EXPECTATIONAL COORDINATION.
THE « EDUCTIVE STABILITY » VIEWPOINT.

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PLAN OF THE TALK.

1- Introduction
   • Rational Expectations Equilibria.
   • The « eductive » stability viewpoint.

2 - Further discussion
   • Conceptual and mathematical insights

3- A quick walk into applications.
   • Finance,
   • Intertemporal macroeconomics.

4- Connections with other viewpoints.
   • Evolutive learning, multiplicity, experiments,
   • incomplete information, herding..
EXPECTATIONAL COORDINATION AND THE RATIONAL EXPECTATIONS HYPOTHESIS.

- Present theory is dominated by the Rational Expectations Hypothesis. *(REH)*
  - Muth: the « relevant economic theory ».
  - Economic agents do not make systematic mistakes
  - They make the « best predictions » / with their information.
  - Remarks: Nash hypothesis different from the rationality hypothesis.

- Since the seventies, the REH has taken over:
  - General equilibrium
  - Macro-economic theory
    - RBC, New keynesian.
  - Finance. Etc..

- Questions.
  - A deus ex machina? A reasonable assumption?
  - A (rather) blind point of economic theory?
AN INFORMAL DISCUSSION: the implicit framework.

- **Sketching the framework**
  - Agents are small (infinitesimal)
  - Concerned with
    - Their own action and
    - An aggregate state of the economy denoted E.
    - (Own action may mean a complex strategy)

- **Fitting a large class of economic models.**
  - Partial equilibrium, E, a number or a probability distribution
  - General Equilibrium E, price vector.
  - Finance, E is a function of the price of an asset.
  - Infinite horizon models of macroeconomics., E is a trajectory
RATIONAL EXPECTATIONS EQUILIBRIA.

- **The REH.**
  - Economic agents do not make systematic mistakes
  - They make the best predictions /with their information.

- **A formal apparatus.**
  - Definition: let $E$ be the aggregate state of the system.
  - If people have beliefs $*B(i,.)$, then the actual state is $F(*B(i,.))$

- **The REE**
  - $E^*$ (in some vector space) is an (Rat. Exp.) equilibrium, iif $E^*$ takes place whenever all believe that it will take place
  - $*B(i,.)=E^*$.
  - $E^*=F(E^*)$

- **The Muth simplified model.**
  - Producers are concerned with the price to-morrow.
  - In the absence of intrinsic uncertainty, $E$ is a number,
  - the price of the crop to-morrow. (Perfect foresight)
  - If not a random variable.
The Rational Expectations Equilibrium.

- The perfect foresight equilibrium:
  - \( p^*/S(p^*) = A - Bp^* \)
  - Agents have perfect foresight

- The rational expectations equilibrium:
  - In Muth « exogenous » processes, f.e on \( A \)
  - Then, the equilibrium is a random variable.
  - Agents have the « correct statistical knowledge » of \( p \).
  - Rational expectations equilibrium REE
THE «EDUCTIVE » STABILITY CRITERION.

« Eductive » robustness.
- A « fundamentalist » global viewpoint:
  - CK of model +rationality $\rightarrow$ CK of equilibrium ? (G 1992)
- An intuitive local counterpart:
  - Elasticity of realisations to expectations.

A formal definition of global stability
- Let $E^*$ (in some vector space) be an (Rat.Exp.) equilibrium.
  - Assertion $A$ : It is CK that agents are rational and know the model
  - Assertion $B$ : It is CK that $E=E^*$
- If $A \Rightarrow B$, Strongly Rational, « eductively » stable equilibrium.

A game theoretical inspiration and interpretation.
- Rationalizability, Dom. Solvability..
- Eductive learning.
THE MUTH MODEL : AN « EDUCTIVE » PROCESS

- A collective « cognitive » process
  - CK : P < p_{max}
  - Evbd knows that
    - S(f) < S(p_{max},f)
    - S < fS(p_{max},f)df
    - p > D^{-1}(0) \ 0 S = p_1
  - Then
  - S > f S(p_1,f)df
- If evbd knows that evbd knows :
  - evbd knows that
  - p < D^{-1}(0) \ 0 S(p_1) = p_2....Etc
- Conclusion \textit{Cvgce iif C<B}
THE « EDUCTIVE » PROCESS : A VARIANT.

- Another (almost) equivalent story:
  - Expectations $Q$,
  - $Q=A-Bp$, Price $=A/B-Q/B$,
  - Realisations $: Q(R) = CA/B - (C/B)Q(e)$

- Hence the process
  - Strategic substitutabilities.
  - Alternate optimism and pessimism.

Strong « eductive » stability $C<B$
EVOLUTIVE LEARNING : AN INSIGHT INTO THE ALGEBRA.

- The equations:
  - Learning equations: $a = \alpha$
  - $p(e,t,t+1) = ap(t) + (1-a)p(e,t-1,t)$
  - Equilibrium equations.
  - $A - Bp(t) = Cp(e,t-1,t)$
  - $p(t) = (1/B)(A-Cp(e,t-1,t))$
  - $p(e,t,t+1) = [-a(C/B) + (1-a)]p(e,t-1,t)$
  - $A(C/B) + a - 1 < 1$

- Conclusions.
  - Small $a$ is good. (inf. learning process)
  - Small $C/B$ is good. (inf system)
  - Captured by the «eductive » viewpoint.
  - Major consequences for standard theory..
Comparison « eductive » and evolutive learning.

\[ p(e,t,t+1) = \alpha p(t) + (1 - \alpha) p(e,t-1,t) \]
LOCAL « EDUCTIVE STABILITY ».

- Local eductive stability: the high tech view.
  - $V(E^*)$ is a small neighbourhood of $E^*$
  - Assertion A implies Assertion B.
    - *Assertion A*: It is CK that $E$ is in $V(E^*)$
    - *Assertion B*: It is CK that $E = E^*$
      - Hypothetical CK of $V$ implies CK of $E^*$ (Eductive learning)

- Local eductive stability: the low tech view.
  - *Can we find a non-trivial nbd of equilibrium s.t if everybody believes that the state will be in it, it will surely be...?*
  - *Implies the high tech conclusion, but refers at most to rationality not CK of rationality*
  - Connection with « evolutive » learning:
    - In the absence of such a neighbourhood, « evolutive » learning is likely to fail for some learning processes.
SOME REMARKS.

- Justification of the REH?
  - Certainly, if unique REE
    - globally eductively stable
    - locally eductively stable.
  - If multiplicity of REE,
    - for some of them, locally stable, (refinement device)
    - But not really a refinement device (may reject all REE).

- Criticisms:
  - It is (too much) 0-1: you accept or reject.
    - Note quite true: sequential decisions in Muth: C/B<T
    - May be better viewed as an index of stability. to evaluate the *likelihood* of rational expectations.
  - It does not provide an alternative when it fails.
    - A broader set/outcomes stressed;
    - The rationalizable equilibria, large.
FURTHER CONCEPTUAL AND MATHEMATICAL INSIGHTS

Based on G-Jara-Moroni (2010)
AGAIN the framework

- Sketching the framework
  - Agents are small (infinitesimal)
  - Concerned with
    - Their own action and
    - An aggregate state of the economy denoted E.
  - Best response mapping
    - $\Gamma(i,E) = B(i,E)\quad B(i,p(X))$, etc..
    - $E^*$ is a RE Equilibrium. $\Gamma(E^*) = \int B(i,E^*)\,di = E^*$
- Fitting a large class of economic models enter it.
  - Partial equilibrium, E, a number or a probability distribution
  - General Equilibrium E, price vector.
  - Finance, E is a function of the price of an asset;
  - Infinite horizon models of macroeconomics., E is a trajectory
**SET-RATIONALIZABILITY 1.**

- The "economic viewpoint" Conjectures on the aggregate state $E$.
- **Point expectations : Cobweb mapping**
  - Def $\Gamma(E) = \int B(i,E) \, di$
  - Cobweb tâtonnement outcome
    $$\subseteq \bigcap_{t \geq 0} \Gamma^t(A)$$
- **Point expectations rationalizable**
  - $Pr(X) = \int B(i,X) \, di$
  - The set of point-rationalizable states $P$, is the largest set $X \subseteq A$ such that:
    $$Pr(X) = X$$
- **Equivalence with the game viewpoint**
- **Probabilistic expectations.**
  - $R(X) = \int B(i,\mathcal{g}(X)) \, di$
  - The set of rationalizable states $R$, is the largest set $X \subseteq A$ such that: $R(X) = X$
- Provides a substitute (equivalent) with the game viewpoint.
**EQUILIBRIA AND RATIONALIZABLE STATES.**

- **The state space:** 1- **Concepts.**
  - $E, \mathbb{C}, \mathbb{P}, \mathbb{R}$
  - $E \subseteq C \subseteq \mathbb{P} \subseteq \mathbb{R}$

- **Properties:**
  - The set of point rationalizable states is non-empty, convex, compact.
  - The set of rationalizable states is non-empty and convex.

- **Definitions and terminology.**
  - $E= \mathbb{C}$, *Iteratively expectationally stable,*
    - (homogenous expectations)
  - $E= \mathbb{P}$, **Strongly Point Rational.**
    - Heterogenous deterministic expectations
  - $E= \mathbb{R}$ **Strongly Rational**
    - Heterogenous probabilistic expectations.
THE LOCAL VIEWPOINT.

- The local transposition.
  - \( a \in V(a^*) \), hypothetical Common Knowledge
  - \( a^* \) is locally iteratively stable...
  - \( a^* \) is locally Strongly point Rational...
  - \( a^* \) is locally strongly rational....

- The connections.
  - \( 3 \Rightarrow 2 \Rightarrow 1 \).
  - 1 weaker than 3

- The equivalence between 2 and 3
  - Reinforcing locally strongly rational in Strictly locally strongly point rational (The contraction \( V-Pr^n(v) \) is strict).
  - Strictly locally strongly rational = locally point rational.
ECONOMIES WITH STRATEGIC COMPLEMENTARITIES.

- **Strategic complementarities** in the state space.
  - 1B, $S$ is the product of $n$ compact intervals in $\mathbb{R}_+$.
  - 2B, $u(i, \cdot, a)$ is supermodular for all $a \in A$ and all $i \in I$.
  - 3B, $\forall i \in I$, the function $u(i, y, a)$ has increasing differences in $y$ and $a$.

- **Properties.**
  - $a_{\text{min}}^*$ and $a_{\text{max}}^*$, smallest and largest equilibria.
  - $a_{\text{min}}^* \leq E \subseteq C \subseteq \mathcal{P} \subseteq \mathbb{R} \leq a_{\text{max}}^*$
  - All these sets but the first are convex. $\mathcal{P} = \mathbb{R}$ ??
  - Strong Rationality, Strong point rationalizability, IE stability.
  - Locally, criteria equivalent.
  - Heterogeneity does not matter so much, neither probabilistic beliefs.
1-DIMENSIONAL STRATEGIC COMPLEMENTARITIES.

- The unique equilibrium is Strongly Rational.
  - The argument.
  - The graal !...

- In case of multiplicity
  - The rationalizable set .
  - Some equilibria are locally strongly rational.
STRATEGIC COMPLEMENTARITIES WITH $A \subseteq \mathbb{R}^2$ AND MULTIPLE EQUILIBRIA.
The local viewpoint
- Hypothetical Common Knowledge of the nbhd.
- If slope >1, the hypothetical CK is dismissed.

Question:
- Is there a nbhd, distinct from the equilibrium s.t. the fact that evby believes in that the outcome is in the nbd implies surely that it will be?

Locally eductively stable equilibrium

It is CK That $Q \subseteq V$
ECONOMIES WITH STRATEGIC SUBSTITUTABILITIES.

- Economies with **Strategic substitutabilities**.
  - 1B, \( S \) is the product of \( n \) compact intervals in \( \mathbb{R}_+ \).
  - 2B, \( u(i, \cdot, a) \) is supermodular for all \( a \in A \) and all \( i \in I \).
  - 3–B', \( \forall i \in I \), the function \( u(i, y, a) \) has **decreasing** differences in \( y \) and \( a \).
- The cobweb mapping \( \Gamma \) is decreasing
- The second iterate of \( \Gamma \), \( \Gamma^2 \) is increasing.

**Results**

- \( a_{\text{min}}^* \) and \( a_{\text{max}}^* \), cycles of order 2 of \( \Gamma \)
- \( C \subseteq P \subseteq R \subseteq [a_{\text{min}}^* + R^n, a_{\text{max}}^* - R^n] \)
- All these sets but the first are convex.
- \( P = R ?? \)
- Uniqueness + no cycle of order 2, Strong Rationality, Strong point rationalizability, IE stability.
- Locally, criteria equivalent.
- Heterogeneity does not matter so much, neither probabilistic beliefs.
Strategic substitutes for \( A = [0, A_{\text{max}}] \subseteq \mathbb{R} \) with unique equilibrium and multiple fixed points of \( \Gamma^2 \)
A VARIETY OF APPLICATIONS.

Expectational fragilities, three snapshots.
1- New financial instruments.

○ The conventional wisdom and its limits.
  • Many new financial instruments (starting from options)
  • The rationale:
    ○ Lower transaction costs (options),
    ○ Improve risk-sharing..
  • Objections.
    ○ How do they affect the transparency/intrinsic information?
    ○ Can they affect plausibility of good expectational coordination?

○ A simple model (Guesnerie-Rochet (1993))
  ○ Problem of inventory decisions (random crop).
  ○ Introduce a new market (futures) available to all.

○ The standard view: à la Friedman
  ○ Speculation is good (robust argument due to MF)
    ○ It decreases the variance of the price of the crop.
Destabilising futures market...?

- The standard view.
  - M>0, indexes the size of the futures market
  - Increasing M is good in terms of volatility of the equilibrium
- The « eductive » stability viewpoint.
  - Eductive Stability : 
    - $C/kN + bkv^2/(N+M) > 2$
    - The set of parameters values involvg stability shrinks with M.
    - For the same parameters values the equilibrium is less stable.
- Destabilising Speculation!
  - Speculation is « de-stabilizing in the sense that it destabilizes expectations
- A potentially very large set applications
THE « EDUCTIVE » PROCESS: THE INVENTORY VARIANT

- An « eductive » story:
  - Expectations $X(e)$,
  - Realisations:
    - $-2kNX(e)/\{bk^2v^2+C\}$

- Results:
  - $N<\{bk^2v^2+C\}/2k$
  - Bad
    - More traders
    - Less risk averse
    - Less uncertainty
    - Less costly..
INVENTORIES WITH FUTURES MARKETS

- M mass of « speculators » : intervene on the market of futures, price $P(f)$, one unit of wheat to morrow.
  - Hedging behaviour from primary traders :
    - $N[p(f)-p(1)]/C = (N+M)[p(2)-p(1)]/bk^2v^2$.
    - Previously $X = X*/\{2+C/kN + bkv^2/N\}$
  - Now : $X = X*/\{2+C/kN + bkv^2/(N+M)\}$
  - Intuition : uncertainty cost born by $N+M$ agents.
  - The variance of prices is decreased

- Eductive stability :
  - $C/kN + bkv^2/(N+M) >2$
  - Intuition. $N(M)$, $N$ decreasing function of $M$. $M>0$ is bad.
AN « EVOLUTIVE » LEARNING MODEL..

- From Brock-Hommes-Wagener: « more hedging instruments may destabilize markets ».
- The model (sketch).
  - Stock $p(0,t) \rightarrow q(t+1,s) = p(0,t+1) + y(s)$, $s=1, \ldots S$, prob. $\gamma$
  - $N$ Arrow securities, $i$ pays the vector $d(i)$, i.e pays 1 in state $i=1, \ldots N<S$, price $p(i,t)$
  - Mean variance utility.
- The demand:
  - $Z(t) = IV(N) [-R(p(0,t) + E\{q(t+1)\}, -Rp(t) + E\{d\}]^t$
  - $V(N)$ prop. cov $(q(t+1)/d)$
  - Steady state.
    - $p(0,*) = (y^* - a\rho Q) / (R-1)$, $\rho$ variance of the stock, $a$ coef risk aversion, $Q$ total amount of the stock.
    - $p(s,*) = (1/R)(\gamma - abQ)$, cov. Vect. $q,d$
AN « EVOLUTIVE » LEARNING MODEL..

- The learning process:
  - Heterogenous expectations away from the RE benchmark.
  - Deviations f(h,t) depends on the type of the traders.
  - Remarks.
    - Along the path, Arrow securities are correctly priced in the REE
    - If beliefs are homogenous, no trade on Arrow securities
  - The fraction of agents following a given strategy depends on its fitness: average profit of the previous period corrected by riskiness
  - The speed of adjustment measured by c>0,
  - c small, the fundamental equilibrium is stable
  - When c becomes large, the fundamental equilibrium is destabilized

- Results.
  - With one more Arrow security, the fundamental equil. Is destabilized earlier for a smaller c!
  - The mechanism: Optimists (resp. pessimists) buy (resp. sell) the stock and hedge with Arrow securities...
AN « EVOLUTIVE » LEARNING MODEL..

- Results.
  - With one more Arrow security, the fundamental equil. is destabilized earlier for a smaller b!
  - The mechanism.
  - With more insurance possibility more hedging and risk taking, and profit if you are on the right side of the opinion, and these strategies through reinforcement mechanisms will attract more followers,
  - And vice versa...
  - More movement of opinion and of prices....

- Other results.
  - « Rational » agents may or may not stabilize the market...
  - Dubious...
Expectational fragilities, three snapshots.
2- Markets as transmitters of information

- The conventional wisdom and its limits.
  - Financial markets transmit well the information
  - Efficient market hypothesis.

- A standard set-up: (f.e Grossman-Stiglitz(198)).
  - Informed, partially informed and uninformed agents interact,
  - A Nash-bayesian (RE) equilibrium risk-sharing.
  - Informed: high demand when the state is good.
  - Non-informed: extract optimally information from prices

- The results:
  - A significant amount of information is transmitted

- The above question:
  - can the plausibility of good expectational coordination
  - be affected by some characteristics of the problem.
Expectational coordination in a Grossman-Stiglitz like model.

- **The set up**: a variant of GS (based on Desgranges.)
  - CARA utility function + Gaussian randomness.
  - Signals with an individual component $s(i) = \theta + b(c) + b(i)$
  - Linear REE: price lin. fct of $s = \int s(i)di$, and noise trading.

- **The equilibrium**.
  - Exists and is unique (mild assumptions)
  - Is it "eductively" stable?
  - CK restr. : linear demand, local restr. on parameters.
  - Obtains iif: $\text{Var}(\theta/p) > \text{Var}(\theta/s(i))$ (indep.i)
  - $\text{Var}(\theta/p)$–computed / equ. price distribution- (indep.p)

- **Related work**
  - Also Desgranges-Geoffard-G., Desgranges-Heinemann, Ben Porath-Heifetz
  - *Aggressive search of information kills "eductive stability"*
  - *Too much information cannot be transmitted!!*

- **Intuition**: You cannot trust the market if the others trust it!
**STANDARD EQUILIBRIUM THEORY**

- **Partial Equilibrium.**
  - Muth model...the ratio of elasticities is relevant.
  - Several crops: cross elasticities matter

- **General Equilibrium.**
  - 2 periods fixed wage model:
    - The keynesian multiplier matters for expectational coordination/
  - Flexible wage.
    - The keynesian multiplier still matters
    - Flexibility is good if it triggers market clearing.

- **Long horizon models.**
  - Even monetary...
A fixed wage two-period model.

- The « eductive process »
  - In both cases, it is enough to guess $Q^*$. 
- Results in the fixed wage model:
  - Consider $h = S(F(Q)/w))$
    - $F$ market clearing price when $Q$ is production.
  - Rationalizable equilibria in $(h(Q_), Q_)$
  - Local Strong rationality:
    - $e(S)/e(D) < m$, in $*$. 
    - $m$ inverse of marginal propensity to consume.
- Intuition:
  - Effect strategic substitutability Muth $e(S)/e(D)$
  - $m$ Keynesian multiplier (effect str.complementarities).
EDUCTIVE STABILITY
WITH FLEXIBLE WAGE : A COUNTEREXAMPLE.

- An example of a « rationalizable equilibrium » which is not walrasian.
- Rule out this to obtain:
  - \( \frac{e(S)}{e(D)} < 4m/l \)
- If the fixed wage equilibrium is SR in an interval of \( w \), then the \( W \) equilibrium is SR.
LONG HORIZON AND EXPECTATIONAL COORDINATION.

- Is the economists practice so bad?
- Long horizon... with short sighted agents.
- Long sighted agents.
Problems with rational expectations in infinite horizon models?

- THE WEB, THE FINANCIAL TIMES SITE. MARCH 3 2009 (W. BUITER)

- In financial markets, and in asset markets, real and financial, in general, today’s asset price depends on the view market participants take of the likely future behaviour of asset prices. ..Today’s asset price depends on today’s anticipation of tomorrow’s price, and tomorrow’s price likewise depends on tomorrow’s expectation of the price the day after tomorrow, etc. *ad nauseam*.

- Since there is no obvious finite terminal date for the universe (few macroeconomists study cosmology in their spare time), *most economic models with rational asset pricing imply that today’s price depend in part on today’s anticipation of the asset price in the infinitely remote future.*
Buiter’s pamphlet: between debate and polemics.

- What can we say about the terminal behaviour of asset price expectations? The techniques of dynamic mathematical optimisation imply that the influence of the infinitely distant future on the programmer’s criterion function today be zero, a ‘transversality condition that is part of the conditions for an optimum. And then a small miracle happens.....

- But in a decentralised market economy there is no mathematical programmer imposing the terminal boundary conditions to make sure everything will be all right.
BUITER’S PAMPHLET: FROM DEBATE TO POLEMICS.

- The common practice .......is evidence of the fatal confusion in the minds of much of the economics profession between shadow prices and market prices and between transversality conditions that are an integral part of the solution to an optimisation problem and the long-term expectations that characterise the behaviour of decentralised asset markets.

- (This amonts to assuming) that there is a friendly auctioneer at the end of time - a God-like father figure - who makes sure that nothing untoward happens with long-term price expectations .....with the present discounted value of terminal asset stocks or financial wealth.
PRESENTATION OF THE POINT.

- In a sense a discussion of the previous pamphlet.
  - I believe that expectations have to be explained (or that the robustness of the REH has to be tested)
  - I agree with Buiter that expectations in infinite horizon models have to be tested (explained).
  - I argue that
    - we have reasonable criteria (stability test) for models where agents have short horizon.
    - But that we have difficulties (conceptual and operational) to test when agents are long-lived.

- Plan.
  - The tests in models with short-lived agents. (Review)
  - Models with long-lived agents. (Work in progress)
(Linear) Infinite horizon models: one-dimensional, one-step forward-looking

- One special category:
  - One step forward-looking, one-dimensional, no memory....
  - $y(t) = a \int u(w)y(e, t+1, w) \, dw$, $f u(w) \, dw = 1$.
  - $y(t) = ay(e, t+1)$

- Perfect foresight equilibria
  - $y(t) = ay(t+1)$. perfect foresight equilibrium..
  - $y(t)=0$, steady state,

- Expectational stability:
  - Determinacy
    - Steady state is determinate: no pfe equilibrium « close » to it.
    - $a<1$...
(LINEAR) INFINITE HORIZON MODELS: ONE-DIMENSIONAL, ONE-STEP FORWARD-LOOKING

- One special category:
  - One step forward-looking, one-dimensional, no memory....
  - $y(t) = a\int [u(w)y(e, t+1, w)] \, dw$, $f(u(w)dw = 1$.
  - $y(t) = ay(e, t+1)$

- Expectational stability: absence of Sunspot Equilibrium.
  - $y(t) = aE(y(t+1)$
  - A sunspot equilibrium (markovian) $y(s), p(s/s), s, s'$
  - $y(s) = a[p(s/s)y(s) + p(s'/s)y(s')]$, idem en $s', s'$
  - no sunspot equilibrium. $a < 1$

- Expectational stability: IE-stability
  - $m < y(t) < M$, common conjecture...
  - IE stability, iteration of the conjecture
  - Eductive stability more demanding if the $u(w)$ do not have the same sign.
(Linear) Infinite Horizon Models: One-Dimensional, One-Step Forward-Looking

- One special category:
  - One step forward-looking, one-dimensional, no memory....
  - \( y(t) = a \int [u(w)y(e, t+1, w)] \, dw, \quad \int u(w) \, dw = 1. \)

- Perfect foresight equilibria
  - \( y(t) = ay(t+1). \) pfe, \( y(t)=0, \) steady state, équilibre de repos,

- Expectational stability:
  - 1- Determinacy.
  - 2- Absence of sunspot equilibrium.
  - 3- Expectational stability: IE-stability

- Conv. of “reasonable” adaptive learning rules.
  - 4- Adaptive, detect cycles of order 2.

- THM:
  - The four criteria are equivalent
  - Extends to non linear systems
  - No « Local » sunspot equilibrium.
  - Local IE stability, local convergence
One special category:
- One step forward-looking, one-dimensional, memory one models.
  \[ y(t) = dy(t-1) + bf[u(w)y(e, t+1, w)] \, dw, \]
  \[ f \, u(w) \, dw = 1. \]
- Implicit axioms, generalisation.

Perfect foresight equilibria
- \[ y(t) = dy(t-1) + by(t+1). \]
  \[ bx^2 - x + d = 0, \]
  \[ bd < 1/4, \text{ real roots, } |b+d| < 1, \text{ saddle point case.} \]

Strongly rational or «eductively» stable equilibrium.
- Hypothetical CK conjectures on growth rates: It is CK that \[ ... < g < ... \]
- Triggers a mental process....
Here a saddle path trajectory
- growth rate $y(t+1) = gy(t)$,
- $g < 1$.
- Explosive trajectory.

Lemma:
- If all agents conjecture that the growth rate is between $g - e$ and $g + e$, then the actual growth rate is between $g + \frac{db}{(1-bg)^2}e + \ldots$ and $g - \frac{db}{(1-bg)^2}e + \ldots$.
Standard Expectational Criteria.

- The Standard expectational criteria.
  - **Determinacy** of
    - trajectories \((C0,1)\), of the « perfect foresight growth rates ».
  - **Iterative Expectational Stability** :
    - Belief (perceived law of motion) : the growth rate is \(g^*+e\),
    - Realisation the growth rate is \(g^*+ke, k<1\).
  - **Absence of sunspot equilibrium**:
    - Without memory : \(x^*, x^{**}\), Markov matrix...
    - Can be defined for « growth rates ».
  - **Reasonable learning rule** :
    - adaptive learning on growth rates that detect cycles of order 2.
- The **equivalence theorems** :
  - The four criteria are equivalent in the one –dimensional case.
  - They pick up the saddle path solution...
MULTIDIMENSIONAL SYSTEMS.

- The model:
  - Reduced form: one step forward-looking, memory one, multi dimensional.
  - $C x(t+1) + x(t) + Dx(t-1) = 0$, matrices or numbers.
  - Game theoretical flesh:
    - $x(t) = Dx(t-1) + B \int Z(w(t)) x(e, t+1, w(t)) dw(t)$.

- Solutions:
  - Perfect foresight trajectories.
  - Perfect foresight dynamics of growth rates, « extended » growth rates.
  - Example: if $g(t) = x(t)/x(t-1)$, $g(t) = -[cg(t+1)g(t)+d]$,
    - If $x(t) = B(t)x(t-1)$, $B(t+1) = -(GB(t)+I)^{-1}D$.

STANDARD EXPECTATIONAL CRITERIA IN INFINITE HORIZON MODELS.

- The Standard expectational criteria.
  - Determinacy of
    - Trajectories (C0,1), of the long term extended growth rate.
  - Iterative Expectational Stability:
    - Belief (perceived law of motion): the growth rate is \( g^* + e \),
    - Realisation the growth rate is \( g^* + ke, k < 1 \).
    - On « extended growth rates »
  - Absence of sunspot equilibrium:
    - Without memory: \( x^*, x^{**} \), Markov matrix...
    - Can be defined for « extended growth rates ».
  - Reasonable learning rule: adaptive learning on growth rates that detect cycles of order 2.

- The equivalence theorem:
  - The four criteria are equivalent in the one-dimensional case
  - The first three are equivalent in the multi-dimensional case.
  - They pick up the saddle path solution...
  - Eductive stability is more demanding in general
    - Heterogeneity of expectations.
LONG HORIZON AGENTS
Weak and strong eductive stability.
LONG LIVED AGENTS IN A RBC CONTEXT.

The model:
- A continuum of households, \( w \in (0,1) \).
- 1 unit of labour, \( k(t,w) \), capital.
- Max \( \mathbb{E} \sum U(c(t,w)) : c(t,w)+k(t+1,w)=(1+r(t))k(t,w)+q(t) \)
- With iso-elastic utility, rate of time preference \( b \):
  - \( c(t,w)^{-\sigma} = b \mathbb{E}(w)[(1+r(t+1)) c(t+1)^{-\sigma}] \)
- Equilibrium conditions:
  - \( r(t) = f'_k(K(t),1)-d \),
  - \( q(t) = f'_l(K(t),1) \).
  - \( K(t+1) = (1-d)K(t)+f(K(t),1)-C(t) \).....( \( C(t) = fc(t,w) \)).

The steady state.
- \( K^*, C^*, r^*, q^* \)
- \( 1 = b(1+r^*) \),
- \( r^* = f'_k(K^*,1)-d, \ q^* = f'_l(K^*,1) \).
- \( C^* = f(K^*,1)-dK^* \)
- \( (dC(t)/C^* = -dr(t+1)/\sigma (1+r^*) + dC(t+1)/C^*) \)
LONG RUN EXPECTATIONAL COORDINATION: WEAK EDUCTIVE STABILITY.

\[ y(t) = a y(t+1) + ab y(t+1) + ab^2 y(t+2) + \ldots \]

\[ y(t) < M \quad \ldots \quad a/(1-b) < 1, \text{ not } a < 1 \text{ but } a+b < 1 \]
LONG RUN EXPECTATIONAL COORDINATION. STONG ECUCTIVE STABILITY

Conjectures on the state of the system induced by beliefs
THE DECISION PROBLEM OF A LONG LIVED AGENT.

- An infinitely lived agent the foll. characteristics:
  - An additive iso-elastic intertemporal utility function:
    - \( U = \frac{1}{1-\sigma} \sum \beta^t (C(t)^{1-\sigma}) \).
    - A sequence of interest rates \( \{r(t)\} \).

- First order conditions: Euler equation.
  - \( C(t)^{-\sigma} = \beta[1 + r(t+1)] (C(t+1))^{-\sigma} \).
  - Looks a one-step ahead problem..
  - Steady state: \( \beta[1 + r^*] = 1 \),

- Beliefs and decision.
  - At period \( s \), \( r(s) = r^* + dr(s) \).
  - \( C(t) = C(0) \beta^{t/\sigma} \prod_{1}^{t} [1 + r(s)]^{1/\sigma} \).
  - \( \left[ \frac{dC(t)}{C^*} \right] = \left[ \frac{dC(0)}{C^*} \right] + (\beta/\sigma) \sum (dr(s)) \).
THE LONG RUN DECISION PROBLEM

Beliefs and decision.
- At period s, \( r^* + dr(s) \).
- \( C(t) = C(0) [\beta^{t/\sigma} \Pi_1 t [1+r(s)]^{1/\sigma} \)
- \( [dC(t)/C^*] = [dC(0)/C^*] + (\beta/\sigma) \sum_t(dr(s)). \)

The welfare lemma:
- At the margin of the * situation, first-order changes in expectations have only second order effects on welfare.
- \( [dC(0)/C^*] + \sum_1 \beta^t [dC(t)/C^*] = 0 \)
- \( [dC(0)/C^*] = S(....dr(s)...) \),
- S sufficient statistics of the future

Provisional conclusions.
- Simple algebra.... Particular case : \( dr(s) = dr', s=2,... \)
- The far future has significant bite on to-day decision.
- Long term plans extremely sensitive to expectations
A SIMPLE RBC MODEL : « EDUCTIVE » STABILITY.

- **Weak « eductive » stability**
  - \[ b^2 \frac{C*f''}{(\sigma)(1-b)} < 1. \]
  - Avec CD, \( a=1/3, \sigma>2/3 \), plausible...
  - The condition is more demanding and the more the more labour supply is elastic

- **Strong « eductive » stability**:
  - Strong « eductive » stability impossible...in two senses.
    - For some beliefs compatible with the initial restriction, the plans of the agents at period 1, for later periods, become incompatible with the initial restriction.
    - If agents maintain restriction on beliefs and the economy goes, then the beliefs (may) become( s) invalidated. (depreciation of capital plays a role)
FURTHER DIRECTIONS OF INVESTIGATION.

- Forward looking « evolutive » learning.
  - People revise their long term sufficient statistics, that depend on the evolution of future capital, on the basis of actual capital (adaptive learning rule, $a$, weight of the observation of capital).
  - You want the system remaining in the neighbourhood and eventually converging...
  - If the weak stability condition holds, and $a$ is close to 1, then the required properties obtain.
    - Note that standard learning in this setting works well with $a=0$.
    - With a close to zero, adaptation is too slow to prevent leaving the cylinder...

- Having the required properties for every $a$, is impossible. Still an impossibility result.
END

- « Eductive » learning somewhere between
  - A shortcut,
  - And an alternative

- Thank You.