

## CORRECTIONS TO J-P. SERRE'S BOOKS

### **Abelian $\ell$ -adic Representations and Elliptic Curves** (second edition, AK Peters, 1988)

- p.I-26, line 3. Replace  $L_m(s - m/2)$  by  $L_m(s + m/2)$ .
- p.III-10, line 3. Replace chap. II, 1.1 by chap.II, 2.1 .
- p.III-10. The Proposition should be labelled Proposition 1.
- p.III-13, line 3. Replace chap.II, 2.2 by chap.II, 2.3 .
- p.III-15, line 3. Replace 2.4 by 2.5 .
- p.III-15, line -5,6. Replace proposition by theorem .
- p.III-15, line -1. Replace Remark 1 by Remark 2 .
- p.III-29, line 3. Replace Here  $K^* = (K \otimes R)^*$  by Here  $K_\infty^* = (K \otimes R)^*$  .
- p.III-39, line 1. Replace Proposition 3 by Proposition 4 .
- p.III-39, line 6. Replace A1 by A2.
- p.III-41, line -5. Replace Proposition 4 by Proposition 5 .
- p.III-48, line 3. Replace Proposition 5 by Proposition 6 .
- p.III-49, line -3. Replace Proposition 6 by Proposition 7 .
- p.III-50, lines 10-11. Replace Prop.5 by Prop.6 and replace Prop.6 by Prop.7 .
- p.III-52, line 2 of th.3. Replace  $G^{\text{al}}$  by  $G^{\text{ab}}$
- p.III-53, line -1. Replace Prop.6 by Prop.7 .
- p.IV-8, line 5. Replace  $u^{12}\Delta'$  by  $u_v^{12}\Delta'$  .
- p.IV-17, line -11. Replace section 4 by section 2.2 .
- p.IV-19, part (c) of Main Lemma. Replace  $G_\ell$  by  $\tilde{G}_\ell$  .
- p.B-7. Reference [80]. Add (=Oe.136) .

### **Algebraic Groups and Class Fields** (Springer-Verlag, 1988)

- Contents, Chapter VI , §34. Replace Map by Application .
- p.17, line -2. Replace ample by very ample .
- p.18, line -8. Replace varietities by varieties.
- p.36, line 4. Replace  $N_Q f$  by  $Nf$ .
- p.36, line 6. After with  $g$  , add ; here  $Nf$  means  $N_{L/K}f$ .

- p.36, line -9. In the displayed formula, replace  $>$  by  $<$ .
- p.108, line -2. Replace Châtalet by Châtelet.
- p.117. The title of §2.7 of Chap.VI should be Review of definitions about coverings .
- p.122, Proposition 11. Replace an isogeny by a separable isogeny.
- p.124, Corollary. Replace isogenies by separable isogenies.
- p.154, line 9. Replace non-published by unpublished.
- p.157. The title of §34 should be Application to the cycle class group .
- p.158, line 5. Replace Frobeniuis by Frobenius.
- p.199, [27]. Replace Tohoku by Tôhoku .
- p.202, [86]. Replace *finis* by *fini*.
- p.202, [90]. Replace algébre by algèbre.

**Local Fields** (Springer-Verlag, third printing, 1979)

[The corrections from p.110 to p.216 have been communicated to me by R. Gualdi.]

- p.14, line 18. Replace  $\mathfrak{p}B = \prod_{\mathfrak{p}|\mathfrak{p}} \mathfrak{P}^{e_{\mathfrak{p}}}$  by  $\mathfrak{p}B = \prod_{\mathfrak{p}|\mathfrak{p}} \mathfrak{P}^{e_{\mathfrak{p}}}$ .
- p.15, last line. Replace  $N : I_A \rightarrow I_B$  by  $N : I_B \rightarrow I_A$ .
- p.31, line 3. Replace  $K$  by  $\hat{K}$ .
- p.63, last line of prop.3. Replace  $e_{L/K}$  by  $e_{L/K'}$ .
- p.63, fifth line of proof of prop.3. Replace  $st$  by  $st$ .
- p.74, line -7. Replace twice  $\phi$  by  $\varphi$ .
- p.75, Lemma 5. Replace  $\phi$  by  $\varphi$ .
- p.110, line 7. Replace  $A \otimes X$  by  $\Lambda \otimes X$ .
- p.113, line 1. Replace  $f(g_0, \dots, g_i)$  by  $f(g_1, \dots, g_i)$ .
- p.174, line 6 of the proof of prop.5. Replace  $b' \in A_F$  by  $b' \in A_{F'}$ .
- p.182, line 4 of §2. The reference to §12 Bourbaki Alg.VIII is relative to the first edition. In the 2012 edition, this § is now §17.
- p.182, line -5. Replace  $\omega$  by  $w$ .
- p.183, line 3. Replace  $K^n$  by  $K^{n^2}$ .
- p.183, line 8. Replace result by results.
- p.185, line 4. Add the letter  $v$  on top of the arrow  $L^* \rightarrow \mathbf{Z}$ .
- p.186, line 4 after the Corollary. Replace Brauer group  $B_K$  of  $\bar{K}$  by Brauer group  $B_{\bar{K}}$  of  $\bar{K}$ .
- p.191, prop. 4 (a). Replace  $w \in K_s^*$  by  $w \in k_s^*$ .
- p.193, line 7. The arrow from  $H^2(\mathfrak{g}, \mathbf{Z})$  to  $H^1(\mathfrak{g}, \mathbf{Q}/\mathbf{Z})$  should go in the other direction.

p.206, line 5. The arrow  $\varphi_a : G \rightarrow \mathbf{Z}/\mathbf{Z}$  should be defined first as a map of  $G$  into the group  $\mu_n$  of the  $n$ -th roots of unity, and then as a map into  $\mathbf{Z}/n\mathbf{Z}$  by choosing a generator of  $\mu_n$  in order to identify  $\mathbf{Z}/n\mathbf{Z}$  and  $\mu_n$ .

p.206, prop.4 (iv). One should assume  $a \neq 1$ .

p.211, line -1. Replace  $\alpha'^\beta$  by  $a'^\beta$ .

p.216, prop.14. Formula (ii) should be  $[a, bb']_v = [a, b]_v + [a, b']_v$ .

p.216, prop.14 (v). Add *If* before  $[a, b]_v = 0$ .

### **Galois Cohomology** (Springer-Verlag, corrected second printing 2002)

p.vii. The title of §1.5 should be Free pro- $p$ -groups.

p.7, line -5. Same correction as above.

p.15, line 14. Replace  $M' \in C_K$  by  $M' \in C'_K$ .

p.36, line 15. Replace idèle classes by ideal classes.

p.49, line 7. Replace  $(A', A'')$ -principal by  $(A, A'')$ -principal.

p.129, line -1. Replace prop.36 by prop.37.

p.134, last line of exerc.3. Replace if the field  $k$  is  $(C_1)$  by if the field  $k$  has the following property :

$(C'_1)$  Every finite family  $f_1, \dots, f_m$  of homogeneous polynomials in  $k[x_1, \dots, x_n]$ , of degrees  $d_1, \dots, d_m$  with  $\sum d_i < n$ , has a non trivial zero in  $k^n$ .

Note that  $(C'_1)$  implies  $(C_1)$ . Whether the converse is true seems to be an open question.

p.194, line 20. Replace a SDNB by an SDNB .

### **Lie Algebras and Lie Groups** (Springer-Verlag, LN 1500, corrected fifth printing, 2006)

p.23, line 16. Replace  $H^2 = \{x, y\}$  by  $H^2 = \{xy\}$ .

p.120, exerc. 2 b). In the denominator of the formula,  $(i + j - k)$  should be  $(i + j - k)!$  .

### **Complex Semisimple Lie Algebras** (Springer-Verlag, 1987)

p.4, line -1. Replace This implication by The implication.

p.13, Lemma 1. Replace nilpotent by invertible.

p.22, line -15. Add a closed parenthesis ) after algebra".

p.32, line 8. Replace  $\beta \in S$  by  $\alpha \in S$ .

p.32, line -9. Replace for all  $\alpha \in s$  by for all  $\alpha \in S$ .

p.49, line 19. Replace shows by means.

p.52, line -6. Add a reference to Chevalley's note "Sur la classification des algèbres de Lie simples et de leur représentations" (CRAS 227 (1948), 1136-1138), where the relations  $(\theta_{ij}), (\theta_{ij}^-)$  are defined, and are used to prove results analogous to those given in the text (but somewhat weaker). Similar results were obtained slightly later by Harish-Chandra in his paper "On some applications of the universal enveloping algebra of a semisimple Lie algebra", TAMS 70 (1951), 28-96. These  $\theta$ -relations should thus be called "Chevalley relations" or "Chevalley-Harish-Chandra relations" - but not "Serre relations", as some people do.

- p.54, line -1. Replace  $\mathfrak{g}$  by  $\mathfrak{g}^\alpha$ .
- p.61, line 10. Replace  $E'_1$  by  $E'_i$ .
- p.68, line 9. Replace  $\mathfrak{g}$  by  $\mathfrak{g}$ .
- p.68, line 17. Replace  $\gamma \in P$  by  $\gamma \in P_1$ .

#### Local Algebra (Springer-Verlag, 2000)

- p.9. Rewrite the Corollary to Proposition 8 as follows :  
*Let  $\mathfrak{p} \in \text{Spec}(A)$ . Suppose  $M \neq 0$ . The following are equivalent :*
  - (1)  $\text{Ass } M = \{\mathfrak{p}\}$ .
  - (2)  $x_M$  is nilpotent for every  $x \in \mathfrak{p}$  and is injective for every  $x \notin \mathfrak{p}$ .
- p.92, line 3. Replace  $M$  if by  $M$  is.

#### A Course in Arithmetic (Springer-Verlag, corrected third printing, 1996)

- p.83, line -13. Replace this line by :  $\geq m^2 - |mn| + n^2 = |m\rho \pm n|^2$ .
- p.91, Proposition 7. Add (Euler) after Proposition 7.
- p.112, lines 7,8. Replace by : "G. Lejeune-Dirichlet - Beweis eines Satzes über die arithmetische Progression, 1837, *Werke* I, 307-312.

#### Trees (Springer-Verlag, corrected second printing, 2003)

- p.19, line 6 (without counting the picture). The sentence " The subtree generated by a set of vertices " should be in italics (it is a subtitle).
- p.63, line 7. Replace  $m + 2s$  by  $m + 2d$ .
- p.65, line -2. Replace prop.26 by prop.25.
- p.68, line 12. The reference to Tits Comptes rendus note should be : "Systèmes générateurs de groupes de congruence", CRAS 283 (1976), 693-695 = Collected Works vol.III, [100].
- p.84, line 5. Replace (twice)  $L_V$  by  $L_v$ .
- p.89, line -14. Replace  $\mu = (G/\Gamma^0)$  by  $\mu(G/\Gamma^0)$ .

p.112, line -10. Replace *apartment* by *sector*.

### Lectures on the Mordell-Weil theorem (Vieweg, third edition, 1997)

p.67, exercise. Remove item e).

p.148, line 5. Remove the word “normal” in “closed normal subgroup  $H$ ”.

p.149, line 19. Replace “corresonding” by “corresponding”.

p.151, line 5. Replace “weakend” by “weakened”.

p.162, Questions 1) and 2). An elliptic curve over  $\mathbf{Q}$  of rank  $\geq 28$  has been constructed by N. Elkies in 2006. Over  $\mathbf{Q}(\mathbf{T})$  (with non constant  $j$  invariant), the record is  $\geq 18$ , and it is also due to Elkies. See e.g. Elkies paper arXiv : 0709.2908.

p.162, line 12. No cap. in “variation”.

p.192, Table of the 13 values of  $j$  ... In the case  $d = -3, f = 3$ , the value of  $j$  is  $2^{15}3.5^3$ , and not  $-2^{15}3.5^3$ .

p.205, reference [12]. Replace “groupes” by “groupe”.

p.207, reference [Se1]. Replace “Quelque” by “Quelques”.

p.207, reference [Se2]. Replace “groups” by “groupes”.

### Collected Works I (Springer Verlag, second printing, 2003)

p.vii. Add to the list of Academies : Russia (2003), Norway (2009), Taiwan (2010), Torino (2010).

Add to the list of doctorates : Oslo (2002), Oxford (2003), Bucharest (2004), Barcelona (2004), Madrid (2006), McGill (2008), TsingHua (2017).

In “Cours dans des universités étrangères”, add the year 2007 to the Harvard list, and also : E.P.F.L. Lausanne (2011), Hsinchu (2009, 2011, 2013), Pohang (2011).

p.xix. The list of “textes non reproduits dans les Oeuvres” should be enlarged, by mentioning the following books :

“Exposés de Séminaires (1950-1999)”, S.M.F. Doc. Math. 1, 2008 ;

(with P. Colmez) “Grothendieck-Serre Correspondence” (bilingual edition), S.M.F. Doc. Math. 2 and A.M.S., 2004 ;

“Lectures on  $N_X(p)$ ”, C.R.C. Press, 2012 ;

(with P. Colmez) “Correspondance Serre-Tate” , S.M.F. Doc. Math. 13-14, 2015 ;

“Finite Groups : An Introduction”, International Press, 2016 ;

(with E. Howe, J. Oesterlé and C. Ritzenthaler) “Rational Points on Curves over Finite Fields”, S.M.F. Doc. Math. 18, 2020.

It should also mention the following papers :

“Cohomological invariants, Witt invariants and trace forms” (notes by Sip Garibaldi), ULS 28, A.M.MS. 2003, 1-100 ;

L’histoire de la “modularity conjecture”, SMF Gazette 91 (2002), 55-57.  
(with M. Rost and J-P. Tignol) La forme trace d’une algèbre simple centrale de degré 4, CRAS 342 (2006), 83-87.

(with V. Chernousov) Estimating essential dimensions via orthogonal representations, J. Algebra 305 (2006), 1055-1070

Bounds for the orders of the finite subgroups of  $G(k)$ , in *Group Representation Theory* (M. Geck, D. Testerman & J Thévenaz, edit.), EPFL Press, 2007, 405-450.

Three letters to Walter Feit on group representations and quaternions, J. Algebra 319 (2008), 549-557.

How to use finite fields for problems concerning infinite fields, *Contemp. Math.* 487, AMS (2009), 183-193.

La vie et l’oeuvre scientifique d’Henri Cartan, *Gazette des mathématiciens* 121 (2009), 65-70.

A Minkowski-style bound for the order of the finite subgroups of the Cremona group of rank 2 over an arbitrary field, *Moscow math. J.* 9 (2009), 183-198.

(with J-L. Nicolas) Formes modulaires modulo 2, CRAS 350 (2012), 343-348 and 449-454.

(with E. Bayer-Fluckiger & R. Parimala) Hasse principle for  $G$ -trace forms, *Izv. Math.* 77 (2013), 5-28.

Un critère d’indépendance pour une famille de représentations  $\ell$ -adiques, *Comment. Math. Helv.* 88 (2013), 541-554.

Bases normales autoduales et groupes unitaires en caractéristique 2, *Transf. Groups* 19 (2014), 643-698.

On the mod  $p$  reduction of orthogonal representations, in *Lie Groups, Geometry, and Representation Theory - a tribute to the life and work of Bertram Kostant* (V.G. Kac & V.L. Popov edit.), Birkhäuser (2018), 527-540.

La vie et l’oeuvre de Jean-Marc Fontaine, *Comptes Rendus Mathématique* 358 (2020), 1045-1046.

La vie et l’oeuvre de John Tate, *Comptes Rendus Mathématique* 358 (2020), 1129-1133.

(with E. Bayer-Fluckiger) Lines on cubic surfaces, Witt invariants and

Stiefel-Whitney classes, Indag. Math. 32 (2021), 920-938.

p.xxii. Add to the list of the Bourbaki seminars :

Complète Réductibilité, 2003/2004, n°**932**, 23 p.

Le groupe de Cremona et ses sous-groupes finis, 2008/2009, n°**1000**, 26 p.

Distribution asymptotique des valeurs propres des endomorphismes de Frobenius [d'après Abel, Chebyshev, Robinson,...], 2017/2018, n°**1146**, 43 p.

p.xxiii. Add a subsection named *Oberwolfach Reports* :

On the values of the characters of compact Lie groups, 2004, 666-667.

BL-bases and unitary groups in characteristic 2, 2005, 37-40.

Coordonnées de Kac, 2006, 1787-1790.

Le groupe quaquaversal, vu comme groupe S-arithmétique, 2009, 1421-1422.

Some aspects of the Sato-Tate conjecture, 2011, 1996-1997.

Cohomological invariants mod 2 of Weyl groups, 2018, 1284-1286.

p.594, lines 8-9. “on ignore si  $V$  est toujours simplement connexe”. This has been settled (positively) by J. Kollár, cf. Bourbaki seminar 905 (June 2002), cor. 3.6.

### **Collected Works II** (Springer Verlag, second printing, 2003)

p.3, line 6. Add “compactes ” before “connexes ” .

p.467, line 14. Add an arrow  $\rightarrow$  in the formula. It should be  $\varphi_2 : \Gamma_q(2) \rightarrow C$ .

p.508, line -6. Replace  $X^n$  by  $x^n$ .

p.714, line 5. Replace 1964 by 1974 in the reference to Deligne’s paper.

### **Collected Works III** (Springer Verlag, second printing, 2003)

p.265, line -13. Replace asymptotique by asymptotique.

p.449, line 2. In formula (6), replace  $\sum$  by  $\prod$ .

p.538, footnote. It seems that Brylinski’s proof needs some repair ; see J-L. Colliot-Thélène, Expo. Math. 23 (2005), 161-170.

### **Collected Works IV** (Springer Verlag, second printing, 2003)

p.358, line -4. Replace facon by façon.

p.378, line -16. Replace de  $G$  by de  $H$ .

p.389, line 8. Replace symétriqu by symétrique.

p.389, line -9. Delete the middle parenthesis in  $(\mathbf{F})_2)^r$ .

p.400, line -13. Replace certaine by certains.

p.406, line -4. Replace 8.5.4 by 6.5.4.

- p.414, line 8. Replace  $x, y \in V$  by  $x \in P, y \in V$ .
- p.446, line -7. Replace  $K_x$  by  $F_x$ .
- p.527. About the canonical structures of the supersingular elliptic curves, see the comments given in *Correspondance Serre-Tate*, vol.II, p.727.
- p.566, formula (104). Replace  $T$  by  $T'$ .
- p.568, line 2. Replace formule (13) by prop.4.
- p.676, line 14. Replace Il reste à traiter le cas 2-adique by :  
Le cas  $p = 2$  a été traité onze ans plus tard par les mêmes auteurs : *Ann. Math.* **172** (2010), 1391-1405.

**Linear Representations of Finite Groups** (Springer-Verlag, corrected third printing, 1986)

- p.11, line 10. Add a period after class function.
- P.15, line 6. Replace  $\chi$  by  $\psi$ .
- p.33, line -18. Delete the letter s at the end of representations.
- p.43. Exercises 5.4 and 5.5 should be labelled 5.5 and 5.6.
- p.50, last line of prop.12. Replace  $\tilde{\rho}(u)$  by  $\tilde{\rho}_i(u)$ .
- p.63, line 18. Replace  $W = \bigoplus_{\chi \in X} W$  by  $W = \bigoplus_{\chi \in X} W_\chi$ .
- p.76, line -1. Delete the extra parenthesis ) after the first term.
- p.86, line -1. In the formula  $P_{M,c_2} = P_{M,c_2}$ , replace the first  $c_2$  by  $c_1$ .
- p.105, line 11. Replace reresentation by representation.
- p.106, line -7. Replace  $C$  by  $\mathbf{C}$ .
- p.110, line 4. Delete the parentheses around  $\text{Cent}.R[G]$ .
- p.110, line -3. Replace ex.12.9 by ex.13.9.
- p.125, line 11. Replace 15.1 by 15.2.
- p.125, line -5. Replace By (a) by By the first case.
- p.132, line 11. Replace  $R_A(G)$  by  $P_A(G)$  and replace  $R_k(G)$  by  $P_k(G)$ .
- p.134, line 10. Replace  $P_K^+(G)$  by  $P_k^+(G)$ .
- p.136, part (c) of Proposition 46. Replace  $A[G]$  by  $k[G]$ .
- p.142, line -11. In the bottom line of the diagram, replace  $P_k(G)$  by  $R_k(G)$ .
- p.143, line 13. Replace  $R_{k'}(G)$  by  $R_{K'}(G)$ .
- p.143, line -9. Replace  $r_p$  by  $r_P$ .
- p.145, line -8. Add a parenthesis ) after  $b \in F$ ; delete the parenthesis ) after of  $E$ .
- p.149, line 9. Replace  $R_G(G)$  by  $R_K(G)$ .
- p.159, line -8. Replace  $\nu_F$  by  $a_G$ .



- p.162, line -7. In the formula, replace  $i \geq \ell$  by  $i \geq 1$ .
- p.163, line -1. Permute  $E$  and  $E'$ .
- p.164, line -10. The letter  $Z$  should be in boldface :  $\mathbf{Z}$ .
- p.168, line 8. Replace  $F_{\mathbf{C}(G):9.1}$  by  $F_{\mathbf{C}(G)} : 9.1$ .

**Exposés de Séminaires 1950-1999** (deuxième édition, augmentée, SMF, 2008)

- p.136, line -6. Replace “ne s’étend par” by “ne s’étend pas”.
- p.146, line -8. Replace “mas” by “mais”.
- p.253, line 10. Replace “fixed point of  $G$ ” by “fixed point of  $g$ ” .

**Cohomological invariants, Witt invariants, and trace forms** (p.1-100 of AMS ULS28, 2004)

- p.46, line -8. Replace “Choose  $\alpha$ ” by “Choose  $\alpha \neq 0$ ”.
- p.60, Lemma 25.12. Replace  $n \equiv i \pmod{2}$  by  $n \not\equiv i \pmod{2}$ , and vice-versa.
- p.70, line 7. Replace “Th.27.14” by “Th.27.15”.
- p.78, line -15. Add “with  $A, B, A^2 - B \neq 0$ ” after “ $\langle A, A^2 - B, (AB(A^2 - B)) \rangle$ ”.

**Grothendieck-Serre Correspondence** (Bilingual edition, SMF-AMS, 2004)

- p.137. In conditions  $C_n$  and  $C'_n$ , replace “ $i$ ” by “ $q$ ”.
- p.168, line -10, French side. Replace “où il est ” by “où 11 est ”.
- p.239. Replace the date of the letter by “1.15.1964” (this mistake was pointed out by R. Steinberg).
- p.288, English side. Replace the first two lines by : “Representations over  $\mathbf{Z}$ , 235-238”.

**Lectures on  $N_X(p)$**  (CRC Press, 2012)

[Many of these corrections were communicated to me by Francesc Fité.]

- p.16, line 13. Replace  $\Sigma_K$  by  $V_K$ .
- p.18, line 15. Replace Exerc.2 by Exerc.1.
- p.19, line -3. In the second term of the formula, add the letter  $m$  in front of  $t^{m-1}$ , as in the first term.
- p.21, line 14. Replace When is the by In that .
- p.25, line 1. Replace §3.3.3.3 by §3.3.3.4.

- p.25, line -1. Replace §3.3.2.2 by §3.3.3.2.
- p.35, line -3. Replace th.4.5 by th.4.6.
- p.36, line -7. Replace cf. [Se 84, p.81] by cf. §4.6.2.
- p.46, line 18. Replace  $\Psi^{k+k'}$  by  $\Psi^{kk'}$ .
- p.57, line 21. Replace 5.2.1. Densities by 5.2.2. Densities.
- p.58, line -8. Replace 5.2.2 by 5.2.3.
- p.60, line -4. Replace twice Aut by Out .
- p.68, line 8. Replace §3.3.2.2 by §3.3.3.2.
- p.68, line 10. Replace §3.3.2.2 by §3.3.3.2.
- p.68, line 13. Replace §3.3.2.3 by §3.3.3.3.
- p.69, line 1. Replace This shows that by Hence.  
After the image , add  $(g_{p,i,n})$ . The modified line should then be :  
Hence  $N_X(p)$  mod  $\ell^n$  depends only on the image  $(g_{p,i,n})$  of  $g_p$  in the
- p.69, line 2. Replace §3.3.2 by §3.3.3.
- p.70, line 5. Replace §3.3.2.2 by §3.3.2.
- p.70, line 7. Replace th.6.2 by th.6.3.
- p.71, line 20. Replace th.6.2 a) by th.6.3.
- p.71, line 22. Replace §3.3.2.1 by §3.3.2. Replace th.6.2 d) by th.6.3 d).
- p.71, line 27. Replace th.6.15 by th.6.17.
- p.80, line 18. Replace §6.1.2 by §6.2.1.
- p.86, lines 16, 18, 20, 24. In each of these four lines there occurs a minus sign which should be replaced by a boldface minus sign “ **−** ”, similar to the one which appears on line 21.
- p.86, line -2. Replace is smooth by is proper and smooth.
- p.92, lines 3, 5, 17. Replace (three times)  $N_p(X)$  by  $N_X(p)$ .
- p.98, line 7. Replace Proposition 7.9 by Proposition 7.10.
- p.119, line 15. Replace Proof of 8.1.4.2 by Proof of 8.1.4.1.
- p.119, line 17. Replace the minus sign by a boldface minus sign “ **−** ”.
- p.119, line 21. Replace uppe by upper.
- p.121, line -11. Replace  $X$  by  $X_1$ .
- p.121, line -10. Replace  $Y$  by  $X_2$ .
- p.121, line -5. Replace §8.5.4 by §8.5.5.
- p.121, line -1. The letter  $Q$  in  $\Gamma_Q$  should be boldface.
- p.122, line 1. The letter  $Q$  in  $\Gamma_Q$  should be boldface.
- p.122, line 4. Replace  $G$  by  $G_X$ .
- p.123, line -9. Insert a blank space before Other axioms.
- p.124, line 16. Replace totaly by totally.
- p.129, line -1. Replace [SS 12] by [Sa 12].

- p.133, line 13. Replace Corollary 7.1.3 by Corollary 7.13.
- p.134, line 2. Add a dot after cf .
- p.136, line 9. Replace there exists  $t \in$  by there exists  $t' \in$ .
- p.140, line -9. Replace §9.4.3 by §9.4.2.
- p.141, line 7. Replace It will be then by It will then.
- p.141, line 11. Replace restrited by restricted.
- p.141, line 20. The letter  $A$  is lacking in front of the formula. The left side of it should be  $A_T^1(f, x) =$ .
- p.142, line 4. Replace  $\varphi$  by  $\psi$ .
- p.143, line -1. Delete the vertical bar on the left, just after  $\frac{1}{|G|}$ . The formula should be  $\frac{1}{|G|} \sum_{g \in G} \dots$
- p.149, reference [EJ 10]. Replace arXiv : 1006.0721 by Geom. Dedicata 159 (2012), 29-40.
- p.149, reference [FKRS 11]. Replace in preparation by Compos. Math.148 (2012), 1390-1442.
- p.151, reference [KP 99]. Replace Jaczorowki by Jaczowski .
- p.152, reference [SS 12]. Replace by :  
[Sa 12] T. Saito, *The discriminant and the determinant of a hypersurface of even dimension*, Math. Research Letters 12 (2012), 855-871.
- p.159, line -5. Replace 3.3.2 by 3.3.3.
- p.161, line 9. Replace 3.3.3.2 by 3.4.1.2.
- p.162, line -1. Replace 3.3.2 by 3.3.3.2.

**Finite Groups - an Introduction, first edition**, International Press, 2016

[The errors pointed out below have been corrected in the revised second edition, published by International Press in January 2022.]

About half of these corrections are taken from the home page of Bjorn Poonen.

Besides B. Poonen and myself, the following people have contributed to this list : Anlong Chua, Peter Mizes, Timothy Ngotiaoco, Ahaan Rungta, Adam Theriault-Shay, Chase Vogeli.

Another half was contributed by J. Sangroniz.

- p.1, *Note*. Replace  $G \times X$  by  $X \times G$ .
- p.1, Definition 1.2. Add “and every  $x \in X$ ” after “every  $g \in G$ ”.
- p.2, lines 4-5. Definition 1.5 : faithful, free, torsor should be boldface (not just their first letters).

- p.2, *Example*. Replace “automorphisms” by “permutations”.
- p.2, line 27. Replace “We have” (before (1.2)) by “By (1.1), we have”.
- p.5, Definition 1.8. Replace “ $i = 1, \dots$ ” by “ $i = 0, \dots$ ”.
- p.8, line 14. Replace that line by “ $H.G_1 = G_1 \times G_2$ . Since both  $H$  and  $G_2$  normalize  $N_1$ , this shows that  $G_1 \times G_2$ ”.
- p.9, lines 11,12,13. Replace “ $(y, 1, v)$ ” by “ $(y, 1, \dots, 1, v)$ ” and replace “ $(xyx^{-1}y^{-1}, 1, 1)$ ” by “ $(xyx^{-1}y^{-1}, 1, \dots, 1)$ ”.
- p.10, exerc.5. Replace  $(H : H') = 3$  by  $(H : H') = 2$ .
- p.10, exerc.7(a). Replace  $x/h$  by  $h/x$ .
- p.11, exerc.11(i). Replace “ $gx_i = y_i$  for  $i = 1, 2$ ” by “ $gx_1 = x_2$  and  $gy_1 = y_2$ ”.
- p.13, exerc.21. In the final sentence, replace  $\mathcal{S}_q$  by  $C_q$  (cyclic group of order  $q$ ).
- p.18, line 8. Replace “imply” by “implies”.
- p.18, proof of prop.2.11(2). Replace  $(G : H)$  by  $(G : S)$ .
- p.19, line 7. Replace  $H$  by  $S$  in the first sentence of §2.4.
- p.24, exerc.3(b), middle of the Hint. Replace “ordre” by “order”.
- p.25, exerc.7(b). The last displayed line should be

$$x_m \mapsto x_m + a_m(x_1, \dots, x_{m-1}),$$

with  $m$  instead of  $n$  each time it occurs.

- p.25, exerc.8(b). Replace “The of  $G$ ” by “The conjugation action of  $G$ ”.
- p.26, exerc.9. Rewrite that exercise as follows :  
Let  $J$  be the set of 5-Sylow subgroups of  $\mathcal{S}_5$ . We have  $|J| = 6$ .
- a) Use the action of  $\mathcal{S}_5$  on  $J$  to show that  $\mathcal{S}_5$  is isomorphic to a transitive subgroup  $H$  of  $\mathcal{S}_6$ .
- b) Use the action of  $\mathcal{S}_6$  on  $\mathcal{S}_6/H$  to define an automorphism of  $\mathcal{S}_6$  which is not inner<sup>1</sup>.
- p.26, footnote 3. Replace the letters  $c, s$  by  $x, y$ , in order to avoid a conflict with the use of “ $c$ ” in exerc. 15(a).
- p.27, exerc.15(b), line 2. Delete one of the two commas after  $c$ .
- p.27, exerc.15(d). In the Hint, replace “ $S$ -conjugation” by “ $S$ -conjugacy”.
- p.27, exerc.15(e). In the Hint, replace twice  $a_1$  by  $a_0$ .
- p.27, exerc.2.16(b). Replace  $q \equiv 3 \pmod{16}$  by  $q \equiv \pm 3 \pmod{8}$ .
- p.28, line 3. Replace “prop.2.4” by “cor.2.15”.
- p.30, line 5. Replace  $|K| \leq 3$  by  $|K| = 2$ .
- p.31, Corollary 3.3. Rewrite the proof of (i) as follows :

Choose an abelian subgroup  $A$  of  $G$  having property (3) of prop.3.2, and take for  $N$  the set of  $a \in A$  with  $a^p = 1$ , where  $p$  is a prime divisor of  $|A|$ .

p.32, line 8. Extend the definition of  $B_i$  to all  $i \geq 0$  by putting  $B_i = 1$  for  $i \geq n$ ; similarly, define  $V_i$  to be 0 for  $i \geq n$ .

p.32, line -8. Replace “condition (2)” by “condition (2)’”.

p.37, Proposition 3.17. Replace “ $p$ -group” by “finite  $p$ -group”.

p.37, footnote. Replace “ $K$ -linear projector  $K' \rightarrow K$ ” by “ $K$ -linear projector of  $K'$  onto  $K$ ”.

p.38, th.3.18 (3). Remove the word “proper”.

p.38. Rewrite the proof of th.3.18 as follows :

*Proof.* (1) follows from the fact that  $G$  is nilpotent, cf. cor.3.12; it implies (2). For (3), it is enough to prove the existence of  $H_2$  with  $H \subset H_2 \subset G$  and  $(G_2 : H) = p$ . To do so, use induction on  $|G|$ . If  $N_G(H) \neq G$ , apply the induction assumption to  $N_G(H)$ . If  $N_G(H) = G$ , choose a subgroup of  $G/H$  of order  $p$  and take for  $H_2$  its inverse image in  $G$ .

p.38, th.3.20 (5). Replace “Two elements” by “Any two elements”.

p.39, line -2. Replace “cor.3.18” by “part (4) of th.3.18”.

p.41, line -5. Replace “ $((a(zb))z)\bar{z}$ ” by “ $((az)z)b\bar{z}$ ”.

p.42, proof of th.3.23. Replace (12 times) “ $i(g)$ ” by “ $i(s)$ ”.

p.43, proof of th.3.27. Replace “the theorem above” by “th.3.26”.

p.44, line 4. Replace “assume” by “Assume now”.

p.44, line 11, proof of prop.3.29. Replace  $t(g)e(g)t(g')e(g') = t(gg').e(g)e(g')$  by  $g.e(g)g'.e(g') = gg'.e(gg')$ .

p.46. Remove exerc.1, since it has already been done in cor.3.3 (ii).

p.46, exerc.2 (iii). Replace “exists” by “exist”.

p.47, lines 1 and 2. Replace “ $x, y$ ” by “ $u, v$ ”.

p.47, line 5. Replace “ $b_1$ ” by “ $b_2$ ”.

p.47, line 7 of exerc. 7. Replace “ $\lambda \in k$ ” by “ $\lambda \in K$ ”.

p.47, line 8 of exerc. 7. Replace “of  $U$ ; this proves a). As for e)” by “of  $\tilde{U}$ ; this proves e). As for a)” .

p.47, exerc. 8 a). Delete the hypothesis that the  $A_i/A_{i+1}$  are abelian. Add : [Hint : Let  $g \in G_n$ . Show that  $g$  acts on  $A_i/A_{i+n+1}$  by  $x \mapsto xz$ , for some  $z \in A_{i+n}/A_{i+n+1}$ . If  $h \in G_1$ , show that  $hgh^{-1}$  acts on  $A_i/A_{i+n+1}$  by  $x \mapsto xh(z)$ . Since  $h$  acts trivially on  $A_{i+n}/A_{i+n+1}$ , conclude that  $g^{-1} \cdot hgh^{-1}$  acts trivially on  $A_i/A_{i+n+1}$ , hence belongs to  $G_{n+1}$ .]

p.48, exerc.13 d). Replace “denotes” by “denote”.

p.53, line 8. Replace “ $C^{n+1}(g, A)$ ” by “ $C^{n+1}(G, A)$ ”.

- p.54, line -6. Replace “obtain” by “obtains”.
- p.58, line 7. Replace by “have  $h'(x) = a \cdot x a^{-1} h(x) = ah(x)a^{-1}$ .”
- p.58, Theorem 4.13. Replace “of  $G$ ” by “of  $E$ ”.
- p.59, line 5 of §4.5.1. Replace “composion” by “composition”.
- p.59, line 4 of §4.5.2. Replace  $\text{Out}(G)$  by  $\text{Out}(A)$ , and  $\text{Aut}(G)$  by  $\text{Aut}(A)$ .
- p.59, th.4.16. Replace “is and only if” by “if and only if”.
- p.59, line -9. Replace “and  $f$ ” by “and  $\varphi$ ”.
- p.60, formula (4.10). Replace “ $e \bullet_f e'$ ” by “ $e *_f e'$ ”.
- p.60, line after Theorem 4.17. Replace “ $H^2(G, A)$ ” by “ $H^2(G, Z(A))$ ”.
- p.61, line 10. Delete the second occurrence of  $A'$ .
- p.61, lines 2-4 of the proof of th.4.20, part II. After “ $E = A.E'$ .”, add “Hence the homomorphism  $E' \rightarrow G$  is surjective; its kernel  $A'$  is equal to  $E' \cap A$  and we have the exact sequence  $1 \rightarrow A' \rightarrow E' \rightarrow G \rightarrow 1$ .”
- p.62, proof of th.4.20, part III, lines 3-4. Replace “Let  $p$  ... its inverse image” by : “We may assume that  $G \neq 1$ . Then, by cor.3.3(i),  $G$  has a nontrivial abelian normal  $p$ -subgroup  $I$  for some prime  $p$ . Let  $\tilde{I}$  be its inverse image”.
- p.62, proof of th.4.20, part III, line 7. Replace “ $I_1$  by  $aI_1a^{-1}$ ” by “ $G_1$  by  $aG_1a^{-1}$ ”.
- p.62, proof of th.4.20, part III, line 10. After “ $G_1$  and  $G_2$  are conjugate”, add “by an element of  $A \cap N$ ”.
- p.63, Note after Proposition 4.23. Replace “ $G_{\psi'}$ ” by “ $\psi'_0(G)$ ” and replace “ $G_{\psi}$ ” by “ $\psi_0(G)$ ”.
- p.65, exerc.2(b). The identity should be “ $\partial_i \partial_j F = \partial_j \partial_{i-1} F$  if  $i > j$ ”.
- p.66, line 8. Replace “ $z^{n/4}$ ” by “ $z^{n/2}$ ”.
- p.66, exerc.5, case (ii) : Replace “agument” by “argument”, “same image” by “same images”, and “than” by “as”.
- p.66, exerc.7(b), Hint. Replace  $H$  by  $A$ .
- p.67, exerc.8. Replace  $(1, z)$  by  $(1, nz)$ .
- p.67, exerc.8(a). Replace  $(g, z)$  by  $(g, 0)$ .
- p.67, titles of exerc.10 and 11. Replace “ $H^2(G, A)$ ” by “ $H^2(G, Z(A))$ ”.
- p.68, line 10. Replace “ $\psi(c)$ ” by “ $\psi(s)$ ” and replace “ $a$ ” by “ $x$ ”.
- p.68, line 17. Replace “ $Z(A)_D/Z(A)^T$ ” by “ $Z(A)_T/Z(A)^D$ ”.
- p.68, lines 12, 13. Replace “ $Z(A)_D$ ” by “ $Z(A)^D$ ” and replace “ $Z(A)^T$ ” by “ $Z(A)_T$ ”.
- p.69, exerc.16(c), Hint. Replace “prop.4.22” by “cor.4.22”.
- p.70, line -14. Delete the letter “ $n$ ” in can be written uniquely  $n$ ”.
- p.71, just before th.5.4. Replace “ th.8.21” by “th.8.62”.

- p.73, proof of th.5.9. In the sentence after the display, replace “th.4.6” by “th.4.20(1)”.
- p.74, proof of prop.5.11(1). Replace “ $p$ -Sylow” by “ $\pi$ -Sylow”.
- p.75, proof of th.5.13. Replace “lemma 5.8” by “cor.5.8”.
- p.79, th.6.6. In (1’), “after “such that”, replace “ $\mathfrak{q}O_K$  is the  $p$ -th power of an ideal of  $O_K$ ” by “ $p$  divides the residue field degree  $[O_K/\mathfrak{q}_K : O_k/\mathfrak{q}]$  for every prime ideal  $\mathfrak{q}_K$  of  $O_K$  lying above  $\mathfrak{q}$ ”.
- p.83, lines -6,-7. Replace (four times) “ $n$ ” by “ $m$ ”.
- p.83, line -5. Replace “ $G$ ” by “ $H$ ”.
- p.84, proof of th.6.13 (1)  $\Rightarrow$  (2). Replace “If  $p$  a prime factor . . .” by “If  $p$  is a prime factor . . .”.
- p.85, comment after the proof of cor.6.14. Replace “property 6.14” by “property  $\mathcal{F}$ ”.
- p.86, line 3. Replace “exerc.6” by “exerc.11”.
- p.86, exercice. 1 d). Add the assumption that  $G$  acts faithfully on  $X$ .
- p.86, exerc.3. Add “, except 18,” after “in this list”. Rewrite the next sentence as “Show that there are two non isomorphic Frobenius groups of order 18; same question for the orders 42 and 48”.
- p.86, exerc.4. Replace “as” by “has”.
- p.94, proof of prop.7.18. On the first line, add “of minimal order” after “odd order  $N < 2000$ ”; delete the product sign in front of “ $p_1^{m_1} \dots p_n^{m_n}$ ”. On the sixth line, replace “number” by “numbers”.
- p.95, line 15. Replace “(mod 5))” by “(mod 5)”.
- p.96, line 11. Replace “ $(c + id)$ ” by “ $(ci + d)$ ”.
- p.96, lines 16 and 22. Replace “ $P$ ” by “ $\mathbf{P}$ ”.
- p.96, line 18. Replace “th.7.15” by “Th.7.15”.
- p.96, line 27. Replace “ $w(2\lambda) = 4\lambda$ ” by “ $w(2\lambda) = 4$ ”, and replace “ $w(4\lambda) = 2\lambda$ ” by “ $w(4\lambda) = 2$ ”.
- p.101, exerc.9(b), Hint. Replace “transfert” by “transfer”.
- p.102, exerc.17 b). Delete the sentence “Hence the action of  $B/U$  on  $\mathbf{P} - \{\infty\}$  is faithful.”
- p.102, exerc.17 d). Replace the second sentence by “Use Th.7.5 to prove the existence of  $w \in G$  such that  $t.wt^{-1} \in D(B) = U$ ; by replacing  $w$  by  $uw$  with a suitable  $u \in U$ , show that one can achieve  $t.wtw^{-1} = 1$ .”
- p.106, second line of the proof of prop.8.5. Delete one of the two commas of “every  $i,$ , ”.
- p.109, line 3. Replace “ $f(\rho_1(g))(x)$ ” by “ $f(\rho_1(g)(x))$ ”.
- p.110, Proof of Proposition 8.16. Replace “By th.8.4” by “By th.8.15”.

p.112, line 1. Replace “ $\chi \in \text{Irr}$ ” by “ $\chi \in \text{Irr}(G)$ ”.

p.118, Example 3. Replace “ $C_p$ ” by “ $C_g$ ”.

p.121, line 9. Replace “th.8.5” by “th.8.22”.

p.121, proof of lemma 8.49. Replace “ $V$ ” by “ $E$ ” (three times).

p.122, line -12. Replace “ $V_0 \otimes_{\mathbf{R}} \mathbf{C}$ ” by “ $\mathbf{C} \otimes_{\mathbf{R}} V_0$ ”.

p.123, Rewrite lines 2 to 8 as follows :

Let us write  $h(x, y)$  as  $\langle x, y \rangle$ . For every  $y \in E$ , the map  $x \mapsto B(x, y)$  is linear; hence there exists a well-defined  $\phi(y) \in E$  such that  $B(x, y) = \langle x, \phi(y) \rangle$ . The map  $\phi : E \rightarrow E$  so defined is antilinear (i.e., such that  $\phi(\lambda x) = \bar{\lambda}\phi(x)$ ) and bijective. Since  $B$  is symmetrical, we have :

$$\langle x, \phi(y) \rangle = \langle y, \phi(x) \rangle \quad (8.19)$$

Hence :

$$\langle \phi(x), \phi(y) \rangle = \langle y, \phi^2(x) \rangle. \quad (8.20)$$

p.124, line 3. Replace “ $\langle \chi^2, 1 \rangle = 0$ ” by “ $\langle \chi^2, 1 \rangle = 0$ ”.

p.124, line 7. Replace “form” by “forms”.

p.125, last line. Replace “on  $G$ ” by “on  $H$ ”.

p.131, exerc. 4 a). Replace “ $|X_s|$ ” by “ $|X_s|$ ”.

p.131, exerc. 4 b). Replace “ $Z_G(g)$ ” by “ $C_G(g)$ ”.

p.131, exerc. 4 c). Replace “methof” by “method”.

p.132, line 5. Replace “ $Z_G(g)$ ” by “ $C_G(g)$ ”.

p.132, exerc.6(c). Replace by “Construct an example of b) such that  $G$  acts transitively both on  $X$  and on  $Y$ ”.

p.134, exerc.13 a). Replace the Hint by “Hint : if  $m$  is not prime to  $|G|$ , the  $m$ -th power map is not surjective. This implies that the action of  $\Psi^m$  on the ring of complex class functions is not injective, hence the same is true for its action on  $R(G)$ .”

p.135, exerc. 19. Replace b) by : b) Assume that  $|G|$  is divisible by  $p$ ; write  $|G|$  as  $p^m q$ , with  $(q, p) = 1$ . Let  $f$  be the function equal to  $q$  on the elements of  $G$  of order  $p$ , and equal to 0 elsewhere. Prove the inclusions  $f \in R'(G) - R(G)$  and  $p^m f \in R(G)$ . Deduce that the index of  $R(G)$  in  $R'(G)$  is divisible by  $p$ .

p.139, line 7. Replace “apply d)” by “apply b)”.

p.139, line 9. Replace “central function” by “class function”.

p.139, line 12. Replace twice “ $\iff$ ” by “ $\implies$ ”.

p.140, line 2. Replace “denotes” by “denote”.

p.140, line -3. Replace “ $\mathbf{GL}_V$ ” by “ $GL(V)$ ”.

p.141, line -2. Replace “ $\langle \tilde{f}, \theta_H \rangle_G$ ” by “ $\langle \tilde{f}, \theta \rangle_G$ ”.



- p.142. Remove the stray symbols +.. after th.9.1.
- p.144, second sentence of §9.1.3. Replace “if  $x$  in a non-zero integer” by “if  $x$  is a non-zero integer”.
- p.146, last line of 9.1.5. The index  $n$  for the symmetric group should be  $m$ .
- p.149, proof of part (2) of prop.9.12. The end of the first line should be “ $N(au) = N(a)$ ”. Replace the second line by : “Similarly,  $ua.(au)^* = uaa^*u^{-1}$ , hence  $N(ua) = \text{Tr}(uaa^*u^{-1}) = \text{Tr}(aa^*) = N(a)$ .”
- p.152, §9.2.7, complement 2. Delete “the” in “the Lefschetz’s principle” .
- p.152, §9.2.7, end of complement 2. Add : “ See also M.J. Larsen & R. Pink, Finite subgroups of algebraic groups, *J. Amer. Math. Soc.* **24** (2011), 1105-1158.”
- p.153, line -12. Replace “commutent” by “commute”.
- p.154, line -6. Replace “wigth” by “with”.
- p.164, line -2. Replace “J.W.C. Cassels” by “J.W.S. Cassels”.
- p.166, line -6. Replace “polyedral” by “polyhedral”.
- p.170, Index, line 5. Add : Adams operations : 8.8.2.
- p.173, add at the end of the letter-p list :  $\Psi^m = m$ -th Adams operation, 8.8.2.
- p.173, add a new line : “ $R'(G)$  : 8.8.1.”

**Rational Points on Curves over Finite Fields** (with contributions by Everett Howe, Joseph Oesterlé and Christophe Ritzenthaler), S.M.F. Doc. Math. 18, 2020.

- p.viii, line 21. Replace “Chapter 6 is about” by “Chapter 7 is about”.
- p.104, line 5. Replace “. Then :” by “, then”.

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J-P. Serre, June 2022