Quenches across phase transitions: the density of topological defects

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In collaboration with

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arXiv : 1012.0417 **J. Stat. Mech. P02032 (2011).**

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The problem

Predict the density of topological defects left over after traversing a phase transition with a given speed.

Out of equilibrium relaxation:

the system does not have enough time to equilibrate to new changing conditions.

Motivation

From the statistical physics perspective

Classical systems with well-known equilibrium phases & transitions.

- Applications in, *e.g.* soft condensed-matter, phase separation.
- Hard problem to solve analytically : non-linear interacting field theory.
- Out of equilibrium dynamics in macroscopic systems with mechanisms for relaxation that are understood.
- Comparison to more complex systems for which the phases and phase transitions are not as well known, *e.g.* glassy systems.

Some open issues mentioned in **orange**

Quantum counterparts mentioned at the end.

Plan of the talk

The problem's definition from the statistical physics perspective

- Canonical setting: system and environment.

- Paradigmatic phase transitions with a divergent correlation length:

  second-order paramagnetic – ferromagnetic transition

  realized by the $d > 1$ **Ising** or $d = 3$ **xy** models.

  Kosterlitz-Thouless disordered – quasi long-range order transit.

  realized by the $d = 2$ **xy** model.

- Stochastic dissipative dynamics: $g = T/J$ is the quench parameter.

- What are the topological defects to be counted ?

Plan of the talk

The analysis

- An instantaneous quench from the symmetric phase:
  - — initial condition (a question of length scales) and evolution.
  - — Critical dynamics and sub-critical coarsening.
  - — Dynamic scaling and the typical ordering length.
- Relation between the growing length and the density of topological defects.
- A slow quench from the symmetric phase:
  - — Dynamic scaling, the typical ordering length, and the density of topological defects.

Corrections to the KZ scaling

Density of topological defects

Kibble-Zurek mechanism for 2nd order phase transitions

The three basic assumptions

- Defects are created close to the critical point.
- Their density in the ordered phase is inherited from the value it takes when the system falls out of equilibrium on the symmetric side of the critical point. It is determined by

  Critical scaling above g_c

- The dynamics in the ordered phase is so slow that it can be neglected.
- results are universal.

and one scaling law

that we critically revisit within 'thermal' phase transitions

Open system

Equilibrium statistical mechanics

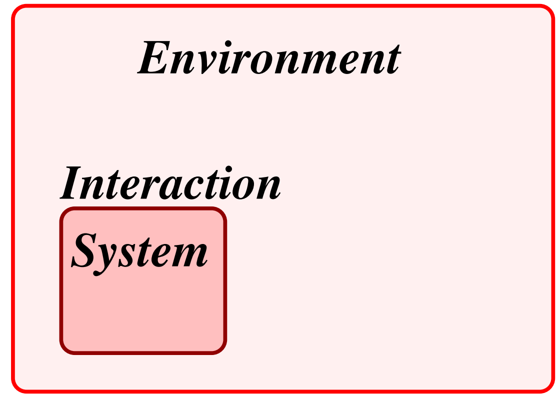
$\mathcal{E} = \mathcal{E}_{syst} + \mathcal{E}_{env} + \mathcal{E}_{int}$

Neglect $\mathcal{E}_{int}$ (short-range interact.)

Much larger environment than system

$\mathcal{E}_{env} \gg \mathcal{E}_{syst}$

Canonical distribution

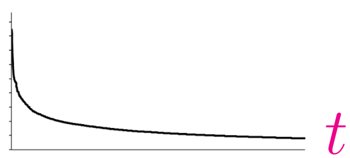


$$P(\{\vec{p}_i, \vec{x}_i\}) \propto e^{-\beta \mathcal{H}(\{\vec{p}_i, \vec{x}_i\})}$$

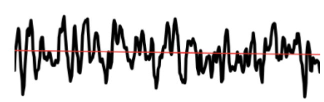
Dynamics

Energy exchange with the environment or thermal bath (dissipation) and thermal fluctuations (noise)

$\mathcal{E}_{syst}(t) \neq$ ct



Zoom



Statement

Defects exist and progressively annihilate even after an instantaneous quench into the symmetry-broken phase.

During the time spent in the critical region and/or in the ordered phase the system evolves and the number of topological defects - be them domain walls, vortices or other - decreases.

How much it does depends on how long it remains close or below the critical point.

Goal

Show these claims using a simple and well-understood system

Find a new scaling law

d-dimensional magnets

Archetypical examples

$$\boxed{\mathcal{H} = -J \sum_{\langle ij \rangle} \vec{s}_i \cdot \vec{s}_j}$$

$J > 0$	Ferromagnetic coupling constant.
$\sum_{\langle ij \rangle}$	Sum over nearest-neighbours on a d-dim. lattice.
$s_i = \pm 1$	Ising spins.
$\vec{s}_i = (s_i^x, s_i^y)$	xy two-component spins.
$\ell^d \vec{\phi}(\vec{r}) = \sum_{i \in V_{\vec{r}}} \vec{s}_i$	Coarse-grained field over the volume $V = \ell^d$
L	Linear size of the system $L \gg \ell$
$T_c > 0$	for $d > 1$ and $L \to \infty$.

Non-conserved order parameter dynamics [*e.g.*, $\uparrow\downarrow$ towards $\uparrow\uparrow$] allowed.

Other microscopic rules - local order parameter conserved, *etc.*

Stochastic dynamics

Open systems

- **Microscopic**: identify the 'smallest' relevant variables in the problem (*e.g.*, the spins) and propose stochastic updates for them, as the Monte Carlo or Glauber rules.

- **Coarse-grained**: write down a stochastic differential equation for the field, such as the effective (Markov) Langevin equation

$$\underbrace{m\ddot{\vec{\phi}}(\vec{r},t)}_{\text{Inertia}} + \underbrace{\gamma_0\dot{\vec{\phi}}(\vec{r},t)}_{\text{Dissipation}} = \underbrace{\vec{F}(\vec{\phi})}_{\text{Deterministic}} + \underbrace{\vec{\xi}(\vec{r},t)}_{\text{Noise}}$$

  with $\vec{F}(\vec{\phi}) = -\delta f(\vec{\phi})/\delta\vec{\phi}$ (see next-to-next slide for f)

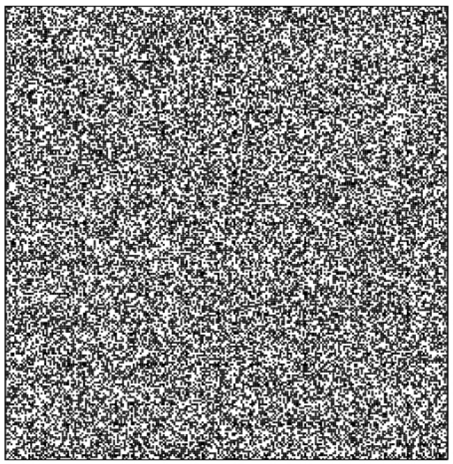
  e.g., time-dependent stochastic Ginzburg-Landau equation

- Stochastic Gross-Pitaevskii equation

Equilibrium configurations

Up & down spins in a $2d$ Ising model

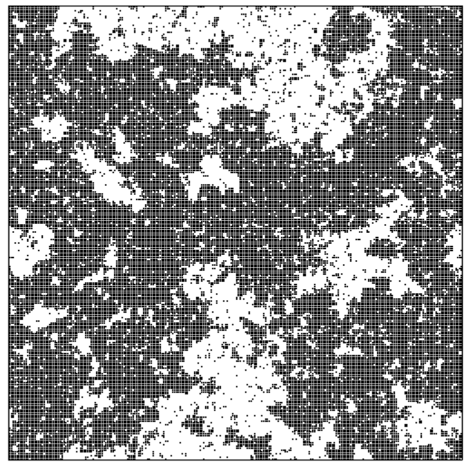
$g \to \infty$



$\langle s_i \rangle_{eq} = 0$

$\phi(\vec{r}) = 0$

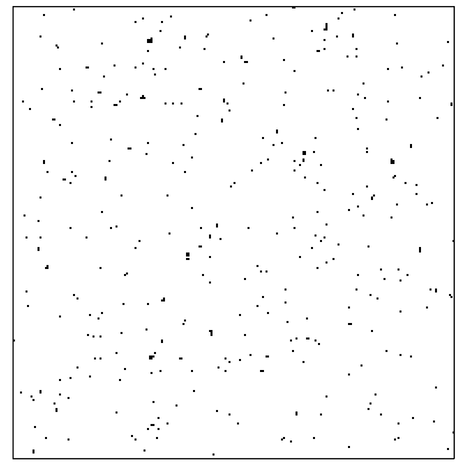
$g = g_c$



$\langle s_i \rangle_{eq} = 0$

$\phi(\vec{r}) = 0$

$g < g_c$



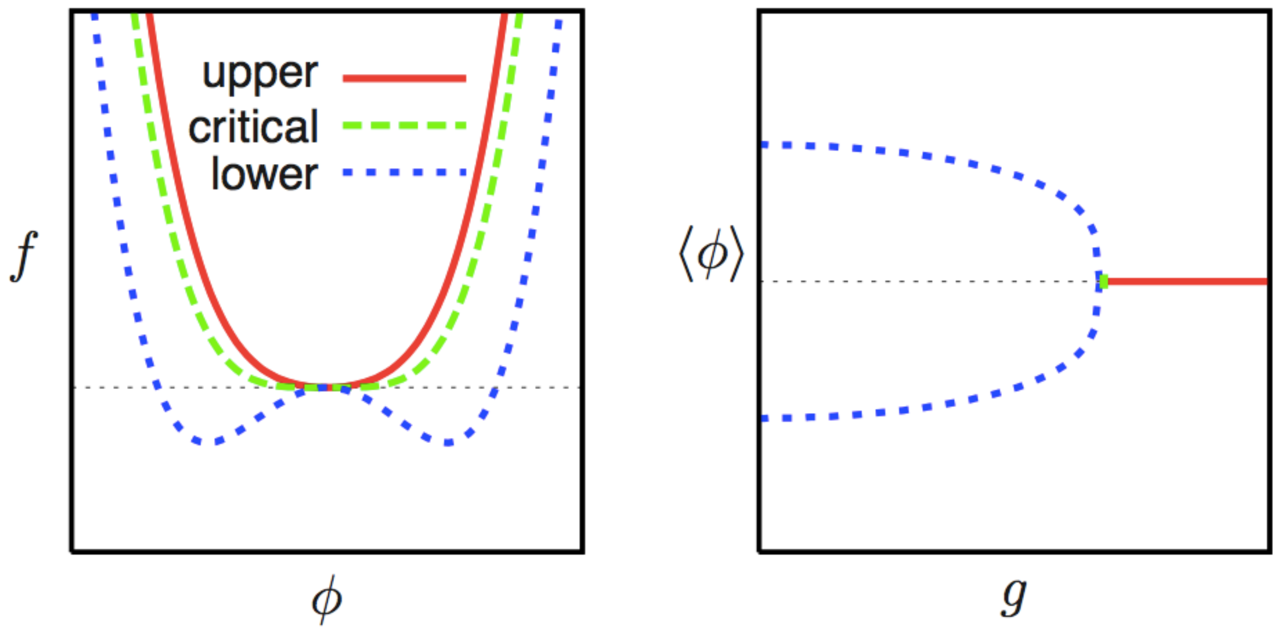
$\langle s_i \rangle_{eq^+} > 0$

$\phi(\vec{r}) > 0$

Coarse-grained scalar field $\phi(\vec{r}) \equiv \frac{1}{V_{\vec{r}}} \sum_{i \in V_{\vec{r}}} s_i$

2nd order phase-transition

Continuous phase trans. with spontaneous symmetry breaking



Ginzburg-Landau free-energy

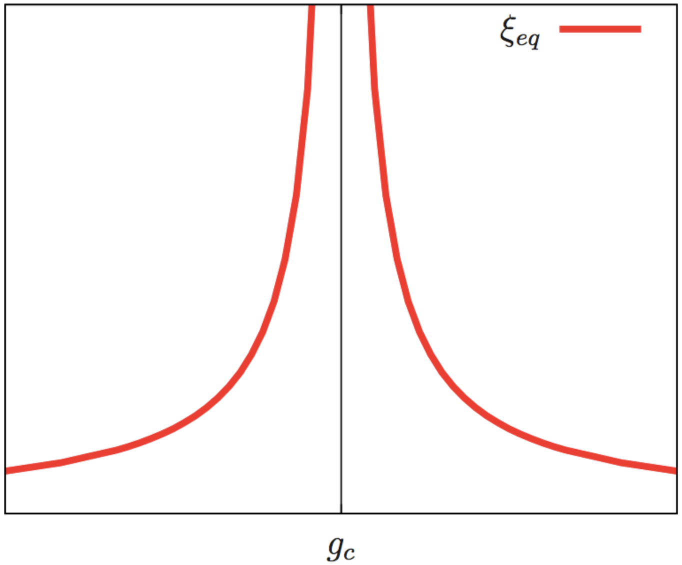
Scalar order parameter

e.g. $g = T/J$ is the control parameter

The eq. correlation length

From the spatial correlations of equilibrium fluctuations

$$C(\vec{r}) = \langle \delta\phi(\vec{r})\delta\phi(\vec{0})\rangle_{eq} \simeq e^{-r/\xi_{eq}(g)}$$



$$\boxed{\xi_{eq}(g) \simeq |g-g_c|^{-\nu} = |\Delta g|^{-\nu}}$$

In KT transitions, ξ_{eq} diverges exponentially on the disordered and it is ∞ in the quasi long-range ordered side of g_c, that is a critical phase, *e..g.* $2d$ xy model.

Topological defects

Definition via one example

Exact, locally stable, solutions to non-linear field equations such as

$$\partial_t^2 \phi(\vec{r},t) - \nabla^2 \phi(\vec{r},t) = -\frac{\delta f[\phi(\vec{r},t)]}{\delta \phi(\vec{r},t)} = -u\phi(\vec{r},t) - \lambda \phi^3(\vec{r},t)$$

$u < 0$ with finite localized energy.

$d = 1$ **domain wall**

$$\phi(x,t) \propto \sqrt{\frac{-u}{\lambda}} \tanh\left(\sqrt{\frac{-u}{\lambda}}\, x\right)$$

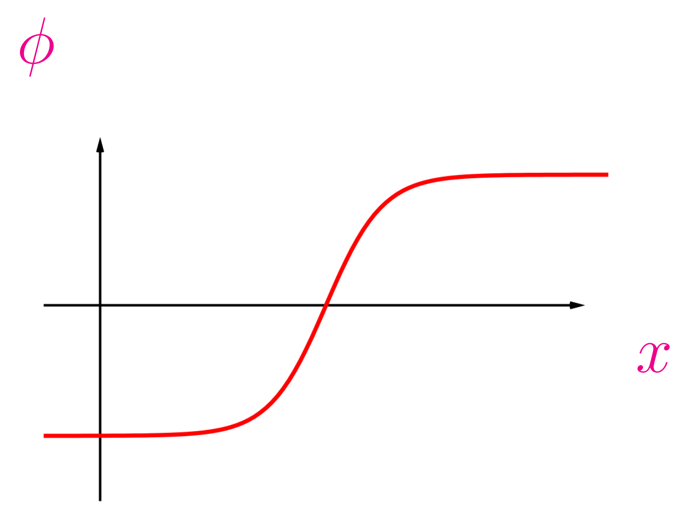
Interface between oppositely ordered

FM regions

Boundary conditions

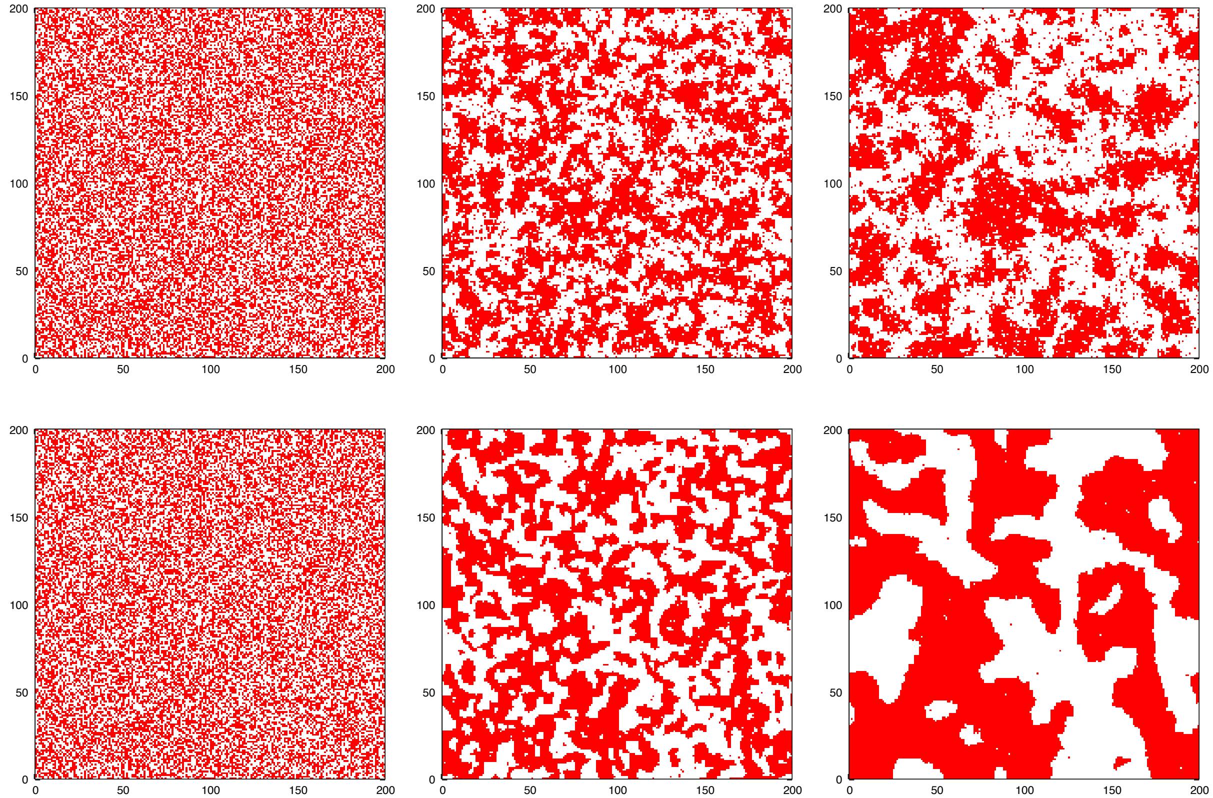
$$\phi(x \to \infty, 0) = -\phi(x \to -\infty, 0)$$

The field vanishes at the center of the wall



$2d$ Ising model

Snapshots after an instantaneous quench at $t = 0$



$g_f = g_c$

$g_f < g_c$

At $g_f = g_c$ **critical dynamics**

At $g_f < g_c$ **coarsening**

A certain number of interfaces or domain walls in the last snapshots.

Statement

In both cases one sees the growth of ‘red and white’ patches and

interfaces surrounding such geometric domains.

More precisely, spatial regions of local equilibrium (with vanishing or non-vanishing order parameter) grow in time and

a growing length $R(t, g)$ can be computed with the help of dynamic scaling.

Instantaneous quench

Dynamic scaling

very early MC simulations **Lebowitz et al 70s** & experiments

One identifies a **growing linear size of equilibrated patches**

$$\boxed{R(t, g)}$$

If this is the only length governing the dynamics, the **space-time correlation functions** should scale with $\mathcal{R}(t, g)$ according to

At $g_f = g_c$ $\quad C(r, t) \simeq C_{eq}(r)\, f_c(\frac{r}{\mathcal{R}_c(t)})$ **proven w/dyn-RG**

At $g_f < g_c$ $\quad C(r, t) \simeq C_{eq}(r) + f(\frac{r}{\mathcal{R}(t,g)})$ **argued & MF**

and the number density of interfaces should scale as

$$\boxed{n(t, g) = N(t, g)/L^d \simeq [R(t, g)]^{-d}}$$

Reviews **Hohenberg & Halperin 77** (critical) **Bray 94** (sub-critical)

Instantaneous quench

Dynamic scaling

very early MC simulations **Lebowitz et al 70s** & experiments

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At $g_f = g_c$ $\qquad C(r,t) \simeq C_{eq}(r)\, f_c(\frac{r}{\mathcal{R}_c(t)})$ $\qquad$ **Scaling fct f_c ✔**

At $g_f < g_c$ $\qquad C(r,t) \simeq C_{eq}(r) + f(\frac{r}{\mathcal{R}(t,g)})$ $\qquad$ **Scaling fct f ?**

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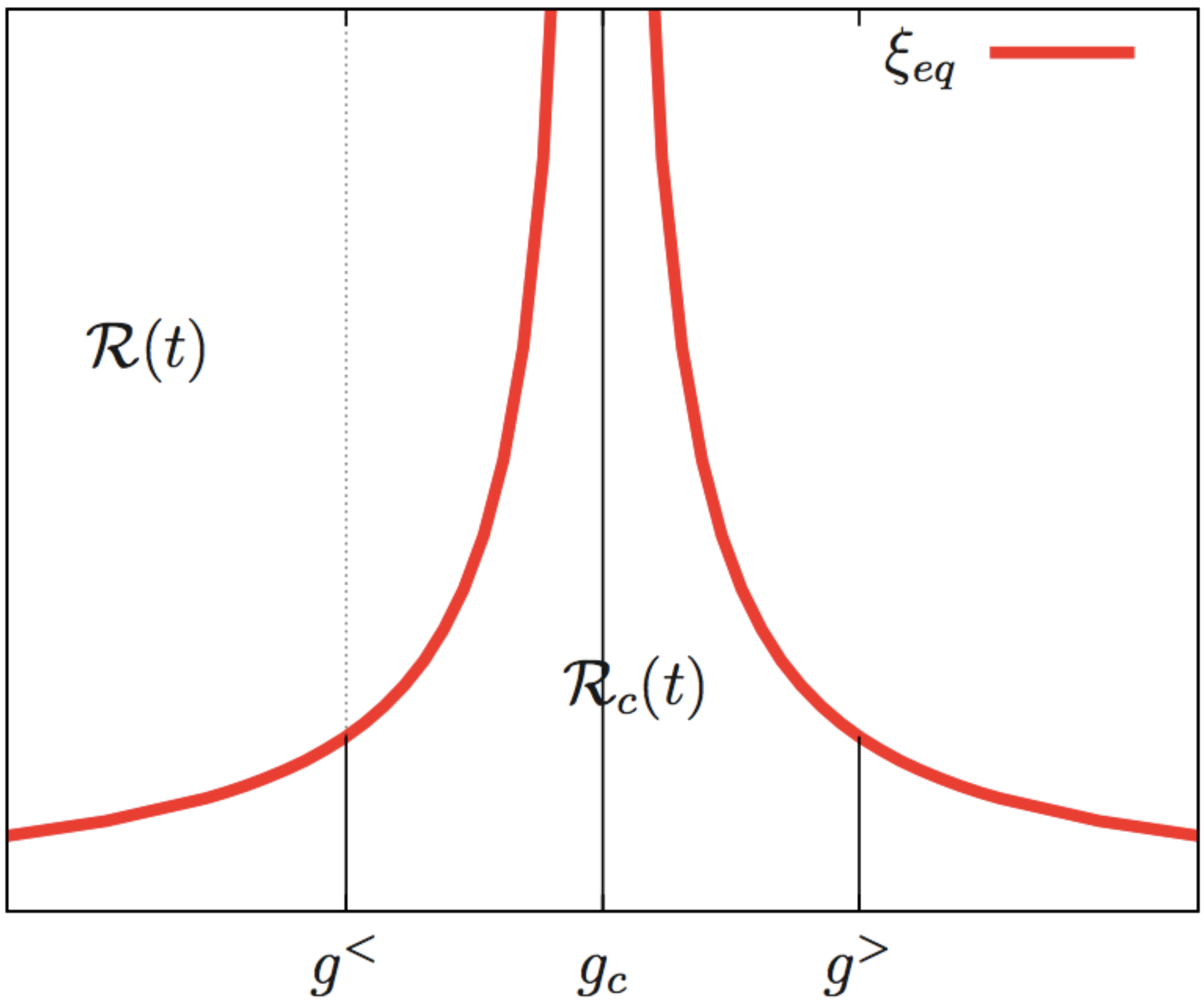
Reviews **Hohenberg & Halperin 77** (critical) **Bray 94** (sub-critical)

Instantaneous quench

Control of cross-overs



Instantaneous quench to $g_c+\epsilon$

Growth and saturation

The **length** grows and saturates

$$R(t,g) \simeq \begin{cases} t^{1/z_c} & t \ll \tau_{eq}(g) \\ \xi_{eq}(g) & t \gg \tau_{eq}(g) \end{cases}$$

with $\tau_{eq}(g) \simeq \xi_{eq}^{z_c}(g) \simeq |g-g_c|^{-\nu z_c}$ the equilibrium relaxation time.

Saturation at $t \simeq \tau_{eq}(g)$ when $\boxed{R(\tau_{eq}(g),g) \simeq \xi_{eq}(g)}$

z_c is the exponent linking times and lengths in **critical dynamics**

e.g. $z_c \simeq 2.17$ **for the 2dIM with NCOP**.

Dynamic RG calculations **Bausch, Schmittmann & Jenssen 80s**.

Instantaneous quench to g_c

Non-stop growth

The **length** grows

$$\boxed{R(t,g) = \mathcal{R}_c(t) \simeq t^{1/z_c} \qquad t \ll \tau_{eq}(g) \to \infty}$$

with $\tau_{eq}(g) \simeq |g - g_c|^{-\nu z_c} \to \infty$ the equilibrium relaxation time.

z_c is the exponent linking times and lengths in **critical dynamics**

e.g. $z_c \simeq 2.17$ **for the 2dIM with NCOP**.

Dynamic RG calculations **Bausch, Schmittmann & Jenssen 80s**.

Instantaneous quench to $g<g_c$

Deep quenches

The **length** grows as

$$\boxed{R(t,g) = \mathcal{R}(t,g) \approx \zeta(g)\, t^{1/z_d} \qquad t \gg \tau_{eq}}$$

with τ_{eq} the equilibrium relaxation time.

Non-conserved scalar order parameter $\boxed{z_d = 2}$

Proven for time-dependent Ginzburg-Landau equation **Allen & Cahn 79** & arguments for lattice models **Kandel & Domany 90, Chayes** *et al.* **95**

Not really a 'formal' proof & even harder for vector order parameter and/or conservation laws

Instantaneous quench to $g < g_c$

Deep quenches

The **length** grows as

$$\boxed{R(t,g) = \mathcal{R}(t,g) \approx \zeta(g)\, t^{1/z_d} \qquad t \gg \tau_{eq}}$$

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Proven for time-dependent Ginzburg-Landau equation **Allen & Cahn 79** & arguments for lattice models **Kandel & Domany 90, Chayes** *et al.* **95**

Weak quench disorder effect on $\mathcal{R}$?

Is there an $\mathcal{R}$ with strong disorder ?

Instantaneous quench to $g_c - \epsilon$

Control of cross-overs

The **length** grows with different laws

$$R(t,g) = \begin{cases} \mathcal{R}_c(t) \approx t^{1/z_c} & t \ll \tau_{eq} \\ \mathcal{R}(t,g) \approx \xi_{eq}^{1-z_c/z_d}(g)\, t^{1/z_d} & t \gtrsim \tau_{eq} \end{cases}$$

with ξ_{eq} and τ_{eq} the equilibrium correlation length and relaxation time.

Crossover at $t \simeq \tau_{eq}(g)$ when $\boxed{R(\tau_{eq}(g), g) \simeq \xi_{eq}(g)}$

Arenzon, Bray, LFC & Sicilia 08

Note that $z_c \geq z_d$

e.g. $z_c \simeq 2.17$ **and** $z_d = 2$ **for the 2dIM with NCOP**

$z_c \simeq 2.13$ **and** $z_d = 2$ **for the 3d xy with NCOP**

Topological defects

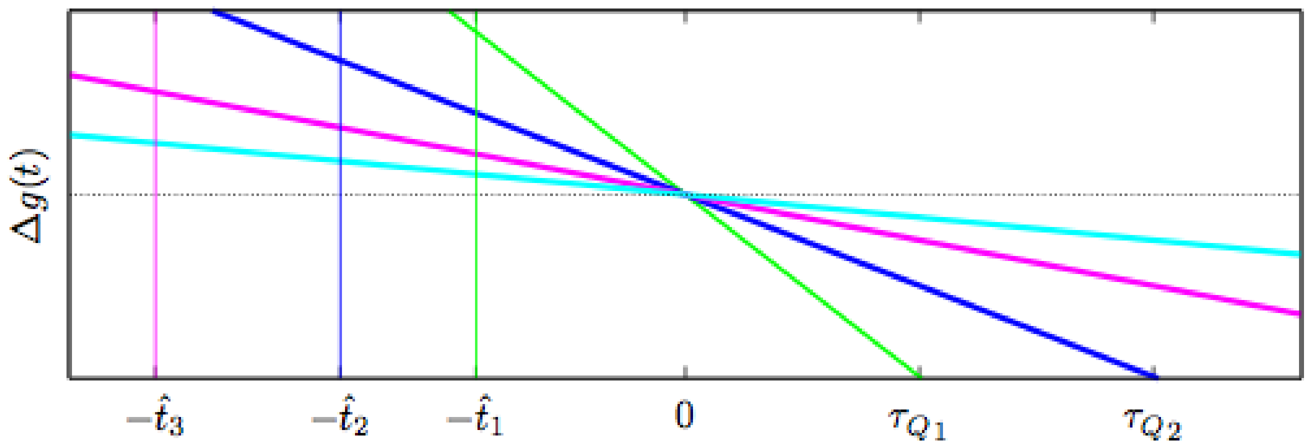
configurations after a sub-critical instantaneous quench

$$\boxed{n(t,g) = N(t,g)/L^d \simeq [R(t,g)]^{-d}}$$

Remember the initial ($g \to \infty$) configuration : germs already there !

Finite rate quenching protocol

How is the scaling modified for a very slow quenching rate ?



$\Delta g \equiv g(t) - g_c = -t/\tau_Q$ with $\tau_{Q1} < \tau_{Q2} < \tau_{Q3} < \tau_{Q4}$

Standard time parametrization $\boxed{g(t) = g_c - t/\tau_Q}$

Simplicity argument: linear cooling could be thought of as an approximation of any cooling procedure $g(t)$ close to g_c.

Zurek's argument

Slow quench from equilibrium well above g_c

The system follows the pace imposed by the changing conditions, $\Delta g(t) = -t/\tau_Q$, until a time $-\hat{t} < 0$ (or value of the control parameter $\hat{g} > g_c$) at which its dynamics are too slow to accommodate to the new rules. The system **falls out of equilibrium**.

$-\hat{t}$ is estimated as the moment when the relaxation time, τ_{eq}, is of the order of the typical time-scale over which the control parameter, g, changes :

$$\tau_{eq}(g) \simeq \left. \frac{\Delta g}{d_t \Delta g} \right|_{-\hat{t}} \simeq \hat{t} \qquad \Rightarrow \qquad \boxed{\hat{t} \simeq \tau_Q^{\nu z_c/(1+\nu z_c)}}$$

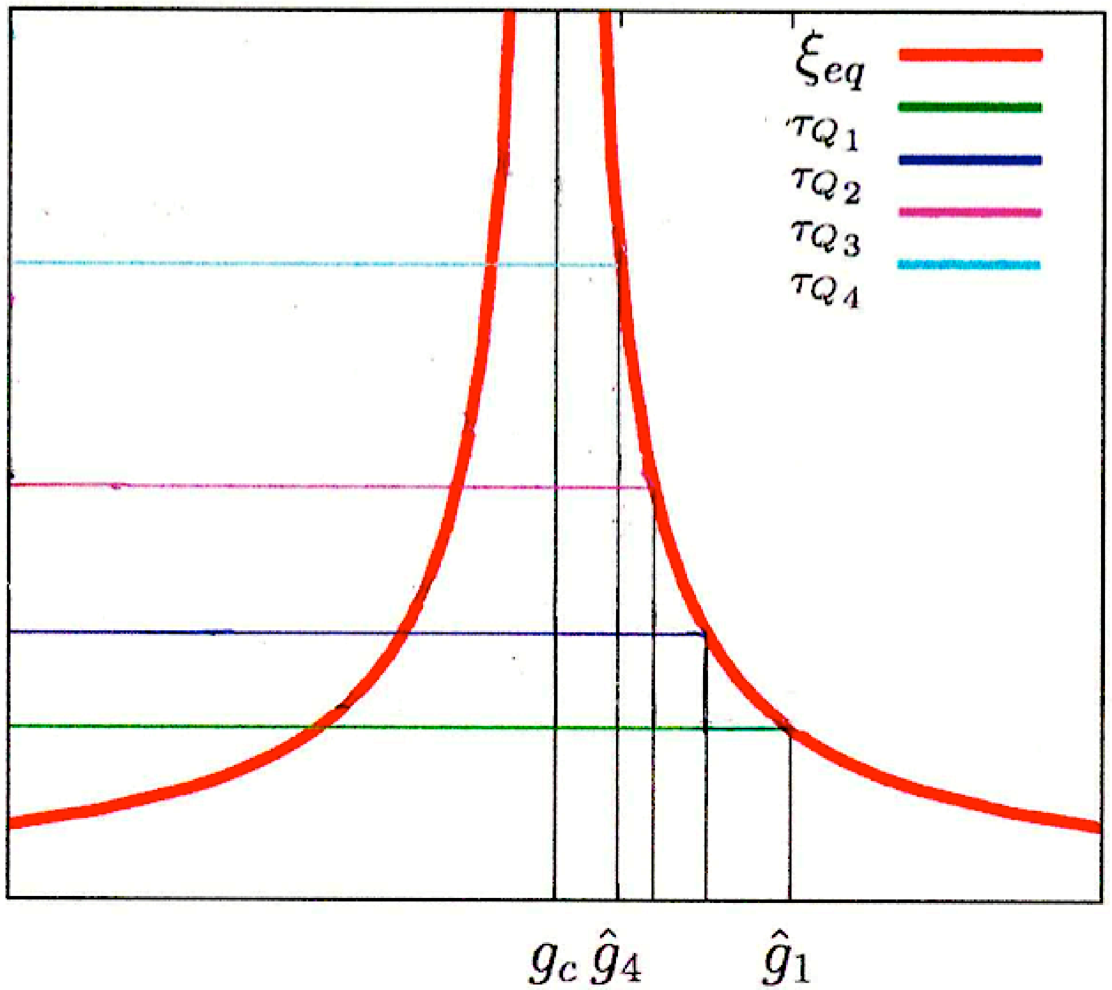
The density of defects is $\boxed{\hat{n}_{\rm KZ} \simeq \xi_{eq}^{-d}(\hat{g}) \simeq (\Delta \hat{g})^{\nu d} \simeq \tau_Q^{-\nu d/(1+\nu z_c)}}$

and the claim is that it gets blocked at this value ever after

Zurek 85

Finite rate quench

Sketch of Zurek's proposal for R_{τ_Q}



Finite rate quench

Critical coarsening out of equilibrium

In the critical region the system coarsens through critical dynamics and these dynamics operate until a time $t^* > 0$ *at which the growing length is again of the order of the equilibrium correlation length,* $R^* \simeq \xi_{eq}(g^*)$.

For a linear cooling a simple calculation yields

$$\boxed{R^* \simeq \zeta \, \hat{R} \simeq \zeta \, \xi_{eq}(\hat{g})}$$

(if the scaling for an infinitely rapid critical quench, $\Delta R(\Delta t) \simeq \Delta t^{1/z_c}$, with $\Delta t = t^* - \hat{t}$ the time spent since entering the critical region holds)

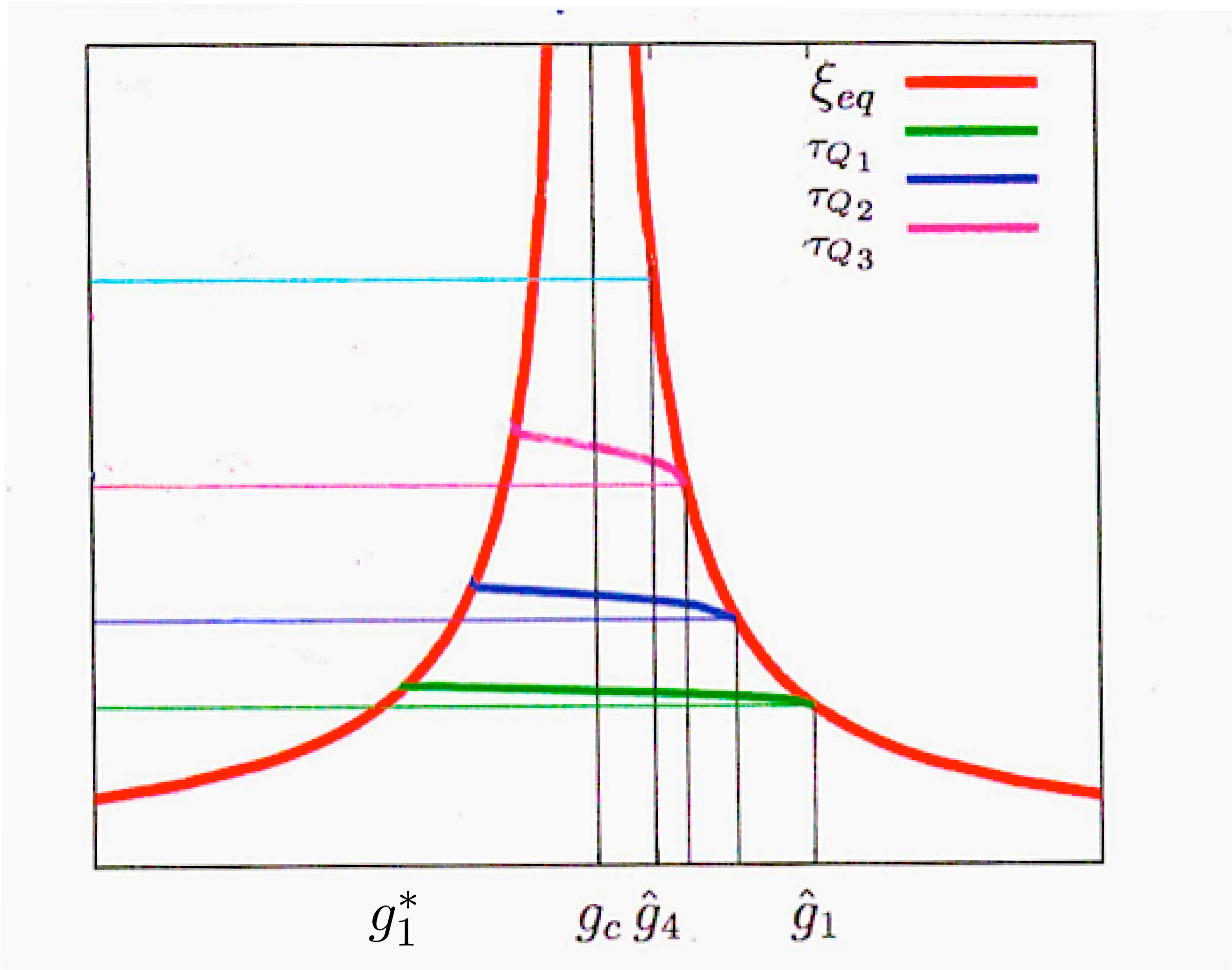
No change in leading scaling with τ_Q.

However, for a non-linear cooling, *e.g.* $\Delta g = (t/\tau_Q)^x$ with $x > 1$,

$$\boxed{R^* \simeq \tau_Q^a \quad \text{with} \quad a > \nu/(1+\nu z_c) \quad \text{for} \quad x > 1}$$

Finite rate quench

Contribution from critical relaxation, $R^*_{\tau Q}$



Finite rate quench

Far from the critical region, in the coarsening regime

In the 'ordered' phase usual coarsening takes over. The correlation length R *continues to evolve and its growth cannot be neglected.*

Working assumption for the slow quench

$$\boxed{R(\Delta t, g(\Delta t)) \qquad \to \qquad \mathcal{R}(\Delta t, g(\Delta t))}$$

with Δt the time spent since entering the sub-critical region at R^*.

∞-**rapid** quench with	$\to$	**finite-rate** quench with
$g = g_f$ held constant		g slowly varying.

Finite rate quench

The two cross-overs

One needs to match the three regimes :

equilibrium, critical and sub-critical growth.

New **scaling assumption** for a linear cooling $|\Delta g(t)| = t/\tau_Q$:

$$\boxed{R(t, g(t)) \simeq \begin{cases} |\Delta g(t)|^{-\nu} & t \ll -\hat{t} \quad \text{in eq.} \\ |\Delta g(t)|^{-\nu(1-z_c/z_d)}\, t^{1/z_d} & t \gtrsim t^* \quad \text{out of eq.} \end{cases}}$$

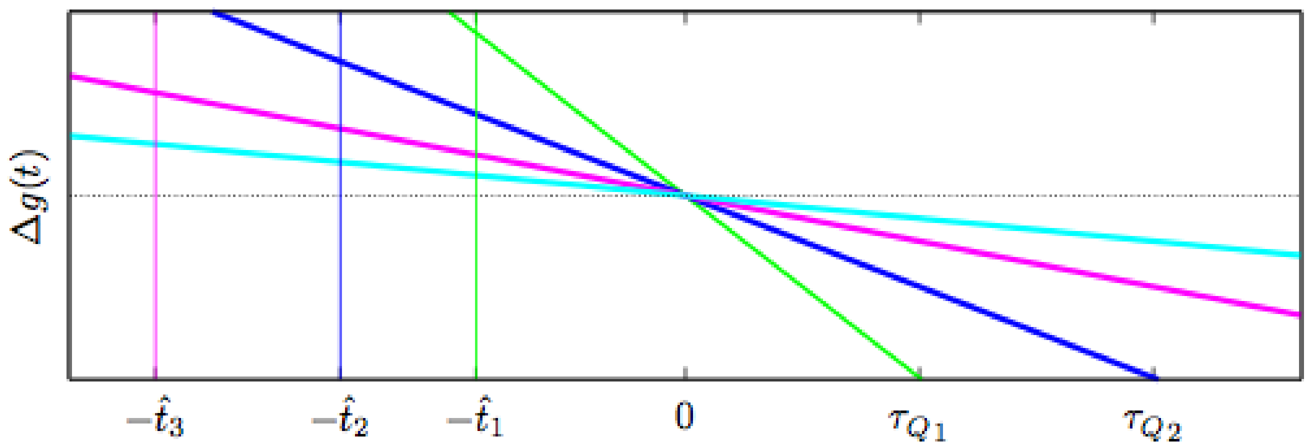
Scaling on both sides of the critical (uninteresting for a linear cooling) region

Crossover at $t \simeq t^* \simeq \tau_Q^{\alpha}$ with $\alpha < 1$ ensured

Finite rate quenching protocol

How is the scaling modified for a very slow quenching rate ?

$$\boxed{R \simeq (t/\tau_Q)^{-\nu+\frac{\nu z_c}{z_d}}\ t^{\frac{1}{z_d}} \simeq |\Delta g|^{-\nu+\frac{1+\nu z_c}{z_d}}\ \tau_Q^{\frac{1}{z_d}}}$$



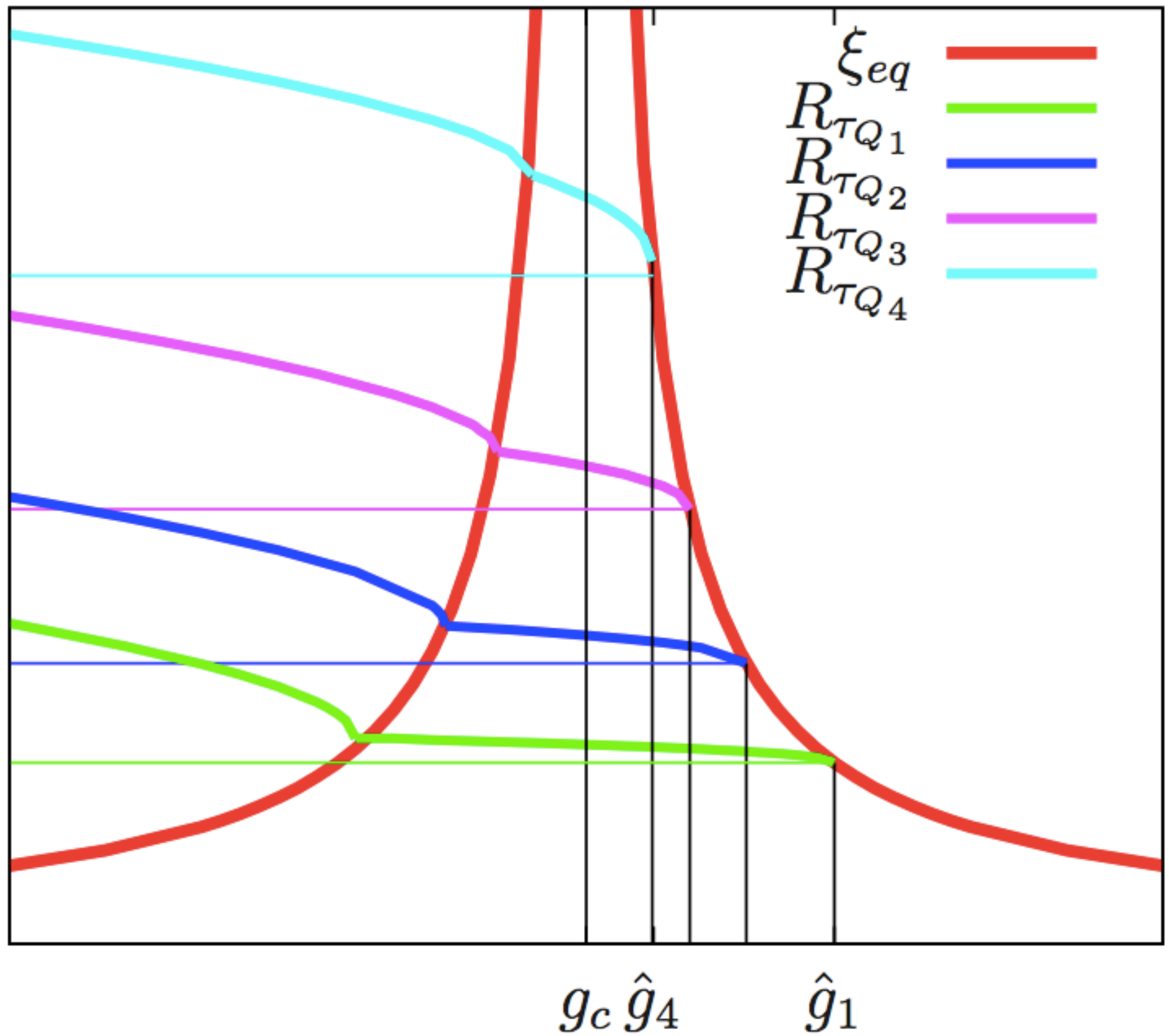
$\Delta g \equiv g(t) - g_c = -t/\tau_Q$ with $\tau_{Q_1} < \tau_{Q_2} < \tau_{Q_3} < \tau_{Q_4}$

R depends on $]t$ and $\tau_Q]$ or on $[\Delta g$ and $\tau_Q]$ independently

R increases with $[\Delta g$ and $\tau_Q]$

Finite rate quench

Sketch of the effect of τ_Q on $R(t, g)$

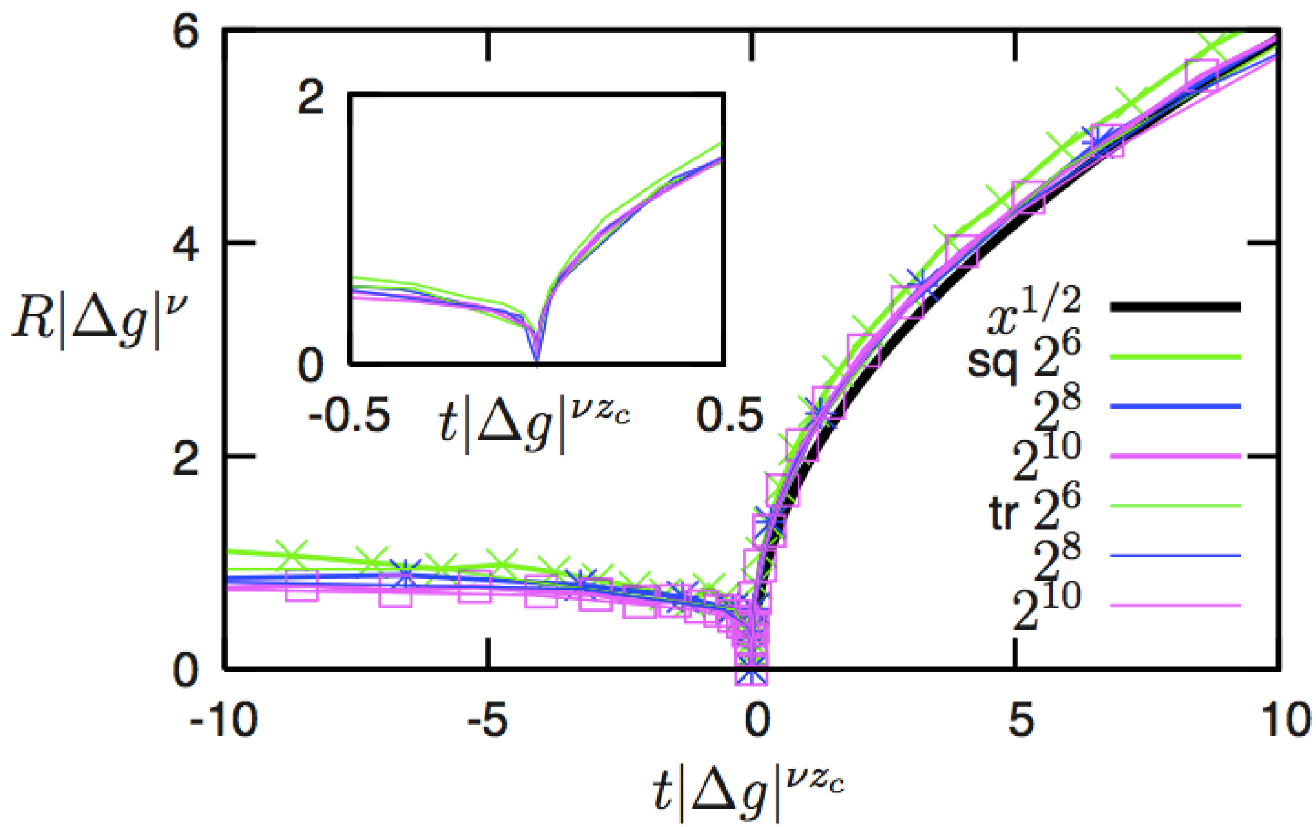


cfr. constant thin lines, **Zurek 85**

Simulations

Test of universal scaling in the 2dIM with NCOP dynamics

$R\ |\Delta g|^{\nu}$ cst $(|\Delta g|^{\nu z_c} t)^{1/z_d}$



$z_c \simeq 2.17$ and $\nu \simeq 1$; the square root ($z_d = 2$) is in black

Also checked (analytically) in the $O(N)$ model in the large N limit.

Number of domain walls

Test of universal scaling in the 2dIM with NCOP dynamics

Dynamic scaling implies

$$n(t, \tau_Q) \simeq [R(t, \tau_Q)]^{-d}$$

with d the dimension of space

Therefore

$$\boxed{n(t, \tau_Q) \simeq \tau_Q^{d\nu(z_c - z_d)/z_d} \; t^{-d[1+\nu(z_c-z_d)]/z_d}}$$

depends on *both* times t and τ_Q.

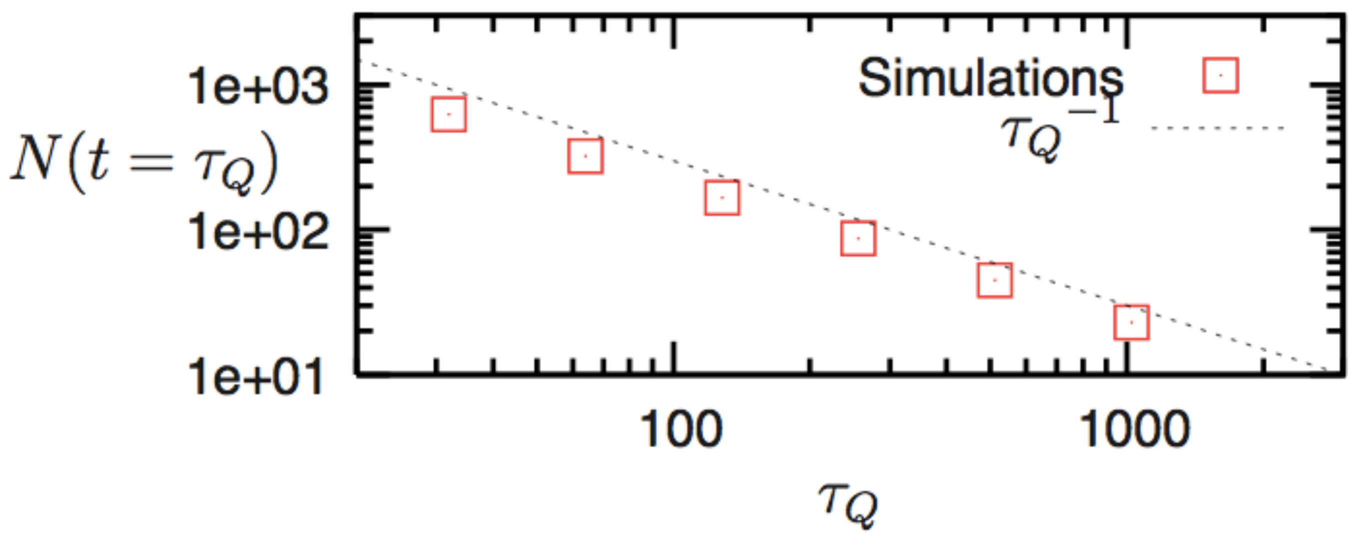
NB t can be much longer than t^* (time for starting sub-critical coarsening) ; in particular t can be of order τ_Q while t^* scales as τ_Q^α with $\alpha < 1$.

Since z_c is larger than z_d this quantity grows with τ_Q at fixed t.

Density of domain walls

At $t \simeq \tau_Q$ in the 2dIM with NCOP dynamics

$$\boxed{N(t \simeq \tau_Q, \tau_Q) = n(t \simeq \tau_Q, \tau_Q) L^2 \simeq \tau_Q^{-1}}$$



while the KZ scaling yields $N_{\rm KZ} \simeq \tau_Q^{-\nu/(1+\nu z_c)} \simeq \tau_Q^{-0.31}$.

Biroli, LFC, Sicilia, Phys. Rev. E 81, 050101(R) (2010)

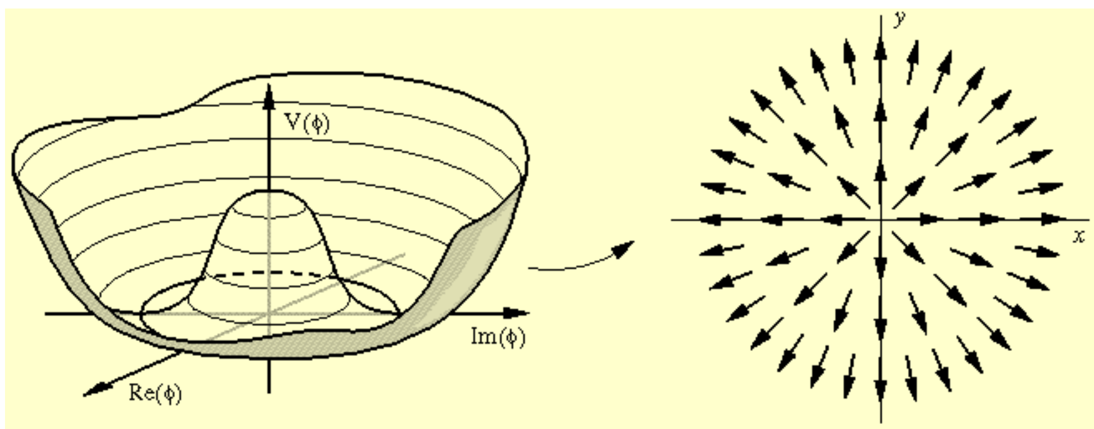
Topological defects

Definition via another example

A vector field

$$\partial_t^2 \vec{\phi}(\vec{r},t) - \nabla^2 \vec{\phi}(\vec{r},t) = -\frac{\delta f[\vec{\phi}(\vec{r},t)]}{\delta \vec{\phi}(\vec{r},t)} = -u\vec{\phi}(\vec{r},t) - \lambda \vec{\phi}(\vec{r},t)\, \phi^2(\vec{r},t)$$

in $d = 2$ for $\vec{\phi} = (\phi^x, \phi^y)$ leads to a two dimensional **vortex**



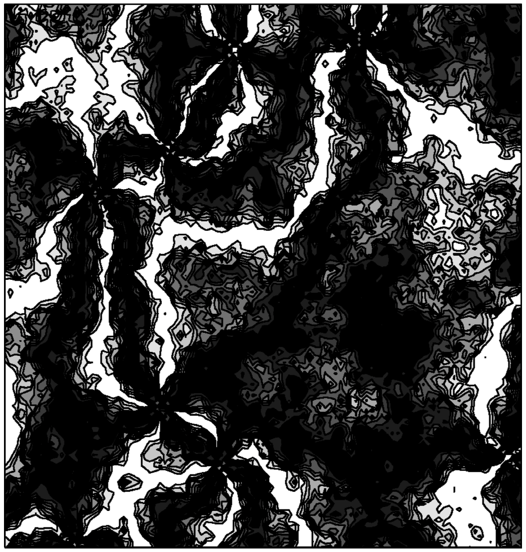
Picture from the Cambridge Cosmology Group webpage

The two-component field turns around a point where it vanishes

Dynamics in the $2d$ XY model

Vortices : planar spins turn around points

Schrielen pattern : gray scale according to $\sin^2 2\theta_i(t)$



After a quench vortices annihilate and tend to bind in pairs

$$\boxed{R(t,g) = \mathcal{R}_c(t) \simeq \zeta(g)\{t/\ln[t/t_0(g)]\}^{1/2}}$$

Pargellis *et al* **92, Yurke** *et al* **93, Bray & Rutenberg 94**

Dynamics in the $2d$ XY model

KT phase transition & coarsening

- The high T phase is plagued with vortices. These should bind in pairs (with finite density) in the low T quasi long-range ordered phase.

- Exponential divergence of the equilibrium correlation length above $T_{\rm KT}$

$$\boxed{\xi_{eq} \simeq a_\xi e^{b_\xi [(T-T_{\rm KT})/T_{\rm KT}]^{-\nu}}} \qquad \text{with} \qquad \nu = 1/2.$$

- Zurek's argument for falling out of equilibrium in the disordered phase

$$\boxed{\hat{\xi}_{eq} \simeq (\tau_Q / \ln^3(\tau_Q/t_0))^{1/z_c}} \qquad \text{with } z_c = 2 \text{ for NCOP.}$$

- Logarithmic corrections to the sub-critical growing length

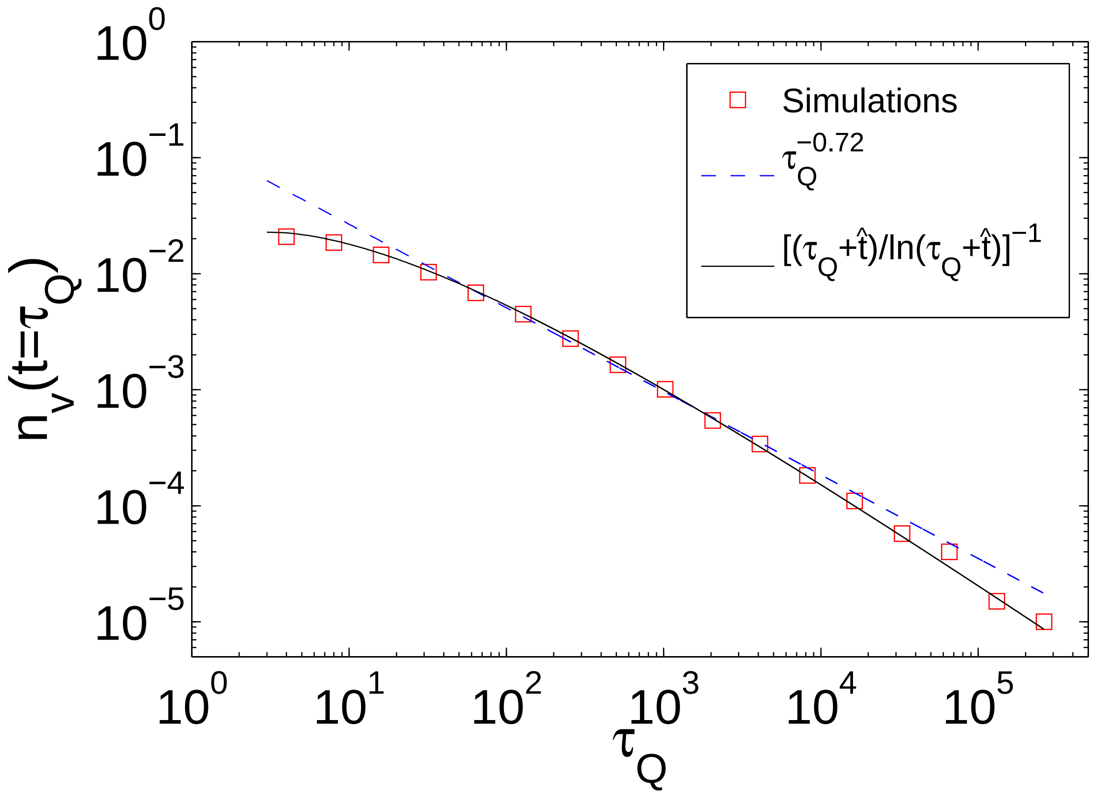
$$\boxed{R(t,T) \simeq \zeta(T) \left[\frac{t}{\ln(t/t_0)}\right]^{1/z_d}}$$

with $z_d = 2$ for NCOP

Dynamics in the $2d$ XY model

KT phase transition & coarsening

$$\boxed{n_v(t \simeq \tau_Q, \tau_Q) \simeq \ln[\tau_Q/\ln^2\tau_Q + \tau_Q]/(\tau_Q/\ln^2\tau_Q + \tau_Q)}$$



Large τ_Q

$$n_v \simeq \frac{\ln \tau_Q}{\tau_Q}$$

while

$$n_{\rm KZ} \simeq \frac{\ln^3 \tau_Q}{\tau_Q}$$

A. Jelić and LFC, J. Stat. Mech. P02032 (2011).

Work in progress

Quench rate dependencies in the dynamics of the

$3d\, O(2)$ relativistic field theory

$$c^{-2}\partial_t^2\psi(\vec{r},t) + \gamma_0\partial_t\psi(\vec{r},t) = \left[\nabla^2 - g(|\psi|^2 - \rho)\right]\psi(\vec{r},t) + \xi(\vec{r},t)$$

and the stochastic Gross-Pitaevskii equation

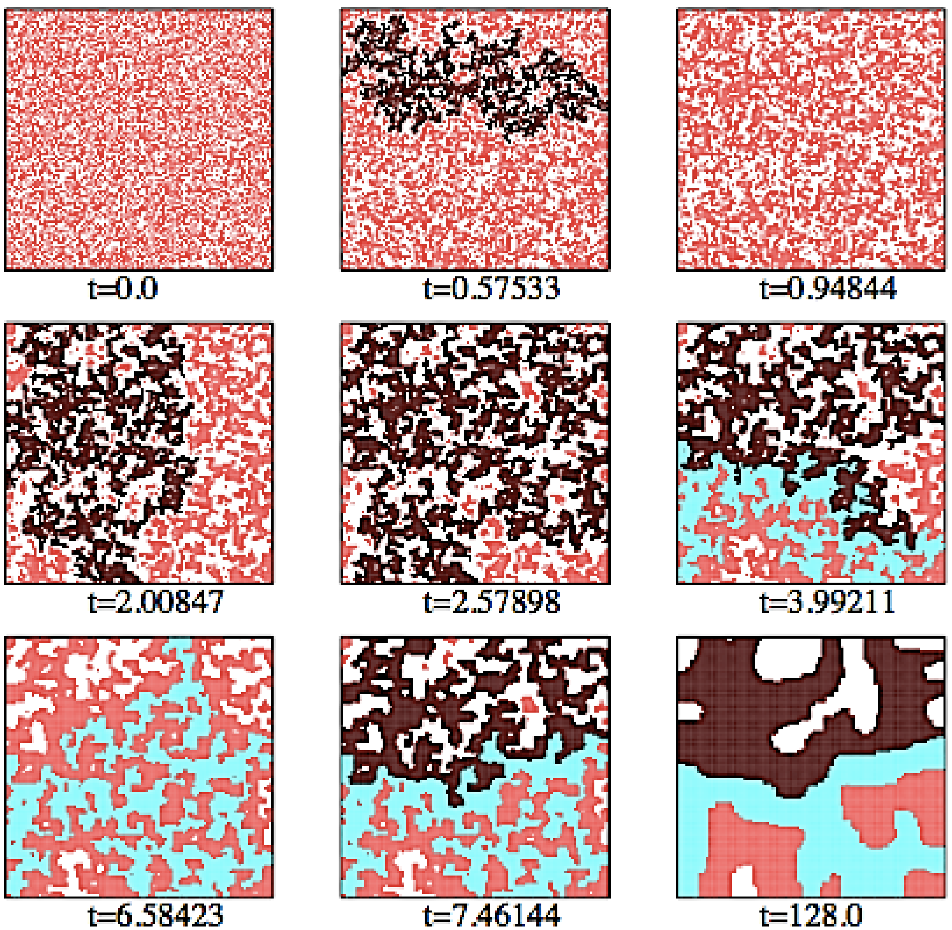
$$(-2i\mu + \gamma_L)\partial_t\psi(\vec{r},t) = \left[\nabla^2 - g(|\psi|^2 - \rho)\right]\psi(\vec{r},t) + \xi(\vec{r},t)$$

$(\psi(\vec{r},t) \in \mathbb{C})$

Study of **vortex lines**.

Kobayashi & LFC

Beyond density of defects



$2d$ Ising & voter models

Finite size effects &

short-time dynamics.

Distributions & geometry

Percolation & fractality

Arenzon, Blanchard, Bray

Corberi, LFC, Picco, Sicilia

& Tartaglia

Krapivsky & Redner

Conclusions

- The criterium to find the time when the system falls out of equilibrium above the phase transition ($-\hat{t}$) is correct ; exact results in the $1d$ Glauber Ising chain **P. Krapivsky, J. Stat. Mech. P02014 (2010).**
- However, defects continue to annihilate during the ordering dynamics ; their density at times of the order of the cooling rate, $t \simeq \tau_Q$, is **significantly lower** than the one predicted in **Zurek 85**.
- Experiments should be revisited in view of this claim (with the proviso that defects should be measured as directly as possible).
- Some future projects : annealing in systems with **other type of phase transitions and topological defects**.
- **Microcanonical quenches.**

Conclusions

Annealing in quantum dissipative systems

Same arguments apply though harder problem since

Quantum environment usually implies non-Ohmic ‘noise’ & non-Markov ‘dissipation’.

- Critical quenches $\mathcal{R}_c(t) \simeq t^{1/z_c}$

Bonart, LFC & Gambassi 11 (classical non-Ohmic)

Gagel, Orth & Schmalian 14 (quantum non-Ohmic)

- Quantum coarsening $\mathcal{R}(t) \simeq t^{1/z_d}$

Rokni & Chandra 04

Aron, Biroli & LFC 08

Slow quenches in a XY quantum spin chain coupled to a bath

Patané, Amico, Silva, Fazio, Santoro 09

Finite rate quenching protocol

Some details

Standard time parametrization $\boxed{g(t) = g_c - t/\tau_Q}$

$g = T/J$ implies $g_{max} \to \infty$ and therefore $t_{min} \to -\infty$

$g_{min} = 0$ $t_{max} = \tau_Q \, g_c$

Zurek's $\tau_{eq}(g) \simeq \left.\dfrac{\Delta g}{d_t \Delta g}\right|_{-\hat{t}} \simeq \hat{t} \quad \Rightarrow$

$$\boxed{\hat{t} \simeq \tau_Q^{\nu z_c/(1+\nu z_c)}}$$

$$\boxed{\Delta \hat{g} \simeq \tau_Q^{-1/(1+\nu z_c)}}$$

$$\boxed{\hat{R} \simeq \tau_Q^{\nu/(1+\nu z_c)}}$$

$2d$IM : $\hat{t} \simeq \tau_Q^a$ with $a \simeq 0.68$, $\Delta \hat{g} \simeq \tau_Q^{-b}$ with $b \simeq 0.31$, $\hat{R} \simeq \tau_Q^c$ with $c \simeq 0.31$.

Finite rate quenching protocol

Some details

Say $\boxed{g(t) = g_c - (t/\tau_Q)^x}$ after $-\hat{t}$ $\quad$ ($t_{max} = \tau_Q^x\, g_c$)

Non-trivial growth within the critical region

$$\boxed{R^* \simeq |\Delta g^*|^{-\nu} \simeq (t^*/\tau_Q)^{-x\nu} \simeq \hat{R} + c\,|t^* - \hat{t}|^{1/z_c}}$$

yields t^* and

$$R^* \simeq \begin{cases} \hat{R} & x < 1 \\ \hat{R}\, \tau_Q^{(x-1)/[x z_c(1+\nu z_c)]} & x \geq 1 \end{cases}$$

$2d$IM and, *e.g.*, $x = 2$: $\quad \hat{R} \simeq \tau_Q^{0.31}$ and $R^* \simeq \tau_Q^{0.39}$

$2d$IM and, *e.g.*, $x \to \infty$: $\quad \hat{R} \simeq \tau_Q^{0.31}$ and $R^* \simeq \hat{R}^{\frac{1+\nu z_c}{\nu z_c}} \simeq \tau_Q^{1/z_c} \simeq \tau_Q^{0.46}$