Particules Élémentaires, Gravitation et Cosmologie
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Théorie des cordes: quelques applications

Cours XV: 1 avril 2011

Can string theory be tested?

EM perturbations in string cosmology

Let us consider the gauge part of the string effective action

$$\Gamma^{EM} = -\frac{1}{4}\int d^4x\sqrt{-g}e^{-\phi_4}F_{\mu\nu}F_{\rho\sigma}g^{\mu\rho}g^{\nu\sigma} \to -\frac{1}{4}\int d^3x d\eta e^{-\phi_4}F_{\mu\nu}F_{\rho\sigma}\eta^{\mu\rho}\eta^{\nu\sigma}$$

where $\exp(\phi_4)$ (containing also a contribution from V_6) is the effective fine structure constant α.

In order to amplify EM perturbation we need a dynamical ϕ_4! Such a feature is absent in conventional models.

The canonical EM potential is $\exp(-\phi_4/2)$ *A* and satisfies:

$$\hat{A}_k'' + \left[k^2 - e^{\phi_4/2}(e^{-\phi_4/2})''\right]\hat{A}_k = 0 \ ; \ \hat{A} = e^{-\phi_4/2}A_k$$

$$\hat{A}_k'' + \left[k^2 - e^{\phi_4/2}(e^{-\phi_4/2})''\right]\hat{A}_k = 0 \; ; \; \hat{A} = e^{-\phi_4/2}A_k$$

The vacuum fluctuations of the EM field (which do exist, see Casimir effect) are amplified by a (k-dependent) factor

$$\frac{\hat{A}_k|_f}{\hat{A}_k|_i} = \sqrt{\frac{e^{\phi_{re}}}{e^{\phi_{ex}}}} = \sqrt{\frac{\alpha_{re}}{\alpha_{ex}}}$$

It is not unconceivable that these amplified vacuum fluctuations may act as seeds for the cosmic magnetic fields that are known to exist at the μ-Gauss level on galactic and intergalactic scales.

A very large increase in α is needed between exit and reentry of the relevant scales in order to have large enough seeds for a "dynamo" mechanism to work.

Putting numbers one finds that a factor of at least 10^{66} is needed between $\alpha_{(now)}$ and $\alpha_{(exit\ of\ g.s.)}$. Sounds huge but is of the same order as the increase in the scale factor one needs in order to solve the usual cosmological puzzles.

In PBB cosmology the growth of the scale factor is related to the growth of ϕ and therefore it is natural to expect the same order of magnitude for both. Comments:

a) the actual spectrum of EM perturbations is always blue-tilted and depends on the behaviour of V_6 during the pre-bang phase: a window on extra dimensions.

b) unfortunately a reliable computation of present magnetic fields from a spectrum of initial seeds is not yet available.

c) One of the distinctive predictions of PBB cosmology!

Axion perturbations

So far we have found only blue spectra. However:

In all known string theories there is a pseudoscalar partner to the dilaton:the universal NS-NS axion σ.

- The pump field for a massless axion is:

$$P_\sigma = \frac{a^2 e^{\phi}}{V_6} = a^2 e^{\phi_4} \quad ; \quad P_\phi = a^2 e^{-\phi_4}$$

- P_σ can be of the inflationary type ($P_\sigma = 1/\eta^2$). One finds:

$$|\delta\sigma_k|^2 = \left(\frac{H^*}{M_P}\right)^2 \left(\frac{\omega}{\omega^*}\right)^{n-1}, 4 - 2\sqrt{3} \sim 0.53 < n < 2$$

($H^* \sim M_s$, $\omega^* = H^* a^*/a_0 \sim 10^{11}$ Hz, σM_P = can.[al] axion field)

The Kasner sphere: $3\alpha^2 + \Sigma \beta_i^{\,2} = 1 = \sin^2\theta + \cos^2\theta$

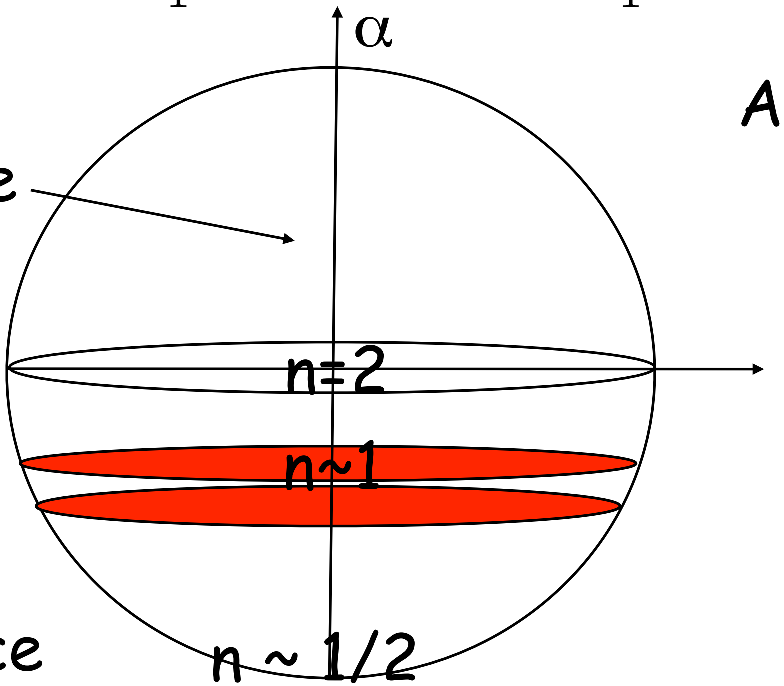
$$ds^2 = -dt^2 + \sum_1^3 (-t)^{2\alpha} dx_i^2 + \sum_1^6 (-t)^{2\beta_i} dy_i^2$$

Contracting 3-d space

Static 3-d space

9-d isotropy, mod. T

Static internal space



Axion-spectrum can be blue, red or flat

Example:

$$n = \frac{4 + 6r^2 - 2\sqrt{3 + 6r^2}}{1 + 3r^2} \; ; \; r \equiv \frac{1}{2}\frac{\dot{V}_6 \; V_3}{V_6 \; \dot{V}_3}$$

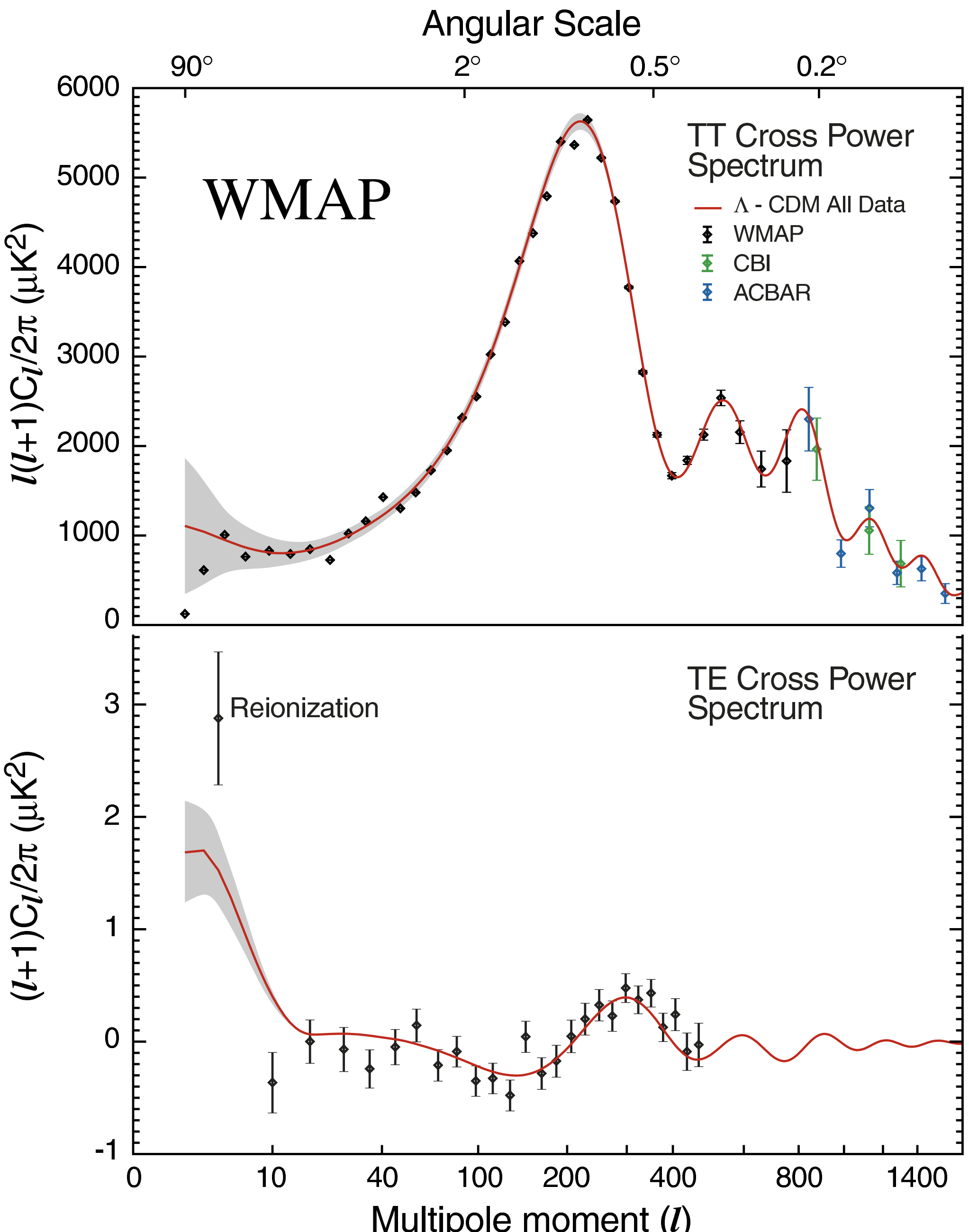
Unfortunately..

- Axion gives isocurvature (entropy) perturbations since its fluctuations do not mix, to first order, with metric pert.s (unlike for dilaton).
- Isocurvature perturbations feed back on curvature to 2nd order but give “wrong” structure of acoustic peaks.

The situation looked quite hopeless for a while, but then a new idea, independent of string cosmology, came out…

THE CURVATON

The axion can do the job by playing the curvaton’s role!


Angular Scale
90°
2°
0.5°
0.2°
6000
5000
4000
3000
2000
1000
0
WMAP
TT Cross Power Spectrum
Λ - CDM All Data
WMAP
CBI
ACBAR
l(l+1)Cl/2π (μK2)
3
2
1
0
-1
Reionization
TE Cross Power Spectrum
(l+1)Cl/2π (μK2)
0
10
40
100
200
400
800
1400
Multipole moment (l)

- If a $V(\sigma)$ is generated when the Universe cools down, and if $\langle\sigma\rangle = \sigma_i$ is not initially at its minimum, axion pert.s induce calculable curvature pert.s. This "curvaton" mechanism needs:
  - ❶ a phase of axion relevance, dominance.
  - ❷ the axion to decay before NS (m_σ > 10 TeV?)
- Conversion efficiency can be computed. Bardeen potential Φ_k (related to curvature pert.) after axion decay:

$$|\Phi_k|^2 = f^2(\sigma_i)\Omega_d^2|\delta\sigma_k|^2 = f^2(\sigma_i)\Omega_d^2\left(\frac{H^*}{M_P}\right)^2\left(\frac{\omega}{\omega^*}\right)^{n-1}$$

where $f(\sigma_i) \sim (4\sigma_i)^{-1}$ (σ_i < 1),

Ω_d is the fraction of critical energy in the axion at decay.

- One then computes the Sachs-Wolfe contribution to the C_l's

$$C_l^{(SW)} = \frac{1}{9\pi} f^2(\sigma_i)\Omega_d^2 \left(\frac{H^*}{M_P}\right)^2 \left(\frac{\omega_0}{\omega^*}\right)^{n-1} \times \frac{\Gamma[l+(n-1)/2]}{\Gamma[l+2-(n-1)/2]}$$

($H^* \sim M_s$, $\omega^* \sim 10^{11}$ Hz $\sim 10^{30}\,\omega_0$, $f(\sigma_i) \sim (4\sigma_i)^{-1}$)

$$\frac{\Gamma[l+\ldots]}{\Gamma[l+\ldots]} \sim \frac{l^{n-1}}{l(l+1)} \Rightarrow l(l+1)C_l \sim l^{n-1}$$

- COBE normalization: $C_2 = (1.09 \pm 0.23)\times 10^{-10}$ gives:

$$(1.09 \pm 0.23)10^{-10} = \frac{1}{54\pi} f^2(\sigma_i)\Omega_d^2 \left(\frac{H^*}{M_P}\right)^2 \left(\frac{\omega_0}{\omega^*}\right)^{n-1}$$

=> acoustic-peaks come out fine provided primordial axion spectrum is nearly flat (n~1).

⇒ Slightly blue spectra ($n > 1$) and/or low (H^*/M_P) preferred

Q: Can we play with $\Omega_d \sim \varepsilon^2$ to allow a higher H^*/M_P ?

It turns out that one gains a factor ε^{-1} at the price of generating a $f_{NL} \sim \Omega_d^{-1} \sim \varepsilon^{-2}$

$$\frac{\Delta T}{T} = \left(\frac{\Delta T}{T}\right)_L + f_{NL} \left(\frac{\Delta T}{T}\right)_L^2$$

=> Given bounds on f_{NL} ($O(10^2)$) we cannot gain much on normalization...

On the contrary, some non Gaussianity is all but unexpected.

NB: (Non)Gaussianity is one of the objectives of PLANCK.

NB': Tensor contribution to CMB (B-polarization) is still completely negligible.

Can String Theory be tested ?

- There has been much discussion, even in the popular literature, on whether string theory can be tested/ falsified, even in principle. History can help.
- The old string theory of the sixties was tested, falsified, abandoned in a matter of a few years.
- With the rescaling of M_s by ~20 orders of magnitude one may have real doubts about whether the new string theory gives testable predictions.
- The old string theory was partly abandoned because of "high-energy" (> few GeV) experiments but also because of its long distance predictions (m=0 particles, long range forces that are absent in hadronic interactions).

Case I: $M_s \ll M_P$

- If the string/quantum gravity scale can be lowered to that of present accelerators the answer to the question is easy.
- Same if there are large extra dimensions through light KK modes etc. (see C. Deffayet's seminars).
- It is fair to say that there is no theoretical motivation for either thing to happen (T-duality argument for $R_c \sim l_s$, GUT scale at ~ 10^{16} GeV...).
- Let us then take the conventional point of view that the string scale is at most a couple of orders of magnitude below the Planck scale.

Case II: $M_s \sim M_{KK} \sim (10^{-2}\text{--}10^{-1})M_P$

Still (at least) 3 ways of testing string theory

1. COSMOLOGY

The Universe is the biggest accelerator of all.

The way string theory modifies GR at short distances should have left marks in the physics of the early Universe.

Its expansion thereafter kindly brings these features to macroscopic (even astrophysical) scales.

Progress in this direction is hampered by the difficulties in solving string theory in the high-curvature/large-coupling regimes.

2. MODEL BUILDING

At low energy, string theory should give an effective unified theory of non-gravitational interactions but with its own strict rules.

Although, through compactification, many 4-dimensional theories are possible, getting just the minimal SM (actually a SUSY extension of it) is highly non trivial.

A string-theory-based SM of elementary particles will probably contain much more structure than the usual SM or MSSM and hence should come with definite predictions.

Technically hard (e.g. implementing SUSY breaking) but progress is being made (see String Phen. conferences).

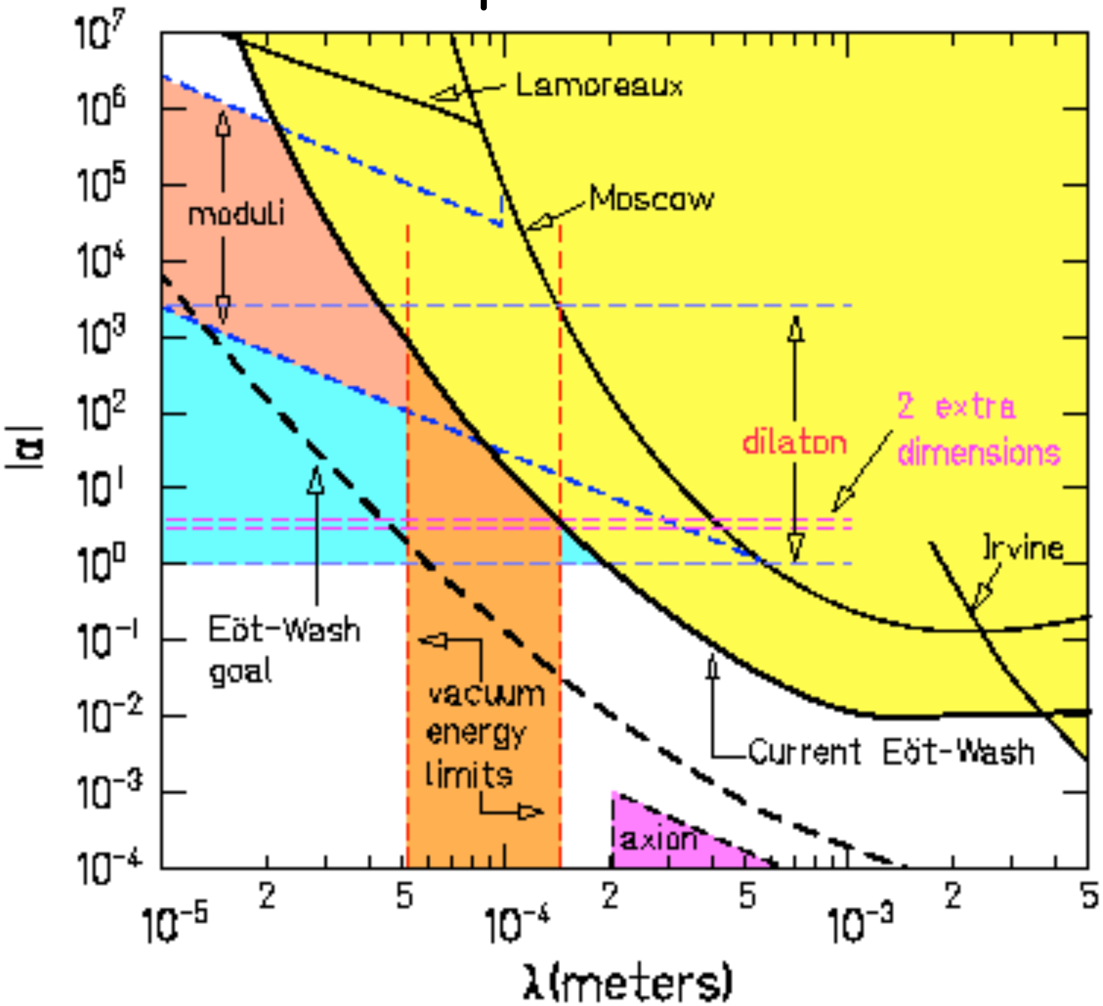
3. NEW LARGE-DISTANCE PHENOMENA

There are no free parameters in string theory: these are replaced by scalar fields whose values provide the «Constants of Nature». The fine-structure constant α and $G_N T$ are fixed by the dilaton and V_6. Other scalars are associated with shape & topology of the 6 extra dimensions whose existence is a robust prediction.

All these fields are massless in perturbation theory (PT) (because of supersymmetry). If they remain massless beyond PT string theory is doomed. If they acquire a small enough mass (after SUSY breaking), they will induce «short-distance» modifications of gravity, threaten the equivalence principle and universality of free-fall, induce space-time variations of the above «constants», etc.

Finding such new phenomena would be a smoking gun for QST. Instead, if one could prove non-perturbatively that some of these massless scalars are there to stay, QST could be falsified very much like its hadronic predecessor.

A very active field of experimental & theoretical research.



Adelberger et al.

One test we already have...

(Don Zagier, private comm.)

- For the first time in history it looks as if physics is asking for entirely new mathematical tools.
- String-theory based intuition, together with non-rigorous reasoning, has led physicists (e.g. E. Witten) to conjecture mathematical results that have been proven later by mathematicians!
- Let's hope that, in turn, new mathematical tools will allow string theory to make progress in those directions that are crucial for its experimental verification.

Some bibliography

On string theory in general

B. Greene: L'Univers élégant (Robert Laffont)

On string cosmology

- G. Veneziano: L'Univers avant le Big Bang (Pour la Science, juin 2004).
- M. Gasperini and G. Veneziano: The pre-big bang scenario in string cosmology, Phys. Rep. 373 (2003).
- M. Gasperini: The Universe before the big bang: cosmology and string theory (Springer).