COLLÈGE DE FRANCE 1530

Chaire de Physique Mésoscopique
Michel Devoret
Année 2008, 13 mai - 24 juin

CIRCUITS ET SIGNAUX QUANTIQUES

QUANTUM SIGNALS AND CIRCUITS

Sixième Leçon / *Sixth Lecture*

VISIT THE WEBSITE OF THE CHAIR OF MESOSCOPIC PHYSICS

http://www.college-de-france.fr

and follow links to:

http://www.physinfo.fr/lectures.html

PDF FILES OF ALL LECTURES WILL BE POSTED ON THIS WEBSITE

Questions, comments and corrections are welcome!

CALENDAR OF SEMINARS

May 13: Denis Vion, (Quantronics group, SPEC-CEA Saclay)
Continuous dispersive quantum measurement of an electrical circuit

May 20: Bertrand Reulet (LPS Orsay)
Current fluctuations : beyond noise

June 3: Gilles Montambaux (LPS Orsay)
Quantum interferences in disordered systems

June 10: Patrice Roche (SPEC-CEA Saclay)
Determination of the coherence length in the Integer Quantum Hall regime

June 17: Olivier Buisson, (CRTBT-Grenoble)
A quantum circuit with several energy levels

June 24: Jérôme Lesueur (ESPCI)
High Tc Josephson Nanojunctions: Physics and Applications

08-VI-3

PROGRAM OF THIS YEAR'S LECTURES

Lecture I: Introduction and overview

Lecture II: Modes of a circuit and propagation of signals

Lecture III: The "atoms" of signal

Lecture IV: Quantum fluctuations in transmission lines

Lecture V: Introduction to non-linear active circuits

Lecture VI: Amplifying quantum signals with dispersive circuits

08-VI-4

LECTURE VI : AMPLIFYING QUANTUM SIGNALS WITH DISPERSIVE CIRCUITS

OUTLINE

1. Main ideas introduced in last lecture, purpose of this lecture
2. Minimal symplectic circuits are maximally efficient
3. Characteristics of amplifiers
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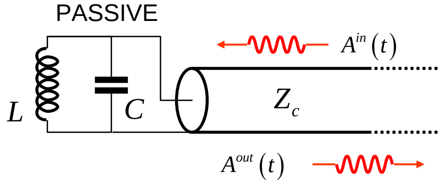
5. Ring modulator implementation of 2-port circuit

08-VI-5

PASSIVE vs ACTIVE LINEAR, DISPERSIVE 1-PORT

Simplest example: LC. Both L and C are dispersive elements (no internal dissipation)

PASSIVE



$\omega_0 = \frac{1}{\sqrt{LC}}$ $\Gamma = \frac{1}{2Z_cC}$

generic frequency

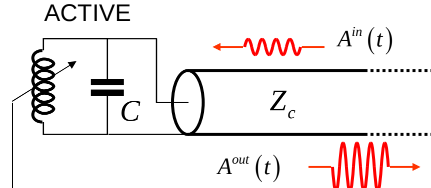
$$\begin{bmatrix} a^{out}[\omega] \\ a^{out}[-\omega] \end{bmatrix} = \begin{bmatrix} r[\omega] & 0 \\ 0 & r[-\omega] \end{bmatrix} \begin{bmatrix} a^{in}[\omega] \\ a^{in}[-\omega] \end{bmatrix}$$

$|r| = 1$

$S^{\dagger}IS=I$

UNITARY=
ORTHOGONAL
+ SYMPLECTIC

ACTIVE



$L(t) = L + \delta L \sin(2\omega_0 t)$

have treated only res. frequency case

$$\begin{bmatrix} a^{out}[\omega_0] \\ a^{out}[-\omega_0] \end{bmatrix} = \begin{bmatrix} r_0 & s_0 \\ s_0^* & r_0^* \end{bmatrix} \begin{bmatrix} a^{in}[\omega_0] \\ a^{in}[-\omega_0] \end{bmatrix}$$

$|r| > 1$

${}^{t}SJS=J$

GENERALIZED S MATRIX

ONLY SYMPLECTIC

ENERGY NOT CONSERVED: POWER GAIN

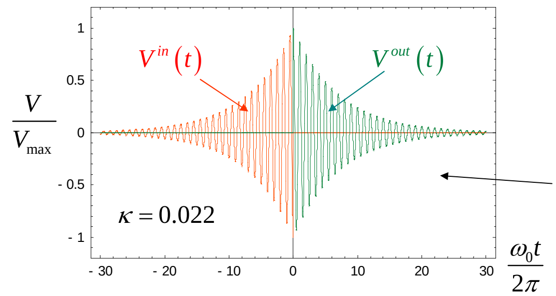
08-VI-6c

PASSIVE S IN ROTATING WAVE APPROXIMATION

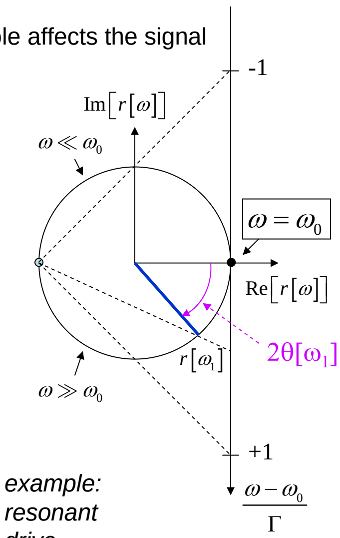
When $\frac{\Gamma}{\omega_0} = \kappa = \frac{1}{2Q} \ll 1$ we can consider only 1 pole affects the signal

$$r[\omega] = e^{i2\theta[\omega]} = -\frac{\omega_0^2 - \omega^2 - 2i\omega\Gamma}{\omega_0^2 - \omega^2 + 2i\omega\Gamma} \simeq \frac{1 - i\frac{\omega - sg(\omega)\omega_0}{\Gamma}}{1 + i\frac{\omega - sg(\omega)\omega_0}{\Gamma}}$$

$$V[\omega] = (1 + r[\omega])V^{in}[\omega] \simeq \frac{2}{1 + i\frac{\omega - sgn(\omega)\omega_0}{\Gamma}} V^{in}[\omega]$$



example: resonant drive

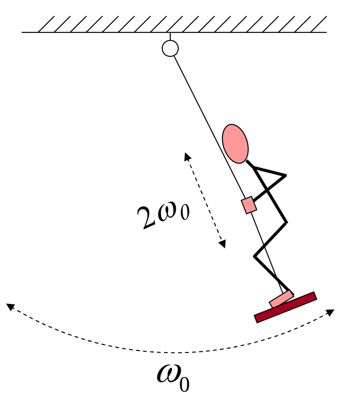


$\tan(\theta[\omega]) = \frac{\omega - \omega_0}{\Gamma}$

08-VI-7b

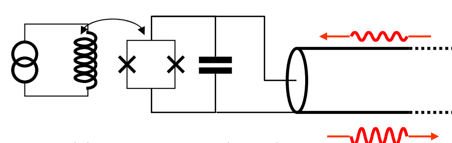
EXAMPLES OF PARAMETRICALLY PUMPED OSCILLATORS

MECHANICAL SYSTEM



ELECTRICAL SYSTEM

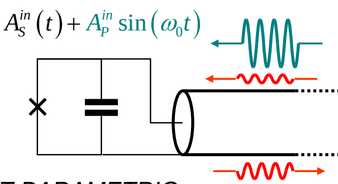
RF FLUX BIASED DC SQUID



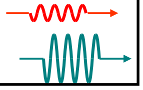
$I_B(t) = I_B + I_P \sin(2\omega_0 t)$

EFFECTIVE PARAMETRIC OSCILLATOR

$A^{in}(t) = A_S^{in}(t) + A_P^{in} \sin(\omega_0 t)$

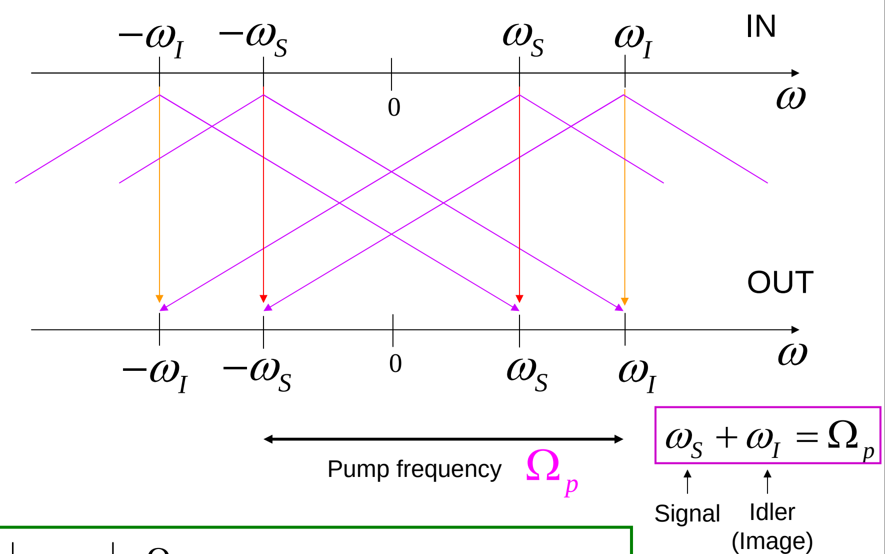


INDIRECT PARAMETRIC PUMPING @ $2\omega_0$



08-VI-8b

IN NON-LINEAR ACTIVE DEVICE, SIGNALS SCATTER BETWEEN DIFFERENT FREQUENCIES



IF $|\omega_S - \omega_I| = \Omega_p$ CONVERSION W/O AMPLIFICATION

08-VI-9a

PURPOSE OF THIS LECTURE: EXPLAIN HOW MINIMAL, ACTIVE DISPERSIVE CIRCUITS PERFORM AMPLIFICATION AT QUANTUM LIMIT

TWO MAIN ISSUES:

GAIN-BANDWIDTH TRADE-OFF

WHAT IS THE GENERALIZED SCATTERING MATRIX OF AN ACTIVE 1-PORT AND 2-PORT, BOTH AS A FUNCTION OF FREQUENCY OF SIGNALS AND AS A FUNCTION OF PUMP STRENGTH?

NOISE

HOW DO QUANTUM FLUCTUATIONS AFFECT THE NOISE OF THE AMPLIFIER? WHAT IS THE MINIMAL AMOUNT OF NOISE ADDED BY THE AMPLIFIER?

IMPLEMENTATION ?

08-VI-10

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08-VI-5b

GENERAL SCATTERING MATRIX

$$\vec{a}^{in} = \begin{bmatrix} a_1^{in}[+\omega_1] \\ a_1^{in}[-\omega_1] \\ a_2^{in}[+\omega_2] \\ a_2^{in}[-\omega_2] \\ \cdots \\ \cdots \\ a_l^{in}[+\omega_l] \\ a_l^{in}[-\omega_l] \end{bmatrix} \qquad \vec{a}^{out} = \begin{bmatrix} a_1^{out}[+\omega_1] \\ a_1^{out}[-\omega_1] \\ a_2^{out}[+\omega_2] \\ a_2^{out}[-\omega_2] \\ \cdots \\ \cdots \\ a_l^{out}[+\omega_l] \\ a_l^{out}[-\omega_l] \end{bmatrix}$$

$$\vec{a}^{out} = S\,\vec{a}^{in}$$

$$\sqrt{\frac{\hbar|\omega_l|}{2}}\, a_l[\omega_l] = A_l[\omega_l]$$

HERE,
WAVE AMPLITUDES
HAVE (PHOTON NB.)$^{1/2}$
DIMENSION
BUT ARE TREATED
CLASSICALLY

$$J = \begin{bmatrix} 0 & 1 & 0 & 0 & .. & .. & 0 & 0 \\ -1 & 0 & 0 & 0 & .. & .. & 0 & 0 \\ 0 & 0 & 0 & 1 & .. & .. & 0 & 0 \\ 0 & 0 & -1 & 0 & .. & .. & 0 & 0 \\ .. & .. & .. & .. & .. & .. & .. & .. \\ .. & .. & .. & .. & .. & .. & .. & .. \\ 0 & 0 & 0 & 0 & .. & .. & 0 & 1 \\ 0 & 0 & 0 & 0 & .. & .. & -1 & 0 \end{bmatrix}$$

POISSON BRACKET:

$$\{u,v\}_{P.B.} = {}^{t}\left[\partial_{\vec{a}} u\right] J \left[\partial_{\vec{a}} v\right]$$

see Goldstein, "Classical Mechanics" (Addison-Wesley1980)

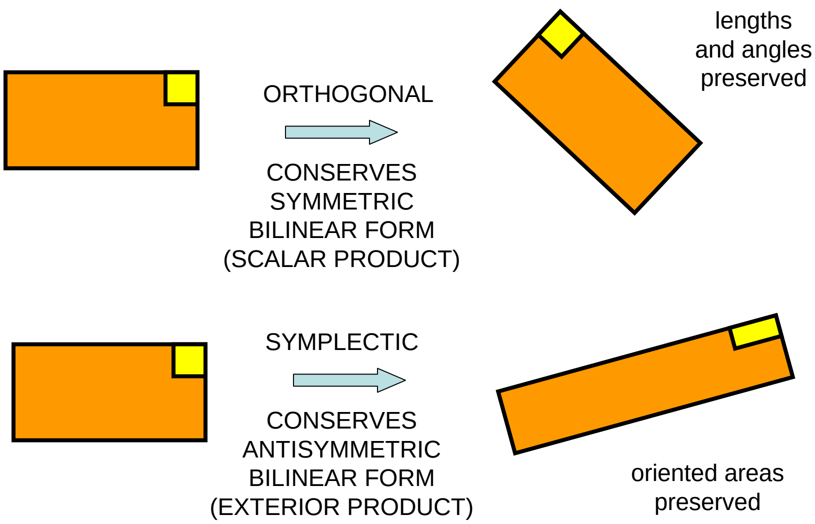
S MATRIX MUST CONSERVE ALL POISSON BRACKETS: **S** IS SYMPLECTIC

$${}^{t}SJS=J$$

SYMPLECTICITY CAN BE UNDERSTOOD AS INFORMATION CONSERVATION

08-VI-11b

TWO TYPES OF MAPS



08-VI-12

SYMPLECTICITY IS A 1ST QUANTIZATION PROPERTY

HOWEVER

IT IS OFTEN EXPRESSED AS THE PROPERTY OF
CONSERVATION OF COMMUTATORS ...

$$\hat{X} = \overleftarrow{x}\cdot\vec{\hat{a}} \qquad \hat{Y} = \overleftarrow{y}\cdot\vec{\hat{a}} \quad\Longrightarrow\quad \left[\hat{X},\hat{Y}\right] = \overleftarrow{x}\cdot \mathrm{J}\cdot\vec{y}\left[\vec{\hat{a}},\overleftarrow{\hat{a}}\right]$$

$$\left[\hat{X}^{out},\hat{Y}^{out}\right] = \left[\hat{X}^{in},\hat{Y}^{in}\right] \quad\Longrightarrow\quad {}^{t}\mathrm{SJS}=\mathrm{J}$$

... WHICH CAN BE MISLEADING!

08-VI-13

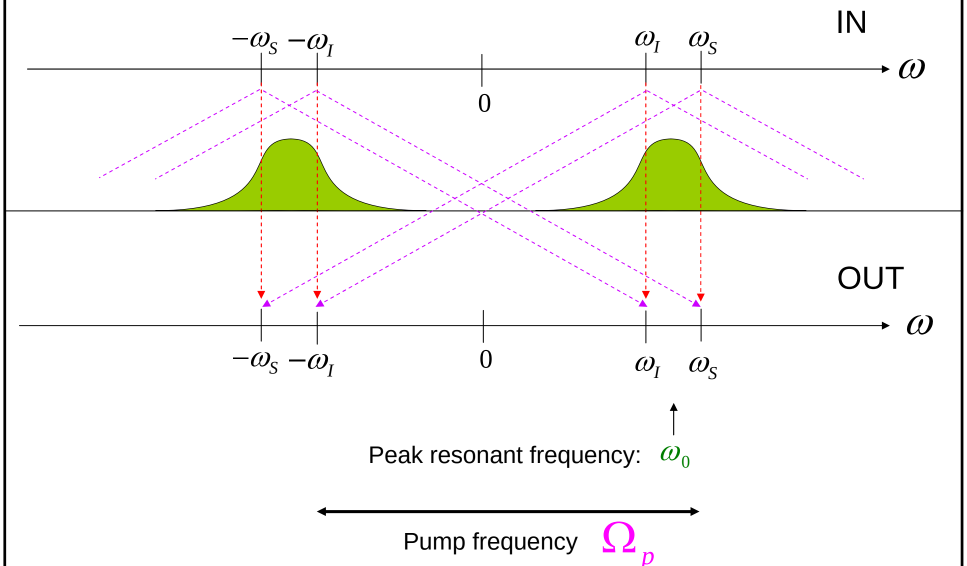
A MAXIMALLY EFFICIENT
SIGNAL PROCESSING FUNCTION
MIX THE MINIMAL NUMBER OF MODES
COMPATIBLE WITH SYMPLECTICITY

AMPLIFICATION: 2 CASES

08-VI-14a

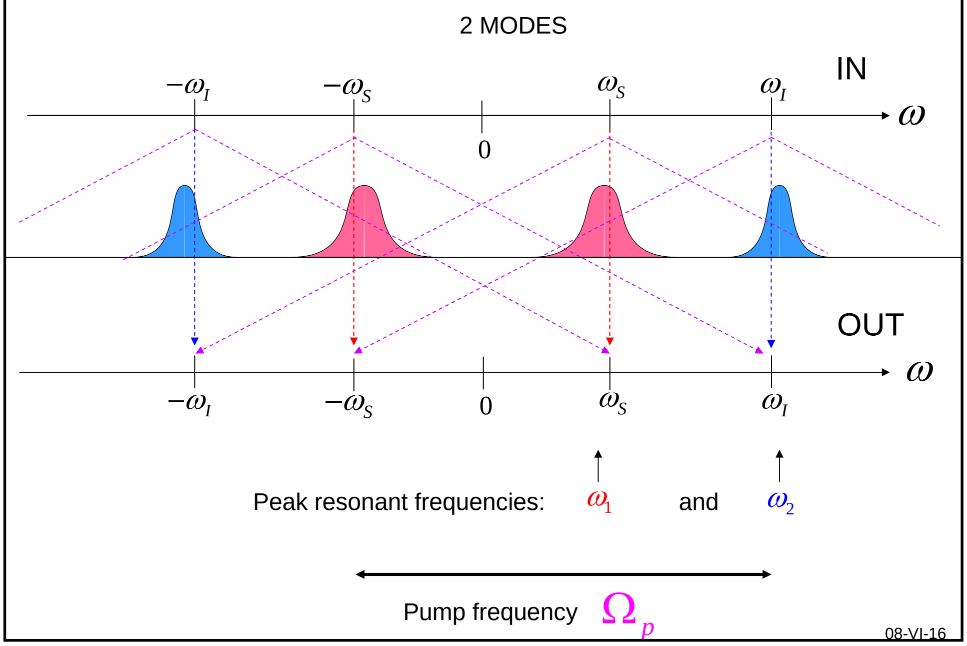
DEGENERATE PARAMETRIC PUMPING

1 MODE



08-VI-15

NON-DEGENERATE PARAMETRIC PUMPING



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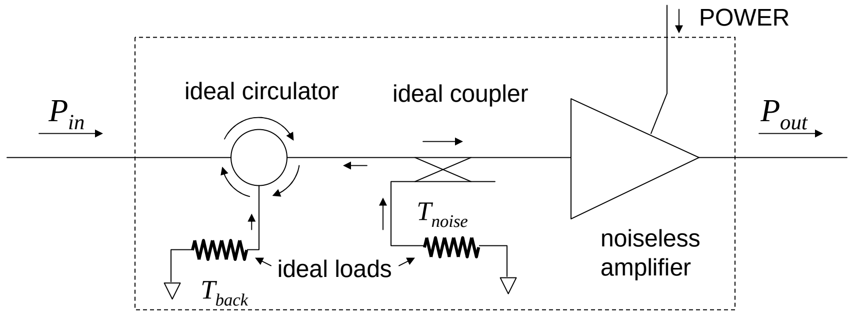
08-VI-5c

LINEAR AMPLIFIER CHARACTERISTICS

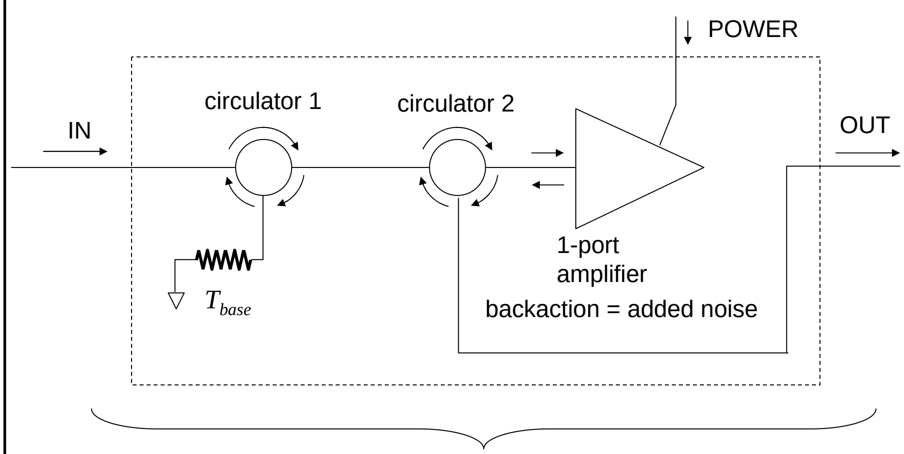


REPRESENTATION OF MATCHED 2-PORT

- Power gain (P_{out}/P_{in})
- Signal bandwidth
- Noise temperature T_{noise}
- Backaction T_{back}
- Dynamic range, intermodulation
- Tuning bandwidth

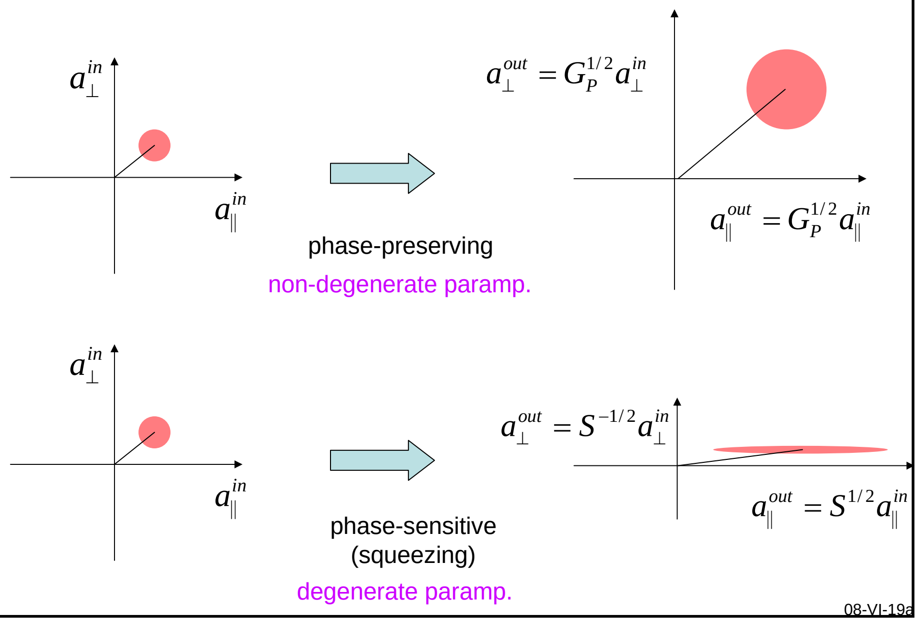
08-VI-17

2-PORT vs 1-PORT AMPLIFIER



08-VI-18

TWO TYPES OF LINEAR AMPLIFIERS



08-VI-19a

CRYOELECTRONIC AMPLIFIERS APPROACHING THE QUANTUM LIMIT

type	$kT_N/(\hbar\omega/2)$	power gain	out-of-band back-action noise	ease of use
HEMT	40-80	25-35dB	small	easy
SQUID	1-2	20-30dB	concern	OK
RF-SET	1-2	15-20 dB	concern	OK
QPC	1	~0dB	very small	difficult

HEMT: High Electron Mobility Transistor, SET: Single Electron Transistor, QPC: Quantum Point Contact

08-VI-20

LARGE GAIN LIMITS

PHASE-PRESERVING: $$k_B T_N \geq \frac{\hbar\omega_s}{2}$$ ← STANDARD QUANTUM LIMIT

(Caves, 1982)

PHASE-SENSITIVE: $$k_B T_N \geq 0$$ NOISELESS

08-VI-21

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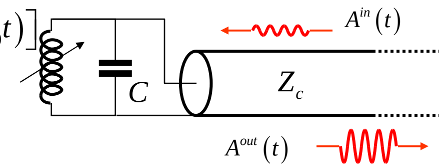
08-VI-5d

SCATTERING MATRIX OF DPA FOR ARBITRARY ω

$L^{-1}(t) = L_0^{-1}\left[1 + 2\varepsilon \sin(2\omega_0 t)\right]$



$\omega_0 = \frac{1}{\sqrt{LC}}$

$\Gamma = \frac{1}{2Z_c C}$

Stiff pump condition

$$\ddot{\Phi} + 2\Gamma\dot{\Phi} + \omega_0^2\Phi - i\omega_0^2\varepsilon\Phi\left(e^{2i\omega_0 t} - e^{-2i\omega_0 t}\right) = 4\Gamma V^{in}(t) \qquad V^{out}(t) = \dot{\Phi} - V^{in}(t)$$

Harmonic balance + RWA ⇨ $$i\omega_0\left[\frac{(\omega_0 - \omega)}{i\Gamma} - 1\right]\Phi[\omega] - i\frac{\omega_0^2\varepsilon}{2\Gamma}\Phi[\omega - 2\omega_0] = 2V^{in}[\omega]$$

After a few steps:

$$\begin{bmatrix} A^{out}[+\omega_S] \\ A^{out}[-\omega_S] \\ A^{out}[+\omega_I] \\ A^{out}[-\omega_I] \end{bmatrix} = \begin{bmatrix} r & 0 & 0 & s \\ 0 & r^* & s^* & 0 \\ 0 & s^* & r^* & 0 \\ s & 0 & 0 & r \end{bmatrix} \begin{bmatrix} A^{in}[+\omega_S] \\ A^{in}[-\omega_S] \\ A^{in}[+\omega_I] \\ A^{in}[-\omega_I] \end{bmatrix}$$

$$r = \frac{1 + \vartheta^2 + \zeta^2}{(1 - i\vartheta)^2 - \zeta^2}$$

$$s = \frac{2\zeta}{(1 - i\vartheta)^2 - \zeta^2}$$

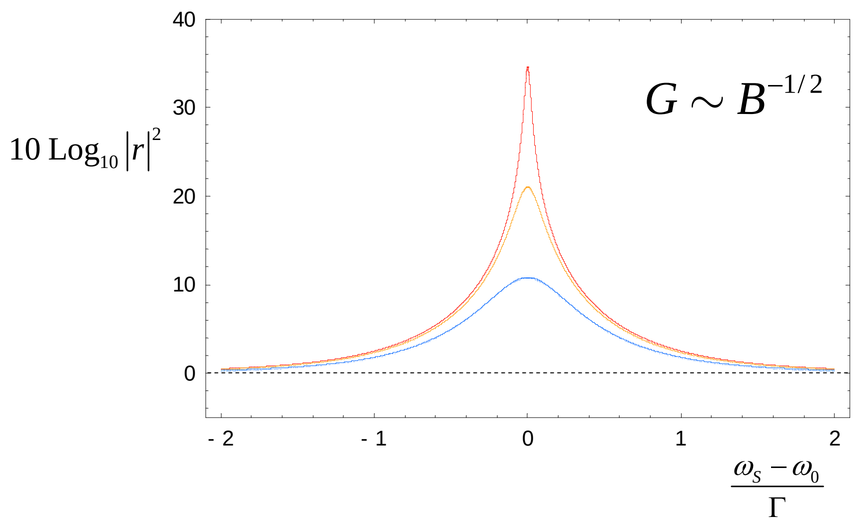
$$\zeta = \varepsilon\frac{\omega_0}{2\Gamma} < 1$$

SYMPLECTICITY: $$\begin{cases} |r|^2 - |s|^2 = 1 \\ rs^* - r^*s = 0 \end{cases}$$

$$\vartheta = \tan\theta = \frac{\omega_S - \omega_0}{\Gamma}$$

08-VI-21d

GAIN-BANDWIDTH COMPROMISE

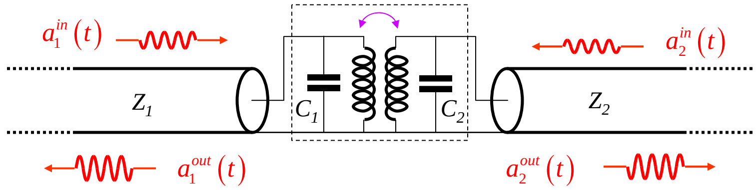


08-VI-22

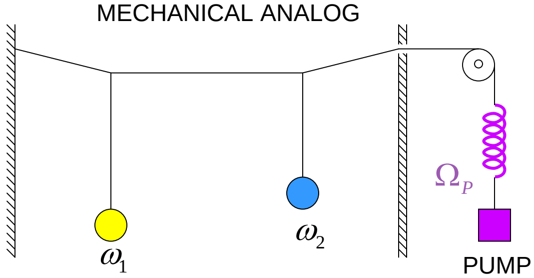
MINIMAL IMPLEMENTATION OF NON-DEGENERATE PARAMETRIC AMPLIFIER

TIME-DEPENDENT COUPLING BETWEEN 2 OSCILLATORS

$$\left(\mathrm{L}^{-1}\right)_{12} = \left(\mathrm{L}_0^{-1}\right)_{12}\left[1 + 2\varepsilon \sin\left(\Omega_P t\right)\right]$$



MECHANICAL ANALOG



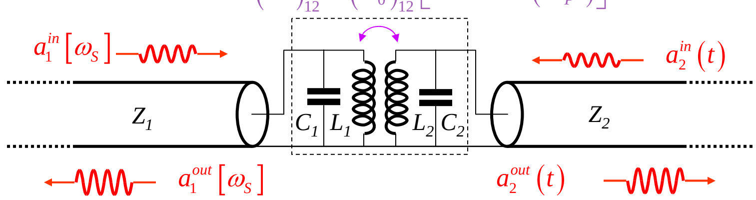
HERE, WAVE AMPLITUDES HAVE (PHOTON NB.)$^{1/2}$ DIMENSION BUT ARE TREATED CLASSICALLY

$$\sqrt{\frac{\hbar\left|\omega_l\right|}{2}}\, a_l\left[\omega_l\right] = A_l\left[\omega_l\right]$$

08-VI-23

SCATTERING MATRIX OF NDPA FOR ARBITRARY ω

$$\left(\mathrm{L}^{-1}\right)_{12} = \left(\mathrm{L}_0^{-1}\right)_{12}\left[1 + 2\varepsilon \sin\left(\Omega_P t\right)\right]$$



$$\omega_{1,2} = \frac{1}{\sqrt{L_{1,2}C_{1,2}}}$$

$$\Gamma_{1,2} = \frac{1}{2Z_{1,2}C_{1,2}}$$

phase-preservation

$$\begin{bmatrix} a^{out}[+\omega_S] \\ a^{out}[-\omega_S] \\ a^{out}[+\omega_I] \\ a^{out}[-\omega_I] \end{bmatrix} = \begin{bmatrix} r_S & 0 & 0 & s \\ 0 & r_S^* & s^* & 0 \\ 0 & s^* & r_I^* & 0 \\ s & 0 & 0 & r_I \end{bmatrix} \begin{bmatrix} a^{in}[+\omega_S] \\ a^{in}[-\omega_S] \\ a^{in}[+\omega_I] \\ a^{in}[-\omega_I] \end{bmatrix}$$

$$r_{S,I} = \frac{\left(1 + i\vartheta_{S,I}\right)\left(1 - i\vartheta_{I,S}\right) + \zeta^2}{\left(1 - i\vartheta_S\right)\left(1 - i\vartheta_I\right) - \zeta^2}$$

$$s = \frac{2\zeta}{\left(1 - i\vartheta_S\right)\left(1 - i\vartheta_I\right) - \zeta^2}$$

$$\zeta = \frac{\varepsilon}{2}\left(\frac{\omega_1\omega_2}{\Gamma_1\Gamma_2}\right)^{1/2} < 1$$

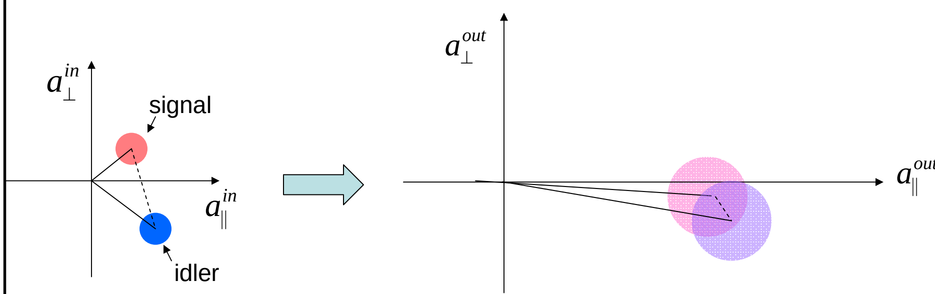
$$\vartheta_{S,I} = \tan\theta_{S,I} = \frac{\omega_{S,I} - \omega_{1,2}}{\Gamma_{1,2}}$$

AMPLIFICATION: $\left|r_S\right|^2 = \left|r_I\right|^2 \geq 1$

SYMPLECTICITY: $\left|r_S\right|^2 - \left|s\right|^2 = \left|r_I\right|^2 - \left|s\right|^2 = 1$

08-VI-24c

SIGNAL AND IDLER CO-AMPLIFICATION FOR NON-DEGENERATE PARAMETRIC AMPLIFIER

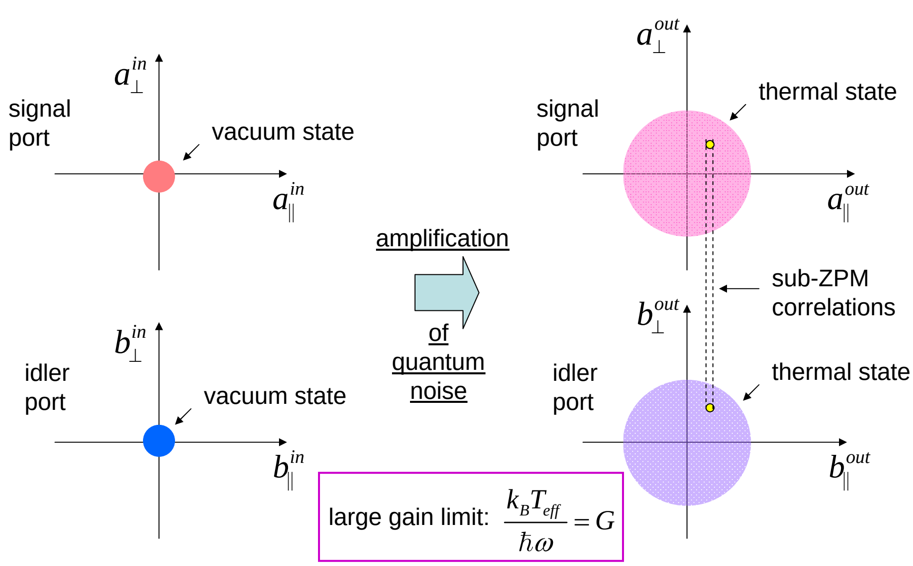


sum of signal and idler is amplified
difference of signal and idler is de-amplified

total volume in phase space is conserved

08-VI-25

SQUEEZING OF QUANTUM FLUCTUATIONS FOR FOR NON-DEGENERATE PARAMETRIC AMPLIFIER



large gain limit: $\frac{k_B T_{eff}}{\hbar\omega} = G$

see "Quantum Squeezing", Drummond and Ficek eds (Springer 2004)

08-VI-26d

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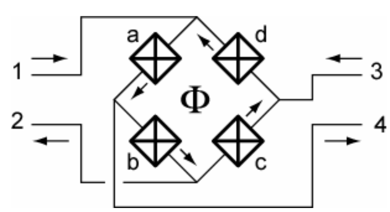
08-VI-5e

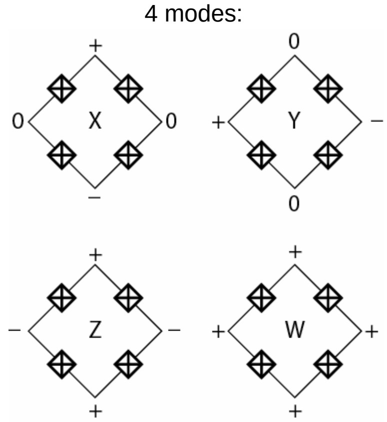
TOWARDS THE PUREST NON-LINEARITY: THE JOSEPHSON RING MODULATOR

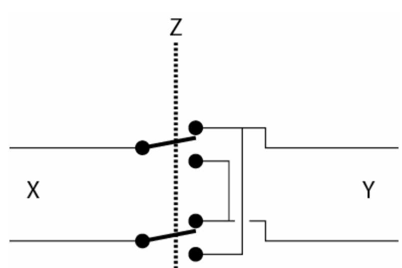
(Bergeal *et al.*, 2008)



4 junctions in a ring threaded by flux

4 modes:





The Z mode can be understood as providing an invertible coupling between the X and Y mode

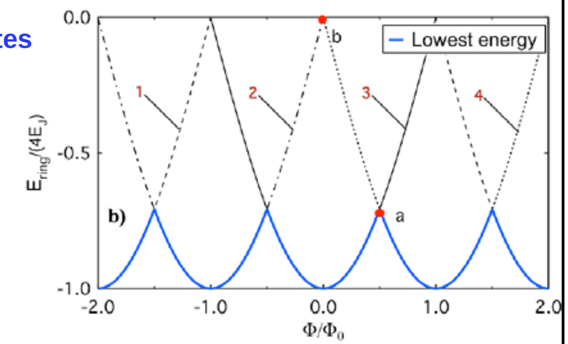
08-VI-27

CONSEQUENCES OF RING SYMMETRY

- **Coexistence of 2 stable states with $4\Phi_0$-periodicity**

junction phases satisfy

$$\begin{cases} \phi_a + \phi_b + \phi_c + \phi_d = \dfrac{2\pi\Phi}{\Phi_0} \\ \sin\phi_a = \sin\phi_b = \sin\phi_c = \sin\phi_d \end{cases}$$



- **Useful non-linearity with minimal number of spurious terms**

At $\Phi = \Phi_0/2$ and for small X, Y, Z:

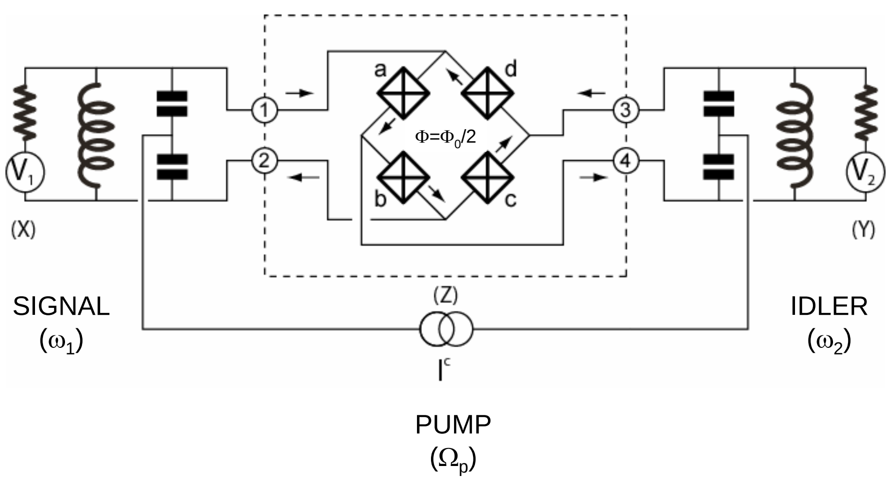
$$E_{ring} = g^{(3)}XYZ + B\left(\frac{X^2}{4} + \frac{Y^2}{4} + \frac{Z^2}{2}\right) + \text{higher order terms}$$

Mix 3 orthogonal modes X, Y, Z (S. Girvin)

Spurious terms only renormalize mode frequencies

08-VI-28

SCHEMATICS OF JOSEPHSON AMPLIFIER BASED ON RING MODULATOR

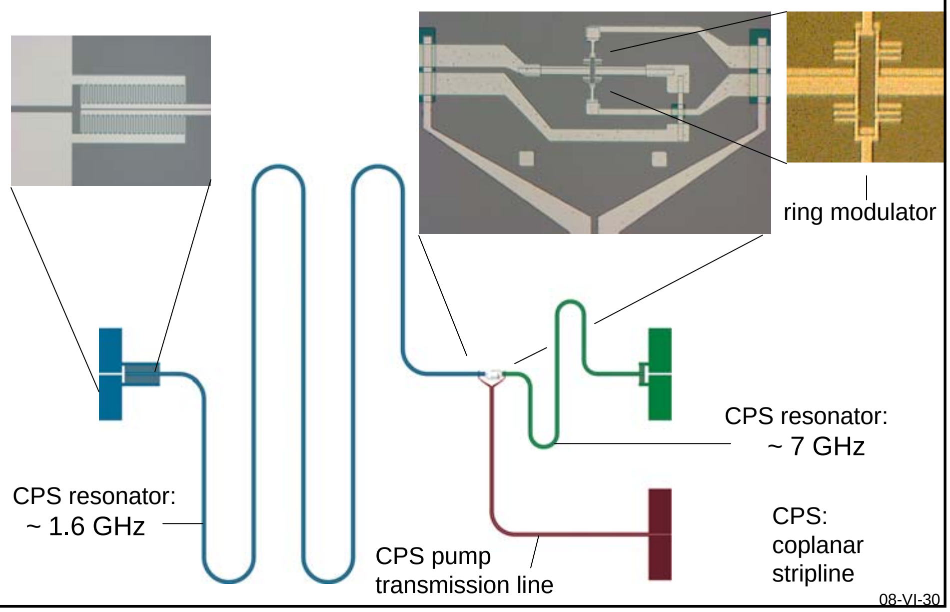


Bergeal et al., Nature Physics 6, 296 (2010)

08-VI-29

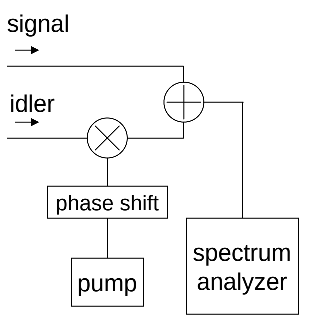
JOSEPHSON PARAMETRIC AMPLIFIER CHIP

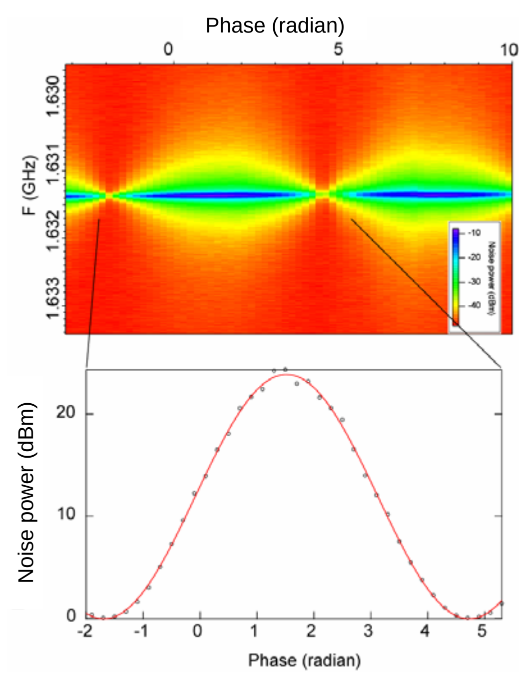
Bergeal et al., Nature 465, 64 (2010)



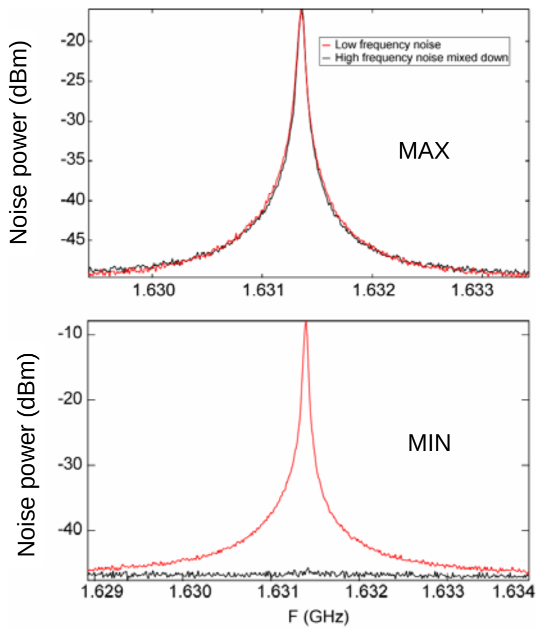
2-MODE NOISE SQUEEZING DATA

(Bergeal *et al.*, in preparation, 2008)





2-MODE NOISE SQUEEZING DATA (CTND)



CONCLUSIONS

A PHASE PRESERVING AMPLIFIER ADDS AT LEAST HALF A PHOTON OF NOISE AT THE SIGNAL FREQUENCY

A PHASE SENSITIVE AMPLIFIER CAN BE NOISELESS

MINIMAL NUMBER OF MODES IN A DISPERSIVE ACTIVE CIRCUIT IS A NECESSARY CONDITION FOR AMPLIFICATION AT THE QUANTUM LIMIT

PARAMETRIC AMPLIFICATION USING 3-WAVE OR 4-WAVE MIXING OF JOSEPHSON JUNCTION IS MINIMAL

JOSEPHSON RING MODULATOR IS A CONVENIENT PURE 3-WAVE MIXING DEVICE FOR NON-DEGENERATE PARAMETRIC AMPLIFICATION

08-VI-30

NEXT YEAR :

1) NON-PERTURBATIVE ASPECTS OF QUANTUM CIRCUITS

2) PERIODICITY OF JOSEPHSON COSINE POTENTIAL AND ITS CONSEQUENCES

3) STRONG COUPLING BETWEEN JUNCTION AND REST OF CIRCUIT

08-VI-31

END OF 2008 COURSE

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