Particules Élémentaires, Gravitation et Cosmologie
Année 2010-'11

Théorie des cordes: quelques applications

Cours I: 4 février 2011

Résumé des cours 2009-'10: première partie

Le CERN annonce que le LHC fonctionnera en 2012

Genève, le 31 janvier 2011. Le CERN a fait savoir ce jour que le LHC fonctionnera jusqu'à la fin de 2012, avec un bref arrêt technique fin 2011.

L'énergie de faisceau sera de 3,5 TeV en 2011. Cette décision a été prise par la Direction du CERN suite à l'atelier de Chamonix, organisé chaque année pour établir le calendrier d'exploitation du LHC, et à un rapport du Comité consultatif du CERN pour les machines, rendu public aujourd'hui.

Elle donne aux expériences LHC de bonnes chances de découvrir une nouvelle physique dans les deux années à venir, avant le long arrêt prévu pour préparer une exploitation de la machine à de plus hautes énergies à compter de 2014.

« Si le LHC continue sur sa lancée et est aussi performant en 2011 qu'en 2010, l'année à venir s'annonce passionnante, a estimé Steve Myers, directeur des accélérateurs et de la technologie au CERN.
Tout porte à croire que nous devrions pouvoir augmenter le taux de collecte de données d'au moins un facteur trois dans le courant de l'année. »

Introduction

In 2004-'05, 2005-'06, 2007-'08, 2008-'09 we have discussed our present theory of the four known fundamental interactions (electromagnetic, weak, strong and gravitational).

That theory consists of two separate pillars:

1. The Standard Model of elementary particles (SMEP);
2. The Standard Model of gravity (SMG), General Relativity.

The SMG is a classical field theory.

The SMEP is a quantum field theory (QFT), a framework combining the principles of Quantum Mechanics and of Special Relativity. It actually belongs to a special class of QFTs, called gauge theories: these appear naturally as the way to describe, in a Lorentz-invariant way, spin-1 massless particles (with 2 physical polarizations).

The SMEP thus contains 8+3+1 = 12 such "gauge bosons" one for each generator of the SM gauge group

$$G = SU(3)xSU(2)xU(1)$$

It also contains 3 gauge couplings, one for each factor in G

The other "actors" in the SMEP are the matter fields. With the exception of the yet-to-be-discovered Higgs boson, a spin zero particle, the matter fields are spin-1/2 Weyl fermions, the smallest non-trivial reps. of the Lorentz group: (1/2, 0) left-handed & (0, 1/2) right-handed fermions.

The fermions belong to a highly reducible representation of G. Furthermore, this rep. is repeated 3 times, giving the well-know 3 families of quarks and leptons.

As for the Higgs the LHC is getting ready for its discovery but still needs to accumulate statistics.

G-rep. for one family of left handed quarks and leptons (the corresp. right-handed antiparticles are in the c.c. rep.)

l.h. ferm's	SU(3)	SU(2)	$U(1)_Y$
(u, d)	3	2	1/6
(ν, e)	1	2	-1/2
u^c	3*	1	-2/3
d^c	3*	1	+1/3
e^c	1	1	+1

plus perhaps a completely sterile heavy neutrino N to give a small mass to the active neutrinos via the see-saw mechanism.

This rep. is fully “chiral” meaning, in physical terms, that we cannot write down any gauge-invariant mass term (a bilinear in l.h. or r.h. fermions). Quark and lepton masses can only appear as a consequence of the spontaneous breaking of the gauge symmetry à la Brout-Englert-Higgs. The same mechanism gives mass to the gauge bosons of SU(2)xU(1) leaving just a massless photon (and 3 massive “intermediate bosons” the $W^{\pm}$ and the Z^0).

For the SU(3) part (the strong interactions) a different (non-perturbative) mechanism (confinement) prevents the existence of free quarks and gluons which, instead, bind into SU(3)-singlets, the hadrons (mesons and baryons) whose mass-scale is controlled by Λ_{QCD} (see '04-'05 and '05-'06 courses).

The SMG, General Relativity, is also based on a local symmetry, general covariance, which implements the equivalence principle and is naturally associated with a tensor field, the gravitational field. Semiclassically, such a field describes a massless particle of spin 2, the graviton, the analogue of the photon for the electromagnetic field.

General covariance, like gauge invariance, removes the unphysical degrees of freedom, in this case of a massless spin-2 particle.

Both the SMEP and the SMG have been tested with increasingly high accuracy for many decades.

The LHC should tell us whether the true SMEP is the minimal one or not... but also solve the mystery of dark matter... discover supersymmetry... find unexpected surprises.

Unfortunately this very successful model is limping.

So far, theorists have been unable to extend to gravity the fully quantum framework that led them to the SMEP: for quantum gravity the UV divergences are too strong!

There are indications that, in order to arrive at a fully consistent quantum theory of gravity, one needs to go beyond the framework of local QFT (loop quantum gravity claims the opposite).

String theory is at present the most promising candidate we have for combining the principles of QM, Gauge Invariance and General Covariance within a fully unified quantum theory of all forces and all elementary particles.

It predicts the existence of massless J=1,2 particles!

Today this is the main physical motivation for studying String Theory (there are also mathematical reasons).

This was not the way string theory came about. It came from an attempt, in the sixties, to describe in an unconventional way the strong interactions. That attempt, as such, failed. However, in the process, a beautifully consistent theoretical framework was constructed which, instead, looked able of addressing the deeper question of how to reconcile gravity and Quantum Mechanics.

Most courses/books in string theory start directly from this end (a top-down approach) arriving at the model that historically led to string theory after much work.

In last year's course we followed an opposite, bottom-up approach.

In the first part of the course we retraced the birth of the so-called Dual Resonance Model (DRM) -and of its interpretation as a string theory- as a candidate theory of strong interactions.

We then discussed some basic properties of quantum strings and why these properties led to abandoning the original goal when QCD came about.

Moreover, QCD explained, a posteriori, why string theory was invented in an apparently "serendipitous" way and why it had remarkable success in explaining some strong interaction phenomena.

The obstinate presence of massless particles, and the absence of point-like "hard" interactions were the main reasons for the death of the hadronic string.

In the second part of last year's course we discussed the modern formulation and reinterpretation of string theory as a quantum theory of all interactions, including gravity.

For lack of time we only discussed general features of the theory postponing its possible applications (to black holes, to cosmology, to strongly coupled gauge theories).

This year we will concentrate on a selection of such applications. However, in view of the amount of material presented last year and in order for the course to be more or less self-contained, we shall devote the first two weeks to a review of what we discussed last year.

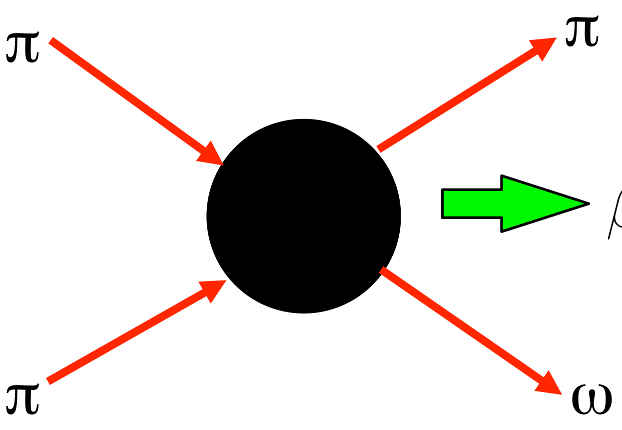
Dual Resonance Models

Strong interactions in the 60s	Strong interactions in the 60s
DHS duality and a bootstrap	A simple, exact solution
DRM: counting states, ghosts, operators, algebras	The no-ghost theorem, loops, D=26

- I gave a brief historical introduction to the birth of the Dual Resonance Model (DRM) as a way to get out of the impasse in which the theory of strong interaction was in the mid sixties.
- QFT methods looked completely inadequate.
- Regge-Chew-Mandelstam theory, coupled to DHS duality (1967), gave rise to a bootstrap program that ended in 1968 with the construction of an explicit closed-form solution (the B-function anzatz) for $\pi\pi \to \pi\,\omega$, soon extended to $\pi\pi \to \pi\pi$ scattering.

$$A(s,t) = \beta\frac{\Gamma(1-\alpha(s))\,\Gamma(1-\alpha(t))}{\Gamma(2-\alpha(s)-\alpha(t))} = \beta\, B(1-\alpha(s), 1-\alpha(t))$$

$$\alpha(t) = \alpha_0 + \alpha' t$$



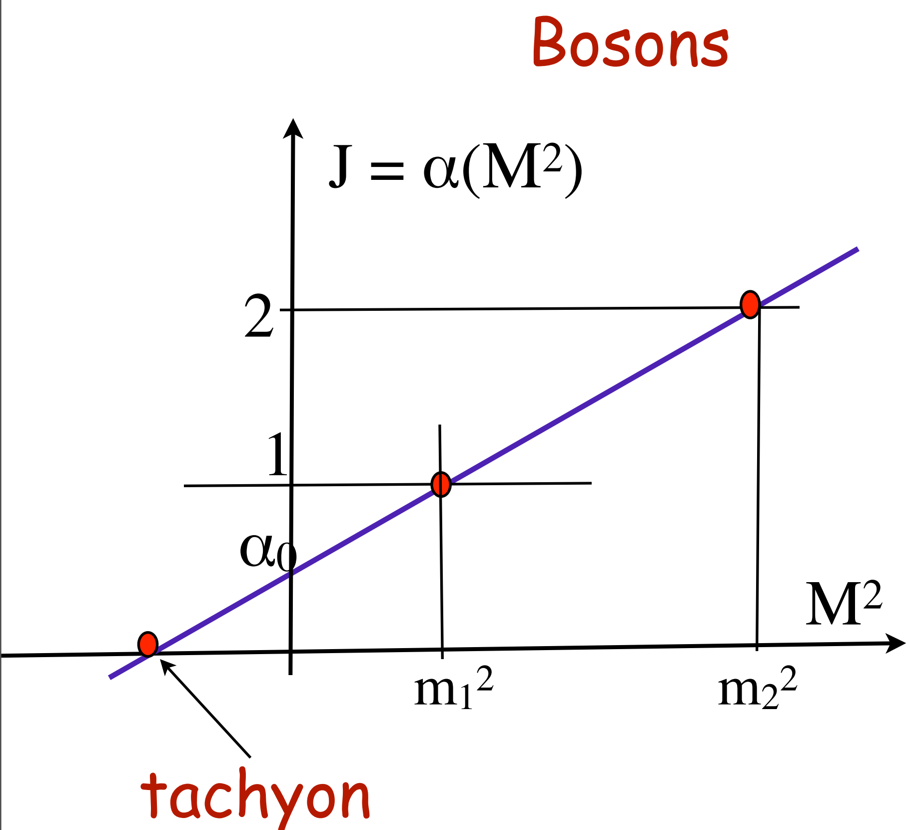
Full x-ing symmetry then implies:

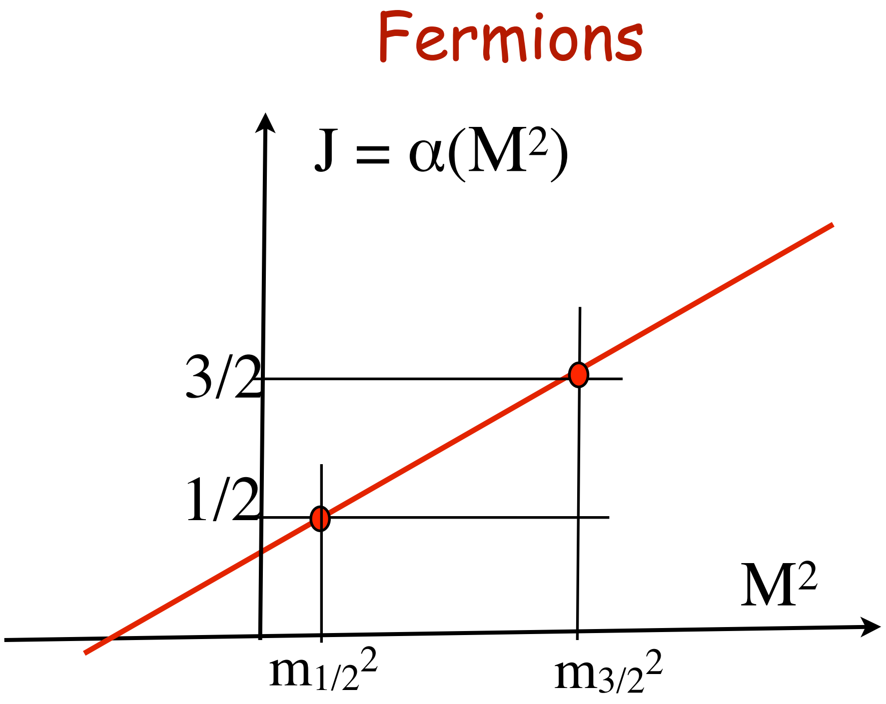
$$\begin{aligned} \beta^{-1}A(s,t) &= B\,(1-\alpha(s), 1-\alpha(t)) \\ &+ B\,(1-\alpha(u), 1-\alpha(t)) \\ &+ B\,(1-\alpha(s), 1-\alpha(u)) \end{aligned}$$

For spinless particles the natural generalization is:

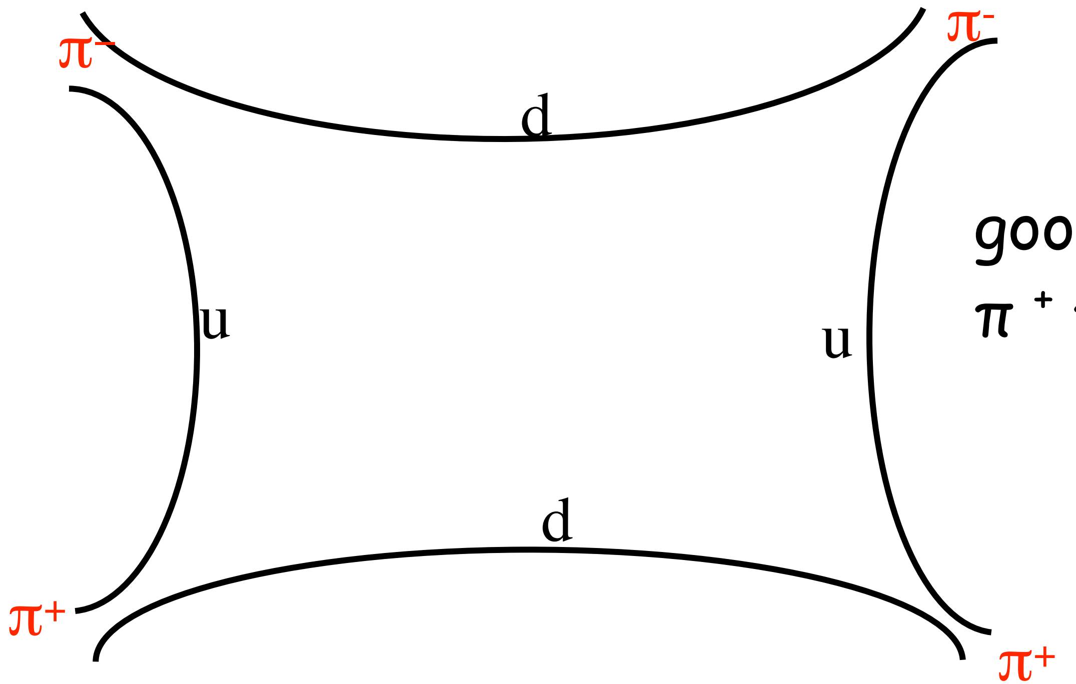
$$\frac{\Gamma\,(1-\alpha(s))\,\Gamma\,(1-\alpha(t))}{\Gamma\,(2-\alpha(s)-\alpha(t))} \to \frac{\Gamma\,(-\alpha(s))\,\Gamma\,(-\alpha(t))}{\Gamma\,(-\alpha(s)-\alpha(t))}$$

Meaning of Regge trajectory $J=\alpha(M^2)$





- We then discussed some properties of the 4-point function, its singularities (corresponding to zero-width resonances in its "planar" channels), its high-energy fixed-t (Regge) behaviour, and its high-energy fixed-angle behaviour.
- The duality properties are easily visualized by drawing duality diagrams. These also suggest the way to add internal quantum numbers for the external particles (via Chan-Paton factors).



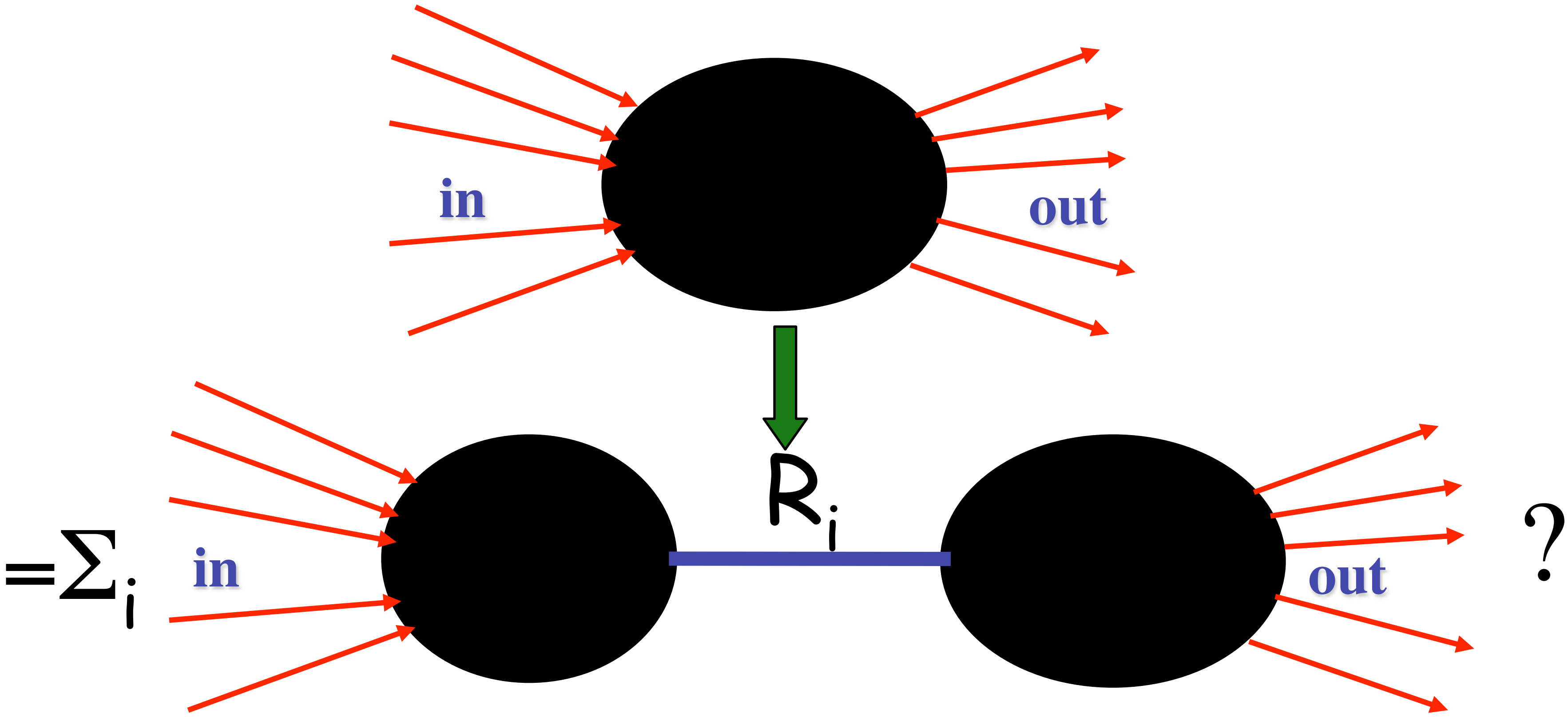
good to describe
$\pi^+ \pi^-$ scattering?

A phenomenologically better model was proposed by Lovelace:

$$
\begin{aligned}
A(\pi^+\pi^- \to \pi^+\pi^-) &= g^2 \frac{\Gamma(1-\alpha(s))\,\Gamma(1-\alpha(t))}{\Gamma(1-\alpha(s)-\alpha(t))} \\
\alpha(t) &= \alpha_\rho(t) \sim 0.5 + 0.9t \ \mathrm{GeV}^{-2}
\end{aligned}
$$

- In order to understand the true spectrum "hidden" below the singularities (poles) of the Beta-function we had to generalize the Beta-function amplitude to a process involving N external particles, the so-called N-point function.
- We then use a fundamental property of single-particle intermediate states: they provide a pole in the scattering amplitude whose location is related to the mass of the particle and whose residue should factorize.

Q: How many terms are needed (in the sum over i) in order to have, **for all in and out states,**



We then gave the N-point-function formula in the convenient form due to Koba & Nielsen:

$$B_N = \int_{-\infty}^{+\infty} \frac{\prod dz_i \theta(z_i - z_{i+1})}{dV_{abc}} \prod_{j>i} (z_i - z_j)^{2\alpha' p_i \cdot p_j}$$

Factorization of the N-point function is made easier by introducing an "operator" formalism (FGV, Nambu).

$$[q_\mu \ , \ p_\nu] = i\eta_{\mu\nu} \ , \quad [a_{n,\mu} \ , \ a^\dagger_{m,\nu}] = \delta_{n,m}\eta_{\mu\nu} \ , \quad \eta_{\mu\nu} = \mathrm{diag}(-1, 1, \ldots, 1)$$

$$(n = 1, 2, \ldots ; \mu = 0, 1, 2, \ldots D-1)$$

$$|\text{generic state}, k_\mu\rangle \sim \prod_{n,\mu} \left(a^\dagger_{n,\mu}\right)^{N_{n,\mu}} e^{iq_\mu k^\mu} |0\rangle$$

- The operator formalism clearly points to the danger that, in order to achieve factorization, one would need to introduce states of negative norm, so-called ghosts, which would then be produced in a scattering process with negative probability.
- Ghost states are obtained by applying the time component of the harmonic oscillator creation operators on the Fock vacuum.
- Fortunately, the full set of harmonic oscillator states is sufficient, but not necessary, to achieve full factorization. Some of the states decouple from the external states (so called spurious states).

The “ghost hunting” project was a “tour de force” that culminated in the proof of a “no-ghost theorem” by R. Brower and by P. Goddard & Ch.Thorn.

At the basis of the theorem was the discovery of the Virasoro operators (needed to construct the spurious/ physical states) and of their algebra (FV+W), the construction of the vertex operators (FV,G) and of an underlying Conformal Field Theory, and, finally, the explicit construction of an infinite set of positive-norm physical (DDF) states.

There was a price to pay for the absence of ghosts: the Regge intercept, α_0, had to be exactly 1 (implying a massless spin one particle and a spin zero tachyon) and the dimensionality of spacetime had to be at most 26.

Exactly for D=26 the physical Hilbert space would be completely spanned by the DDF states corresponding to harmonic oscillators in (D-2)=24 dimensions.

Meanwhile, C. Lovelace had shown that loops were consistent with unitarity only for D=26.

Counting now the number of physical states leads to a limiting (Hagedorn) temperature due to its exponential growth with M.

$$T_H = \frac{1}{2\pi\sqrt{\alpha'}}\sqrt{\frac{6}{D-2}} \rightarrow \frac{1}{4\pi\sqrt{\alpha'}} \quad \text{if} \quad D = 26$$

For D=26 and α_0=1 the model looked consistent except for the presence of a "tachyon" ($M^2 = -1/\alpha'$).

And it contained no fermions! They were added already in the DRM but it's easier to discuss them within string theory.