Particules Élémentaires, Gravitation et Cosmologie
Année 2010-'11

Théorie des cordes: quelques applications

Cours XII: 18 mars 2011

Cosmology, inflation, and string theory

Hot Big Bang cosmology

Einstein's equations, together with the cosmological principle (assumption of a homogeneous, isotropic Universe at large scales) and present observations (e.g. the redshift), lead to a very simple model known as Hot Big Bang (HBB) cosmology.

Its geometry is described by the Friedmann-Lemaître-Robertson-Walker (FLRW) metric:

$$
\begin{aligned}
ds^2 &= g_{\mu\nu} dx^\mu dx^\nu = -dt^2 + a^2(t) \left[\frac{dr^2}{1 - K r^2} + r^2 d\Omega^2 \right] \\
d\Omega^2 &= d\theta^2 + sin^2\theta d\phi^2 \quad ; \quad K = 0, \pm 1
\end{aligned}
$$

It contains a scale-factor a(t), telling us how physical distances depend on (cosmic-proper) time, and a discrete parameter (K = 0, 1, -1) giving at any given time the spatial geometry (flat, closed, open) with curvature ${}^{(3)}R \sim K/a^2(t)$.

a(t) is related to the redshift by $(1+z) = a(t_0)/a(t_s)$. Its evolution is determined by the energy & pressure content of the universe via the two Friedman equations:

$$H^2 + \frac{K}{a^2} = \frac{8\pi G}{3}\rho \; ; \; \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) \qquad H(t) \equiv \frac{\dot{a}}{a}$$

implying: $\dot{\rho} = -3H(\rho + p) = -3H\rho(1+w) \;\; ; \;\; w \equiv \frac{p}{\rho}$

For standard matter with $\rho + 3p > 0$ this leads to a scale factor that goes to zero at a finite time in our past, conventionally called t=0.

At t=0, curvature and energy density diverge, forcing the physical interpretation of t=0 as the beginning of time.

Critical density and fractions

Introducing $$\rho^{(cr)} \equiv \frac{3H^2}{8\pi G} = \sum_i \rho_i + \rho_K \ ; \ \rho_K = -\frac{3K}{8\pi G a^2}$$

$$\Omega_i \equiv \frac{\rho_i}{\rho^{(cr)}} \qquad\qquad \dot{\rho}_i = -3H\rho_i(1+w_i)$$

The 1st Friedman equation:

$$H^2 + \frac{K}{a^2} = \frac{8\pi G}{3}\rho = \frac{8\pi G}{3}\sum_i \rho_i$$

can be rewritten in the simple form:

$$\Omega \equiv \sum_{i \neq K} \Omega_i = 1 - \Omega_K$$

NB: A spatially flat Universe is equivalent to $\Omega = 1$

Successes of HBB cosmology

1. The cosmic microwave background

(Penzias and Wilson 1965)

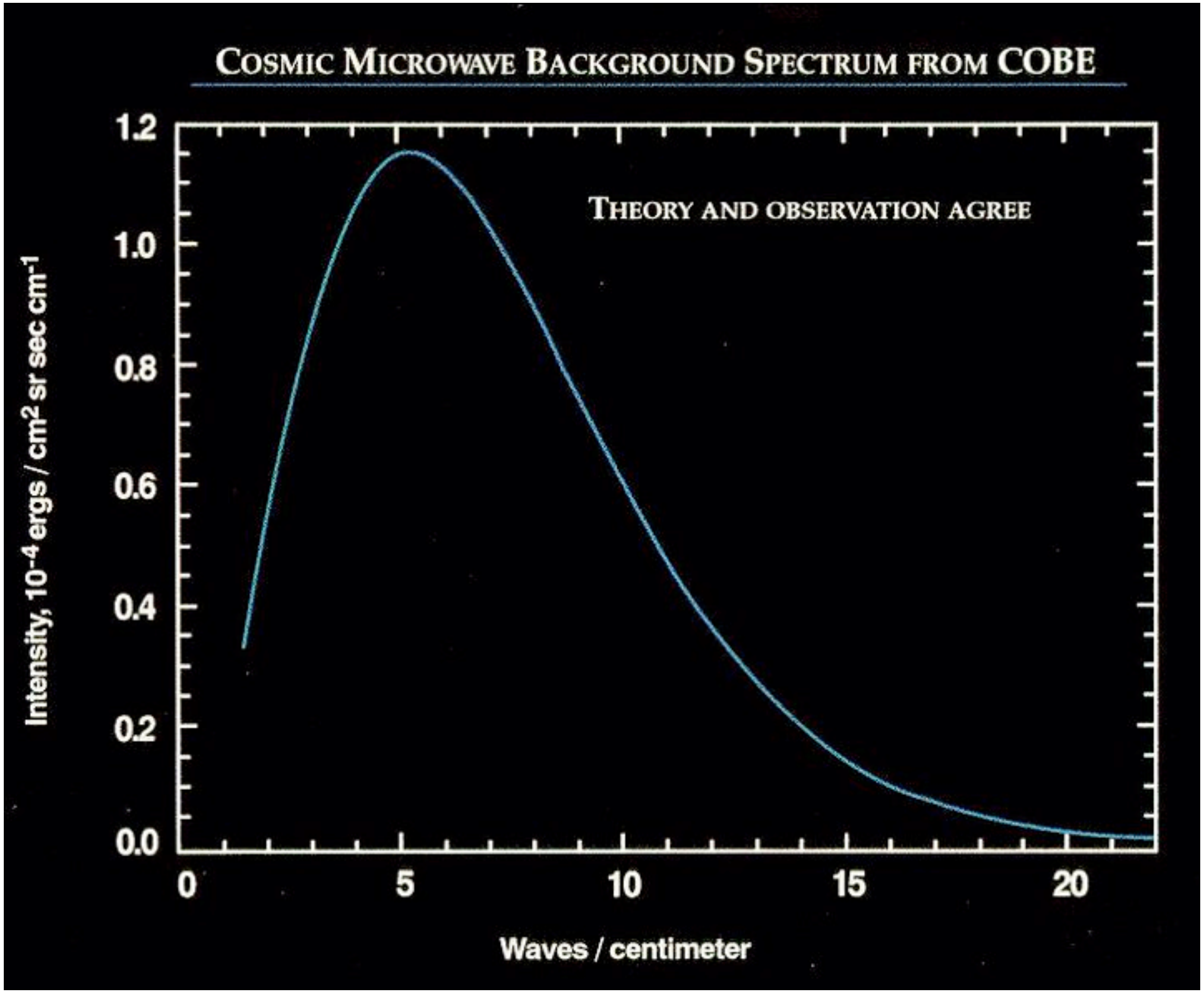
Since the 1940s, Gamow and coll. had realized that the Universe should now be filled with a black-body spectrum of electromagnetic radiation.

The first theoretical estimate (~1950) for the present temperature was 5K in quite good agreement with the first determination of 3.5±1.0 K.

Today, the CMB spectrum is the best Planck spectrum known in Nature. Its average temperature is 2.725±0.002K.

Predicting the CMBR and its temperature was the first clear success of HBB cosmology!

$$dn(\nu) = \frac{8\pi\nu^2 d\nu}{\exp(h\nu/k_B T) - 1}$$

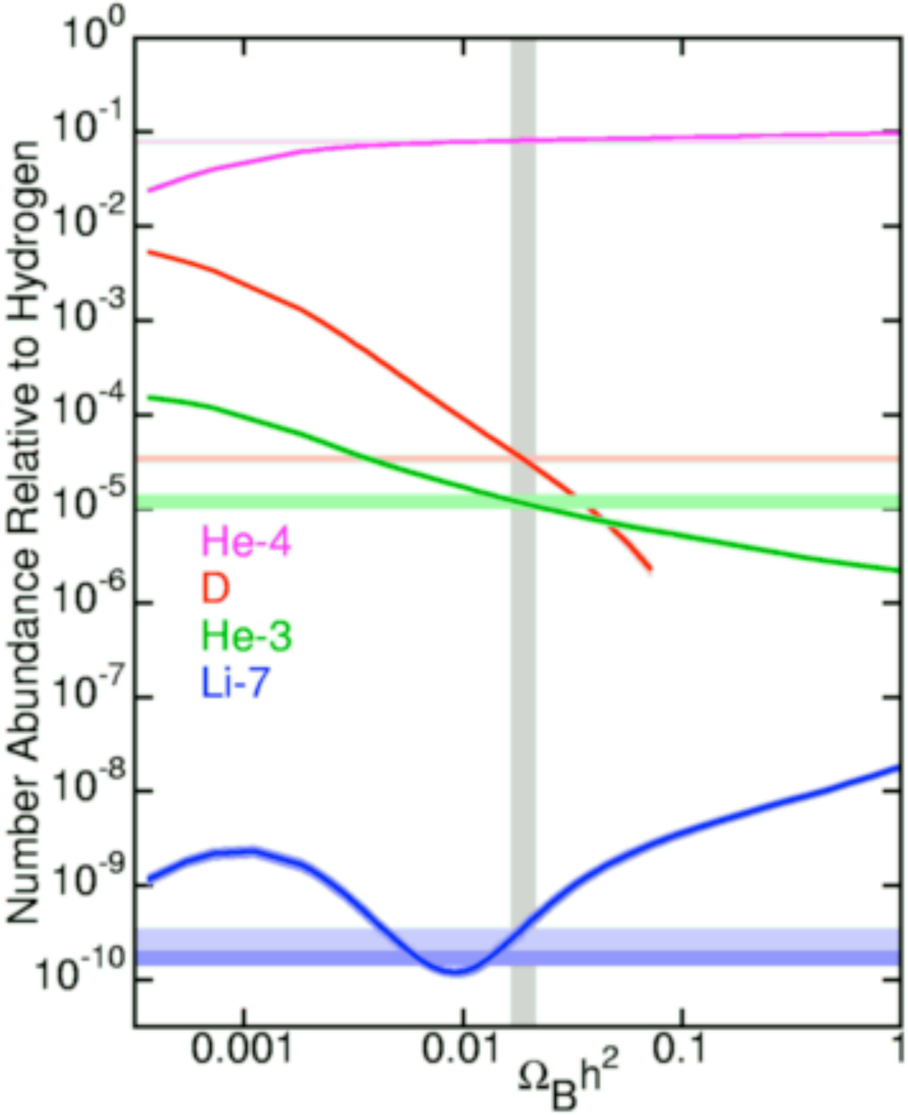


2. Primordial (BB) nucleosynthesis

A second big success of HBB cosmology is that it provides a mechanism (BBN) for producing light nuclei*) (d, He, Li, ..) out of protons and neutrons.

Temperatures of order 10^{10}K are needed for this to happen. The success of BBN is not just qualitative: we know the physics of the underlying processes, we can calculate the relative abundances of those light elements and compare them with the data.

*) Heavier elements are believed to be produced much later in very hot and dense stars, like supernovae.



Comparison with data

Horizontal bands correspond to experimental bounds;

Vertical band to allowed range for $\Omega_B \sim 0.021\ h^{-2}$

$$H(t_0) \equiv H_0 = 100\ h\ \text{km}\ s^{-1}\text{Mpc}^{-1} \quad ; \quad h \sim 0.72 \pm 0.05$$

Shortcomings of HBB cosmology

1. Flatness problem

We know that, today, $|\Omega_K|$ cannot exceed 0.1. On the other hand Ω_K evolves in time according to:

$$\Omega_K(t) = \Omega_{K,0}\ \frac{a_0^2}{a^2}\ \frac{H_0^2}{H^2} = \Omega_{K,0}\ \left(\frac{\dot{a}_0}{\dot{a}(t)}\right)^2 \sim \Omega_{K,0}\ \left(\frac{t}{t_0}\right)^{\frac{2(1+3w)}{3(1+w)}}$$

and increases with t for a decelerated expansion ($w > -1/3$).

=> $|\Omega_K| < 10^{-32}$ at BBN & $< 10^{-60}$ at $t = t_P \sim 10^{-43}$ sec.

Q: Why should the Universe start with such a small spatial curvature w.r.t. the total space-time curvature?

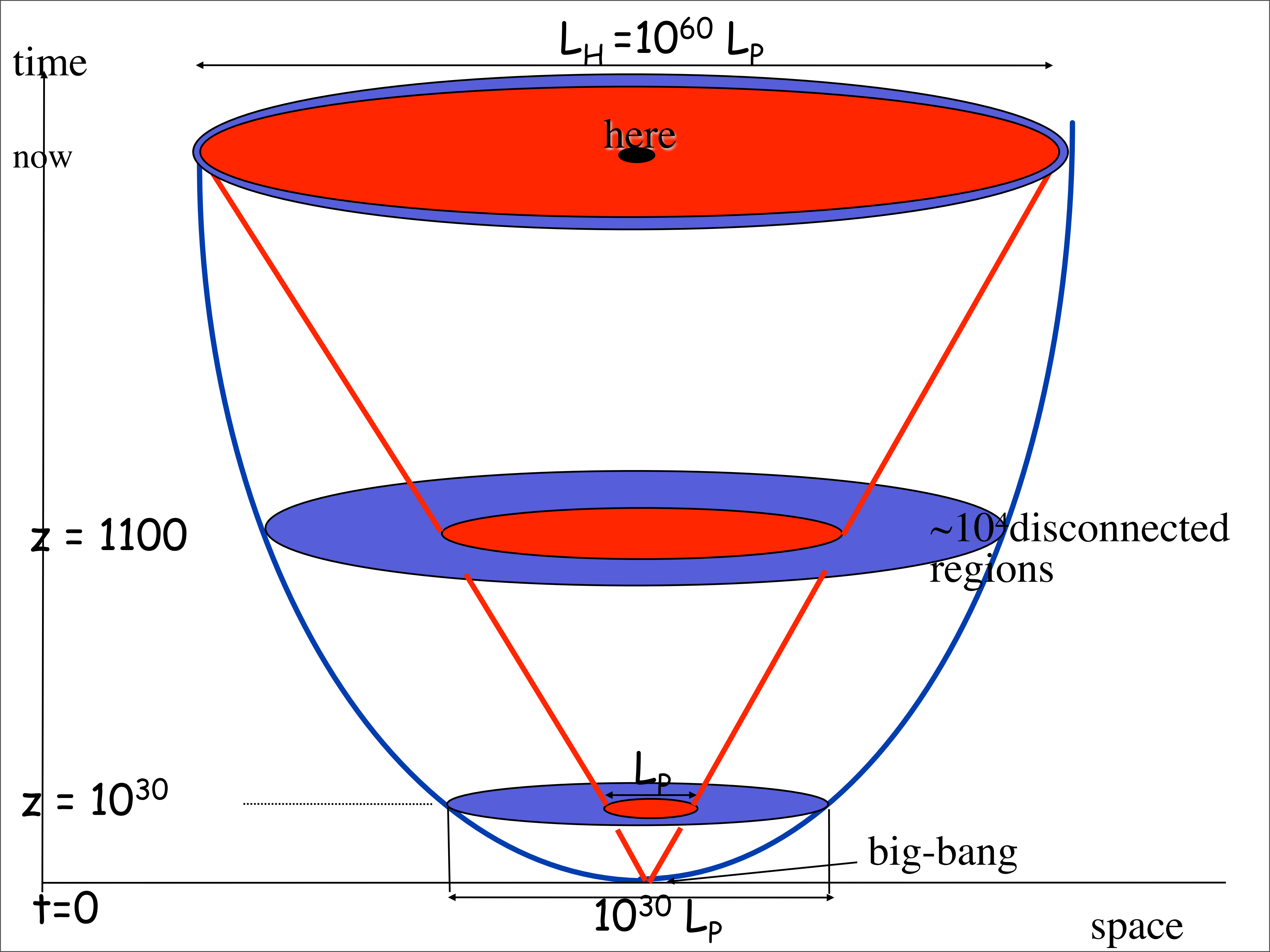
NB: A similar result holds for the contribution of spatial gradients. It had to be infinitesimal in the early Universe in order not to dominate today.

2. Homogeneity problem

The CMB comes to us today, basically undisturbed (just redshifted) from the time of recombination (or last scattering, when atoms formed and the Universe became transparent to photons). This happened at $z = z_{rec} \sim 1100$ i.e. when the Universe we can observe today was 1100 times smaller.

This size should be compared with another scale, the horizon, which is the distance traveled by light from $t=0$ till t_{rec}.

For standard HBB cosmology this second length scale is much smaller than the size of the Universe. The ratio is about 30 at recombination and can be as large as 10^{30} if we go back to $t = t_P \sim 10^{-43}$ sec (see picture).



$L_H = 10^{60}\ L_P$
time
now
here
$z = 1100$
$\sim 10^4$ disconnected regions
L_P
$z = 10^{30}$
big-bang
$t=0$
$10^{30}\ L_P$
space

By causality (finite c), primordial inhomogeneities can only be washed out over distances of the order of the horizon, while at recombination our Universe consisted of about 10^4-10^5 causally disconnected regions.

The puzzle is that the CMB temperature was(is) the same in each one of those causally disconnected region (directions).

Clearly, the reason why in the past the Universe was larger than the horizon is, again, that w > -1/3:

$$\frac{(a/a_0)}{(t/t_0)} = \left(\frac{t}{t_0}\right)^{\frac{2}{3(1+w)}-1} = \left(\frac{t}{t_0}\right)^{-\frac{1+3w}{3(1+w)}}$$

3. Origin of large-scale structure (LSS)

The Universe, even if homogeneous on very large scales, has large (and to an even larger extent small) scale structures: clusters of galaxies, galaxies, stars, ...

In HBB cosmology there is no explanation for LSS. In order to explain today's structures one has to start with some tiny inhomogeneities to be put by hand on top of the LFRW Universe.

In other words the HBB model tends to give either too much or too little LSS. Another fine-tuning problem.

The obvious solution: acceleration!

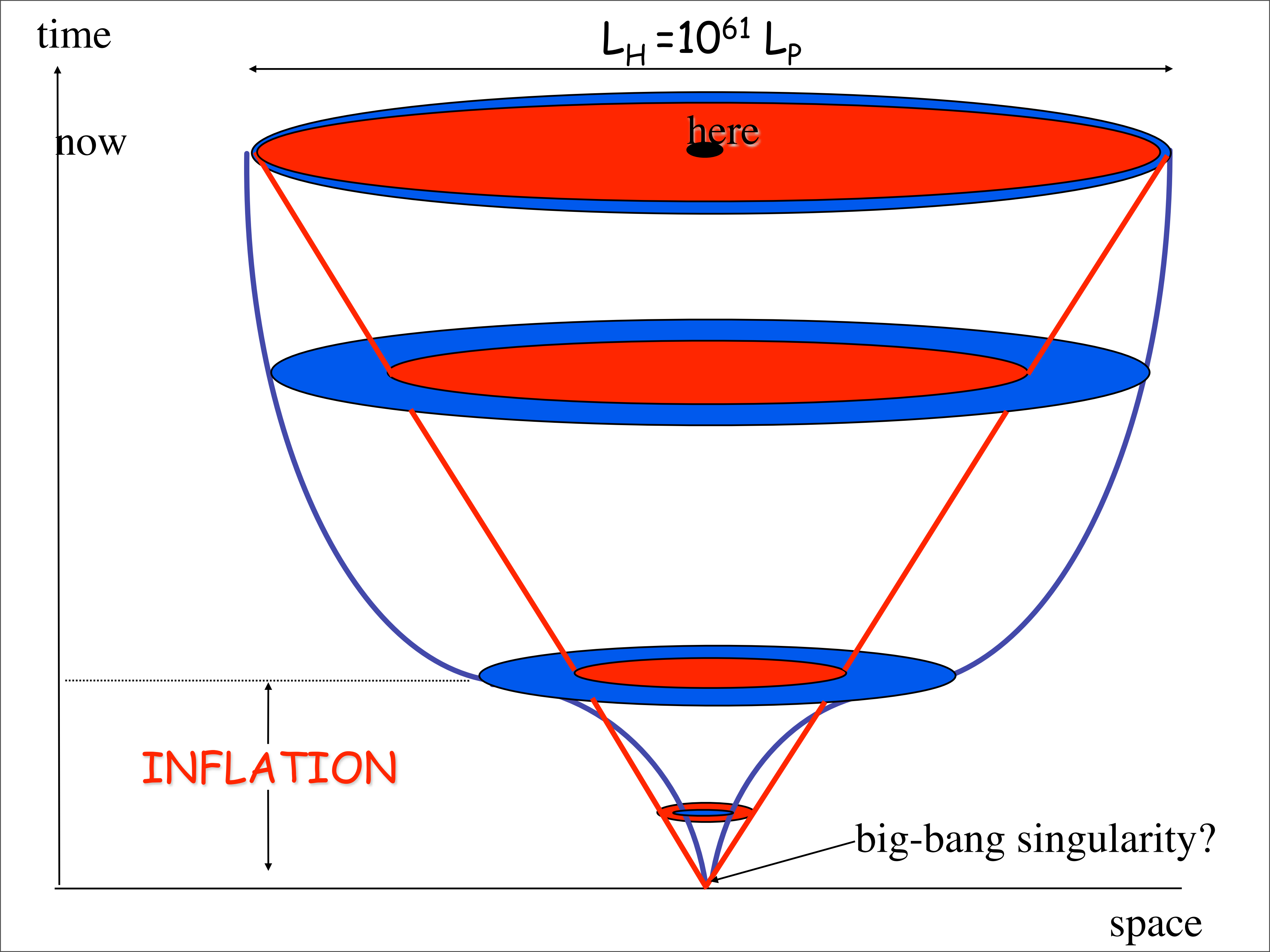
From the preceding discussion it is clear that an obvious solution to our puzzles is to insert a sufficiently long period of accelerated expansion, called inflation. One demands:

$$\frac{(a_f H_f)}{(a_i H_i)} = \frac{\dot{a}_f}{\dot{a}_i} \geq e^{N_{\min}}$$

If $N > N_{min} \sim 60$ inflation turns a generic initial Universe into a very (spatially) flat one since a^{-2} goes down faster than H^2.

Thus, $\Omega = 1$ is a generic prediction of inflation. Also, initial inhomogeneities are stretched to scales larger than our present Horizon.

The homogeneity problem is also solved since, in the far past, our visible Universe was inside a single Hubble patch (picture).



time
$L_H = 10^{61} L_P$
now
here
INFLATION
big-bang singularity?
space

Who provides the acceleration?

Ordinary matter, thanks to gravitational attraction, resists the expansion, decelerates it. In order to accelerate the expansion we need a "fluid" with $\rho + 3\,p < 0$ (negative enough pressure).

Quite amazingly it is relatively easy to "invent" such fluids. A positive cosmological constant is the simplest example (in fact was invented by Einstein for a similar purpose) but it's hard to get rid of. A more interesting choice is the potential energy of a nearly homogeneous and constant scalar field, called the inflaton. It has almost the same equation of state as a cosmological constant: $w \sim -1$ ($p \sim -\rho$).

At some point the inflaton starts changing rapidly in time and inflation stops. The inflaton's potential energy has to be dissipated, heating up the Universe (otherwise no BBN!).

Inflation's bonus: a quantum origin of LSS

One of the greatest bonuses of inflation is that, besides providing a mechanism for erasing initial inhomogeneities and spatial curvature, it can also generate a calculable (within a given inflationary model) amount of primordial perturbations.

As we shall discuss the reason for this "miracle" is quantum mechanics. Indeed, while the wavelength of any primordial classical perturbation gets stretched beyond our horizon by inflation, quantum mechanics keeps acting throughout inflation continuously generating new short-scale perturbations. When amplified and stretched to present cosmological scales by inflation they may well give rise to all the structures we see in the sky.

Shortcomings of standard inflation

Inflation is a very interesting paradigm but looks to be short of a truly satisfactory theory:

1. One needs a special kind of potentials in order to keep the inflaton nearly constant for a long time (slow-roll conditions);
2. Initially, the inflaton has to be away from the minimum of its potential and has to be "fairly homogeneous" (i.e. over several Hubble lengths);
3. It is difficult (but perhaps not impossible?) to identify the inflaton with some (fundamental or effective) scalar field already present in models of particle physics.

Can QST help?

There have been several attempts to incorporate standard (i.e. slow-roll) inflation in QST*). It seems that slow-roll inflation is not a natural outcome of string theory (although it may be possible to get realistic inflaton potentials with some amount of fine-tuning). We shall instead ask the question:

What is the most natural cosmology that emerges from QST?

Let us start from the field equations that follow from the effective action of string theory at tree level (small g_s) and small curvature (i.e. neglecting higher-derivative terms).

****

*) Incidentally: classical strings (i.e. cosmic strings) have been shown to fail as a model for LSS.

$$\Gamma_{eff} = -\int \frac{d^D x}{l_s^{D-2}} \sqrt{-G} e^{-\phi} \left[\frac{4(D-10)}{3 l_s^2} + R(G) - \partial_\mu \phi \partial^\mu \phi + \frac{1}{12} H^2 \right]$$

Here $G_{\mu\nu}$ is the string-frame metric $H = dB$ and $\phi = 2\Phi$ of previous formulae. If $D \neq 10$ we have no chance to get a low-curvature solution and thus we shall limit ourselves to $D = 10$:

$$\Gamma_{eff} = -\int \frac{d^{10} x}{l_s^8} \sqrt{-G} e^{-\phi} \left[R(G) - \partial_\mu \phi \partial^\mu \phi + \frac{1}{12} H^2 \right]$$

We limit ourselves to these massless fields representing a universal sector in all string theories (we shall briefly discuss later what happens if we add other backgrounds).

We allow the extra dimensions to be dynamical (unfrozen).

We work in the "string frame" (fixed l_s, varying l_P) but physical consequences are frame-independent.

Homogeneous (Bianchi I) equations

It is straightforward to write down the field equations for a homogeneous (for simplicity Bianchi I) universe:

$$ds^2 = -dt^2 + \sum_i a_i^2(t)(dx^i)^2 \quad ; \quad \phi = \phi(t) \quad ; \quad B = 0$$

They take the simple form:

$$(\dot{\bar{\phi}})^2 - \sum_i H_i^2 = 0 \quad ; \quad \dot{H}_i - \dot{\bar{\phi}} H_i = 0 \quad ; \quad H_i \equiv \frac{\dot{a}_i}{a_i} \quad ; \quad \dot{\bar{\phi}} \equiv \dot{\phi} - \sum_i H_i$$

where the so-called shifted dilaton is defined by:

$$\bar{\phi} = \phi - \frac{1}{2}\log(\mathrm{det} G_{ij})$$

and satisfies, as a consequence,

$$\ddot{\bar{\phi}} - (\dot{\bar{\phi}})^2 = 0 \Rightarrow \frac{d}{dt} e^{-\bar{\phi}} = \mathrm{constant}$$

Generalized Kasner solutions

In the absence of other sources the equations of Bianchi I string cosmology can be easily solved. One finds:

$$a_i(t) = (\pm t)^{p_i} \quad ; \quad \phi(t) = -(1 - \sum_i p_i) \log(\pm t) + \text{const.} \quad ; \quad \sum_i p_i^2 = 1$$

These reduce to the usual Kasner cosmology if we impose a constant dilaton. Note however that, unlike for pure Kasner, one can have a perfectly isotropic cosmology for a non-trivial dilaton:

$$a_i(t) = t^{\pm \frac{1}{\sqrt{d}}} \quad ; \quad \phi(t) = -(1 \mp \sqrt{d}) \log t \quad ; \quad t > 0 \quad ; \quad d \equiv D - 1 = 9$$

and similarly for t < 0.

Also note the interesting possibility of flipping arbitrarily the signs of the Kasner exponents if we do not freeze the dilaton.

Scale-factor duality

This last feature is related to an interesting symmetry of the string-cosmology equations under inversion of any individual scale factor $a_i(t)$, provided we keep the shifted dilaton invariant.

Indeed, under $a_i(t) \to 1/a_i(t)$, $H_i(t) \to - H_i(t)$, but our two independent equations go into themselves under this change.

This symmetry, mapping solutions into new (and generically inequivalent) ones has been called scale-factor duality (SFD) and is closely connected to T-duality (although the latter is a true symmetry of the theory). It also holds if we add stringy matter.

If the $B_{\mu\nu}$ field is turned on, the discrete (Z_2^9) SFD symmetry becomes a continuous $O(9,9;R)$ symmetry closely connected to Narain's $O(n,n;R)$ group of (generically inequivalent) compactifications of n space dimensions (see last year's course).

Only a subgroup leaves the physics invariant.

The pre-big bang scenario

The so-called pre big bang scenario in string cosmology is deeply rooted on SFD (a stringy symmetry) combined with the (more standard) invariance of the cosmological equations under T, the time reversal operation t -> -t. The combination SFDxT clearly acts on an individual scale factor as follows:

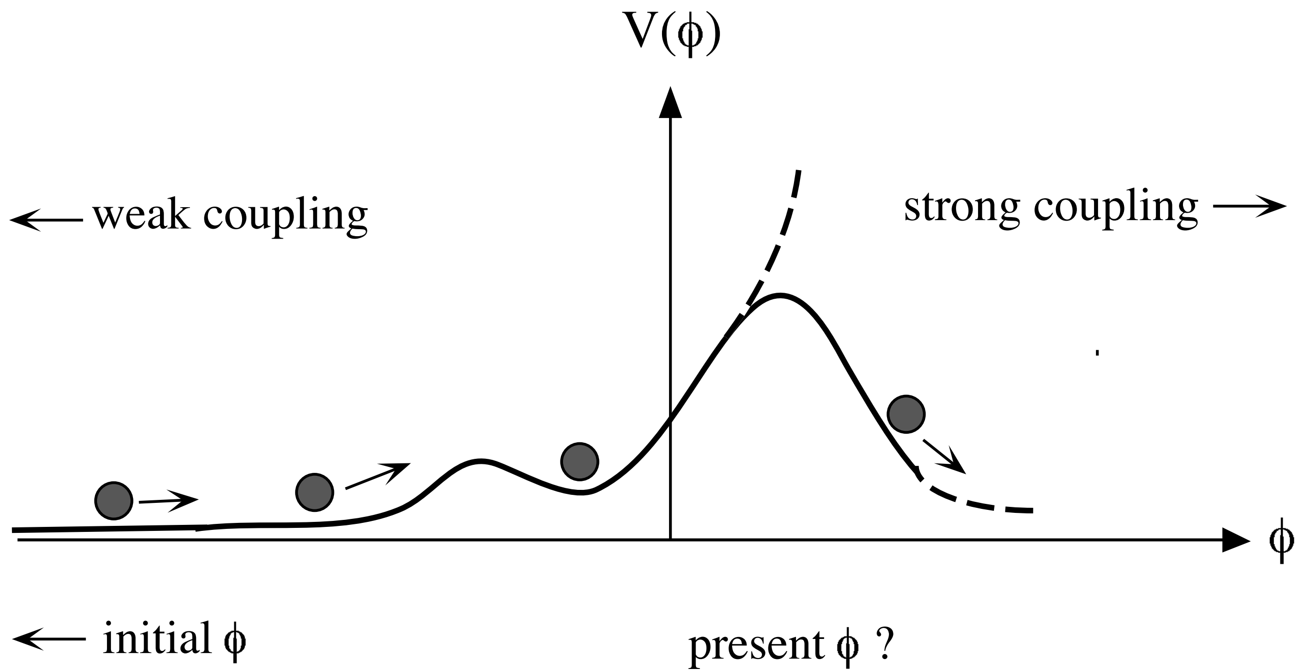
$$a_i(t) \to \tilde{a}_i(t) \equiv a_i^{-1}(-t) \Rightarrow \tilde{H}_i(-t) = H_i(t) \ ; \ \dot{\tilde{H}}_i(-t) = -\dot{H}_i(t)$$

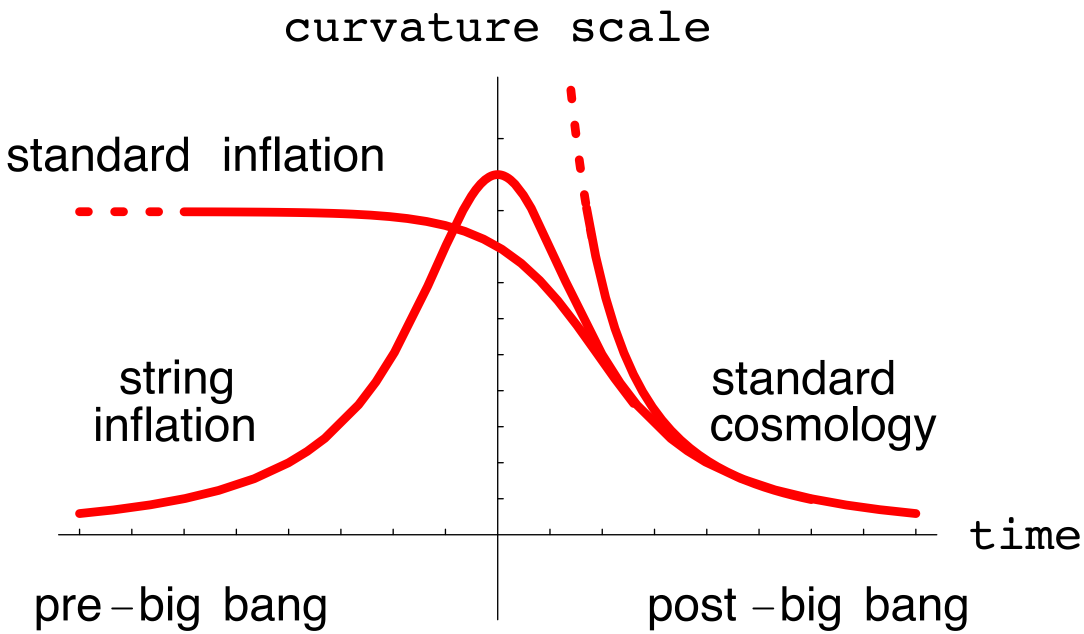
Therefore, given a standard FLRW cosmology (an expanding & decelerating Universe at t > 0), SFDxT associates to it another expanding, but now accelerating, cosmology at t < 0. Can we put together these two SFDxT-related cosmologies?

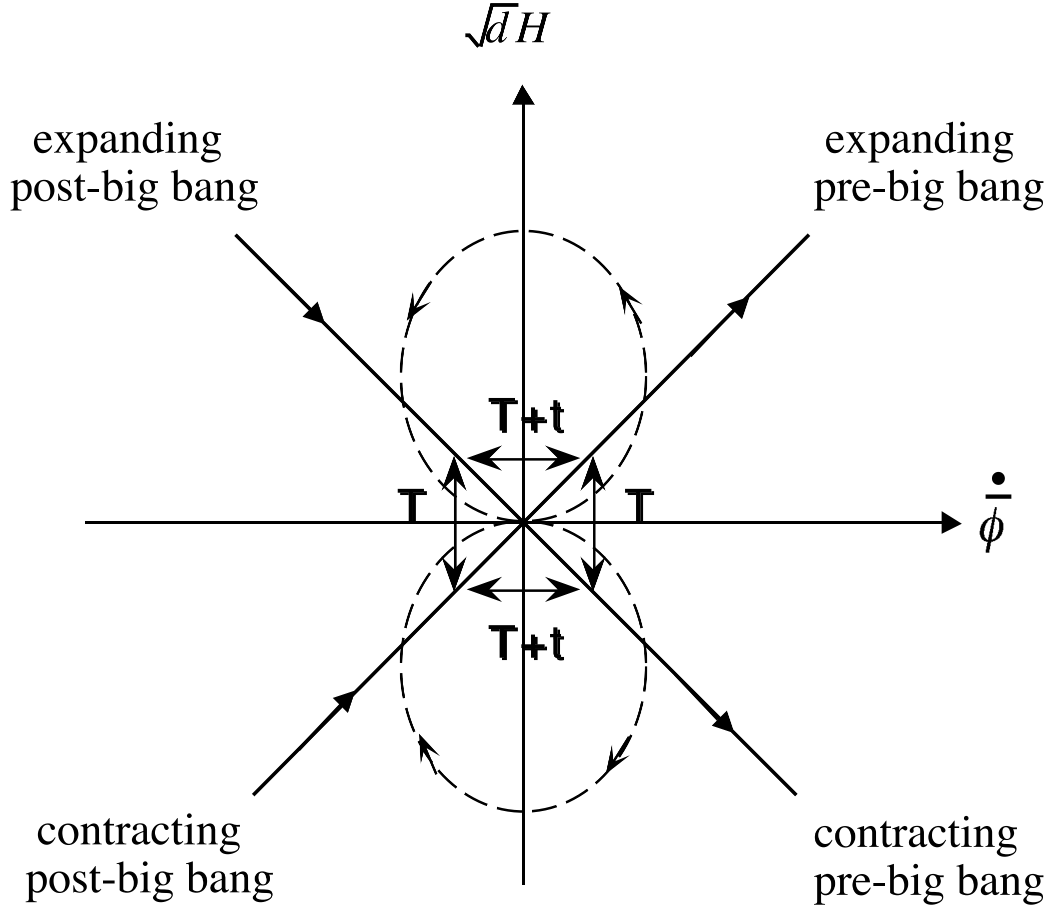
If the answer is yes we may have a new scenario in which a long "dual" phase at t < 0 preceded the standard FLRW phase possibly solving the shortcomings of the latter.

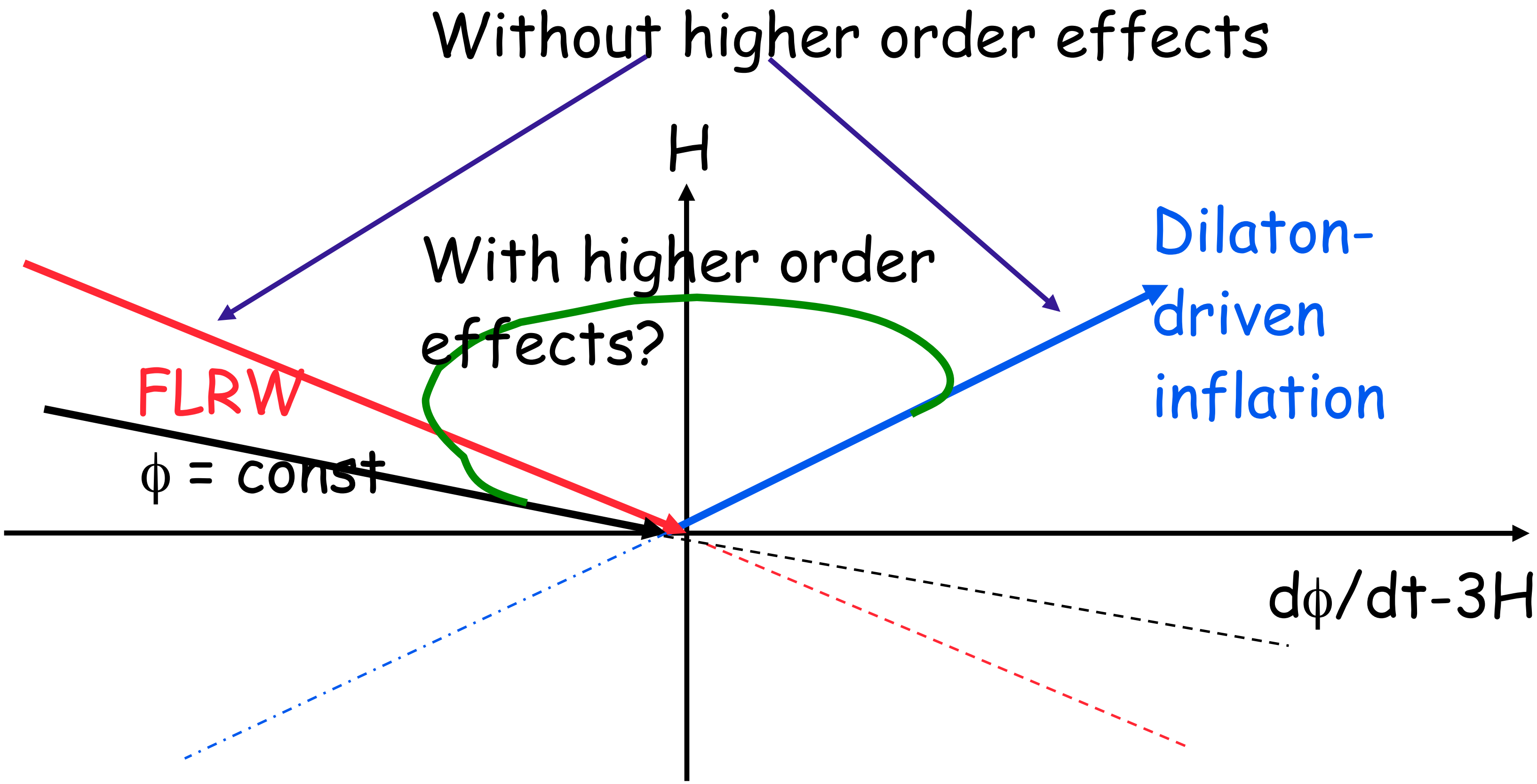
Diagrams illustrating PBB idea

(GV '91, Gasperini & GV '93)




curvature scale
standard inflation
string
inflation
standard
cosmology
time
pre – big bang
post – big bang


$\sqrt{d}H$
expanding
post-big bang
expanding
pre-big bang
T+t
T
T
$\dot{\bar{\phi}}$
T+t
contracting
post-big bang
contracting
pre-big bang


Without higher order effects
H
With higher order
effects?
Dilaton-
driven
inflation
FLRW
ϕ = const
dϕ/dt-3H