equiv x_5 + 2\pi m R \; ; \; p_5 = n\frac{\hbar}{R} \; ; \; q = n\frac{l_P}{R} \; ;$$

$$m_4^2 = m_5^2 + p_5^2 = m_5^2 + n^2\left(\frac{\hbar}{R}\right)^2$$

The stringy version of KK

In string theory, for a generic value of R, the gauge symmetry is actually U(1)xU(1). The reason is that both $G_{\mu 5}$ and $B_{\mu 5}$ give rise to gauge bosons. General covariance and invariance under gauge transformations of B both become ordinary gauge transformations.

For the U(1) coming from G we can identify the charge with the momentum in the 5th dimension.

Which is the charge associated with the U(1) of B?

By looking at the vertex operator for the $B_{\mu 5}$ gauge boson, we discover that its associated "charge" is winding!

Since the points X_5 and $X_5 + 2\pi\, m\, R$ are now identified, there is nothing wrong for a closed string to obey:

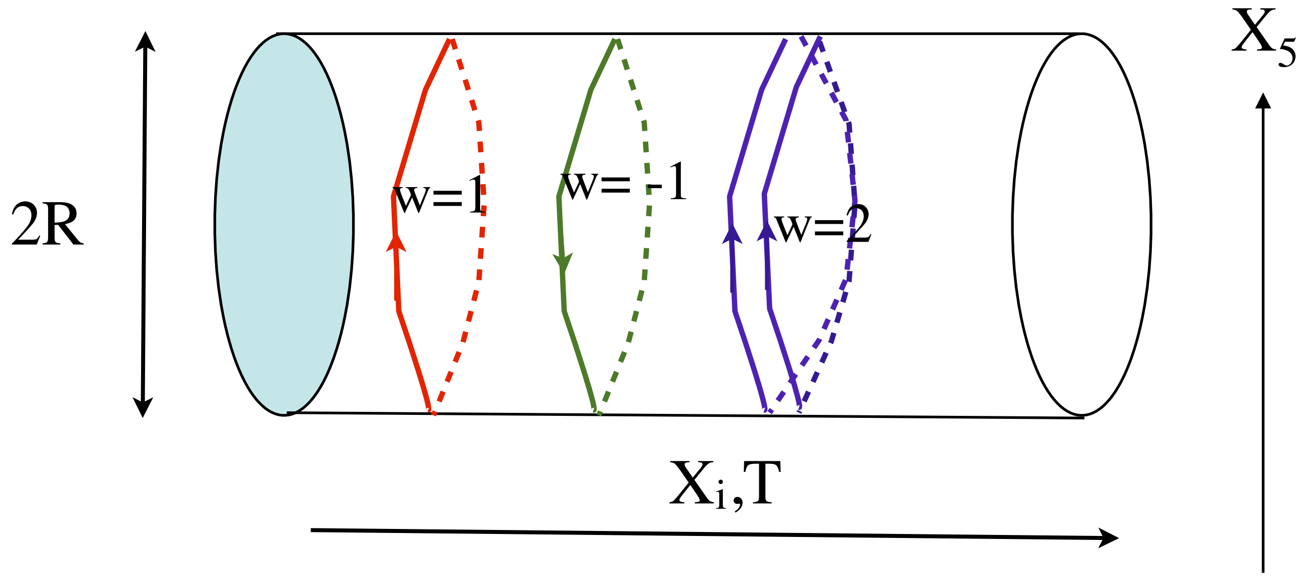
$$X_5(\sigma=\pi) = X_5(\sigma=0) + 2\pi\, w\, R.$$

It simply means that the closed string winds around the compact direction w-times!

Winding costs energy because of the string tension.

The new boundary condition is easily implemented in the general solution by adding a "winding term".

NB: neither point particles, nor open (N) strings can wind!



$$X_5(\sigma,\tau) \quad = \quad q_5 + 2n\alpha'\frac{\hbar}{R}\tau + 2wR\sigma$$
$$+ \quad \frac{i}{2}\sqrt{2\alpha'}\sum_{n=1}^{\infty}\left[\frac{a_{n,5}}{\sqrt{n}}e^{-2in(\tau-\sigma)} - \frac{a^{\dagger}_{n,5}}{\sqrt{n}}e^{2in(\tau-\sigma)}\right]$$
$$+ \quad \frac{i}{2}\sqrt{2\alpha'}\sum_{n=1}^{\infty}\left[\frac{\tilde{a}_{n,5}}{\sqrt{n}}e^{-2in(\tau+\sigma)} - \frac{\tilde{a}^{\dagger}_{n,5}}{\sqrt{n}}e^{2in(\tau+\sigma)}\right]$$

Mass shell conditions for the closed bosonic string:

$$M^2 = \frac{n^2}{R^2} + \frac{w^2 R^2}{\alpha'^2} + \frac{2}{\alpha'}(N + \tilde{N} - 2) \; ; \; N - \tilde{N} + nw = 0$$

$$N = \sum_{n,\mu} n N_{n,\mu} \quad ; \quad \tilde{N} = \sum_{n,\mu} n \tilde{N}_{n,\mu}$$

For generic R the only m=0 physical states are of the

$$N = \tilde{N} = 1 \; ; \; n = w = 0 \; ; \; \text{i.e.} \;\; a^\dagger_{1\mu} \tilde{a}^\dagger_{1\nu} |0\rangle$$

Without compactification there are just $(D-2)^2$ physical states: a graviton, a dilaton, an antisymmetric tensor.

After compactification these split into a graviton, a dilaton, an antisymmetric tensor in (D-1) dimensions, two (D-3)-component vectors, and 1 scalar, the "radion".

However, if we choose a particular value for R:

$$R = R^* \equiv \sqrt{\hbar\alpha'} = \frac{l_s}{\sqrt{2}}$$

there are new ways of getting massless states

$$n = w = \pm 1 \ ; \ N = 0, \tilde{N} = 1 \quad \text{or}$$

$$n = -w = \pm 1 \ ; \ N = 1, \tilde{N} = 0$$

These 4 massless vectors together with the 2 previous ones are the gauge bosons of an SU(2)xSU(2) gauge group. The 4 new gauge bosons carry momentum and winding and are therefore themselves charged, a characteristic of non-abelian gauge theories.

At R = R* there are also (4+4) additional massless scalars for a total of 10. One of them (the "radion") plays the role of a Higgs field that breaks SU(2)xSU(2) down to U(1)xU(1) away from R = R*.

T-duality (for closed strings)

Classically a winding number is an integer while momentum in the compact direction is not quantized. There is no possible symmetry between winding and momentum.

At the quantum level momentum is quantized in units of h/R. As a result, a symmetry appears between winding and momentum i.e. if we exchange n & w and, at the same time, R into $l_s^2/2R$. The point of enhanced gauge symmetry is precisely the fixed point of this T-duality transformation!

Therefore the inequivalent compactifications are labelled by the range $R > R^*$ so that, effectively, there is a minimal compactification radius $R=R^*$.

The second string revolution (Polchinski, 1995)

T-duality for open strings

Recall the boundary conditions for open strings:

$$X'_\mu \delta X^\mu(\sigma = 0) = X'_\mu \delta X^\mu(\sigma = \pi) \quad ; \quad (\text{no sum over } \mu)$$

For closed strings T-duality corresponds to a canonical transformation exchanging X' with P.

Neumann boundary conditions correspond to setting X'=0 at the ends of the open string, while Dirichlet boundary conditions mean δX=0, which amounts to setting P = 0.

It looks therefore highly reasonable that, for open strings, T-duality simply changes the boundary conditions from N to D, and vice versa.

Unlike closed strings, open string are not "self T-dual": they come in two kinds which are T-dual to each other!

We can choose N or D boundary conditions independently for each string coordinate.

If we set Dirichlet BC for a certain number n of spatial directions, the ends of such strings are only free to move in the remaining (D-n) directions. These span (D-n)-dimensional hyperplanes immersed in the full spacetime.

Such hyperplanes are called D-branes (D for Dirichlet) or, more precisely, Dp-branes, where p = (D-n-1) is the number of spatial dimensions of the hyperplane to which one should add time in order to arrive at p+1 = D-n.

Summarizing: the hypersurface on which the ends of our D-strings can move is (p+1)-dimensional. Such open strings have (p+1) Neumann and n Dirichlet directions.

D-strings & compactification

Let's compactify one dimension, x_5. The X_5 coordinate of an N-string is given by:

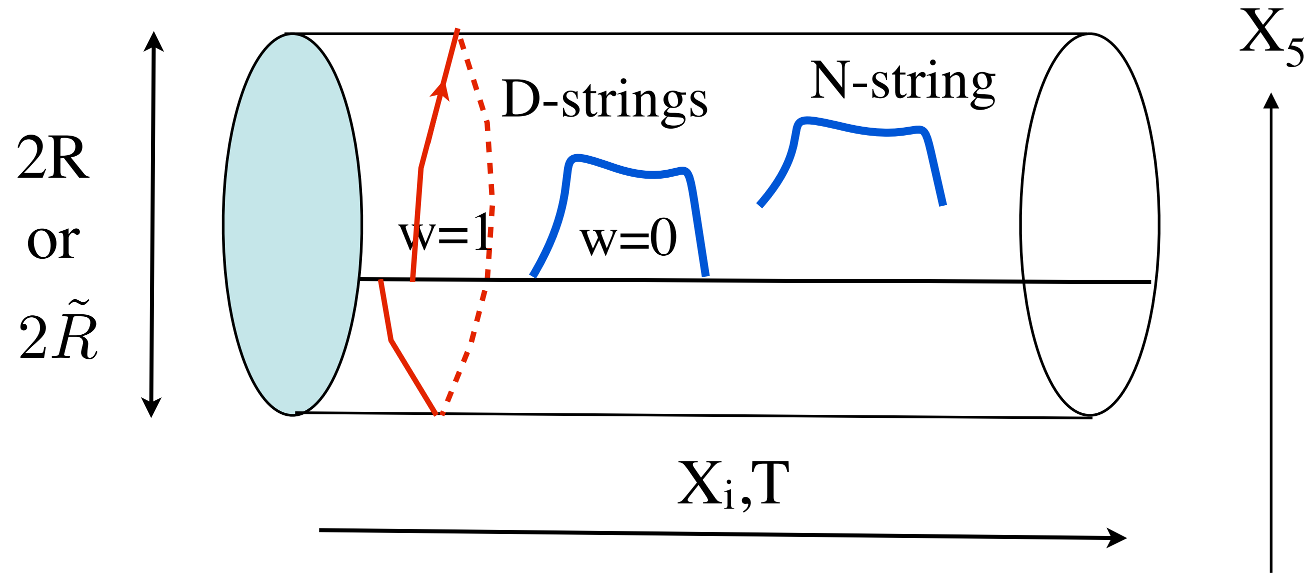
$$X_5^{(N)}(\sigma,\tau) = q_5 + 2n\alpha' \frac{\hbar}{R}\tau + i\sqrt{2\alpha'}\sum_{n=1}^{\infty}\left[\frac{a_{n,5}}{\sqrt{n}}e^{-in\tau} - \frac{a_{n,5}^{\dagger}}{\sqrt{n}}e^{in\tau}\right] cos(n\sigma)$$

T-duality interchanges P_5 and TX'_5. The result is simply:

$$X_5^{(D)}(\sigma,\tau) = q_5 + 2w\tilde{R}\sigma + i\sqrt{2\alpha'}\sum_{n=1}^{\infty}\left[\frac{a_{n,5}}{\sqrt{n}}e^{-in\tau} - \frac{a_{n,5}^{\dagger}}{\sqrt{n}}e^{in\tau}\right] sin(n\sigma)$$

The D-string winds around the dual circle w-times! D-strings can wind!

NB: T-dual N and D-strings move/wind around dual circles!

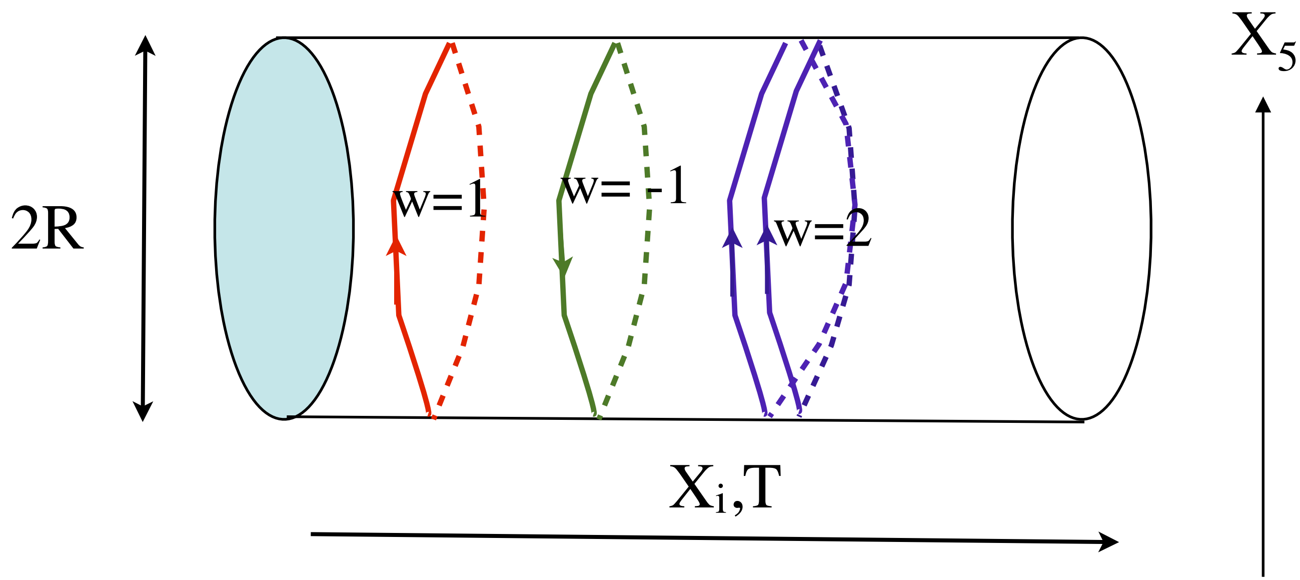


$$X_5^{(N)}(\sigma,\tau) = q_5 + 2n\alpha' \frac{\hbar}{R}\tau + i\sqrt{2\alpha'}\sum_{n=1}^{\infty}\left[\frac{a_{n,5}}{\sqrt{n}}e^{-in\tau} - \frac{a_{n,5}^{\dagger}}{\sqrt{n}}e^{in\tau}\right] cos(n\sigma)$$

$$X_5^{(D)}(\sigma,\tau) = q_5 + 2w\tilde{R}\sigma + i\sqrt{2\alpha'}\sum_{n=1}^{\infty}\left[\frac{a_{n,5}}{\sqrt{n}}e^{-in\tau} - \frac{a_{n,5}^{\dagger}}{\sqrt{n}}e^{in\tau}\right] sin(n\sigma)$$

$$X_5^{(D)}(\sigma=\pi,\tau) = X_5^{(D)}(\sigma=0,\tau) + 2\pi\tilde{R}\ w$$

to be compared with the closed string case:



$$X_5(\sigma,\tau) = q_5 + 2n\alpha' \frac{\hbar}{R}\tau + 2wR\sigma$$
$$+ \frac{i}{2}\sqrt{2\alpha'}\sum_{n=1}^{\infty}\left[\frac{a_{n,5}}{\sqrt{n}}e^{-2in(\tau-\sigma)} - \frac{a^{\dagger}_{n,5}}{\sqrt{n}}e^{2in(\tau-\sigma)}\right]$$
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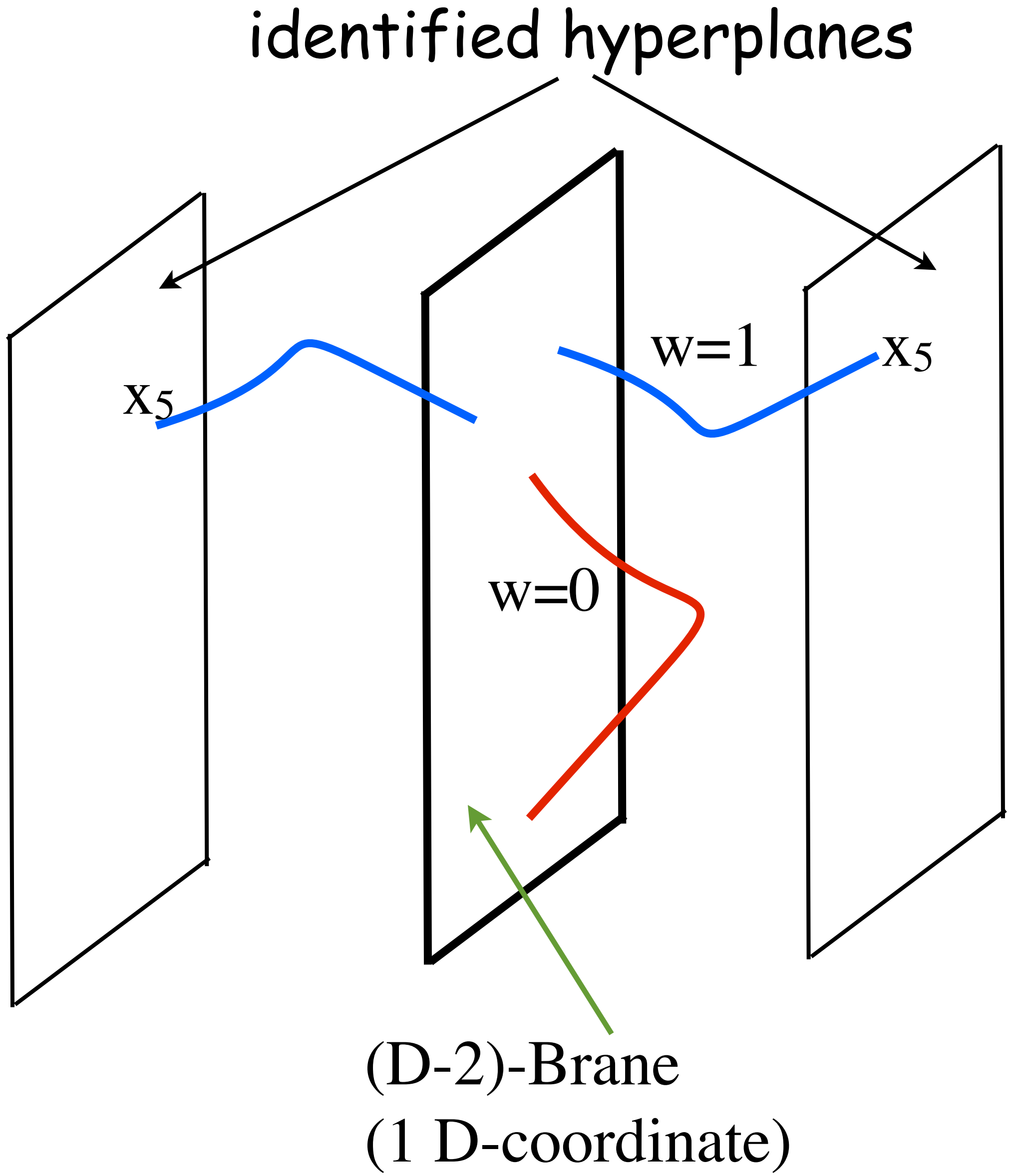


identified hyperplanes
x_5
w=1
x_5
w=0
(D-2)-Brane
(1 D-coordinate)

For an open bosonic D-string the mass shell condition reads:

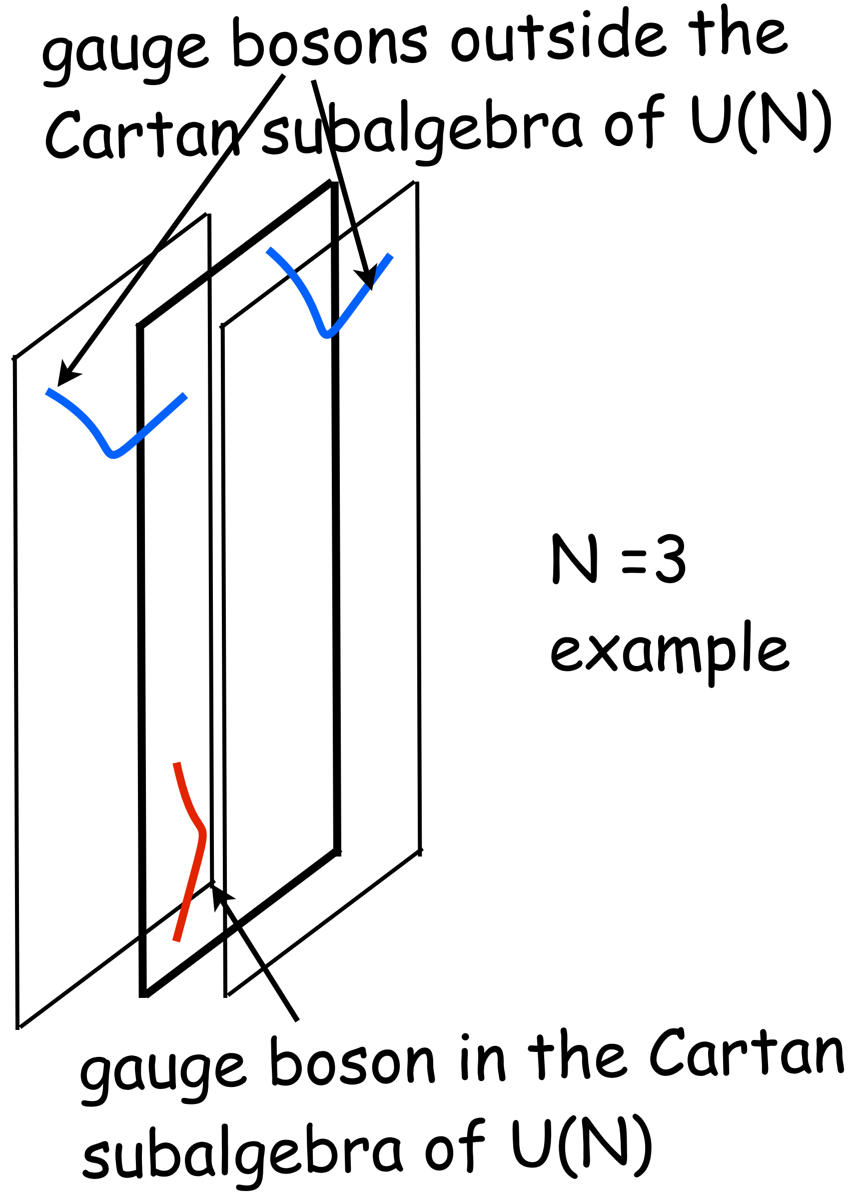
$$L_0 = 1 \Rightarrow M^2 = \frac{w^2 R^2}{\alpha'^2} + \frac{1}{\alpha'}(N-1)$$

For generic R the massless states are given by n=w=0, N=1, i.e. by the state $a^\dagger_{1\mu}|0\rangle$.
If the index of the oscillator is not 5 this is a gauge boson stuck on the brane (with (D-3) physical components); if the index is 5 it's a massless scalar also confined to the brane.

The presence of the brane breaks (spontaneously) translation invariance in the 5th direction. The massless scalar is the Nambu-Goldstone boson of that broken symmetry and describes the possible local deformations of the brane itself!

When N branes overlap we get N^2 massless J=1 bosons since each end can be on any of the N coincident branes without generating mass -> the gauge bosons of a U(N) gauge theory.

There are also massless scalars corresponding to the distances between the branes. They play the role of Higgs fields giving mass to the gauge bosons of the spontaneously broken generators (the mass being proportional to the separation).



The Dp-brane action

The low-energy effective action that describes the dynamics of a single D-brane is quite simple. It was invented long ago by Born and Infeld and used by Dirac, the DBI (Dirac-Born-Infeld) action.

$$\frac{1}{\hbar}S_p = -c_p l_s^{-(d+1)} \int d^{p+1}\xi e^{-\Phi} \left[-\det\left(G_{ab} + B_{ab} + \pi l_s^2 F_{ab}\right)\right]^{1/2}$$

where c_p is a known p-dependent number and G_{ab} , B_{ab} , are the induced metric and B on the brane's world sheet:

$$G_{ab} = \frac{\partial X^\mu}{\partial \xi^a}\frac{\partial X^\nu}{\partial \xi^b} G_{\mu\nu}(X(\xi)) \qquad B_{ab} = \frac{\partial X^\mu}{\partial \xi^a}\frac{\partial X^\nu}{\partial \xi^b} B_{\mu\nu}(X(\xi))$$

while F_{ab} is the gauge field strength of its associated U(1).

The scalar fields X^μ describe the transverse fluctuations of the brane. After using invariance under reparametrization of the (p+1) ξ^a coordinates the physical scalar degrees of freedom are only (D-p-1).

Another observation concerns the dependence of the brane action from the dilaton. Instead of the overall $\exp(-2\Phi)$ of the effective closed-string action we get here a factor $\exp(-\Phi)$.

This is exactly what we should expect since the open-string coupling is $\exp(\Phi)$ while the closed-string coupling is $\exp(2\Phi)$.

The brane Universe. Since the ends of open strings are stuck on the brane, if all the SM particles are open strings we may be living on a 3-brane and only gravity and other gravitational-like forces would feel the full dimensionality of spacetime (and are modified at short distances).

D-branes as classical solutions

From an open-string viewpoint D-branes are hypersurfaces on which open strings end but D-branes also emerge as classical solutions of the string effective action when we add all the massless bosonic fields contained in Type IIa or IIb superstring theories. Of crucial importance are the RR forms present in such theories since, as it turns out, D-brane are "charged" under those fields (i.e. they are sources for the forms). A p-brane couples naturally to a (p+1)-form potential. Thus, Type IIa, having odd forms, gives rise to even-p-branes, the opposite being the case for Type IIb.

The solutions are relatively simple: metric, dilaton and the relevant forms only depend on the transverse distance from the brane. If wrapped they can give rise to "black-branes".

This observation is important for the black-hole entropy calculation to be discussed in next week's seminar.

D=11 supergravity and M-Theory

D=10 is surprisingly close to D=11 which was known for sometime to be the maximal number of dimensions in which consistent interacting supersymmetric theories can be constructed (otherwise massless particles of spin higher than 2 are needed and put several problems).

D=11 supergravity was studied for quite sometime even before the 1984 GS revolution with the hope to solve the UV problems of quantum gravity. The bosonic part of its action looks as follows:

$$S = \frac{1}{2\kappa_{11}^2}\left[\int d^{11}x\sqrt{-g_{11}}\;\left(R_{11} - \frac{1}{2}F_4^2\right) - \frac{1}{6}\int A_3 \wedge F_4 \wedge F_4\right]$$

Thus, in D=11, one has just two fields: the metric and a 3-form potential (with the associated 4-form field strength).

$$S = \frac{1}{2\kappa_{11}^2}\left[\int d^{11}x\sqrt{-g_{11}}\,\left(R_{11} - \frac{1}{2}F_4^2\right) - \frac{1}{6}\int A_3 \wedge F_4 \wedge F_4\right]$$

Let us see what this theory becomes when the 11th dimension is a circle of radius R. We proceed as before defining:

$$ds_{11}^2 = g_{\mu\nu}dx^\mu dx^\nu + e^{2\sigma}(dx^{11} + A_\mu dx^\mu)^2$$

The effective action in D=10 turns out to be:

$$S_{10} = -\tfrac{\pi R}{2\kappa_{11}^2}\int d^{10}x\sqrt{-g_{10}}\left(2e^\sigma R_{10} + e^{3\sigma}F_2^2 + e^{-\sigma}F_3^2 + e^\sigma \tilde{F}_4^2\right)$$
$$+ \text{Chern Simons terms},$$

where F_2 is the 2-form associated with A_μ, while F_3 and F_4 follow from the dimensional reduction of the D=11 F_4.

Thus, at the D=10 level we have the metric, a scalar, a 1-form, a 2-form and a 3-form potential. This is the set of bosonic massless fields of the D=10 Type IIa superstring! Indeed the low energy actions fully coincide after some field redefinition.

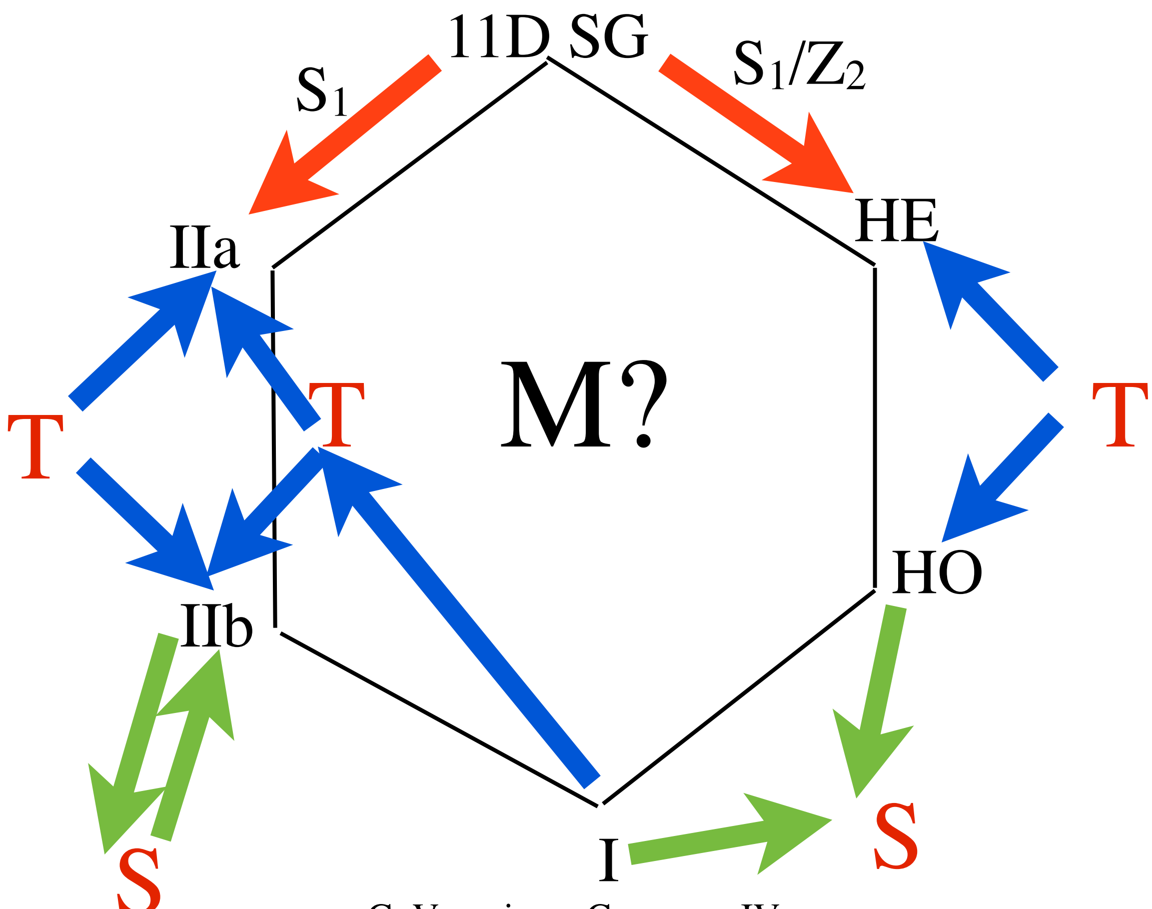
This was the first indication that D=11 supergravity may have something to do with D=10 superstrings!

The 5 string theories and D=11 SUGRA can be put at the corners of a hexagon. A "web of dualities" connects them as different limits of one and the same (yet largely unknown) theory, called M-theory.

For lack of time I only show the final picture, details to be found in last year's last lecture.

Six theories in search of a Mother

We still don't know which is the common Mother of all these theories. But it has a name: M-Theory!



(Tentative) plan of 2010-'11 course

Date	9h45-10h45	11h-12h
04/02	Summary of 2009-'10 course	Summary of 2009-'10 course
11/02	Summary of 2009-'10 course	Summary of 2009-'10 course
18/02	(Semi)classical black holes	Black holes & branes
25/02	Black holes & strings	TPE string collisions as a GE
04/03	TPE string collisions as a GE	TPE string collisions as a GE
11/03	HE string-brane collisions	HE string-brane collisions
18/03	String-inspired cosmologies	String-inspired cosmologies
25/03	String-inspired cosmologies	String-inspired cosmologies
01/04	Brane-cosmology	Brane-cosmology