Particules Élémentaires, Gravitation et Cosmologie
Année 2010-'11

Théorie des cordes: quelques applications

Cours X: 11 mars 2011

Transplanckian scattering in QST:
IV. The strong-gravity regime

Outline

- Identification of the relevant diagrams
- Resumming classical corrections via an effective action
- Two-body scattering at $b \neq 0$: a numerical solution
- The axisymmetric case: comparison with CTS criteria
- Graviton spectra below and near criticality
- What happens above criticality?

D=4 hereafter

IV: Small-angle graviton (GW) emission

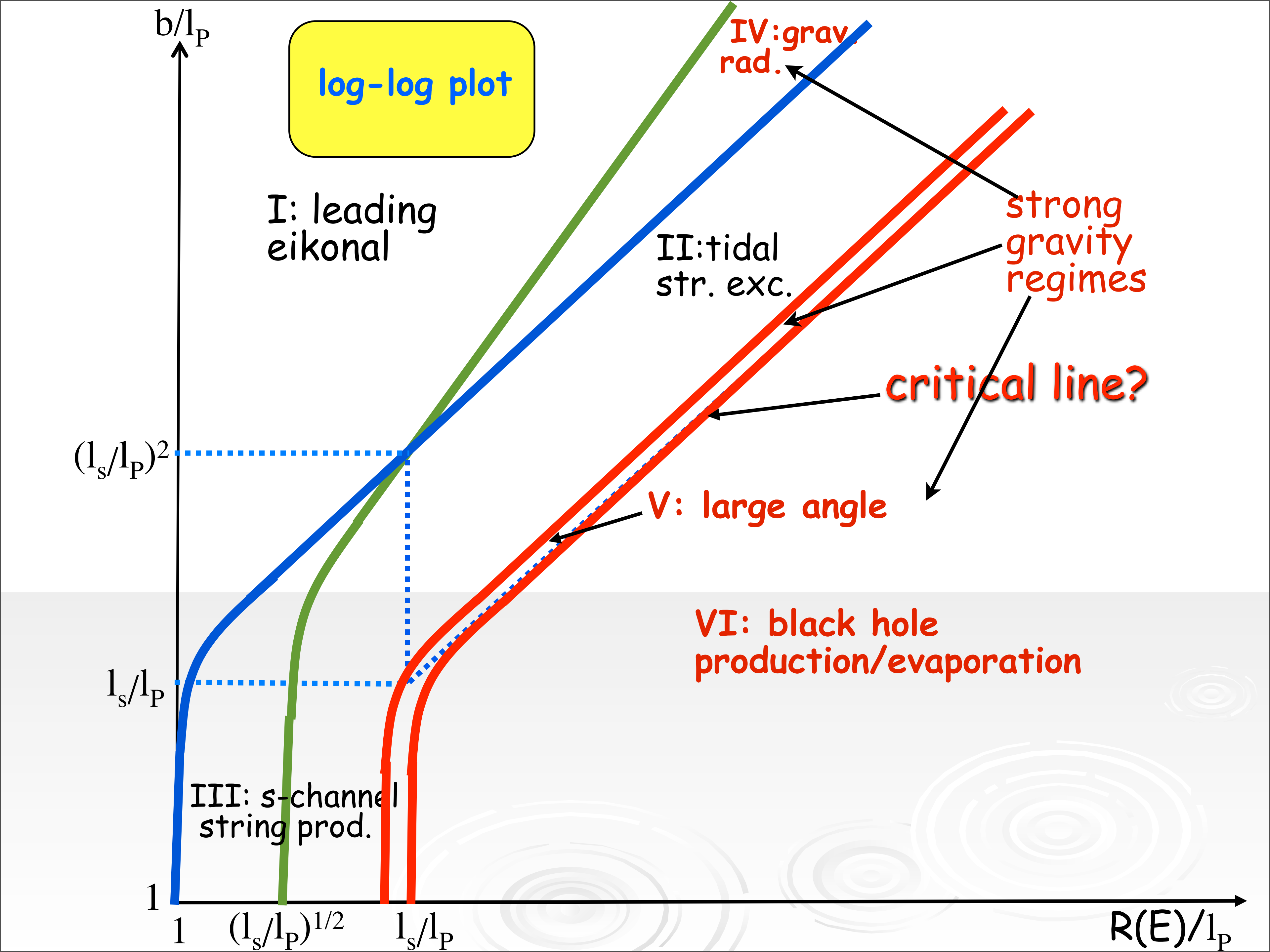
=> Classical corrections to leading eikonal

$$S(E,b) \sim exp\left(i\frac{A_{cl}}{\hbar}\right) \sim exp\left(-i\frac{Gs}{\hbar}(logb^2 + O(R^2/b^2) + \cancel{O(l_s^2/b^2)} + \cancel{O(l_P^2/b^2)} + \ldots)\right)$$

V: Large-angle inelastic scattering

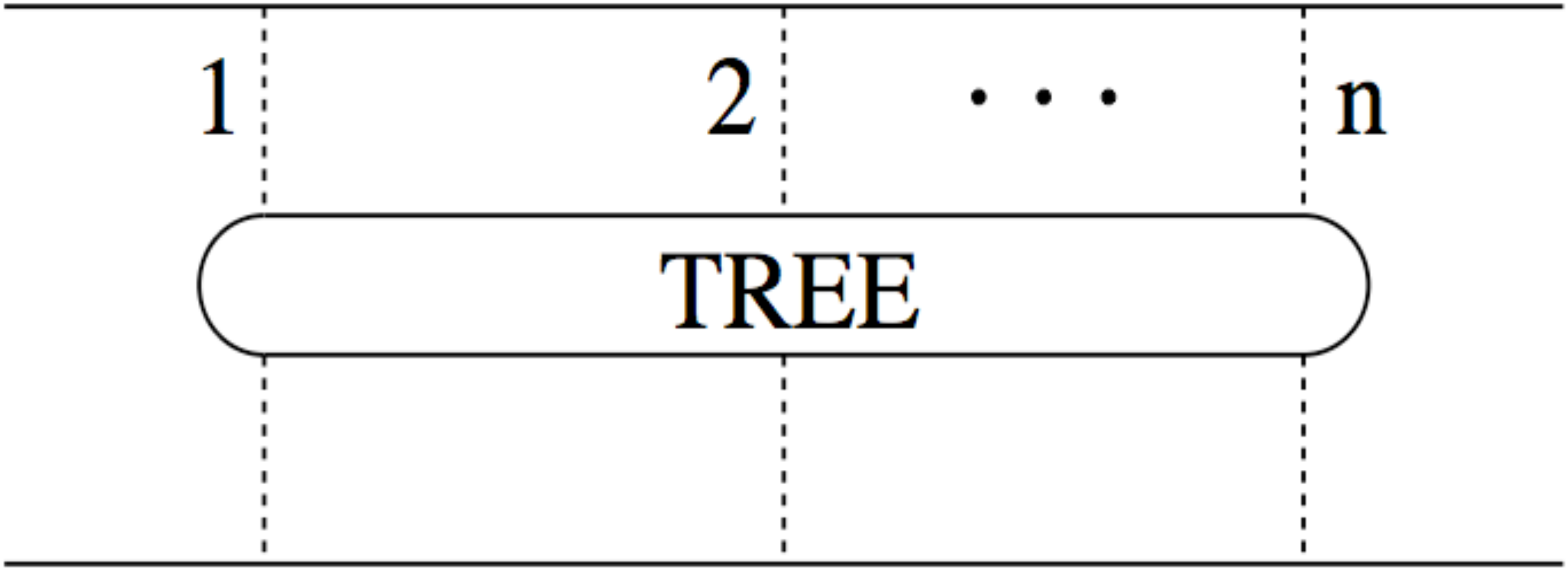
VI: Collapse?

=> Resumming classical corrections


b/l_P
log-log plot
IV:grav. rad.
I: leading eikonal
II:tidal str. exc.
strong gravity regimes
critical line?
(l_s/l_P)^2
V: large angle
VI: black hole production/evaporation
l_s/l_P
III: s-channel string prod.
1
1
(l_s/l_P)^1/2
l_s/l_P
R(E)/l_P

Classical corrections characterized by absence of h.

Not surprisingly, they are related to tree diagrams once the coupling to the external energetic particles is replaced by a classical source.



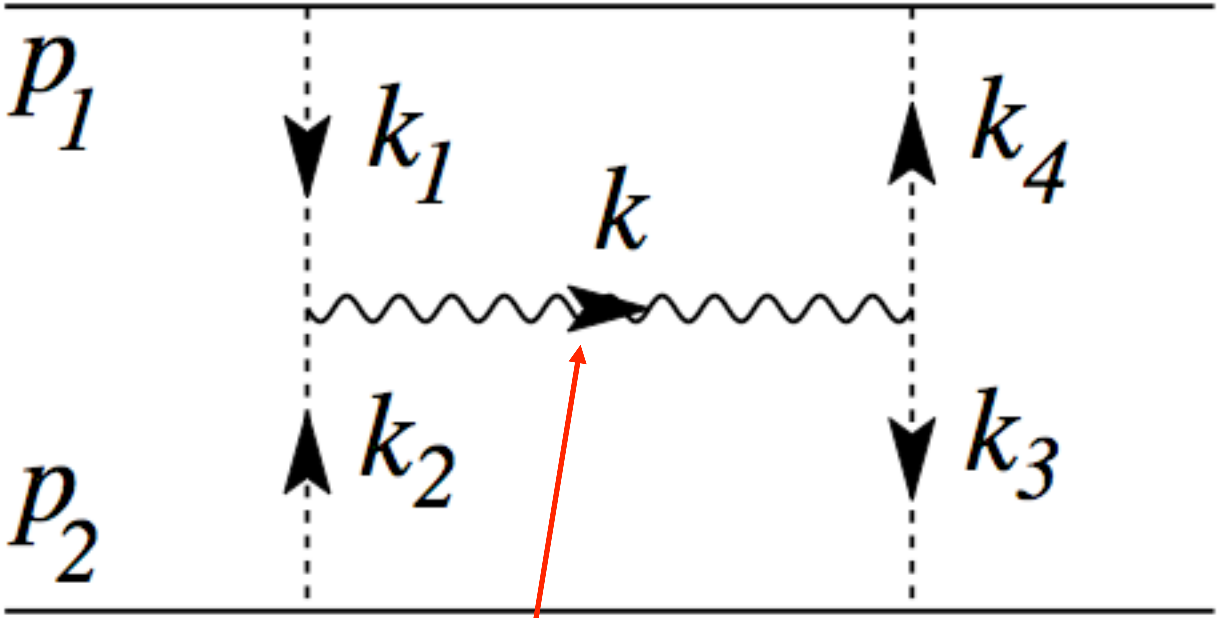
The exponent (the "phase") is given by connected trees

Power counting for connected trees:

$$\delta(E,b) \sim G^{2n-1} s^n \sim Gs\ R^{2(n-1)} \rightarrow Gs\ (R/b)^{2(n-1)}$$

If gravitons do not interact we get the leading eikonal.

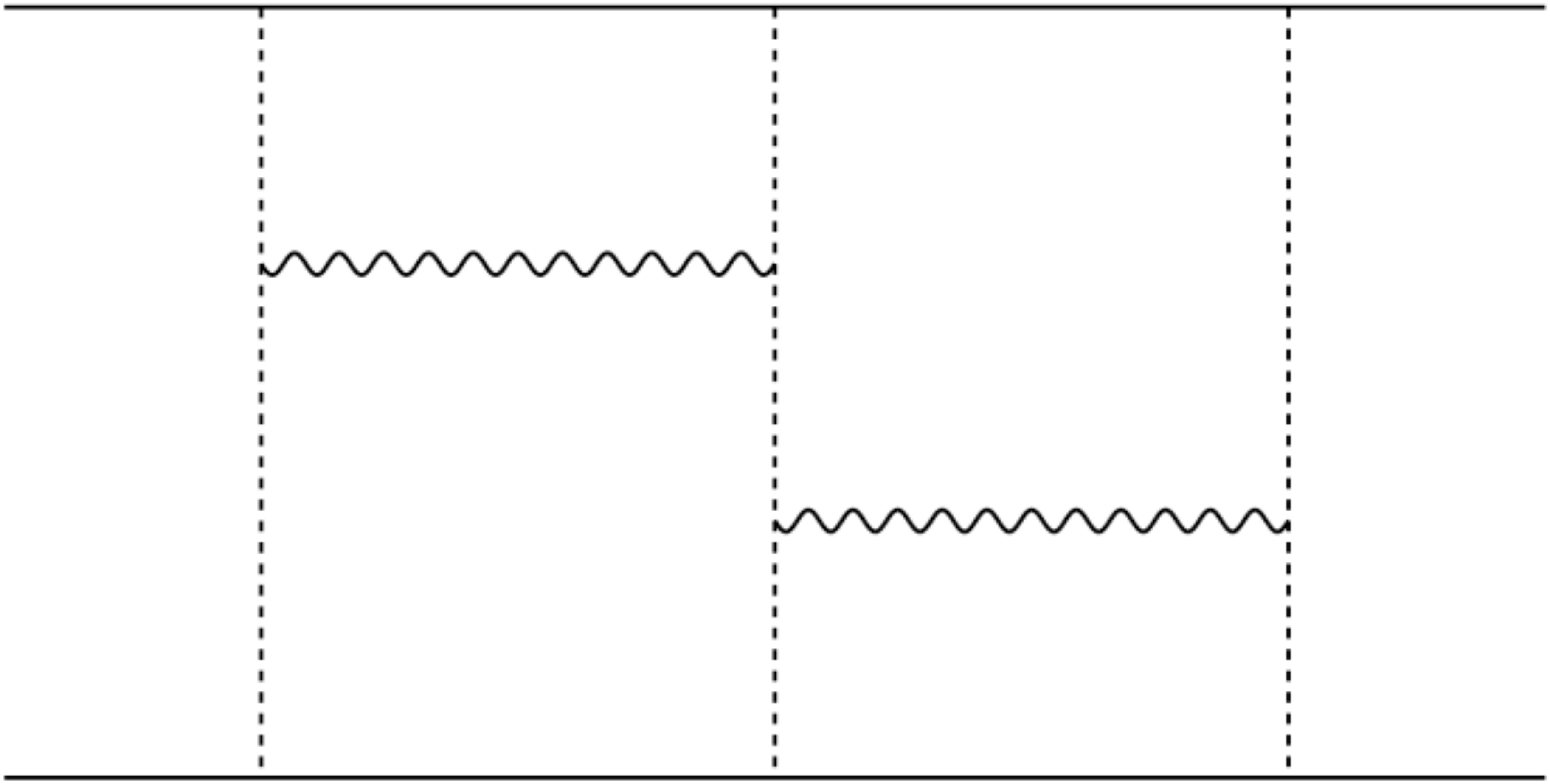
Next to leading order: the H diagram



$$\sim G^3 s^2 = Gs\ G^2 s = GsR^2 \to Gs\ (R/b)^2$$

One of the produced graviton's polarizations ("TT") is IR-safe the other ("LT") is not.

NNL-order



$$\sim G^5 s^3 = Gs\ G^4 s^2 = GsR^4 \to Gs\ (R/b)^4$$

Reduced effective action & field equations

There is a D=4 effective action generating the leading diagrams (Lipatov, ACV '93) but the resulting equations are too complicated. Drastic simplification: "freeze" the longitudinal dynamics => a D=2 effective action.

Neglecting the IR-unsafe (LT) polarization, it contains:
a and $\bar{a}$, representing the longitudinal (++ and --) components of the gravitational field, coupled to the corresponding components of the E-M tensor;
ϕ, representing the TT graviton-emission field.
Besides source and kinetic terms there is a derivative coupling of a, $\bar{a}$ and ϕ.

The action (generalized to extended null sources):

$$\frac{\mathcal{A}}{2\pi G s} = \int d^2x \left[a(x)\bar{s}(x) + \bar{a}(x)s(x) - \frac{1}{2}\nabla_i\bar{a}\nabla_i a\right]$$
$$+ \frac{(\pi R)^2}{2}\int d^2x \left(-(\nabla^2\phi)^2 + 2\phi\nabla^2\mathcal{H}\right) ,$$

$-\nabla^2\mathcal{H} \; \equiv \; \nabla^2 a \; \nabla^2\bar{a} - \nabla_i\nabla_j a \; \nabla_i\nabla_j\bar{a}$, & corresponding eom

$$\nabla^2 a + 2s(x) \; = \; 2(\pi R)^2(\nabla^2 a \; \nabla^2\phi - \nabla_i\nabla_j a \; \nabla_i\nabla_j\phi), \quad \bar{a}(x) = a(b-x)$$

$$\nabla^4\phi = -(\nabla^2 a \; \nabla^2\bar{a} - \nabla_i\nabla_j a \; \nabla_i\nabla_j\bar{a})$$

Semiclassical approximation: solve eom and compute the classical action on the solution. At leading order in R/b and for s(x) = δ(x), $\bar{s}$(x) = δ(x-b), we recover the leading eikonal. Iterative procedure possible but not so useful for analytic study!

Numerical solutions

(G. Marchesini & E. Onofri, 0803.0250)

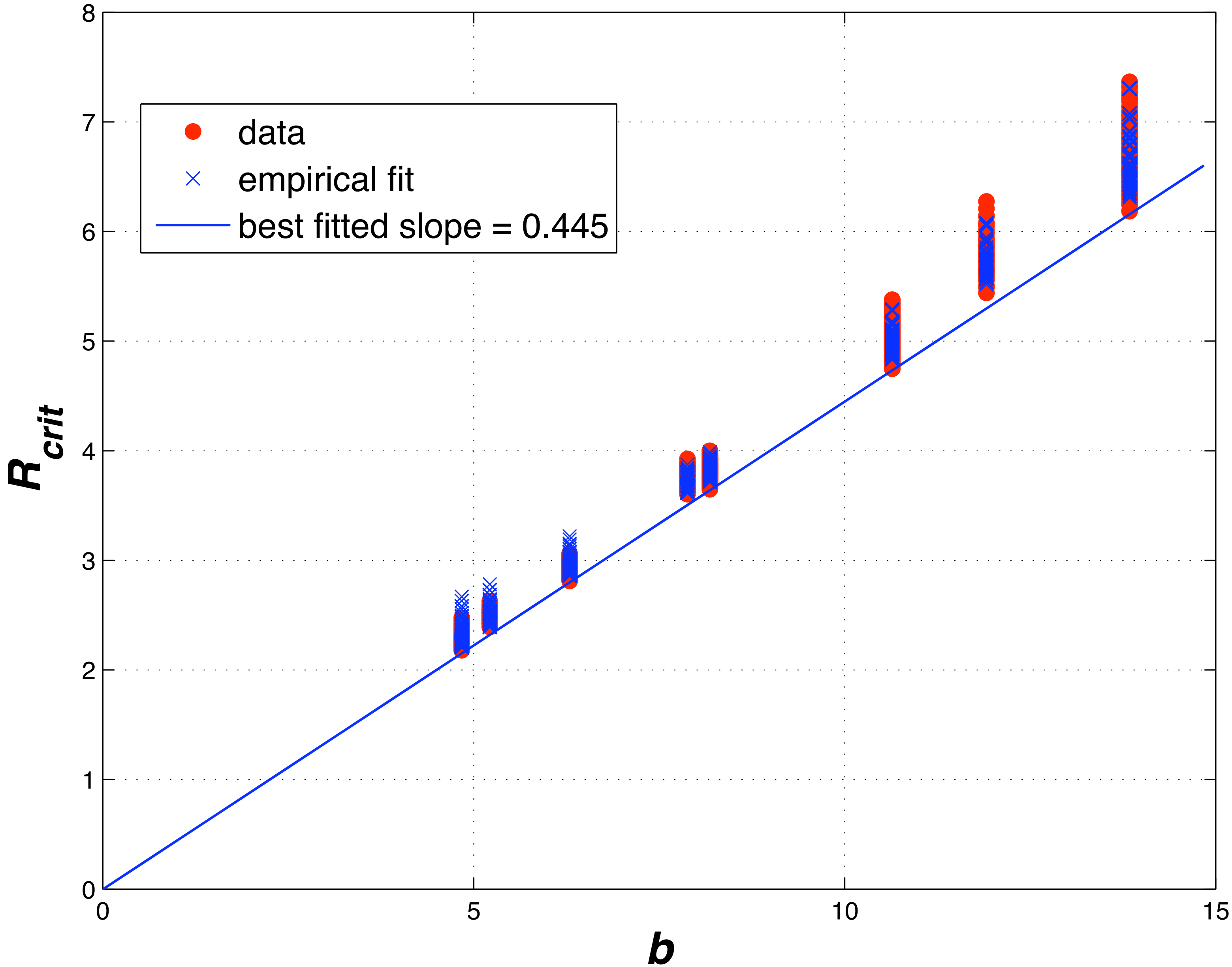
Solve directly PDEs by Fast FT methods in Matlab

Result: real solutions only exist for:

$$b > b_c \sim 2.28R$$

Compare with Eardely-Giddings's CTS lower bound:

$$b_c > 0.80R$$



data
empirical fit
best fitted slope = 0.445
R_{crit}
b
0
1
2
3
4
5
6
7
8
0
5
10
15

For an analytic approach we turn to

Axisymmetric Solutions

(J. Wosiek & G.V. '08)

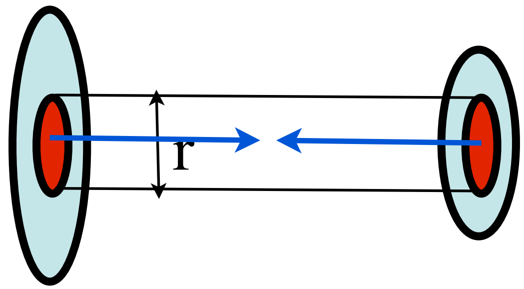
Central beam-beam collisions

A very rich problem in spite of its restrictive symmetry:

1. The two beams contain several parameters: total energy, shapes (same or different) & we can look for critical surfaces in their multi-dim.^{al} space.
2. The CTS (KV) criterion is simple (see below).
3. Numerical results are coming (see below).
4. Two major simplifications occur in ACV eqns:

- PDEs become ODEs.
- The IR-singular polarization is just not produced.

The relevant parameters



$R_1(r)=4GE_1(r)$ $\quad$ $R_2\,(r)=4GE_2(r)$

Axisymmetric action, eqns ($t=r^2$)

$$\frac{\mathcal{A}}{2\pi^2 G s} = \int dt\,[a(t)\bar{s}(t) + \bar{a}(t)s(t) - 2\rho\dot{\bar{a}}\dot{a}]$$
$$- \frac{2}{(2\pi R)^2}\int dt(1-\dot{\rho})^2$$

$$\rho = t\left(1 - (2\pi R)^2\dot{\phi}\right) \qquad \pi\int^t dt' s_i(t') \;=\; R_i(t)/R$$

$$\dot{a}_i = -\frac{1}{2\pi\rho}\frac{R_i(r)}{R}$$

$$\ddot{\rho} = \frac{1}{2}(2\pi R)^2\dot{a}_1\dot{a}_2 = \frac{1}{2}\frac{R_1(r)R_2(r)}{\rho^2} \qquad \rho(0)=0 \;\;;\;\; \dot{\rho}(\infty)=1$$

2nd order ODE w/ Sturm-Liouville-like b. conditions.

CTS criterion
(KV gr-qc/0203093)

If there exists an r_c such that

$$R_1(r_c)R_2(r_c) = r_c^2$$

we can construct a CTS and therefore expect a BH to form.

Theorem (VW08): whenever the KV criterion holds the ACV field equations do not admit regular (at r=0) real solutions. Thus:

KV criterion ==> ACV criterion

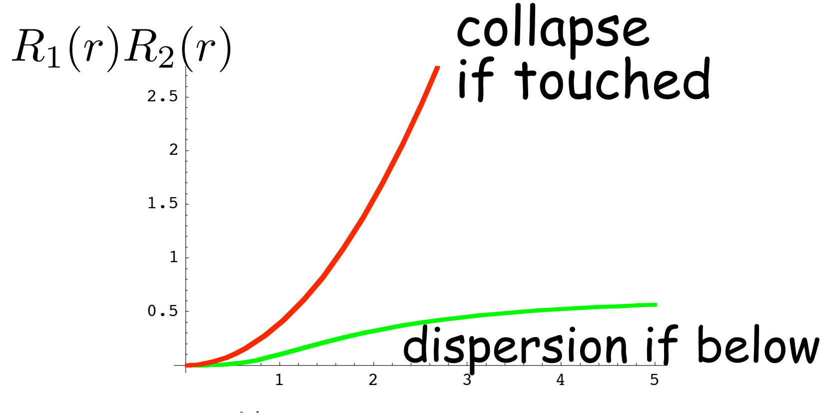
but of course not the other way around!

A sufficient criterion for dispersion
(P.-L. Lions, private comm.)

If $$R_1(r)R_2(r) \leq \frac{8}{27}\frac{r^4}{(1+r^2)^2}\left[1+\frac{1}{2}\left(1-\frac{log(1+r^2)}{r^2}\right)\right]^2$$

the ACV eqns do admit regular, real solutions.

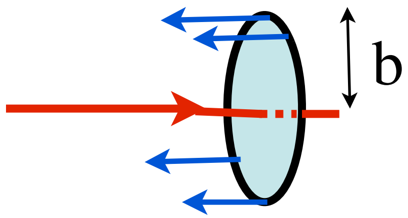
To summarize



clearly, there is room for improvement...

Examples

Example 1: particle-scattering off a ring



Can be dealt with analytically:

$$\ddot{\rho} = \frac{R^2}{2\rho^2}\Theta(r^2 - b^2)$$

$$\begin{aligned} \rho &= \rho(0) + r^2\dot{\rho}(0) \quad , \quad (r < b) \\ \dot{\rho} &= \sqrt{1 - R^2/\rho} \quad , \quad (r > b) \end{aligned}$$

Since ρ(0) =0:

$$\rho(b^2) = b^2\dot{\rho}(b^2) = b^2\sqrt{1 - R^2/\rho(b^2)}$$

This (cubic) equation has positive real solutions iff

$$b^2 > \frac{3\sqrt{3}}{2}R^2 \equiv b_c^2$$

$(b/R)_c \sim 1.61$

CTS: $(b/R)_c > 1$

Example 2: Two hom. beams of radius L.

The equation for ρ becomes

$$\ddot{\rho}(r^2) = \frac{R^2}{2\rho^2}\Theta(r-L) + \frac{R^2 r^4}{2L^4\rho^2}\Theta(L-r)$$

We can compute the critical value numerically:

$$\left(\frac{R}{L}\right)_{cr} \sim 0.47$$

It is compatible with (and close to) the CTS upper bound of KV:

$$\left(\frac{R}{L}\right)_{cr} < 1.0$$

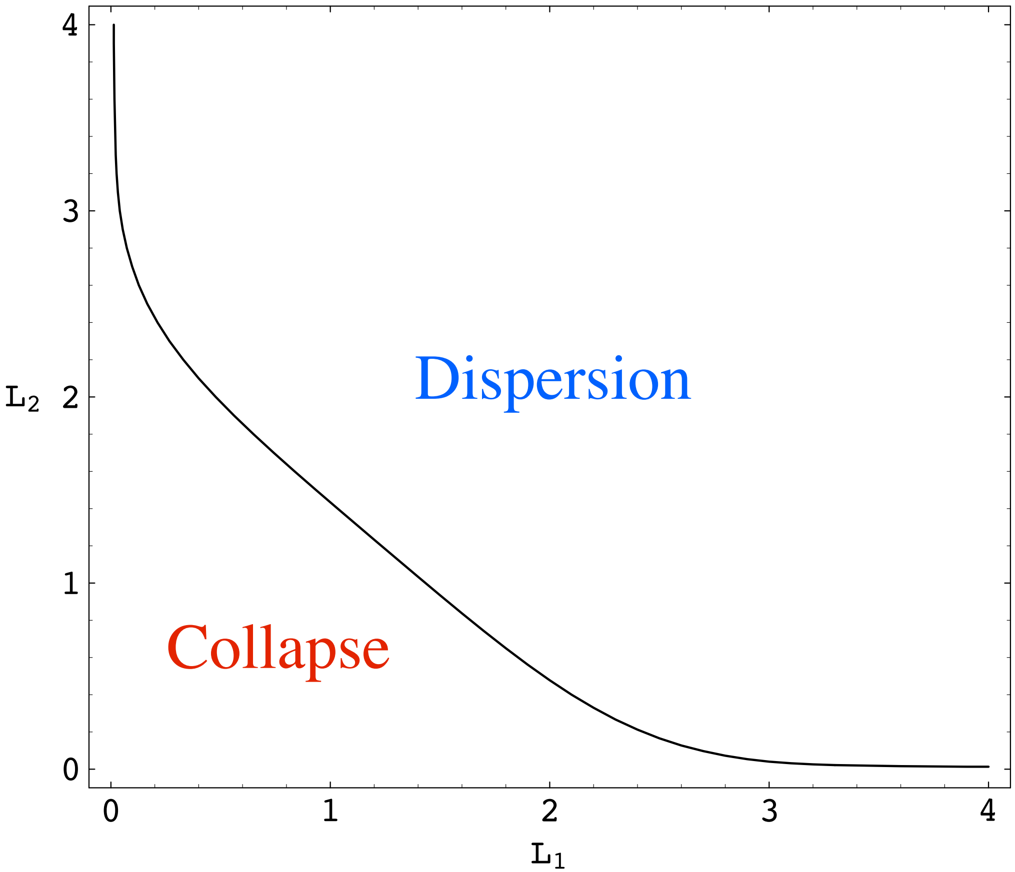
Example 3: Two different Gaussian Beams

(GV & J.Wosiek '08)

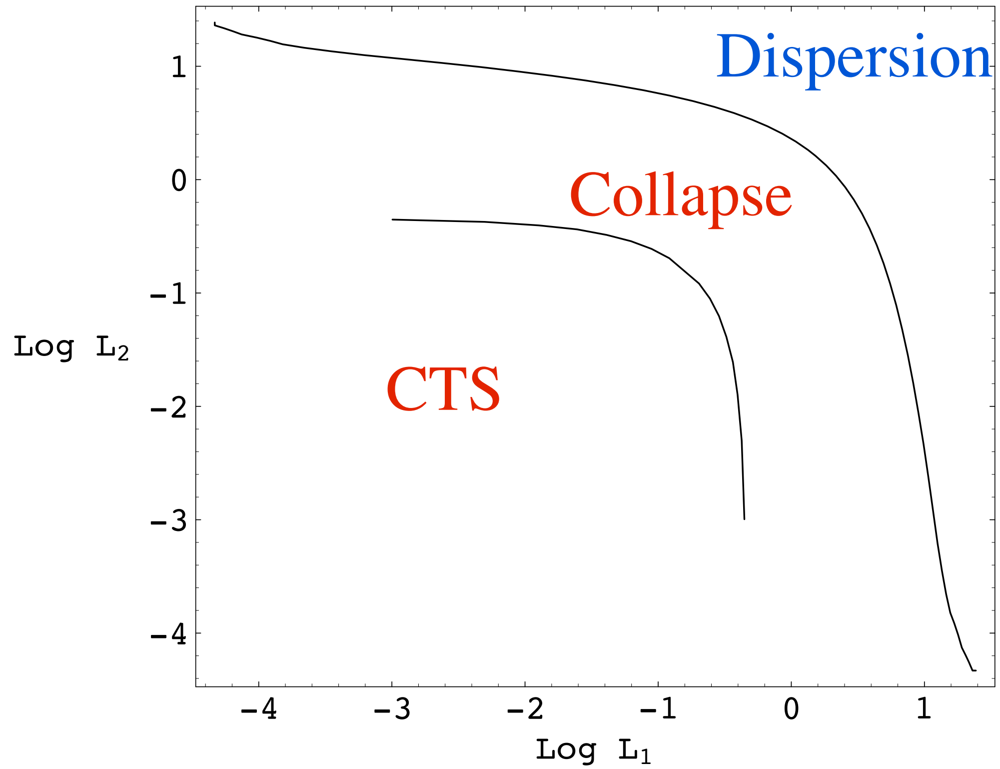
Two extended sources with the same fixed total energy and Gaussian profiles centered at r=0 with different widths L_1 and L_2:

$$s_i(t) = \frac{1}{2\pi L_i^2} exp\left(-\frac{t}{2L_i^2}\right) \quad , \quad \frac{R_i(t)}{R} = 1 - exp\left(-\frac{t}{2L_i^2}\right)$$

The critical line in the (L_1 , L_2) plane can be compared with the one coming from the CTS criterion.



4
3
L_2 2
1
0
0
1
2
3
4
L_1
Dispersion
Collapse


Dispersion
Collapse
CTS
Log L2
Log L1
1
0
-1
-2
-3
-4
-4
-3
-2
-1
0
1

An amusing coincidence?

In 0908.1780 Choptuik & Pretorius analyzed a "similar" situation numerically (relativistic central collision of two solitons of fixed mass and transverse size).
BH formation occurs at a critical γ_c (i.e. R_c) which is a factor 2 or 3 below the naive CTS value.

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The above results are encouraging but real control over the different approximations is lacking, in particular on the freezing of longitudinal dynamics.
This is probably at the origin of some puzzles we find in connection with gravitational radiation at $b \gg R$.