Particules Élémentaires, Gravitation et Cosmologie

Année 2009-'10

Théorie des Cordes: une Introduction

Cours XII: 19 mars 2010

Symétries conventionnelles et non dans l'espace cible

- Emergence of field-theoretic symmetries
- The stringy version of KK
- Target-space duality for closed strings

Field-theoretic symmetries from QST

We have already remarked that the effective action satisfies some local spacetime symmetries. In particular it is invariant under GCT in spacetime (the principle underlying GR) and under gauge transformations of the B field.

What is the reason for these symmetries to emerge? No one had put them in: we only imposed Lorentz invariance in flat spacetime. However, a graviton (a photon for open strings) emerged from quantization and GCT-invariance looks necessary for describing its interactions.

Even more simply, the B-field gauge transformation changed the 2D-action by a total derivative.

Can we understand the emergence of these symmetries at a deeper level? A formal argument goes as follows.

Consider, for example, the string action in a $G_{\mu\nu}$ background:

$$S = -\frac{T}{2}\int d^2\xi\sqrt{-\gamma}\gamma^{\alpha\beta}(\xi)\partial_\alpha X^\mu(\xi)\partial_\beta X^\nu(\xi)G_{\mu\nu}(X(\xi))$$

It is invariant under a GCT of X^μ(X^μ--> $Y^\mu(X)$) accompanied by the corresponding GCT of $G_{\mu\nu}$, $G_{\mu\nu}$--> $G'_{\mu\nu}$. What matters in the quantum theory is the functional integral over X^μ at fixed $G_{\mu\nu}$, seen as a functional of the latter.

If the integration measure is itself invariant under X-->Y(X) then the functional integral is invariant under G-->G' since:

$$Z(G_{\mu\nu}) = \int [dX^\mu]\dots e^{-S(X^\mu,G_{\mu\nu})} = \int [dX^\mu]\dots e^{-S(Y^\mu,G'_{\mu\nu})}$$
$$= \int [dY^\mu]\dots e^{-S(Y^\mu,G'_{\mu\nu})} = Z(G'_{\mu\nu})$$

This is even easier to see using a Hamiltonian path integral where one integrates over X and P. A GCT is a particular canonical transformation. In infinitesimal form:

$$\delta X^\mu = \xi^\mu(X) \quad ; \quad \delta P_\mu = -\frac{\partial \xi^\nu}{\partial X^\mu} P_\nu$$

The gauge transformation of B (B--> B + dΛ) is also associated with a canonical transformation:

$$\delta X^\mu = 0 \quad ; \quad \delta P_\mu = (\Lambda_{\mu,\nu} - \Lambda_{\nu,\mu}) X'^\nu$$

It is easy to check that the constraints are invariant under these transformations provided G, B are also transformed.

If the Liouville measure dXdP is invariant (it is classically under ANY CT) then invariance of Z(G,B) again follows.

For open strings, symmetry of the effective action under spacetime gauge transformations (corresponding to the global symmetry introduced via Chan-Paton factors) formally follows from a similar canonical transformation.

However, there is no general rule that classical canonical transformations become quantum symmetries, meaning that there is a unitary transformation associated with them. More often than not the integration measure is not invariant (if defined in such a way as to give finite results) and one has "anomalies". The question of what is the full symmetry group underlying QST is still unanswered although the huge degeneracy of states in QST hints towards a huge symmetry.

Consider for instance the "stringy" GCT transformation:

$$X^\mu \to X^\mu + \xi^\mu_{\rho\sigma}(X)\gamma^{\alpha\beta}\partial_\alpha X^\rho \partial_\beta X^\sigma$$

Is this a symmetry of string theory? It would imply relations among scattering of particles of different mass and spin. It turns out to be difficult to check this claim...

So far, the symmetries we have discussed are common to QFT and QST, except that in QFT they are imposed from the start in order to describe massless spinning particles while in QST they “emerge” (since the massless spinning particles are also “emergent”).

There are, however, new local symmetries in string theory that have no field-theoretic equivalent. Many of them appear to be related to compactification of the extra dimensions.

We have already mentioned that gauge symmetries can emerge in QFT from the Kaluza-Klein (KK) mechanism when some of the spacetime dimensions are taken to be compact.

What happens to the KK mechanism in QST?

KK in QFT

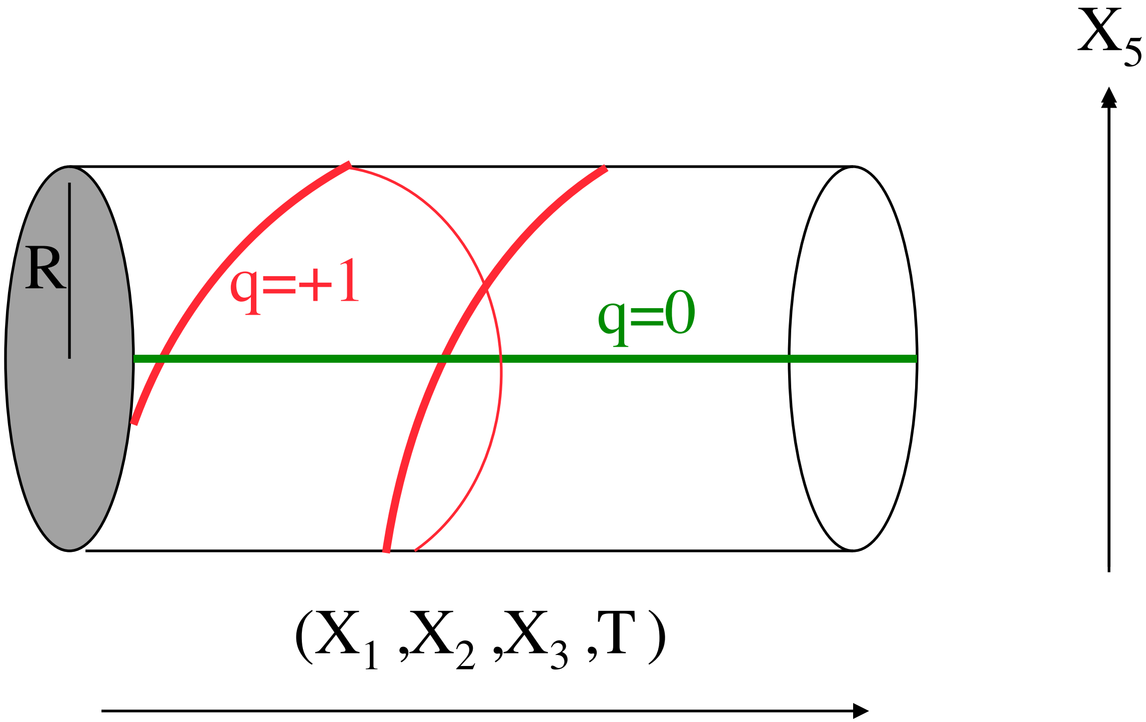
(one toroidal dimension)

In the original KK theory the extra dimension of space is a circle of radius R. The e.m. potential A_μ becomes, essentially, the $g_{\mu 5}$ component of the 5-dimensional metric, while g_{55} plays the role of a scalar field associated with the proper radius of the circle (the "radion"). More precisely, by writing the 5D metric as

$$ds_5^2 = g_{\mu\nu}dx^\mu dx^\nu + e^{2\sigma}(dx^5 + A_\mu dx^\mu)^2$$ we find

$$S = \frac{1}{2\kappa_5^2}\int d^5x\sqrt{-g_5}\; R_5 \Rightarrow \frac{\pi R}{\kappa_5^2}\int d^4x\sqrt{-g_4}\; e^\sigma \left[R_4 - \frac{e^{2\sigma}}{4}F_{\mu\nu}F^{\mu\nu}\right]$$

giving $$l_P^2 \equiv 16\pi G = \frac{l_5^3}{2\pi\rho}\; ; \;\alpha_4 = \frac{l_P^2}{\rho^2}\; ; \;\rho \equiv e^\sigma R$$



$$x_5 \equiv x_5 + 2\pi m R \; ; \; p_5 = n\frac{\hbar}{R} \; ; \; q = n\frac{l_P}{R} \; ;$$

$$m_4^2 = m_5^2 + p_5^2 = m_5^2 + n^2\left(\frac{\hbar}{R}\right)^2$$

NB. Typically, charged particles have masses O(1/R) but there can be "zero modes".

The stringy version of KK
(5 = compact dimension)

In string theory, for a generic value of R, the gauge symmetry is actually U(1)xU(1). The reason is that both $G_{\mu 5}$ and $B_{\mu 5}$ give rise to gauge bosons. General covariance and invariance under gauge transformations of B both become ordinary gauge transformations when the GCT (or B-gauge) parameter is chosen to be independent of the x_5 coordinate.

In a QFT context we can add by hand a B-field and get the extra U(1) gauge invariance. The problem is to find what the charge wrt this new U(1) means.

While for the U(1) coming from G we can identify the charge with the momentum in the 5th dimension, for the U(1) of B we cannot find a 5-dimensional meaning for its associated charge.

In string theory (where the B-field is inevitable) we will have instead a very nice interpretation. Let us see how.

Recall the boundary conditions for the closed bosonic string in flat space and in the ON gauge:

$$X'_\mu \delta X^\mu(\sigma = 0) = X'_\mu \delta X^\mu(\sigma = \pi) \quad ; \quad (\text{no sum over } \mu)$$

Strict periodicity of X_5 clearly satisfies the b.c. However, since now the points X_5 and $X_5 + 2\pi\, m\, R$ are identified, there is nothing wrong with imposing, instead,

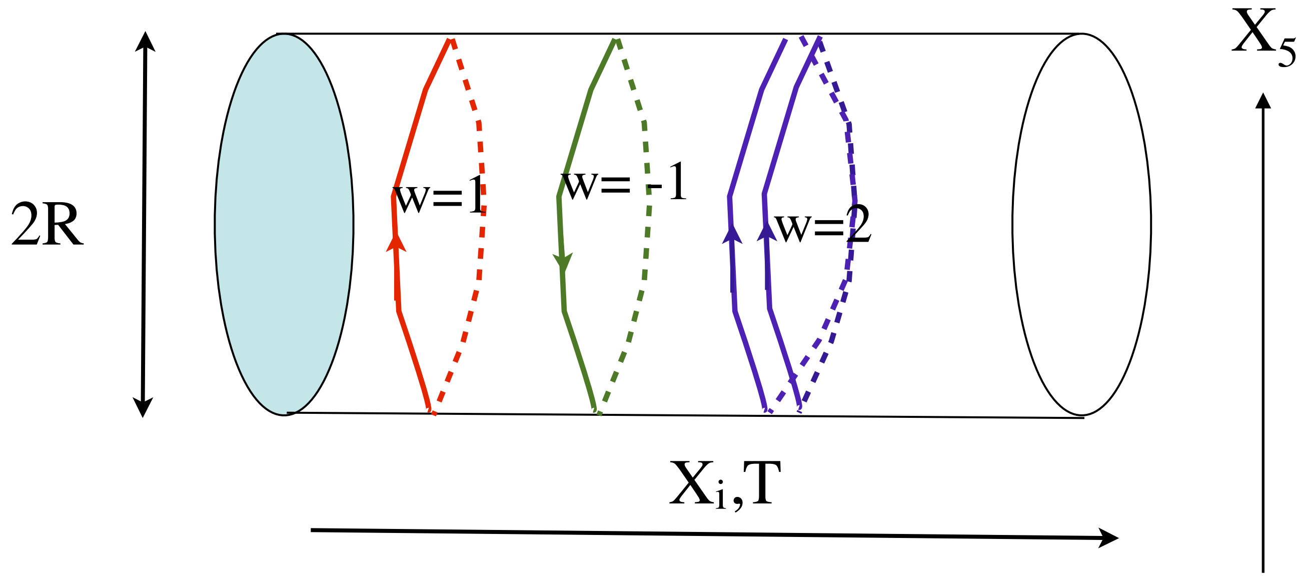
$X_5(\sigma = \pi) = X_5(\sigma = 0) + 2\pi\, w\, R$.

It simply means that the closed string winds around the compact direction w-times! Note that winding is a topological property of closed strings which has no analogue for points or open strings.

And, of course, winding costs energy because of the string tension.

The new boundary condition is easily implemented in the general solution by adding a "winding term".

NB: neither point particles, nor open strings can wind!



$$X_5(\sigma,\tau) = q_5 + 2n\alpha' \frac{\hbar}{R}\tau + 2wR\sigma$$
$$+ \frac{i}{2}\sqrt{2\alpha'}\sum_{n=1}^{\infty}\left[\frac{a_{n,5}}{\sqrt{n}}e^{-2in(\tau-\sigma)} - \frac{a^\dagger_{n,5}}{\sqrt{n}}e^{2in(\tau-\sigma)}\right]$$
$$+ \frac{i}{2}\sqrt{2\alpha'}\sum_{n=1}^{\infty}\left[\frac{\tilde{a}_{n,5}}{\sqrt{n}}e^{-2in(\tau+\sigma)} - \frac{\tilde{a}^\dagger_{n,5}}{\sqrt{n}}e^{2in(\tau+\sigma)}\right]$$

For the closed bosonic string the mass shell conditions are:

$$L_0 = 1 \quad \Rightarrow \quad M^2 = \left(\frac{\hbar n}{R} + \frac{wR}{\alpha'}\right)^2 + \frac{4}{\alpha'}(N-1)$$

$$\tilde{L}_0 = 1 \quad \Rightarrow \quad M^2 = \left(\frac{\hbar n}{R} - \frac{wR}{\alpha'}\right)^2 + \frac{4}{\alpha'}(\tilde{N}-1)$$

$$M^2 = \frac{n^2}{R^2} + \frac{w^2R^2}{\alpha'^2} + \frac{2}{\alpha'}(N + \tilde{N} - 2) \ ; \ N - \tilde{N} + nw = 0$$

Let us find which of the string states have zero mass. For a generic R it is quite clear that the only zero-mass states are given by:

$$N = \tilde{N} = 1 \ ; \ n = w = 0 \ ; \ \text{i.e.} \quad a^\dagger_{1\mu}\tilde{a}^\dagger_{1\nu}|0\rangle$$

In the absence of compactification these are just $(D-2)^2$ physical states: a graviton, a dilaton, an antisymmetric tensor.

$$N = \tilde{N} = 1 \; ; \; n = w = 0 \; ; \; \text{i.e.} \;\; a^{\dagger}_{1\mu}\tilde{a}^{\dagger}_{1\nu}|0\rangle$$

With compactification these $(D-2)^2$ massless states split into a graviton, a dilaton, an antisymmetric tensor in (D-1) dimensions (giving $(D-3)^2$ states), two (D-3)-component vectors, and 1 scalar, the "radion".This is just what the QFT analysis would suggest.

However, by looking at the vertex operator for the $B_{\mu 5}$ gauge boson, we discover that its "charge" is winding!

At zero momentum the vertex operator of this field is a total derivative that gives zero, unless there is winding:

$$\partial_+ X^\mu \partial_- X^5 - \partial_+ X^5 \partial_- X^\mu = \partial_-(X^5 \partial_+ X^\mu) - \partial_+(X^5 \partial_- X^\mu)$$

Something quite remarkable happens, however, if we choose a particular value for R:

$$R = R^* \equiv \sqrt{\hbar\alpha'} = \frac{l_s}{\sqrt{2}}$$

In this case there are ways of getting massless strings on top of those we had for generic R:

$$n = w = \pm 1 \ ; \ \ N = 0, \tilde{N} = 1 \quad \text{or}$$

$$n = -w = \pm 1 \ ; \ \ N = 1, \tilde{N} = 0$$

These are 4 massless vectors, two left and two right-moving. Together with the 2 previous ones they are the 6 gauge bosons of an SU(2)xSU(2) gauge group w/ the two factors corresponding to left and right-moving states. Note that the 4 new gauge bosons carry momentum and winding and are therefore themselves charged, a characteristic of non-abelian gauge theories.

The above solutions also provide 4 massless scalars (when we take the oscillator in the 5th direction).

Actually there are 4 more massless scalars corresponding to taking the oscillator vacuum and ($n = \pm 2$, $w=0$) or ($n=0$, $w = \pm 2$).

The total number of massless scalars is thus 10. Leaving a singlet dilaton aside, they form a (3,3) representation of

SU(2)xSU(2). The radion corresponds to a particular direction in both SU(2)'s and plays the role of a Higgs field that breaks spontaneously SU(2)xSU(2) down to U(1)xU(1) away from the special point $R = R^*$.

The mass of the gauge bosons corresponding to the broken generators is linear in ($R-R^*$).

But the surprises are not over...

T-duality (for closed strings)

Winding and momentum appear on a similar footing in the expression for X_5 and for M^2. However, while even in the classical theory winding number is an integer, classically, momentum in the compact direction is not quantized. At the classical level there is no possible symmetry between winding and momentum.

At the quantum level single-valuedness of the wave function forces momentum to be quantized in units of 1/R. Then, all of a sudden, a symmetry appears between winding and momentum if we exchange n and w and, at the same time, we change R into $l_s^2/2R$. Note that the point of enhanced gauge symmetry is precisely the fixed point of this T-duality transformation!

T-duality is thus based on QM. It could not be otherwise since R-->1/R needs a length scale and there is no length scale in CST! It's yet another miracle of QST!

We conclude that the inequivalent compactifications are labelled by the range $R > R^*$ so that, effectively, there is a minimal compactification radius $R=R^*$. For $R < R^*$ momentum modes decouple and it is the winding that provides the light spectrum with which we probe space.

Furthermore, precisely at $R=R^*$, a non-abelian symmetry emerges.

There are reasons to believe that, dynamically, string theory has a preference for such a value of R. Indeed, if some non-perturbative dynamics generates a potential for the radion field and T-duality is respected, the potential will have an extremum (minimum?) at the self-dual value of R.

Possibly there will be two preferred values of R, R = infinity and $R = R^*$. They could correspond to our 3 dimensions of space and to the compactified ones...

T-duality as a canonical transformation

Can we find a canonical transformation that gives T-duality? The answer is yes, but illustrates the danger of trusting blindly canonical transformations at the quantum level.

Consider, in flat spacetime, the canonical transformation (NB: no minus sign needed with X' and P):

$$X'_5 \to \alpha P_5/T \; ; \; P_5 \to \alpha^{-1} T X'_5$$

and the constraints: $P \cdot X' = 0 \; ; \; P^2 + T^2 X'^2 = 0$

The first is left invariant by the canonical transformation for any value of α; the second is not unless α=1. Since:

$$\frac{P^2}{T^2} = \dot{X}^2 \sim 4n^2\hbar^2\alpha'^2 R^{-2} \quad ; \quad X'^2 \sim 4w^2R^2$$

interchanging the two is equivalent to interchanging n and w and to replacing R by $l_s^2/2R$. Only one classical canonical transformation leads to a quantum unitary transformation!