Particules Élémentaires, Gravitation et Cosmologie
Année 2010-'11

Théorie des cordes: quelques applications

Cours IX: 4 mars 2011

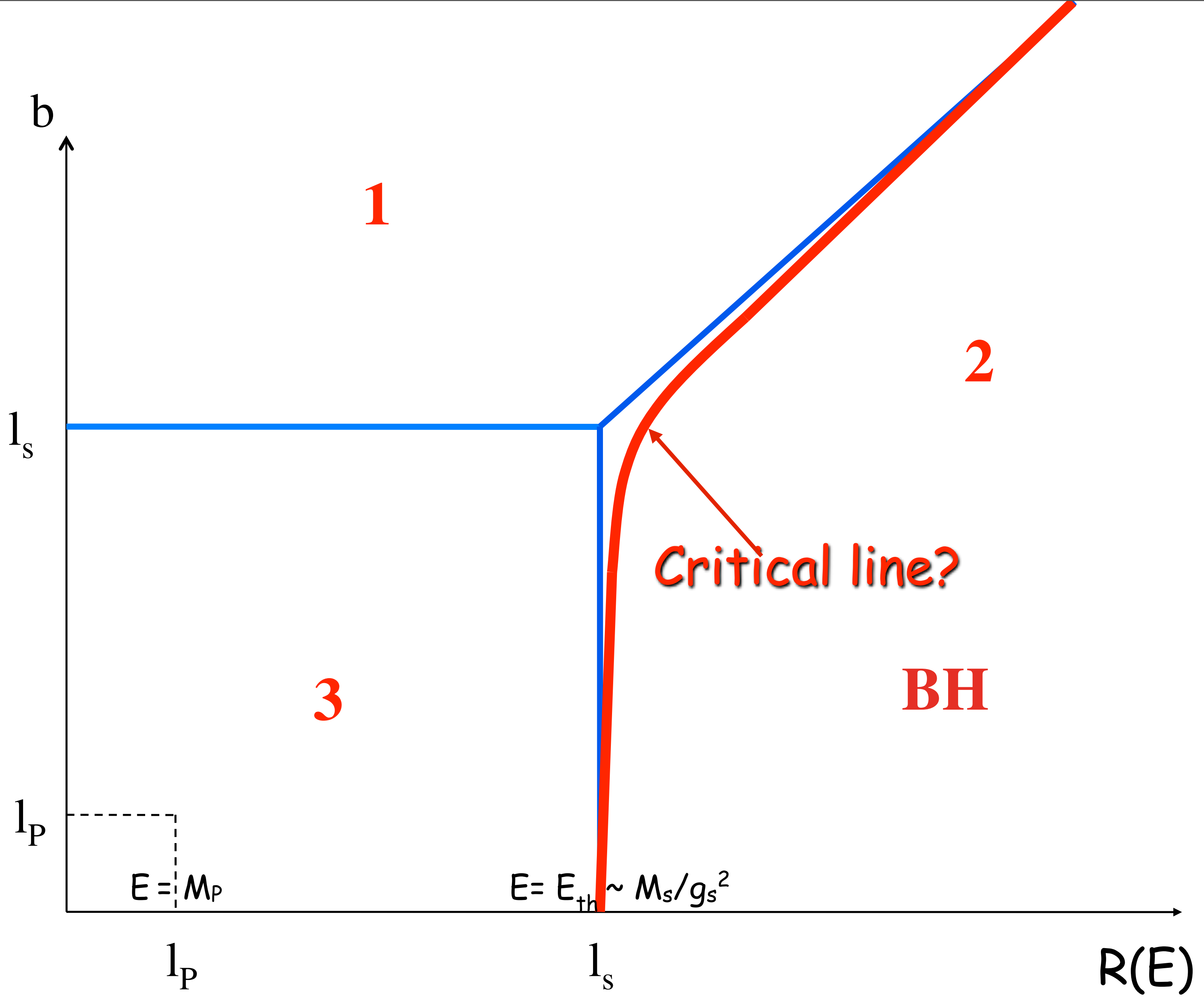
Transplanckian scattering in QST:
III. Stringy effects

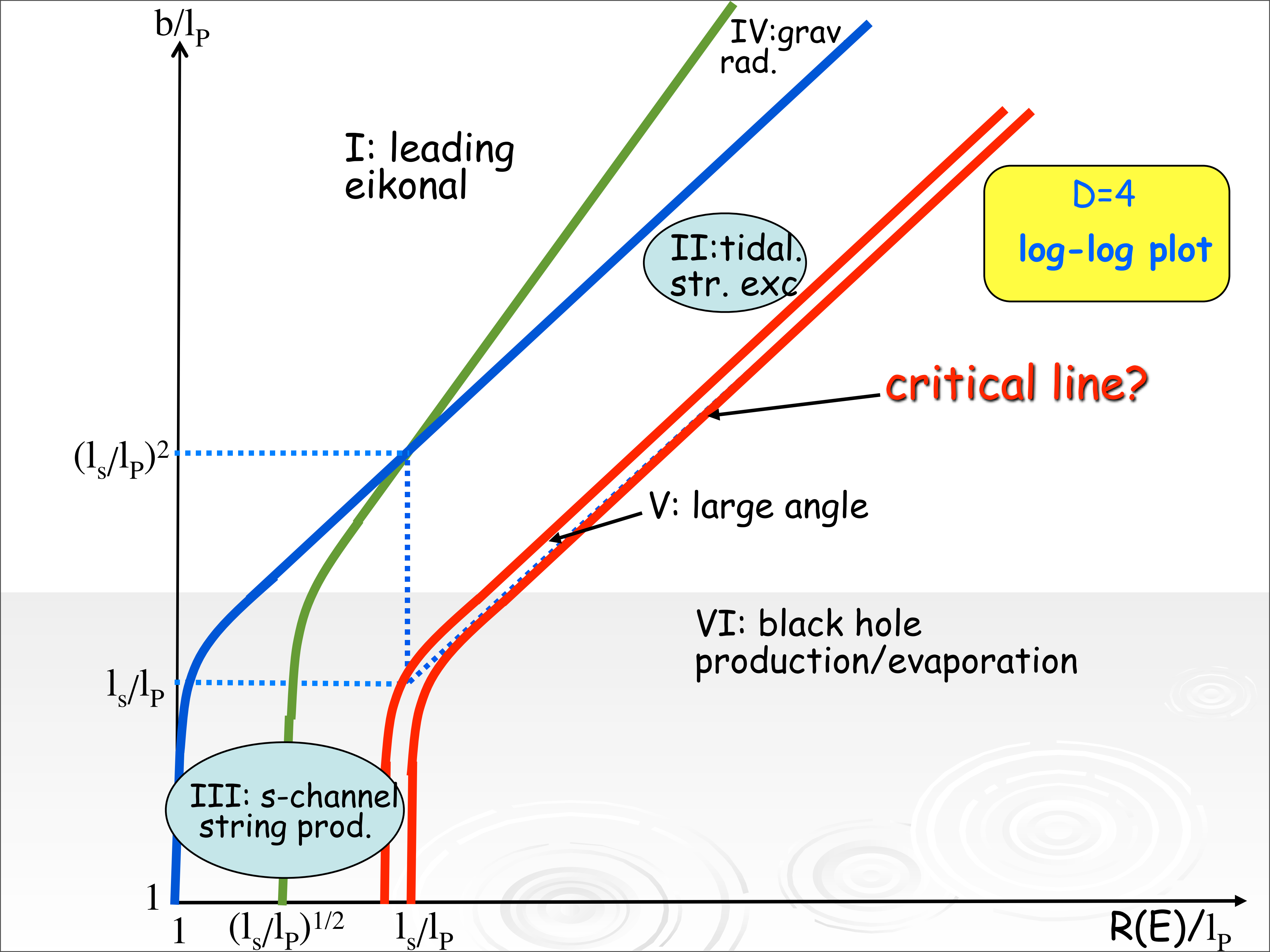
Two kinds of string effects/corrections

$$S(E,b) \sim exp\left(i\frac{A_{cl}}{\hbar}\right) \quad ; \quad \frac{A_{cl}}{\hbar} \sim \frac{Gs}{\hbar} c_D b^{4-D}\left(1 + \cancel{O((R/b)^{2(D-3)})} + O(l_s^2/b^2) + \cancel{O((l_P/b)^{D-2})} + \dots\right)$$

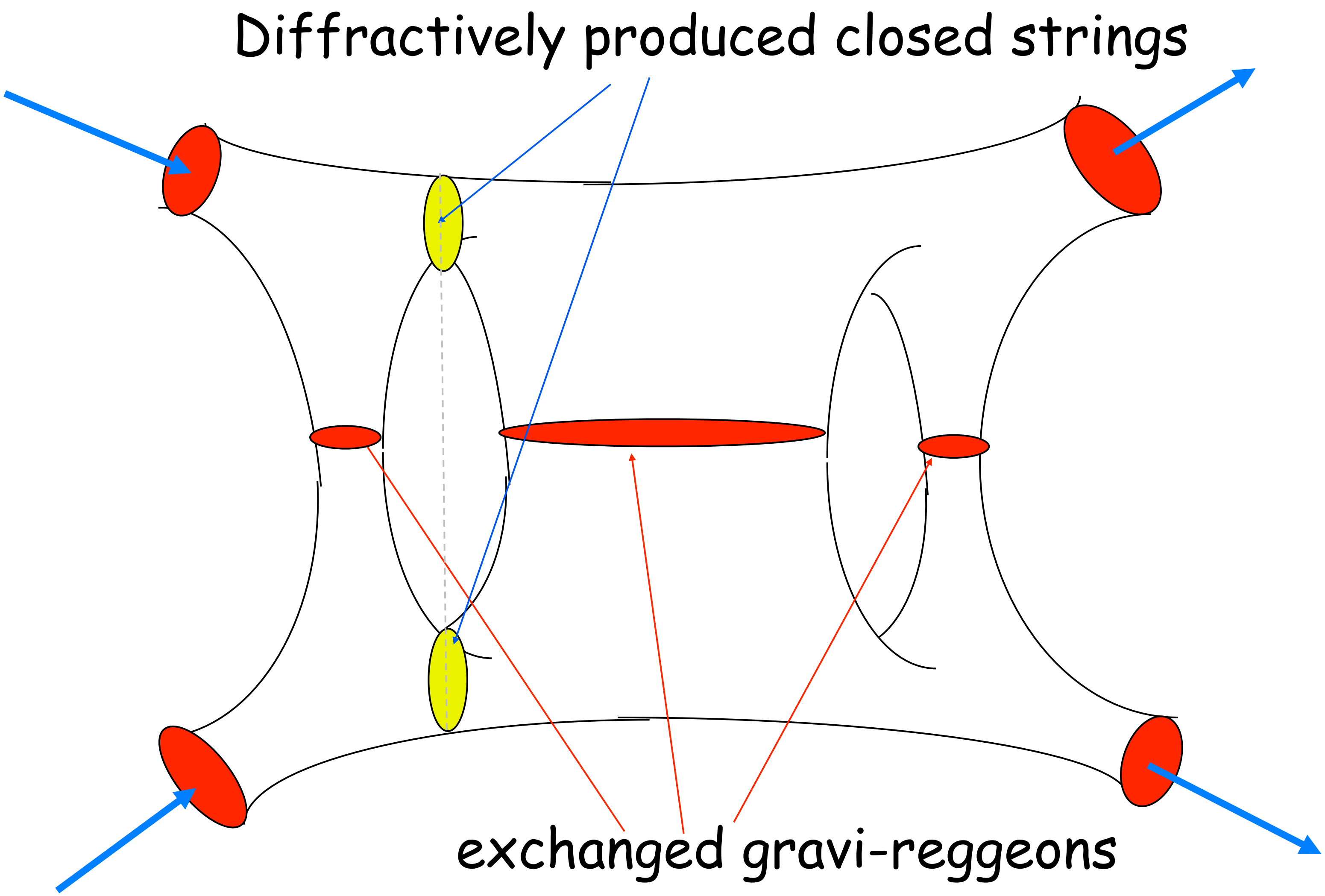
1. Graviton exchanges can excite one or both incoming strings. Similar to diffractive excitation in hadron-hadron collisions, but here rather interpreted as tidal-excitations. Relevant even in region 1 (see subregion II) because of Im $\delta \neq 0$.

2. Because of good old (DHS) duality even a single graviton exchange does not give a real phase shift. The imaginary part is due to closed-strings formation in s-channel and lacks the graviton pole at t=0. Hence, its contribution is exponentially damped at large b. It is only relevant in region 3 (and III).

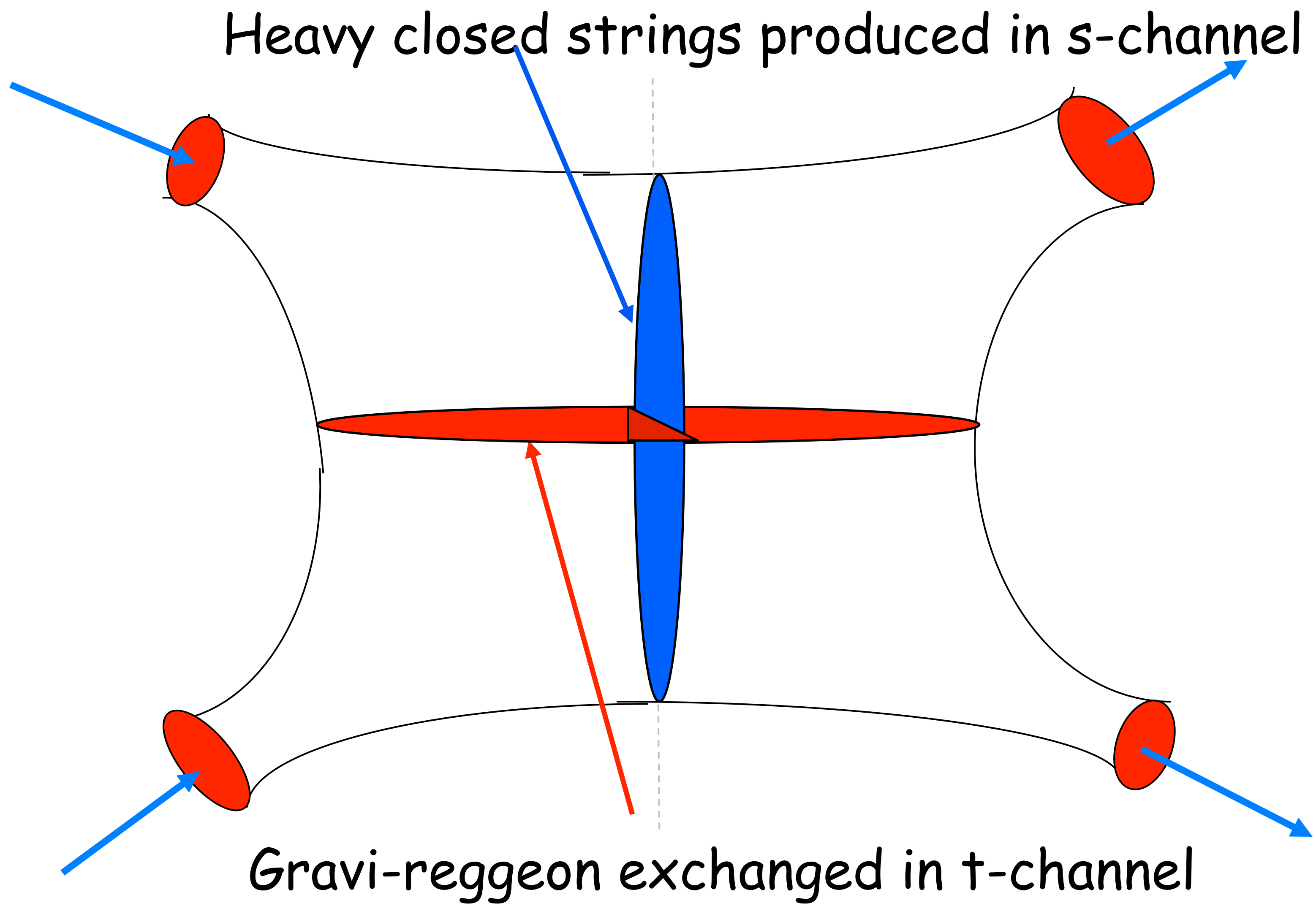

b
1
2
l_s
Critical line?
3
BH
l_P
E = M_P
E= E_{th} ~ M_s/g_s^2
l_P
l_s
R(E)



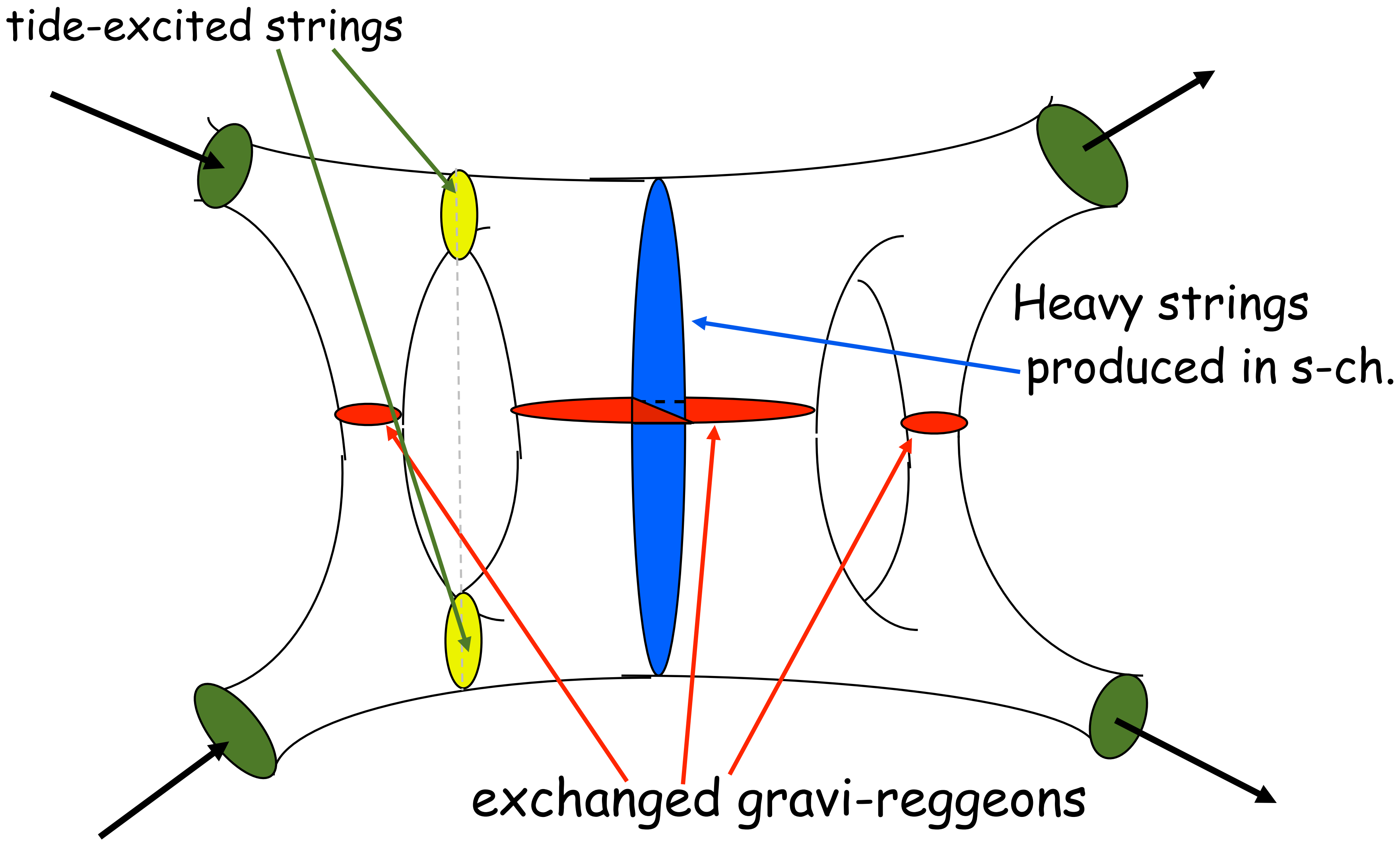
b/l_P
IV:grav rad.
I: leading eikonal
II:tidal. str. exc
D=4
log-log plot
critical line?
$(l_s/l_P)^2$
V: large angle
VI: black hole production/evaporation
l_s/l_P
III: s-channel string prod.
1
1
$(l_s/l_P)^{1/2}$
l_s/l_P
R(E)/l_P


Diffractively produced closed strings
exchanged gravi-reggeons

Im A is due to closed strings in s-channel (DHS duality)



Two examples of string corrections (controlled by l_s)



II: String excitation at large b (ACV '87)

When a string moves in an AS metric it suffers tidal forces as a result of its finite size (Giddings '06).

When does the phenomenon start? Recall:

$$\theta_1 \sim G_D \; E_2 \; b^{3-D} \;\Rightarrow \Delta\theta_1 \sim G_D \; E_2 \; l_s \; b^{2-D}$$

"String bits" sitting at different b are deflected differently: this spread in θ induces an excitation mass:

$$M_1 \sim E_1 \Delta\theta_1 \sim G_D \; s \; l_s \; b^{2-D} = M_2 \equiv M_{ex.}$$

But quantum strings only get excited if

$$M_{ex.} \geq M_s = \hbar l_s^{-1} \Rightarrow b \leq b_D \sim \left(\frac{G s l_s^2}{\hbar}\right)^{\frac{1}{D-2}}$$ ... as in ACV '87.

Below this critical value of b the initial massless strings get excited and we can compute the excitation spectrum. Interestingly, the spectrum increases like the string density of states up to $M_{ex.}$ and then falls off exponentially:

$$\frac{d\sigma_{inel}}{dM} \sim \exp\left(\frac{M}{M_s}\right) \quad ; \quad M < M_{ex.}$$
$$\sim \exp\left(-\frac{M^2}{M_{ex.}^2}\right) \quad ; \quad M >> M_{ex.}$$

Correspondingly, the elastic cross section is suppressed:

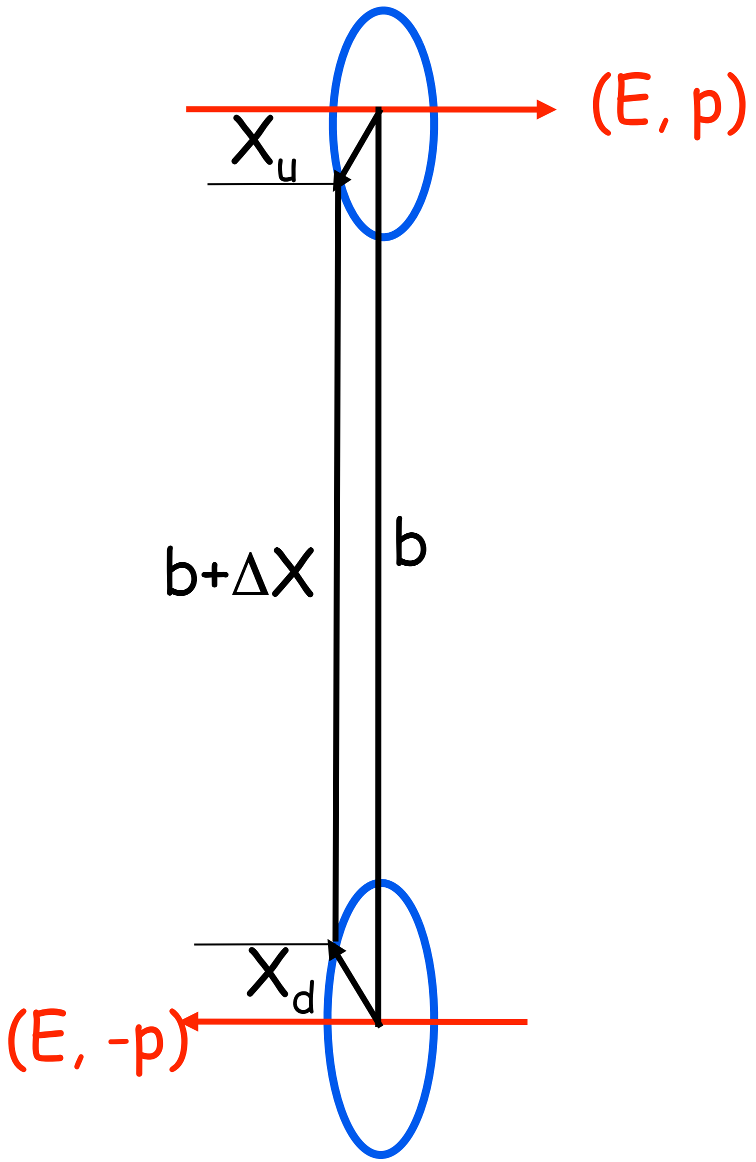
$$\sigma_{el} \sim \exp(-S(M_{ex})) \sim \exp(-\frac{M_{ex}}{M_s}) \sim \exp(-\frac{Gs}{\hbar}\frac{l_s^2}{b^{D-2}})$$

The precise way in which this phenomenon is accounted for is quite interesting (ACV 1987): the leading eikonal formula gets modified by the simple replacement:

$$e^{2i\delta(b)} \to \frac{1}{4\pi^2}\int_0^{2\pi} d\sigma_u \int_0^{2\pi} d\sigma_d : e^{2i\delta(b+\hat{X}_u(\sigma_u,\tau=0)-\hat{X}_d(\sigma_d,\tau=0))} :$$

$$\delta(E,b) = \int d^{D-2}q \frac{A_{tree}(s,t)}{4s}\, \mathrm{e}^{-iqb} \ , \quad s = E^2, \quad t = -q^2$$

where $X_{u,d}$ are string operators at the collision time (τ = 0). In other words, the process occurs at different values of b for different "bits" of the string and the whole process is subject to the quantum mechanical vibrations of the string itself.
In pictures:


(E, p)
X_u
b+ΔX
b
X_d
(E, -p)

Finally, the result can be interpreted in the spirit of 't Hooft's derivation of the leading eikonal.

Consider the quantization of a string moving in the effective AS (or GAS) shock-wave metric produced by the other string:

$$ds^2 = -dudv + \phi(\mathbf{x})\delta(u)du^2 + d\mathbf{x}^2 \ ; \ \Delta_T\phi(\mathbf{x}) = -16\pi G_D \rho_s(\mathbf{x})$$

using a light-cone gauge in which τ is proportional to the u-coordinate of the shock wave ($U(\sigma, \tau) = \alpha' p_2^+ \tau$).

$$S = \int d\sigma d\tau \partial X^\mu \bar{\partial} X^\nu G_{\mu\nu}(X) \rightarrow \int d\sigma d\tau \partial X^\mu \bar{\partial} X^\nu \eta_{\mu\nu} + \int d\sigma p_2^+ \phi(X_T^i)$$

This just reproduces the result of the explicit calculation.

NB: Strictly speaking, a GAS is not a consistent background for a string... only seen as an effective metric...

The string-size dominated regime

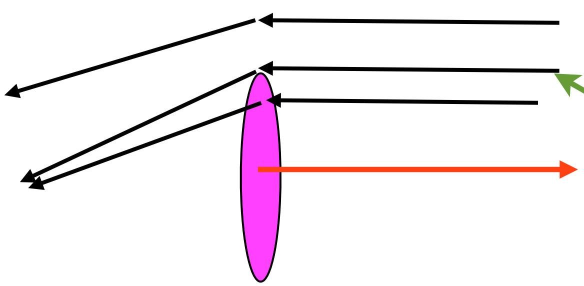
The phase shift, which diverges at b=0 in the point-like limit, gets regularized by string-size effects. At b < l_sY:

$$\mathrm{Re}\delta \sim -\frac{G_D s b^2}{(Y l_s)^{D-2}} \text{ ‘; } \quad Y = \sqrt{\log(\alpha' s)}$$

The saddle point condition now gives a different deflection:

$$\theta = G_D\ \rho\ b\ ;\ \rho = \frac{E}{(Y l_s)^{D-2}}$$

corresponding to deflection from a disc of transverse size ~ Yl_s: maximal θ reached for b ~ l_s (border between I & III).



$$\theta < \theta_{max} \sim \left(\frac{R}{l_s}\right)^{D-3} \ll 1$$

The region with $\theta > \theta_{max}$ is the one where GMO and ACV can be compared with amazing agreement ($q \sim \theta$ E):

$$
\begin{aligned}
A_{GMO}(s,\theta) &\sim \quad exp\left(-l_s q \sqrt{\log(1/\theta^2)\log(1/g_s^2)}\right) \\
A_{ACV}(s,\theta) &\sim \quad exp\left(-l_s q \sqrt{\log(\alpha' s)\log(1/g_s^2)}\right)
\end{aligned}
$$

to be compared to $\quad exp(\alpha' t \log(\alpha' s))$ vs. $exp(\alpha' t \log(1/\theta^2))$

of tree-level fixed t vs. fixed, small θ

This is also the regime in which one can argue in favour of a Extended Uncertainty Principle (GV, Gross, ACV) in string theory preventing one from testing scales smaller than l_s:

$$\Delta x \geq \frac{\hbar}{\Delta p} + \alpha' \Delta p \geq \sqrt{\alpha' \hbar} = l_s$$

Absorption due to string production

The second important effect is that δ picks up an imaginary part corresponding to the above-mentioned duality of graviton exchange in string theory.

$$\mathrm{Im}\delta \sim \frac{G_D\ s\ l_s^2}{(Yl_s)^{D-2}} \mathrm{e}^{-b^2/b_I^2}\ ,\ b_I^2 \equiv l_s^2 Y^2\ ,\ \ Y = \sqrt{\log(\alpha' s)}$$

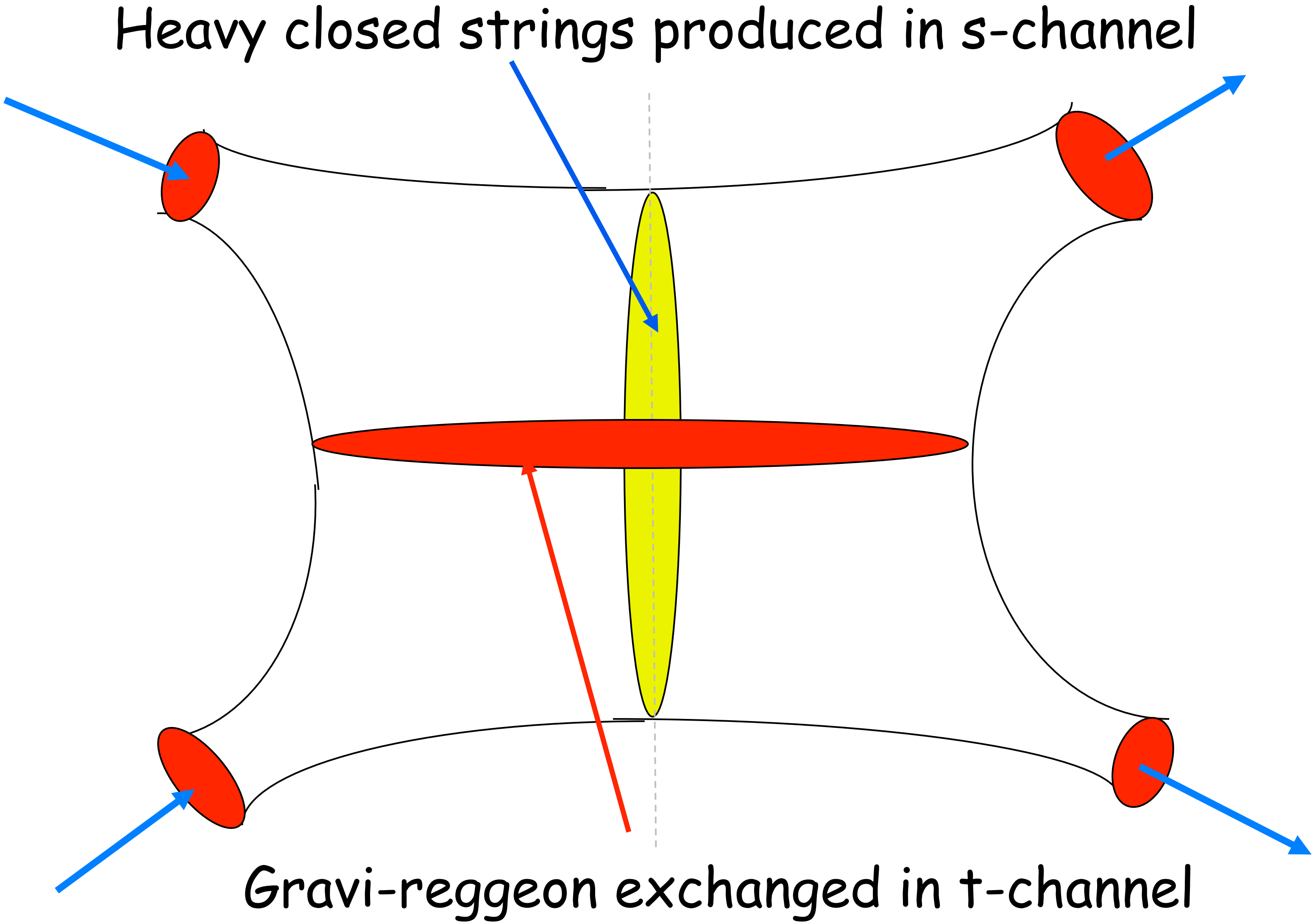
Because of Im δ≠0, $|S_{el}|$ is suppressed as exp(-2 Im δ):

$$\sigma_{\mathrm{el}} \sim \exp(-4\mathrm{Im}\delta) = \exp\left[-\frac{G_D\ s\ l_s^2}{(Yl_s)^{D-2}}\right] \equiv \exp\left[-\frac{s}{M_*^2}\right]$$

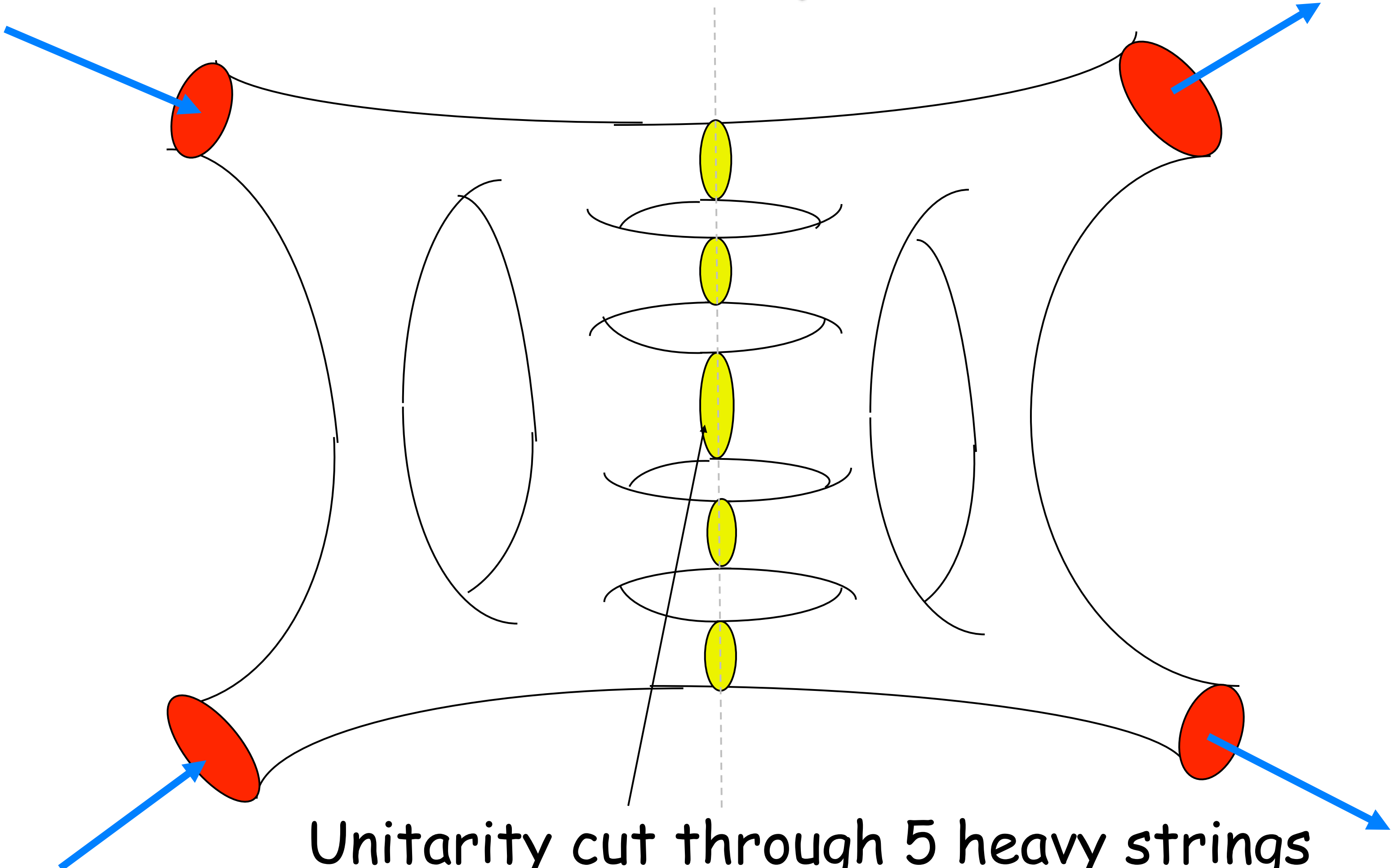
$$M_* = \sqrt{M_s M_{th}} \sim M_s g_s^{-1}$$ (Cf. tidal abs. $\exp(-\frac{Gs}{\hbar}\frac{l_s^2}{b^{D-2}})$)

At $E = E_{th} = M_s/g_s^2$ $\quad \sigma_{\mathrm{el}} \sim \exp(-g_s^{-2}) \sim \exp(-S_{sh})$

Im A is due to closed strings in s-channel (DHS duality)



Typical final state to saturate unitarity (as a result of exponentiation)



Unitarity cut through 5 heavy strings

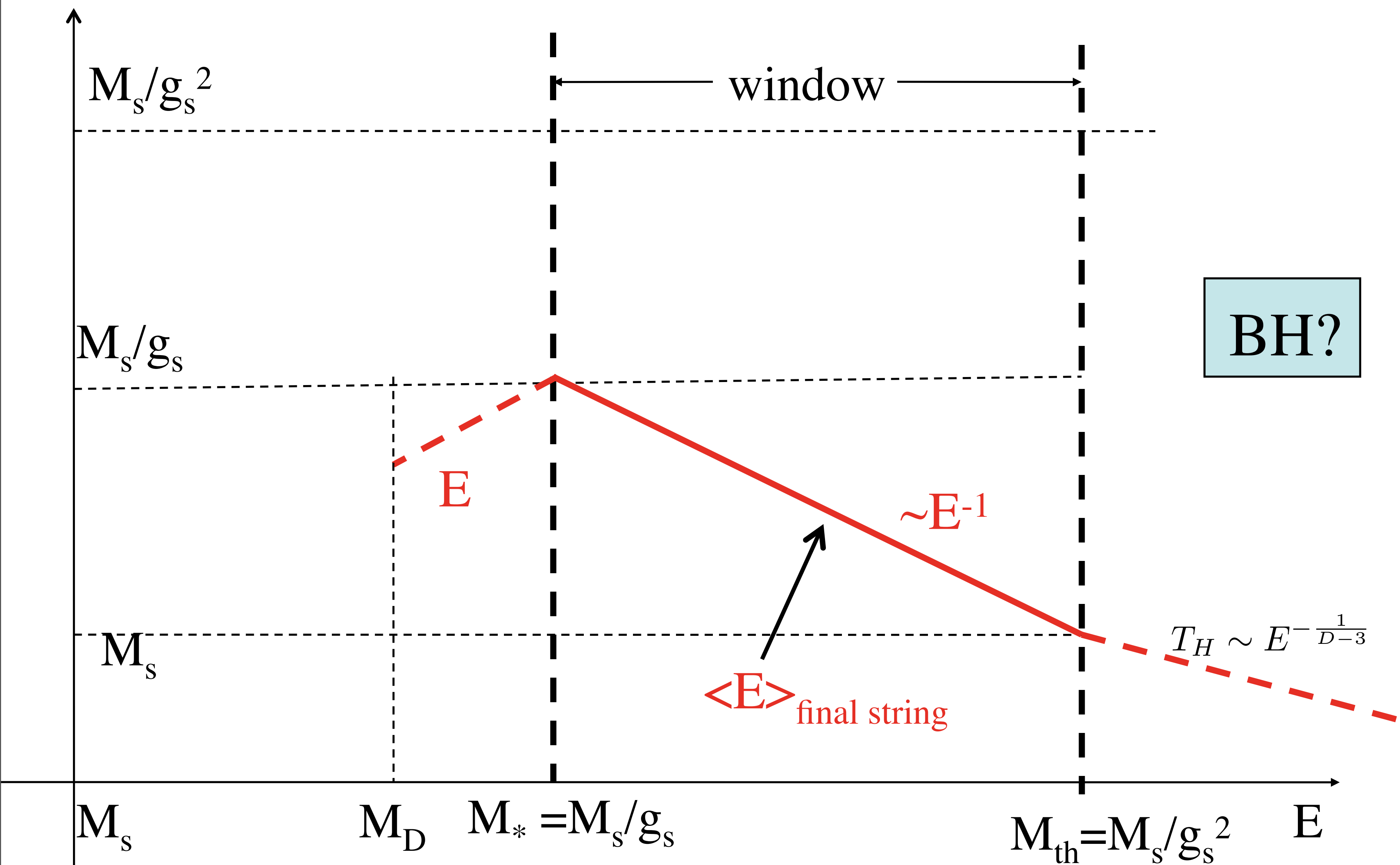
Also: $\langle N_{\mathrm{CGR}}\rangle = 4\mathrm{Im}\delta = \dfrac{G_D\ s\ l_s^2}{(Yl_s)^{D-2}} = O\left(\dfrac{s}{M_*^2}\right)$ and thus:

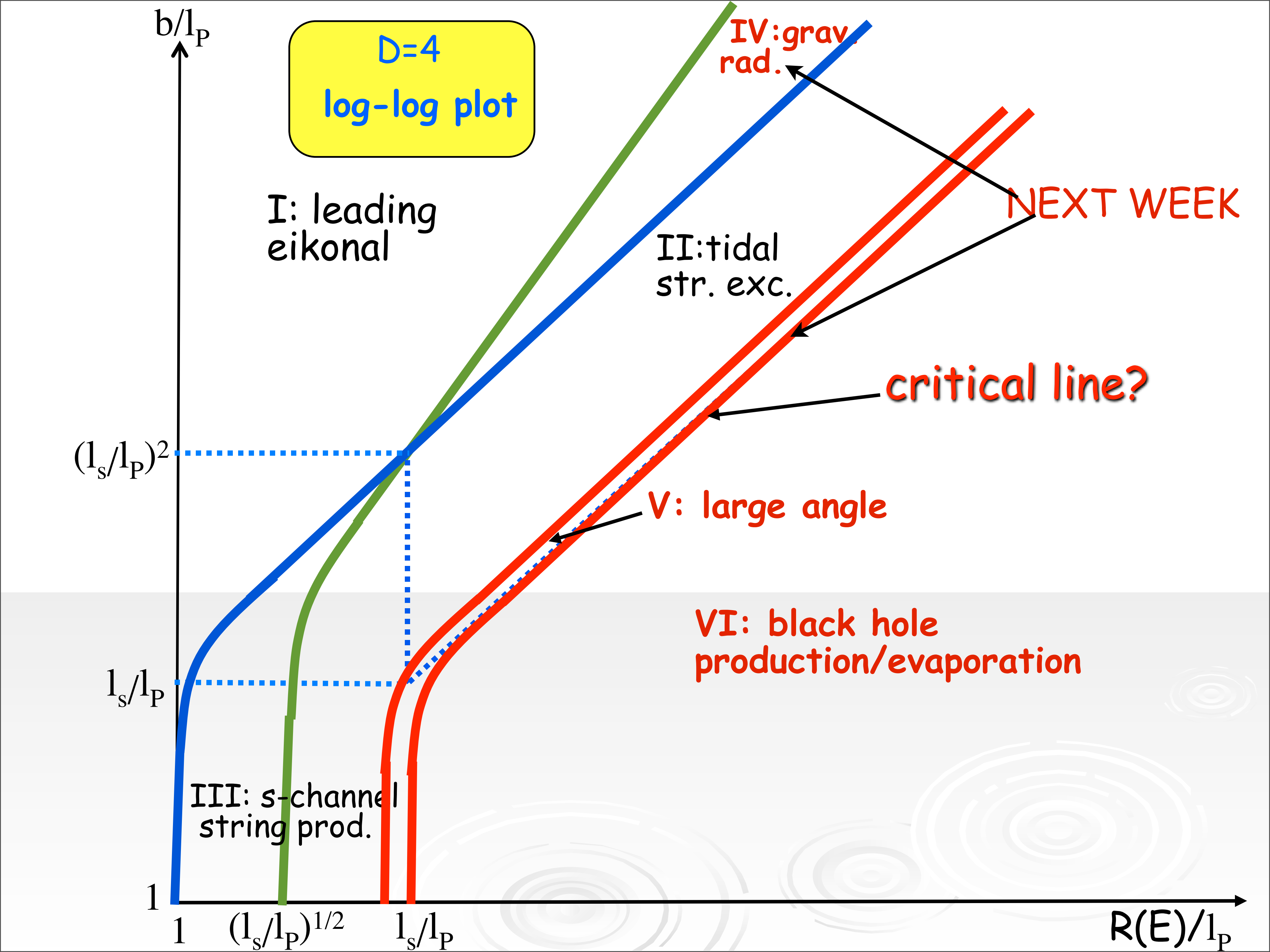
$$\langle E\rangle_{\mathrm{CGR}} = \frac{\sqrt{s}}{\langle N_{\mathrm{CGR}}\rangle} \sim M_s Y^{D-2}\left(\frac{l_s}{R_S}\right)^{D-3} \sim T_{\mathrm{eff}} \equiv \frac{M_*^2}{E} = \frac{M_s^2}{g_s^2 E}$$

We have thus found that final-state energies obey a sort of «anti-scaling» law!

This antiscaling is unlike what we are familiar with in HEP.

It is however similar to what we expect in BH physics!
In particular: For D=4, $T_{eff} \sim T_{Haw}$ even at $E < E_{th}$.
This could be a very interesting signal of strong gravity taking place below (not far from) the threshold of BH production!


M$_s$/g$_s$^{2}
window
M$_s$/g$_s$
BH?
E
~E^{-1}
M$_s$
$T_H \sim E^{-\frac{1}{D-3}}$
<E>$_{\text{final string}}$
M$_s$
M$_D$
M$_*$ =M$_s$/g$_s$
M$_{th}$=M$_s$/g$_s$^{2}
E


b/l_P
D=4
log-log plot
I: leading eikonal
II:tidal str. exc.
IV:grav. rad.
NEXT WEEK
critical line?
V: large angle
VI: black hole production/evaporation
III: s-channel string prod.
(l_s/l_P)^2
l_s/l_P
1
1
(l_s/l_P)^1/2
l_s/l_P
R(E)/l_P