The Value of Real and Financial Risk Management*  

Marcel Boyer,† M. Martin Boyer‡ and René Garcia§  

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Abstract  
We characterize a firm as a nexus of projects with their associated cash flows. Production and operations activities and real risk management activities distribute cash flows over states of nature and time periods, leading to a transformation possibility frontier similar to a production function. We show how changes in the price of risk affect the value maximizing levels of real activities. The typical separation between management activities concerning production and operations on one hand and real risk on the other is a source of efficiency gains but causes coordination problems. Financial risk management creates value by alleviating these coordination problems. It also allows a firm to meet cash flow-at-risk or value-at-risk constraints at no cost.  
JEL Classification: G22, G31, G34  
Keywords: Risk Management, Firm Value, Coordination, Value at Risk.

Résumé  
Nous caractérisons une entreprise comme un ensemble de projets avec les flux monétaires qui y sont associés. Les activités de production et d’exploitation de même que les activités de gestion des risques réels distribuent ces flux entre les divers états de la nature et périodes. Il en résulte une frontière des possibilités de transformation des flux similaires à une fonction de production. Nous montrons comment les changements dans le prix du risque modifient les niveaux des activités réelles qui maximisent la valeur de l’entreprise. La séparation usuelle entre les activités de gestion concernant d’une part la production et l’exploitation et d’autre part les risques réels est à la source de gains d’efficience mais elle cause des problèmes de coordination. La gestion des risques financiers crée de la valeur en atténuant ces problèmes de coordination. Elle permet également à une entreprise de respecter des contraintes de flux monétaires à risque ou de valeur à risque sans coût.  
JEL Classification: G22, G31, G34  
Mots clés: Gestion des risques, Valeur de la firme, Coordination, Valeur à risque.

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†Bell Canada Professor of industrial economics, Université de Montréal, and Fellow of CIRANO and CIREQ (marcel.boyer@cirano.qc.ca).  
‡CEFA Associate Professor of finance and insurance, HEC-Montréal, Université de Montréal, and Fellow of CIRANO (martin.boyer@hec.ca).  
§Hydro-Québec Professor of risk management and mathematical finance, Université de Montréal, and Fellow of CIRANO and CIREQ (rene.garcia@umontreal.ca).


1 Introduction

The objective, function and value of financial risk management remain debated issues. In 1993, the Group of 30 \(^1\) recommended that market and credit risk management should be functions conducted independently of the day-to-day operations of a firm. Twenty years earlier, Mehr and Forbes (1973) argued the exact opposite. Today, Holton (2004) claims that the risk management function within a firm is too close to operations.

The modern academic view of risk management or hedging activities is mainly financial in nature and does not involve operations. However, chief risk officers are increasingly working alongside chief operating officers to maximize firm value, thus making risk management a central function. We propose a model where risk management activities are put alongside operations with the explicit goal of maximizing firm value.

In perfect financial markets, hedging risk cannot increase firm value. Smith and Stulz (1985) discuss a hedging irrelevance proposition as an extension of the leverage irrelevance theorem of Modigliani and Miller (1958). In the absence of market inefficiencies, investors can undo any financial transaction undertaken by a firm so that firm value is independent of the risk management strategy (Titman, 2002). Put differently, a firm cannot create value by hedging risks since investors bear the same cost of risk as the firm. Therefore, taxes, financial distress costs and agency costs are the reasons most frequently invoked to justify hedging in a value-maximizing firm. A firm facing a convex tax schedule increases its value by reducing the variability of its taxable earnings through risk management. Hedging can lower the cost of financial distress since it reduces the probability of unfavorable left-tail outcomes. If hedging makes earnings less volatile, it lessens the information asymmetry and reduces agency costs (managers vs. shareholders, shareholders vs. bondholders, internal vs. external finance).\(^2\)

\(^1\)The Group of 30 is a private not-for-profit international body composed of senior representatives of industry, government and academia. See their 1993 study Special Report on Global Derivatives at www.group30.org.

\(^2\)Several other reasons are found in the literature. Risk management can facilitate optimal investment and add value to a firm by favoring more stable free cash flows when the cost of external financing is higher than the cost of funding projects internally. Hedging may help retain valuable large shareholders or get stakeholders to make firm-specific investments. Stulz (2003) provides a systematic review of the various theoretical justifications for risk management within a firm. For the convexity of the tax schedule, see Main (1983), Smith and Stulz (1985), Graham and Smith (1999), Graham and Rogers (2002) and Graham (2003), as well as MacKay and Moeller (2003) for the case of general cost convexity. For the lower expected cost of bankruptcy or financial distress, see Booth et al. (1984), Smith and Stulz (1985), Block and Gallagher (1986), Mayers and Smith (1990), Nance et al. (1993), Geczy et al. (1997), and Bodnar et al. (1998). For a better assessment of the performance of executives, see DeMarzo and Duffie (1995) and Breeden and Vishwanathan (1996). For improving the investment decisions and for better planning of a firm’s capital needs, see Mayers and Smith (1987), Bessembinder (1991), Doherty and Smith (1993), Lessard (1991),
Our representation of operations and risk management within a firm supports the view that financial risk management can add value even in a world with no taxes, no financial distress costs, and no information asymmetry. We start by describing a firm as a nexus of projects\(^3\) with their associated cash flows. Projects are characterized by their respective level of expected cash flows and risk, which captures the correlation structure between the cash flows and risk factors faced by a firm. In this context, the object of production and operations management (POM) is to raise expected cash flows while the object of real risk management (RRM) is to lower risk. Based on this representation, we derive an efficient frontier in a space with expected cash flows and risk as coordinates, representing the set of efficient combinations or portfolios of feasible projects and activities selected by the real asset management (POM and RRM) functions of a firm.

Given the market prices of risk associated with risk factors, one can find the value of all feasible combinations of projects and arrive at the combination that maximizes firm value. Note that there is still no role for financial risk management in maximizing value. However, as the market prices of risk change, a firm must adjust its portfolio of projects to achieve a new optimal position on its transformation possibility frontier. Coordination between the different real functions in adapting to changes in the market prices of risk will typically be difficult and costly. Movement towards the new optimal combination may lead to disagreements between the POM and the RRM units when expected cash flows must be reduced or risk increased. This is where financial risk management contributes to firm value by bringing flexibility. Coordination is facilitated at little or no cost and provides a large saving with respect to real costs that will otherwise be incurred. Even in a world with no taxes, no bankruptcy or financial distress costs and no agency conflicts between the different classes of stakeholders, this value-adding role for financial risk management has definite empirical implications. We provide new testing grounds on the value of financial risk management for an organization.

Empirical evidence supporting the traditional reasons for corporate risk management remains rather weak. For example, the evidence presented by Graham and Smith (1999) is not very supportive of the tax savings argument. With respect to bankruptcy and financial distress costs, larger firms should need less risk management than smaller firms because their expected bankruptcy costs are relatively lower. However, larger firms seem to hedge more through the use of derivatives than

\(^3\)Projects may in some contexts correspond to the different business units or to different sets of activities within the organization.
smaller firms. Our coordination argument is compatible with this evidence. Larger corporations and corporations where more executives are involved in investment decisions will be heavier users of financial management instruments. Multinational firms and conglomerates, which have a more diverse project mix than single-industry single-country firms, should be greater users of derivatives.

Our setup establishes a definite link between hedging activities and changes in market parameters, namely the risk free rate, market volatility and the expected market returns. The use of financial instruments to hedge risk will be more pronounced when the efficient frontier is weakly concave since a small movement in the market parameters will then lead to an important adjustment in the optimal combination of projects. Therefore, a direct test will be to relate a measure of the concavity of the efficient frontier to the use of financial instruments across firms.

Another implication concerns firms subject to regulatory or self-imposed cash flow-at-risk (CaR) or value-at-risk (VaR) constraints. We show that a firm can, through appropriate financial risk management operations, meet these financial constraints without affecting its value maximizing activities and therefore its market value. In other words, CaR and VaR constraints can be met without changing the optimal mix of real activities. Hence, we can predict that, because of the VaR and CaR constraints they face, firms in sectors such as financial services, energy, gambling, utilities, and the like, will be among the heaviest users of derivatives and other financial risk management instruments.

The remainder of the paper is organized as follows. We present the model of the efficient frontier in Section 2. Section 3 discusses coordination problems between RRM and POM activities and stresses the important role that financial risk management plays in alleviating these coordination problems. In Section 4, we perform a comparative statics analysis relative to changes in parameters of the market price of risk. In Section 5, we spell out empirical implications of the model and discuss several issues related to the actual implementation of our approach to financial and real risk management in firms. We also extend the basic framework to an intertemporal setting. We conclude in Section 6.

4 See Bodnar et al. (1998), Nance et al. (1993), and Geczy et al. (1997). An exception is Mayers and Smith (1990).
5 Block and Gallagher (1986) and Booth et al. (1984) argue that larger firms engage in more financial risk management because of the large fixed costs involved to hedge financial risks. Therefore, it is less efficient for small firms to invest in risk management. Our argument is based on the large costs involved in reorganizing real activities relative to the small cost of using financial derivatives.
2 The Model

2.1 Preliminaries

A firm is defined as a nexus of projects representing all real activities, such as those related to investment and production, and giving rise to a transformation possibility frontier for cash flows. This frontier is the envelope of all feasible vectors of cash flows over states of nature and time periods obtainable from all projects characterizing and identifying the firm as an economic entity. Hence, it accounts for all human, technological, contractual, legal and other constraints facing a firm. In the short term, a firm can modify its overall distribution of cash flows over states and time periods and switch from one distribution to another within its feasibility set by changing its project portfolio. In the long term, a firm can modify its feasibility frontier by changing constraints underlying the transformation possibility set, generally through technological and organizational innovations such as mergers, acquisitions and divestitures, and patent initiation.

If a firm can change its operations or increase its flexibility to significantly reduce its risk without changing expected cash flows, its market value will increase as the given expected cash flows will be discounted at a lower rate. Rather than characterizing a firm by a quasi-fixed and exogenously given risk measure, we see a firm as choosing, within its feasibility set, a portfolio of projects to obtain a distribution of cash flows that maximizes its value given the market prices of risk. We therefore approach risk management from the general viewpoint of the economics of the firm and the economics of organizations rather than from the usual financial perspective. We summarize the activities of a firm through the generated cash flows over states of nature and time periods.

In Figure 0, we characterize a firm by two blocks, real asset management (RAM) and financial risk management (FRM). The first block is broken down into production and operations management on one hand, and real risk management on the other. All activities within a firm, such as project selection or self-protection, can be described along these two dimensions. Financial risk management is purposely set apart and involves all transactions carried out through the purchase or the sale of financial instruments.

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6 As noted by Stulz (2004, p.42), a firm can become more flexible so that it has lower fixed costs in a cyclical downturn. This greater flexibility translates into a lower beta.

7 In so doing, we develop a model of the firm in the spirit of the early contributions of Fama and Miller (1972) and Cummins (1976).
Our modeling of POM, RRM and FRM activities allows us to address their relationship from a new perspective. The relative independence of these functions in a typical firm is a potential source of inefficiency. We will see that, in various situations, restoring efficiency may require different and sometimes rather unorthodox ways and means. The critical role of organizational design in this context is twofold: to assign the POM and RRM responsibilities within the RAM function and to design a coordination strategy between the RAM and FRM functions within the firm as a whole.8

Assuming the existence of multiple sources of risk, we see the POM function as maximizing a firm’s expected cash flows for a given level of overall risk, and the RRM function as minimizing risk for a given level of expected cash flows. The optimal mix of POM and RRM activities maximizes the value of the firm given its possibility frontier and the prices of risk. Changes in the determinants of these prices of risk, namely the factors’ risk premia and volatility, and the risk-free rate, can affect the optimal portfolio of feasible projects characterizing a firm.

More importantly, when difficult coordination problems exist between the RRM and POM functions, FRM activities precisely and explicitly facilitate the choices and adjustments undertaken by RRM and POM functions to maximize firm value.

2.2 The possibility frontier and the market prices of risk factors

A firm is a technology by which cash flows \( cf_{st}^p \) related to various projects \( p \in \{1, 2, \ldots, P\} \) defining a firm as an economic entity are distributed over or transformed between different states \( s \) and periods \( t \), with \( s \in \{1, 2, \ldots, S\} \) and \( t \in \{1, 2, \ldots, T\} \), under technological, legal, or contractual constraints. The transformation possibility frontier of firm \( j \) (i.e., the envelope of all feasible cash flow vectors) given its information set \( \Omega_0 \) at time \( t = 0 \) can be represented as

\[
G_j (cf_{s1}, \ldots, cf_{st}, \ldots, cf_{ST} \mid \Omega_0) = 0, \tag{1}
\]

where \( cf_{st} \) is the aggregate cash flow over all projects \( p \) in state \( s \) and period \( t \). The envelope of all feasible cash flow vectors is concave.

A firm modifies cash flows through changes in its portfolio of projects. Characteristics of the cash flow aggregate vector lead to an evaluation of the firm on the financial markets. Given its technological possibilities represented by (1), a firm chooses the mix of POM and RRM activities to

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8The organizational design is particularly crucial for corporate governance rules, which allocate imputability, responsibility and liability within the organization. A related issue is that of allocating economic capital to different divisions, products, or lines of business.
reach the aggregate vector of cash flows that maximizes its value. Hence, the frontier \( G_j (\cdot) = 0 \) must be understood as a frontier in real terms, that is, emerging from the POM and RRM activities. We later discuss the representation of financial risk management (FRM) activities in this framework.

For presentation clarity, we now describe a multifactor model with \( N \) risk factors.\(^9\) We assume, for simplicity, constant expected cash flows per period, \( E_s(cf_{st}) = E_j, \forall t, \) and an infinite number of periods. Constant expected cash flows are discounted at a rate of return given by:

\[
ER_j = R_F + \sum_{i=1}^{N} \beta_{ji} (ER_i - R_F)
\]

where \( ER_i \) is the expected return on risk factor \( i \), \( R_F \) is the risk free rate, and \( \beta_{ji} \) is the measure of risk with respect to the \( i \)-th factor.

In such a setting, firm value is simply:

\[
V_j = \frac{E_j}{ER_j}
\]

Expressed in terms of cash flows, the security market line or hyperplane (2) takes the form:

\[
E_j = V_j ER_j = V_j R_F + \sum_{i=1}^{N} V_j \beta_{ji} (ER_i - R_F)
\]

where \( V_j \beta_{ji} \) measures the risk of the firm’s cash flows with respect to the \( i \)-th factor:

\[
V_j \beta_{ji} = V_j \frac{COV (R_j, R_i)}{\text{Var} (R_i)} = \frac{COV (V_j R_j, R_i)}{\text{Var} (R_i)} = \frac{COV (cf_j, R_i)}{\sigma_i^2}.
\]

We can rewrite (4) as

\[
E_j = V_j R_F + \sum_{i=1}^{N} \rho_{ji} \sigma_j \left( \frac{ER_i - R_F}{\sigma_i} \right),
\]

or

\[
V_j = \frac{1}{R_F} \left[ E_j - \sum_{i=1}^{N} \rho_{ji} \sigma_j \left( \frac{ER_i - R_F}{\sigma_i} \right) \right],
\]

where \( \sigma_j \) measures the volatility of the firm’s cash flows and \( \sigma_i \) measures the volatility of the market return on the \( i \)-th risk factor. The value of a firm depends, in this context, only on \( E_j \) and the scaled correlations \( SCOR_{ji} = \rho_{ji} \sigma_j \) between a firm’s cash flows and market returns on the different risk factors.

Relative to valuing a firm, the variables \( E_j \) and \( SCOR_{ji} \equiv \rho_{ji} \sigma_j, \ i \in \{1,2,\ldots,N\} \), are the \( N + 1 \) sufficient statistics of all projects within a firm. The transformation possibility frontier

\(^9\)We assume that the factors have been orthogonalized so that their mutual covariances are zero.
(1) can therefore be rewritten in terms of $E_j$ and $SCOR_{ji}$ as the envelope of all feasible points $(E_j, SCOR_{j1}, \ldots, SCOR_{jN})$:

$$H_j (E_j, SCOR_{j1}, \ldots, SCOR_{jN}) = 0.$$  

We will work with this representation of a firm’s technology.\textsuperscript{10}

Defining a firm’s feasibility set in terms of expected cash flows $E_j$ and the $N$ scaled correlation values $SCOR_{ji}$ has several advantages. First, it allows the value of RRM and POM activities to be measured from their capacity to move a firm toward or along the frontier $H_j(\cdot) = 0$ in the $(E_j, SCOR_{j1}, \ldots, SCOR_{jN})$-space. A change in the mix of POM and RRM activities will usually generate a change of value. Second, it allows proper aggregation of risks at the firm level by establishing a functional relationship between risk factors and cash flows for the many projects or business units. Identifying risk factors that are common to the various projects and accounting for the dependencies between them is an important function in a firm, which can fall under the responsibility of a central unit or delegated to various units. Both organizational forms may result in inefficient real risk management. Selecting the better one is an important function of the chief operations officer, the chief risk officer or the chief executive officer.

### 2.3 The value of the firm

The value of a firm is generated by a mix of POM and RRM activities with specific roles. Operations management is intent on maximizing expected cash flows for given levels of risk measured by the scaled correlations of a firm’s cash flows with the $N$ different risk factor returns, thereby contributing to firm value. Real risk management is intent on minimizing such scaled correlations for a given level of expected cash flows, thereby also contributing to firm value. Both groups of activities thus contribute to the overall objective of maximizing value. In this framework, the primary responsibility of higher level executives is to ensure that a firm’s decision making process brings it not only on its frontier but also at the optimal point on that frontier.

For further simplicity, let us assume that there is a single risk factor, namely the market portfolio

\textsuperscript{10}To draw, in practice, the efficient frontier for a given firm, one needs the set of cash flows associated with the numerous projects defining the firm along with the scaled correlations between the firm’s cash flows and the returns on risk factors.
risk. With $\text{SCOR}_j M = \rho_j M \sigma_j$, we can write (6) and (7) as:

$$E = VR_F + V \beta (ER_M - R_F) = VR_F + \text{SCOR}_M \left( \frac{ER_M - R_F}{\sigma_M} \right),$$  \hspace{1cm} (9)

$$V = \frac{1}{R_F} \left[ E - \text{SCOR}_M \left( \frac{ER_M - R_F}{\sigma_M} \right) \right].$$ \hspace{1cm} (10)

From (9), we observe that $\beta \leq [\geq] 1$ as $\text{SCOR}_M \leq [\geq] V \sigma_M$. We can illustrate the problem of a firm in the $(E, \text{SCOR}_M)$-space as in Figure 1, where each dot represents a potential project with a $(E, \text{SCOR}_M)$ pair of coordinates. All projects a firm can undertake are represented in that space where the frontier is constructed as the minimum level of risk obtainable for a given level of expected cash flows (see Merton, 1972).

[Insert Figure 1]

We can represent iso-value lines as in Figure 2. By definition, an iso-value line represents combinations of $E$ and $\text{SCOR}_M$ giving the same market value. From (10), iso-value lines are linear and parallel, with slope equal to the market price of risk

$$\frac{E (R_M) - R_F}{\sigma_M}.$$ \hspace{1cm} (11)

[Insert Figure 2]

The value $V$ attached to a given iso-value line can be obtained by discounting the zero-$\text{SCOR}$ expected cash flow level ($C_1$ and $C_2$ in Figure 2) at the risk-free rate $R_F$: $V_1 = C_1 / R_F$, $V_2 = C_2 / R_F$. Firm value increases in the North-West direction.

The combination of expected cash flows $E$ and scaled correlation between cash flows and market returns $\text{SCOR}_M$ that maximizes firm value is the combination at which the efficient frontier reaches the highest iso-value line. For that combination (point $A_2$ on Figure 2), the usual tangency condition holds:

**Proposition 1:** To maximize its value, a firm must equate its marginal rate of substitution, the rate at which it can substitute POM and RRM activities while remaining on its efficient frontier, to the market price of risk:

$$- \frac{\partial (OM)}{\partial (RM)} = \frac{\partial E}{\partial \text{SCOR}_M (cf_j, R_M)} \bigg|_{H(E, \text{SCOR}_M) = 0} = \frac{E (R_M) - R_F}{\sigma_M}. \hspace{1cm} (12)$$

\footnote{We will drop the index of firm $j$ when the context is clear and no confusion is possible.}
At point $A_1$, however, the scaled correlation can be reduced without affecting expected cash flows because point $A_1$ is not located on the efficient frontier. A firm’s POM and RRM strategies and policies are not efficient if they bring it to a situation such as point $A_1$. By better managing its real risk to reduce the scaled correlation of its cash flows, or by better managing operations to increase expected cash flows, a firm is able to increase its value.

If there are two risk factors ($N = 2$), then we have two prices of risk $\frac{E(R_1) - R_F}{\sigma_1}$ and $\frac{E(R_2) - R_F}{\sigma_2}$ and two scaled correlations between cash flows and returns on the risk factors, $\text{SCOR}_1 = \rho_{j1}\sigma_j$ and $\text{SCOR}_2 = \rho_{j2}\sigma_j$. A firm maximizes its value at the point of tangency between the efficient (hyper)frontier and the highest reachable iso-value hyperplane.

\[
\begin{align*}
-\frac{\partial E}{\partial \text{SCOR}_1(c_{fj}, R_1)} & \bigg|_{H(E, \text{SCOR}_1, \text{SCOR}_2) = 0} = \frac{E(R_1) - R_F}{\sigma_1} \tag{13} \\
-\frac{\partial E}{\partial \text{SCOR}_2(c_{fj}, R_2)} & \bigg|_{H(E, \text{SCOR}_1, \text{SCOR}_2) = 0} = \frac{E(R_2) - R_F}{\sigma_2} \tag{14} \\
\frac{\partial \text{SCOR}_2(c_{fj}, R_2)}{\partial \text{SCOR}_1(c_{fj}, R_1)} & \bigg|_{H(E, \text{SCOR}_1, \text{SCOR}_2) = 0} = \frac{E(R_1) - R_F}{E(R_2) - R_F} \left( \frac{\sigma_2}{\sigma_1} \right) \tag{15}
\end{align*}
\]

### 2.4 The case of non-valued risks

We have assumed until now that all the risk factors have a market price, so that firm value maximization is achieved at the optimal tangency point between the iso-value hyperplane and the possibility frontier. When the market does not value some risks that are nevertheless taken into consideration by a firm, the valuation problem is different.

We can illustrate this situation with two risk factors: the first is valued by the market and is represented by the market portfolio while the second is managed by the firm at some cost but is diversifiable for an outside investor so that its market value is zero. At what optimal level should a firm manage this non-valued risk? Each level of non-valued risk corresponds to a projected transformation possibility frontier in the space expected value – market-valued risk, namely $H(E, \text{SCOR}_M \mid \text{SCOR}_{NV}) = 0$, where $\text{SCOR}_{NV}$ is the level of non-valued risk taken or assumed by a firm. Under some reasonable assumptions about the non-valued risk (including the existence of a unique global maximum), there is one best or maximal transformation possibility frontier in the space expected value – market-valued risk, namely $H(E, \text{SCOR}_M \mid \text{SCOR}^{*}_{NV}) = 0$. The tangency point between the highest iso-value line and this maximal frontier gives the maximal

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market value of a firm.12

3 Independence and Coordination

Developments in the previous sections dealt mainly with real asset management. This section covers the role of financial risk management. A firm maximizes its value by finding a mix of POM and RRM activities in order to reach the highest feasible iso-value line given its cash flow transformation possibility frontier. To do so will require some level of coordination between real risk management, intent on minimizing the $SCOR$-value for a given level of expected cash flows $E$, and production and operations management, intent on maximizing the $E$-value for a given level of risk $SCOR$. We will show that the role of financial risk management is to facilitate adjustments in POM and RRM activities in order to modify the portfolio of projects following changes in the market price of risk.

For our purposes, to capture the coordination problem, we assume that the real risk manager and the production and operations manager must agree before a project is chosen, added or abandoned. Hence, when a firm must move from an optimal set of activities to a new optimal set of activities following a change in the market price of risk, a significant coordination problem arises. The real risk manager will tend to oppose increases in risk ($SCOR$) whereas the production and operations manager will tend to oppose reductions in expected cash flows ($E$). Such a representation of the inherent conflict between RRM and POM functions is admittedly extreme but it characterizes in a simplified yet realistic way the difficulties encountered when various managers need to coordinate their choices to maximize value. Agreement between several function before a project can be undertaken is quite generic. For instance, once a division manager or a business decision support group have produced the cash flow scenarios characterizing a proposed investment project, the project will generally be analyzed and assessed by a risk evaluation committee and sometimes by the board itself: only projects supported by the different instances can be implemented.13 The above simplified formulation is intended to represent such coordination procedures.14

12 A parallel can be drawn with the production function using a non-valued or zero-cost input, such as water or air. If production affects the quality of this input, there will be an optimal amount of activity, say in terms of quantity of pollutants rejected, that will be compatible with maximizing profit. Similarly, there will an optimal amount of non-valued risk that a firm should take or assume in order to maximize its market value in the space expected value–market-valued risk. In so doing, a firm optimally manages this non-valued risk.

13 One can also think of coordination problems as being driven by organizational inertia, which emerges as different groups (or management functions) acquire quasi-veto rights on some changes in the activities of a firm. See Hannan and Freeman (1984) and Boyer and Robert (2006).

14 The compensation structure of a chief operating officer in large corporations includes bonuses that are usually
3.1 Value creating coordination: increasing \textit{SCOR} for a given $E$

A firm may find itself at a certain point on its efficiency frontier to the left of the optimal mix of POM and RRM activities. This sub-optimal mix is represented as point $A_1$ in Figure 3a.

If the POM manager continues trying to increase $E$ for a given \textit{SCOR}, while the RRM manager keeps working to reduce \textit{SCOR} for a given $E$, the firm as a whole finds itself trying to move in an infeasible North-West direction. The way out of this efficient but not value maximizing combination of POM and RRM activities is for the RRM manager to let \textit{SCOR} increase above its current level, providing the POM manager with some leeway to increase $E$. In so doing, the RRM manager must momentarily \textit{destroy value}, by letting \textit{SCOR} increase given $E$, giving the POM manager the flexibility to ultimately increase firm value.

3.2 Value creating coordination: reducing $E$ for a given \textit{SCOR}

Similarly, a firm may find itself at a certain point on its efficiency frontier to the right of the optimal mix of POM and RRM activities, such as $A_2$ in Figure 3a. As before, if the POM manager continues trying to increase $E$ for a given \textit{SCOR}, while the RRM manager keeps working to reduce \textit{SCOR} for a given $E$, the firm as a whole finds itself trying to move in a direction that would take it above its efficiency frontier. The way out of this efficient but suboptimal mix of activities is for the POM manager to let $E$ decrease below its current level to give the RRM manager the possibility to reduce \textit{SCOR}. In so doing, the POM manager must \textit{destroy value} at least momentarily to allow an increase in the performance of the RRM manager, which in the end will increase firm value above its current level.

linked to cash flow performance targets and less so to risk measures. Even option-based compensation rewards managers for cash flow performance to the possible detriment of real risk management activities. With respect to the compensation of real risk managers, Gable and Sinclair-Desgagné (1997) and Sinclair-Desgagné (1999) offer an audit like procedure to assess managerial performance in the context of environmental (real) risk management and control. When the excellent cash flow performance of a manager warrants a bonus, an environmental audit is performed to check whether or not performance has been achieved to the detriment of proper risk management. If no, the manager obtains the bonus. If yes, the manager may be penalized (negative bonus). Hence, cash flow performance and risk management are made more congruent.
3.3 Flexibility and value creation through financial risk management

We have thus far posited that, in our framework with no taxes, no financial distress costs or transaction costs of bankruptcy, and no agency problems, value is created within a firm only through its choice of real projects and activities. This means that maximal value is created only through an optimal mix of RAM activities that blend both POM and RRM concerns. As the market price of risk changes, the optimal combination of expected cash flows and scaled correlations also changes, thus generating significant coordination problems as both the real risk manager and the production and operations manager must agree on changes. Financial risk management creates value by alleviating this coordination problem.

Consider Figure 3b. Suppose a firm’s optimal mix is initially at $A_2$ but because of a change in the market price of risk, the new optimal mix is at $A_0$. Suppose, moreover, that the POM manager is unwilling or unable to destroy positive net present value projects (moving down) to provide the RRM manager with enough flexibility to reach point $A_0$. How can financial risk management help in this process?

Consider the iso-value line that goes through point $A_2$. This line is, by definition, lower than the iso-value line tangent to the possibility frontier at point $A_0$. The slope of iso-value lines is the price of risk, that is, the price at which one can exchange $SCOR$ risk for expected cash flows on the financial market. Therefore, under conditions of perfect financial markets, a firm can enter into financial transactions to move from $A_2$ to any point on the relevant iso-value line. These movements along the iso-value line, for example to point $B$, are done at no cost but do not affect firm value since financial transactions are not creating value.

The advantage of moving a firm’s $(E,SCOR)$ combination to point $B$ is that the RRM and POM managers are given the flexibility to move the firm to $A_0$ without going on a path of value destruction. Value is thus created not through financial risk management per se but through real activities that would have been difficult and costly to coordinate without financial risk management.

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15 This is clearly reminiscent of Proposition III in Modigliani and Miller (1958, page 288): “... the cut-off point for investment in the firm ... will be completely unaffected by the type of security used to finance the investment.”
16 The analysis is similar if we start at point $A_1$. The RRM manager is unwilling to create risk and destroy value in order to give the POM manager enough flexibility to reach point $A_0$.
17 In a manner similar to an individual’s portfolio choice under the two-fund separation approach.
What then is the value of financial risk management? In and of itself, the value is zero. The value of financial risk management comes from the fact that it gives a firm sufficient flexibility to attain a new mix of risk and expected cash flows that would have been otherwise impossible or very difficult and costly to achieve. Moving from \( A_2 \) to \( A_0 \) requires abandoning [accepting] some projects with positive [negative] net present value given the \( SCOR \)-coordinate at \( A_2 \), hence the opposition of the POM manager to those changes. Similarly, moving from \( A_1 \) to \( A_0 \) requires abandoning [accepting] some projects that are risk reducing [increasing] given the \( E \)-coordinate at \( A_1 \), hence the opposition of the RRM manager to those changes. But at \( B \) with the new \( E \) and \( SCOR \) coordinates, changes in the project mix can now be agreed upon by both managers.

When market imperfections prevent a firm from trading \( SCOR \) for expected cash flows \( E \) at the constant market price of risk, the analysis still holds but with a twist. Trading expected cash flows \( E \) for \( SCOR \) can then be done not on the straight iso-value lines but on a nonlinear price manifold that goes through point \( A_2 \) and remains below the straight iso-value line through \( A_2 \). The analysis remains basically the same: the financial risk manager can find a \( B \)-like point along this manifold so that RRM and POM managers can more easily coordinate and move the firm to \( A_0 \).

### 3.4 An application to \( VaR \) and \( CaR \) constraints

The flexibility offered by financial risk transactions also allows a firm to obey some regulatory or self-imposed constraints on acceptable losses with a certain probability. For instance, a cash flow-at-risk constraint imposes the requirement that the cash flow shortfall \( E(cf) - cf \) will surpass a desired level \( (CaR) \) with a given probability \( \alpha \): \( \Pr [E(cf) - cf > CaR] = \alpha \). These constraints, when binding, are usually perceived as preventing the maximization of firm value. Every \((E, SCOR)\) combination can be associated with a \( CaR \) value. \( Iso-CaR \) curves, that is curves on which all points have the same \( CaR \) value, can be drawn. On Figure 4, the \( CaR \) value at point \( A_H \) is the same as at point \( D \). Let us identify this curve as \( CaR_H \) and suppose that a firm is required to satisfy that \( CaR_H \) level.

[Insert Figure 4]

A firm’s value is not maximized at point \( A_H \) since the iso-value line through \( A_H \) lies below the iso-value line through \( A_L \), the value maximizing point. The project mix in \( A_L \) is certainly attainable given the possibility set of the firm, but \( CaR_L \), the \( iso-CaR \) curve through point \( A_L \),
does not satisfy the constraint. As a result, the difference in firm value between $C_L/R_F$ and $C_H/R_F$ represents the cost of the CaR constraint.

In a perfect capital market, a firm is always able to trade zero-value financial contracts at no cost to move along the iso-value line whose slope is the market price of risk. Then, such a movement with financial instruments along the iso-value line going through $A_L$ can bring the firm to point $D$, which satisfies the CaR requirement. At point $D$, firm value is equal to $C_L/R_F > C_H/R_F$ since point $D$ lies on the same iso-value line as $A_L$. Again, value is not created by financial risk management per se. It simply makes a firm obey a CaR constraint while keeping its optimal mix of real activities. This would have been unfeasible without the use of financial instruments. A firm keeps its value while moving from $A_L$ to $D$ on the same iso-value line but it satisfies the required CaR level.

Therefore, CaR constraints should have no impact on the market value of firms provided there is access to an efficient capital market. A firm should instruct its real asset managers (POM and RRM) to maximize its value and then ask the financial risk manager to use financial transactions to satisfy the CaR requirement. Consequently, financial risk managers in industries with a binding CaR regulation, such as the financial services industry, will be asked to purchase zero net present value financial contracts that reduce a firm’s risk and expected cash flows (typically from $A_L$ to $D$ in Figure 4b) in order to attain the risk-return constraint set by the regulatory body, at no cost in terms of value.

4 Comparative Statics Analysis of Changes in the Underlying Drivers of the Market Price of Risk

At this point, we assume, for graphical representation, that the single risk factor is the market risk factor so that the parameters of the iso-value lines are $E(R_M)$, $\sigma_M$, and $R_F$. A firm’s technology, transforming scaled correlation $SCOR$ between a firm’s cash flows and market returns into expected cash flows $E$, is represented by the efficiency frontier (8), which can be rewritten as follows with $\rho_M \equiv \rho_{jM}$ and $\sigma \equiv \sigma_j$:

$$H(E, SCOR_M) = H(E, \rho_M \sigma; Z)$$

(16)

where $Z$ is a vector of exogenous variables. As we will illustrate, a change in the market portfolio expected return or volatility rotates the iso-value lines, while a change in the risk-free rate both
rotates and translates the iso-value lines.

4.1 Change in $E(R_M)$ or $\sigma_M$: rotation in iso-value lines

Let us consider the impact of changes in $E(R_M)$ and $\sigma_M$ on the optimal POM-RRM balance. A decrease in the market portfolio expected return as well as an increase in market return volatility reduce the slope without changing the intercept of iso-value lines, generating a clockwise rotation with the intercept as anchorage point. Changes in $E(R_M)$ or $\sigma_M$ have no impact on a firm’s transformation technology and thus on the efficient frontier. Both a decrease in $E(R_M)$ and an increase in $\sigma_M$ reduce the price of risk and shift the value maximizing point, the tangency point between the efficiency frontier and the maximal iso-value line, to the right as illustrated in Figure 5.

[Insert Figure 5]

**Proposition 2:** A firm reacts to an increase in the volatility of market returns, which reduces the price of risk, by modifying its RRM and POM activities to increase both its expected cash flows $E$ and scaled correlation $SCOR$ (from $A_1$ to $A_2$ in Figure 5).

**Proposition 3:** A firm reacts to an increase in the expected market return, which increases the price of risk, by modifying its RRM and POM activities to reduce both its expected cash flows $E$ and scaled correlation $SCOR$ (from $A_2$ to $A_1$ in Figure 5).

Following an increase in $\sigma_M$, a firm undertakes financial risk management FRM activities to move in the North East direction along the new iso-value line. It takes on more risk in a zero-value exchange for higher expected cash flows to save on coordination costs. In the case of an increase in $E(R_M)$, movement along the new iso-value line is in the South West direction so as to lower risk and expected cash flows, again to save on coordination costs.

Will firms react more to changes in the volatility $\sigma_M$ or in the expected value $E(R_M)$ of market returns? Because firms react, in fact, to changes in the price of risk, we can focus exclusively on changes in this price. A change in $\sigma_M$ has a smaller marginal impact than a change in $E(R_M)$ in the sense that the $\sigma_M$-elasticity of the price of risk is less in absolute value than its $E(R_M)$-elasticity. Indeed,

$$\varepsilon_{\sigma_M} = \frac{\partial \frac{E(R_M)-R_F}{\sigma_M}}{\partial \sigma_M} \left( \frac{\sigma_M}{E(R_M)-R_F} \right) = -1$$
\[
\varepsilon_{R_M} = \frac{\partial E(R_M) - R_F}{\partial E(R_M)} \left( \frac{E(R_M)}{\sigma_M} \right) = \frac{E(R_M)}{E(R_M) - R_F} > 1.
\]

As a result, the composition of a firm’s project portfolio will vary more, everything else being equal, following a 10% reduction (say from 20% to 18%) in market volatility than following a 10% increase (say from 10% to 11%) in the expected market return. Indeed, the market price of risk increases by 10% in the former case and by more than 10% in the latter.

4.2 Change in \( R_F \): rotation and translation of the iso-value lines

An increase in the risk-free rate reduces the price of risk (11) so that the iso-value lines rotate clockwise while, as the zero correlation cash flows are discounted at a higher rate than before, the intercept moves up. These movements are illustrated in Figures 6 and 7.

Assume that the zero-\( SCOR \) level of cash flows \( C_1^{\text{new}} \) is such that, under the new risk-free rate \( R'_F \), firm value is equal to the value at \( C_1 \) under the original risk-free rate \( R_F \): \( C_1^{\text{new}} / R'_F = C_1 / R_F = V \). This means that the value assigned to all combinations \( (E, SCOR) \) on the iso-value line going through \( C_1^{\text{new}} \) and parallel to the iso-value line going through \( A_2 \) and \( C_2 \) is equal to the value assigned to all combinations \( (E, SCOR) \) on the original iso-value line crossing \( C_1 \) since, by construction, the new iso-value line going through \( C_1^{\text{new}} \) has the same value as the old iso-value line going through \( A_1 \) and \( C_1 \). Because their slopes are different, those iso-value lines intersect at some point. Interestingly, we know exactly where this point lies: it is where \( SCOR = V \sigma_M \), that is, where \( \beta = 1 \) since from (9), we know that the value of a firm whose \( \beta \) equals 1 is independent of the risk free rate. Hence, an increase in the risk free rate reduces the price of risk, generating a clockwise rotation with the \( \beta = 1 \) point as anchorage point, as illustrated in Figures 6 and 7.

Suppose a firm is initially at point \( A_1 \). Following the risk-free rate increases, the new optimal project mix is given by point \( A_2 \) where both \( E \) and \( SCOR \) are higher: the reduction in the price of risk induces firms to move towards riskier portfolios of projects.

How does the firm value at \( A_2 \), following an increase in the risk-free rate, compare to the value at \( A_1 \) under the old risk-free rate? The increase in \( R_F \) affects firm value, directly at \( A_1 \) and indirectly through adjustments in the project mix. Because the iso-value line crossing \( C_1^{\text{new}} \) may be above
or below the iso-value line going through $A_2$ and $C_2$, firm value may increase or decrease with an increase in the risk free rate.

This means that when a firm’s current project portfolio is such that $SCOR_j = V_j\sigma_M$, then a change in the risk free rate will have no impact on firm value under a constant project mix, that is, when the firm does not change its portfolio of projects, hence its $\beta$.

For firms with an original $\beta > 1$ or similarly a $SCOR > V\sigma_M$, a reduction in the price of risk combined with the upward translation of the iso-value lines imply that firm value at the original optimal point $A_1$ is now on a higher iso-value line. A fortiori, when a firm optimally adjusts its project portfolio, its value increases even further, as illustrated in Figure 6. The new optimal point $A_2$ is then necessarily above the iso-value line $C_1^{new}$ indicating that for those firms, the increase in the risk-free rate causes an increase in firm value, both directly (at $A_1$) and indirectly by inducing changes in the mix of RAM activities that generate increases in both the expected cash flow and scaled correlation.

For firms with an original $\beta < 1$, as depicted in Figure 7, rotation of the iso-value lines due to a reduction in the price of risk combined with their upward translation by the amount $C_1^{new} - C_1$ imply that firm value at the original optimal point $A_1$ is now on a lower iso-value line. The new optimal point $A_2$, which represents a higher $SCOR$ and a higher $E$, may be above or below the iso-value line $C_1^{new}$ depending on the curvature of the transformation possibility frontier $H(E, SCOR)$. The illustration in Figure 7 corresponds to a case where an increase in the risk-free rate causes a drop in firm value, even after accounting for optimal adjustments in RRM and POM activities that generate increases in both expected cash flows and scaled correlation.

**Proposition 4:** An increase in the risk-free rate induces an increase in both the risk taken by a firm and its expected cash flows. Firm value increases if its original beta is larger than 1. When the original beta is less than 1, the direct effect is to lower firm value; a firm can alleviate and even reverse this direct effect by optimally adjusting its portfolio of projects.

To summarize, variations in the price of risk induced by changes in market expected return or volatility impact similarly, either positively or negatively, all firms in the economy (Propositions 2 and 3). This is not the case for a variation in the risk free. Following an increase in the risk free rate, firms with a market beta smaller than one (or alternatively, $SCOR < V\sigma_M$) could see their market
value increase or decrease, whereas firms with a market beta greater than one unambiguously see their market value increase. Firms that are originally relatively more risky benefit from an increase in the risk free rate, while the relatively less risky may suffer. This is true even if all firms move toward a riskier project mix (higher SCOR and higher $E(R_M)$). Financial risk management plays the same role, however, as with changes in $\sigma_M$ and $E(R_M)$.

5 Empirical Implications and Implementation issues

5.1 Predictions

Despite the arguably abstract nature of this financial and real risk management model, it conveys several empirical implications. We look at four testable implications, which are either associated with frontier effects or with the use of financial instruments, namely derivatives.

If a firm’s frontier is weakly concave, small changes in the market price of risk will induce relatively important changes in the optimal mix of projects. Firms in this situation are more likely to use financial risk management operations to facilitate coordination between their POM and RRM managers. At the other end of the spectrum, a firm with a strongly concave (or even kinked) possibility frontier will not have much use for financial operations since a change in the market price of risk will not have much impact on its optimal combination of SCOR and expected cash flows $E$. Because obtaining a consensus is more likely to be difficult when changes in the project mix are important, corporations with a very diverse potential project mix are more likely to gain by engaging in financial risk management. We are most probably talking here about conglomerates and multinational firms, as well as firms with significant growth options. A first prediction of our model is therefore that conglomerates and multinational firms, that have a more diverse project mix than single-industry single-country firms, as well as firms with significant growth options, will be heavier users of derivatives. Indeed, Geczy et al. (1997) find that firms with extensive foreign exchange-rate exposure (like multinational firms) are more important users of derivatives; He and Ng (1998) maintain the same in the case of conglomerates; and Nance et al. (1993) find that firms with significant growth options use more derivatives.

Our model has also predictions related to variations in the risk free rate (Figures 6 and 7). A rate increase will push firms towards adopting riskier project portfolios with higher expected cash flows. The corresponding impact on market value will differ across firms, depending on the beta
value of the initial project mix with respect to 1. Therefore, a second prediction of our model is that, following an increase in the risk free rate, all firms will take on a riskier project mix with higher expected cash flows. Moreover, firms with a $\beta$ larger than unity will see their value increase and firms with $\beta$ smaller than unity will see their value increase or decrease. A caveat is that this prediction deals with new equilibrium points, which may be reached more or less rapidly by different firms.

Our model suggests that firms facing more difficult coordination problems between their POM and RRM managers could benefit the most from financial risk management. If coordination is difficult to achieve, that is, if both objectives of POM managers (increasing expected cash flows) and of RRM managers (reducing risk) must be met before a project is undertaken, then it will be important for the financial risk manager to reshape the firm’s cash flows. By selling or purchasing zero-value financial contracts, a financial risk manager will give POM and RRM managers the necessary flexibility to implement the overall best value maximizing strategy for the corporation.

What factors would make coordination problems more likely and important? We see two: size and delegation. A third prediction of our model is therefore that larger corporations and corporations with a more delegated investment decision structure (more executives involved in project selection) will be greater users of financial instruments. Larger corporations are more likely faced with more challenging coordination problems simply because of their wider dispersion of real assets and extensive distribution of responsibilities. Indeed, Nance et al. (1997), Mian (1996) and Graham and Rogers (2002) have shown that financial risk management procedures and products, such as forwards, futures, swaps, and options, are more common in larger firms while almost inexistent in small firms. These empirical regularities contradict current theories, in which the value of financial risk management is based upon the reduction of the cost of financial distress, but are clearly compatible with the predictions of our model. Another test would be to compare corporations

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18 The value will increase if the marginal rate of substitution, the rate at which a firm can substitute POM and RRM activities while remaining on its efficient frontier, is relatively insensitive to the increase in risk.

19 See also the results from the Wharton-Chase survey (1995) and the Wharton-CIBC Wood Gundy survey (1996) as mentioned in Stulz (1996, page 9): “Whereas 65% of companies with a market value greater than $250 million reported using derivatives, only 13% of the firms with market values of $50 million or less claimed to use them.”

20 Stulz (1996) writes: “The primary emphasis of the [corporate risk management] theory is on the role of derivatives in reducing the variability of corporate cash flows and, in so doing, reducing various costs associated with financial distress. The actual corporate use of derivatives, however, does not seem to correspond closely to the theory. For one thing, large companies make far greater use of derivatives than small firms, even though small firms have more volatile cash flows, more restricted access to capital, and thus presumably more reason to buy protection against financial trouble. Perhaps more puzzling, however, is that many companies appear to be using [financial] risk management to pursue goals other than variance reduction.”
where the number of executives who have a say in project approval and selection is large with corporations where that number is small. Because financial risk management is more valuable for corporations that have major coordination problems, our model predicts also that firms with a larger number of executives involved in project selection will use more financial risk management techniques. We are not aware of any evidence to that respect.

Finally, our model leads to predictions regarding the use of financial hedging instruments by firms subject to regulated or self-imposed financial constraints, such as value-at-risk or cash flow-at-risk constraints. Indeed Stulz (1996) proposes “a somewhat different goal for corporate risk management – namely the elimination of costly lower-tail outcomes – that is, designed to reduce the expected costs of financial troubles while preserving a company’s ability to exploit any comparative advantage in risk-bearing it may have” (page 8, emphasis in original).21 We showed (Figure 4) that financial risk management could, through the use of zero-value contracts, allow firms to meet those constraints without sacrificing firm value. A fourth prediction of our model is therefore that, because they are typically subject to stringent financial constraints of the VaR and CaR types, firms in sectors such as financial services, energy, gambling, and utilities, will be among the heavier users of derivatives and other financial risk management instruments. The reason for this significant use of financial risk management procedures and products is clearly different from that proposed by Stulz. Growth in the securitization market suggests that firms in the financial services sector use financial instruments more than manufacturing firms.

5.2 Data and information acquisition

In deriving the transformation possibility frontier between the expected value of projects and their risk, we have assumed away technical or informational issues. Such issues could prevent a chief executive officer from implementing the necessary trade-offs. We will sketch below the main obstacles, in particular issues related to incomplete and asymmetric information, indivisibility and transaction costs.

A first obvious problem is the significant data collection implied by the dimension of the problem. Projects are numerous in a firm and obtaining the corresponding cash flows over time is no small task. The information collected is also likely to lack precision. Therefore, the frontier may be

21Stulz’s surprising proposal came in reaction to “… the popularity of a practice known as ‘selective’ as opposed to ‘full-cover’ hedging. […] Such a practice seems inconsistent with modern risk management theory.”
derived under imprecise and potentially incomplete information, and uncertainty will prevail as to its exact position. This uncertainty will directly affect determination of the optimal mix of production and risk management activities.

A parallel with mean-variance optimization in asset allocation will help us gauge the extent of the problem. It is well known in this literature that small changes in the assumed distribution of asset returns often imply large changes in the optimized portfolio. Many portfolios may be statistically as efficient as the ones on the efficient frontier. Several statistical solutions have been proposed to account for the variability of the efficient frontier (see Michaud, 1998) and to increase the stability of the optimal portfolio (Jagannathan and Ma, 2003). Beyond these statistical solutions, one can mitigate the uncertainty associated with a detailed computation of intertemporal cash flows by aggregating projects among various organizational units. This will make the problem of gathering data generally easier given the accounting system already in place and facilitate the optimization process.

Asymmetric information could also prevent a firm from attainng the project mix that maximizes its value. Adverse selection and moral hazard problems can impede the process of gathering information at every level of a firm’s hierarchy (see Williamson 1967). Managers may propose projects that have been selected on criteria other than maximizing firm value. The collection of projects from which the frontier is drawn may not, therefore, be the right one and the final mix of projects will be suboptimal. Solutions for these problems are the usual incentive schemes that will help elicit the right information.

Another important difficulty in drawing up a possibility frontier for a firm lies in the indivisibility of real assets. In portfolio theory with infinitely divisible financial assets, it is always possible to be arbitrarily close to the efficient point on the frontier. With real activities, some projects must be undertaken completely or not at all. A numerical search for the optimal mix of activities has to proceed differently, but it is still possible to arrive at a frontier. It will not have the smooth appearance that we drew in our graphs but it will keep its optimality property. Similarly, some constraints may be imposed on the minimal size of projects in deriving the optimal frontier.

Transaction costs may explain why a firm does not want to continuously change the optimal mix of projects. For example, premature termination of a project may involve penalties in terms of labor compensation or legal fees. A change in the optimal mix may also be postponed because of fixed costs associated with the disposal of fixed assets. Incorporating these transaction costs in
portfolio choice is an extremely difficult theoretical and computational issue. Only partial solutions with specific cost structures, often unrealistic, are available. Transaction costs associated with a change of policy are just one example of sunk or irreversible costs. When a project in under way, managers may induce some changes that will affect its future cash flows. Therefore, the flexibility given to managers to modify the course of projects over time, as new information becomes available, is another potentially important source of costs.

Given these considerations, it is clear that the level of organizational inertia \(^{22}\) will directly affect the capacity of a firm to adapt to market changes and ultimately its market value. Organizations that are characterized by a high degree of reliability and accountability, which are themselves rooted in standardization and routines, may lose value by their inability to rapidly solve the coordination problems we have illustrated earlier.

### 5.3 An Intertemporal Framework

Until now, we have sidestepped the problem of computing the present value of intertemporal cash flows by assuming a flat term structure and a constant risk measure over time. In our simple framework, the transformation possibility frontier did not change over time. In a more realistic setting where risk and return change over time, we need to compute at each point in time, say \(t\), an efficient frontier \(H_t(E_t, SCOR_t) = 0\), where \(E_t\) and \(SCOR_t\) group all the conditional expected values and scaled correlations. The extension to an intertemporal framework can be set in an Arrow-Debreu type economy or in a world with a general stochastic discount factor. In such intertemporal extensions, the price of risk and the price of time will play a role in the marginal trade-offs the firm will engage in, both across states of nature and periods.

In a recent paper, Alexander and Pézier (2003) propose a similar approach to aggregating market and credit risks in large complex financial firms. They identify risk factors that are common to various business activities and account for the dependencies between the different risk factors across business lines to evaluate the aggregate risk of the firm. They set up their factor model in terms of present value of profit and loss over the relevant time horizon. They never mention how to compute the present value but their model implies that discounting is done with the term structure of interest rates taken as given. A fully integrated intertemporal framework needs to treat time

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\(^{22}\)Hannan and Freeman (1984) develop such an evolutionary model of inertia in organizations. See also Boyer and Robert (2006) who develop a model that addresses the optimal level of inertia in an organization.
and risk together.

To be as general as possible, we need not specify a linear risk model. We can rely on the existence of a stochastic discount factor, say $m_{t,T}$, which gives the value in $t$ of a cash flow in $T$, in the absence of arbitrage opportunities. The value in $t$ of any project within a firm with associated cash flows $C_{t+1}, \cdots, C_T$ from $t + 1$ to $T$ is then given by:

$$P_t = E_t[m_{t,t+1}C_{t+1} + \cdots + m_{t,T}C_T]$$

(17)

By the covariance formula, this can be further decomposed as:

$$P_t = E_t[m_{t,t+1}]E_t[C_{t+1}] + \text{Cov}_t(m_{t,t+1}C_{t+1})$$

$$+ \cdots$$

$$+ E_t[m_{t,T}]E_t[C_T] + \text{Cov}_t(m_{t,T}C_T)$$

(18)

We can group these expressions into two distinct blocks, one for products of expectations, the other for covariances:

$$P_t = EV_t + COV_t$$

(19)

with:

$$EV_t = E_t[m_{t,t+1}]E_t[C_{t+1}] + \cdots + E_t[m_{t,T}]E_t[C_T]$$

(20)

$$COV_t = \text{Cov}_t(m_{t,t+1}C_{t+1}) + \cdots + \text{Cov}_t(m_{t,T}C_T)$$

The expectation terms $E_t[m_{t,T}]_{T=t+1}^T$ provide the prices of zero-coupon bonds for corresponding horizons $\tau = t + 1, \cdots, T$. An efficiency frontier can then be defined in terms of $(E_t, COV_t)$ as before, but now the frontier will change at each period depending on the evolution of the term structure of interest rates and of the risk measures embedded in the stochastic discount factors. Since all quantities have been discounted at time $t$ accounting for both the values of time and risk in cash flows over time and states of nature the iso-value lines will have a slope of one. Of course the analysis of the trade-offs between expected cash flows and risk or between different risks becomes
more involved but remains possible once a specific content is given to the stochastic discount factor through a model.

When the stochastic discount factor is modeled as in the CAPM or the multifactor model described in section 2, we saw that the trade-offs can be expressed between expected cash flows and scaled correlations. A separation of parameters leading to the use of scaled correlations cannot be obtained with the general specification in (17). More structure should be put in the stochastic discount factor to arrive at a similar separation. For example, one can extend the factor model described earlier to a dynamic factor model where the betas will change over time, assuming, for simplicity, that the term structure of interest rates is flat. Different dynamic economic models will imply different frontiers but each firm will likely identify the main risk factors over which it wants to assess the riskiness of its future cash flows.

Another dynamic consideration relates to the use of real options in the estimation of a firm’s transformation frontier. Management of real options is a process aimed at actively reducing exposition to downside risk and promoting exposition to upside opportunities. From the real options viewpoint, projects are not simply sequences of uncertain cash flows over time but rather sets of compound American options. Indeed, managing risks and more generally creating value in a firm requires the identification, creation and management of the real options embedded in the portfolio of potential projects, which define a firm as an economic entity. The contribution of higher level managers to the value of a firm may lie mainly in creating and exercising real options.23

6 Conclusion

The current theoretical and empirical literature on risk management does not integrate the risk management function within the boundaries of a corporation. Instead risk management, and especially financial risk management, is viewed as a special purpose function rather than an integral part of decision making. We deviate from this traditional perception by incorporating risk management in a firm’s technology that allows to transform cash flows between states of nature and

23 As pointed out by The Economist (August 12, 1999): “To evaluate potential projects, [firms] almost invariably have to resort to a theory of corporate finance called the ‘Capital Asset Pricing Model’ (CAPM). Yet real-life managers tend not to like this model, for the simple reason that it ignores the value of real-life managers. So they might welcome some recent academic work. In the ivory tower, they are talking about ditching the CAPM for a rival, called ‘real options theory’. Managers are therefore placed at the very core of the use of real options. More fundamentally, the flaw in the CAPM is that it implicitly assumes that when firms buy new assets, they hold these passively for the life of the project.”
To do so, we define an efficient frontier in terms of expected cash flows ($E$) and risk as measured by the scaled correlation ($SCOR$) between cash flows and market returns, making it possible to more explicitly study the role and impact of parameters affecting the market price of risk. Moreover, we are able to characterize the relative impact of each factor underlying the market price of risk on a firm’s project mix related to operations and real risk management. In so doing, we present real risk management as a fundamental function of a firm. Like any other function in a corporation, its goal is to contribute to the maximization of firm value within the feasible set of cash flow transformation possibilities. In other words, real risk management is an input in a firm’s value function, just as labor and capital are inputs in the production function. New insights on the value of real risk management are obtained.

The approach we adopted to study risk management emphasized two important sets of considerations within a firm: first, the relative levels of independence and coordination that must take place between a firm’s real risk management and operations management units; and second, the search for efficiency (being on the frontier of possibilities) and optimality (being at the right point on that frontier) that determines the observed values of expected cash flows ($E$) and scaled correlation ($SCOR$). Both the production and operations management unit and the real risk management unit seek to increase firm value, but they each have power only over a subset of possible tools. As such, reaching the possibility frontier and finding the best possible point on the frontier given the market conditions are two different programs in maximizing firm value. The role of financial risk management is fundamentally to facilitate the coordination between the real risk management function and the production and operations management function in a context where real asset management activities are decentralized. Although financial risk management transactions on financial markets generate no value directly (they are zero-NPV transactions), they do generate flexibility and therefore contribute indirectly to the value of a firm. This is a new and important role for financial risk management, which is present even in a world without taxes, bankruptcy or financial distress costs, or agency problems. Access to micro data sets on firms could lead to the estimation of risk-reward frontiers, that is, frontiers expressed in terms of risk ($SCOR$) and expected cash flows. Our approach also has several empirical implications, some direct, some indirect, that can be tested based on accounting and market data for firms, aggregate market data and data on the use of financial instruments to hedge risk.
References


FIGURE 0. Production and operation management (POM), real risk management (RRM) and financial risk management (FRM) in the firm.
FIGURE 1. Efficient frontier given the portfolio of projects available to the firm.

FIGURE 2. Efficient frontier and value maximization of the firm given the price of risk.
FIGURE 3a. Coordination problems.

FIGURE 3b. Value of financial risk management.
FIGURE 4. The value of using financial instruments for a firm constrained by CaR requirements.

FIGURE 5. Impact of an increase in $\sigma_M$ or a decrease in $E(R_M)$. 
FIGURE 6. Change in the project mix following a change in the risk free rate when $\beta > 1$.

FIGURE 7. Change in the project mix following a change in the risk free rate when $\beta < 1$. 