Particules Élémentaires, Gravitation et Cosmologie
Année 2010-'11

Théorie des cordes: quelques applications

Cours III: 11 février 2011

Résumé des cours 2009-'10: troisième partie

For a popular book on the SM and string theory:

Brian Greene, "L'Univers élégant"

(Editions Robert Laffont, 1999)

From last slide of last week.....

Up to the 1984 paper by Green and Schwarz, one knew about 3 (apparently) consistent (no ghost, no tachyon) string theories: Type I, IIA, IIB. All had SUSY.

Two of them (I and IIB) had chiral fermions and in Type I one could even add a large gauge symmetry like SO(16) (a leading candidate "GUT" for unifying SU(3)xSU(2)xU(1))

... but this is not yet the end of the story.

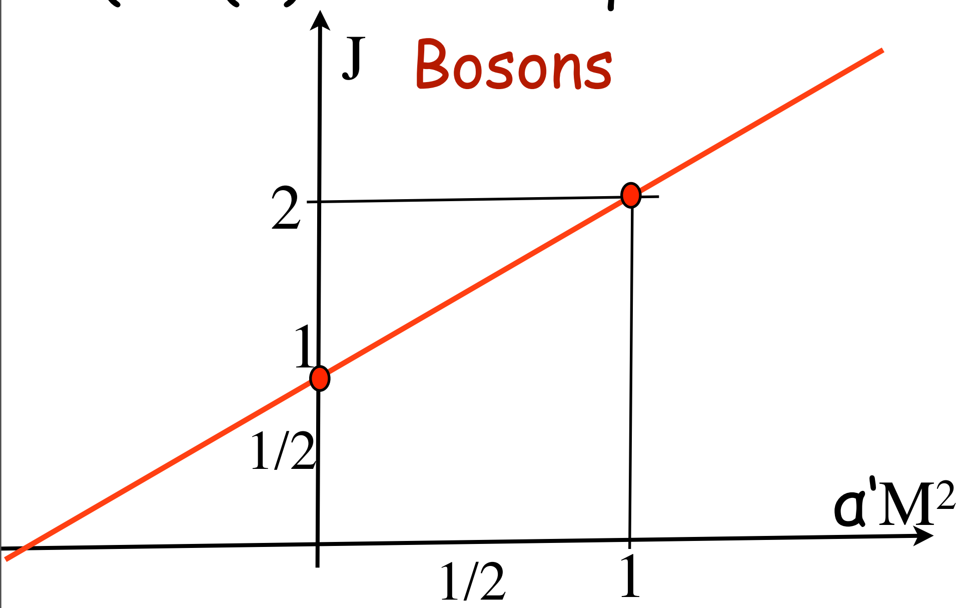
Let us first recall the massless spectra of each of these theories.

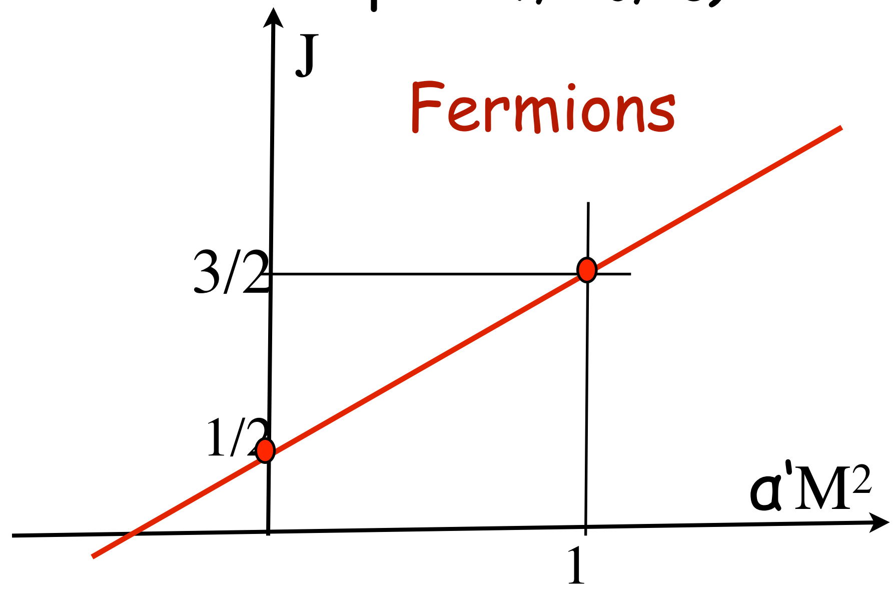
Type I (open sector)

Bosons: a massless vector with (D-2) = 8 physical components. It belongs to the 8_v rep. of SO(8).

Fermions: a Majorana-Weyl spinor in D=10. Normally a D=10 spinor has $2^{D/2} = 2^5 = 32$ components, but the MW conditions bring them down to 8. It belongs to the 8_c or 8_s rep. of SO(8) (depending on the eigenvalue of γ_{11}).

(SO(8) has 3 inequivalent 8-dimensional reps: 8_v, 8_c, 8_s)





Type IIA (non chiral)

One takes opposite chirality for left and right movers:

$(8_v+8_c)\times(8_v+8_s) = 8_v\times 8_v + 8_c\times 8_s + 8_v\times 8_s + 8_c\times 8_v =$

$(1+35_v+28)_{NS\text{-}NS} + (8_v+56_v)_{R\text{-}R} + (8_c+56_c)_{NS\text{-}R} + (8_s+56_s)_{R\text{-}NS}$

Two NS vectors lead to a scalar (the dilaton), a symmetric 2-index tensor (the graviton) and a 2-form ($B_{\mu\nu}$).

Two R-spinors give a vector C_1 and a 3-form C_3 (with 8x7x6/3! = 56 components).

The NS-R & R-NS give 2 gravitinos and 2 dilatinos of opposite chirality.

Type IIB (chiral)

$(8_v+8_c)x(8_v+8_c) = 8_vx8_v + 8_cx8_c + 8_vx8_c + 8_cx8_v =$

$(1+35_v+28)_{NS\text{-}NS} + (1+28+35_c)_{R\text{-}R} + (8_s+56_s)_{NS\text{-}R} + (8_s+56_s)_{R\text{-}NS}$

In words: NS-NS as in Type IIA. Two R-spinors give a scalar C_0, a 2-form C_2 (with 8x7/2! = 28 components) and a self-dual 4-form C_4 (with 8x7x6x5/2x4! = 35 components). The NS-R & R-NS give 2 gravitinos and 2 dilatinos of the same chirality.

Closed string sector of Type I (chiral)

It coincides with a particular subsector of Type IIB:

$(1+35_v)_{NS\text{-}NS} + (28)_{R\text{-}R} + (8_s+56_s)_{NS\text{-}R+R\text{-}NS}$

dilaton, graviton, C_2 (of GS anomaly cancellation!) and chiral fermions.

The first string revolution

End of a dream, zero-slope limit and the SS proposal	Loops in QFT and QST
The GS breakthrough, and the first revolution	Strings in non-trivial backgrounds: effective action

We first discussed the phenomenological shortcomings of string theory (in particular its softness) and how it could not stand the competition of QCD.

We then considered the zero-slope (or low-energy) limit of string theory. Gauge and gravitational interactions (as described by gauge theories and GR) emerge as effective low-energy approximations.

This motivated Scherk and Schwarz's 1974 proposal that string theory should rather be considered as an extension of the SM and of GR for the description of the elementary particles appearing in those theories, the gauge bosons, the graviton, the fermions etc.

The proposal did not find a resonance in the scientific community for about 10 years.

One problem with it was that it looked impossible to have a string theory with chiral fermions in D=4 (as demanded by the SMEP) and yet unaffected by gauge and/or gravitational anomalies.

The situation changed drastically in 1984 when M. Green and J. Schwarz found that all those anomalies cancel in Type I string theory if one took SO(32) as gauge group (see seminar #4 by PDV).

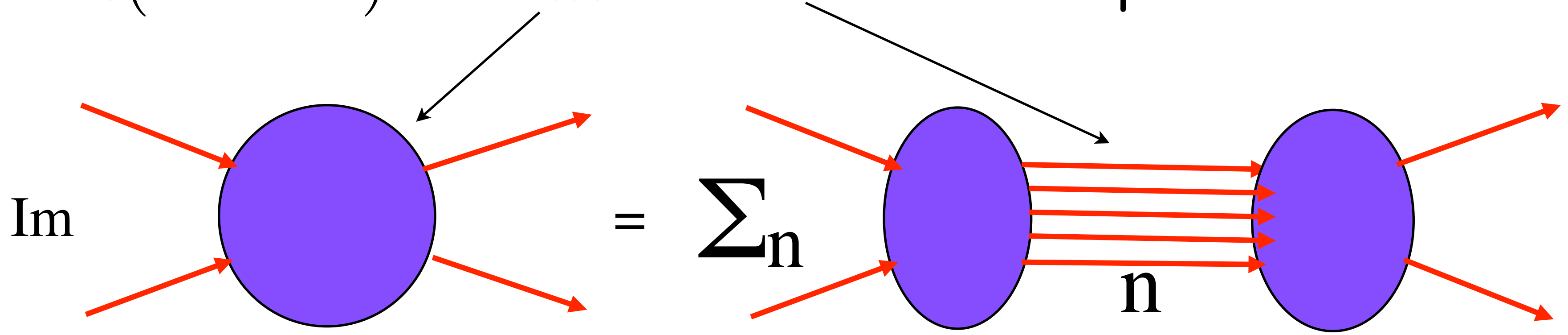
The anomalies that Green and Schwarz managed to cancel are well-known in QFT (they come at loop level) but take a "new look" in string theory since also loops do.

Loops in QFT

In QFT loops come out naturally from its formalism . Physically, loops are needed to ensure unitarity of the S-matrix. Writing S=1+iT unitarity ($S^+ S = 1$) gives:

$$i(T^\dagger - T) = 2ImT = T^\dagger T$$

In pictures:



Even if the blobs on the rhs are tree diagrams this will generate loops for the lhs.

How do loops appear in string theory? The quantum fields are NOT some spacetime fields in D = 10 but rather the string coordinates X^μ, ψ^μ and the 2D metric $\gamma_{\alpha\beta}$, all seen as functions of the two world-sheet coordinates (what is usually referred to as 1st quantization).

In QFT books, in order to go over to a relativistic quantum theory where real and virtual particle production is allowed, one abandons 1st quantization techniques and perform a so-called 2nd quantization. The coordinates x^μ become c-numbers while the fields $\varphi_i(x^\mu)$ become operators.

If we try to do the same in QST we end up with String Field Theory, a QFT involving an infinite number of spacetime fields, one for each state of the string.

There have been attempts to construct such a theory, with some interesting conceptual results, but also a lot of technical complications.

It turns out that in QST, at least in perturbation theory, one can introduce the equivalent of QFT's loops while staying all the time within a 1st quantization framework.

This amounts to working with a finite number of quantum fields in D=2, an immense simplification. How is this possible?

Consider a Feynman path integral approach to string quantization starting from a Polyakov-like action:

$$
\begin{aligned}
Z &\sim \int .. \int [d\gamma_{\alpha\beta}(\xi)][dX^\mu(\xi)][d\psi^\mu(\xi)] exp(-S_P) \\
S_P &= -\frac{T}{2} \int d^2\xi \sqrt{-\gamma}\gamma^{\alpha\beta}(\xi)\partial_\alpha X^\mu(\xi)\partial_\beta X^\nu(\xi) G_{\mu\nu}(X(\xi)) + \ldots
\end{aligned}
$$

and look first at the integral over the 2-metric $\gamma_{\alpha\beta}$.

At first sight such integral should be trivial since 2D reparametrization plus Weyl invariance should allow to gauge-fix completely $\gamma_{\alpha\beta}$. This is certainly true locally but there is a "global obstruction".

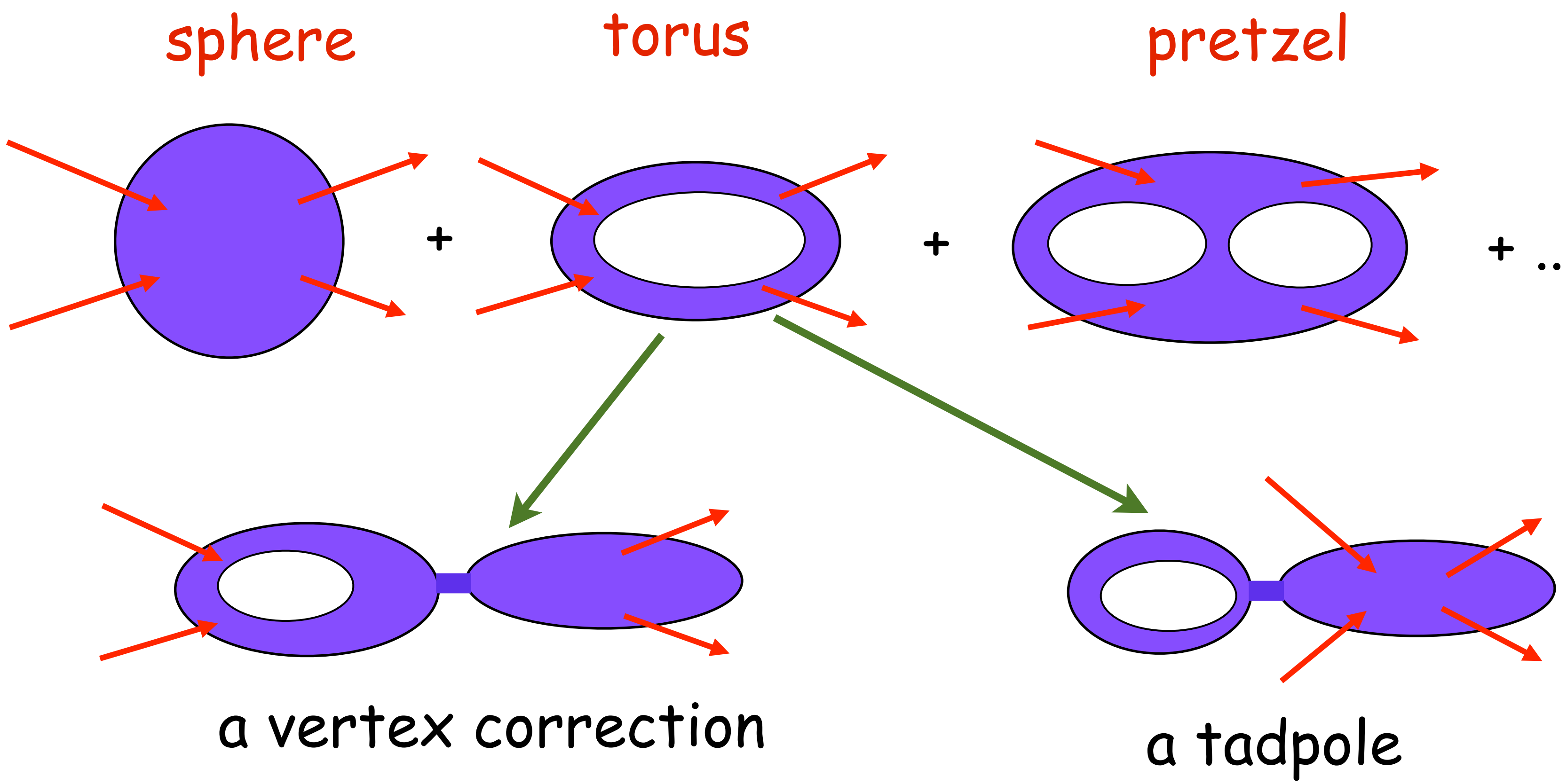
A well-known theorem states that :

$$\frac{1}{4\pi}\int d^2\xi\sqrt{-\gamma}R(\gamma) = 2(1-g)$$

where g is the genus of the 2D Riemann surface (g = 0 for the sphere, g = 1 for the torus, etc.) whose geometry is given by $\gamma_{\alpha\beta}$. Fixing globally $\gamma_{\alpha\beta}$ would mean fixing the genus of the surface!

Instead, the functional integral over the 2D metric naturally splits into a sum of functional integrals each representing Riemann surfaces of a given genus g. Precisely this sum over g corresponds to the loop expansion in QFT! QST has managed to introduce QFT's loops without invoking any 2nd quantization!

Loop expansion for closed string collisions

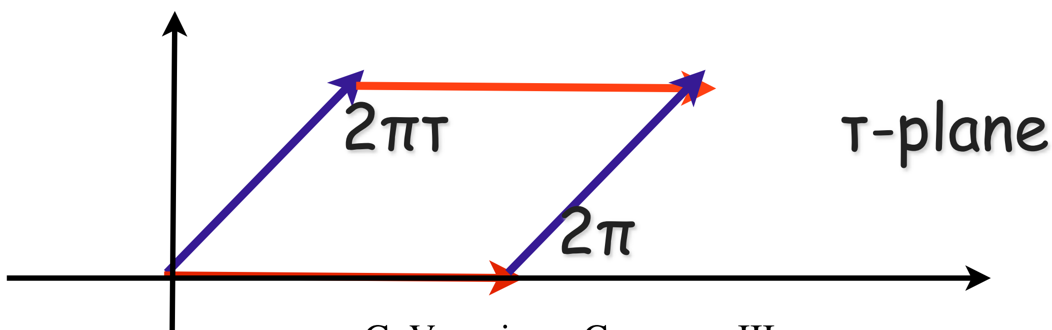


Closed strings attach at points on the Riemann surface. These are just the Koba-Nielsen variables z_i over which one had to integrate in the DRM.

Modular Invariance

Things are actually more complicated. For a given g, one has to find out what are the integration variables after gauge fixing.

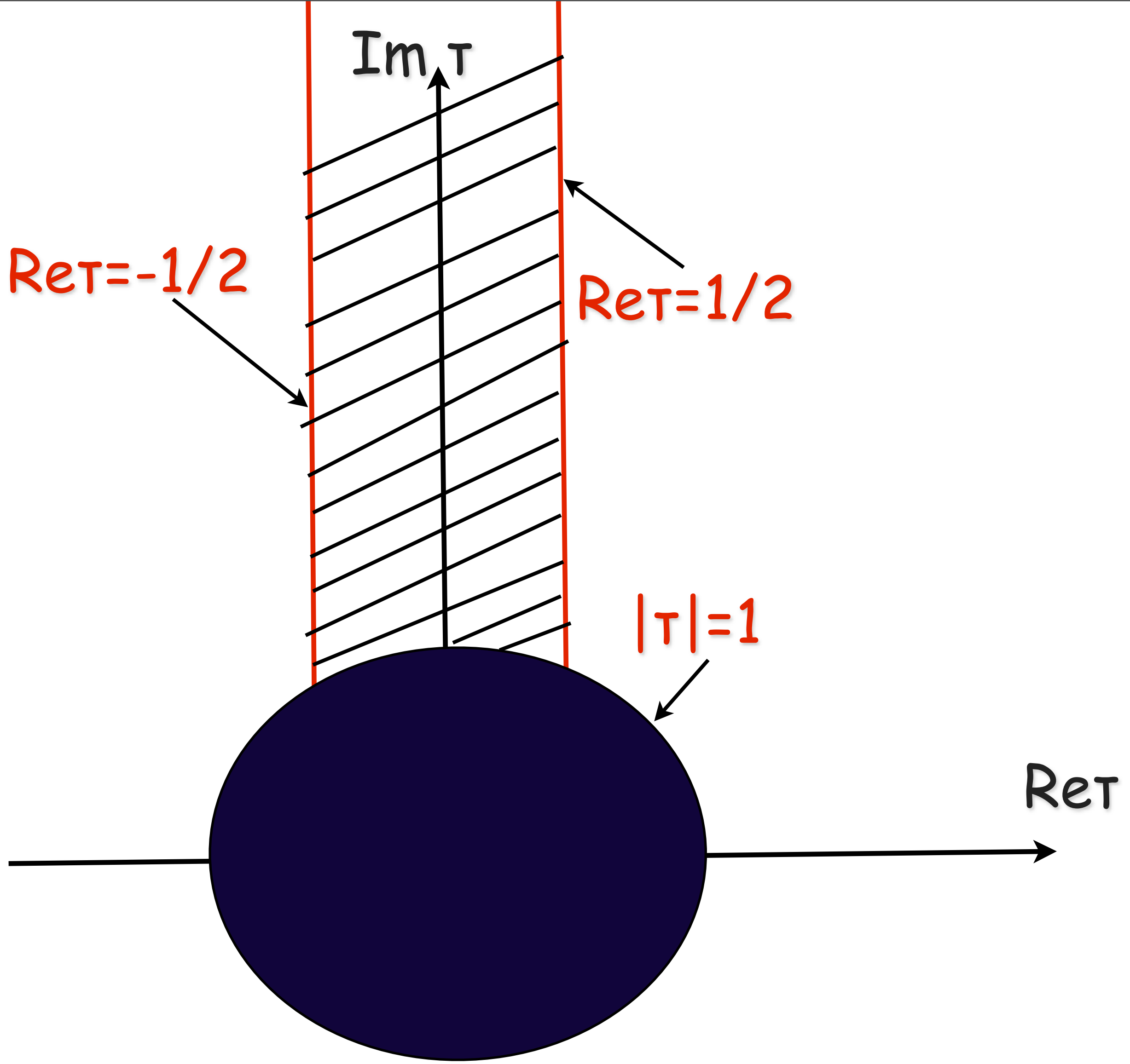
1. For the sphere there is a residual invariance under projective $O(2,1)$ transformations that allows to fix 3 KN coordinates (exactly what we had in the DRM!).
2. For $g = 1$ (torus) there is still an integration over the complex parameter τ that characterizes each torus.
3. For $g > 1$ there is an integration over $3(g-1)$ complex parameters that characterize the Riemann surface.



For the torus (one loop), there is still a discrete group of transformations that leaves the torus invariant. This is the group of modular transformations:

$$\tau \to \frac{p\ \tau + q}{r\ \tau + s} \quad ; \quad p, q, r, s \in Z \quad ; \quad ps - qr = 1$$

Such a transformation maps the same torus in the complex τ-plane an infinite number of times leading (again!) to an infinite result if we were to integrate over the whole complex plane. We should only take one region e.g. the so-called fundamental region. This region nicely avoids the point τ = 0 that turns out to be associated (in a naive QFT limit) with the UV region. This is how, technically, string theory avoids UV infinities!



Fundamental region for the torus (shaded)

Modular invariance is as essential for the consistency of string theory as Weyl and reparametrization invariance (they are all parts of the gauge invariances of ST). As it turns out, imposing modular invariance at the one-loop level eliminates the gauge and gravitational anomalies (also one-loop effects!) that the GS mechanism cancels by a brute-force calculation (see seminar #4 of PDV).

The search for consistent QSTs is therefore reduced to the problem of finding theories that respect modular invariance.

This is how the two consistent heterotic string theories have been found!

The Heterotic String

The heterotic string starts from the observation that, for closed strings, one can impose different conditions on left and right movers. What happens if we try to combine a superstring theory for right-movers with a bosonic string for left-movers?

Consistency with 2D-anomaly cancellation requires D=10 for the right movers and D=26 for the left-movers. How can we make sense of such a situation? The answer is to use the compactification idea for the 16 = 26-10 extra left-moving bosonic coordinates and to go to $O(l_s)$ compactification radii.

Consistency with modular invariance constrain the lattice of left-momenta to be euclidean, even ($p_L^2 = 4n/\alpha'$) and self-dual.

Such lattices are rare. They only exist for d=8n, but, fortunately for us, d=16!

In fact, in d=8 there is only one even self-dual lattice:

$$\Gamma_8 : (n_1, n_2, \ldots n_8) \text{ or } (n_1 + \frac{1}{2}, n_2 + \frac{1}{2}, \ldots n_8 + \frac{1}{2}) \text{ with } \sum_i n_i \text{ even}$$

It has 240 vectors of length^2 = 2 and is related to the exceptional group E_8. In d=16 there are two distinct lattices. They give rise to the 2 consistent heterotic strings. Their light spectrum contains massless vectors (from the k_L^2= 0,2 states, see next lecture), the Lorentz index being carried by the right-moving part, the gauge label by the left movers. They fill either the adjoint representation of SO(32) or the one of E_8xE_8, both of dimensionality 496 (= 2x(240+8)).

HO and HE spectra (chiral)

Quantum numbers are given by multiplying the left and right moving quantum numbers (Lorentz, gauge):

Bosons: $[(8_v, 1)+(1, 496)] \times (8_v, 1) = (1+28+35, 1) + (8_v, 496)$

Fermions: $[(8_v, 1)+(1, 496)] \times (8_c, 1) = (8_s+56, 1) + (8_c, 496)$

Interestingly, for the SO(32) case the above supersymmetric spectrum coincides with the one of the SO(32) Type I string (this is no longer true for the massive states).

In conclusion, we arrived, so far, at the definition of 5 consistent (no ghost, no tachyon, modular invariant) string theories. They are all supersymmetric, live in D=10, and some of them can lead to chiral fermions in D=4 after compactification (= phenomenologically interesting).

Bosonic strings in non-trivial backgrounds

For a string in a pure metric background we have:

$$S_G = -\frac{T}{2}\int d^2\xi\sqrt{-\gamma}\gamma^{\alpha\beta}(\xi)\partial_\alpha X^\mu(\xi)\partial_\beta X^\nu(\xi)G_{\mu\nu}(X(\xi))$$

Can other background fields can interact with the string? All we have to require is to preserve the local WS symmetries at the quantum level. Let us proceed by analogy with the point-particle case.

A charged point-particle couples naturally to a vector potential, a 1-form (without even invoking a 1D-metric):

$$S_A^{point} = q \int d\tau \; \dot{x}^\mu(\tau) \; A_\mu(x(\tau)) = q \int dx^\mu(\tau) \; A_\mu(x(\tau))$$

This action is invariant under the gauge transformation A--> A + dΛ

In perfect analogy, a string naturally couples to a 2-form $B_{\mu\nu} = -B_{\nu\mu}$ without invoking a 2D-metric:

$$S_B = -\frac{T}{2} \int d^2\xi \epsilon^{\alpha\beta} \partial_\alpha X^\mu(\xi) \partial_\beta X^\nu(\xi) B_{\mu\nu}(X(\xi))$$

with $\varepsilon^{\alpha\beta}$ the Levi-Civita symbol in D=2. This action is invariant under B --> B + dΛ where Λ is a 1-form.

This generalized to p-branes... see later.

Can we write anything else that satisfies classically the 2D local symmetries, and in particular Weyl invariance? The only possibility appears to be:

$$S_\Phi = \frac{1}{4\pi}\int d^2\xi\sqrt{-\gamma}R(\gamma)\Phi(X(\xi))$$

but only if the field $\Phi(x)$, called the dilaton, is a constant.

As already discussed:

$$\frac{1}{4\pi}\int d^2\xi\sqrt{-\gamma}R(\gamma) = 2(1-g)$$

Thus, if Φ is constant, $S_\Phi = 2\Phi(1-g)$; if it isn't, S_Φ is non-trivial and classically not Weyl-invariant.

Let's put anyway all 3 terms together and write the action for a string in a metric, antisymmetric and dilaton background:

$$S = -\frac{T}{2}\int d^2\xi\sqrt{-\gamma}\left[\partial_\alpha X^\mu \partial_\beta X^\nu \left(\gamma^{\alpha\beta}G_{\mu\nu} + \frac{\epsilon^{\alpha\beta}}{\sqrt{-\gamma}}B_{\mu\nu}\right) - \frac{1}{2\pi T}R(\gamma)\Phi\right]$$

Under what conditions for the background fields G, B, and Φ can we satisfy the conditions of 2D-rep. and Weyl invariance at the quantum level?

This is, in general, a highly non trivial problem. We know one solution: Minkowski spacetime, vanishing B, and constant Φ, provided that D takes a critical value (D=26, 10).

This is the string we have been discussing so far with just one small additional insight.

When inserted in the (Euclidean) path integral the above action will weight the contribution of a Riemann surface of genus g with a factor $\exp(-2\Phi(1-g))$ hence with an extra factor $\exp(2\Phi)$ for each extra string loop. Therefore $\exp(2\Phi)$ plays, in QST, the same role that α plays in QED. It is the loop-counting parameter.

In order to look for more general solutions we resort to perturbation theory around the "trivial" backgrounds.

We expand the background fields around a particular point x. This gives terms containing derivatives of the backgrounds and which are cubic, quartic etc. in the string coordinates.

In terms of a 2-dimensional field theory we go from a free theory to an interacting one where the effective coupling is l_s/L, with L the typical length scale of the geometry*) . New contributions to the anomaly will come as a power expansion in $(l_s/L)^2 \sim \alpha'$.

This method is referred to as the α′ expansion. The conditions for having no anomaly are written as the vanishing of some β-functions (by analogy with QFT).

*) $l_s \equiv \sqrt{2\alpha' \hbar}$ is the so-called string length parameter a fundamental length characterizing QST.

A non trivial calculation leads to:

$$\beta^{\Phi} = \frac{D - D_c}{3} + l_s^2 \left(\partial_\mu \Phi \partial^\mu \Phi - \frac{1}{2} D_\mu D^\mu \Phi - \frac{1}{24} H_{\mu\nu\rho} H^{\mu\nu\rho} \right) + O(l_s^4) = 0$$

$$\beta^{G}_{\mu\nu} = l_s^2 \left(R_{\mu\nu} + \frac{1}{4} H_{\mu\rho\sigma} H_\nu^{\ \rho\sigma} - 2 D_\mu D_\nu \Phi \right) + O(l_s^4) = 0$$

$$\beta^{B}_{\mu\nu} = l_s^2 \left(D^\rho H_{\mu\nu\rho} - 2 \partial^\rho \Phi H_{\mu\nu\rho} \right) + O(l_s^4) = 0 \quad ; \quad H_{\mu\nu\rho} = \partial_\mu B_{\nu\rho} + \text{cyclic}$$

We can now see the meaning of $D=D_c$. If $D \neq D_c$, there is no solution with nearly constant backgrounds.

The effective action of QST

A very interesting property of the β-function equations is that they correspond to the eom that follow from an effective action . Up to the order we have considered:

$$\Gamma_{eff} = -\left(\frac{1}{l_s}\right)^{D-2} \int d^D x \sqrt{-G} e^{-2\Phi} \left[\frac{4(D-D_c)}{3l_s^2} + R(G) - 4\partial_\mu \Phi \partial^\mu \Phi + \frac{1}{12} H^2 + \ldots\right]$$

1. The dilaton appears through an overall factor multiplying something that can only depend on its derivatives. This is as expected since, if Φ is constant, the only dependence on Φ must be an overall factor exp (-2Φ(1-g)).
2. Γ_{eff} contains no arbitrary dimensionless parameters and just one dimensionful one, l_s.
3. Γ_{eff} is general covariant and also invariant under B--> B+dΛ. Indeed, B only enters through its field strength H = dB.

The two meanings of Γ_{eff}

The effective action actually has two distinct meanings.

1. It generates (as eom) the conditions to be satisfied by the background fields in order to preserve the 2D local symmetries of string theory.

2. Γ_{eff} can be used to compute the couplings of various massless particles and their scattering amplitudes as an expansion in powers of energy (Cf. zero-slope limit).

It is amazing that these two concepts get related in string theory.

A theory of gravity but not Einstein's!

In D dimensions, the analogue of the Einstein-Hilbert action takes the form:

$$\frac{1}{\hbar} S_{EH} = \left(\frac{1}{l_P}\right)^{D-2} \int d^D x \sqrt{-g(x)} \left(\Lambda - \frac{1}{2} R(g)\right) \quad ; \quad 8\pi G_N \hbar \equiv l_P^{D-2}$$

while in QST we found:

$$\Gamma_{eff} = -\left(\frac{1}{l_s}\right)^{D-2} \int d^D x \sqrt{-G} e^{-2\Phi} \left[\frac{4(D - D_c)}{3 l_s^2} + R(G) - 4\partial_\mu \Phi \partial^\mu \Phi + \frac{1}{12} H^2 + \dots\right]$$

QST gives a scalar-tensor theory of a Jordan-Brans-Dicke kind! Like JBD can be tested and contradicted!

The two expansions of Γ_{eff}

We have seen how quantization produces potential anomalies that have a natural expansion in powers of l_s.

We have also seen that integration over the 2D metric produces another expansion in powers of $\exp(2\Phi)$.

Γ_{eff} encodes both effects and thus has a double expansion:

$$\begin{aligned}\Gamma_{eff} &= -\left(\frac{1}{l_s}\right)^{D-2}\int d^Dx\sqrt{-G}e^{-2\Phi}\left[\frac{4(D-D_c)}{3l_s^2}+R(G)-4\partial_\mu\Phi\partial^\mu\Phi+\frac{1}{12}H^2+O(l_s^2)\right]\\ &+ \left(\frac{1}{l_s}\right)^{D-2}\int d^Dx\sqrt{-G}\,[\ldots]+O(e^{2\Phi})\end{aligned}$$

One expansion has a QFT analogue. The other does not and has the virtue of making the former much better defined!