COLLÈGE
DE FRANCE
1530

Chaire de Physique Mésoscopique
Michel Devoret
Année 2011, 10 mai - 21 juin

AMPLIFICATION ET RETROACTION QUANTIQUES

QUANTUM AMPLIFICATION AND FEEDBACK

Première Leçon / *First Lecture*

Transparents des leçons disponibles à http://www.physinfo.fr/lectures.html

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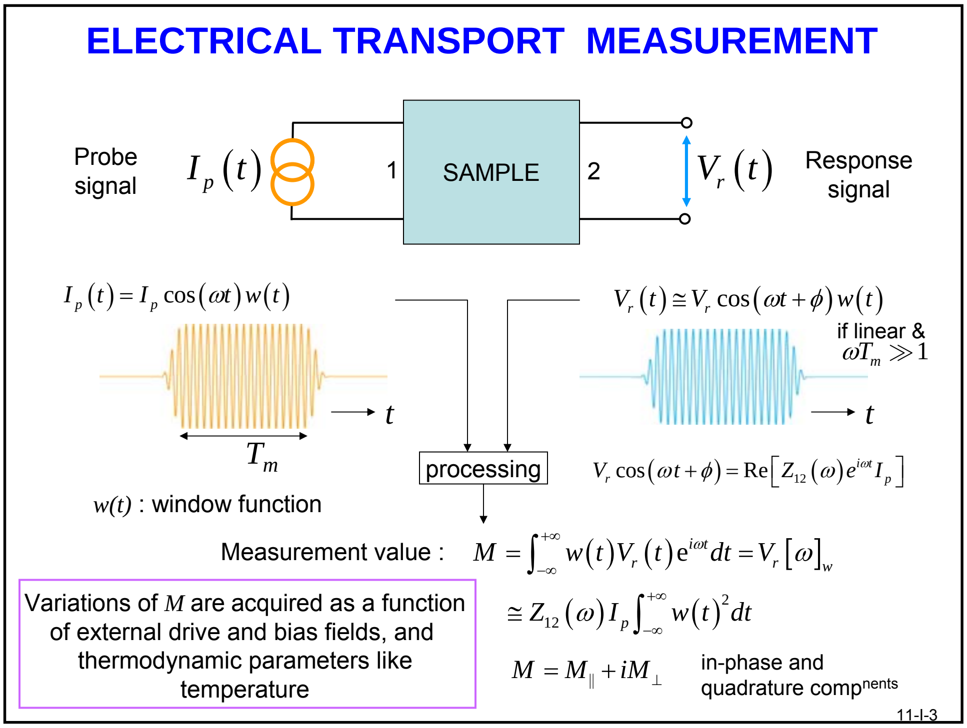
LECTURE I : INTRODUCTION TO AMPLIFICATION AND FEEDBACK OF ENGINEERED QUANTUM SYSTEMS

CONTENTS

1. Measurements, noise and amplification
2. Caves' theorem, link with detection in quantum optics
3. Are amplifiers and photomultipliers equivalent?
4. Principle of Josephson parametric amplifiers
5. Using quantum amplifiers in mesoscopic physics
6. Measurement based-feedback



ELECTRICAL TRANSPORT MEASUREMENT

Probe signal $I_p(t)$ 1 SAMPLE 2 $V_r(t)$ Response signal

$I_p(t) = I_p \cos(\omega t)\, w(t)$

$\longrightarrow t$

T_m

w(t) : window function

$V_r(t) \cong V_r \cos(\omega t + \phi)\, w(t)$

if linear & $\omega T_m \gg 1$

$\longrightarrow t$

processing

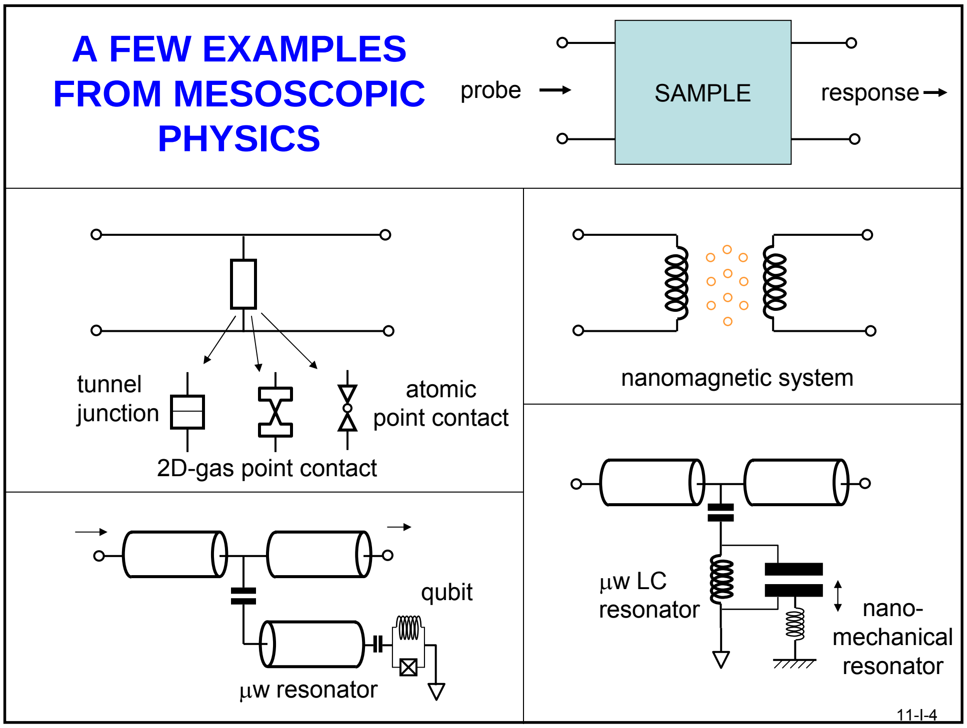
$V_r \cos(\omega t + \phi) = \mathrm{Re}\left[Z_{12}(\omega) e^{i\omega t} I_p \right]$

Measurement value : $M = \int_{-\infty}^{+\infty} w(t) V_r(t) \mathrm{e}^{i\omega t} dt = V_r[\omega]_w$

$\cong Z_{12}(\omega) I_p \int_{-\infty}^{+\infty} w(t)^2 dt$

$M = M_\parallel + i M_\perp$ in-phase and quadrature comp^{nents}

Variations of M are acquired as a function of external drive and bias fields, and thermodynamic parameters like temperature



A FEW EXAMPLES FROM MESOSCOPIC PHYSICS

probe → SAMPLE response →

tunnel junction

2D-gas point contact

atomic point contact

nanomagnetic system

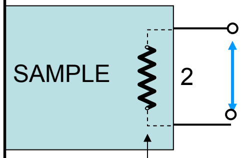
qubit

μw resonator

μw LC resonator

nano-mechanical resonator

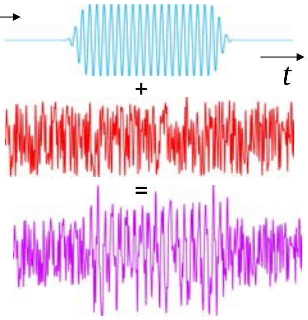
NOISE REDUCES THE INFORMATION EXTRACTED FROM MEASUREMENT



$R = \mathrm{Re}\left[Z_{22}(\omega)\right]$

$$V_2(t) = V_r(t) + V_N(t)$$

$$M = \underset{\substack{\uparrow \\ \text{deterministic}}}{V_r[\omega]_w} + \underset{\substack{\uparrow \\ \text{stochastic}}}{V_N[\omega]_w}$$



3 origins for noise: a) probe, b) sample, c) processing.

signal energy: $E_S = \frac{1}{R}\int_{-\infty}^{+\infty} V_r(t)^2 dt \cong \frac{2}{R}\left|V_r[\omega]_w\right|^2$

noise energy: $E_N = \frac{2}{R}\left|V_N[\omega]_w\right|^2$

signal to noise: $SNR = \frac{E_S}{E_N}$

Shannon information: $\mathcal{I} = \ln_2\left(1 + \frac{E_S}{E_N}\right)$ (strict validity requires noise be gaussian)

$SNR = 1 \Rightarrow$ 1 bit of information in value of M

THE NOISE OF ONE PHYSICIST........
.... MAY BE THE SIGNAL OF ANOTHER

courtesy Binarywolf and Dan Prober

noise spectral density:

$$S_{VV}[\omega] = \lim_{T_m \to \infty} \left\langle \left|V_N[\omega]_w\right|^2 \right\rangle / \int_{-\infty}^{+\infty} w(t)^2 dt$$

$$\left\langle V_N[\omega_1] V_N[\omega_2] \right\rangle = 2\pi\delta(\omega_1 + \omega_2) S_{VV}[\omega_1]$$

Stationarity $V_N[\omega] = \int_{-\infty}^{+\infty} V_N(t) \mathrm{e}^{i\omega t} dt$

$$V_N[-\omega] = V_N[+\omega]^*$$

Wiener-Kinchin theorem: $S_{VV}[\omega] = \int_{-\infty}^{+\infty} \mathrm{e}^{i\omega t} \left\langle V_N(t+\tau) V_N(t) \right\rangle d\tau$

In thermal equilibrium at T: $S_{VV}[\omega] = \hbar\omega\left(1 + \coth\frac{\hbar\omega}{2k_B T}\right)\mathrm{Re}\left[Z_{22}(\omega)\right]$

Fluctuation-Dissipation theorem :
1-to-1 relation between spectral density
and response function (here impedance).

$\hbar\omega \ll k_B T \Rightarrow S_{VV}[\omega] = 2k_B T\,\mathrm{Re}\left[Z_{22}(\omega)\right]$ Johnson Nyquist

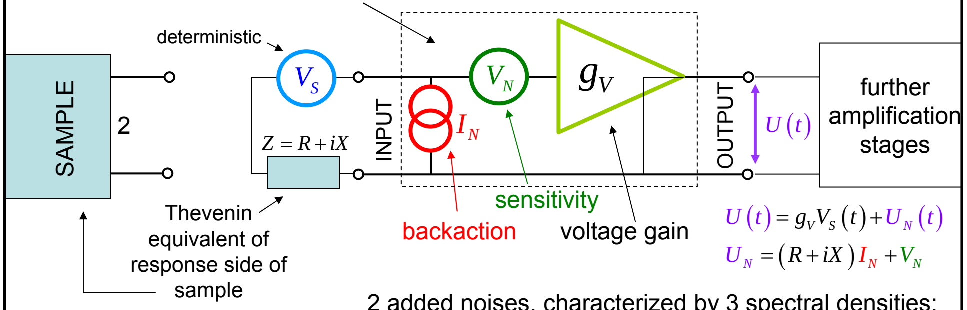
$\hbar\omega \gg k_B T \Rightarrow S_{VV}[\omega] = \left|\hbar\omega\right| + \hbar\omega$

Out-of-equilibrium, noise can reveal useful information on steady-state properties

AMPLIFIERS REMOVE NOISE IN PROCESSING STAGE BUT ADD NOISE OF THEIR OWN

Noise in measurement has 3 origins: a) lack of control of probe
b) sample itself
c) processing ← cause removed by amplification

Idealized representation of op-amp type amplifier:



$$U(t) = g_V V_S(t) + U_N(t)$$
$$U_N = (R + iX) I_N + V_N$$

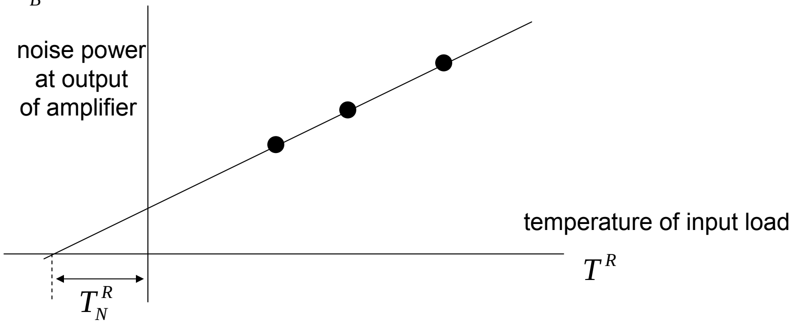
2 added noises, characterized by 3 spectral densities:

$$S_{II}[\omega], S_{VV}[\omega], S_{IV}[\omega]$$

Total noise energy: $$E_N^Z = \frac{2}{R}\left(|Z|^2 S_{II}[\omega] + S_{VV}[\omega] + 2\,\mathrm{Re}\left[Z(\omega) S_{IV}[\omega]\right]\right)$$

NOISE TEMPERATURE

$T_N^R = \frac{E_N^R}{4k_B}$: equivalent temperature of load giving the same output noise

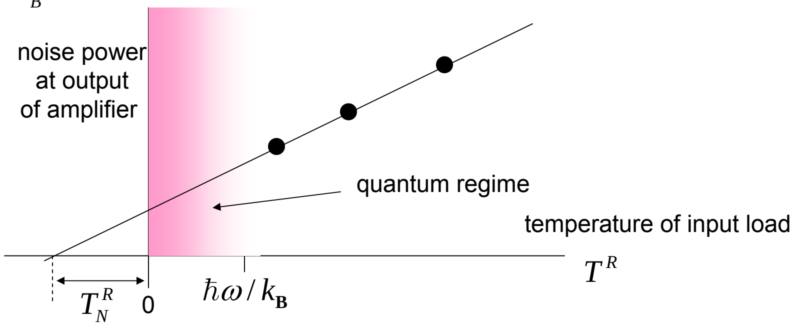


$$k_B T_N^{R,X} = \frac{(R^2 + X^2)}{2} S_{II} + \frac{1}{2R} S_{VV} + \mathrm{Re}[S_{IV}] - \frac{X}{R}\mathrm{Im}[S_{IV}]$$

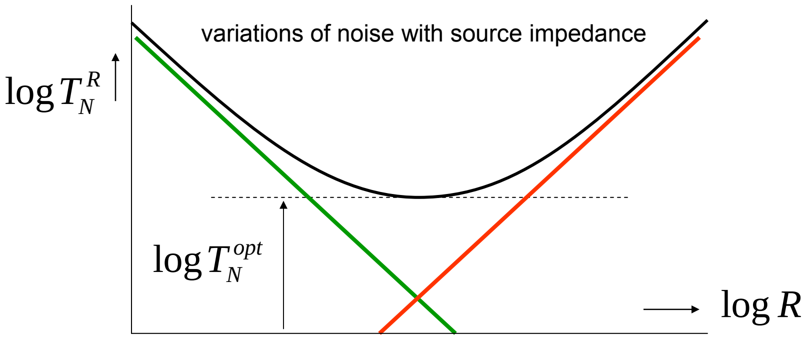
Optimize X : $$k_B T_N^R = \frac{R}{2} S_{II} + \frac{1}{2R} S_{VV} - \frac{|\mathrm{Im}[S_{IV}]|^2}{2RS_{II}} + \mathrm{Re}[S_{IV}] \qquad X^{opt} = \frac{\mathrm{Im}[S_{IV}]}{S_{II}}$$

NOISE TEMPERATURE

$T_N^R = \dfrac{E_N^R}{4k_B}$: equivalent temperature of load giving the same output noise



OPTIMUM NOISE TEMPERATURE OF AMPLIFIER



$$R_{opt} = \sqrt{\frac{S_{VV}}{S_{II}} - \frac{\left|\mathrm{Im}[S_{IV}]\right|^2}{(S_{II})^2}} = R_N$$

noise resistance of amplifier

T_N^{opt} is intrinsic to the amplifying process itself

Results are easily generalized to finite input impedance amplifiers

THE QUANTUM LIMIT OF AMPLIFICATION

Optimal noise energy : $k_B T_N^{opt} = \sqrt{S_{VV} S_{II} - \left[\operatorname{Im} S_{IV}\right]^2} - \operatorname{Re} S_{IV}$

QUANTUM LIMIT $\boxed{k_B T_N^{opt} \geq \frac{\hbar\omega}{2}}$ (*Caves, 1982*)

The minimum noise energy is equivalent to half-a-photon at the signal frequency.

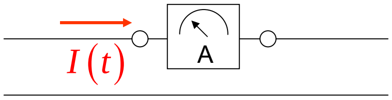
Relevant in RF mesoscopic measurements: f =10GHz implies T_N > 250mK >> T_{base}

Can be understood as a generalization of the Fluctuation-Dissipation Theorem to active systems (*Clerk et al., 2010*)

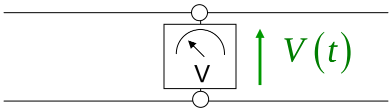
THE PROOF OF THIS THEOREM IS NOT CONSTRUCTIVIST.
HOW DOES ONE REACH THIS LIMIT IN PRACTICE, FOR A USEFUL DEVICE?

CAVEAT: IT IS POSSIBLE TO PERFORM OTHER KINDS OF AMPLIFICATION WITHOUT NOISE

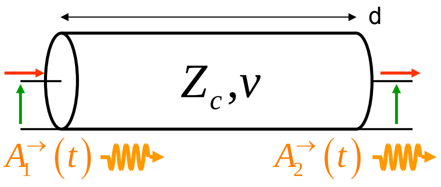
DIFFERENT DESCRIPTIONS OF A SIGNAL



CURRENT



VOLTAGE



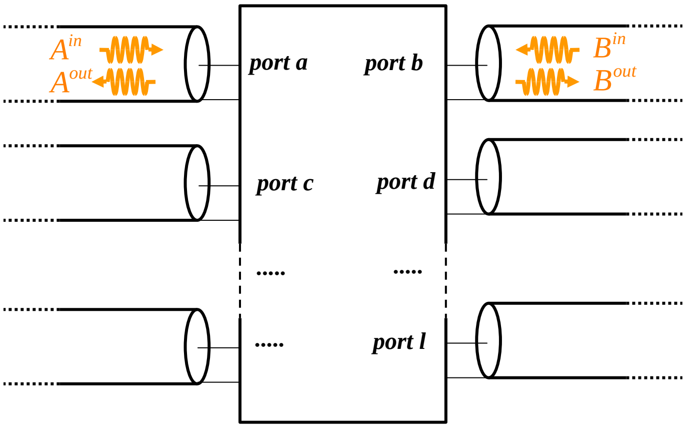
WAVE AMPLITUDE

$$A^{\rightleftarrows}(t) = \frac{V(t)}{2\sqrt{Z_c}} \pm \frac{\sqrt{Z_c}\, I(t)}{2}$$

$$A_2^{\rightleftarrows}\left(t \pm d/v\right) = A_1^{\rightleftarrows}(t)$$ dimension: (watts)$^{1/2}$

$\left|A^{\rightarrow}\right|^2 - \left|A^{\leftarrow}\right|^2 = P^{\rightarrow} - P^{\leftarrow} = P$: total energy flux through a section of line

SPATIAL MODE DESCRIPTION: SIGNAL PORTS



notation: port index vs signal amplitude symbol

REPRESENTATIONS OF A LINEAR, PHASE-PRESERVING AMPLIFIER

OP-AMP

$$\begin{bmatrix} V_b \\ I_a \end{bmatrix} = \begin{bmatrix} g_V & z_{\text{output}} \\ y_{\text{input}} & g_I \end{bmatrix} \begin{bmatrix} V_a \\ I_b \end{bmatrix} + \begin{bmatrix} g_V V_N \\ I_N \end{bmatrix}$$

ADMITTANCE

$$\begin{bmatrix} I_a \\ I_b \end{bmatrix} = \begin{bmatrix} Y_{aa} & Y_{ab} \\ Y_{ba} & Y_{bb} \end{bmatrix} \begin{bmatrix} V_a \\ V_b \end{bmatrix} + \begin{bmatrix} I_{aN} \\ I_{bN} \end{bmatrix}$$

SCATTERING

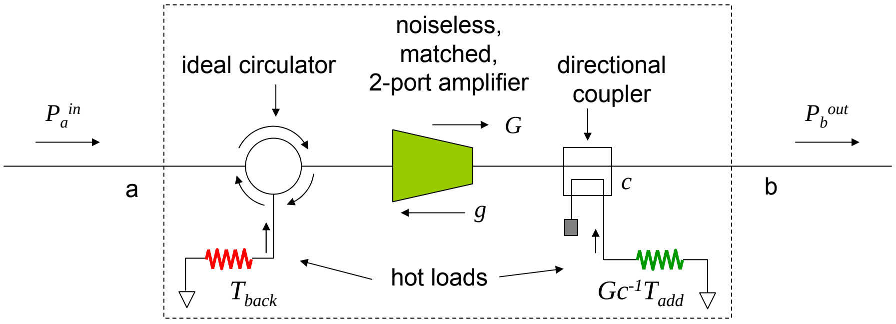
$$\begin{bmatrix} A^{out} \\ B^{out} \end{bmatrix} = \begin{bmatrix} r_{aa} & t_{ab} \\ t_{ba} & r_{bb} \end{bmatrix} \begin{bmatrix} A^{in} \\ B^{in} \end{bmatrix} + \begin{bmatrix} A_N^{out} \\ B_N^{out} \end{bmatrix}$$

Noise: $\begin{cases} S_{AA}[\omega] \\ S_{BB}[\omega] \\ S_{AB}[\omega] \end{cases}$

$|t_{ba}|^2 = G(\omega)$: Power gain

MICROWAVE AMPLIFIER CHARACTERISTICS

REPRESENTATION OF MATCHED 2-PORT AMPLIFIER



- Forward power gain ($G = P_b^{out}/P_a^{in}$)
- Added noise temperature $T_{add} = S_{BB}/Gk_B = T_N$
- Backaction noise temperature $T_{back} = S_{AA}/Gk_B$
- Correlation temperature $T_{corr} = |S_{AB}|/k_B$

- Directionality $D = G/g$
- Signal bandwidth BW
- Tuning bandwidth
- Dynamic range

Commercial HEMT amplifiers: T_N ~5K, BW ~4GHz@10GHz

CLASSIFICATION OF AMPLIFIERS

"PHASE-INSENSITIVE" AMPLIFIERS :

Phase preserving : $$B^{out} = \sqrt{G}A^{in} + B_N^{out}$$

Phase conjugating : $$B^{out} = \sqrt{G}\left(A^{in}\right)^* + B_N^{out}$$

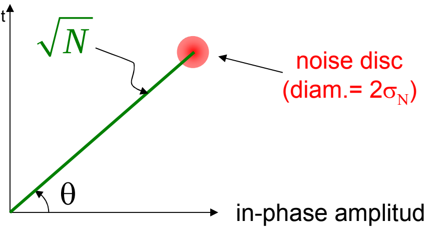
Gain is independent of signal phase

"PHASE-SENSITIVE" AMPLIFIERS :

$$\begin{aligned} B^{out} &= \sqrt{H}A^{in} + \sqrt{K}\left(A^{in}\right)^* + B_N^{out} \\ &= \sqrt{G_{\parallel}}A_{\parallel}^{in} + i\sqrt{G_{\perp}}A_{\perp}^{in} + B_N^{out} \end{aligned}$$

Gain depends on signal phase

GEOMETRIC REPRESENTATION OF SIGNAL MODE



noise disc
(diam.= $2\sigma_N$)

out-phase amplitude comp$^{\text{nent}}$

$$\mathrm{Im}\left[a_\mu\right] = a_\mu^\perp$$

μ : mode index
(spatial and temporal)

in-phase amplitude comp$^{\text{nent}}$

$$\mathrm{Re}\left[a_\mu\right] = a_\mu^\parallel$$

N = signal mode energy in photon number
θ = signal mode phase

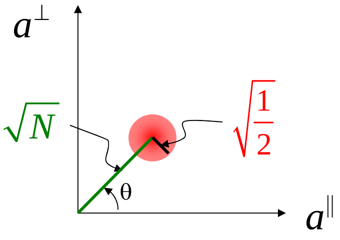
FRESNEL VECTOR ⇨ FRESNEL "LOLLYPOP"

Classical $A_\mu \to$ Quantum $a_\mu = \dfrac{A_\mu}{|\hbar\omega|^{1/2}} \to \hat{a}_\mu$

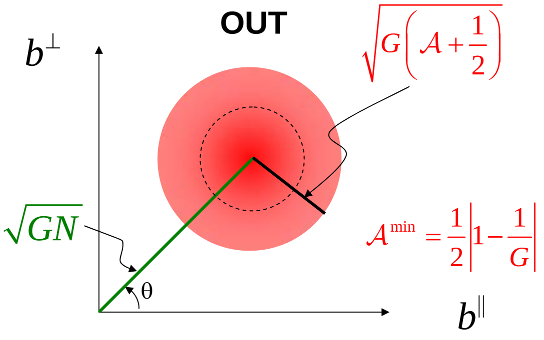
Thermal equilibrium $\sigma_N = \sqrt{\dfrac{1}{2}\coth\dfrac{\hbar\omega_\mu}{2k_BT}}$

QUANTUM LIMITED AMPLIFICATION WITH A LINEAR, PHASE-<u>PRESERVING</u> AMPLIFIER

IN



OUT

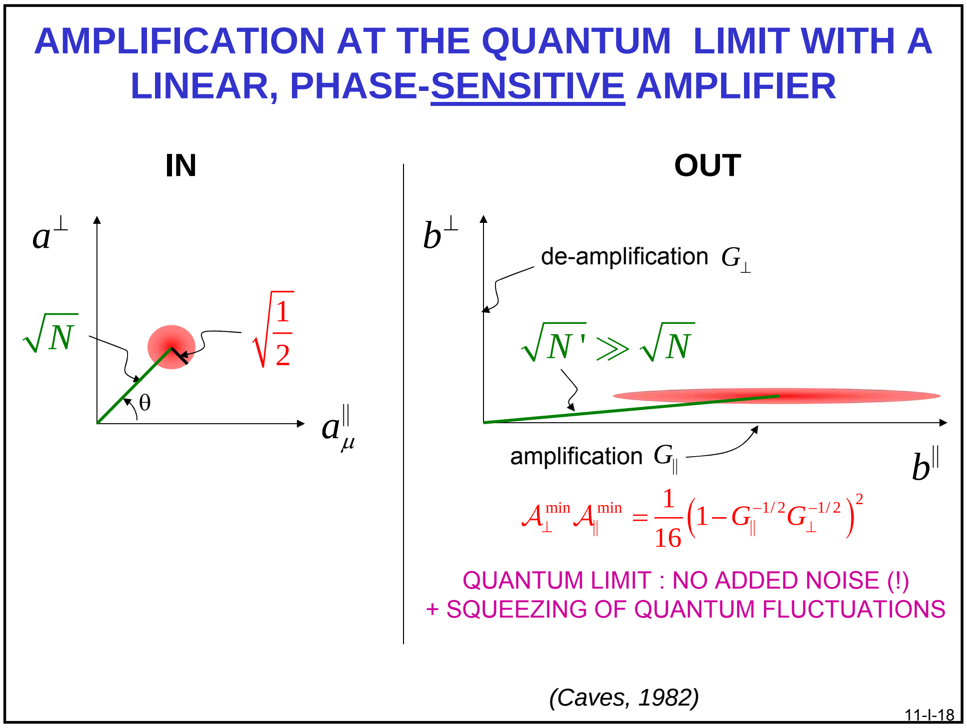


$$\mathcal{A}^{\min} = \frac{1}{2}\left|1-\frac{1}{G}\right|$$

STAND$^{\text{ARD}}$ QUANTUM LIMIT: AMPLIFIER ADDS ONLY ANOTHER ½ PHOTON OF NOISE !
MINIMUM REQUIRED BY HEISENBERG PRINCIPLE FOR A MEASUREMENT OF BOTH QUADRATURES

$\mathcal{A}^{\text{HEMT's}} \approx 20$ $\mathcal{A}^{\text{JPA's}} < 1$

Shimoda, Takahasi and Townes, J. Phys. Soc. Jpn. 12, 686 (1957); Haus and Mullen, Phys. Rev. 128, 2407 (1962); Caves, Phys. Rev. D 26, 1817 (1982)

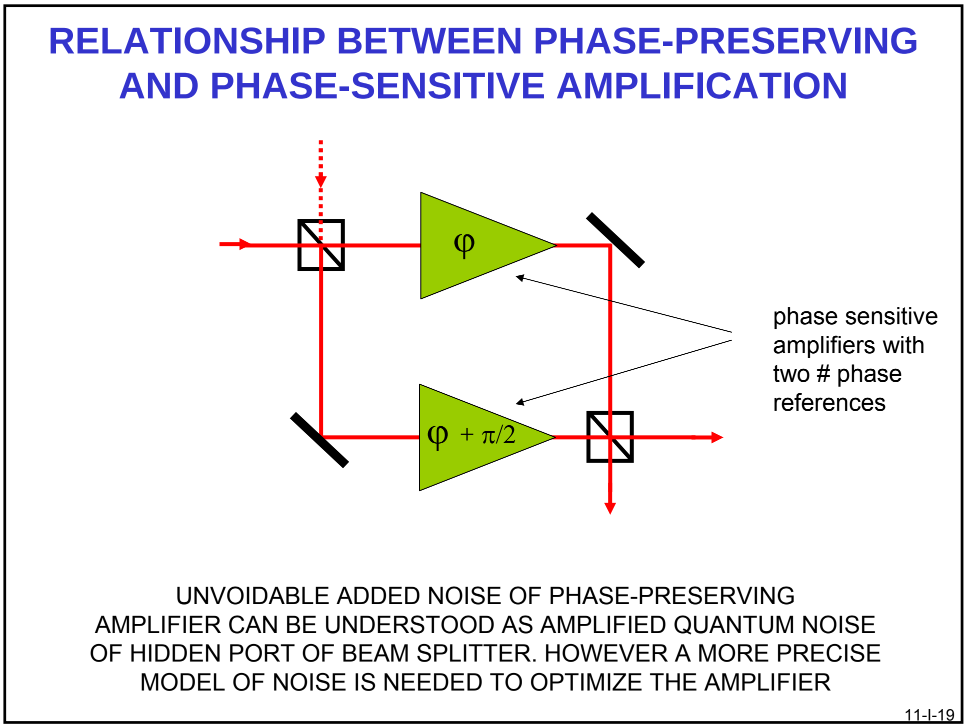


AMPLIFICATION AT THE QUANTUM LIMIT WITH A LINEAR, PHASE-SENSITIVE AMPLIFIER

$$\mathcal{A}_\perp^{\min}\mathcal{A}_\parallel^{\min} = \frac{1}{16}\left(1-G_\parallel^{-1/2}G_\perp^{-1/2}\right)^2$$

QUANTUM LIMIT : NO ADDED NOISE (!)
+ SQUEEZING OF QUANTUM FLUCTUATIONS

(Caves, 1982)



RELATIONSHIP BETWEEN PHASE-PRESERVING AND PHASE-SENSITIVE AMPLIFICATION

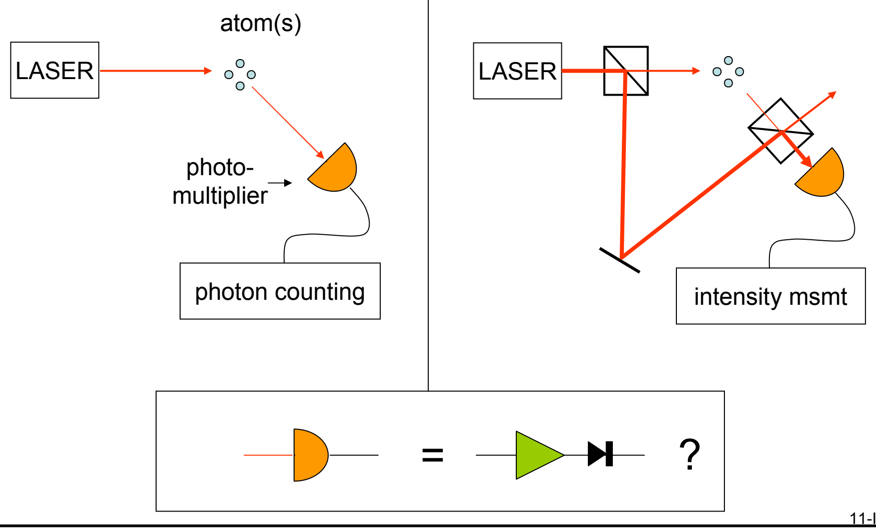
phase sensitive amplifiers with two # phase references

UNVOIDABLE ADDED NOISE OF PHASE-PRESERVING AMPLIFIER CAN BE UNDERSTOOD AS AMPLIFIED QUANTUM NOISE OF HIDDEN PORT OF BEAM SPLITTER. HOWEVER A MORE PRECISE MODEL OF NOISE IS NEEDED TO OPTIMIZE THE AMPLIFIER

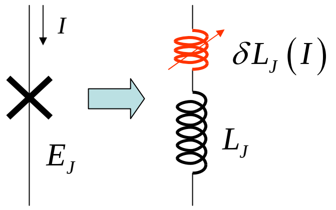
LINK WITH QUANTUM OPTICS

DIRECT DETECTION

HOMODYNE DETECTION



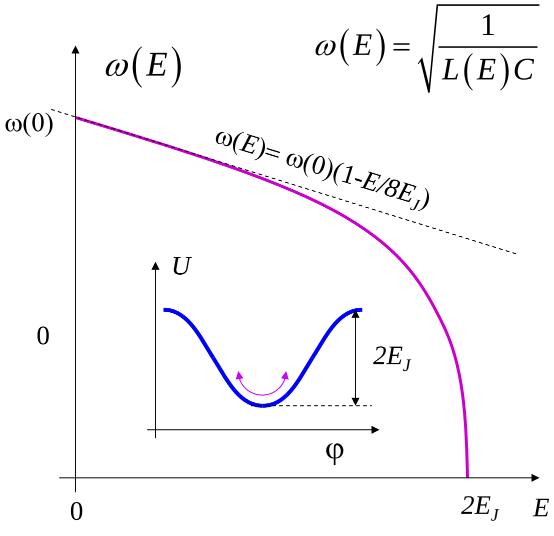
PARAMETRIC AMPLIFIER RESOURCE: JUNCTION NON-LINEAR INDUCTANCE



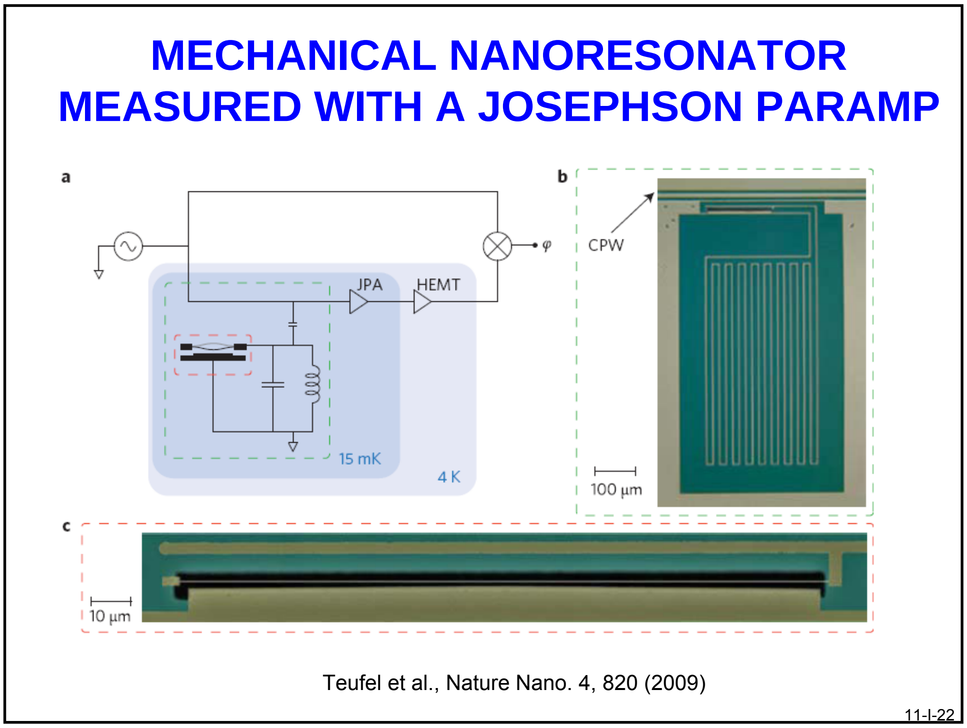
$$L_J = \frac{\hbar^2}{(2e)^2 E_J}$$

$$\delta L_J = L_J\left[\left(1-\frac{I^2}{I_0^2}\right)^{-1} - 1\right]$$

$$= \frac{1}{2} L_J \frac{I^2}{I_0^2} + O\left(\frac{I^4}{I_0^4}\right)$$

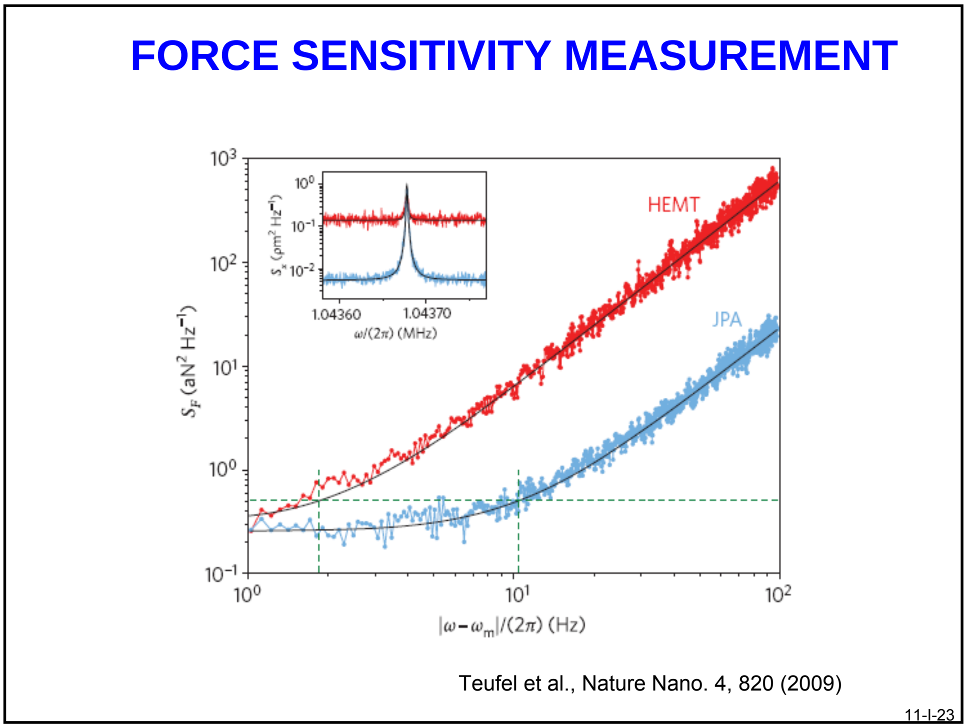


$$\omega(E) = \sqrt{\frac{1}{L(E)C}}$$



MECHANICAL NANORESONATOR MEASURED WITH A JOSEPHSON PARAMP

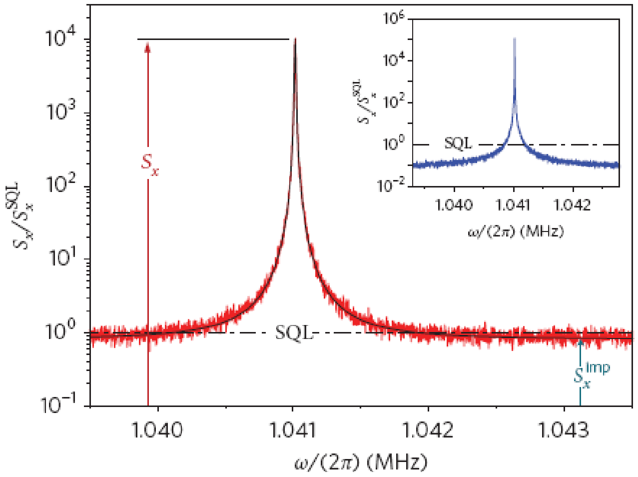
Teufel et al., Nature Nano. 4, 820 (2009)



FORCE SENSITIVITY MEASUREMENT

Teufel et al., Nature Nano. 4, 820 (2009)

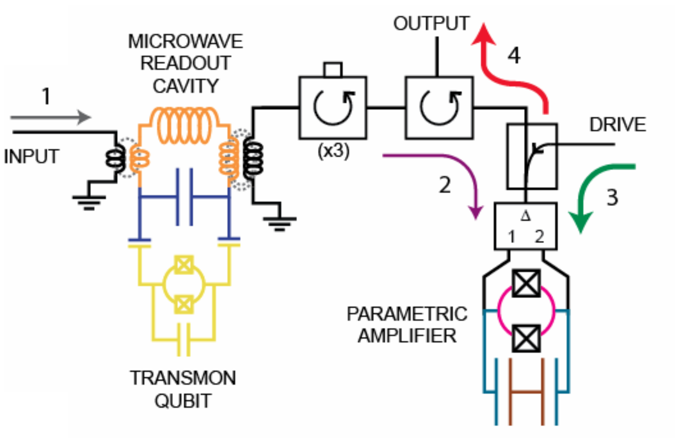
OBSERVATION OF MOTION IMPRECISION OF NANOMECHANICAL RESONATOR BELOW THE STANDARD QUANTUM LIMIT



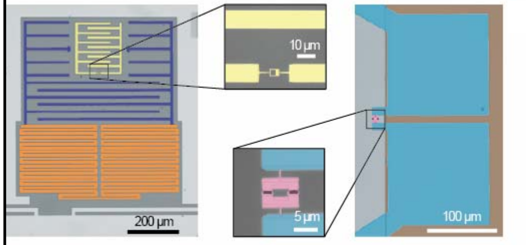
Teufel et al., Nature Nano. 4, 820 (2009)

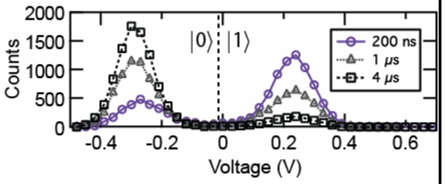
READOUT OF A SUPERCONDUCTING QUBIT WITH A JOSEPHSON PARAMP

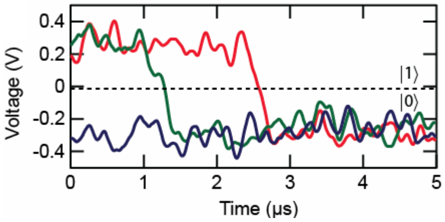
Vijay, Slichter and Siddiqi, Phys. Rev. Lett. **106**, 110502 (2011)

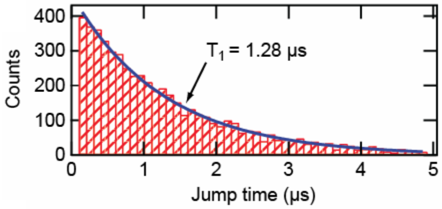


OBSERVATION OF QUANTUM JUMPS IN A CONDENSED MATTER SYSTEM





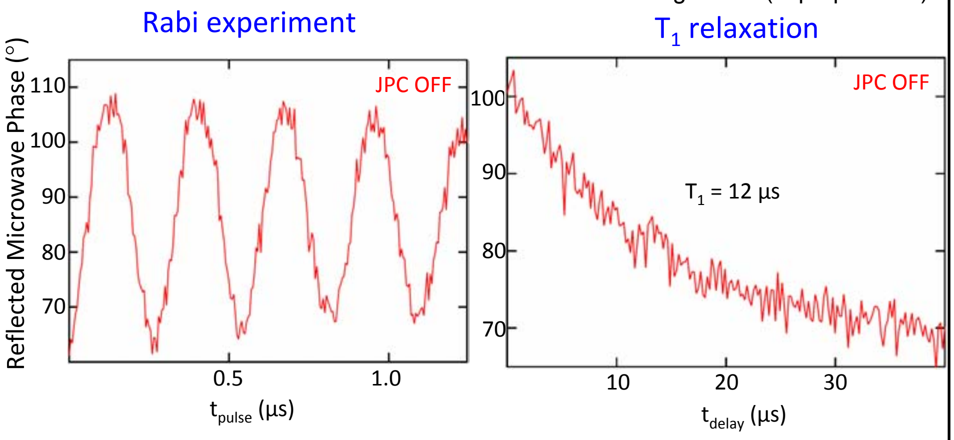




Vijay, Slichter and Siddiqi, Phys. Rev. Lett. **106**, 110502 (2011)

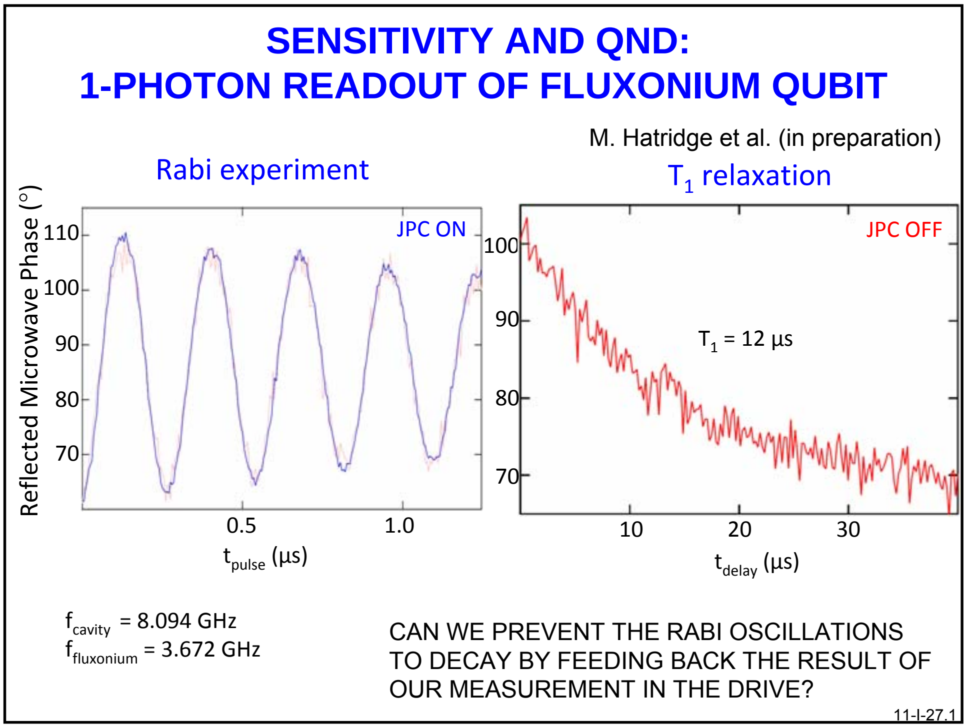
SENSITIVITY AND QND: 1-PHOTON READOUT OF FLUXONIUM QUBIT

M. Hatridge et al. (in preparation)


Rabi experiment

T_1 relaxation

f_{cavity} = 8.094 GHz
$f_{fluxonium}$ = 3.672 GHz



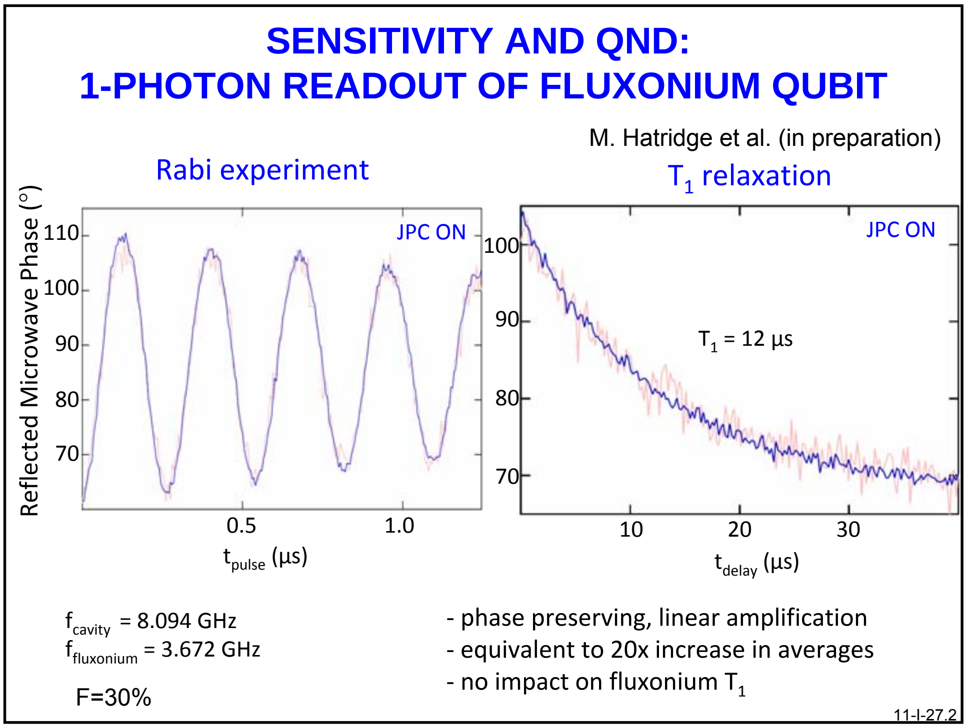
SENSITIVITY AND QND:
1-PHOTON READOUT OF FLUXONIUM QUBIT

M. Hatridge et al. (in preparation)

f_{cavity} = 8.094 GHz
$f_{fluxonium}$ = 3.672 GHz

CAN WE PREVENT THE RABI OSCILLATIONS TO DECAY BY FEEDING BACK THE RESULT OF OUR MEASUREMENT IN THE DRIVE?

11-I-27.1



SENSITIVITY AND QND:
1-PHOTON READOUT OF FLUXONIUM QUBIT

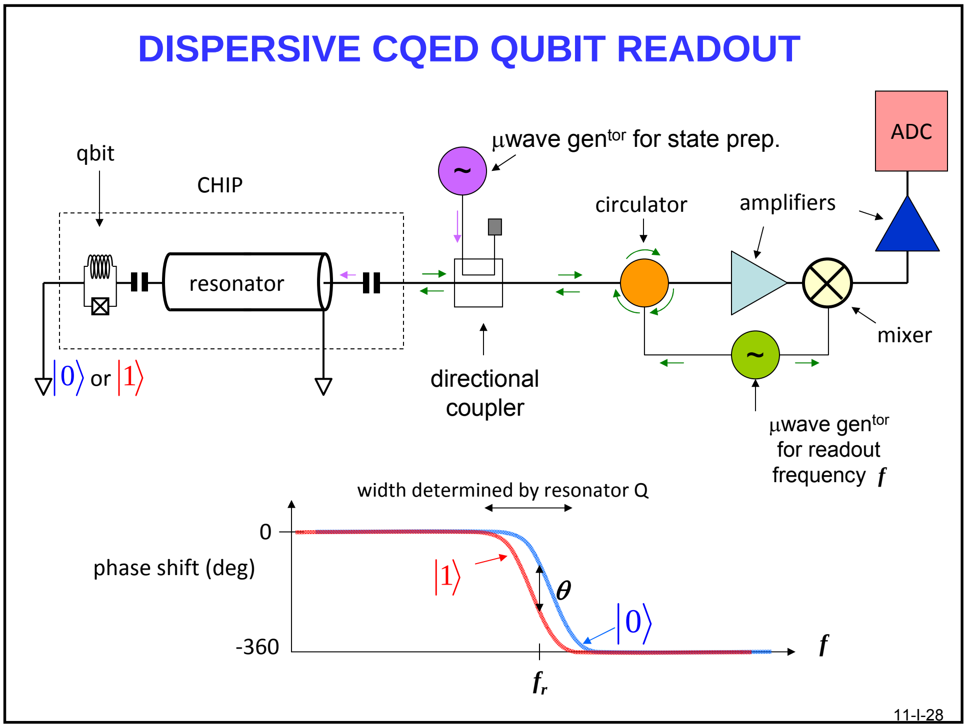
M. Hatridge et al. (in preparation)

f_{cavity} = 8.094 GHz
$f_{fluxonium}$ = 3.672 GHz

F=30%

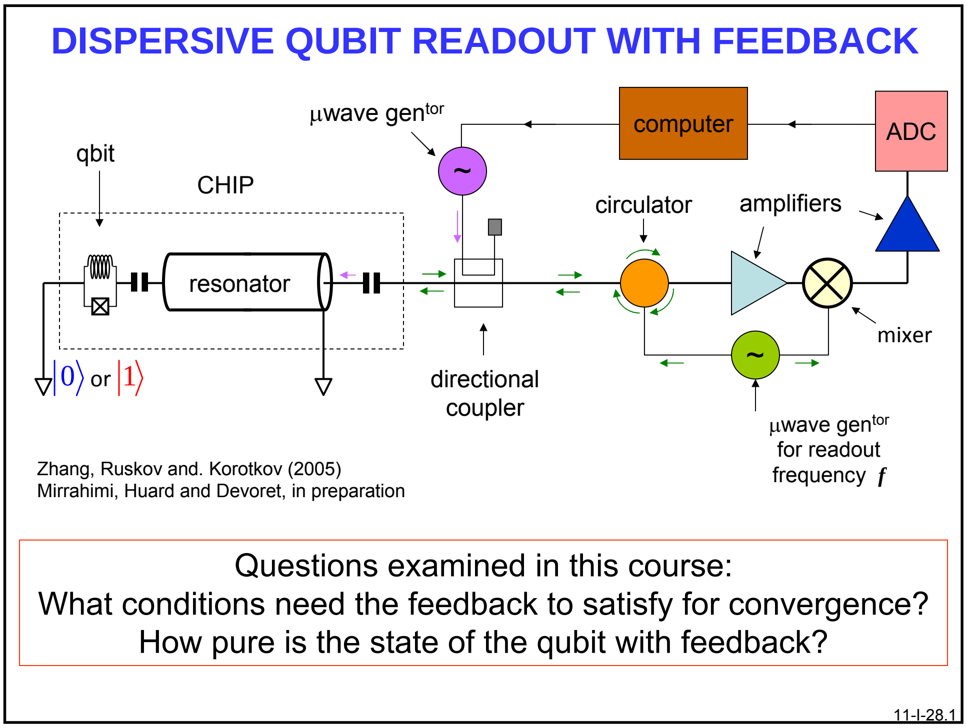
- phase preserving, linear amplification
- equivalent to 20x increase in averages
- no impact on fluxonium T_1

11-I-27.2



DISPERSIVE CQED QUBIT READOUT

11-I-28



DISPERSIVE QUBIT READOUT WITH FEEDBACK

Zhang, Ruskov and. Korotkov (2005)
Mirrahimi, Huard and Devoret, in preparation

Questions examined in this course:
What conditions need the feedback to satisfy for convergence?
How pure is the state of the qubit with feedback?

11-I-28.1

PROGRAM OF THIS YEAR'S LECTURES

Lecture I: Introduction to quantum-limited amplification and feedback

Lecture II: How do we model out-of-equilibrium non-linear quantum systems?

Lecture III: How do we optimize the parametric amplifier characteristics while maintaining its noise at the quantum limit?

Lecture IV: What are the minimal requirements for an active circuit to be fully directional and noiseless?

Lecture V: Can continuous quantum measurements be viewed as a form of Brownian motion?

Lecture VI: How can we maintain a dynamic quantum state alive?

Please note that there will be no lecture on May 24

SELECTED BIBLIOGRAPHY

Books and series of lectures

Braginsky, V. B., and F. Y. Khalili, "Quantum Measurements" (Cambridge University Press, Cambridge, 1992)
Clarke, J. and Braginsky, A. I., eds., "The SQUID Handbook" (Wiley-VCH, Weinheim, Germany, 2006)
Esteve, D., Raimond, J-M., and Dalibard J., "Quantum Entanglement and Information Processing" (Elsevier, Amsterdam, 2004)
Gardiner, C.W. and Zoller, P. "Quantum Noise" (Springer, Berlin, 2004)
Haroche, S., Lectures at College de France, 2011
Haroche, S. and Raimond, J-M., "Exploring the Quantum" (Oxford University Press, 2006)
Nielsen, M. and Chuang, I., “Quantum Information and Quantum Computation” (Cambridge, 2001)
Walls, D.F., and Milburn, G.J. "Quantum Optics" (Springer, Berlin, 2008)
Wiseman, H.M. and Milburn, G.J., "Quantum Measurement and Control" (Cambridge, 2011)

Articles

Blais A., Gambetta J., Wallraff A., Schuster D. I., Girvin S., Devoret M.H., Schoelkopf R.J., Phys. Rev. (2007) A 75, 032329
Clarke, J. and Wilhelm, F. K., "Superconducting quantum bits". Nature **453**, 1031–1042 (2008).
Clerk A. A., Devoret M. H., Girvin S. M., Marquardt F., and Schoelkopf R. J., "Introduction to Quantum Noise, Measurement and Amplification", Rev. Mod. Phys. **82**, 1155 (2010).
Devoret, M. H., Wallraff A., and Martinis J. M., e-print cond-mat/0411174

Schoelkopf, R.J., and Girvin, S.M., “Wiring up quantum systems,” Nature **451**, 664 (2008).
R. Vijay, M. H. Devoret, and I. Siddiqi, “Invited Review Article: The Josephson bifurcation amplifier,” Review of Scientific Instruments **80**, 111101 (2009)
R. Vijay, D. H. Slichter, and I. Siddiqi, “Observation of Quantum Jumps in a Superconducting Artificial Atom,” Phys. Rev. Lett. **106**, 110502 (2011).
Q. Zhang, R. Ruskov, and A.N. Korotkov "Continuous quantum feedback of coherent oscillations in a solid-state qubit" Phys. Rev. **B 72**, 245322 (2005).

CALENDAR OF SEMINARS

May 10: Fabien Portier, SPEC-CEA Saclay
The Bright Side of Coulomb Blockade

May 17, 2011: Jan van Ruitenbeek (Leiden University, The Netherlands)
Quantum Transport in Single-molecule Systems

May 31, 2011: Irfan Siddiqi (UC Berkeley, USA)
Quantum Jumps of a Superconducting Artificial Atom

June 7, 2011: David DiVicenzo (IQI Aachen, Germany)
Quantum Error Correction and the Future of Solid State Qubits

June 14, 2011: Andrew Cleland (UC Santa Barbara, USA)
Images of Quantum Light

June 21, 2011: Benjamin Huard (LPA - ENS Paris)
Building a Quantum Limited Amplifier from Josephson Junctions and Resonators

June 21, 2011 (3pm): Andrew Cleland (UC Santa Barbara, USA)
How to Be in Two Places at the Same Time ?

END OF LECTURE