COLLÈGE DE FRANCE
1530

Chaire de Physique Mésoscopique
Michel Devoret
Année 2008, 13 mai - 24 juin

CIRCUITS ET SIGNAUX QUANTIQUES

QUANTUM SIGNALS AND CIRCUITS

Troisième Leçon / *Third Lecture*

VISIT THE WEBSITE OF THE CHAIR OF MESOSCOPIC PHYSICS

http://www.college-de-france.fr

and follow links to:

http://www.physinfo.fr/lectures.html

PDF FILES OF ALL LECTURES WILL BE POSTED ON THIS WEBSITE

Questions, comments and corrections are welcome!

CALENDAR OF SEMINARS

May 13: Denis Vion, (Quantronics group, SPEC-CEA Saclay)
Continuous dispersive quantum measurement of an electrical circuit

May 20: Bertrand Reulet (LPS Orsay)
Current fluctuations : beyond noise

June 3: Gilles Montambaux (LPS Orsay)
Quantum interferences in disordered systems

June 10: Patrice Roche (SPEC-CEA Saclay)
Determination of the coherence length in the Integer Quantum Hall Regime

June 17: Olivier Buisson, (CRTBT-Grenoble)
A quantum circuit with several energy levels

June 24: Jérôme Lesueur (ESPCI)
High Tc Josephson Nanojunctions: Physics and Applications

PROGRAM OF THIS YEAR'S LECTURES

Lecture I: Introduction and overview

Lecture II: Modes of a circuit and propagation of signals

Lecture III: The "atoms" of signal

Lecture IV: Hamiltonian vs scattering description of circuits

Lecture V: Non-linear circuit elements: length and energy
scales of superconductivity

Lecture VI: Amplifying quantum signals with dispersive circuits

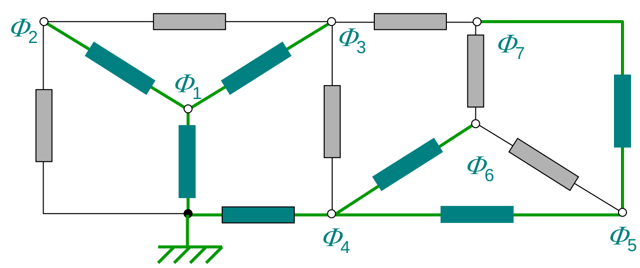
LECTURE III : THE "ATOMS" OF A SIGNAL

OUTLINE

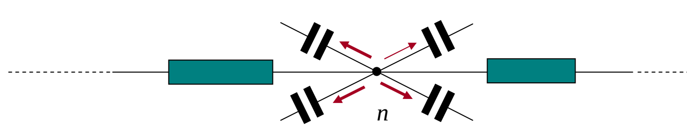
1. Introduction, purpose of this lecture
2. Quantum operators of a transmission line
3. Wavelets and temporal modes
4. Quantum states of the line

REVIEW OF LAST LECTURE: NODE VARIABLES

INTRODUCED
NODE FLUXES



NODE CHARGES
ARE CONJUGATE
TO NODE FLUXES

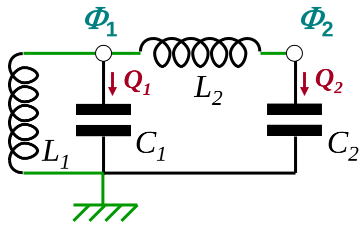


$$\hat{Q}_n = \sum_{\substack{\text{capacitive} \\ \text{branches} \\ \text{exiting from } n}} \hat{Q}_\beta$$

for each node:

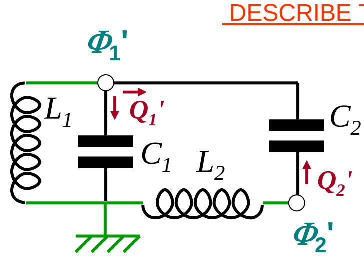
$$\boxed{\left[\hat{\Phi}_n, \hat{Q}_n\right] = i\hbar}$$

REVIEW OF LAST LECTURE (CONT'ND): HAMILTONIAN OF CIRCUIT



$$\hat{H}\left(\hat{\Phi}_1,\hat{Q}_1;\hat{\Phi}_2,\hat{Q}_2\right)=\frac{\hat{Q}_1^2}{2C_1}+\frac{\hat{\Phi}_1^2}{2L_1}+\frac{\hat{Q}_2^2}{2C_2}+\frac{\left(\hat{\Phi}_2-\hat{\Phi}_1\right)^2}{2L_2}$$

ATTENTION: TWO VERY DIFFERENT-LOOKING HAMILTONIANS CAN DESCRIBE THE SAME CIRCUIT!



$$\hat{H}\left(\hat{\Phi}_1',\hat{Q}_1';\hat{\Phi}_2',\hat{Q}_2'\right)=\frac{\left(\hat{Q}_1'-\hat{Q}_2'\right)^2}{2C_1}+\frac{\hat{\Phi}_2'^{\,2}}{2L_1}+\frac{\hat{Q}_2'^{\,2}}{2C_2}+\frac{\hat{\Phi}_2'^{\,2}}{2L_2}$$

08-III-7a

NORMAL MODES OF A LINEAR CIRCUIT

Hamiltonian of a linear circuit has quadratic form:

$$\hat{H}\left(\hat{\Phi}_1,\hat{Q}_1;\hat{\Phi}_2,\hat{Q}_2;...;\hat{\Phi}_n,\hat{Q}_n;...;\hat{\Phi}_N,\hat{Q}_N\right)=\frac{1}{2}\sum_{n=1}^{N}\sum_{p=1}^{N}C_{np}^{-1}\hat{Q}_n\hat{Q}_p+L_{np}^{-1}\hat{\Phi}_n\hat{\Phi}_p$$

Introduce normal coordinates preserving canonical commutation relations:

$$\hat{P}_\mu=\sum_n \mathcal{A}_{\mu n}\hat{Q}_n \qquad \left[\hat{X}_{\mu_1},\hat{P}_{\mu_2}\right]=i\hbar\delta_{\mu_1\mu_2}$$

$$\hat{X}_\mu=\sum_n \mathcal{B}_{\mu n}\hat{\Phi}_n \qquad \left[\hat{X}_{\mu_1},\hat{X}_{\mu_2}\right]=0$$

$$\left[\hat{P}_{\mu_1},\hat{P}_{\mu_2}\right]=0$$

such that:

$$\hat{H}=\frac{1}{2}\sum_\mu\left(\hat{P}_\mu^2+\omega_\mu^2\hat{X}_\mu^2\right)$$

Dimension of P_μ $=[P_\mu]=\left(\frac{[\text{Power}]}{[\text{Frequency}]}\right)^{1/2}=[\text{voltage}]\times[\text{capacitance}]^{1/2}$

08-III-8b

NORMAL MODE OF A CIRCUIT (CTND)

Introduce mode annihilation and creation (ladder) operators:

$$\hat{a}_\mu = \frac{1}{\sqrt{2\hbar}}\left(\sqrt{\omega_\mu}\hat{X}_m + \frac{i}{\sqrt{\omega_\mu}}\hat{P}_\mu\right) \qquad \hat{X}_\mu = \sqrt{\frac{\hbar}{2\omega_\mu}}\left(\hat{a}_\mu + \hat{a}_\mu^\dagger\right)$$

$$\hat{a}_\mu^\dagger = \frac{1}{\sqrt{2\hbar}}\left(\sqrt{\omega_\mu}\hat{X}_\mu - \frac{i}{\sqrt{\omega_\mu}}\hat{P}_\mu\right) \qquad \hat{P}_\mu = \sqrt{\frac{\hbar\omega_\mu}{2}}\frac{\left(\hat{a}_m - \hat{a}_\mu^\dagger\right)}{i}$$

Nice trick we will use often: $\boxed{\hat{a}_{m=\mu} = \hat{a}_\mu;\ \hat{a}_{m=-\mu} = \hat{a}_\mu^\dagger}$ $\quad \mu \in \mathbb{N}, m \in \mathbb{Z}$

Then <u>all</u> the commutation relations can be summarized by

$$\left[\hat{a}_{m_1}, \hat{a}_{m_2}\right] = \mathrm{sg}\left(m_1 - m_2\right)\delta_{m_1+m_2} \quad \text{where} \quad m_1, m_2 \in \mathbb{Z}$$

And the hamiltonian can be written as:

$$\boxed{\hat{H} = \sum_{\mu=1}^{N} \hbar\omega_\mu \hat{a}_\mu^\dagger \hat{a}_\mu = \sum_{m=1}^{N} \hbar\omega_m \hat{a}_{-m}\hat{a}_m}$$

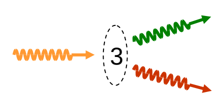
(zero-point energy has been substracted)

08-III-9b

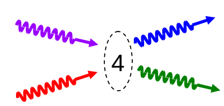
ANHARMONICITY

Non-quadratic potential generates extra terms in the hamiltonian of the form:

$\hbar\, g^{(3)}_{m_1 m_2 m_3} a_{m_1} a_{m_2} a_{m_3}$ CUBIC ANHARMONICITY



$\hbar\, g^{(4)}_{m_1 m_2 m_3 m_4} a_{m_1} a_{m_2} a_{m_3} a_{m_4}$ QUARTIC ANHARMONICITY



For the moment, in this course, we suppose the anharmonicity is weak in the sense:

$$g_{\mu_1\mu_2\mu_3} \ll \omega_{\mu_1}, \omega_{\mu_2}, \omega_{\mu_3}$$

$$g_{\mu_1\mu_2\mu_3\mu_4} \ll \omega_{\mu_1}, \omega_{\mu_2}, \omega_{\mu_3}, \omega_{\mu_4}$$

However, the <u>intrinsic non-linearity</u> of the system can remain strong in the sense:

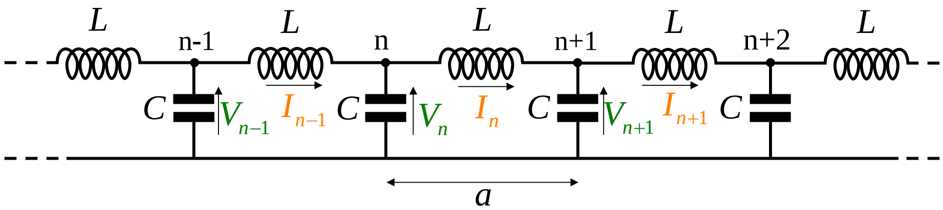
EXCITATIONS BREED FASTER THAN THEY DIE

$$\boxed{\Gamma\text{'s} \ll g\text{'s}}$$

Here the Γ's are damping rates of system modes

08-III-10b

REVIEW OF LAST LECTURE (CONT'ND): TRANSMISSION LINES



Dynamical equations:

$$V_n - V_{n+1} = L\frac{d}{dt}I_n$$

$$I_{n-1} - I_n = C\frac{d}{dt}V_n$$

Continuum limit:

$$\frac{V_{n+1}-V_n}{a} \to \frac{\partial V}{\partial x}$$

$$\frac{I_{n+1}-I_n}{a} \to \frac{\partial I}{\partial x}$$

$$\frac{C}{a} \to C_\ell \; ; \; \frac{L}{a} \to L_\ell$$

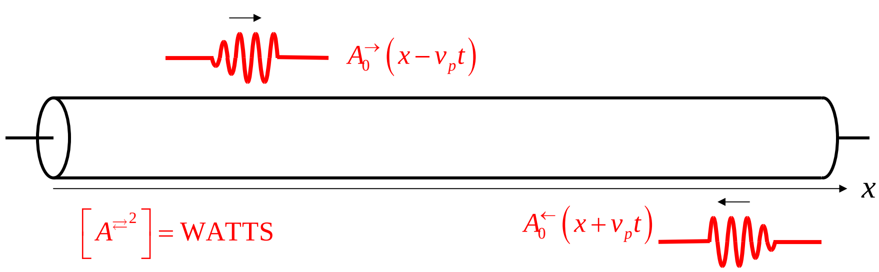
Field equations:

$$\frac{\partial V}{\partial x} = -L_\ell \frac{\partial I}{\partial t}$$

$$\frac{\partial I}{\partial x} = -C_\ell \frac{\partial V}{\partial t}$$

08-III-11b

PROPAGATING WAVE AMPLITUDES



$$V = V^{\rightarrow} + V^{\leftarrow} = \sqrt{Z_c}\left[A^{\rightarrow}(x,t) + A^{\leftarrow}(x,t)\right]$$

$$I = I^{\rightarrow} - I^{\leftarrow} = \frac{1}{\sqrt{Z_c}}\left[A^{\rightarrow}(x,t) - A^{\leftarrow}(x,t)\right]$$

$$\frac{\partial}{\partial x}A^{\rightleftarrows} = \mp\frac{1}{v_p}\frac{\partial}{\partial t}A^{\rightleftarrows}$$

solution:

$$\boxed{A^{\rightleftarrows}(x,t) = A_0^{\rightleftarrows}(x \mp v_p t)}$$

$$Z_c = \sqrt{\frac{L_\ell}{C_\ell}} = \frac{V^{\rightarrow}}{I^{\rightarrow}} = \frac{V^{\leftarrow}}{I^{\leftarrow}}$$

characteristic impedance

$$v_p = \sqrt{\frac{1}{L_\ell C_\ell}}$$

propagation velocity

HOW DO WE QUANTIZE THIS DISTRIBUTED ELEMENT SYSTEM?

08-III-12b

OUTLINE

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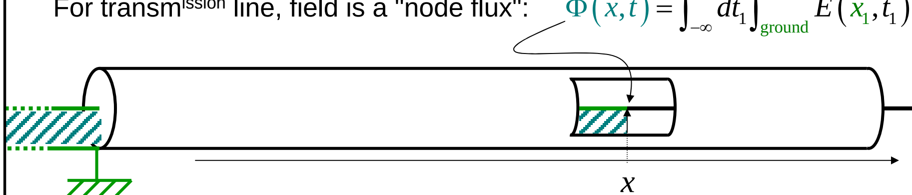
08-III-5b

LAGRANGIAN OF FREE SCALAR FIELD

For transmission line, field is a "node flux": $\Phi(x,t) = \int_{-\infty}^{t} dt_1 \int_{\text{ground}}^{x} \vec{E}(x_1,t_1)\,dx_1$



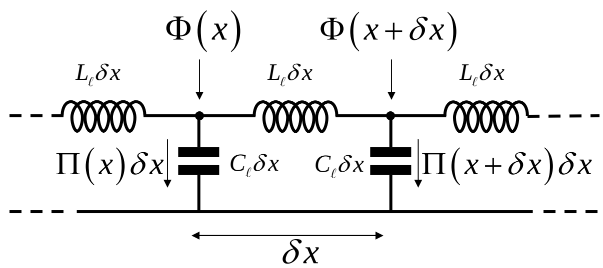
Lagrangian density: $$\mathcal{L}(x) = \frac{C_\ell}{2}\left(\frac{\partial\Phi}{\partial t}\right)^2 - \frac{1}{2L_\ell}\left(\frac{\partial\Phi}{\partial x}\right)^2$$

Momentum density: $$\Pi(x) \equiv \frac{\partial L}{\partial(\partial\Phi/\partial t)} = C_l\frac{\partial\Phi}{\partial t}$$ = CHARGE DENSITY ON WIRE

Commutation relations: $$\left[\hat{\Phi}(x_1),\hat{\Pi}(x_2)\right] = i\hbar\delta(x_1 - x_2)$$
$$\left[\hat{\Phi}(x_1),\hat{\Phi}(x_2)\right] = \left[\hat{\Pi}(x_1),\hat{\Pi}(x_2)\right] = 0$$

08-III-13c

WHAT DOES THIS MEAN?



$$\left.\begin{array}{l}\left[\Phi(x),\Pi(x)\delta x\right]=i\hbar \\ \left[\Phi(x),\Pi(x+\delta x)\delta x\right]=0\end{array}\right\} \xrightarrow[\delta x\to 0]{} \left[\hat{\Phi}(x_1),\hat{\Pi}(x_2)\right]=i\hbar\delta(x_1-x_2)$$

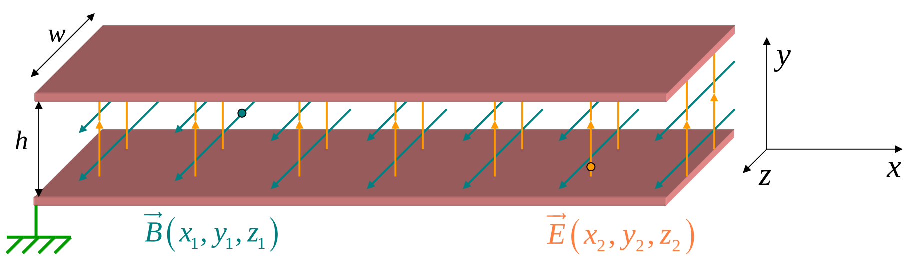
Hamiltonian gives energy density as a function of field and conjugate momentum:

$$\hat{H}=\int_{-\infty}^{+\infty}dx\left\{\frac{1}{2C_\ell}\left[\hat{\Pi}(x)\right]^2+\frac{1}{2L_\ell}\left[\nabla\hat{\Phi}(x)\right]^2\right\}$$

08-III-14b

COMPATIBILITY WITH STANDARD QED

For simplest geometry, consider stripline waveguide:



Flux between strips:

$$\hat{\Phi}(x_1)=\int_{-\infty}^{x_1}dx\int_{y_b}^{y_b+h}dy\ \hat{B}_z(x,y,z_1)$$

Strip charge per unit length:

$$\hat{\Pi}(x_2)=\varepsilon_0\int_{z_f}^{z_f+w}dz\ \hat{E}_y(x_2,y_2,z)$$

Commutation relations between field operators in standard QED:

$$\left[\hat{B}_z(x_1,y_1,z_1),\hat{E}_y(x_2,y_2,z_2)\right]=\frac{i\hbar}{\varepsilon_0}\frac{\partial}{\partial x_1}\delta(x_1-x_2)\delta(y_1-y_2)\delta(z_1-z_2)$$

$$\Rightarrow \boxed{\left[\hat{\Phi}(x_1),\hat{\Pi}(x_2)\right]=i\hbar\delta(x_1-x_2)}\quad \text{OK!}$$

08-III-15c

08-III-16c

FOURIER DOMAIN REPRESENTATION

$$\hat{\Phi}[k] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} dx\, \hat{\Phi}(x)\, \mathrm{e}^{-ikx}$$

$$\hat{\Phi}(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} dk\, \hat{\Phi}[k]\, \mathrm{e}^{+ikx}$$

$$\hat{\Phi}^{\dagger}[k] = \hat{\Phi}[-k]$$

NEW OPERATORS $\hat{\Phi}[k]$ & $\hat{\Pi}[k]$ ARE NOT HERMITIAN !

$$\hat{\Pi}[k] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} dx\, \hat{\Pi}(x)\, \mathrm{e}^{-ikx}$$

$$\hat{\Pi}(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} dk\, \hat{\Pi}[k]\, \mathrm{e}^{+ikx}$$

$$\hat{\Pi}^{\dagger}[k] = \hat{\Pi}[-k]$$

Commutation relations:

$$\left[\hat{\Phi}[k_1], \hat{\Phi}[k_2]\right] = \left[\hat{\Pi}[k_1], \hat{\Pi}[k_2]\right] = 0$$

$$\left[\hat{\Phi}[k_1], \hat{\Pi}[k_2]\right] = i\hbar\delta(k_1 + k_2)$$

Hamiltonian has now independent mode form:

$$\hat{H} = \frac{1}{2C_\ell} \int_{-\infty}^{+\infty} dk \left\{ \hat{\Pi}[-k]\hat{\Pi}[k] + \omega^2(k) C_\ell^2 \hat{\Phi}[-k]\hat{\Phi}[k] \right\}$$

with $\omega^2(k) = \dfrac{k^2}{C_\ell L_\ell} = v_p^2 k^2$

FIELD LADDER OPERATORS

$$\hat{a}[k] = \frac{1}{\sqrt{2\hbar}} \left(\sqrt{\omega(k) C_\ell}\, \hat{\Phi}[k] + \frac{i}{\sqrt{\omega(k) C_\ell}} \hat{\Pi}[k] \right)$$

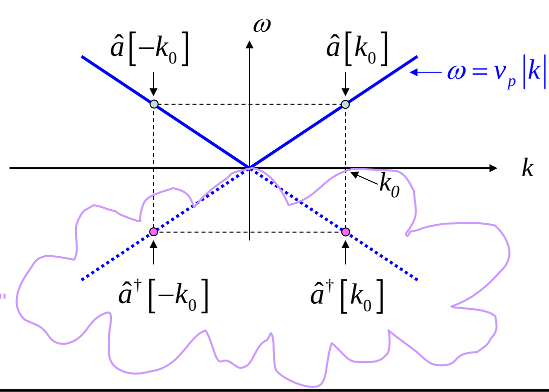
$$\hat{a}^{\dagger}[k] = \frac{1}{\sqrt{2\hbar}} \left(\sqrt{\omega(k) C_\ell}\, \hat{\Phi}[-k] - \frac{i}{\sqrt{\omega(k) C_\ell}} \hat{\Pi}[-k] \right)$$

$$\hat{\Phi}[k] = \sqrt{\frac{\hbar}{2\omega(k) C_\ell}} \left(\hat{a}[k] + \hat{a}^{\dagger}[-k] \right)$$

$$\hat{\Pi}[k] = \sqrt{\frac{\hbar\omega(k) C_\ell}{2}} \frac{1}{i} \left(\hat{a}[k] - \hat{a}^{\dagger}[-k] \right)$$

SIMILAR BUT NOT IDENTICAL TO STANDING MODE LADDER OPERATORS

Dispersion Relation:



IT IS CONVENIENT TO INTRODUCE A SORT OF "MIRROR" DISPSION RELTION

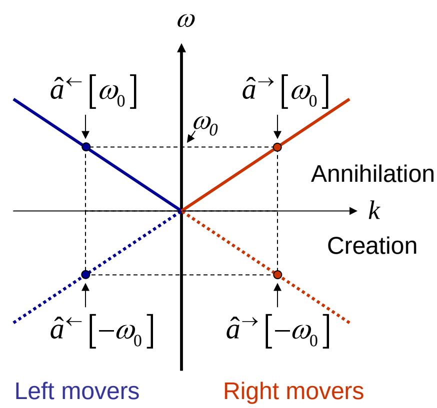
08-III-17c

FIELD LADDER OPERATORS IN FREQ$^{\text{UENCY}}$ DOMAIN

Introduce new notation extending frequencies to negative values, for both pos$^{\text{itive}}$ and neg$^{\text{ative}}$ k:

$$\hat{a}^{\rightarrow}[-\omega_0] = \hat{a}^{\dagger}\left[+\omega(k_0>0)\right]$$
$$\hat{a}^{\leftarrow}[-\omega_0] = \hat{a}^{\dagger}\left[+\omega(-k_0<0)\right]$$

[Courty, Grassia and Reynaud, Europhys.Lett. 46 (1999) 31]



Makes sense since:

$$i\frac{d}{dt}\hat{a}^{\dagger}\left[|\omega|\right] = -|\omega|\hat{a}^{\dagger}\left[|\omega|\right]$$
$$i\frac{d}{dt}\hat{a}\left[|\omega|\right] = +|\omega|\hat{a}\left[|\omega|\right]$$

Commutation relations:

$$\left[\hat{a}^{\rightleftarrows}[\omega_1], \hat{a}^{\rightleftarrows}[\omega_2]\right] = \mathrm{sg}(\omega_1-\omega_2)\,\delta(\omega_1+\omega_2)$$
$$\left[\hat{a}^{\rightarrow}[\omega_1], \hat{a}^{\leftarrow}[\omega_2]\right] = \left[\hat{a}^{\leftarrow}[\omega_1], \hat{a}^{\rightarrow}[\omega_2]\right] = 0$$

Hamiltonian:

$$\hat{H} = \int_0^{\infty} \hbar\omega\, d\omega \left\{\hat{a}^{\leftarrow}[-\omega]\hat{a}^{\leftarrow}[\omega] + \hat{a}^{\rightarrow}[-\omega]\hat{a}^{\rightarrow}[\omega]\right\} \qquad \hat{H}|\mathrm{vac}\rangle = 0$$

08-III-18c

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08-III-5c

08-III-19d

WAVELET BASIS

Choose a complete orthonormal set of functions of time with both time and frequency locality forming a discrete basis (wavelet):

$$\int_{-\infty}^{+\infty} \psi^*_{m_1 p_1}(t)\,\psi_{m_2 p_2}(t)\,dt = \delta_{m_1 m_2}\delta_{p_1 p_2}$$

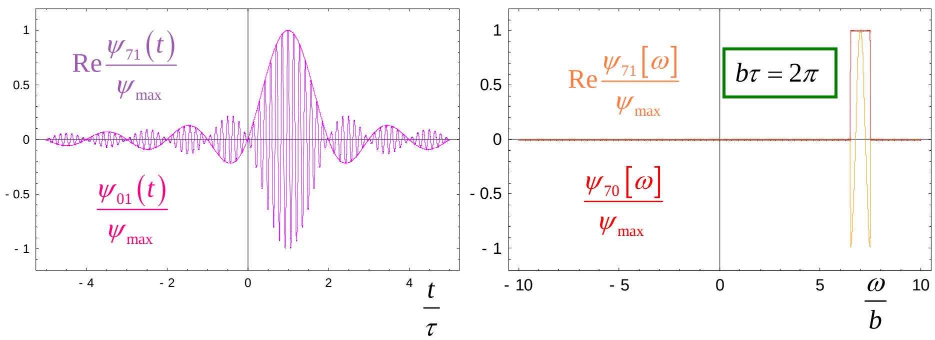
WAVELETS HAVE TWO INDICES
m specifies frequency
p specifies time

For instance, Shannon "bandpass" wavelets based on sinc function:

$$\psi_{mp}(t) = \sqrt{b}\,\exp[-imbt]\frac{\sin[b(t-p\tau)/2]}{b(t-p\tau)/2}$$

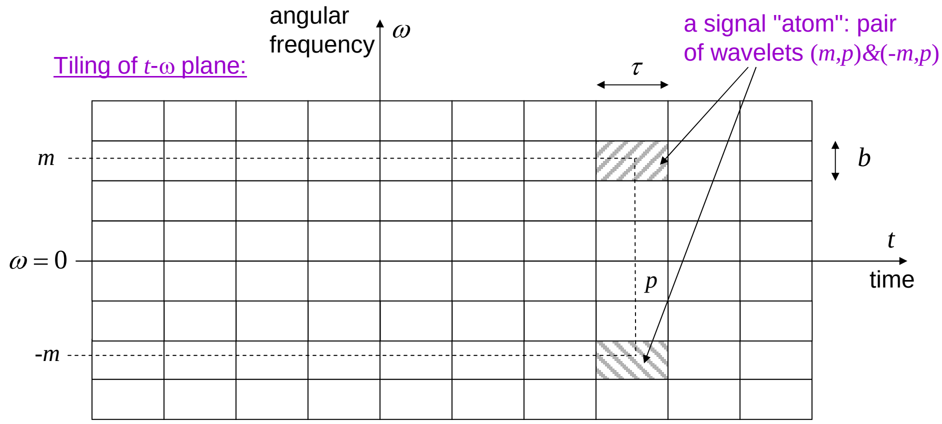
$$\psi_{mp}[\omega] = \frac{e^{ip\tau\omega}}{\sqrt{b}} \text{ for } \omega \in \left[\left(m-\frac{1}{2}\right)b, \left(m+\frac{1}{2}\right)b\right]$$

$$\psi_{mp}[\omega] = 0 \quad \text{elsewhere}$$



08-III-20a

TEMPORAL MODES



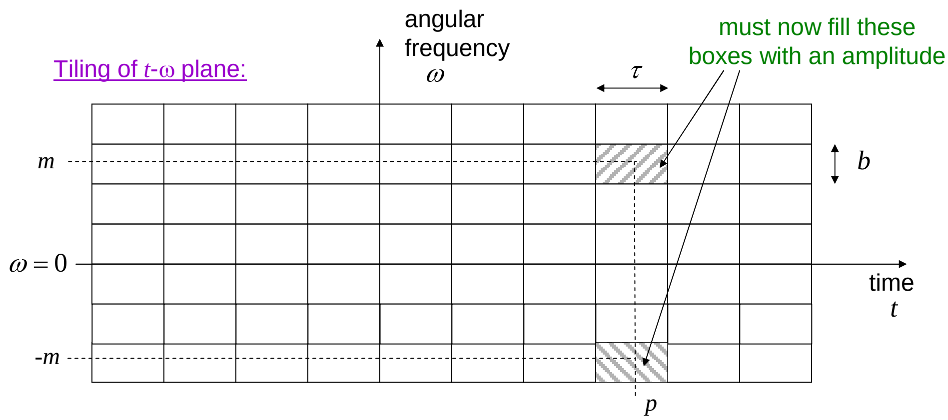
Each pair of conjugate basis functions correspond to a pair of real basis functions

$$\left[\psi_{mp}(t)+\psi_{-mp}(t)\right]/2 = \mathrm{Re}\left[\psi_{mp}(t)\right] = \text{ in-phase basis component (position)}$$

$$\left[\psi_{mp}(t)-\psi_{-mp}(t)\right]/2i = \mathrm{Im}\left[\psi_{mp}(t)\right] = \text{ quadrature basis component (momentum)}$$

THE RF PHOTON FINALLY APPEARS!

Tiling of t-ω plane:



The non-singular representation of photons:

$$\hat{a}_{mp} = \int_{-\infty}^{+\infty} d\omega\, \psi^*_{mp}[\omega]\,\hat{a}[\omega] \qquad \hat{a}^\dagger_{mp} = \hat{a}_{-mp}$$

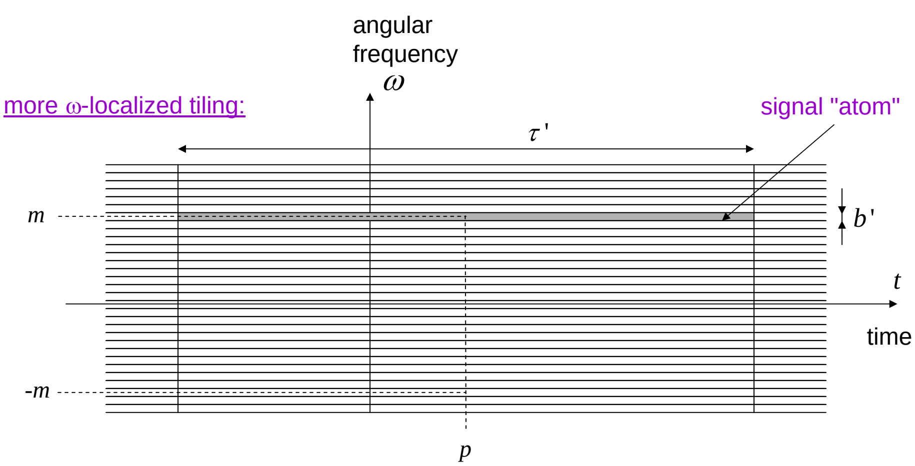
THE "AMPLITUDE" OF THE CONTENT OF THE BOX IS THE PHOTON OPERATOR

$$\boxed{\left[\hat{a}_{m_1 p_1}, \hat{a}_{m_2 p_2}\right] = \mathrm{sg}\left(m_1 - m_2\right)\delta_{m_1+m_2}\,\delta_{p_1-p_2}}$$

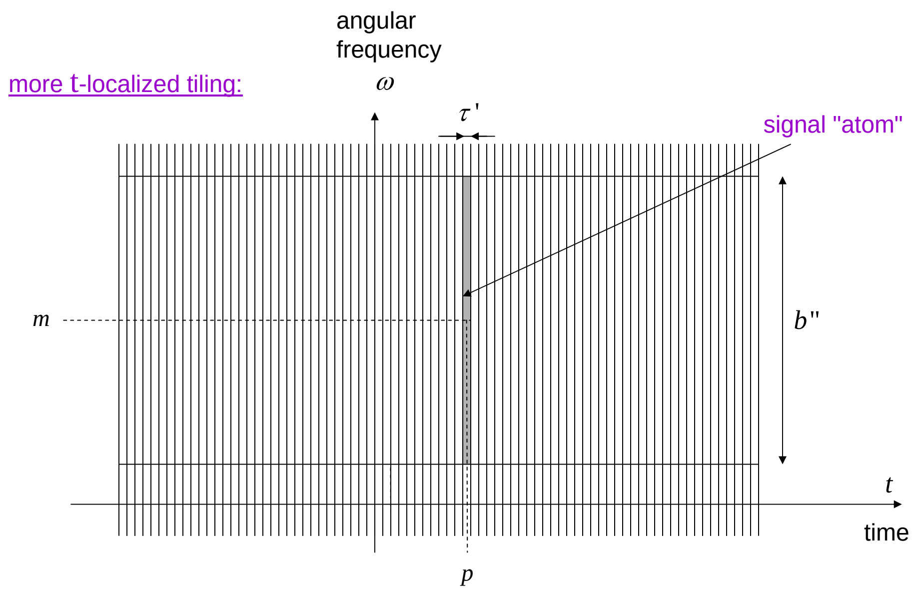
where $m_1, m_2, p_1, p_2 \in \mathbb{Z}$

OTHER POSSIBLE MODE BASIS

more ω-localized tiling:

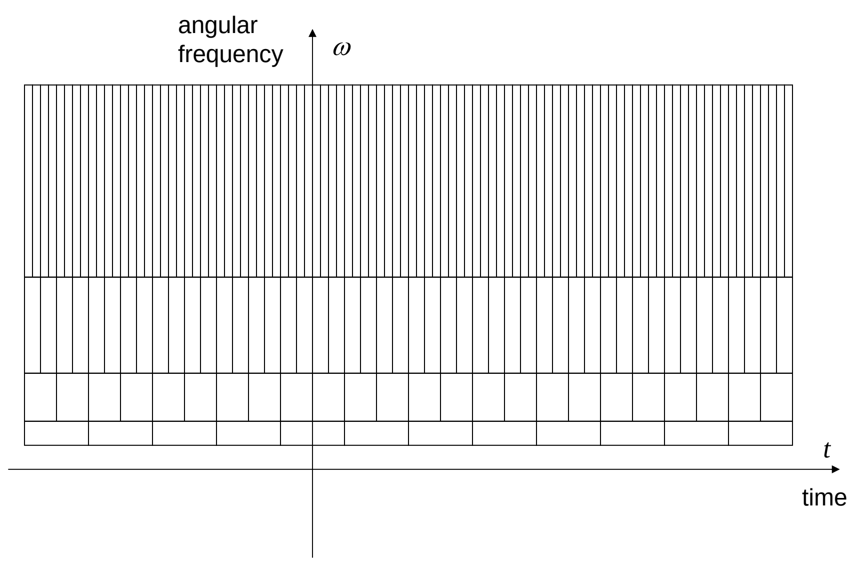


OTHER POSSIBLE MODE BASIS



08-III-23

SCALING TILING



Mallat, S. M., "A Wavelet Tour of Signal Processing" (Academic Press, San Diego, 1999)

08-III-24

TILING OF TIME-FREQUENCY PLANE (CTND)

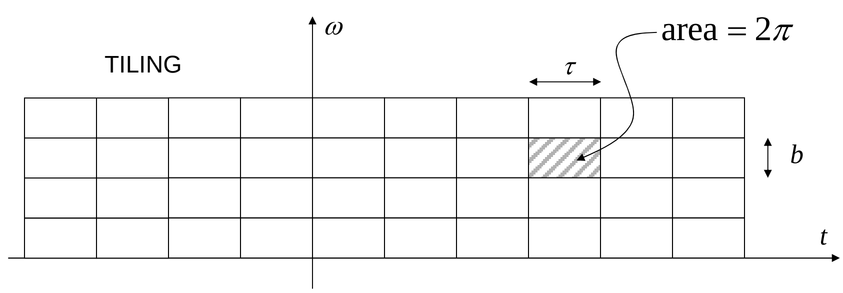
A musical score has similarities with tiling of time-frequency plane. However, shape, rather than width, code for duration. Also, tempo is not usually linked to pitch.

08-III-25

08-III-26a

STEP TILE vs UNCERTAINTY BOX

TILING

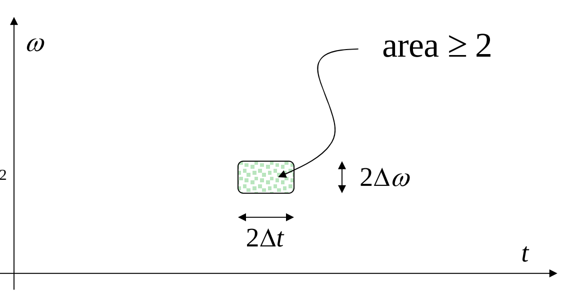


UNCERTAINTY

$$\Delta t = \left[\int_{-\infty}^{+\infty} \left(t-\bar{t}\right)^2 \left|\psi_\mu\left(t\right)\right|^2 dt\right]^{1/2}$$

$$\Delta\omega = \left[\int_{-\infty}^{+\infty} \left(\omega-\bar{\omega}\right)^2 \left|\psi_\mu\left[\omega\right]\right|^2 d\omega\right]^{1/2}$$

$$\Delta t \Delta\omega \geq 1/2$$



MINIMAL UNCERTAINTY → OVERCOMPLETENESS

ARBITRARINESS OF TEMPORAL MODE BASES

Like coordinate systems, mode bases have a part of arbitrariness,

But some bases take better advantage of system specificities.

We will see that sources and detectors
often determine preferred wavelet bases.

08-III-27

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08-III-5d

STATE OF THE LINE

Each mode $\mu = \{(m,p),(-m,p)\}$ can be excited with an arbitrary number of photons

We obtain the general photon state:

$$\left| n_1, n_2, ..., n_\mu, ... \right\rangle = \prod_\mu a_{-\mu}^{n_\mu} \left| vac \right\rangle$$

Most general pure state of the line:

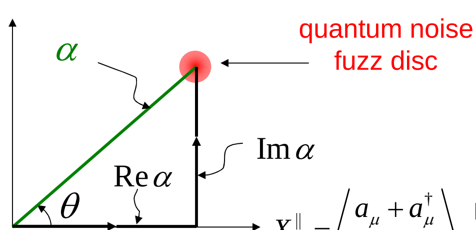
$$\left|\Psi\right\rangle = c_1 \left| n_1^1, n_2^1, ..., n_\mu^1, ... \right\rangle + c_2 \left| n_1^2, n_2^2, ..., n_\mu^2, ... \right\rangle + ... + c_\Omega \left| n_1^\Omega, n_2^\Omega, ..., n_\mu^\Omega, ... \right\rangle$$

08-III-28a

GEOMETRIC REPRESENTATION OF AMPLITUDE OF SEMI-CLASSICAL (GLAUBER) SIGNAL MODE

$$X_\mu^\perp = \left\langle \frac{a_\mu - a_\mu^\dagger}{2i} \right\rangle$$

QUADRATURE COMPNENT



$$X_\mu^\parallel = \left\langle \frac{a_\mu + a_\mu^\dagger}{2} \right\rangle$$

IN-PHASE COMPNENT

α = signal mode complex amplitude

$|\alpha|^2$ = signal mode mean energy in photon number

θ = signal mode mean phase

$$a_\mu \left|\alpha\right\rangle = \alpha \left|\alpha\right\rangle$$

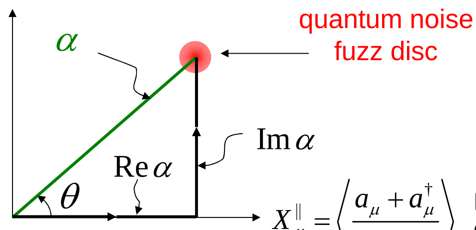
$$\left|\alpha\right\rangle = e^{\alpha a_\mu^\dagger - \alpha^* a_\mu} \left|0\right\rangle$$

08-III-29

GEOMETRIC REPRESENTATION OF AMPLITUDE OF SEMI-CLASSICAL (GLAUBER) SIGNAL MODE

$$X_\mu^{\perp} = \left\langle \frac{a_\mu - a_\mu^\dagger}{2i} \right\rangle$$

QUADRATURE COMPONENT



$$X_\mu^{\parallel} = \left\langle \frac{a_\mu + a_\mu^\dagger}{2} \right\rangle$$

IN-PHASE COMPONENT

α = signal mode complex amplitude

$|\alpha|^2$= signal mode mean energy in photon number

θ = signal mode mean phase

$$a_\mu |\alpha\rangle = \alpha |\alpha\rangle$$

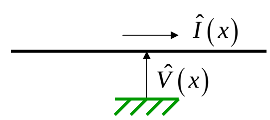
$$|\alpha\rangle = e^{\alpha a_\mu^\dagger - \alpha^* a_\mu} |0\rangle$$

Fresnel vector ⇨ Fresnel "lollypop"

08-III-29a

BACK TO VOLTAGES AND CURRENTS

Want expressions for physical variables:



Flux:

$$\hat{\Phi}(x,t) = \sqrt{\frac{\hbar Z_c}{4\pi}} \int_{-\infty}^{+\infty} \frac{d\omega}{\sqrt{|\omega|}} \left\{ \hat{a}^{\rightarrow}[\omega] e^{-i\omega(t-x/c)} + \hat{a}^{\leftarrow}[\omega] e^{-i\omega(t+x/c)} \right\}$$

Voltage:

$$\hat{V}(x,t) = \hat{V}^{\rightarrow}(x,t) + \hat{V}^{\leftarrow}(x,t)$$

Current:

$$\hat{I}(x,t) = \hat{I}^{\rightarrow}(x,t) - \hat{I}^{\leftarrow}(x,t)$$

$$\boxed{\begin{aligned} \hat{V}^{\rightleftarrows}(x,t) &= \sqrt{\frac{\hbar Z_c}{4\pi}} \int_{-\infty}^{+\infty} \frac{d\omega}{\sqrt{|\omega|}} i\omega \hat{a}^{\rightleftarrows}[\omega] e^{-i\omega(t\mp x/c)} \\ \hat{I}^{\rightleftarrows}(x,t) &= \sqrt{\frac{\hbar}{4\pi Z_c}} \int_{-\infty}^{+\infty} \frac{d\omega}{\sqrt{|\omega|}} i\omega \hat{a}^{\rightleftarrows}[\omega] e^{-i\omega(t\mp x/c)} \end{aligned}}$$

Characteristic scales for $Z_c \sim 50\Omega$:

$$\sqrt{\hbar Z_c} \sim 0.036\Phi_0 \qquad \sqrt{\frac{\hbar}{Z_c}} \sim 9e$$

08-III-30b

THERMAL STATE OF LINE

$$\frac{1}{2}\left\langle\left\{a[\omega_1],a[\omega_2]\right\}\right\rangle_T = N_T\left(|\omega_1|\right)\delta\left(\omega_1+\omega_2\right)$$

$$N_T\left(|\omega|\right)=\frac{1}{2}\coth\left(\frac{\hbar|\omega|}{2k_BT}\right)$$

$$\left\langle\left\{\hat{V}^{\rightleftarrows}(t_1),\hat{V}^{\rightleftarrows}(t_2)\right\}\right\rangle = -\left\langle\frac{\hbar Z_c}{4\pi}\int_{-\infty}^{+\infty}d\omega_1\frac{\omega_1}{\sqrt{|\omega_1|}}\mathrm{e}^{-i\omega_1 t_1}\int_{-\infty}^{+\infty}d\omega_2\frac{\omega_2}{\sqrt{|\omega_2|}}\mathrm{e}^{-i\omega_2 t_2}\left\{\hat{a}^{\rightleftarrows}[\omega_1],\hat{a}^{\rightleftarrows}[\omega_2]\right\}\right\rangle$$

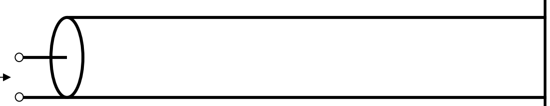
$$=\frac{\hbar Z_c}{4\pi}\int_{-\infty}^{+\infty}d\omega\;\omega\mathrm{e}^{-i\omega(t_1-t_2)}\coth\left(\frac{\hbar\omega}{2k_BT}\right)$$

$$\left\langle\left\{\hat{V}^{\rightleftarrows}[\omega_1],\hat{V}^{\rightleftarrows}[\omega_2]\right\}\right\rangle = 2\breve{S}_{V^{\rightleftarrows}V^{\rightleftarrows}}[\omega_1]\delta\left(\omega_1+\omega_2\right)$$

$$\breve{S}_{V^{\rightleftarrows}V^{\rightleftarrows}}[\omega]=\frac{Z_c}{4}\hbar\omega\coth\left(\frac{\hbar\omega}{2k_BT}\right)$$

$\longleftarrow$ $P = k_B T B$ in classical regime

$$\boxed{\breve{S}_{VV}[\omega]=Z_c\hbar\omega\coth\left(\frac{\hbar\omega}{2k_BT}\right)}$$



NEXT LECTURE:

EXPLORE FURTHER LINK BETWEEN DISSIPATION
AND FLUCTUATIONS IN QUANTUM REGIME
USING EXAMPLE OF DAMPED LC CIRCUIT