Can Nuclear Power Supply Clean Energy in the Long Run?

A Model with Endogenous Substitution of Resources

by

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Abstract

This paper models develops a model of nuclear energy with endogenous substitution over time among polluting nonrenewable resources. We find that nuclear power can reduce the cost of generating clean energy significantly. However, continued expansion of nuclear capacity at historical rates is likely to cause a scarcity of uranium and make nuclear costlier than other energy sources within some decades. However, renewables such as solar, wind and biomass, clean coal and next generation nuclear technologies may supply significant amounts of clean energy late this century. The cost of generating low carbon energy increases sharply if global carbon concentration targets are set at 450 ppm instead of 550 ppm. A policy implication is that current political and regulatory impediments to the expansion of nuclear power generation may prove to be costly if large volumes of clean energy need to be supplied over a short period of time.

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Running Head: Can Nuclear Power Supply Clean Energy in the Long Run?

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1. Introduction

Nuclear power accounts for a sixth of all electricity production globally. Seventeen countries depend on it for at least a quarter of their electricity (World Nuclear Association, 2003). The United States has 103 plants that generate 20% of its electricity. France has 56 of them that account for 80% of electricity supply. Global nuclear generation capacity has exhibited double digit growth in recent years and continues to grow rapidly in the developing countries. About 36 new reactors are under construction. China which has 9 plants, expects to build 30 more in the next 15 years.

Even though the developed countries have not built any new nuclear plants for some time, there is a resurgence of interest in nuclear power as a clean alternative to polluting fossil fuels. The current movement towards a binding international treaty along the lines of the failed Kyoto Treaty has also revived interest in non-carbon energy alternatives. Limiting the use of carbon-emitting fossil fuels such as coal, oil and natural gas which currently account for 85% of global energy consumption will mean increased use of nuclear energy, since hydro and renewable energy sources can not supply large volumes of baseload power. In the U.S., nuclear power has been used to replace coal to meet standards set by the Clean Air Act, especially in the Northeast.[2]

This paper develops a long run global model of energy substitution to examine the role of nuclear power as a source of clean energy supply. The economic modelling of nuclear power presents several methodological challenges. Major energy resources such as oil, gas and coal are nonrenewable, and their cost of extraction must increase with cumulative depletion. But nuclear

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[2] "Most of the avoided carbon dioxide emissions over the last 20 years have come from nuclear power," according to a U.S. Department of Energy official (Moniz, 1999).

power is strictly not a renewable energy source. Its major input uranium is nonrenewable. In next generation nuclear technologies, the output (reprocessed uranium and plutonium) may be re-used as input. We explicitly model the recycling of materials in the nuclear fuel cycle. We consider several scenarios – no growth in nuclear and continuation of past growth trends as well as cost reductions and technological change both in the nuclear industry and in conventional and renewable energy sectors. These cases are examined with and without environmental regulation in the form of a cap on atmospheric carbon concentration.[3]

There are relatively few studies of the long-run economics of nuclear energy. Nordhaus (1973) pioneered the endogenous substitution approach in partial equilibrium to examine the market allocation of scarce resources over time and accounted for limited uranium resources. Cropper (1980) has examined a theoretical model of the trade-offs between fossil fuels and nuclear energy. Most energy models tend to assume the availability of nuclear energy at given prices, but do not account for the uranium used, which turns out to be a critical issue, as we see in this paper.

A major finding is that nuclear power can play a significant role in the transition from fossil fuels to clean renewable resources. Its share of energy supply peaks around 2055 but remains significant for several decades after that. However, the rising cost of uranium and high capital costs of building new nuclear plants will ultimately make it costlier relative to new coal technologies and renewables. Only major developments in nuclear technology such as fast breeder reactors could supply a significant share of energy in the long run, i.e., in the second half

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[3] The regulation we consider is a quota on the stock of emissions. It can be thought of as a target carbon concentration in the atmosphere. We do not explicitly model an agreement such as the Kyoto Protocol because even though the stated objective of the Intergovernmental Panel on Climate Change (IPCC) is a stabilization of pollution concentration, the actual agreement is subject to political considerations (some countries may be exempt from targets for a period) and the agreement may itself be cast in terms of emissions limits mainly to begin a period during which the institutional mechanisms are put in place. For the IPCC's atmospheric stabilization goals, see IPCC (2001).

of this century. Without these new nuclear technologies, the problem of waste accumulation becomes critical. Nuclear power may help us reduce atmospheric carbon, but will give rise to a new problem of storing significant amounts of toxic waste.[4]

We find that a model with endogenous substitution among energy resources leads to a lower estimate of the shadow price of carbon, at least in the near term. Most estimates in the literature suggest a range of $100-500/ton of carbon by 2050 and up to $1000 by 2100.[5] We get a price of $18/ton of carbon in 2050 rising to nearly $300 in the year 2100. These figures are substantially lower than in other studies, suggesting that nuclear power may have an important role in reducing the price of carbon. A policy implication is that current political and regulatory constraints to the expansion of nuclear power may result in a significant increase in the cost of producing clean energy.

Section 2 introduces a simple theoretical framework with resource depletion and environmental regulation. Section 3 summarizes the main elements of the empirical model with details and data provided in an Appendix. Section 4 discusses the simulation results. Section 5 concludes the

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[4] However, the future contribution of nuclear energy is highly dependent on whether new technologies such as fast breeders come into play or not. If these technologies are widely deployed, then they will occupy a significant position in the energy mix. Current projections by groups such as the ten-country Generation IV International Forum (USDOE 2002) suggests that next generation nuclear technologies (Generation IV) could begin to be deployed in during the period 2030-2050 (see page 13, USDOE, 2002). The evolution of these technologies will depend on several factors including public perception of safety and proliferation issues as well as research and development expenditures. These technologies are not yet economical because of the low price of uranium (partly due to use of uranium from dismantled weapons systems) but increased input and recycling costs in the future are expected to make them economically feasible compared to standard nuclear reactors. It is hard to find specific probabilities regarding when FBR technologies will become widely available, but expert opinion suggests the time frame 2030-2050 as the likely period when they may be deployed in response to rising input and recycling costs for traditional technologies.

[5] e.g., see Fischer and Morgenstern (2005), Nordhaus (2007), Edenhofer *et al* .(2006), and Clark *et al.*, (2007). All dollar figures are in 2000 US dollars, unless stated otherwise.

paper.

2. A Dynamic Model with a Cap on the Stock of Emissions

In this section we extend the basic Hotelling (1931) model with environmental regulation imposed in the form of a ceiling on the stock of pollution. Such a ceiling may be thought of as a target carbon concentration in the atmosphere (e.g., 550 parts per million). We assume one demand, one polluting nonrenewable resource (say, coal) and a "clean" backstop resource (call it solar energy). The main conclusion here is that regulation of the stock may lead to the joint use of the two resources before a complete transition to the clean fuel.

Let the instantaneous utility at time t generated by energy consumption $q(t)$ be given by $u(q(t))$ which is assumed to be strictly increasing and concave in q. Both coal and solar are assumed to be perfect substitutes, so $q(t) = x(t) + y(t)$ where $x(t)$ and $y(t)$ are their respective consumption rates. Define $X(t)$ as cumulative extraction of coal. Then we must have $\dot{X}(t) = x(t).$ The unit extraction cost is given by $c_x(X)$. It increases with cumulative extraction at an increasing rate. This is a plausible assumption which suggests that the cost of extraction may increase as deeper or more inaccessible resources are tapped. Let the aggregate known reserves of coal be denoted by $\bar{X}$.

By scaling appropriately, we can assume that each unit of coal generates one unit of pollution (e.g., carbon). Denote $Z(t)$ to be the stock of carbon at time t, with $Z(0)$ as the initial stock. Pollution increases $Z(t)$, but a portion declines naturally at an assumed rate $\alpha > 0$. That is, the growth of the carbon stock is given by $\dot{Z}(t) = x(t) - \alpha Z(t)$. Define the exogenous ceiling on the stock of carbon to be $\bar{Z}$ with $Z(0) < \bar{Z}$. Then we can define $\bar{x}$ as the maximum consumption rate of coal if $Z(t)$ equals its ceiling $\bar{Z}$, i.e., $\bar{x} = \alpha \bar{Z}$, and by $\bar{p}$ the corresponding marginal utility, so that $\bar{p} = u'(\bar{x})$.

Finally, let c_y be the constant unit cost of the abundant solar energy. Let $\bar{y}$ be the extraction rate for which the marginal utility equals the unit cost of solar, i.e., $u'(\bar{y}) = c_y$. The social planner chooses extraction rates of the two resources to maximize welfare as follows:

$$\max_{\{(x(t),y(t)\}} \int_0^\infty \left\{u\left(x+y\right) - c_x(X)x - c_y y\right\} e^{-\rho t} dt$$

subject to the two differential equations $\dot{X}(t)$ and $\dot{Z}(t)$, and given values of $\bar{X}, Z(0)$ and $\bar{Z}$. The above model can be solved using standard optimal control techniques. We do not explicitly solve it here. The complete model characterization is available from the authors and for a similar model with constant resource extraction costs, see Chakravorty *et al.* (2006). Here we provide the basic intuition. There are two shadow prices in this model, one representing the scarcity rent of the nonrenewable and the other the shadow price of a unit of carbon stock, the latter being negative since pollution decreases welfare. When coal is used, its marginal benefit must equal its total marginal cost, which includes the unit cost of extraction c_x, the scarcity rent and the externality cost. When solar is used, the energy price must equal its extraction cost c_y. Both the shadow costs of resources and pollution grow with time as in standard dynamic models.[6] Finally at the end of the planning horizon, the value of the resource and pollution stocks must also go to zero.

If the cost of the backstop c_y is higher than the maximum extraction cost $\bar{c}_x$ it is obvious that all the coal will be exhausted. Then each unit of coal may have a differential rent as well as a scarcity rent. Suppose $c_y < \bar{c}_x$. It can be shown that there may be only three solutions, if we assume that the cap on the stock of pollution must bind, at least over some interval of time. If not, we are in a pure Hotelling world. The solution that matches with the empirical model in the rest of this paper is shown in Fig. 1. The polluting fossil fuel is used until the ceiling is hit, and exactly at that instant, the clean backstop becomes economical. Both resources are used at constant rates until coal is exhausted. Beyond this point, only solar supplies energy and the stock of pollution decreases from the regulated level to zero.

The curve MC_A represents the unit extraction cost plus the shadow cost of coal over time absent environmental regulation. This is the Hotelling model with no pollution. Coal is consumed from the beginning until time Γ, when it is exhausted and the backstop solar is used at rate $\bar{y}$. The curve MC_B represents the marginal cost of coal with the ceiling constraint, and includes its extraction cost and the shadow price plus the shadow price of pollution, i.e., the right hand side of equation (2). MC_B increases to equal the cost of the backstop c_y at time t_1. At this time, the stock of carbon also reaches the ceiling. However, at price c_y, demand is too high to be satisfied only by the nonrenewable without violating the ceiling, hence some backstop must be used. From t_1 to

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[6] As we will see in the empirical section, higher discount rates imply higher shadow prices of carbon in the future.

t_2 the pollution level is at its maximum. The extraction rate of coal is the maximal rate $\bar{x}$, and the marginal costs MC_B and c_y are equal. Additions to the stock of carbon exactly equal the natural decay, $\bar{x} = \alpha \bar{Z}$. Coal gets exhausted at t_2 and solar supplies all energy. The ceiling is not binding from time t_2, and the stock of carbon declines gradually to zero. Beginning from t_2 the shadow price of carbon is zero, and MC_B is higher than c_y.

Equilibrium quantities are also shown in Fig. 1. The dashed curve corresponds to the pure Hotelling path without regulation. Resource extraction declines to y_c at time Γ, followed by use of the backstop. The solid lines show resource use under regulation. Regulation initially slows down the extraction rate of coal until Γ, but extends the time period during which it is used, since cumulative demand in both cases must equal the initial stock. Two other solutions can arise depending on parameter values, although we do not detail them here. If the backstop solar is costly, there may be only coal use at the ceiling, followed by a phase with rising coal prices but the pollution stock strictly below the ceiling, and finally a transition to the clean backstop resource. Or the backstop may become economical exactly when the ceiling period ends, and at that instant, coal also gets exhausted. Since $c_y < \bar{c}_x$, exhaustion implies that there is coal that is costlier to exploit than the backstop, which remains in the ground. The main point of the above framework is to show that when a ceiling is imposed on the stock of pollution, extraction may increase for a time, then stay at the ceiling when both the fossil fuel and the clean resource are used simultaneously until the former is completely exhausted.

3. The Simulation Model with Fossil Fuels and Nuclear Power

In this section, we apply the framework outlined above but with several nonrenewable resources and demands, nuclear technology with recycling of materials and backstop resources. We outline the main economic features of the model and provide details of the model and data in the Appendices. The supply side of the model is shown in a schematic in Fig. 2. Primary energy is provided by two types of resources – nonrenewables, namely, crude oil, coal, natural gas and uranium; and renewables - biomass, wind and solar. These resources can be used to produce electricity or refined petroleum products.

In the electricity sector**,** we assume that existing fossil fuel-based power plants will not be replaced by the same designs because of their poor efficiency and low environmental performance. Rather, they will be progressively phased out so that their current capacity is exogenously decreased, i.e. their production is reduced to zero within 30 years.[7] New electricity units from gas and coal will be supplied by more efficient and cleaner plants, if they are competitive relative to other energy sources. These new gas and coal plants use combined cycle (NGCC and IGCC) technology (see IPCC, 2005). [8] They could also be endowed with scrubbers for controlling carbon emissions, if cost effective. These plants are called CCS plants (Carbon Capture and Storage). Refined petroleum products can only be supplied by the three fossil fuels as well as biomass. If crude oil is expensive, transportation energy can be provided by liquefaction of coal, gas or biomass. Gas, coal and backstops can also be used directly (combustion) as secondary energy sources.

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[7] This is reasonable because electric plants generally have a lifetime of 30 or so years.

[8] Natural Gas Combined Cycle (NGCC) plants are the new standard for gas power stations in North America and Europe. Integrated Gasification Combined Cycle (IGCC) is considered to be the leading technology candidate for electricity production with coal (see MIT, 2007).

Final energy demand is divided into transportation, industry and residential/commercial. The energy consumed in the industry and residential/commercial sectors is modeled as a convex combination of electric and non-electric energy as in Manne *et al.* (1995), with a CES specification that accounts for imperfect substitutability between the two inputs. Non-electric energy supply is also CES and is produced from oil, gas, coal and the backstop when the latter is economical. The energy consumed by the transportation sector can be supplied either by refined petroleum or by a perfectly substitutable backstop in the form of cars powered by solar-powered fuel cells. The sector-specific backstops are entirely carbon-free and renewable. They take the form of fuel cells powered by hydrogen, which in turn is produced by solar-thermal technology.[9]

The three final energy sectors are characterized by independent demands that are a function of energy prices and income. Following Chakravorty et al. (1997), generalized Cobb-Douglas demand functions for each sector are given as $D_j = A_j P_j^{\alpha_j} Y^{\beta_j}$, where α_j and β_j are respectively the price and income elasticities for demand in sector j, A_j is the sector-specific technical coefficient, P_j is the price of delivered energy in sector j , and Y is global GDP which is non-stationary. GDP increases exogenously over time at a declining rate as in Nordhaus and Boyer (2000).

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[9] The backstop costs have two components consisting of the cost of producing hydrogen for a solar thermal plant (by solar thermo-chemical water splitting) plus the cost of the fuel cell device specific to each sector, as discussed in more detail in the Appendix. In general the model results were found not to be very sensitive to the backstop price mainly because other resources such as nuclear and biomass energy emerge as cheaper options in the future.

Nuclear technology is optimized by choosing the amount of energy produced by conventional Light Water Reactor (LWR) technology. Technical breakthrough in the nuclear sector is modelled by assuming that Fast Breeder Reactor (FBR) technology is available.[10] The nuclear model is embedded in the general model of substitution across resources and demands.

All conversion processes from resources to the two secondary energy sectors (electric and non-electric) incur costs of conversion and losses, such as in electricity transmission. We include investment as well as operation and maintenance costs in the transformation of one form of energy into another, e.g., coal into electricity or crude oil into refined petroleum products. These investment costs decline with accumulated experience, as in Goulder and Mathai (2000) and van der Zwaan *et al.* (2002). Operation and maintenance costs are kept constant over time. Extraction costs for the nonrenewable resources in our model – oil, coal, gas and uranium are assumed to rise with cumulative extraction. The functional form is taken from Nordhaus and Boyer (2000). Cost data are adapted from Rogner (1997). Intra-marginal resource units will accrue Ricardian rents and may accrue scarcity rents if they are completely exhausted. Crude oil extraction costs range from $20-200 per barrel ($3.5-35/GJ). Initial gas, coal and uranium extraction costs are respectively $2.5/GJ ($2.63/MBtu), $1.5/GJ ($0.05/ton of coal) and $0.05/GJ ($20/kg of uranium). If the stocks of gas, coal and uranium were to approach exhaustion, the cost of coal extraction would go up by a factor of 4, and for gas and uranium by a factor of 7. Conversion costs for each resource into each demand are added to these extraction costs.

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[10] LWR is the nuclear technology most commonly used. It uses uranium and produces a significant volume of waste. The FBR is generally viewed as a next generation nuclear technology with higher capital costs, prototypes of which are operational. It uses uranium and plutonium and recycles a larger portion of the waste**.** See Generation IV International Forum: http://www.gen-4.org/ and Appendix for further details of the technology.

The model works as follows. The combustion of fossil fuels releases carbon into the atmosphere. Nuclear power is carbon free. LWR technology uses uranium ore as input. FBR technology uses a mix of several inputs, including wastes from LWR production.[11] The algorithm chooses the least cost energy supply for each sector.[12] The two nuclear technologies enjoy complementarities in materials use and waste recycling and may be deployed jointly. Unlike for fossil fuels, production of nuclear energy creates the need for costly reprocessing and storage of wastes which must be included in the total marginal cost of nuclear energy. In models where only LWR technology is available, nuclear waste does not have economic value so its shadow price is zero. However, waste has economic value as an input in FBR operation. Consumer plus producer surplus is maximized subject to the technological relationships and stock dynamics. The discount rate is assumed to be 5%.[13]

We consider several scenarios, described as follows:

A. *Stagnation in Nuclear Capacity with No Environmental Regulation*: This model is run with the fossil fuels and renewable resources shown in Fig. 2. But the nuclear capacity is fixed at current levels.[14] There is no environmental regulation in the form of a cap on carbon emissions. Even though current trends towards building new plants suggests that nuclear capacity is expected to grow in the near future, we run this scenario mainly to demonstrate how zero growth in nuclear power affects the utilization of fossil fuels and carbon emissions.

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[11] Mori (2000) describes a similar nuclear fuel cycle that allows for waste recycling as well.

[12] Adjustment lags are imposed by providing a lower bound on the endogenous rate of decline for each technology. This smoothens the transition in energy supply, as in Manne et al. (1995). For example, electricity production from any given type of plant can only decrease at most by 5% per year. Transitions among non-electric technologies such as a switch from oil to biomass in the production of refined petroleum products could be faster and are capped at 10% per annum.

[13] Newell and Pizer (2003) advocate a low discount rate, 5% or below, for long-run policy analyses.

[14] Nuclear electricity generation in year 2000, the start year of our model, was 9.25EJ or 17% of global electricity generation.

B. *Stagnation in Nuclear Capacity with Environmental Regulation:* Here the goal is to show how regulation may affect a carbon standard without growth in nuclear capacity. This scenario imposes a carbon target of 550 parts per million (ppm) on Model A. Later we perform sensitivity analysis with alternative caps of 450 and 650 ppm as has been done in other studies (e.g., Manne and Richels, 2002).[15] This scenario may represent a policy environment in which nuclear power generation makes no headway yet emissions must be controlled.

C. *Expansion in Nuclear Capacity with No Environmental Regulation:* This is the case when nuclear capacity grows at a business-as-usual pace. We follow the International Atomic Energy Agency projections for nuclear capacity growth until 2050 (IAEA, 2001, p.21) and extrapolate thereafter. Annual nuclear capacity is assumed to grow in our model by 2.5% until 2020 and by 5% until 2050. Overall, capacity increases by about 35% by 2020 and by a factor of 6 by 2050. This increase is in line with Intergovernmental Panel on Climate Change (IPCC) scenarios discussed by Toth and Rogner (2005) who conclude that the share of nuclear capacity will increase rapidly and represent up to 30 to 40% of total primary energy use by 2100.[16] This model captures a pro-nuclear policy environment. However, only LWR technology is modelled and we do not assume that FBR deployment is feasible in this scenario.

D. *Expansion in Nuclear Capacity with Environmental Regulation:* This case imposes a carbon standard of 550 ppm on Model C. Between models C and D, the purpose is to see how the carbon standard may affect the transition to conventional nuclear power.

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[15] Current CO_2 concentration levels are approximately 390 ppm. A target of 550 ppm is expected to produce some warming but without catastrophic effects (Hoffert *et al.*, 2002).

[16] This is a conservative estimate. Nuclear energy production has grown by a factor of 12 between 1973 and 2000, which is equivalent to an annual average increase of about 12% although from a smaller base (IEA, 2001). An MIT (2003) study assumes that nuclear capacity will increase by a factor of 3 by 2050. We examine the effect of a lower (50%) rate of increase later in the paper.

E. *Growth in Nuclear Capacity with availability of FBR Technology, No Regulation:* This scenario assumes that advances in FBR technology will allow significant adoption of this technology along with standard LWR plants. We assume the same aggregate capacity expansion rates as in the above cases. However, because of proliferation issues relating to the large scale adoption of plutonium based reactors, we introduce an aggregate cap on the amount of electricity that can be derived from the nuclear sector. This is set at 10 times the current level of nuclear energy production, as in van der Zwaan (2002). The effect of a higher cap is examined in the sensitivity analysis section.

F. *Growth in Nuclear Capacity with availability of FBR Technology and Environmental Regulation:* This is Model E with a carbon cap.[17]

4. Model Results

Energy use: Table 1 summarizes the results from models A to F. A common feature of all these runs is that the proportion of aggregate energy supplied by oil and natural gas does not vary significantly across the spectrum. The share of oil is about 30-32% of aggregate energy in 2050 dwindling to almost zero in the year 2100. Similarly the share of natural gas in aggregate energy supply is quite robust - within 18-21% across all scenarios and diminishes to an 8-9% share by 2100. This implies that regulation and the availability of other technologies do not affect the high degree of comparative advantage of oil and gas resources.

Coal shares decline from supplying almost half of all energy under no regulation and no nuclear expansion (model A) in 2050 to about a third when nuclear capacity expands or a carbon cap is

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[17] In summary, model A represents a stagnating nuclear sector, B a growing nuclear sector and C is nuclear with FBR. Models D, E and F are corresponding models with a 550 ppm cap.

imposed (models C through F). The share of nuclear power in aggregate energy rises from the current 2% to about 14% in a pro-nuclear scenario (models C through F). FBR proves to be competitive beyond 2065 (Models E and F).[18] Nuclear supplies almost a quarter of all energy supplied by the year 2100 (Model F).[19]

Gas replaces oil in power generation in the medium term, and supplies up to 45% of total primary energy consumption in 2030 before being replaced by coal (not shown in Table). Biomass or coal-based fuels progressively substitute for oil in the production of petroleum products depending on whether a carbon cap is in place or not. Almost no oil is used by the end of the century because its steady depletion causes a rise in oil extraction costs by a factor of four (Table 1). Nuclear also plays a minor role unless new generation technologies come into play. Without environmental regulation, renewable energy also remains a marginal player, consisting entirely of hydropower.

The introduction of a carbon target decreases aggregate energy consumption because of the added cost of meeting the carbon cap. The share of electricity in the final energy mix increases from 20% to about 33% in the medium term, and higher in the longer term. This occurs partly because the cost of electricity has a bigger investment component than non-electric energy, so it is less sensitive to a rise in fuel costs due to resource depletion. Under a carbon cap, electricity also gains market share because cheaper low carbon substitutes are available in electricity generation, than say in transportation.

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[18] Bunn *et al.* (2005) also conclude that recycling nuclear wastes would remain too expensive for at least the next 50 years.

[19] Other studies (Mori and Saito, 2004, Toth and Rogner, 2006) have predicted that nuclear energy could supply up to a third of aggregate energy.

Because electricity is the most important sector in terms of the potential for substitution of low carbon fuels, we next discuss which fuels will emerge as important players under the various scenarios (Fig. 3). The left hand side panels show the scenarios with no carbon cap. Notice that the bulk of future electricity supplies come from new coal fired generation and nuclear when the model allows for growth in nuclear capacity.[20] Existing coal fired generation and electricity from natural gas decline rapidly as these units are phased out over time. They are replaced by modern coal plants which are more efficient, and their efficiency increases over time from learning-by-doing. The renewable sector is not economical without a carbon cap. In the medium term, coal and nuclear (when permitted) dominate but in the long run, nuclear and renewable energies (biomass and wind) are economical. Nuclear supplies almost half of all electricity in 2060 before finally decreasing to zero. Under environmental regulation, nuclear, coal-fired units with scrubbers (CCS) and renewables supply the bulk of electricity in the long run. If only standard LWR technology is available, nuclear is phased out in the medium run because uranium becomes expensive with depletion.[21] However, new generation FBR technology can continue to supply nuclear power by recycling nuclear waste. FBR replaces coal powered CCS generation and to a lesser extent, renewables.

Final energy use is derived from various conversion processes (e.g. from coal to electricity or from crude oil to gasoline) and accounts for their efficiency or transformation losses. Table 1 shows primary energy use. But any question about end-use energy prices relates to final energy

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[20] See Radetzki (2000) for an analysis of the coal-nuclear trade-off in new power generation.

[21] Other studies (e.g., Rothwell and van der Zwaan, 2003) also conclude that LWR technology is not sustainable in the long run, although without a formal modeling approach.

use, i.e. the price paid by the consumer. Recall from Fig. 2 that each of the three end-use sectors consumes a bundle of secondary fuels.

Figure 4 gives the deviation in sectoral end-use prices and final energy consumption relative to Models A and B, where nuclear capacity is assumed to remain constant over time. It shows that allowing for more nuclear, either more LWR capacity or together with FBR, unambiguously decreases final energy prices. In turn, final energy use increases. The differences in prices and energy use are obviously larger in 2100 than in 2050. By 2050, as long as the carbon constraint is not too tight, substitution among resources in the power sector occurs at a relatively low cost. LWR does not have a big advantage because other low-carbon power generation sources are also available. The introduction of FBR makes a significant difference (especially in a scenario without a climate target such as model E) as it reduces the need to tap into more expensive technologies that lack cumulative experience such as advanced IGCC plants. This is shown by the tall shaded bar in industrial and other uses for the year 2100. In carbon-constrained scenarios, the introduction of nuclear power plays a smaller role in final energy use, especially in the longer term.

Waste, Emissions and Carbon Concentration: The competitiveness of LWR technology for power generation and the exhaustion of uranium resources lead to a significant accumulation of nuclear wastes, as seen from Fig. 5 (Model C). Because of reprocessing, waste production is a lot lower under FBR technology than under LWR despite increased nuclear electricity generation. Accumulated nuclear wastes with both LWR and FBR (model E) are 84% lower than with LWR only (Model C). There is a trade-off between the production of toxic wastes and carbon. Even without a carbon cap, the expansion of nuclear capacity provides carbon-free electricity so that carbon-intensive fossil fuels can be used in other sectors such as transportation. From model A to C, carbon emissions decline from 14 to 11.3 billion tons in 2050, but they catch up later at about 25 billion tons[22] as nuclear power from LWR becomes expensive (see Table 1).

Fig. 6 shows carbon emissions per unit energy. Emissions decline in the short run in model A but go up ultimately because there is no nuclear expansion and coal must provide electricity and refined petroleum products. Nuclear expansion (model C) lowers emissions per unit energy in the short run but they catch up with model A in the long run. A carbon cap (model B) leads to a sharp drop in emissions around 2060 when carbon-free electricity becomes competitive. With growth in nuclear capacity, this drop occurs much earlier (models C, D). In general the carbon intensity of energy production is driven mainly by the electricity sector because other sectors have limited substitution potential.

The heavy dependence on coal in the nuclear stagnation scenario raises cumulative carbon emissions. The carbon concentration (see Fig. 7) reaches a level of 720 ppm in the year 2100 and 884 ppm in 2150, orders of magnitude that are expected to cause significant damages (Alley *et al.,* 2003). The expansion of nuclear power allows for a slowdown in the increase of atmospheric carbon concentration. Adoption of FBR technology reduces the carbon concentration to 650 ppm in year 2100 (Model E). With a carbon cap, emissions decline and the ceiling is attained ten years ahead in time, in 2090 (check models B and D in fig. 7).

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[22] This is several times more than current annual emissions of about 7.35 billion tons.

The Cost of Meeting Carbon Caps: The effect of meeting the carbon cap on consumer surplus is shown in Table 2. Since models A,C,E do not include carbon caps and successively allow for additional technologies or capacity expansion, the net economic surplus increases going from A to E. For the same reason, models B,D,F must also exhibit increasing surplus. However, it is not clear *ex-ante* how imposing a carbon cap *and* allowing for new energy supply options such as nuclear power will affect the economic surplus. For example, surplus declines when a carbon cap is imposed (A to B) but increased nuclear capacity more than compensates for this reduction (model C). Going from model A to F, surplus actually increases (by 0.14%) – nuclear technology more than compensates for the cost of meeting the carbon cap. These numbers may seem small but a 1% reduction in energy costs translates roughly into a trillion dollars in present value terms.

Emissions decline significantly under a 450 cap (Fig. 8) and 2050 emissions need to be approximately at the same level as in 2000. By 2050, primary energy use must decline by 25%, also pointed out by Clarke *et al.* (2007). The more stringent the carbon cap, the higher is the price of carbon and the policy cost[23] of meeting clean carbon objectives (see Figs. 9 and 10). The shadow price of carbon and the policy costs are much lower for the 650 ppm target. In fact the graphs show that costs rise disproportionately as we move from 650 to a 450 ppm target.[24] They

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[23] We compute the policy costs by running each of the three models B, D, and F with three alternative carbon targets, set at 650, 550 and 450 ppm respectively. Then for each date, we plot the shadow cost of carbon against the corresponding emission reduction target relative to the baseline (see Ellerman and Decaux, 1998). This gives a rising marginal abatement cost curve for each model at each date. We integrate the area below the curve by interpolating between each target data point to obtain total abatement cost at each date. The discounted sum of abatement costs for all time steps yields the overall policy cost. This procedure is explained in detail in Ellerman and Decaux (1998).

[24] Our results are comparable to those of Gerlagh and van der Zwaan (2006) even though they use a general equilibrium model and thus account for macroeconomic adjustments**.** Their costs of stabilization to 450 ppm range from $800-1100 billion and $100 billion for a 550 ppm target. Our respective figures are $800 and $200 billion. The differences may be due to calibration – our baseline carbon emissions peak at around 12GtC in 2065, while theirs remain below 11GtC. The cost of achieving a 450 ppm target is much larger than the one for 550, as confirmed by numerous studies (Edenhofer *et al.*, 2006, Nordhaus, 2007,

also show that allowing for expansion of nuclear capacity helps achieve climate targets at a significantly lower cost, and newer generation nuclear technologies may further reduce these costs.

Sensitivity Analysis: In this section we examine the sensitivity of the results to changes in cost and policy parameters. Only models D and F are used, as shown in the left most column of Table 3. Under a 450 ppm cap, oil and natural gas take a higher share of the fuel supply in 2050 and coal a lower share as expected. The 450 ppm scenarios are the only ones where renewables gain significant market share – 12-15% of the energy mix by 2050 and 78% by 2100 when no FBR expansion is feasible (not shown). This suggests that a strict control of atmospheric carbon concentration will essentially imply that either renewables or the next generation nuclear technologies will likely be the primary fuel, analogous to the role coal plays today. The share of electricity also increases to almost half of total energy supply, because of the relative availability of low carbon options in that sector (see bottom panel of Fig. 8).

Table 3 also shows the effect of changes in discount rates, nuclear investment costs, nuclear capacity and parameters for technical progress. The medium run competitiveness of LWR technology is not affected by changing the discount rate. A lower discount rate favors future investments in capital intensive technologies such as wind power. Low discount rates also imply a slower pace of increase of the carbon shadow price (see equation (5)). A high discount rate tends to delay the introduction of capital intensive options such as renewable energy, so that the cost of carbon reductions is higher in the long run ($576/ton in Table 3).

Clarke *et al.*, 2007). Because of climate inertia, the stabilization at 450ppm requires significant emission declines by 2030 and a rapid transformation of the energy supply mix.

A slower (50%) growth rate of nuclear capacity additions slows down nuclear power penetration. Nuclear production shifts to the future and peaks in 2095. This results in an earlier and costlier introduction of wind energy, and thus a higher cost of carbon. A higher (by 50%) nuclear capacity increases the share of nuclear power generation and decreases the shadow cost of carbon. Alternative investment costs for LWR and FBR do not change the results in a significant way. Carbon costs are only slightly affected ($287-322/ton) in Model D. Variations in FBR investment cost affect the levelized cost of FBR technology, and lead to small changes in the aggregate surplus for Model F, since FBR technology only appears in the long run.

To assess the effects of fast technical progress, we assume an across-the-board doubling of learning rates in all technologies. These cost reductions benefit other clean fuels (such as solar energy) and completely remove nuclear power plants from the energy mix. Carbon concentration stabilizes at levels below 500 ppm leading to a zero shadow price of carbon in 2100 (Table 3). Reducing the cost of CCS by 50% does not change results appreciably (see Table 3). Nuclear still plays a key role and is then replaced at a slightly earlier time with CCS plants. When we allow for a higher cap in the change in generation capacity of an energy source (the cap raised from 5% to 10%), results are preserved. Except that in the 550 ppm case, coal with CCS becomes the dominant technology after 2050. In the 450 ppm model, coal is taken out of the power mix, and replaced in the short run with natural gas and nuclear, and in the longer run with carbon-free sources of energy such as wind and biomass with CCS.

Finally, increasing the availability of uranium – doubling the quantity available at each cost alters the results only marginally (not shown), suggesting that uranium depletion, although important in raising the cost of nuclear power, is not the critical limiting factor for LWR expansion. The prospects for increased LWR power generation are also hampered by significant investment costs. Nuclear is ultimately replaced by coal which becomes cheaper due to learning effects.[25]

5. Concluding Remarks

This paper applies a model with price-induced substitution across resources to examine the role of nuclear power in reducing global warming. The cost of fossil fuels and uranium, the main input in nuclear power generation, rises with depletion. The main insight is that nuclear power can help us switch quickly to carbon free energy, but in the long run, large scale adoption of nuclear power will be hindered by the rising cost of uranium and the problem of waste disposal. Only significant new developments such as the availability of new generation nuclear technology that is able to recycle nuclear waste may lead to a steady state where nuclear energy plays an important role. If expansion of nuclear capacity occurs at historical rates, uranium producers could engage in cartel-like behavior since the ore is found mainly in four countries, fewer than for crude oil.[26]

In the long run, renewable energies such as biomass and wind become economical and supply a major portion of energy. But significant supplies also come from clean coal technologies. The availability of new nuclear technologies such as Fast Breeders reduces the dependence on clean coal. Meeting carbon concentrations of 550 ppm is modestly costly but a 450 ppm target implies a rapid ramp-up in terms of clean energy use in the coming decades (by 2050). This significantly

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[25] In most models, 95% of the uranium is depleted by the year 2100. Only 50-60% of the oil and gas is depleted and 10-20% of coal. Only high cost uranium ore remains unexploited.

[26] About 75% of known world reserves are found in Australia, Canada, Kazakhstan and South Africa.

raises the cost to the economy. The cost of carbon jumps up from $18 to $150/ton in 2050. This is somewhat lower than predictions by other studies such as the DICE model of Nordhaus (2007) which predicts a 450 ppm carbon price of $250/ton in 2050.[27]

Going from a freeze on further expansion of nuclear power to a continued expansion of nuclear power at historical rates, the shadow price of carbon declines by almost 50%. This suggests that political constraints on continued expansion of nuclear power are likely to result in a significantly higher cost of reducing carbon. However this price is not sensitive to whether new nuclear technologies such as fast breeders become available or not, since these technologies play a role in the distant future, and those benefits must be discounted to the present.

The shadow price of carbon plays an important part in determining which abatement options may be feasible as well as the size of a global permit market. Lower carbon prices may suggest that such a market may be smaller than expected, with lower benefits relative to no trading. The damage to economies that may be potential buyers of carbon, such as the United States or China, may be smaller than currently estimated. Similarly, potential benefits to sellers of permits such as Russia and Ukraine may be correspondingly lower.

The model results are quite robust to changes in cost parameters. However, the results are sensitive to the choice of the discount rate. A lower discount rate favors capital intensive technologies with relatively low operation and maintenance costs such as wind power. Renewable energy technologies become economical earlier leading to a lower cost of carbon and lower

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[27] Clarke *et al.* (2007) report carbon prices in the order of $500/ton in 2050 and higher.

aggregate emissions. Across-the-board higher learning rates also benefit technologies such as solar energy because they have a lower floor cost. Nuclear power quickly becomes redundant in this scenario.

There are several restrictive assumptions in the model which could be relaxed in future work. We have abstracted from considering adjustment costs. Adding nuclear capacity in the form of a new plant or additions to an existing facility takes several years because of licensing and safety permitting procedures. We have assumed frictionless additions to capacity. We have modelled adjustment lags by imposing a cap on capacity expansion. Because nuclear energy becomes expensive in the long run, explicit modelling of adjustment costs may not make a big difference to the results, although that needs to be checked in future work. Adjustment costs will delay energy transitions between sectors and favor sectors with low adjustment costs such as fossil fuels and solar energy.

It is important in future work to consider other technologies that may be better candidates than fast breeders. As an anonymous referee points out, LWRs may exhibit significant efficiency and safety improvements. Other technologies such as gas-cooled and heavy water reactors may have more potential than the stylized FBR technology modelled in this paper. The increased risk of proliferation concerning the use of plutonium in FBRs may mean that only certain countries will be allowed to build FBR plants. Although the capacity restrictions in our model may, to some extent, mimic such constraints, ideally a multi-region model may be able to show how differential nuclear expansion in developed and developing economies could affect the attainment of clean carbon targets and the structure of a global carbon market, especially in a post-Kyoto world.

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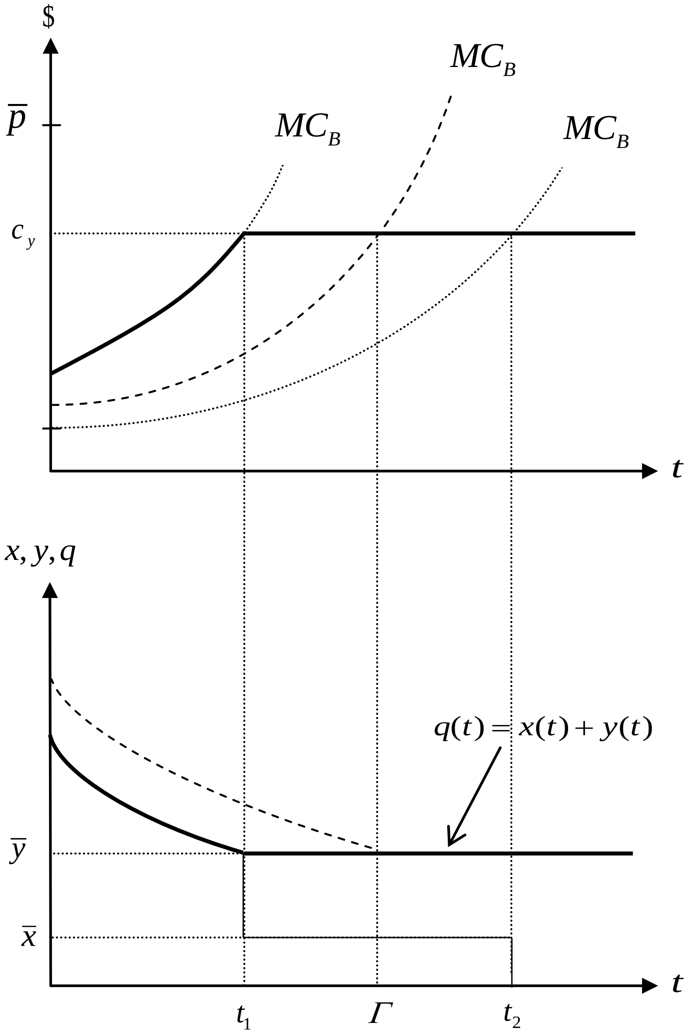


Fig. 1: Both the Polluting Fossil Fuel and the Clean Renewable are used at the Ceiling

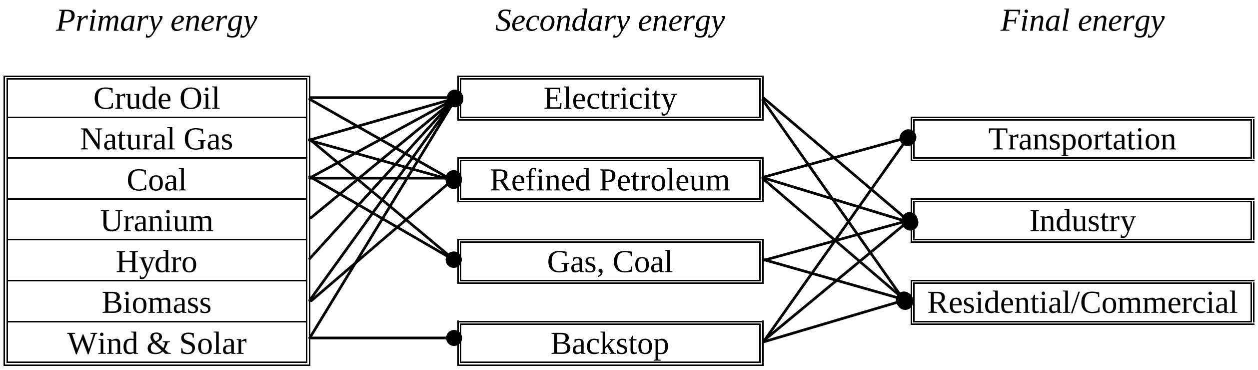


Fig. 2. Schematic of the Energy Model

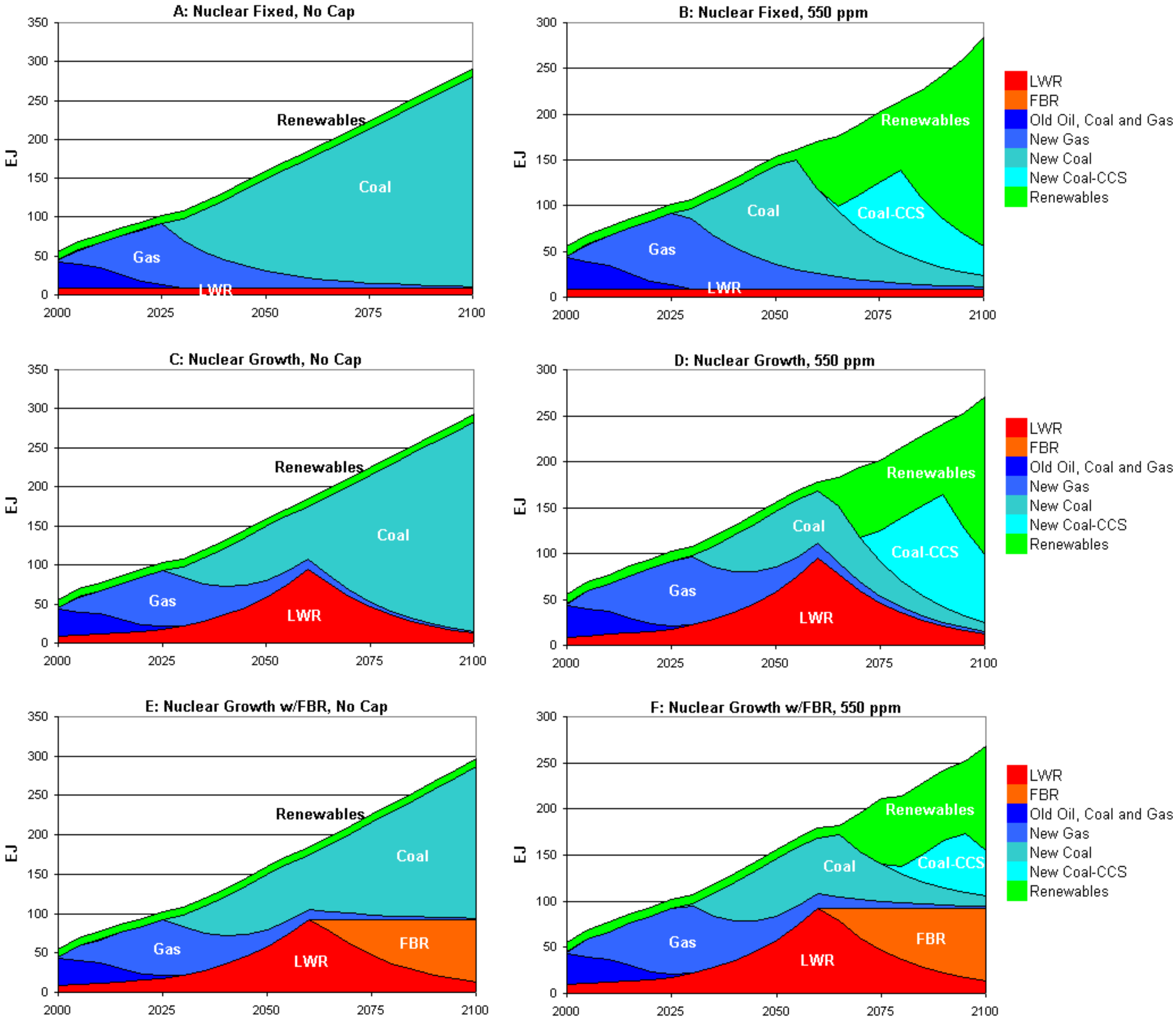


Fig. 3. Electricity Supply under Alternative Scenarios

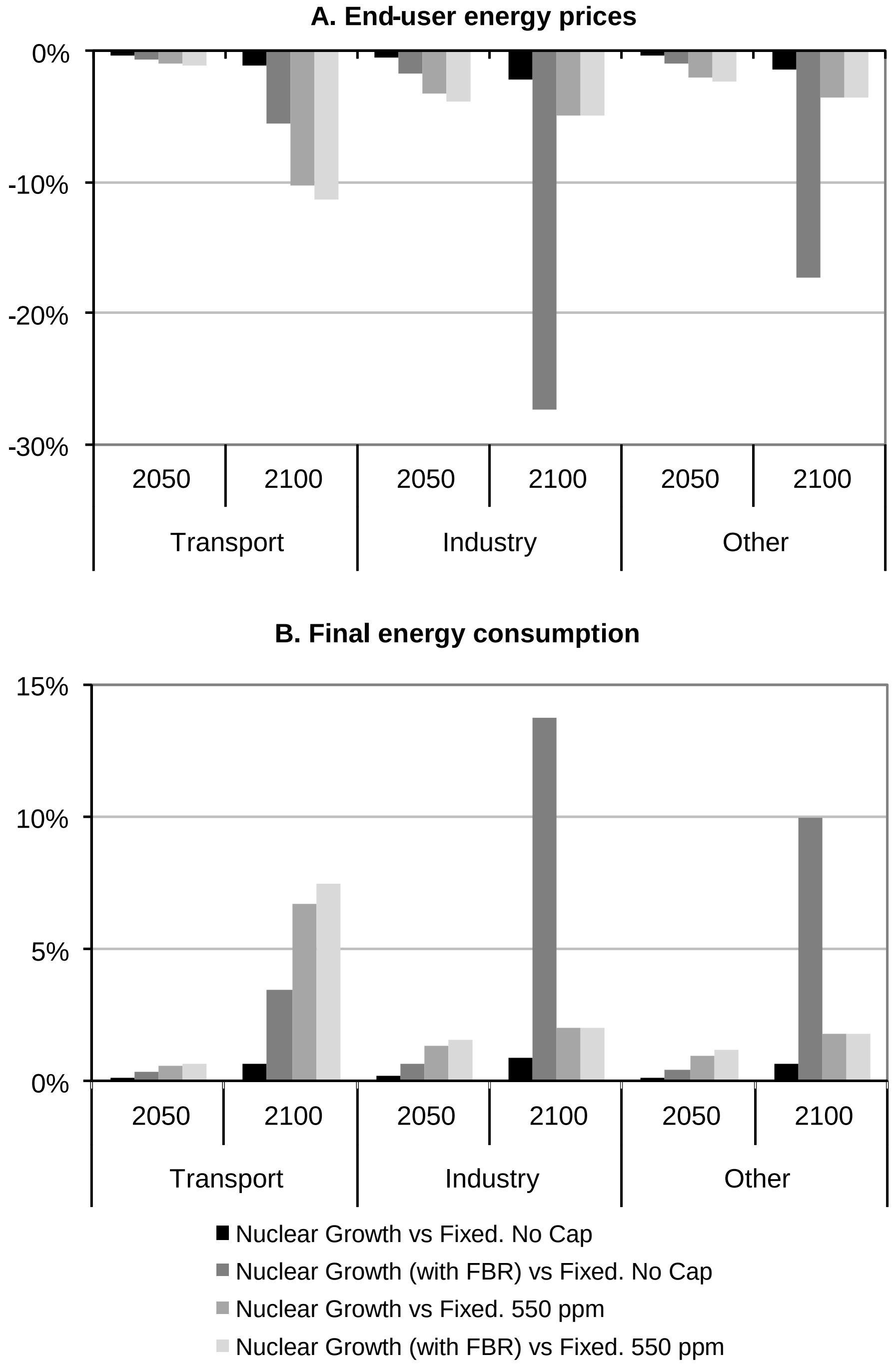


Fig.4. End Use Energy Prices and Sectoral Energy Consumption

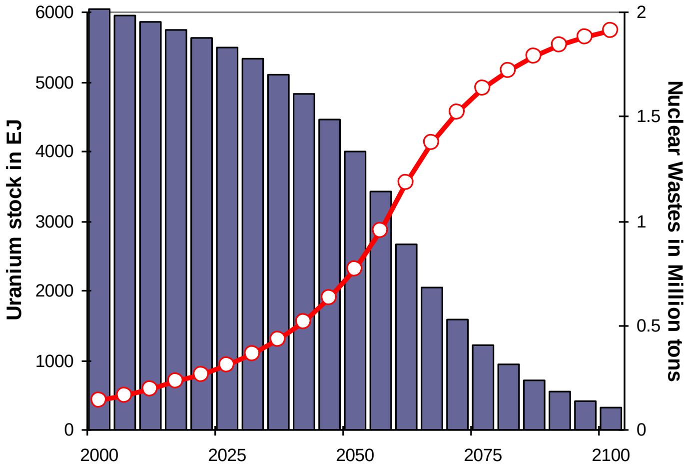


Fig. 5. Depletion of Uranium Stock (bars) and Cumulative Stock of Nuclear Waste (circles) in Model C

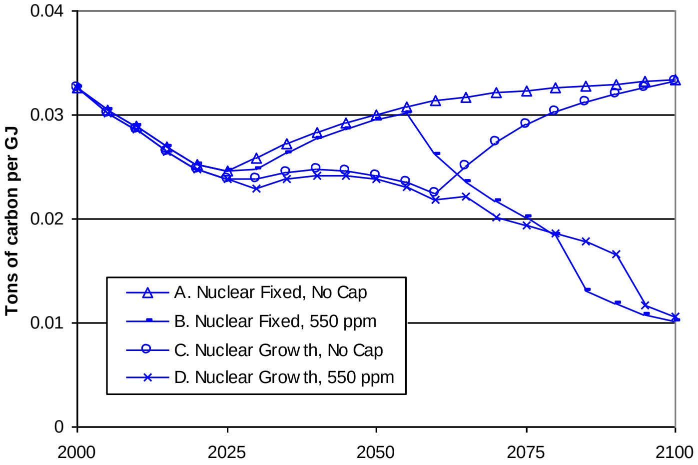


Fig. 6. Carbon intensity of final energy (emissions per unit energy)

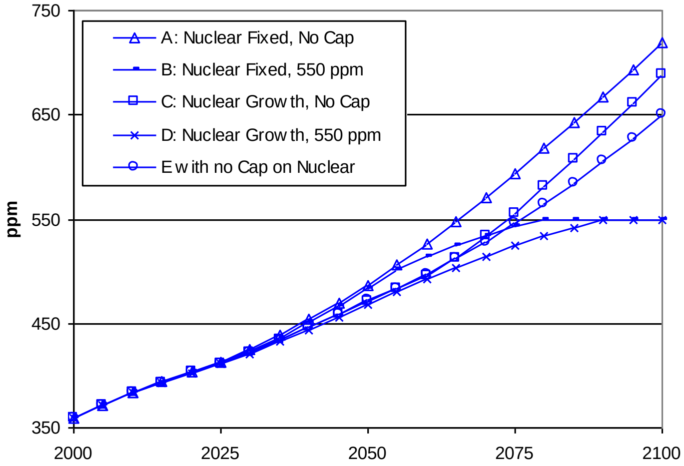
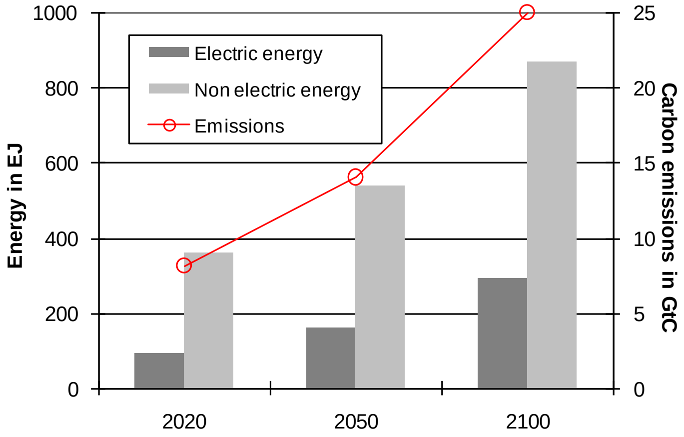
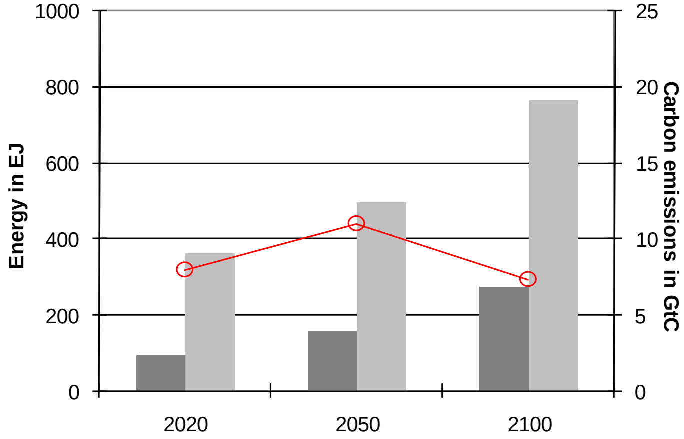


Fig. 7: Time Path of Carbon Concentration

A. Nuclear Fixed, No Cap



D. Nuclear Growth, 550 ppm



D. Nuclear Growth, 450 ppm

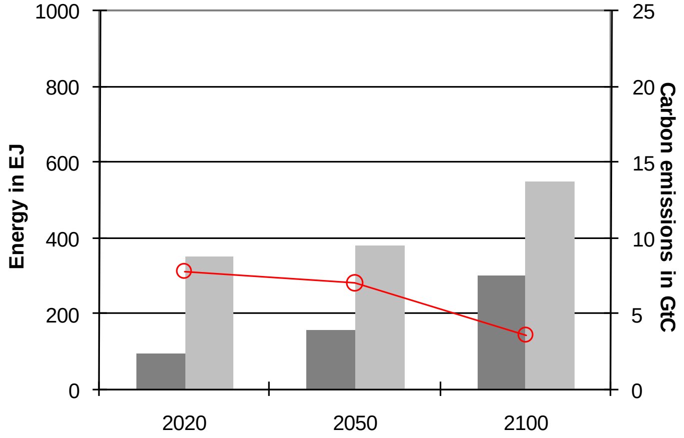


Fig. 8: Electric/Non-Electric Energy Use (bars) and Carbon Emissions (circles) for Selected Models

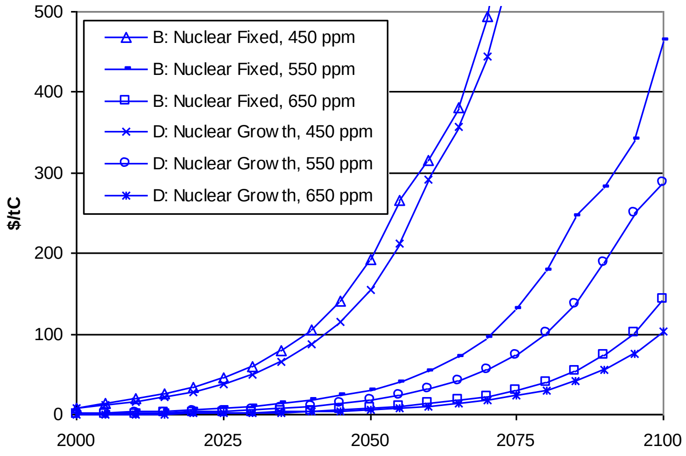


Fig. 9. Shadow Price of Carbon

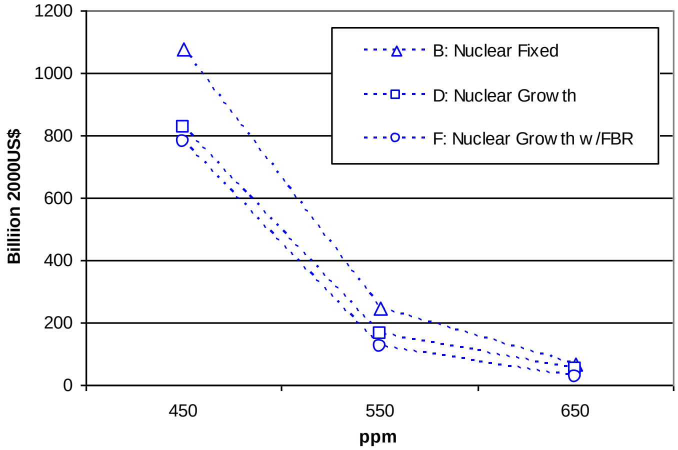


Fig. 10. Costs of achieving different climate targets

Table 1. Energy Mix, Carbon Emissions and Shadow Prices

			Model A	Model B	Model C	Model D	Model E	Model F
Primary energy use (EJ)		2050	699	678	663	650	664	651
		2100	1159	1024	1165	1034	1222	1043
Share of primary energy by fuel	Oil	2050	30%	30%	31%	32%	31%	32%
		2100	1%	1%	1%	0%	1%	0%
	Gas	2050	18%	20%	19%	21%	19%	21%
		2100	9%	8%	9%	9%	9%	9%
	Coal	2050	49%	46%	34%	32%	34%	32%
		2100	88%	31%	88%	42%	69%	36%
	Nuclear	2050	2%	2%	14%	14%	14%	14%
		2100	1%	1%	2%	2%	21%	24%
	Renewables	2050	1%	1%	2%	2%	2%	2%
		2100	1%	58%	1%	46%	1%	30%
Share of electricity in final energy		2000	20%	20%	20%	20%	20%	20%
		2050	33%	32%	32%	32%	32%	32%
		2100	38%	43%	38%	39%	38%	48%
Carbon emissions (GtC)		2050	14.01	13.46	11.31	10.95	11.34	10.99
		2100	25.03	6.71	25.06	7.27	20.94	7.27
Shadow price of carbon ($/tC)		2000	-	2	-	1	-	1
		2050	-	30	-	18	-	17
		2100	-	464	-	287	-	276

Table 2. Energy Production Costs and Net Surplus (in billion $)

	Model A	Model B	Model C	Model D	Model E	Model F
Discounted energy costs	103620	102860	103129	102598	103017	102332
	-	-0.74%	-0.48%	-1.00%	-0.59%	-1.26%
Discounted net surplus	346409	345579	347108	346529	347319	346904
	-	-0.24%	0.20%	0.03%	0.26%	0.14%

Table 3. Summary of Sensitivity Analysis(1)

		Nuclear share in electricity generation		Shadow carbon cost $/tC		Discounted net surplus relative to Model A
		2050	2100	2050	2100	
Model D						
	450ppm	38%	4%	156	1107	99.46%
Discount rate	2%	39%	5%	34	135	307.58%
	8%	38%	5%	9	576	58.66%
50% Nuclear capacity	450 ppm	16%	17%	173	1078	99.25%
	550 ppm	15%	19%	24	345	99.91%
Doubling of learning rate	450 ppm	5%	0%	122	667	98.25%
	550 ppm	5%	0%	0	0	98.52%
LWR investment cost	1600 $/kW	37%	5%	18	287	100.09%
	2000 $/kW	37%	3%	21	322	99.92%
CCS investment cost 50% lower	450 ppm	38%	5%	154	1087	99.52%
	550 ppm	37%	5%	17	267	100.08%
Transition between energy sources capped at 10%	450 ppm	38%	1%	151	1127	99.54%
	550 ppm	37%	1%	18	283	100.08%
Model F						
	450ppm	38%	31%	156	1106	99.59%
FBR investment cost	1850 $/kW	37%	34%	17	276	100.15%
	2600 $/kW	37%	34%	17	276	100.13%
150% Nuclear capacity	450 ppm	38%	47%	152	1112	99.69%
	550 ppm	37%	52%	13	244	100.22%

(1) All runs include a 550 ppm target unless otherwise stated.

Appendix A. Modeling Details and Data

The Energy Model

In this section we provide the detailed specification of the energy model presented in Fig.2. Primary energy is obtained from two types of resources: exhaustible resources namely oil, gas, coal and uranium; and renewable energy resources, biomass, wind and solar. Primary energy is then transformed into secondary energy in the form of electricity, refined petroleum products and backstop energy. These resources plus coal and gas, in turn, are consumed by three final sectors: Transportation, Industry and Residential/Commercial, indexed by $j \in \{T, I, RC\}$.[28]

The energy demand in the final sector j, denoted by $D_j(t)$ at date t is given by $D_j(t) = A_j . P_j(t)^{\alpha_j} Y(t)^{\beta_j}$ where α_j and β_j are respectively the price and income elasticities for demand in sector j, A_j is the sector-specific technical coefficient, P_j is the price of delivered energy in sector j, and Y is global GDP which is non-stationary. GDP increases exogenously over time at a declining rate. Since all variables are a function of time, we omit writing the time subscript when convenient. Energy consumed in the transportation sector, D_T, can be supplied either by refined petroleum products, $dOilP_T$, or by a perfectly substitutable backstop and can be written as $D_T = dOilP_T + dBackstop_T$ where the subscript T denotes the transportation sector. The energy consumed in the Industry and Residential/Commercial sectors, respectively D_I and D_{RC}, are represented by a convex combination of electric, $dElec_{j\in\{I,RC\}}$, and non-electric energy, $dNElec_{j\in\{I,RC\}}$. We use the calibrated form of a CES production function (see Rutherford, 2002) to account for imperfect substitutability between the two inputs[29]

$D_{j\in\{I,RC\}} = \overline{Y}_j \left[\theta \left(\frac{dElec_j}{\overline{Elec_j}} \right)^{1-\rho} + (1-\theta) \left(\frac{dNElec_j}{\overline{NElec_j}} \right)^{1-\rho} \right]^{\frac{1}{1-\rho}}$ where parameters $\overline{Y}_j, \overline{Elec_j}$, $\overline{NElec_j}$ and θ are calibrated against observed data, and ρ is the inverse of the elasticity of substitution. Electricity can be generated by plants indexed by et using resources as shown in Fig.2. The demand-supply balance can be written as $\sum_{j\in\{I,RC\}} dElec_j = \left[\sum_{et} Elprod_{et} \right] \times Eloss$ where $dElec_j$ is the supply of electricity to sector j,

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[28] Coal and gas can also be directly transformed into refined petroleum products.

[29] A similar distinction between electric and non-electric energy has been made by Manne *et al.* (1995) in the MERGE model.

$Elprod_{et}$ is the electricity generated by plant et and $(1-Eloss)$ is the fraction of electricity lost through the distribution grid.

Sectoral non-electric consumption comes from the direct use of petroleum products, $dOilP_{j\in\{I,RC\}}$, gas, (denoted by $dGas_{j\in\{I,RC\}}$), and coal ($dCoal_{j\in\{I,RC\}}$). A CES functional form is used. Because the bulk of fuel substitution is expected to occur in the electricity sector, the modeling approach we adopt focuses on electricity. Non-electric secondary energy is modeled to essentially maintain current trends in energy use, with only a modest degree of substitutability. A CES specification allows us to retain the composition of the fuels if the relative prices across inputs do not change appreciably. In order to allow for a rapid switch towards carbon-free non-electric energy, we sum the CES bundle to a perfectly substitutable backstop. The sectoral non-electric consumption supply of oil products satisfies the global demand $dNElec_{j\in\{I,O\}}$:

$$\begin{aligned} dNElec_{j\in\{I,RC\}} = {} & \overline{NElec_{j\in\{I,RC\}}}\left[\theta_{Liq}\left(\frac{dOilP_{j\in\{I,RC\}}}{\overline{OilP_{j\in\{I,RC\}}}}\right)^{1-\rho_N}+\theta_{Gas}\left(\frac{dGas_{j\in\{I,RC\}}}{\overline{Gas_{j\in\{I,RC\}}}}\right)^{1-\rho_N}+\theta_{Coal}\left(\frac{dCoal_{j\in\{I,RC\}}}{\overline{Coal_{j\in\{I,RC\}}}}\right)^{1-\rho_N}\right]^{\frac{1}{1-\rho_N}} \\ & +dBackstop_{j\in\{I,RC\}} \end{aligned}$$

Oil products can either be supplied by refined oil, called $refoil$, or by perfectly substitutable synthetic fuels obtained from liquefaction of coal, gas or biomass. The aggregate supply of oil products satisfies the global demand $\sum_{j\in\{I,RC\}} dOilP_j$: $\sum_{j\in\{I,O\}} dOilP_j(t) = [refoil(t)+coal(t)+gas(t)+bio(t)]\times NEloss$

where the fraction $NEloss$ accounts for transformation and distribution losses.

Nonrenewable Resource Supply

Each energy transformation process (e.g., coal to electricity) incurs specific investment and operation and maintenance costs. We assume that the investment cost function follows some endogenous reduction according to accumulated experience, i.e., through learning-by-doing (such as in Goulder and Mathai, 2000, van der Zwaan *et al.*, 2002). The cost of investment[30] for plant et denoted by $invc_{et}$, is written as

$invc_{et}(t) = \alpha_{et}\left[\int_0^t Elprod_{et}(s).ds\right]^{-lr_{et}}$ where α_{et} is a scale parameter and lr_{et} the learning rate for

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[30] Investment costs are annualized using a capital recovery factor $crf_{et}(t) = \frac{(1+\rho)^{lf_{et}}\rho}{(1+\rho)^{lf_{et}}+1}$, lf_{et} being the life of the plant and ρ the discount rate.

technology et (see OECD, 2000, Goulder and Mathai, 2000). Operation and maintenance costs denoted by $O\,\&\,M_{et}$ are assumed to be constant over time.

The extraction cost of the nonrenewable resources, namely oil, gas, coal and uranium indexed by $i \in \{O,G,C,U\}$, are denoted by $c_{i\in\{O,C,G,U\}}$ and depend on the cumulative extraction at date t. The functional form for $c_{i\in\{O,C,G,U\}}$ is based on Nordhaus and Boyer (2000):

$c_{i\in\{O,C,G,U\}}(t) = \xi_1 + \xi_2\left[\left(\int_0^t x_i(s)ds\right)/\overline{X_i}\right]^{\xi_3}$ where $\overline{X_i}$ is the initial resource stock given by $\int_0^\infty x_i(s)ds \le \overline{X_i}$. The cost for biomass feedstock is assumed to be constant, suggesting that there is no opportunity cost of land. The levelized cost of generating electricity by plant et, defined by $El\cos t_{et}$ is expressed in \$/unit of energy and consists of the fuel cost $c_{i\in\{O,C,G,U\}}$, the operation and maintenance cost $O\,\&\,M_{et}$ and the investment cost $invc_{et}$. It is computed using the formula

$El\cos t_{et} = \dfrac{c_{i|et}}{\eta_{et}} + \dfrac{\text{O\&M}_{et} + invc_{et}}{Ldf_{et}}$ where η_{et} and Ldf_{et} are the efficiency and load factors for plant et.

Similar calculations are done for non-electric costs, although not shown here.

Calibration Procedure for Demand

The exogenous projection for GDP is the same for all the models and is in line with the IPCC B2 scenario (Nakicenovic *et al.*, 2000), as depicted in Appendix Fig. A1. World GDP is \$333 trillion (in 2000 dollars) in 2100 and reaches \$464 trillion in 2150. The corresponding population projection is also shown. Sectoral world energy consumption in the base year D_j is extracted from IEA data (2002). The rate of GDP growth rate is assumed to be 3.2% initially, decreasing at 0.1% per annum and reproduces the IPCC B2 scenario mentioned above. Sectoral energy prices P_j are not available and thus need to be calibrated. The available data only provides sectoral prices for electricity, oil products, gas and coal at the country level. We thus use IEA price data (2001) to compute average prices that are weighted by country indigenous consumption for each fuel and sector. Base year world prices P_j are in turn computed as weighted averages of the various relevant fuel prices for each demand sector. Long run price and income elasticities for each sector are taken from Barker (1995). Finally, in order to reproduce the base year energy demands, the parameter A_j is obtained from $A_j = D_j(t_0)/\left[P_j(t_0)^{\alpha_j}Y(t_0)^{\beta_j}\right]$. All demand parameters are summarized in

Appendix Table A1.

Energy Data

The parameters of the resource supply curves ξ_1, ξ_2 and ξ_3 as well as resource endowments $\overline{X_i}$ are shown in Table A2. These resources include known unconventional reserves (e.g., oil and gas in shales and tar sands). Atmospheric concentrations are computed using carbon emission rates from Nordhaus and Boyer (2000), after adjusting for the different time intervals in our model.[31] Cost data for electric and non-electric technologies is shown in Tables A3 and A4.[32] While the learning progress of mature technologies is known with higher accuracy, the lack of data hampers estimation of learning rates for more speculative technologies. We have chosen the following estimates that reflect standard assumptions in the literature (see Edenhofer at al., 2006): learning rates are set at 10% for electric technologies, except for solar (15%) and 5% for non-electric technologies, except for hydrogen components (10%).

Nuclear Data

Aggregate estimated reserves of uranium ore, including those already discovered are estimated to be nearly 14.38 million tons (OECD, 2004).[33] The actual cumulative production of nuclear power since the technology was deployed now exceeds 34,000 TWh (1TWh=10^9 KWh). This implies that approximately one thousand tons of plutonium and 0.1 million tons of fissile waste have been produced, including discharged uranium and other fission by-products.[34] These values are used as initial stocks. Since reprocessed uranium is only used for mixed oxide fuels not considered in the paper, its initial stock is assumed to be zero.

LWR technology is modelled on the European Pressurized Reactor (EPR) with a capacity of 1450 MW, producing 11.46 TWh of power annually. The spent fuel discharge consists of 19.132 tons of uranium, 0.271 tons of plutonium, 0.0417 tons of minor actinides and 1.369 other tons of fission products (see Charpin, 2000). After reprocessing and cooling, each TWh of electrical energy generates 23 kg of

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[31] The algorithm is run on 5 year intervals, since reprocessing of the spent fuel takes approximately 5 years. Since Nordhaus and Boyer use 10 year intervals, we adjusted their emission rates to correspond to our 5 year intervals.

[32] Since the backstop technology is electricity from fuel cells and the costs are at best speculative at this time, we assume conservative figures that are within the range proposed by Kypreos (2008) who estimates a cost of \$15/GJ and the "optimistic" projection by Giacona (2007) of \$38.9/GJ. (1 GJ = 277.8 KWH). We take this cost to be \$33.8/GJ.

[33] Our estimates, computed independently, are similar to those developed by an interdisciplinary MIT (2003) study (16 million tonnes).

[34] During this period, 1.2 million tons of depleted uranium have been stockpiled (OECD, 2004).

plutonium and 120.5 kg of wastes.[35]

FBR technology is based on the European Fast Reactor (EFR) with a capacity of 1000 MW, producing 8.76 TWh of power. This representative plant requires 11.7 tons of uranium and 1.5 tons of plutonium annually which are combined to form a mixed oxide fuel. The spent fuel discharge consists of 10.4 tons of uranium, one ton of fission products and 0.3 tons of plutonium, which is recycled back into the plant. [36]

Long-run cost estimates for nuclear power are obtained from NEA (1994 and 2002). We have simplified the specification of the technology and regrouped some stages whose costs are low or which involve a simple transformation of products without any storage. The cost of reprocessing or storing joint products such as reprocessed uranium from LWR plants which can be used in FBR technology are suitably apportioned between the two technologies. For simplicity, we assume constant returns to scale technologies and unit costs that are fixed over time. It is likely that technological change and the costs of labor, capital and materials may alter relative costs over time. It is difficult to predict these changes *ex ante,* but we partly address this issue by applying across the board technology-induced cost reductions.

The unit cost of extraction of uranium oxide and its conversion to uranium hexafluoride is assumed to be $60/kg of uranium. The separation and enrichment stage involves processes that add significant value to the mineral.[37] The cost of enrichment is taken as $80/kg of uranium. The fuel fabrication stage also represents a significant part of the fuel cycle cost and depends largely on the type of reactor. It is assumed to be $250/kg for LWR fuels, and a high $2,500/kg for FBR fuels, partly because of additional safety measures associated with the handling of large amounts of plutonium. The unit cost of reprocessing spent fuel is assumed to be $700/kg for LWR and $2,000/kg for FBR.

Investment costs represent the largest component of total costs in electricity generation. They are assumed to be $1800/kW for LWR, and $2100/kW for FBR. The disposal cost of depleted uranium is taken as $3.5/kg. The cost of interim storage of plutonium is a high $1,000/kg, due to its toxicity. The cost of conditioning of the waste and long-term geological storage is assumed to depend on whether or not wastes are recycled. We use $400/kg for Models C and D and $100/kg for Models E and F. Table A5 provides a summary of the cost estimates.

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[35] LWR waste production decreases with FBR operation because of reprocessing of spent fuels.

[36] Further details on the energy content of fissile material are available in tabular form from the authors and from Hore-Lacy (2003).

[37] Separation produces a large quantity of stockpiled depleted uranium. Recall that this stock is waste in a LWR operation, but is an important source of uranium for FBR technology.

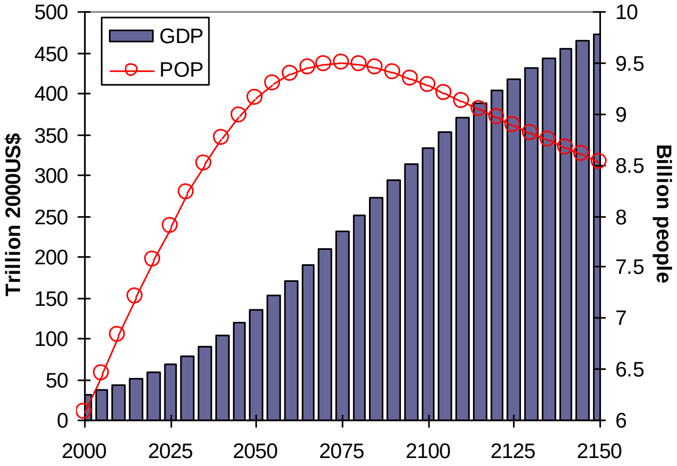

Fig. A1. Gross World Product (Left axis) and corresponding Population Projections (Right axis)

Table A1. Sectoral Demand Parameters and Base Year Calibration

	Energy Prices (1)	Energy consumption (2)	Weighted Prices	Price elasticity	Income elasticity	Constant parameters
	\$/GJ	EJ	\$/GJ	α_j	β_j	A_j
Transportation	18.01	71.06	18.01	-0.6	0.7	0.28598
Refined Petroleum	18.01	71.06				
Backstop	-	-				
Industry		81.41	7.07	-0.4	0.6	0.35622
Electricity	17.21	19.27				
Refined Petroleum	5.29	24.36				
Gas	4.32	20.56				
Coal	1.53	17.23				
Backstop	-	-				
Other		74.48	15.97	-0.5	0.5	1.67783
Electricity	27.21	25.52				
Petroleum products	11.79	20.11				
Gas	8.47	23.87				
Coal	11.24	4.98				
Backstop	-	-				

(1) Source: Retails prices for selected countries, IEA (2001).

(2) Source: Total final consumption from IEA (2002).

Table A2. Parameters for Resource Supply Functions(1)

		Oil	Gas	Coal	Uranium
Resource cost for base year (\$/GJ)	ξ_1	3.50	2.50	1.50	0.05
Parameter	ξ_2	100	100	20	0.5
Parameter	ξ_3	5	5	2	1.5
Resource endowment (EJ)	X_i	20013	24618	261466	6040 (2)

(1) Source: Adapted from Rogner (1997).

(2) The uranium endowment corresponds to the amount of energy that can be obtained from LWR without recycling of nuclear materials.

Table A3. Cost Data for Electric Technologies(1) ,(2)

	Lifetime	Efficiency	Load factor	Investment cost for base year	Investment floor cost	O&M cost	Energy cost for base year[1]	
	Years			$/kW	$/GJ	$/GJ	$/GJ	cents/kWh
Old Oil	20	0.30	0.65	1000	1000	2.59	19.11	6.88
Old Gas	20	0.33	0.65	1200	1200	2.16	15.23	5.48
Gas NGCC	20	0.56	0.65	450	350	0.44	6.91	2.49
Gas NGCC-CCS	20	0.47	0.65	1100	750	0.92	11.13	4.01
Old Coal	30	0.37	0.65	1050	1050	1.92	9.96	3.59
Coal IGCC	30	0.46	0.85	1500	1100	1.81	9.03	3.25
Coal IGCC-CCS	30	0.38	0.85	2100	1500	2.85	12.44	4.48
Biomass IGCC	30	0.40	0.75	2400	1100	1.59	16.21	5.84
Hydro	50	0.39	0.45	2850	2850	1.69	14.61	5.26
Wind	20	0.33	0.30	1200	500	1.26	12.43	4.47
Solar PV	20	0.20	0.30	4000	500	1.54	37.90	13.64

(1) Computed with a 5% discount rate. Initial extraction costs for gas, coal, and biomass are: $3.5, $2.5, $1.5 and $3 /GJ, respectively.

(2) Data source: NGCC and IGCC plants: IEA (2006). Others: IEA (2005).

Table A4. Cost data for non-electric technologies(1) ,(2)

	Lifetime	Efficiency	Load factor	Investment cost for base year	Investment floor cost	O&M cost	Energy cost for base year[1]
	Years			$/kW	$/GJ	$/GJ	$/GJ
Synthetic oil products							
Coal-to-liquids	30	0.65	0.80	2000	1000	3.22	11.24
Gas-to-liquids	30	0.53	0.90	1500	1000	2.59	10.9
Biomass-to-liquids	30	0.65	0.80	1150	750	3.22	11.35
Backstops							
Solar thermal-H2	20	0.30	0.35	4500	1000	1.1	33.82
Transp. - Fuel cell-H2	20	0.40	0.85	5500	3000	6.43	20.02
Industry - Fuel cell-H2	20	0.40	0.85	3500	500	8.13	18.72
Other - Fuel cell-H2	20	0.40	0.85	3500	500	6.43	17.02

(1) Computed using a 5% discount rate. Initial extraction costs are same as in Table A3.

(2) Data source: MIT (2007) and Williams *et al.* (2006) for synthetic fuel costs. Backstop costs are extracted from Barreto and Kypreos, (2004).

Table A5. Unit Costs for the Nuclear Technology (1)

Cost parameters		*LWR*	*FBR*
Conversion	m	5	5
Enrichment	m^S	80	-
Fuel Fabrication	m_f^L, m_f^F	250	2500
Investment	v^L, v^F	1800	2100
Processing	m_R^L, m_R^F	700	2,000
Depleted Uranium Storage	s_{U_D}	3.5	-
Reprocessed Uranium Storage	s_{U_R}	60	60
Plutonium Storage	s_{Pu}	1500	1500
Waste Disposal	s_W	400	100

(1) All costs in $/kg, except enrichment cost which are in $/SWU (i.e. Separative Work Unit) and investment costs in $/kW.

Supplementary Appendix B. The Nuclear Model with Recycling of Materials

Introduction

Uranium is the main raw material used in the generation of nuclear power. Almost three quarters of the world's uranium reserves are found in four countries.[38] In the Light Water Reactor (LWR), which is the most common technology used, mined uranium ore is enriched from 0.7% to 3.5%.[39] Uranium fissions to produce heat which is converted into steam that drives a turbine and produces electricity. The spent fuel contains most of the original uranium and some plutonium. This recovered uranium can be reprocessed, enriched and mixed with the plutonium in the spent fuel to produce a mixed oxide fuel that can be put into long term storage or reprocessed. We also consider a modern nuclear technology, the Fast Breeder Reactor (FBR), about 20 prototypes of which are in operation. These reactors are more efficient in using uranium. They use plutonium as base fuel but also produce it as waste. The FBR can extract approximately 60 times more energy from each ton of uranium than the conventional LWR. However, its higher capital costs and the present low price of uranium makes the FBR uneconomical.[40] About 434 nuclear reactors are in service globally, representing an installed production capacity of 351 Gigawatts (GW).

Elements of the Model

The simplified nuclear model we use is briefly described here (see NEA,1994, 2002 and MIT, 2003). Natural uranium is enriched for use in a LWR plant or used directly in a FBR plant. Production of nuclear power from LWR technology is assumed to be a linear function of the enriched uranium input. The enrichment process creates large quantities of depleted uranium, which cannot be used in the LWR. A key difference between the two technologies is the existence of joint products: several by-products from LWR production, the most important of which is plutonium, are used as inputs into FBR production. The LWR technology produces three different by-products: fissile waste which must be treated and stored, and plutonium and reprocessed uranium, both of which can be used in FBR reactors. These complementarities in material flows are shown in Fig. B1.[41]

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[38] These reserves are recoverable at uranium prices of up to $80/kg. Current prices are about $30/kg. At substantially higher prices, seawater could be tapped for large amounts of the metal.

[39] To facilitate comparison, weapons programs require uranium enrichment of over 90%.

[40] This low price is partly due to the availability of weapons grade uranium and plutonium from military stockpiles of the US and the former Soviet Union. This higher grade uranium is blended down to provide reactor fuel. It currently provides almost 15% of the world's annual uranium supply.

[41] This model is highly simplified. Several important issues are not considered. For example, there may be trade-offs between enriching uranium and using uranium oxide. Different types of uranium may need separate enrichment facilities because of the potential for poisoning of the material by actinides and other chemicals. We thank an anonymous referee for bringing this to our attention.

Consider a single deposit of low grade uranium ore starting at point A in the figure. This natural uranium could be enriched for use in a LWR plant or used directly without enrichment in a FBR facility. Define u_E^L as the instantaneous flow of natural uranium that is enriched and used in a LWR plant. Enriching the ore leads to the separation of uranium into enriched uranium (u_E^L) and depleted uranium (u_D^L). Let these ratios be ε and $1-\varepsilon$, respectively, with $0<\varepsilon<1$. Then $u_E^L = \varepsilon u_N^L$ and $u_D^L = (1-\varepsilon)u_N^L$. Let q^L be the instantaneous production of energy (electricity) from LWR technology. We assume that it is a linear function of enriched uranium $u_E^L = \alpha^L q^L$. The LWR technology produces three different by-products - fissile waste which cannot be re-used and must be stored; plutonium, and reprocessed uranium. The last two can be re-used in the FBR. The amount of plutonium produced by LWR technology is denoted by Pu^L and is assumed to be proportional to the instantaneous production rate q^L, i.e., $Pu^L = \beta^L q^L$. The amount of reprocessed uranium is similarly given by $u_R^L = \xi^L q^L$. The volume of wastes w^L generated by LWR technology is $w^L = \gamma^L q^L$, where $\alpha^L, \beta^L, \gamma^L$ and ξ^L are given positive coefficients.

Let q^F be the corresponding production of energy from FBR technology. Again, we assume this to be a linear function of reprocessed uranium, denoted by $u_{Ri}^F = \alpha^F q^F$, where the subscript i denotes input. The unique feature of FBR technology is that it can reuse part of the plutonium produced. Therefore the choice of the breeding ratio, i.e., the input-output ratio of plutonium, denoted by μ^F is endogenous. Thus the input of plutonium is given by $Pu_i^F = \beta^F q^F$ and the output (denoted by subscript o) by $Pu_o^F = \mu^F \beta^F q^F$. The uranium and plutonium inputs in FBR must be used in fixed proportion k. Their complementarity is described by the relationship $\frac{u_{Ri}^F}{Pu_i^F} \equiv k = \frac{\alpha^F}{\beta^F}$. The output of reprocessed uranium from FBR technology is denoted by u_{Ro}^F.[42] Its proportion is given by $u_{Ro}^F = \xi^F q^F$. Let w^F represent the amount of waste generated by the FBR technology. Then $w^F = \gamma^F q^F$. Again, $\alpha^F, \beta^F, \gamma^F$ are positive constants. In summary, FBR technology uses reprocessed uranium and plutonium as inputs, and produces energy, reprocessed uranium, plutonium and waste fissile material.

In summary, natural uranium is enriched before use in a LWR plant. This process increases the proportion

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[42] The uranium input and output also need to be used in fixed proportions, satisfying the condition: $u_{Ri}^F(t) \equiv u_{Ro}^F(t)(\frac{\alpha^F}{\xi^F})$.

of fissile uranium which sustains the chain reaction in a LWR reactor. The process of enrichment also generates large quantities of depleted (lower grade) uranium, which needs to be stockpiled, and has little economic value.[43] Part of the waste material from LWR reactors can be put to use in FBR reactors, producing yet more plutonium which can be used again.

Stock Dynamics

We consider five distinct stocks of resources: natural uranium (in the ground), depleted uranium, reprocessed uranium, stockpiled plutonium, and nuclear wastes. The stock of uranium ore in the ground, $U_N(t)$ is enriched and declines by the quantity extracted for LWR, $u_N^L(t)$ given by $\dot{U}_N(t) = u_N^L(t).$ The stock of depleted uranium $U_D(t)$ is augmented by the depleted uranium which is rejected from the enrichment process $u_D^L(t) = (1-\varepsilon)u_N^L(t)$ given by $\dot{U}_D(t) = (1-\varepsilon)u_N^L(t)$. The stock of reprocessed uranium $U_R(t)$ is augmented by the reprocessed uranium $u_R^L(t)$ from LWR and $u_{Ro}^F(t)$ from FBR, and reduced by the quantity $u_{Ri}^F(t)$ to be used in FBR, and is given as $\dot{U}_R(t) = u_R^L(t) - u_{Ri}^F(t) + u_{Ro}^F(t).$ The stock of plutonium $Pu(t)$ is augmented by the quantity $\beta^L q^L(t)$ out of the LWR plant, minus the FBR input $\beta^F q^F$, and augmented by the plutonium created by FBR technology, $\mu^F \beta^F q^F$ with $\mu^F > 1$. Now define Δ as the time lag between the date at which the plutonium flow is extracted from the reactor and the date at which it is reintegrated into the plutonium stock for re-use, caused by the need to reduce the temperature of the mineral and other processing tasks. This is given by

$$\dot{Pu}(t) = Pu_o^F(t) - Pu_i^F(t) + Pu_o^F(t) = \beta^L q^L(t) - \beta^F q^F(t) + \beta^F \mu(t-\Delta) q^F(t-\Delta).$$

Finally, the flow of wastes from the two technologies, w^L and w^F are aggregated as follows: $\dot{W}(t) = w^L(t) + w^F(t).$ We assume zero radioactive decay of the nuclear waste because of the relatively short time horizon of the model.

Nuclear Cost Functions

Let m denote the average extraction cost of natural uranium. For the purpose of writing this model, we assume it is constant. In the empirical model, this cost increases with cumulative extraction as explained

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[43] By contrast, this depleted uranium, together with natural ore, may have economic value if it was used in FBR technology, along with plutonium. However, our test runs suggest that this option proves uneconomical given that enough uranium can be recycled to sustain FBRs supply with closed fuel cycle.

previously. The total extraction cost is mu_N^L. Let m^S be the unit enrichment (separation) cost of uranium used in LWR. Then total enrichment cost equals $m^S u_N^L$. This enriched uranium is packaged and assembled before use as an input in LWR production, at an average cost of m^L. Therefore, the total preparation cost of LWR uranium is $m^L u_E^L = m^L \varepsilon u_N^L$. The average cost of fuel reprocessing for LWR technology is denoted by m_R^L, so that the total cost is $m_R^L \left[\beta^L + \gamma^L + \xi^L\right] q^L$. Finally, the LWR reactor incurs an *in situ* operating cost of $v^L q^L$. Let m_f^F and m_R^F denote the average preparation and reprocessing cost of FBR fuel, respectively. Then the total FBR fuel fabrication cost is $m_f^F \left[u_{Ri}^F + r_i^F\right]$ and the total fuel reprocessing cost is $m_R^F \left[\mu\beta^F + \gamma^F + \xi^F\right] q^F$. The operating cost of FBR technology is given by $v^F q^F$. Each unit of depleted uranium is stockpiled at an average annual cost of storage s_{U_D}, so that the total storage cost is $s_{U_D}.U_D$. Similarly, let the respective annual unit cost of storage for reprocessed uranium and plutonium be s_{U_R} and s_{Pu} so that the corresponding storage costs are $s_{U_R} U_R$ and $s_{Pu} Pu$. Finally the annual unit cost of storage for reprocessed uranium is s_W so that the total cost is given by $s_W \left[\gamma^L q^L + \gamma^F q^F\right]$.

Optimization of the Nuclear Model

Production of nuclear energy is optimized by choosing the instantaneous amount of power generated by the two technologies, $q^L(t)$ and $q^F(t)$ and the breeding ratio $\mu^F(t)$, to maximize a social surplus function, net of total costs. Denote the instantaneous gross surplus as $S(t) = S(q^L(t) + q^F(t))$. With a constant social rate of discount δ, we have

$$
\begin{aligned}
\underset{\{q^L(t)\},\{q^F(t)\},\{\mu^F(t)\}}{Max} \int_0^\infty \{S(q^L(t) + q^F(t)) - \quad & m\left[u_N^L(t)\right] \; - \; \left[m^S + \varepsilon m_f^L\right] u_{Nt}^L(t) \\
- \quad & m_R^L \left[\beta^L + \gamma^L + \xi^L\right] q^L(t) - v^L q^L(t) \\
- \quad & m_f^F \left[u_{Ri}^F(t) + r_i^F(t)\right] \\
- \quad & m_R^F \left[\mu(t)\beta^F + \gamma^F + \xi^F\right] q^F(t) - v^F q^F(t) \\
- \quad & s_{U_D} U_D(t) - s_{U_R} U_R(t) - s_{Pu}.Pu(t) \\
- \quad & s_W \left[\gamma^L q^L(t) + \gamma^F q^F(t)\right]\} e^{-\delta t} dt
\end{aligned}
$$

subject to

$$\dot{U}_N(t) = -u_N^L(t), \qquad U_{N_0} > 0\ given,\ U_N(t) \ge 0$$

$$\dot{U}_D(t) = u_D^L(t), \qquad U_{D_0} > 0\ given,\ U_D(t) \ge 0$$

$$\dot{U}_R(t) = u_R^L(t) - u_{Ri}^F(t) + u_{Ro}^F(t), \qquad U_{R_0} > 0\ given,\ U_R(t) \ge 0$$

$$\dot{P}u(t) = r_{ot}^L(t) - r_i^F(t) + \mu^F(t-\Delta)r_i^F(t-\Delta), \quad P_{u_0} > 0\ given,\ P_u(t) \ge 0$$

$$\dot{W}(t) = \gamma^L q^L(t) + \gamma^F q^F(t), \qquad W_0 > 0\ given,\ W(t) \ge 0$$

$$\overline{\mu}^F - \mu^F(t) \ge 0 \quad \text{and} \quad \mu^F(t) - \underline{\mu}^F \ge 0,$$

$$q^C(t) \ge 0,$$
$$q^F(t) \ge 0,$$

$$u_N^L(t) = \varepsilon^{-1}\alpha^L q^L(t) \ge 0,$$
$$u_D^L(t) = (1-\varepsilon)\varepsilon^{-1}\alpha^L q^L(t) \ge 0,$$
$$r_{ot}^L(t) = \beta^L q^L(t) \ge 0, \qquad r_i^F(t) = \beta^F q^F(t) \ge 0,$$
$$u_{Rt}^L(t) = \xi^L q^L(t) \ge 0, \qquad u_{Ro}^F(t) = \xi^F q^F(t) \ge 0,$$

$$u_{Ri}^F(t) \equiv r_i^F(t).\frac{\alpha^F}{\beta^F} = \alpha^F q^F(t),$$

$$u_{Ri}^F(t) \equiv u_{Ro}^F(t)\left(\frac{\alpha^F}{\xi^F}\right).$$

The necessary conditions are available separately from the authors.

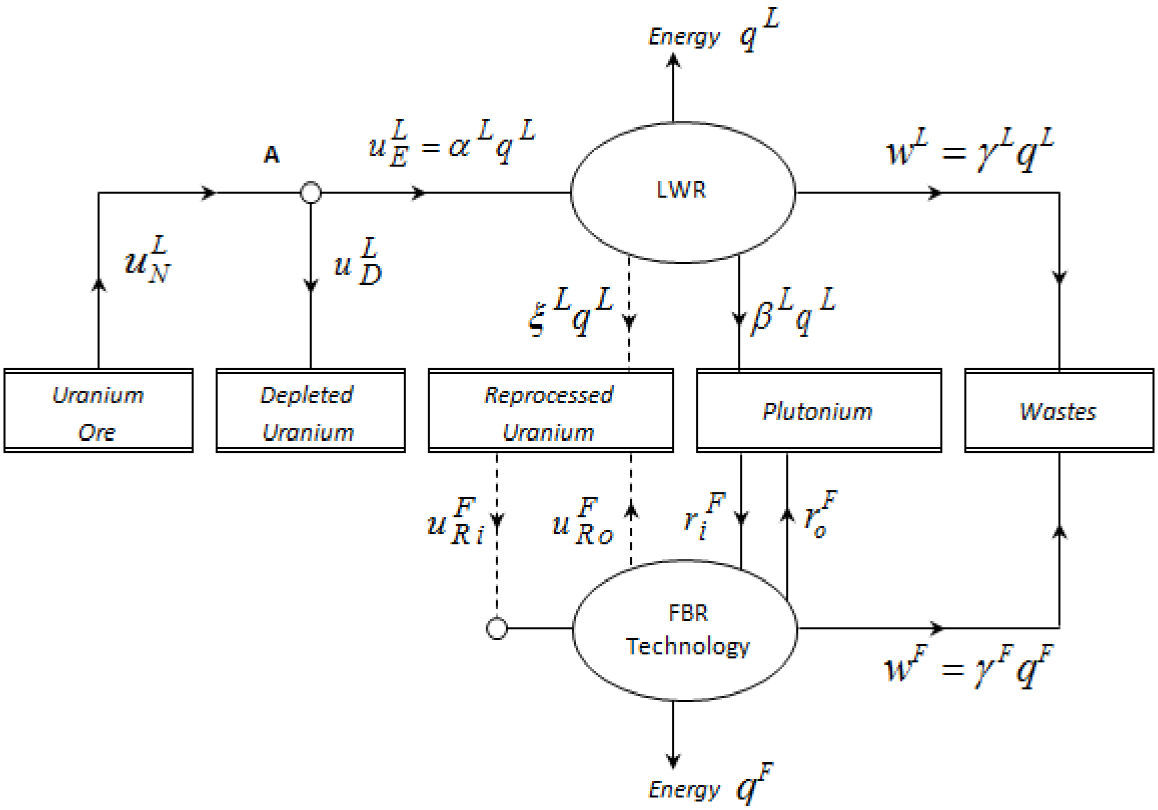

Fig. B1: Flow of Materials in the Nuclear Cycle