Particules Élémentaires, Gravitation et Cosmologie
Année 2010-'11

Théorie des cordes: quelques applications

Cours XI: 11 mars 2011

Transplanckian scattering in QST:
V. String-brane collisions

Particle Spectra: an "energy crisis"?

Within our approximations the produced gravitons give the following spectrum for gravitational wave (GW) emission:

$$\frac{dE_{gr}}{d^2k\ d\omega} = Gs\ R^2\ \exp\left(-|k||b| - \omega\frac{R^3}{b^2}\right)\ ;\ \frac{Gs}{\hbar}\frac{R^2}{b^2} \gg 1$$

$$\frac{E_{gr}}{\sqrt{s}} = \int d^2k\ \frac{d\omega}{\sqrt{s}}\frac{dE_{gr}}{d^2k\ d\omega} = G\sqrt{s}\ \frac{R^2}{b^2}\ \frac{b^2}{R^3} \sim 1$$

The fraction of energy emitted in GWs is O(1) already for $Gs/h\ (R/b)^2 = O(1)$ i.e. already at $R/b \ll 1$ (since $Gs/h \gg 1$).

Q: Is this puzzling as a CGR expectation? A is related to Q':

Q': What is the frequency cutoff on the GWs emitted in an ultra-relativistic small angle 2-particle collision?

The GR answer seems to be unknown. My guess: $\omega_{max} \sim 1/R$.

This would rather give:

$$\frac{dE_{gr}}{d^2k\ d\omega} = Gs\ R^2\ exp(-|k||b| - \omega R) \Rightarrow \frac{E_{gr}}{\sqrt{s}} \sim \frac{R^2}{b^2}$$

so that the fraction of energy emitted in GW becomes O(1) only when the gravitational-collapse regime is approached. If this were the correct answer some of the ACV approximations (e.g. on the longitudinal dynamics) would have to be revised.

Independently of this: it looks that, while for b >> R gravitons are produced at small angles, as b approaches b_c ~ R their distribution becomes more and more isotropic with fast growth of multiplicity, <n> ~ Gs/h, and (again!) with characteristic energies O(h/R ~T_H).

Near & below b_c

Within the ACV07 framework the regime around $b = b_c$ can be studied. For $b \sim b_c$ the on-shell action behaves as follows:

$$\frac{A - A_c}{Gs} = \sqrt{3}\left(1 - \frac{b^2}{b_c^2}\right) + \frac{2\sqrt{2}}{3}\left(\frac{b^2}{b_c^2} - 1\right)^{3/2}$$

Below b_c the elastic amplitude picks up an extra absorption (besides the one contained in A_c from graviton emission) as if some new channels had opened up.
Q: Do these correspond to the formation of BHs?
A: We don't know. We lack understanding this unitarity deficit. This feature may be instead a consequence of our drastic approximations.
Another reason for turning to a simpler problem...

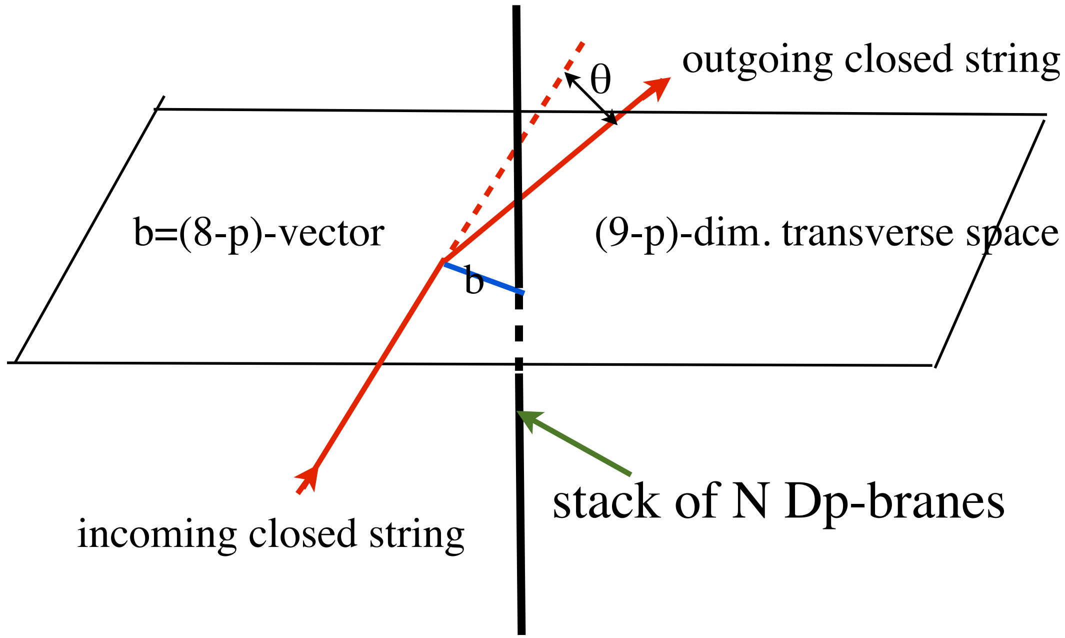
String-brane collisions: A simpler problem?

Consider a stack of N coincident D-p branes immersed in 10-dimensional spacetime.
Denote by x^α (α = 0, 1, 2,..p) the coordinates along the branes and by x^i (i= 1, ..., 9-p) those orthogonal to them (corresponding to D.b.c.) with the branes at $x^i = 0$.

Take now a massless closed string (e.g. a graviton or a dilaton) moving in the bulk and scattering on the stack of branes.
The brane system breaks translational invariance in the Dirichlet directions but preserves translation and Lorentz invariance along the branes. Consequently, we can choose a frame in which the string comes from (and goes out in) a direction orthogonal to the branes.

String scattering off a stack of N Dp-branes

G. D'Apollonio, P. Di Vecchia, R. Russo & G.V.
(1008.4773 & in progress); C. Monni and W. Black (to appear)



Actually the classical c.m. motion takes place in a plane. The deflection angle is the angle between the initial and final directions.

The classical geometry produced by a stack of D-branes is known. It is accompanied, in general, by a non-trivial dilaton Φ and by a (p+1)-form potential $C_{012\ldots p}$.

$$ds^2 = \frac{1}{\sqrt{H(r)}}\left(\eta_{\alpha\beta}dx^\alpha dx^\beta\right) + \sqrt{H(r)}\left(\delta_{ij}dx^i dx^j\right) ,$$

$$\mathrm{e}^{\phi(x)} = g\left[H(r)\right]^{\frac{3-p}{4}} \quad , \qquad C_{01\ldots p}(x) = \frac{1}{H(r)} - 1 ,$$

$$H(r) = 1 + \left(\frac{R_p}{r}\right)^{7-p} \quad , \qquad R_p^{7-p} = \frac{gN(2\pi\sqrt{\alpha'})^{7-p}}{(7-p)\Omega_{8-p}} \quad , \qquad \Omega_n = \frac{2\pi^{\frac{n+1}{2}}}{\Gamma(\frac{n+1}{2})}$$

Trivial as r -> ∞, singular at r=0 (except for p=3).

We take g << 1, N >> 1, keeping gN (hence R_p/l_s) finite.

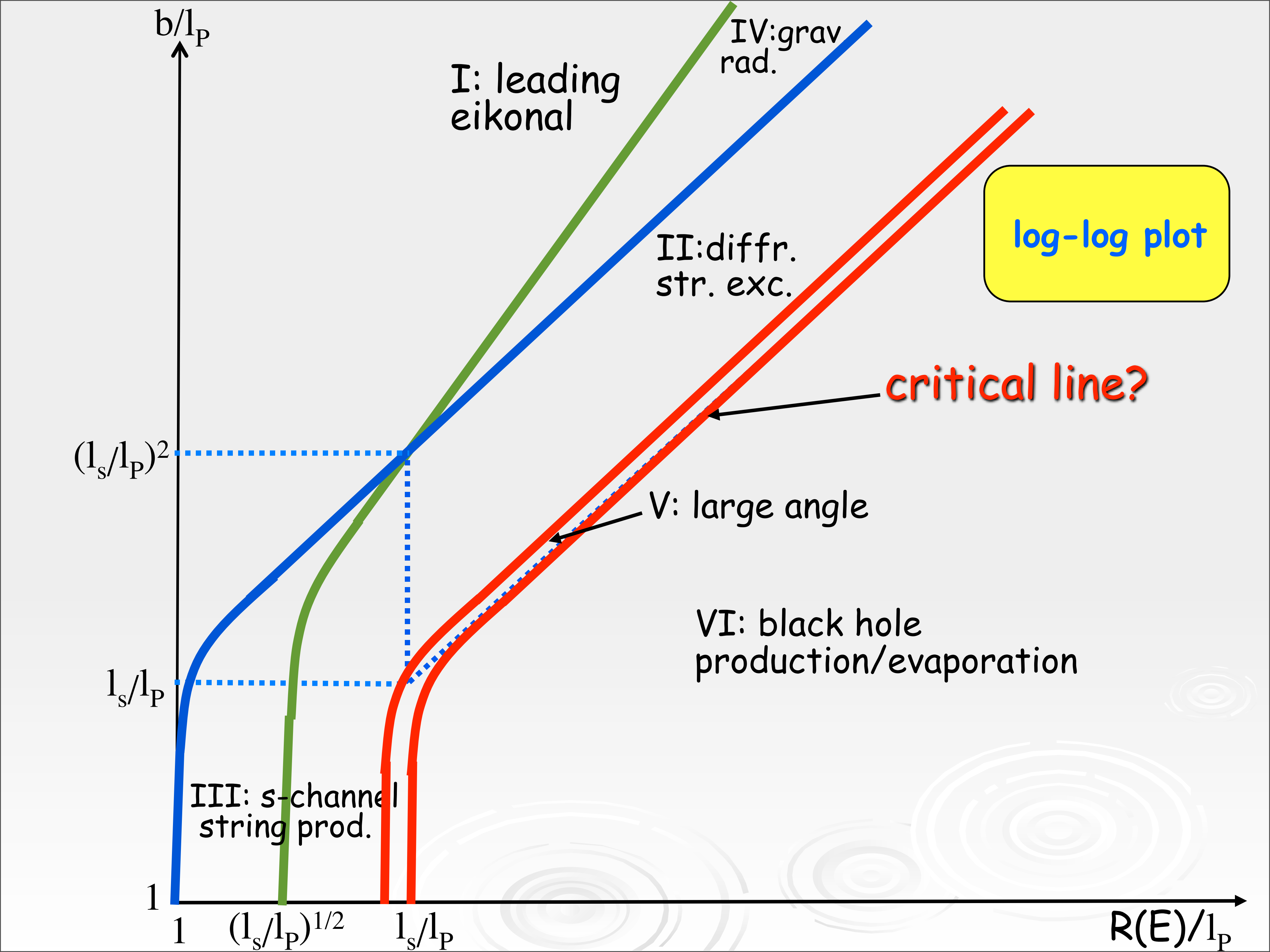
We also take the energy of the closed string very large and keep R_p/b (b= impact parameter) finite.

Relevant scales

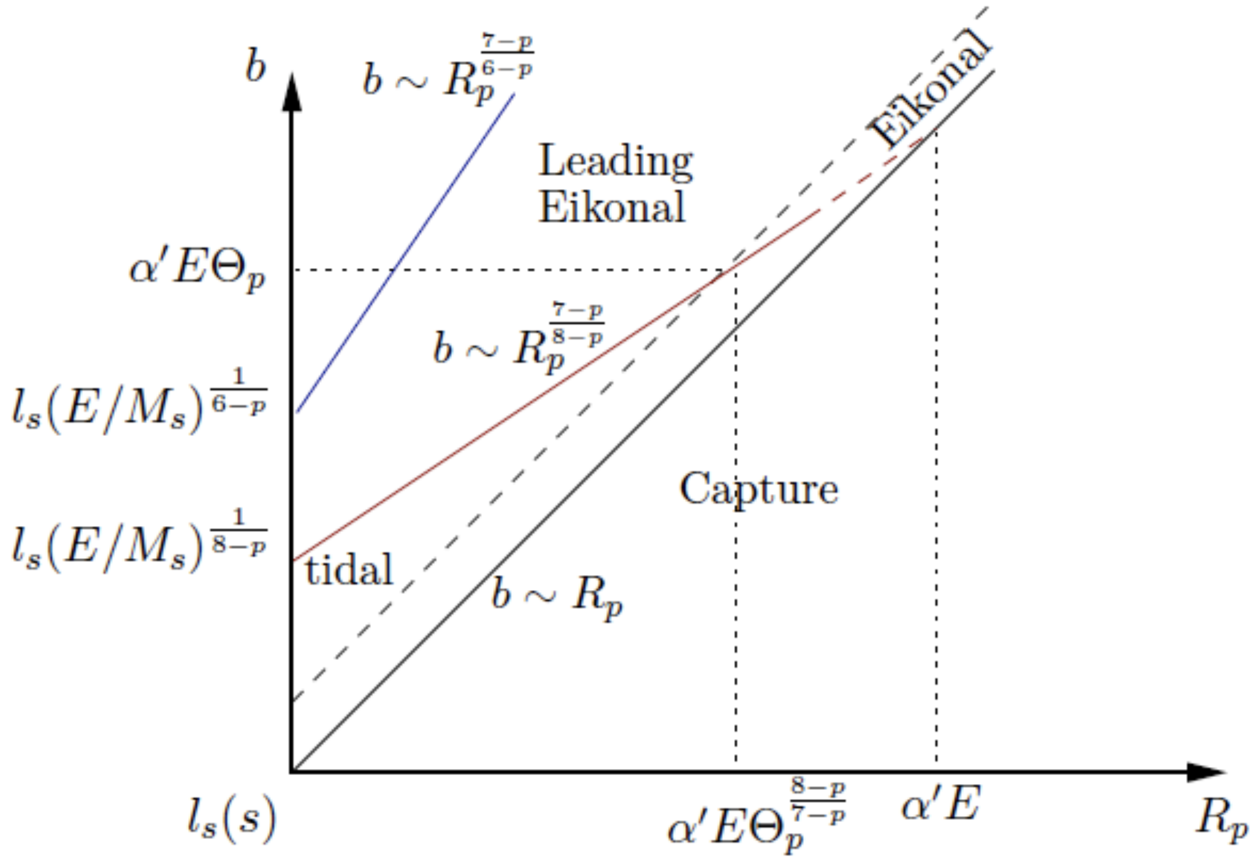
- The relevant scales are:
  - The impact parameter, $b = J/E$, of the incoming string;
  - The scale R_p of the (expected) emerging geometry;
  - The string scale l_s. Ratio R_p/l_s can be tuned by varying $g\,N$ (with $g \ll 1$, $N \gg 1$).

- These 3 length scales lead to a phase diagram resembling that of ACV (w/ collapse --> capture).

NB. Actually, like in the case of string-string collisions, we are not assuming any metric: calculations are done in flat spacetime & in the presence of N-Dp-branes (introduced via the boundary state formalism). The non-trivial metric emerges from the calculation.



b/l_P
IV:grav rad.
I: leading eikonal
log-log plot
II:diffr. str. exc.
critical line?
(l_s/l_P)^2
V: large angle
VI: black hole production/evaporation
l_s/l_P
III: s-channel string prod.
1
1
(l_s/l_P)^{1/2}
l_s/l_P
R(E)/l_P



At very high E, gravity (i.e graviton exchange) dominates. The dilaton and the RR form give subleading contributions. Yet we can neglect closed-string loops (below an E_{max} that goes to ∞ with N). Each graviton propagator must have one end on the branes (in order to get a factor N enhancement).

Comments & results

•Easier than 2-particle collisions: even at high energy the closed string acts as a probe of the brane-induced geometry (this can be changed...).

•At tree and one-loop level an effective classical geometry emerges through the deflection formulae satisfied at the saddle point of b-integral.

• Unlike in ACV this can be done reliably to next-to-leading order in the deflection angle (extension to all orders also looks possible)

•More explicitly: a non trivial calculation of a subleading term in the one-loop diagram gives:

Next to leading deflection angle for various values of p.

$$\theta_0 \sim \frac{16}{5}\frac{R^7}{b^7} + \frac{7\cdot 9\cdot 11\cdot 13}{2^{11}}\pi\frac{R^{14}}{b^{14}} \ , \qquad \theta_1 \sim \frac{15}{16}\pi\frac{R^6}{b^6} + \frac{5\cdot 7\cdot 9\cdot 11}{2^{10}}\pi\frac{R^{12}}{b^{12}} \ ,$$

$$\theta_2 \sim \frac{8}{3}\frac{R^5}{b^5} + \frac{5\cdot 7\cdot 9}{2^7}\pi\frac{R^{10}}{b^{10}} \ , \qquad \boxed{\theta_3 \sim \frac{3}{4}\pi\frac{R^4}{b^4} + \frac{3\cdot 5\cdot 7}{2^6}\pi\frac{R^8}{b^8} \ ,}$$

$$\theta_4 \sim 2\frac{R^3}{b^3} + \frac{15}{16}\pi\frac{R^6}{b^6} \ , \qquad \theta_5 \sim \frac{1}{2}\pi\frac{R^2}{b^2} + \frac{3}{8}\pi\frac{R^4}{b^4} \ ,$$

$$\theta_6 \sim \frac{R}{b} + O\left(\frac{R^3}{b^3}\right)$$

Agree to that order with exact formulae. Example of p=3:

$$\theta_3 = 2\sqrt{1+k^2}\int_0^1 dt \frac{1}{\sqrt{(1-k^2t^2)(1-t^2)}} - \pi = 2\sqrt{1+k^2}K(k) - \pi \ ,$$

$$k^2 = -1 + \frac{1-\sqrt{1-4\beta^4}}{2\beta^4} \ , \quad \beta \equiv R/b$$

•Tidal force effects can also be computed and come out in complete agreement with what one would obtain (to leading order in R_p/b and l_s/b) by quantizing the string in the D-brane metric (see next slide).

•These effects become relevant below $b = b_D \gg l_s, R_p$.

$$b_D^{8-p} = \frac{\pi}{2}\alpha' \sqrt{\pi s}(7-p)\frac{\Gamma\left(\frac{8-p}{2}\right)}{\Gamma\left(\frac{7-p}{2}\right)} R_p^{7-p}$$

•Tidal excitation spectrum has been double checked even for an initial massive string by C. Monni and W. Black (to appear).

Metric near null geodesic in suitable (AS-like) coordinates:

$$ds^2 = 2du d\hat{v} + \sum_{a=1}^{p} d\hat{x}_a^2 + \sum_{i=1}^{7-p} d\hat{y}_i^2 + d\hat{y}_0^2 + \mathcal{G}(u, \hat{x}^a, \hat{y}^i, \hat{y}^0) du^2 \quad ,$$

$$\mathcal{G} = \frac{\partial_u^2 \sqrt{\alpha}}{\sqrt{\alpha}} \sum_{a=1}^{p} \hat{x}_a^2 + \frac{\partial_u^2(\sqrt{\beta} r \sin\bar{\theta})}{\sqrt{\beta} r \sin\bar{\theta}} \sum_{i=1}^{7-p} \hat{y}_i^2 + \frac{\partial_u^2 \sqrt{\beta r^2 - b^2 \alpha}}{\sqrt{\beta r^2 - b^2 \alpha}} \hat{y}_0^2$$

$$\equiv \mathcal{G}_x \sum_{a=1}^{p} \hat{x}_a^2 + \mathcal{G}_y \sum_{i=1}^{7-p} \hat{y}_i^2 + \mathcal{G}_0 \, \hat{y}_0^2 \, .$$

$$du = \pm \frac{\beta dr}{C} \, , \qquad C(r) = \sqrt{\frac{\beta(r)}{\alpha(r)} - \frac{b^2}{r^2}} \qquad \beta(r) = 1/\alpha(r) = \sqrt{H(r)}$$

& σ-model for string fluctuations in suitable gauge ($u = \alpha' E \tau$):

$$S - S_0 = \frac{E}{2} \int_0^{2\pi} \frac{d\sigma}{2\pi} \int_{-\infty}^{+\infty} du \, \mathcal{G}(u, X^a(\sigma, u/\alpha' E), Y^i(\sigma, u/\alpha' E), Y^0(\sigma, u/\alpha' E))$$

$$\to \frac{E}{2} \int_0^{2\pi} \frac{d\sigma}{2\pi} \int_{-\infty}^{+\infty} du \left(\mathcal{G}_x(u) \sum_{a=1}^{p} X_a^2(\sigma, 0) + \mathcal{G}_y(u) \sum_{i=1}^{7-p} Y_i^2(\sigma, 0) + \mathcal{G}_0(u) Y_0^2(\sigma, 0) \right)$$

$$\equiv \frac{E}{2} \int_0^{2\pi} \frac{d\sigma}{2\pi} \left(c_x \sum_{a=1}^{p} X_a^2(\sigma, 0) + c_y \sum_{i=1}^{7-p} Y_i^2(\sigma, 0) + c_0 Y_0^2(\sigma, 0) \right) \, ,$$

Coming soon

(DDRV, to appear)

• Absorption of the string by the brane can be studied in some regimes (typically $l_s > R_p$) . It becomes important at $l_s > b$, in analogy with ACV, but with the crucial difference that the incoming energy now goes into open-string excitations of the D-brane system.

• We have definite hopes to be able to resum classical corrections and to study the S-matrix in the strong gravity (i.e. classical capture) regime ($b < R_p$) for which a precise unitary description of the system's evolution is highly non-trivial (impossible in ordinary scattering from an external field?).

Summary of last 4 lectures

- TPE string-string collisions in flat spacetime are an ideal theoretical lab. for studying several conceptual issues (Cf. inf. paradox) arising from interplay of QM and gravity within a fully consistent framework.

- We have been able to reproduce classical expectations (grav. deflection, tidal effects) and to extend them within a unitarity-preserving semiclassical description.

- When string-size effects dominate we found no evidence for BH formation but, instead, a softening of the final state resembling Hawking radiation.

- In the regime of strong gravitational fields our successes are still limited. Amusingly, a drastic approximation of the dynamics appears to reproduce at the semiquantitative level expectations based on CTS collapse criteria.

- No solid conclusion can be drawn without more work. Some features of the present approach may not survive a more complete treatment (e.g. on long.[al] dynamics).

- A general pattern appears to emerge where, at the quantum level, the transition between the dispersive and the collapse phase is smoothed out by QM.

- As some critical value of the impact parameter is approached the nature of the final state smoothly changes from that characteristic of a dispersive state to one reminiscent of Hawking's radiation (very high multiplicity and low energies O(h/R)).

- TPE string collisions off D-branes seem to offer a new tool to study all these issues within an easier set up. In particular, there are definite hopes to treat the strong-gravity (capture) regime non-perturbatively.

Future prospects

- By going beyond the simplest (infinite, coincident) brane system we should be able to turn the stack into a black hole and to study its interaction with closed strings within a unitary framework.

- Finally, for p=3 we should be able to make connection with the AdS/CFT correspondence, a duality, conjectured by J. Maldacena, between gravity in Anti-de Sitter spacetime (the metric near r=0 for p=3) and a 4-dimensional supersymmetric Conformal Field Theory (N=4 Super Yang Mills) living on the branes.