What Can Rational Investors Do About Excessive Volatility and Sentiment Fluctuations?

Bernard Dumas
INSEAD, Wharton, CEPR, NBER

Alexander Kurshev
London Business School

Raman Uppal
London Business School, CEPR

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Our objective

- Agents in financial markets claimed to exhibit behavior that deviates from rationality – **overconfidence** leading to "**excessively volatility**"
- Suppose a Bayesian, intertemporally optimizing investor ("**smart money**") operates in this financial market:
- We wish to understand:
  1. What **investment strategy** this investor will undertake?
  2. What effect this strategy will have on **equilibrium prices**?
  3. Whether this will ultimately eradicate the source of **excess volatility**?
- We do this by building an **equilibrium model of investor sentiment**.

What we do: Contribution

1. **Model**: Equilibrium of financial market with two populations:
   - Bayesian (rational) learners; Imperfect (irrational) Bayesian learners
   - Extend model in Scheinkman and Xiong (2004)
     (**general equilibrium, risk averse agents, shortsales allowed**)
2. **Effect on prices, volatility and correlation**
   - A few rational investors are not enough to eliminate the effect of irrational traders
3. **Optimal portfolios**
   - Profit from predictability, but more sophistication is needed
4. **Survival of irrational traders** (Kogan-Ross-Wang-Westerfield; Yan)
   - Their rate of impoverishment is quite slow

Model: Output and information structure

- **Exogenous process for aggregate output**
  - Output uncertainty: first source of risk (δ shock)

$$\frac{d\delta_t}{\delta_t} = f_t dt + \sigma_\delta dZ_t^\delta,$$

  - Expected value of rate of growth of dividends f is stochastic

$$df_t = -\zeta\left(f_t - \overline{f}\right)dt + \sigma_f dZ_t^f; \quad \zeta > 0,$$

- **Expected growth rate is not observed** by any investor; investors continuously form (filter) estimates of it, based on δ and a signal s:

$$ds_t = f_t dt + \sigma_s dZ_t^s,$$

Population A is deluded

- **Group A: Irrational traders**
  - They believe steadfastly that
    - $\star$ innovations in signal have correlation $\phi \geq 0$ with innovations in f, when, in fact, true correlation is **zero**

$$ds_t = f_t dt + \sigma_s \phi dZ_t^f + \sigma_s \sqrt{1 - \phi^2} dZ_t^s.$$

  - They overreact to signal and cause excess volatility in stock market
  - Otherwise, behave optimally
  - Degree of irrationality captured by a single parameter: ϕ
- **Group B: Rational traders** ("smart money").

Result of filtering (in terms of B's Wiener)

$$d\widehat{f}_t^A = \left[-\zeta\left(\widehat{f}^A - \overline{f}\right) + \left(\frac{\gamma^A}{\sigma_\delta^2} + \frac{\phi\sigma_s\sigma_f + \gamma^A}{\sigma_s^2}\right)\left(\widehat{f}_t^B - \widehat{f}^A\right)\right] dt + \frac{\gamma^A}{\sigma_\delta^2}\sigma_\delta dW_{\delta,t}^B + \frac{\phi\sigma_s\sigma_f + \gamma^A}{\sigma_s^2}\sigma_s dW_{s,t}^B$$

$$d\widehat{f}_t^B = -\zeta\left(\widehat{f}^B - \overline{f}\right) dt + \frac{\gamma^B}{\sigma_\delta} dW_{\delta,t}^B + \frac{\gamma^B}{\sigma_s} dW_{s,t}^B.$$

- Group A is called "**overconfident**" because the steady-state variance of f as estimated by Group A, γ^A, decreases as ϕ rises.
- Group A has more **volatile beliefs** than Group B because conditional variance of $\widehat{f}^A$ monotonically increasing in ϕ.
- **Difference of opinion**: $\widehat{g} \triangleq \widehat{f}^B - \widehat{f}^A$
  So, $\widehat{g} > 0$ implies Group B **relatively optimistic** compared to Group A.

Sentiment

- **Change from B to A's probability measure** given by η:

$$\frac{d\eta_t}{\eta_t} = -\widehat{g}\left(\frac{dW^B_{\delta,t}}{\sigma_\delta} + \frac{dW^B_{s,t}}{\sigma_s}\right).$$

- η is a measure of **sentiment** – shows how Group A over- or under-estimates the probability of a state relative to Group B.

- Girsanov's theorem tells how current disagreement gets encoded into η:
  - For instance, if A is currently comparatively optimistic ($\widehat{f}^A > \widehat{f}^B$), Group A views positive innovations in δ as more probable than B.
  - This is coded by Girsanov as positive innovations in η for those states of nature where δ has positive innovations.

Diffusion matrix of state variables

- Four state variables $\left\{\delta, \eta, \widehat{f}^B, \widehat{g}\right\}$.
  Driven by only two Brownians, W_δ^B and W_s^B because f is unobserved.

$$
\begin{array}{r}
\delta \cdots \\
\eta \cdots \\
\widehat{f}^B \cdots \\
\widehat{g} \cdots
\end{array}
\begin{bmatrix}
\delta\sigma_\delta > 0 & 0 \\
-\eta\frac{\widehat{g}}{\sigma_\delta} & -\eta\frac{\widehat{g}}{\sigma_s} \\
\frac{\gamma^B}{\sigma_\delta} > 0 & \frac{\gamma^B}{\sigma_s} > 0 \\
\frac{\gamma^B - \gamma^A}{\sigma_\delta} \geq 0 & \frac{\gamma^B - \left(\phi\sigma_s\sigma_f + \gamma^A\right)}{\sigma_s} \leq 0
\end{bmatrix}.
$$

- **Two distinct effects of imperfect learning:**
  1. Instantaneous: $\widehat{g}$ has nonzero diffusion, so disagreement is stochastic.
  2. Cumulative: $\widehat{g}$ affects diffusion of η, so disagreement drives sentiment.

Objective functions

- Market is assumed complete; use static formulation of dynamic problem
- Problem of Group B:

$$\sup_c \mathbb{E}^B \int_0^\infty e^{-\rho t} \frac{1}{\alpha} \left(c_t^B\right)^\alpha dt,$$

subject to the static budget constraint:

$$\mathbb{E}^B \int_0^\infty \xi_t^B c_t^B dt = \overline{\theta}^B \mathbb{E}^B \int_0^\infty \xi_t^B \delta_t dt,$$

- Group A's problem under B's measure

$$\sup_c \mathbb{E}^B \int_0^\infty \eta_t \times e^{-\rho t} \frac{1}{\alpha} \left(c_t^A\right)^\alpha dt,$$

subject to the static budget constraint:

$$\mathbb{E}^B \int_0^\infty \xi_t^B c_t^A dt = \overline{\theta}^A \mathbb{E}^B \int_0^\infty \xi_t^B \delta_t dt.$$

Complete-market equilibrium

- **Definition**: An equilibrium is a price system and a pair of consumption-portfolio processes such that
  1. investors choose their optimal consumption-portfolio strategies, given their perceived price processes;
  2. the perceived security price processes are consistent across investors;
  3. commodity and securities markets clear.

- The aggregate resource constraint is:

$$\delta_t = c_t^A + c_t^B$$

$$\delta_t = \left(\frac{\lambda^A \xi_t^B e^{\rho t}}{\eta_t}\right)^{\frac{1}{\alpha-1}} + \left(\lambda^B \xi_t^B e^{\rho t}\right)^{\frac{1}{\alpha-1}}.$$

Pricing measure and consumption-sharing rule

$$\xi_t^B = e^{-\rho t}\delta_t^{\alpha-1}\overbrace{\left[\left(\frac{\eta_t}{\lambda^A}\right)^{\frac{1}{1-\alpha}}+\left(\frac{1}{\lambda^B}\right)^{\frac{1}{1-\alpha}}\right]^{1-\alpha}}^{\text{power average of beliefs}}$$

$$c_t^A = \delta_t \times \omega(\eta_t) \qquad c_t^B = \delta_t \times (1-\omega(\eta_t))$$

$$\omega(\eta_t) \triangleq \underbrace{\frac{\left(\frac{\eta_t}{\lambda^A}\right)^{\frac{1}{1-\alpha}}}{\left(\frac{\eta_t}{\lambda^A}\right)^{\frac{1}{1-\alpha}}+\left(\frac{1}{\lambda^B}\right)^{\frac{1}{1-\alpha}}}}_{\text{absolute risk tolerance of } A \text{ to total absolute risk tolerance}}$$

- **Linear** consumption-sharing rule because same degree of risk aversion.
- **Stochastic slope** because of the improper use of signal by Group A.

Solving for equilibrium

- Can solve for pricing measure and consumption as a function of δ_t, and current value of change of measure, η_t.

$$\xi_t^i = \delta_0^{\alpha-1} \exp\left(-\int_0^t r dt - \frac{1}{2}\int_0^t \left\|\kappa^i\right\|^2 dt - \int_0^t \left(\kappa^i\right)^\intercal dW^i\right).$$

- Given the constant multipliers λ^A and λ^B, and given exogenous process for δ and η, we have now characterized the complete-market equilibrium.

- To relate the Lagrange multipliers λ^A and λ^B to initial endowments. requires the calculation of the wealth of each group.

Securities markets implementation of complete-market equilibrium

- Financial securities available:
  1. Equity, which is a claim on total output
  2. Consol bond
  3. Instantaneously riskless bank deposit

- The equilibrium price of a security, with payoff $\in \{1,\ \delta_u,\ c_u^B\}$:

$$\text{Price}\left(\delta, \eta, \widehat{f}^B, \widehat{g}, t\right) \triangleq \mathbb{E}^B_{\delta,\eta,\widehat{f}^B,\widehat{g}} \int_t^\infty \frac{\xi_u^B}{\xi_t^B} \times \text{payoff}\, du.$$

Computing expected values to obtain prices and wealth

- To compute equity and bond prices and wealth, need the **joint conditional distribution** of η_u and δ_u, given $\delta_t, \eta_t, \widehat{f}_t^A, \widehat{g}_t$ at t.

- Not easy to obtain joint distribution but its **characteristic function** $\mathbb{E}^B_{\widehat{f}^B,\widehat{g}}\left[\left(\frac{\delta_u}{\delta}\right)^{\varepsilon}\left(\frac{\eta_u}{\eta}\right)^{\chi}\right]; \varepsilon, \chi \in \mathbb{C}$ can be obtained in closed form.

- Three effects:

  1. Effect of growth and variance of δ

  2. Effect of variance of η ($\varepsilon = 0$)

  3. Effect of correlation between δ and η

Results
The interest rate

- **Average belief**

$$\widehat{f}^M \triangleq \widehat{f}^A \times \omega(\eta) + \widehat{f}^B \times (1 - \omega(\eta)) .$$

- Holding $\widehat{f}^M$ fixed, $\widehat{g}$ represents the effect of pure **dispersion of beliefs**

- The **rate of interest** can then be written as:

$$\begin{aligned} r\left(\eta, \widehat{f}^M, \widehat{g}\right) &= \rho + (1-\alpha)\, \widehat{f}^M - \frac{1}{2}(1-\alpha)(2-\alpha)\,\sigma_\delta^2 \\ &\quad - \frac{1}{2}\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{1}{\sigma_\delta^2} + \frac{1}{\sigma_s^2}\right)\widehat{g}^2 \times \omega(\eta) \times [1 - \omega(\eta)] . \end{aligned}$$

- The interest rate is **increasing** in $\widehat{f}^M$ (for all α) and $\widehat{g}$ (for $\alpha < 0$).

Market Prices of Risk

- The market prices of risk in the eyes of Population B and A are:

$$\kappa^{B}(\eta,\widehat{g}) = \begin{bmatrix} (1-\alpha)\,\sigma_\delta \\ 0 \end{bmatrix} + \widehat{g} \times \omega(\eta) \times \begin{bmatrix} \frac{1}{\sigma_\delta} \\ \frac{1}{\sigma_s} \end{bmatrix},$$

$$\kappa^{A}(\eta,\widehat{g}) = \begin{bmatrix} (1-\alpha)\,\sigma_\delta \\ 0 \end{bmatrix} - \widehat{g} \times [1-\omega(\eta)] \times \begin{bmatrix} \frac{1}{\sigma_\delta} \\ \frac{1}{\sigma_s} \end{bmatrix}.$$

- Under agreement ($\widehat{g} = 0$), the prices of risk include a reward for output risk W_δ, but zero reward for signal risk W_s.

- With disagreement, investors realize that probability measure of other population will fluctuate randomly. Hence, require a risk premium for vagaries of others.

Benchmark Parameter Values

- The parameter values that we specify are based on estimation of models similar to ours in Brennan-Xia (2001).

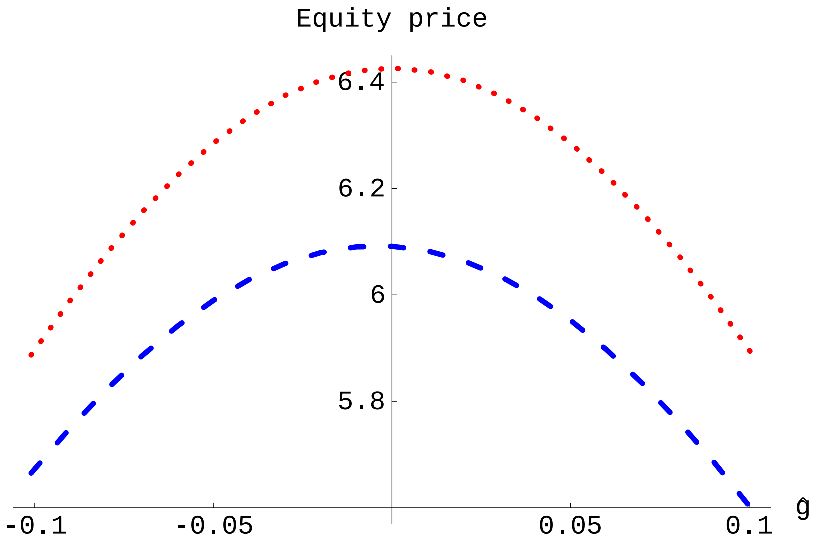
Name	Symbol	Value
Parameters for aggregate endowment and the signal		
Long-term average growth rate of aggregate endowment	$\bar{f}$	0.015
Volatility of expected growth rate of endowment	σ_f	0.03
Volatility of aggregate endowment	σ_δ	0.13
Mean reversion parameter	ζ	0.2
Volatility of the signal	σ_s	0.13
Parameters for the agents		
Agent A's correlation between signal and mean growth rate	ϕ	0.95
Agent B's correlation between signal and mean growth rate	—	0
Agent A's initial share of aggregate endowment	λ^B/λ^A	1
Time-preference parameter for both agents	ρ	0.20
Relative risk aversion for both agents	$1-\alpha$	3

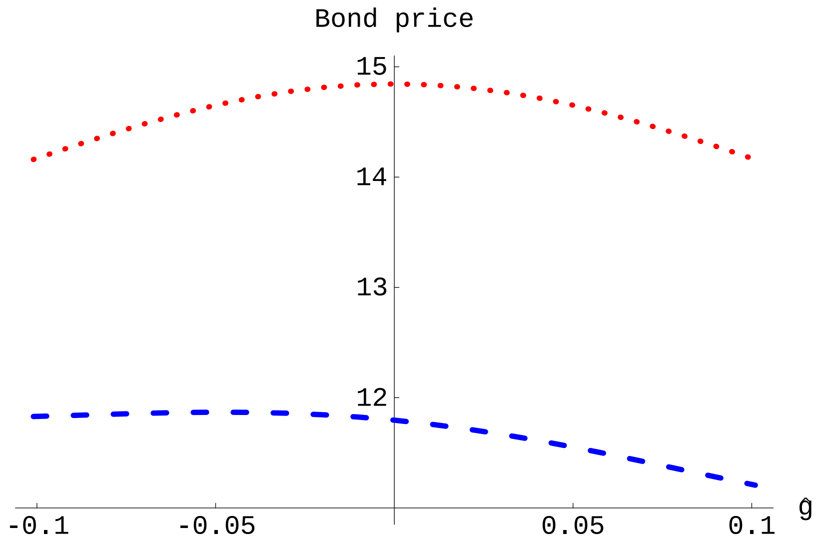
Plots

- All plots have on the x-axis
  - Either $\widehat{g}$ measuring **disagreement**.
  - Or, ω measuring **relative size** of irrational group.

- All plots have two curves for rationality and **irrationality**:
  - A red-dotted curve representing the case of $\phi = 0.00$
  - A blue-dashed curve representing the case of $\phi = 0.95$

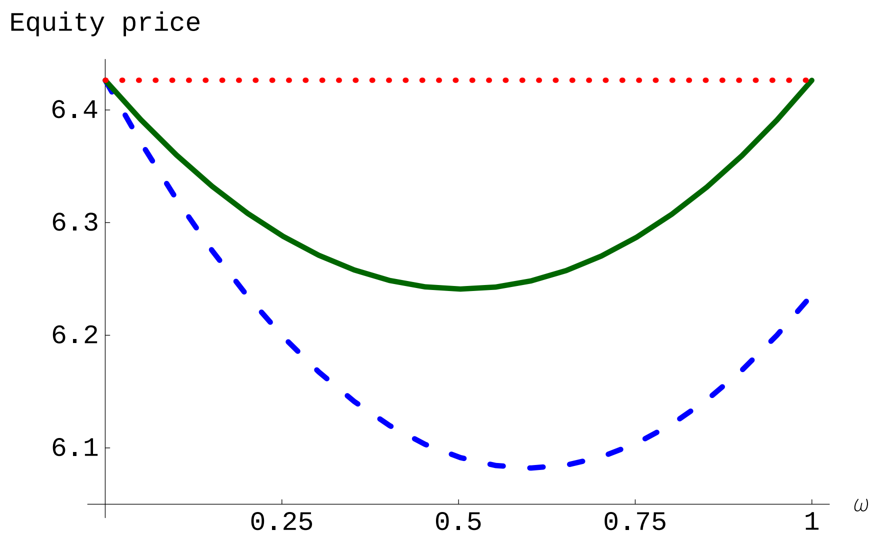
Prices: Effect of irrationality and disagreement

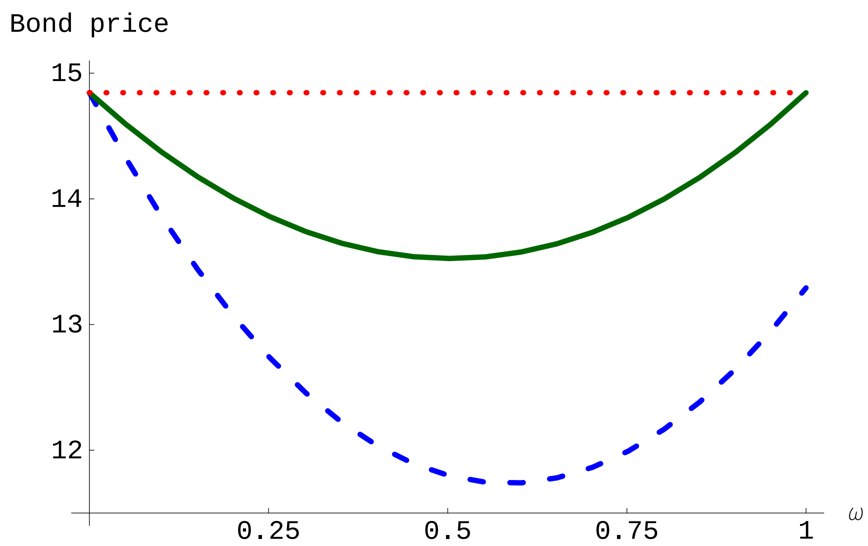




- Irrationality leads to a drop in prices of equity and bonds.
- Prices decrease with disagreement.

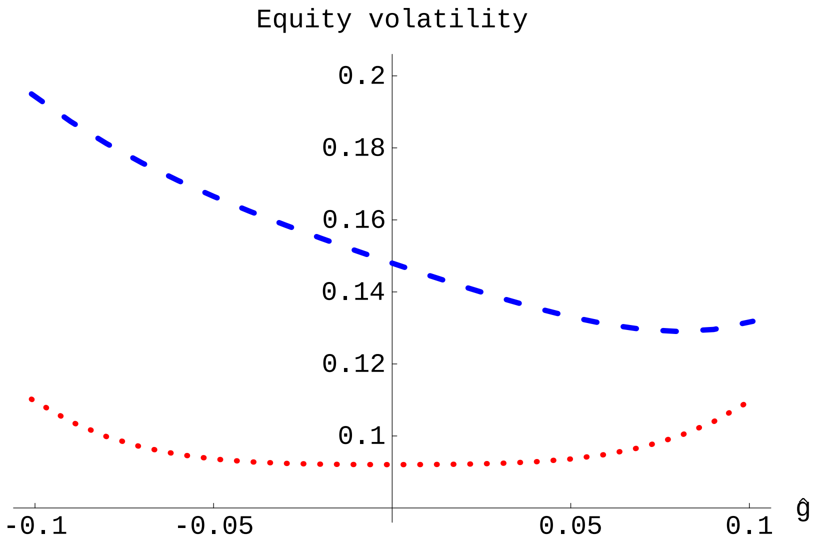
Prices: Effect of heterogeneity

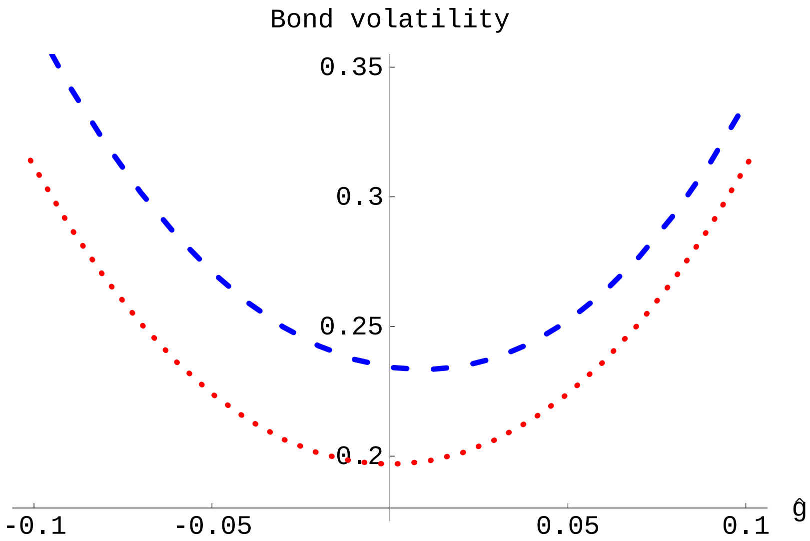




- Even modest population of irrational traders makes sizable difference.
- Heterogeneity increases further the drop in prices.

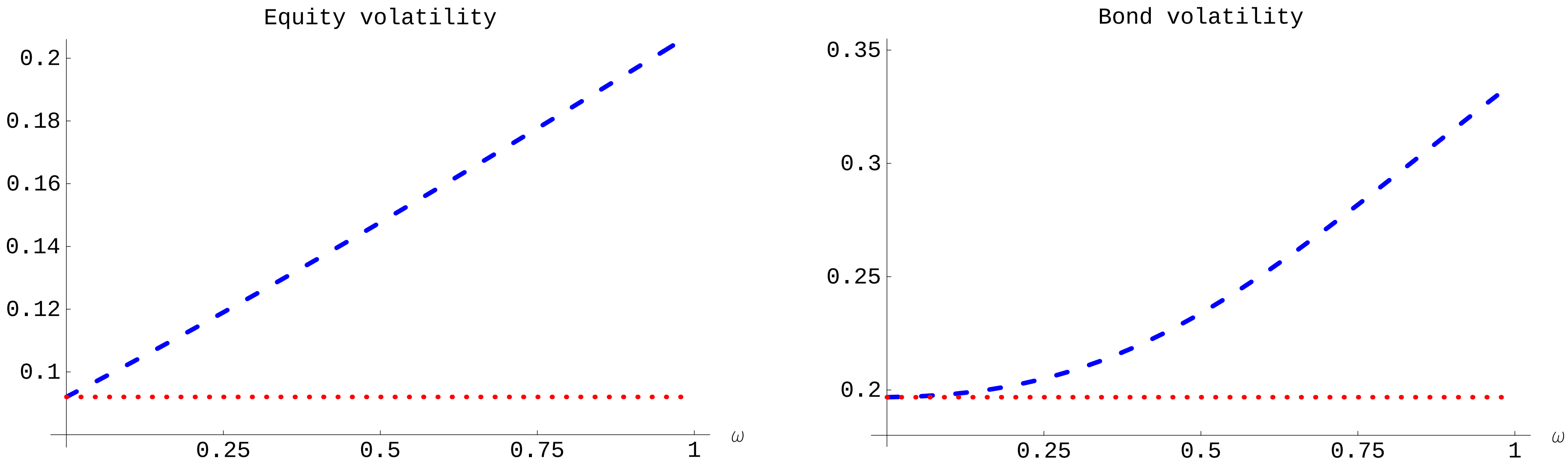
Volatilities : Effect of irrationality and disagreement





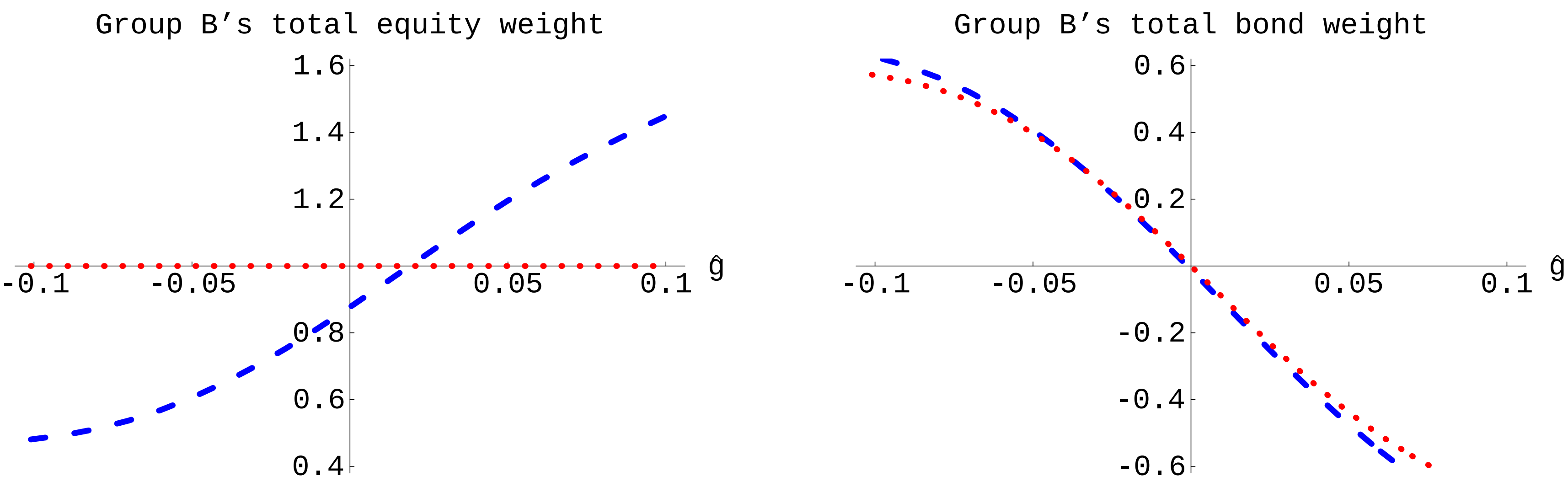
- Dispersion of beliefs and presence of irrational traders increase volatility (same is true for correlation)

Volatilities : Effect of heterogeneity



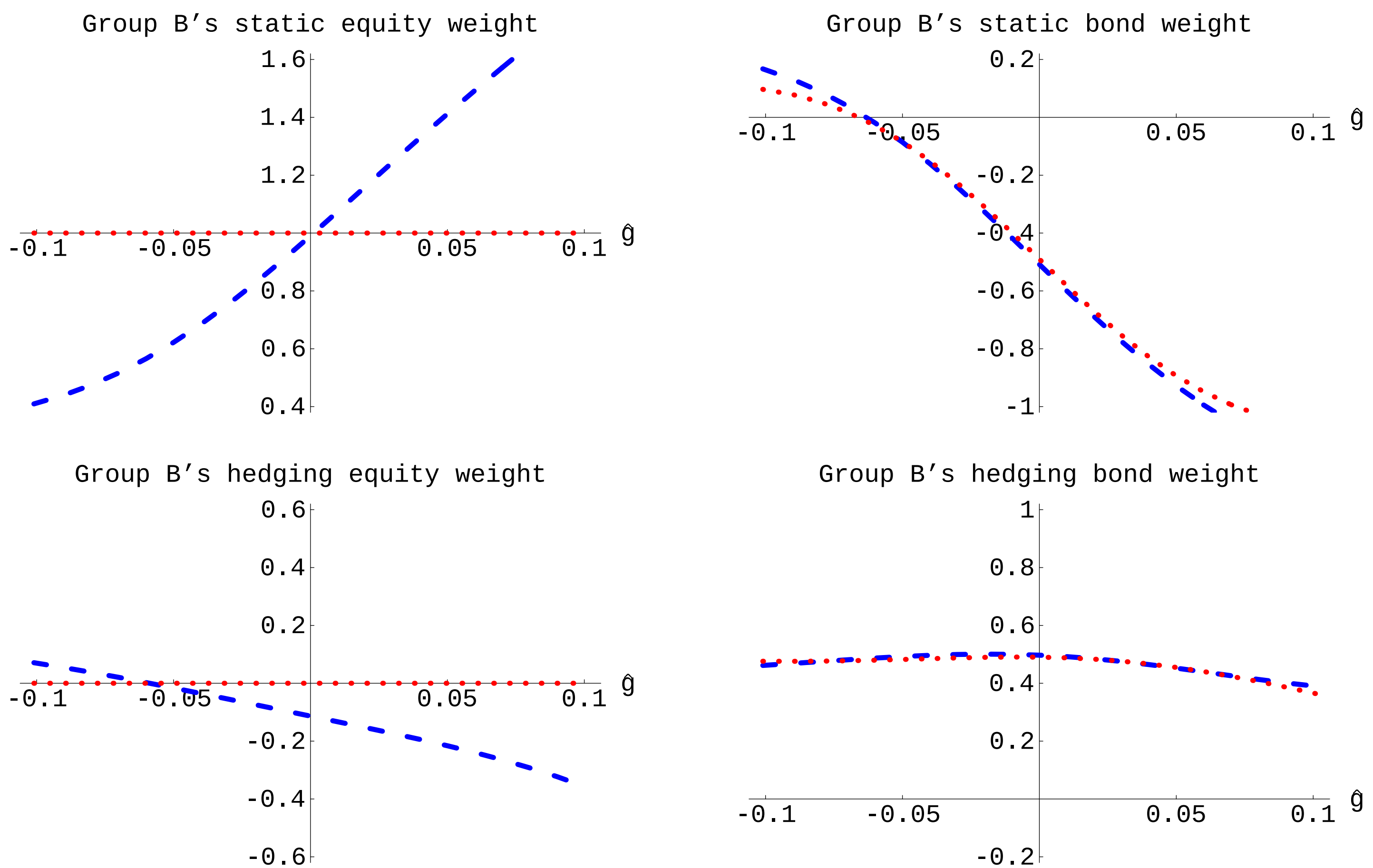
- Presence of a few rational investors not sufficient to drive down volatility.

Portfolio of Group B: Total



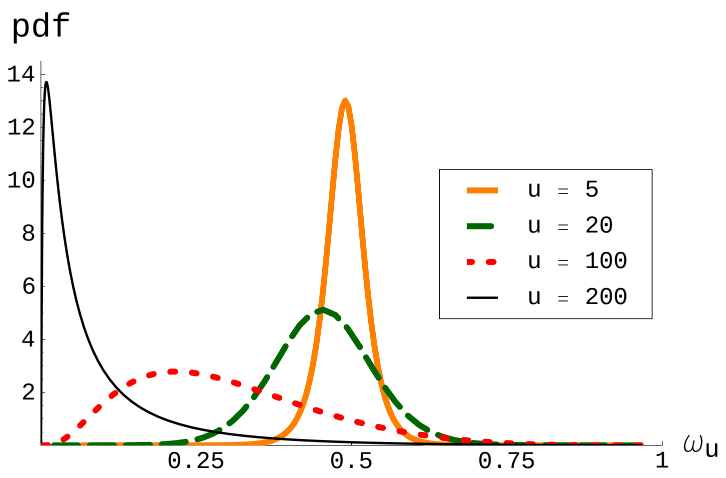
- If rationality ($\phi = 0$) and agreement ($\widehat{g} = 0$): 100% in equity, 0% in bonds because both investors identical
- If rationality but $\widehat{g} \neq 0$, B still 100% in equity and speculates on future growth with only bond
- Under irrationality, B holds **less equity** than he/she would in a rational market, (unless wildly optimistic). Scared of noise.

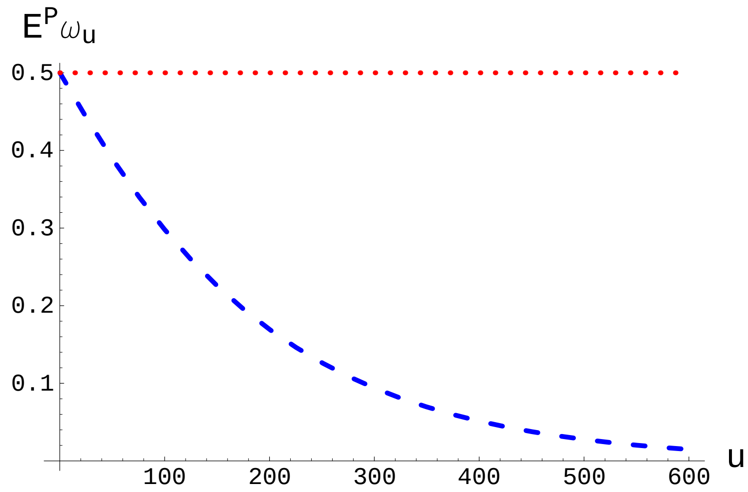
Portfolio of Group B: Static and Intertemporal Hedging



- Intertemporal hedge driven mostly by desire to hedge $\widehat{g}$ fluctuations

Survival of Population A—Irrational agents





- This figure shows expected value of Population A's consumption share as a function of time measured in years.
- This is survival of traders who are fickle: sometimes overoptimistic, sometimes overpessimistic

Conclusions

- We have modeled excessive volatility arising from
  - excessive fluctuations of anticipations of irrational investors
- Even a modest-sized irrational population makes quite a difference
- What rational investor can do:
  - Take positions on current differences in beliefs
  - Hedge against future revisions in:
    - Market's beliefs
    - Their own beliefs
  - Bonds are useful instruments in doing so
- Irrational traders survive a long time
  - Excessive volatility is not easy to "arbitrage"
  - Excessive volatility, if it is there, is likely to remain