Particules Élémentaires, Gravitation et Cosmologie
Année 2010-'11

Théorie des cordes: quelques applications

Cours VII: 25 février 2011

Transplanckian scattering in QST:
I. Motivations & classical considerations

Motivations

We have seen that it is conceivable for QST to provide a microscopic interpretation of BH entropy.

Q: Would this completely solve the information puzzle?

A: Not really! It does not tell us HOW information is preserved and encoded.

If the final state contains the information it cannot be exactly thermal:

Q: How close is it to a thermal Hawking spectrum?

Q: What happens to conserved global QN? Can baryon number be conserved in such a process?

GE as a theoretical Lab.

As in the early days of QM and Relativity we may get information on important conceptual issues by considering Thought (Gedanken) Experiments (GE).

For the problem at hand the simplest GE appears to be the transplanckian-energy (TPE) collision of two light (massless) strings initially prepared in a pure state $|a,t_0\rangle$.

According to QM a state evolves in time according to the action of a unitary operator U. A pure state remains pure:

$$|a,t_0\rangle \to U(t_0,t)|a,t_0\rangle = e^{-iH(t-t_0)}|a,t_0\rangle = |a,t\rangle$$

$$\langle b,t_0|a,t_0\rangle = 0 \Rightarrow \langle b,t|a,t\rangle = 0$$

For t_0 (t) going to $-\infty$ ($+\infty$) U becomes the S-matrix.

Why transplanckian?

In string theory we do not expect BHs to exist below a critical mass that we now rename M_{th}:

$$M_{th} \equiv g_s^{-2} M_s = g_s^{\frac{6-2D}{D-2}} M_D$$

This scale is parametrically larger than M_D (the Planck mass in D-dimensions) if the string coupling g_s is small. We shall consider TPE scattering of massless superstrings at very small string coupling.

This is still a highly non-trivial problem since loop (g_s) corrections turn out to be enhanced at high energy (as they are at large M in the BH-entropy problem).

A phenomenological motivation for studying TPE collisions?

We may hope of finding signatures of string/quantum gravity @ LHC:

- In KK models with large extra dimensions;
- In brane-world scenarios; in general:
- If the true Quantum Gravity scale is O(few TeV)

NB: In the most optimistic situation the LHC will be quite marginal for producing BH, let alone semiclassical ones

Q: Can there be some precursors of BH behaviour even below the expected BH-production threshold?

Example of large extra dimensions

"Large"(i.e. sub-mm size) extra dimensions are phenomenologically allowed if only gravity feels them. Upon compactification from D=10 to D=4:

$$\Gamma_{eff} = -l_s^{-8}\int d^{10}x\sqrt{-G_{10}}e^{-2\Phi}\left[R(G_{10})+\ldots\right]$$
$$\sim -V_6\, l_s^{-8}e^{-2\Phi}\int d^4x\sqrt{-G_4}\left[R(G_4)+\ldots\right]$$

Comparing with EH action we now get:

$$l_P^2 \sim g_s^2\frac{l_s^8}{V_6}\ ;\ M_P^2 \sim g_s^{-2}\,\frac{V_6}{l_s^6}\,M_s^2\ ;\ V_6 \equiv \int d^6x\sqrt{G_6}$$

For a given value of M_P we can lower the string and the true quantum-gravity scale M_{10}. Reason: Newton's law changes below R_c and gravity becomes strong at $r \gg l_P$!

Brief review of GR collapse criteria (D=4)

There are many analytic as well as numerical GR results on whether some given initial data should lead to gravit.[al] collapse or to a completely dispersed final state.

The two phases would be typically separated by a critical hypersurface in the parameter space of the initial states.

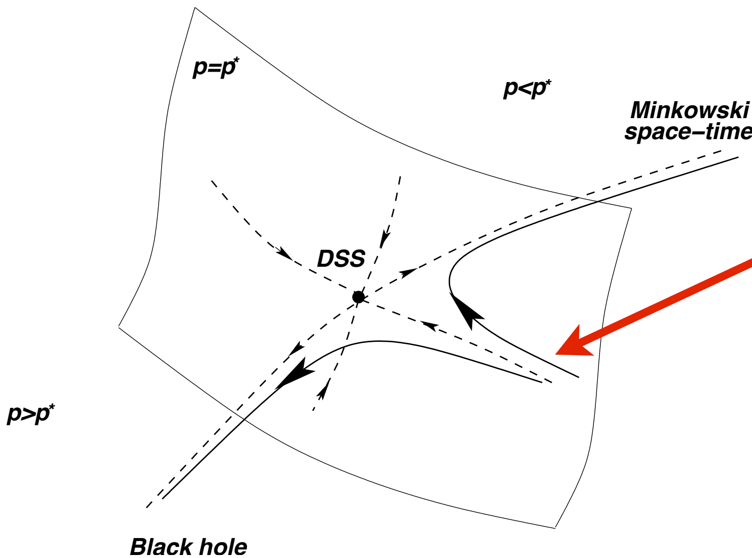


Figure 1: Phase space picture of the critical gravitational collapse.

from C. Gundlach's review ('02)

The approach to criticality resembles that of phase transitions (order, crit. exp. ...)

For pure gravity Christodoulou & Klainerman ('93) have found a region on the dispersion side of the critical surface;

Regions on the collapse side have been found for spherical symmetry by Christodoulou ('91, ...) and, numerically, by Choptuik and collaborators ('93, ...'09);

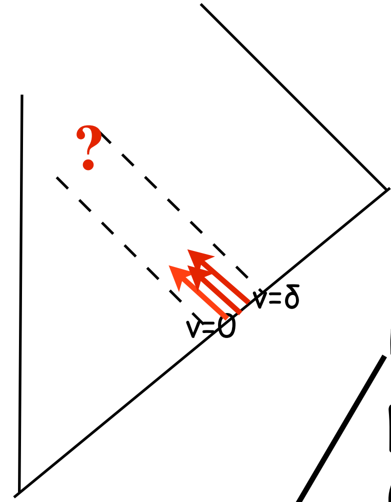
In 0805.3880, Christodoulou identified another such region in which a lower bound on (incoming energy)/(unit adv. time) holds uniformly over the full solid angle; (see also Klainerman & Rodnianski, 0912.5097, 1002.2656).

In 0908.1780 Choptuik and Pretorius have obtained new numerical results for a highly-relativistic axisymmetric situation.

A useful (sufficiency) criterion for collapse is the identification of a Closed Trapped Surface (CTS) at a certain point in the system's evolution.

D. Christodoulou, gr-qc 0805.3880
(slightly simplified)

Penrose diagram
(see 2009 course)



incoming GW-energy (G=1)
per unit advanced time
(v ~ t+r) & solid angle

$$\text{If} \quad M(\theta, \phi, \delta) = \int_0^{\delta} dv \frac{d\mathcal{M}(v, \phi, \delta)}{dv\ dcos\theta\ d\phi} \geq \frac{k}{8\pi} \quad \text{for all} \quad \theta, \phi$$

then a CTS forms with $R \geq k - O(\delta) \quad (\text{provided } R > 0)$

Classical expectations

based on the construction of

Closed Trapped Surfaces

in two-body scattering

(DC's criterion not so useful for this problem)

The Aichelburg-Sexl metric (D generic)

The Aichelburg-Sexl (AS) metric is the shock-wave metric generated by a point-like massless source carrying an energy E.

It can be obtained (an interesting "exercise") by boosting the Schwarzschild metric along, say, the z-direction with a boost parameter γ, and by letting γ -->∞, M--> 0 in such a way as to keep the energy E = γ M fixed.

In suitable coordinates it reads (u = t-z, v = t +z):

$$ds^2 = -dudv + \phi(\mathbf{x})\delta(u)du^2 + d\mathbf{x}^2 \ ; \ \Delta_T \phi(\mathbf{x}) = -16\pi G_D E \delta^{D-2}(\mathbf{x})$$

In D≠ 4: $$\phi(\mathbf{x}) \sim \frac{G_D E}{(D-4)} \, \mathbf{r}^{4-D} \ ; \ r = \sqrt{\mathbf{x}^2}$$

In D= 4: $$\phi(\mathbf{x}) = -4GE \log(\mathbf{x}^2)$$

(Generalized) AS metrics

A generalized AS metric corresponds to a massless source whose total energy E is spread over the transverse plane while it is still a δ-function in the direction of motion (a "pancake"). It is also the metric of a beam of massless particles moving in the same direction and with the same $x^{\pm} = (v,u)$. In the original AS coordinates it reads:

$$ds^2 = -dudv + \phi(\mathbf{x})\delta(u)du^2 + d\mathbf{x}^2 \ ; \ \Delta_T \phi(\mathbf{x}) = -16\pi G_D \rho_s(\mathbf{x})$$

which can be easily solved since we know the Green's function. In other coordinates it becomes free of δ-functions:

$$ds^2 = -dUdV + H_{ik}H_{jk}dX^i dX^j \quad ; \quad H_{ij} = \delta_{ij} + \frac{1}{2}\nabla_i \nabla_j \phi(\mathbf{X}) U \Theta(U)$$

Geodesics in GAS metrics

Modulo some subtleties in dealing with δ and θ-functions, it is quite straightforward to compute the trajectories of massless test particles in a GAS metric. The main features of these geodesics are:

1. A deflection making initially parallel geodesics converge (lensing!). Deflection angles are related to gradients of ϕ
2. A shift (jump) in v (with no shift in u) controlled by ϕ itself.
3. Amusing result: for a homogeneous beam of size L all parallel geodesics with a fixed v and b < L converge at the same space-time point after hitting the shock wave: a perfect gravitational lens!

Collapse criteria for the collision of two shock waves

Consider two GAS shock waves colliding head on. One is at $u = t - z = 0$ (moving to the right), the other at $v = t + z = 0$ (moving to the left). They collide at $t = z = 0$. At $t < 0$ the metric is simply a superposition of the two GAS metrics:

$$ds^2 = -dU\,dV + \left[H^{(1)}_{ik} H^{(1)}_{jk} + H^{(2)}_{ik} H^{(2)}_{jk} - \delta_{ij}\right] dX^i dX^j$$

At $t > 0$ the general problem becomes very difficult (the infinite, homogeneous wavefront case can be reduced to quadratures). However, we can use the above expression to find out whether a CTS can be constructed at $t = 0^-$.

This is the method followed by Eardley-Giddings (for AS) and by Kohlprath & GV (for GAS).

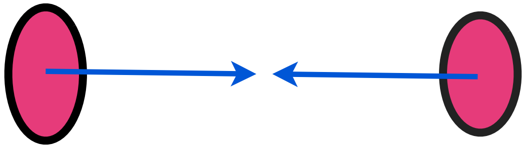
Small sample of results

- Point-particle collisions:

1. b=0: Penrose ('74) : $M_{BH} > E/\sqrt{2} \sim 0.71E$
2. b≠0: Eardley & Giddings ('02), one example:

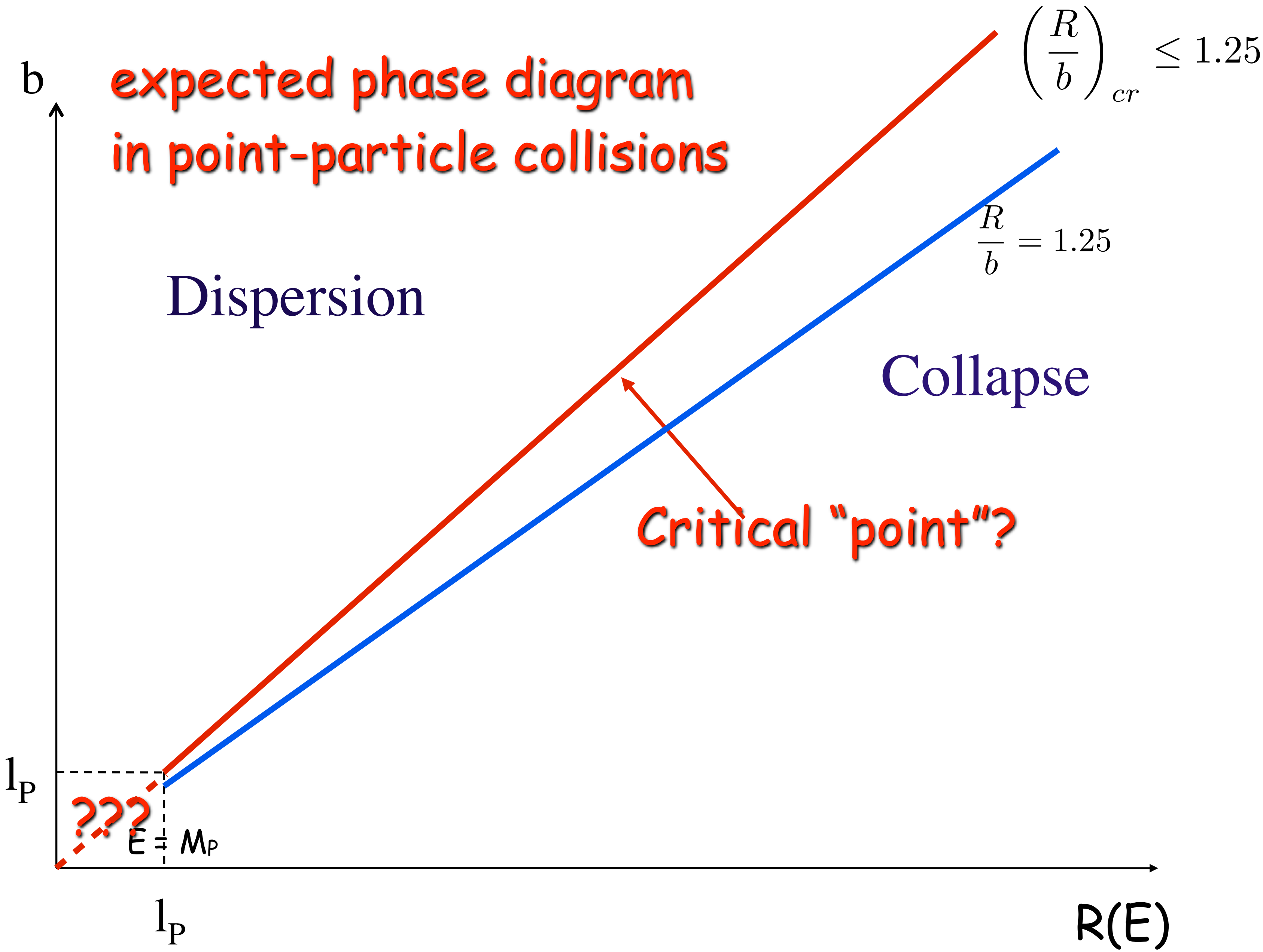
$$\left(\frac{R}{b}\right)_{cr} \leq 1.25 \qquad (R = 2G\sqrt{s} = 4GE_1 = 4GE_2)$$

➢ Extended sources:



- Kohlprath & GV ('02), one example: central collision of 2 homogeneous null discs of radius L:

$$\left(\frac{R}{L}\right)_{cr} \leq 1$$


expected phase diagram
in point-particle collisions
b
$\left(\frac{R}{b}\right)_{cr} \leq 1.25$
$\frac{R}{b} = 1.25$
Dispersion
Collapse
Critical "point"?
l_P
???
$E = M_P$
l_P
R(E)

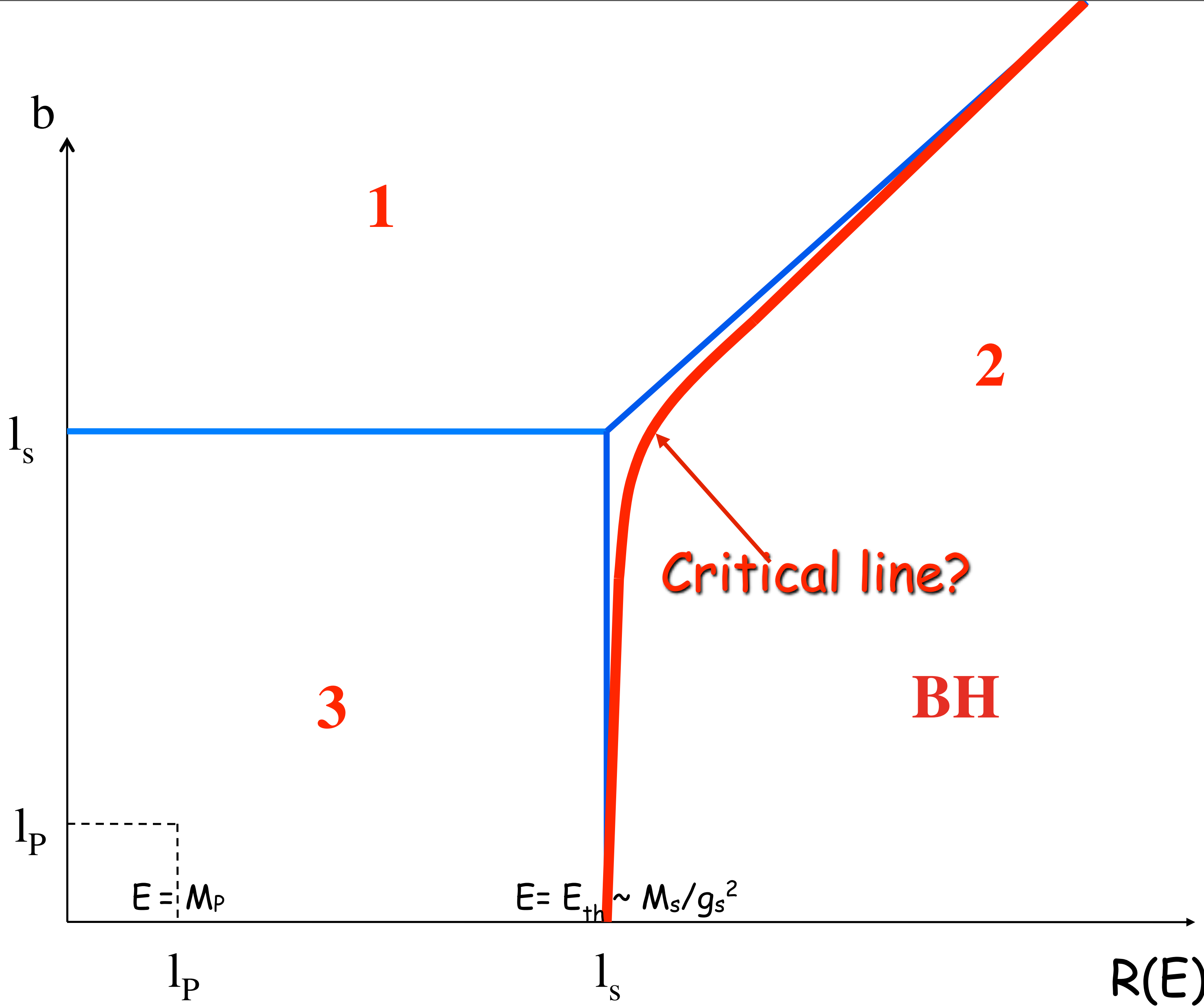
3 broad-band regimes in transplanckian superstring scattering

The string length parameter l_s plays the role of the beam size!

3 length scales: b, R and l_s =>

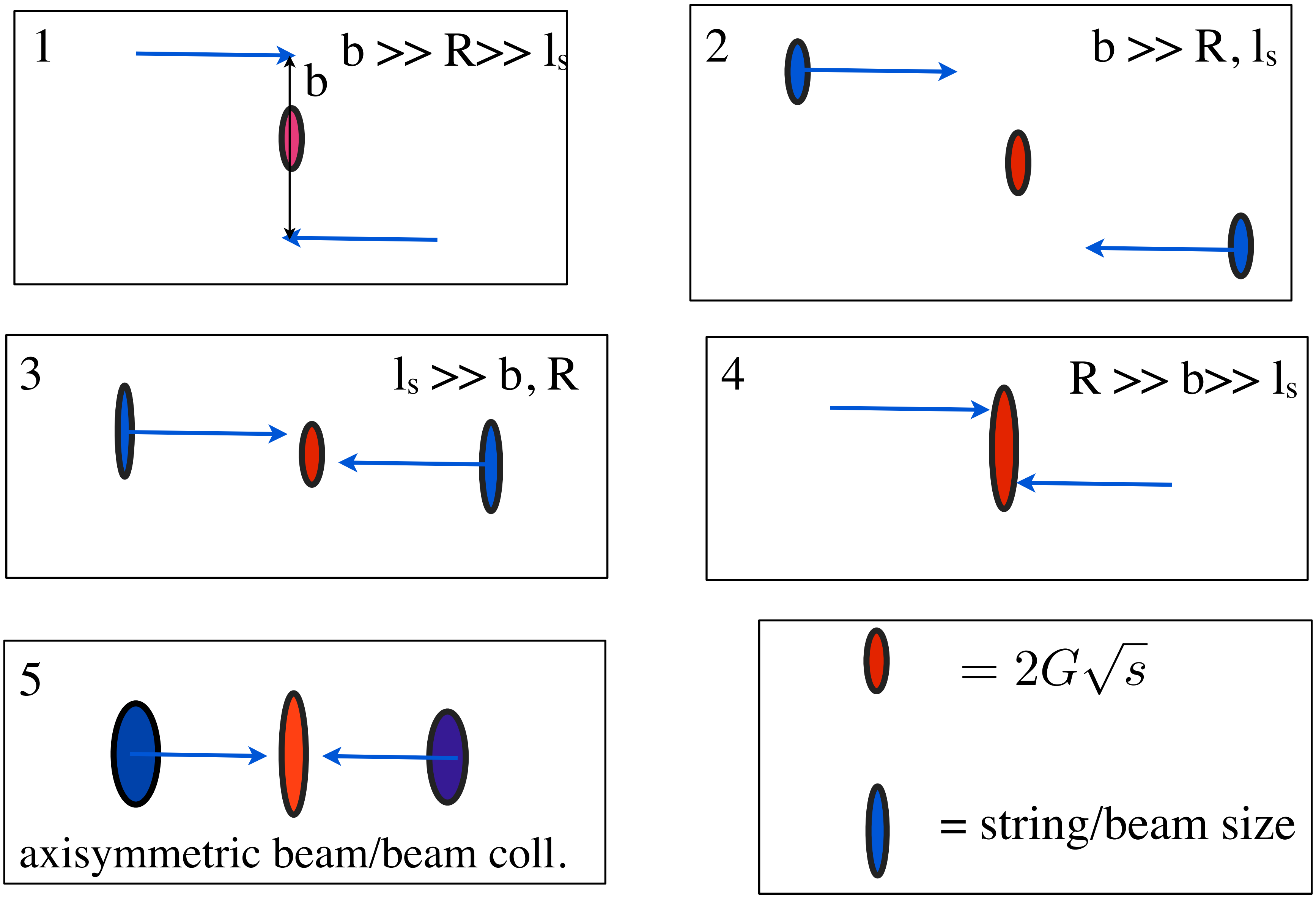
1) Small angle scattering ($b \gg R, l_s$)
2) Large angle scattering ($b \sim R > l_s$), collapse ($b, l_s < R$)
3) Stringy ($l_s > R, b$)

They will become ~ 6 narrow-band regimes



b
1
2
l_s
Critical line?
3
BH
l_P
$E = M_P$
$E = E_{th} \sim M_s/g_s^2$
l_P
l_s
R(E)

Various regimes in string-string collisions



The quantum problem

We can prepare pure initial states that correspond, roughly, to the classical data (J~bE).

- Does a unitary S-matrix (evolution operator) always describe the evolution of the system ?
- If yes, does such an S-matrix develop singularities as one approaches a critical (parameter-space) surface?
- If yes, what happens in its vicinity? Does the nature of the final state change as one goes through it?
- Is there a relation between the classical and quantum critical surfaces?
- What happens to the final state deep inside the collapse region? Does it resemble at all Hawking's thermal spectrum for each initial pure state?