Particules Élémentaires, Gravitation et Cosmologie
Année 2009-'10

Théorie des Cordes: une Introduction

Cours XIV: 26 mars 2010

M-théorie et unification

- Heterotic string theories
- Supergravity in D=11
- Dualities among string theories
- Six theories in search of a Mother
- Conclusion, next year.

The Heterotic String

The heterotic string starts from the observation that, for closed strings, one can impose different conditions on left and right movers. What happens if we try to combine a superstring theory for right-movers with a bosonic string for left-movers?

Consistency with 2D-anomaly cancellation requires D=10 for the right movers and D=26 for the left-movers. How can we make sense of such a situation? The answer is to use the compactification idea for the 16 = 26-10 extra left-moving bosonic coordinates and to go to $O(l_s)$ compactification radii.

Consistency with modular invariance will constrain the lattice of the quantized left-momenta to be even and self-dual, but, given that there is no right-moving part, the lattice now has to be also Euclidean. And even means now $p_L^2 = 4n/\alpha'$.

Such lattices are rare. They only exist for d=8n, but, fortunately for us, d=16!

In fact, in d=8 there is only one even self-dual lattice:

$$\Gamma_8 : (n_1, n_2, \dots n_8) \text{ or } (n_1 + \frac{1}{2}, n_2 + \frac{1}{2}, \dots n_8 + \frac{1}{2}) \text{ with } \sum_i n_i \text{ even}$$

It has 240 vectors of $\text{length}^2 = 2$. In d=16 there is either a trivial extension or just 2 copies of the same. These two possibilities give rise to the 2 consistent heterotic strings. Their light spectrum contains massless vectors (from the $k_L^2=0,2$ states), the Lorentz index being carried by the right-moving part, the gauge label by the left movers. They fill either the adjoint representation of SO(32) or the one of $E_8 \times E_8$, both of dimensionality 496 (16+480).

Incidentally, one arrives at exactly the same conclusion by using a property of D=2 QFT known as fermionization (or bosonization). In D=2, one compact left-moving bosonic coordinate (like X_5) is equivalent to 2 left-moving fermionic coordinates. In our case the 16 left-moving bosons give rise to 32 left-moving fermions and SO(32) comes out very simply ($E_8 \times E_8$ only after some extra work!).

Heterotic spectra

As for Type II strings, the quantum numbers of the massless spectrum of the heterotic strings is given by multiplying the left and right-moving (Lorentz, gauge) quantum numbers:

Bosons: $[(8_v,1)+(1,496)]\times(8_v,1) = (1+28+35, 1) + (8_v, 496)$

Fermions: $[(8_v,1)+(1,496)]\times(8_c,1) = (8_s+56, 1) + (8_c, 496)$

Interestingly, for the SO(32) case the above supersymmetric spectrum coincides with the one of the SO(32) Type I string (this is no longer true for the massive states).

In conclusion, we arrived, so far, at the definition of 5 consistent (no ghost, no tachyon, no anomalies, modular invariant) string theories. They are all supersymmetric, live in D=10, and some of them can lead to chiral fermions in D=4 after compactification (= phenomenologically interesting).

Supergravity in D=11

D=10 is surprisingly close to D=11 which was known for sometime to be the maximal number of dimensions in which consistent interacting supersymmetric theories can be constructed (otherwise massless particles of spin higher than 2 are needed and put several problems).

D=11 supergravity was studied for quite sometime even before the 1984 GS revolution with the hope to solve the UV problems of quantum gravity. The bosonic part of its action looks as follows:

$$S = \frac{1}{2\kappa_{11}^2} \int d^{11}x \sqrt{-g_{11}} \left(R_{11} - \frac{1}{2} F_4^2 \right) - \frac{1}{6} \int A_3 \wedge F_4 \wedge F_4$$

Thus, in D=11, one has just two fields: the metric and a 3-form potential (with the associated 4-form field strength).

$$S_{11} = \frac{1}{2\kappa_{11}^2}\int d^{11}x\sqrt{-g_{11}}\,\left(R_{11} - \frac{1}{2}F_4^2\right) - \frac{1}{6}\int A_3 \wedge F_4 \wedge F_4$$

Let us see what this theory becomes when the 11th dimension is a circle of radius R. We proceed as before defining:

$$ds_{11}^2 = g_{\mu\nu}dx^\mu dx^\nu + e^{2\sigma}(dx^{11} + A_\mu dx^\mu)^2$$

The effective action in D=10 turns out to be:

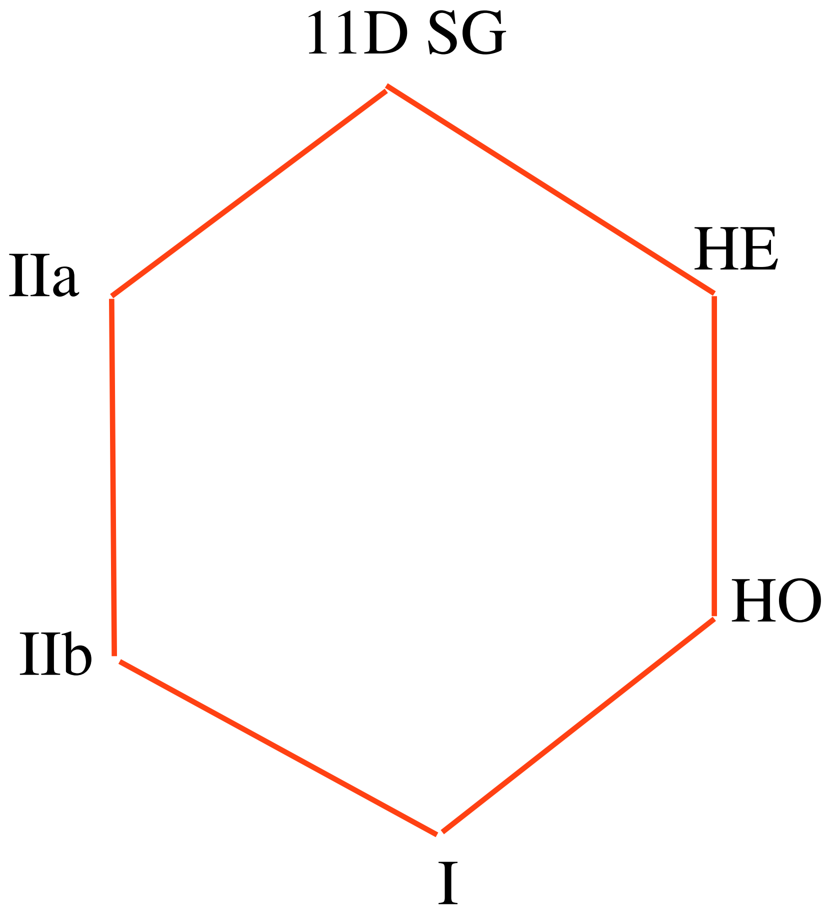
$$S_{10} = -\frac{\pi R}{2\kappa_{11}^2}\int d^{10}x\sqrt{-g_{10}}\left(2e^\sigma R_{10} + e^{3\sigma}F_2^2 + e^{-\sigma}F_3^2 + e^\sigma \tilde{F}_4^2\right) + \text{C.S. terms}$$

where F_2 is the 2-form associated with A_μ, while F_3 and F_4 follow from the dimensional reduction of the D=11 F_4 (the tilde meaning that we have added a little A_1^F_3 piece).

Thus, at the D=10 level we have the metric, a scalar, a 2-form, a 3-form and a 4-form. But this looks like a "déjà vu": it's the set of bosonic massless fields of the D=10 Type IIa superstring! Indeed the low energy actions fully coincide after some field redefinition (as it should, since D=10 supersymmetry is a very strong constraint).

This was the first indication that D=11 supergravity may have something to do with D=10 superstrings!

Let us now look at the 6 theories so far discussed and let us put them at the corners of a hexagon. A "web of dualities" appears to connect them all as different limits of one and the same (yet largely unknown) theory, called M-theory (sometimes people refer to D=11 SG as M-theory but that's just its low-energy limit). In the rest of this lecture I will mention very qualitatively the nature of this web...


11D SG
HE
IIa
HO
IIb
I

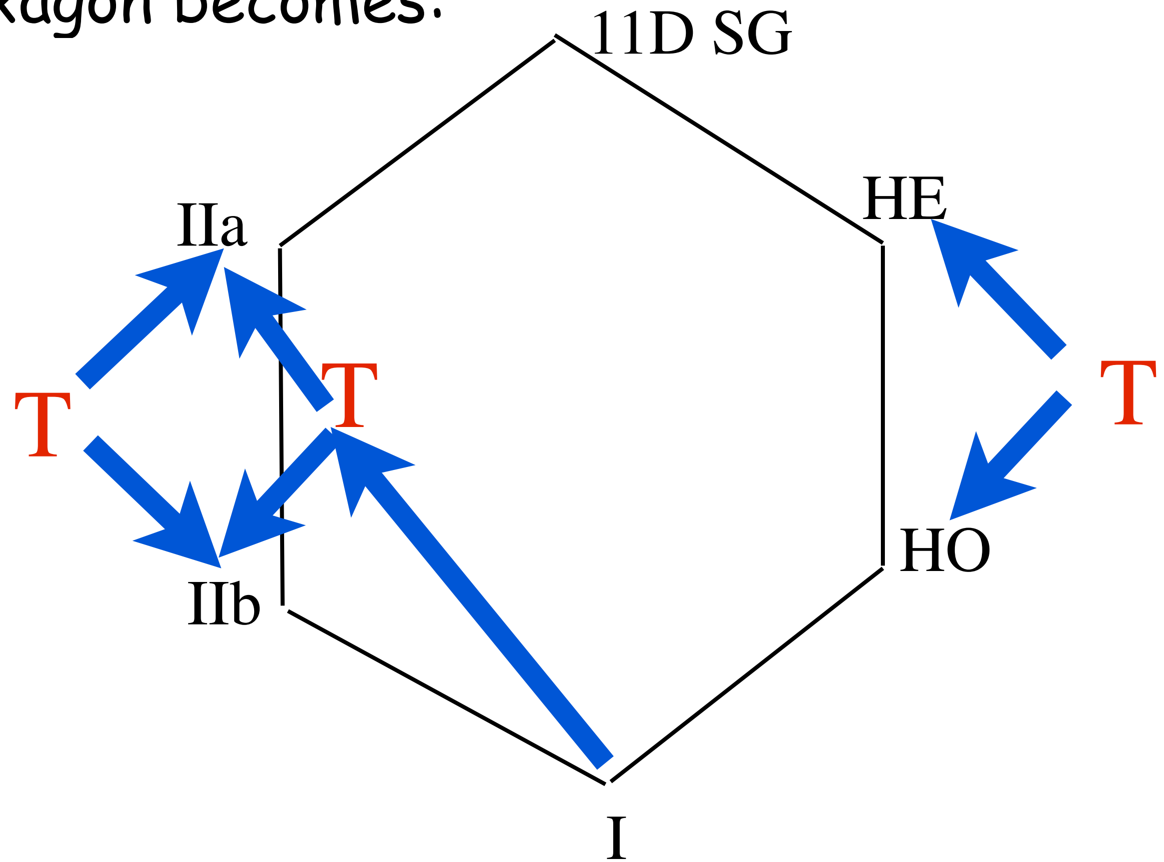
T-dualities among string theories

1. There is a T-duality connection between IIa and IIb. If we consider both theories with one compact coordinate, say X_9, and we perform a T-duality transformation we find that the odd-n RR forms of IIa go into the even-n RR forms of IIb and vice versa according to a simple rule:

$$(C_9, C_\mu)_A \to (C, C_{\mu 9})_B \; ; \; (C_{\mu\nu 9}, C_{\mu\nu\rho})_A \to (C_{\mu\nu}, C_{\mu\nu\rho 9})_B \; \ldots$$

2. There is also a T-duality connection between HO and HE. Let us compactify d of the 9 coordinates that are common to left and right movers. The consistent compactifications are now given by even-self dual Lorentzian lattices connected to one another by a non-compact "Narain" group O(d, 16+d). Unlike the isolated even self dual Euclidean lattices, these are now connected by a continuos set of consistent (but inequivalent) theories and are equivalent thanks to the discrete T-duality subgroup of O(d, 16+d).

3. There is a more subtle T-duality between IIa,b on one side and Type I on the other. That sounds a priori very surprising since Type I has an SO(32) gauge symmetry while Type II has no gauge interactions. Indeed, T duality connects Type I to Type II provided one adds to the latter a consistent set of branes (and then open strings attached to them). Our previous exagon becomes:



S-dualities among string theories

T-duality is a perturbative (although believed exact) symmetry of string theory in the sense that it holds order by order in the string coupling.

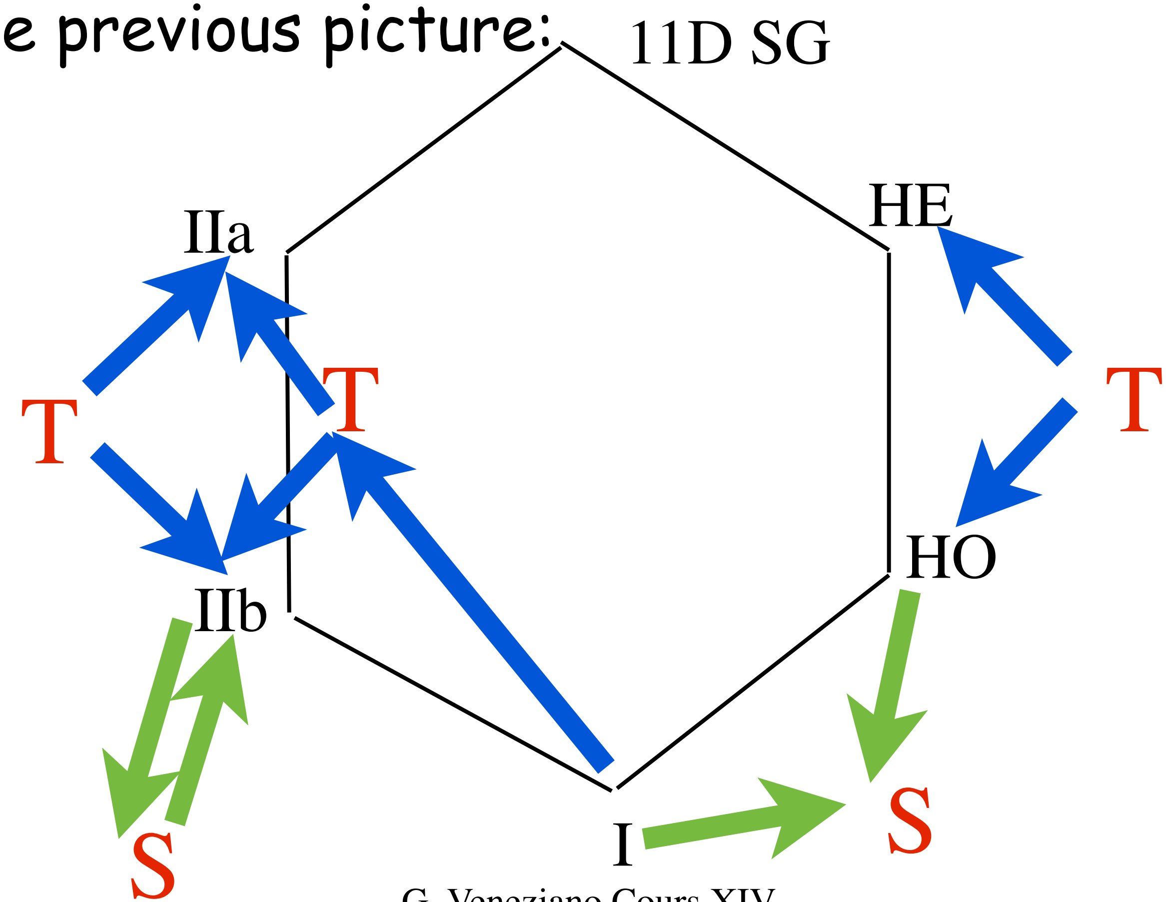
A qualitatively different symmetry, called S-duality, connects instead a strongly coupled theory to a weakly coupled one. As such checking explicitly its validity is much harder (basically one needs non-renormalization theorems due to supersymmetry in order to do so).

S-duality is a close relative of electric-magnetic duality already known in some QFT's which admit electric and magnetic charges. Since Dirac's quantization condition fixes the product of the two, large electric charge means small magnetic charge and viceversa. A Z_2 group exchanges them.

In string theory the Z_2 group is extended to an $SL(2,Z)$ group acting on $S = C_0 + i\exp(-2\Phi)$ where C_0 is a pseudoscalar.

In other words S-duality puts together in a single group the transformation $g^2 \to 1/g^2$ and a shift of C_0 meaning that C_0 is a periodic variable (Cf. the gauge coupling and the vacuum angle in QCD).

There are strong arguments suggesting that the IIb theory is self-dual and that Type I is S-dual to HO. We thus complete the previous picture:



From D=11 to D=10

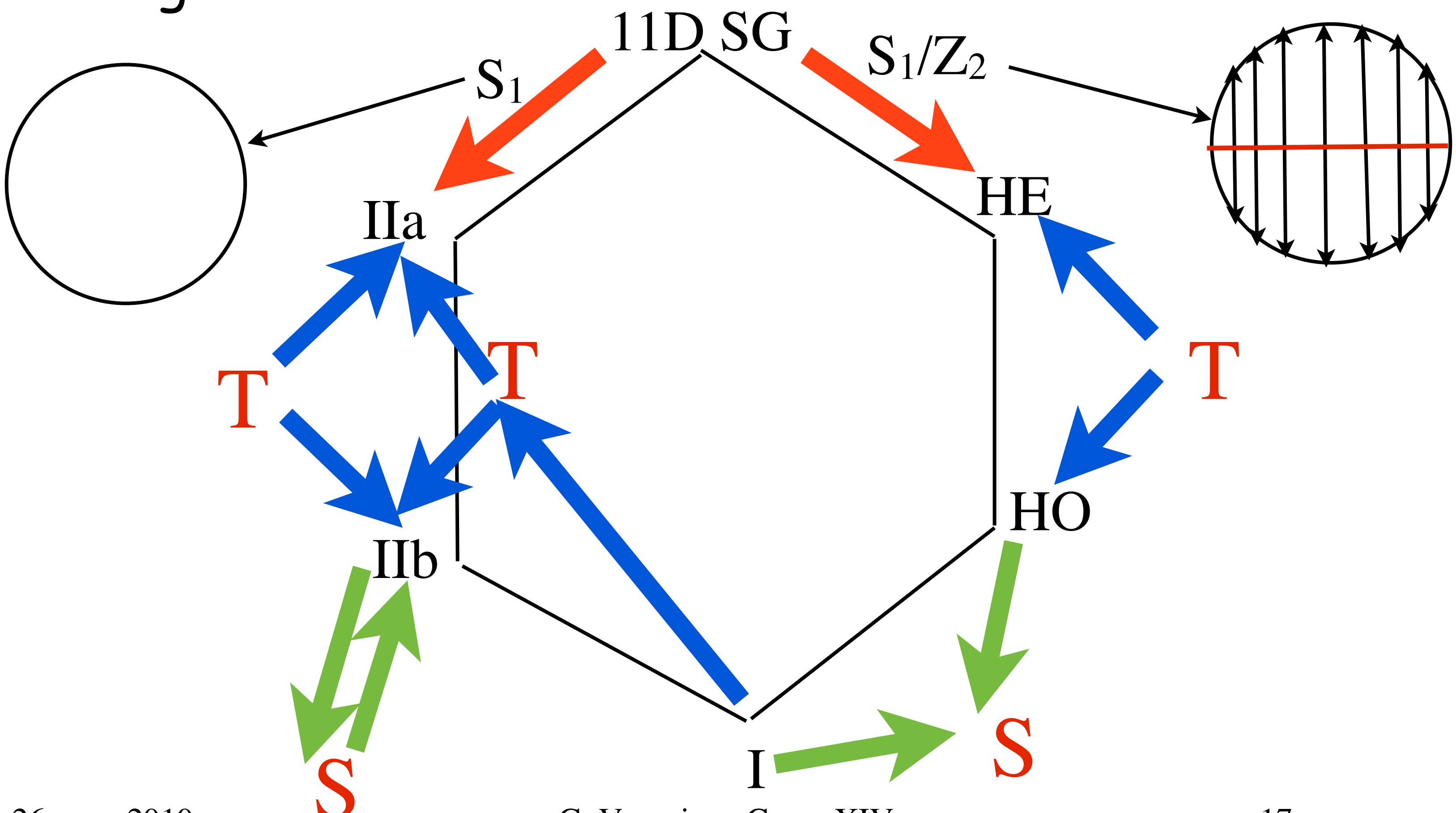
The final "miracle" is the connection between a D=11 and two D=10 theories, without use of dimensional reduction.

Recall that in the D=11 SUGRA action there was no dilaton but only a metric and a 3-form potential.

$$S_{11} = \frac{1}{2\kappa_{11}^2} \int d^{11}x \sqrt{-g_{11}} \left(R_{11} - \frac{1}{2} F_4^2 \right) - \frac{1}{6} \int A_3 \wedge F_4 \wedge F_4$$

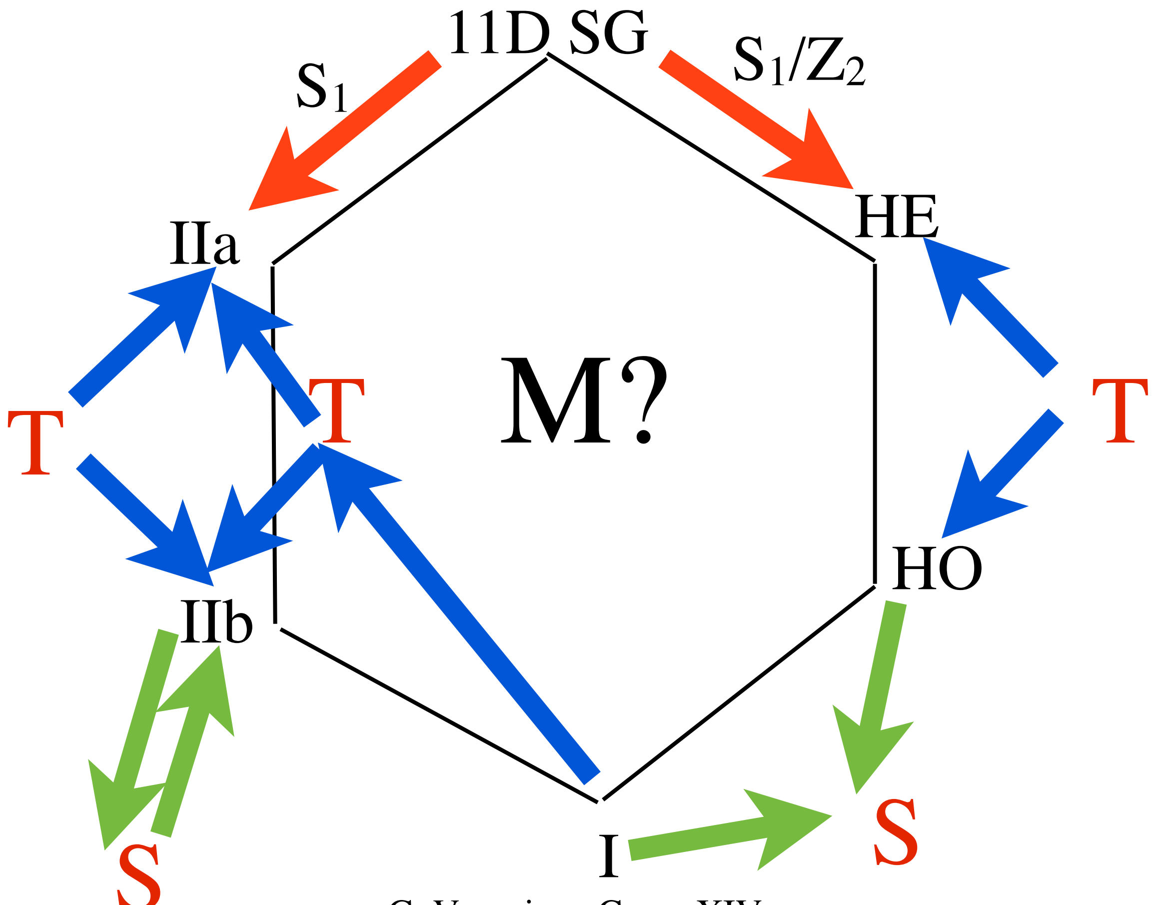
What is at work is a mixture of T and S-duality. The role of the dilaton (the string coupling) is played by the size of the 11th dimension. At weak coupling (or in perturbation theory) one does not "see" the 11th dimension but, as one goes to strong coupling, an 11th dimension opens up and, at least at low energy, the D=10 theory is easy to describe in terms of D=11 supergravity.

The direct connection is between D=11 Supergravity and either IIA theory (we had already seen some similarity) or HE. In the former case the 11[th] dimension describing the strong coupling limit is a circle S_1. In the case of HE is S_1/Z_2, i.e. basically a segment at whose ends the two E_8s of HE lie. We thus get:



Six theories in search of a Mother

Unfortunately we still don't know which is the common Mother of all these theories. It's most likely 11-dimensional but, in some corners of parameter space, it could be totally different, e.g. the QM of some large-N matrices from which spacetime itself would emerge away from those corners.



Conclusion and Outlook

This year we have gone through the history of string theory and gave a general introduction to how it developed from some simple hadron phenomenology into a huge branch of theoretical physics.

The language of QST is very new and its concepts often challenge one's intuition. As a consequence, it is not an easy subject to learn... nor to teach. Many QFT theorist are completely ignorant about QST (and many young QST theorists about QFT?).

The present formulation of QST is not as advanced as that of QFT: it resembles QFT before Feynman, with its old-fashioned perturbation theory. It may take still many decades before we arrive at a fully satisfactory formulation of QST and develop the necessary tools to solve it away from some particularly lucky situations. But the stakes can be hardly overestimated.

Next year?

Next year, unless some fantastic news come out from experiments (LHC, dark matter searches, PLANCK ...), we will continue with some applications of string theory (after giving a few more details on the post 1995 developments).

The menu is long and we will have to make some choices:

1. Black hole entropy from counting string microstates.
2. Black holes from high-energy string collisions.
3. Gauge-gravity duality of the AdS/CFT type.
4. String and brane-inspired cosmologies.
5. String and brane-inspired GUTs.
6. .....   Suggestions are very welcome!!