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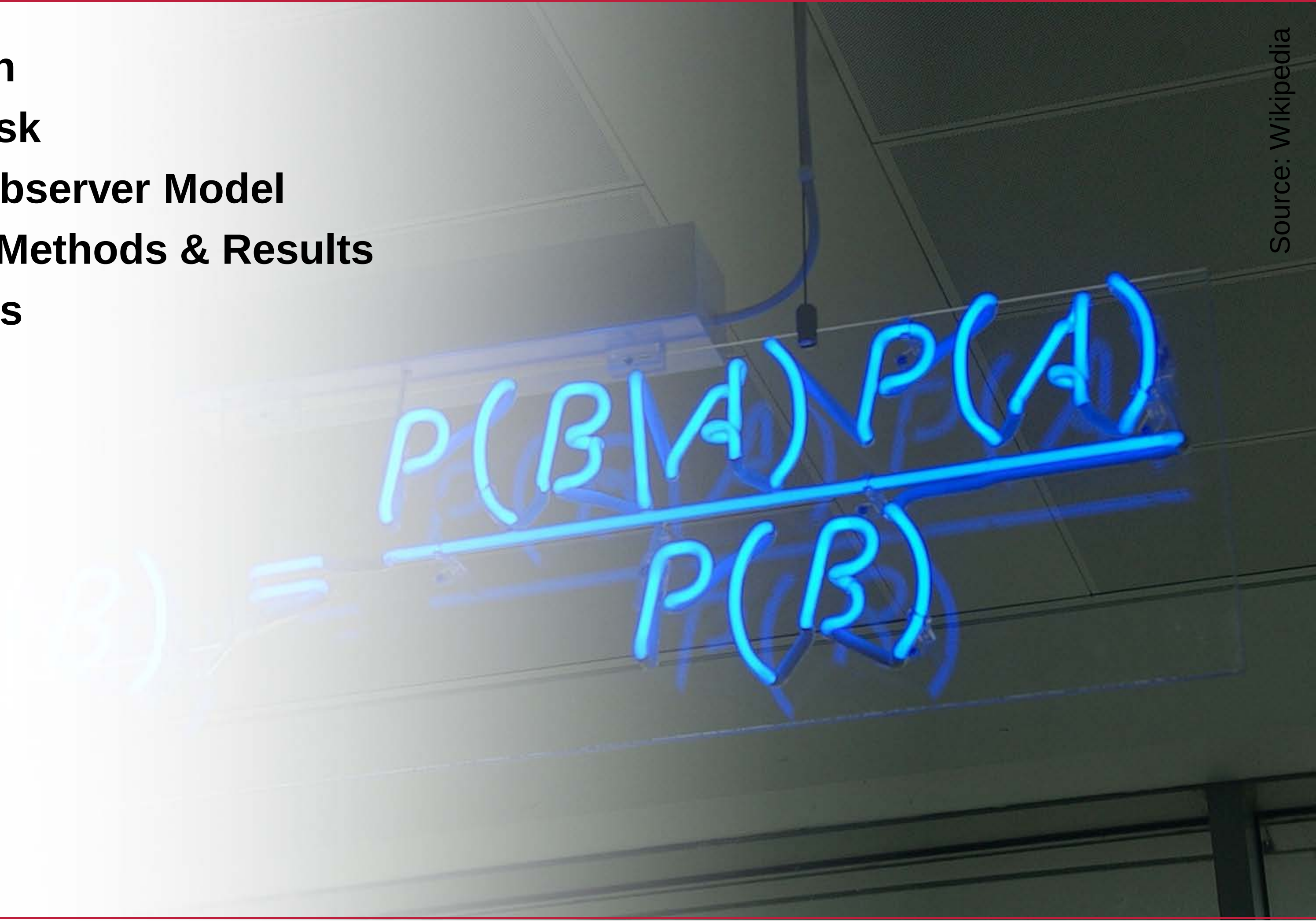
Institut für Nachrichtentechnik

A Computational Analysis of the Bayesian Brain

Antonio Kolossa, Bruno Kopp, Tim Fingscheidt, 10.09.2015

Outline

1. Introduction
2. Urn-Ball Task
3. Bayesian Observer Model
4. Evaluation Methods & Results
5. Conclusions



Source: Wikipedia

1 Introduction
The Bayesian Brain Hypothesis

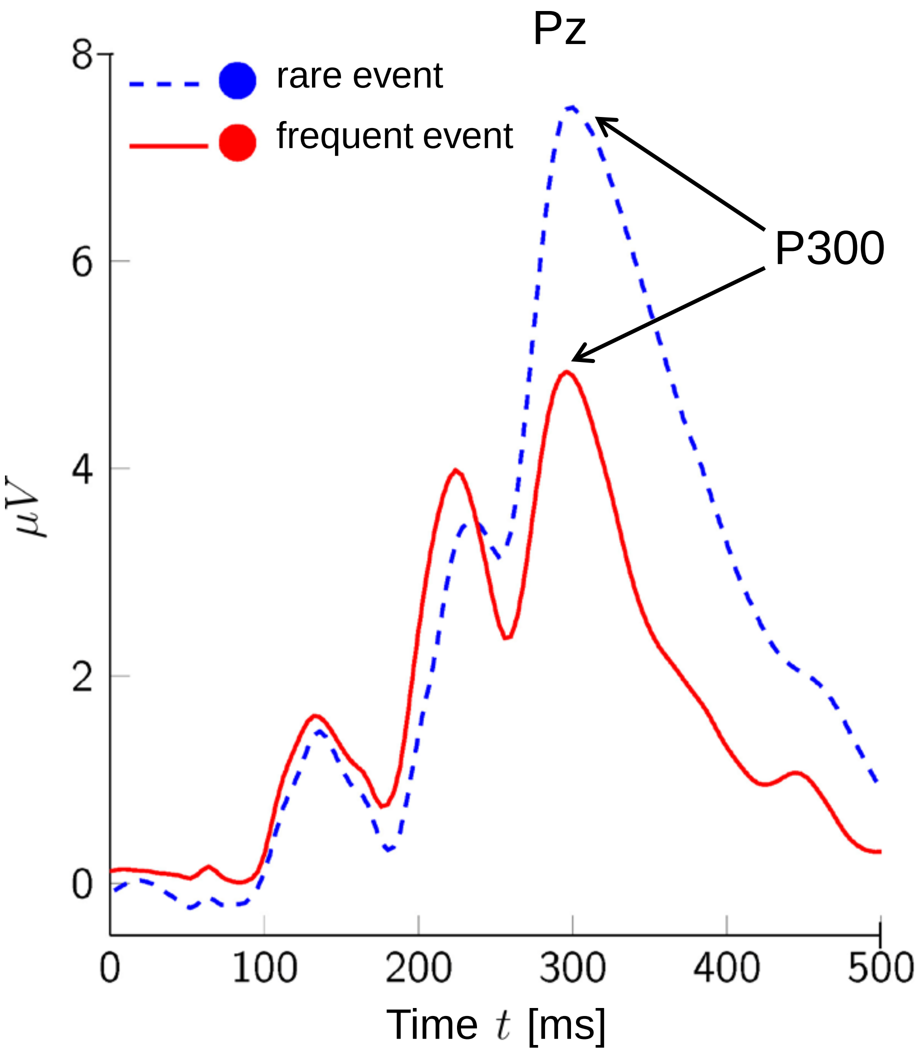
The brain integrates information in a Bayes-optimal manner:

- Only few empirical studies
- Underlying neural processes unknown

Event-related potentials (ERPs) represent neural computations (e.g., the P300 represents a cortical surprise response)

Processing theories of ERPs:

- Context updating model
- Free-energy principle



1 Introduction
The Bayesian Brain and the P300

Context updating model [1]:

- P300 reflects the updating of an internal model of the environment
- Deployment of attention, setting of priorities, assigning probabilities to observations
- Purely conceptual model

Free-energy principle [2]:

- P300 reflects inference and learning about the causes of observations
- Minimization of surprise over future observations
- Inference is well-formulated in statistical terms

[1] Donchin E, Coles MG (1988) Is the P300 component a manifestation of context updating? *Behavioral and Brain Sciences* 11:357–427.

[2] Friston KJ (2005) A theory of cortical responses. *Philosophical Transactions of the Royal Society B: Biological Sciences* 360:815–836.

1 Introduction
The Present Study

Goal:

Computational modeling of trial-by-trial P300 amplitude fluctuations in order to test the Bayesian brain hypothesis

Means:

- Urn-ball task which is directly related to Bayes‘ theorem (Section 2)
- Bayesian observer model (Section 3)
- Bayesian updating and predictive suprise as response functions (Section 3)
- Probability weighting (group level and individual-subject level) (Section 3)
- Bayesian model selection (Section 4)

Kolossa A, Kopp B, Fingscheidt T (2015) A computational analysis of the neural basis of Bayesian inference. *NeuroImage* 106:222–237.

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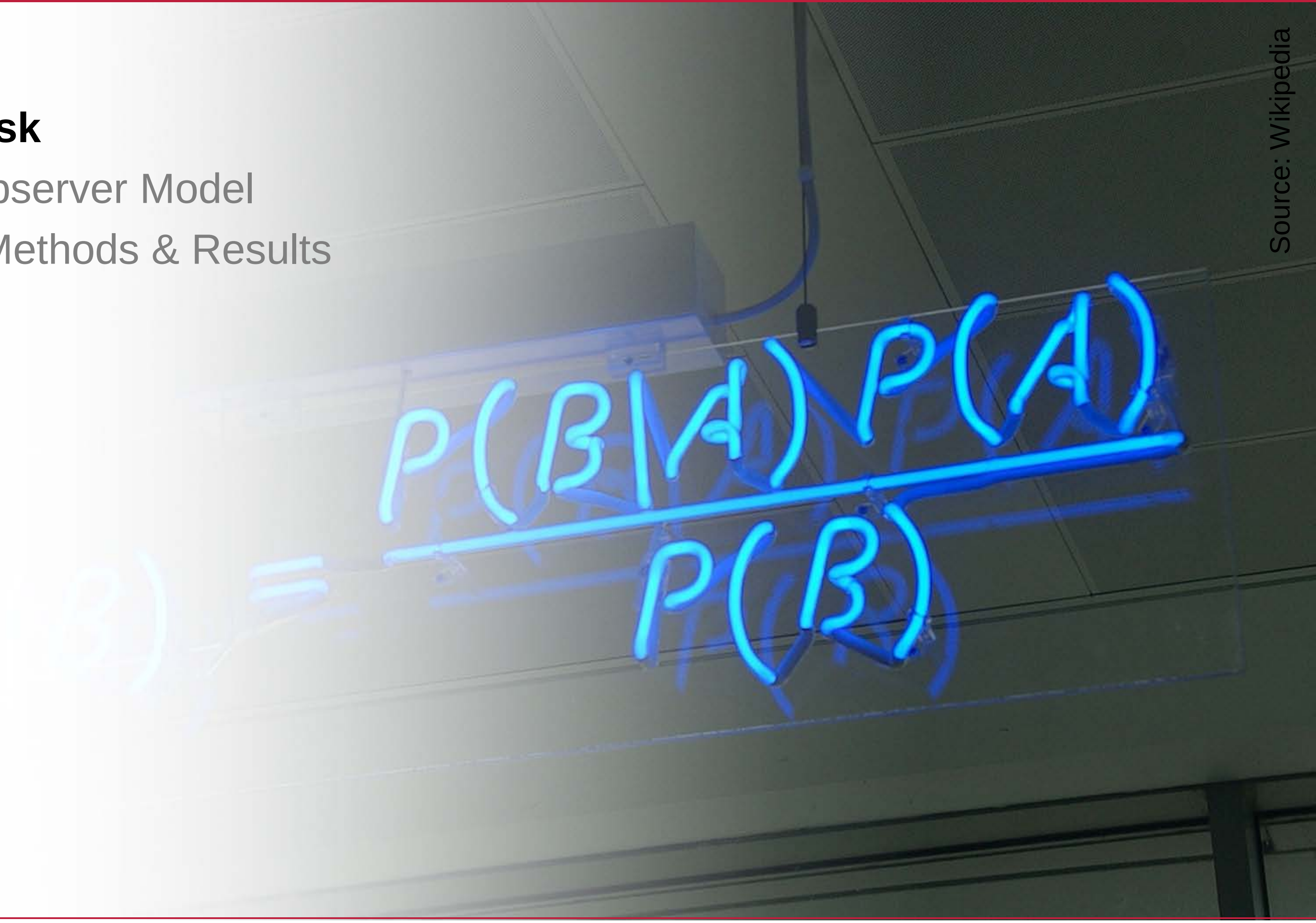


Source: Wikipedia

2 Urn-Ball Task
Bayes' Theorem

Bayes' formula $$\mathrm{P}(u|b) = \frac{\mathrm{P}(b|u)\mathrm{P}(u)}{\mathrm{P}(b)}$$

$u \in \mathcal{U}$ hypotheses
$b \in \mathcal{B}$ observation

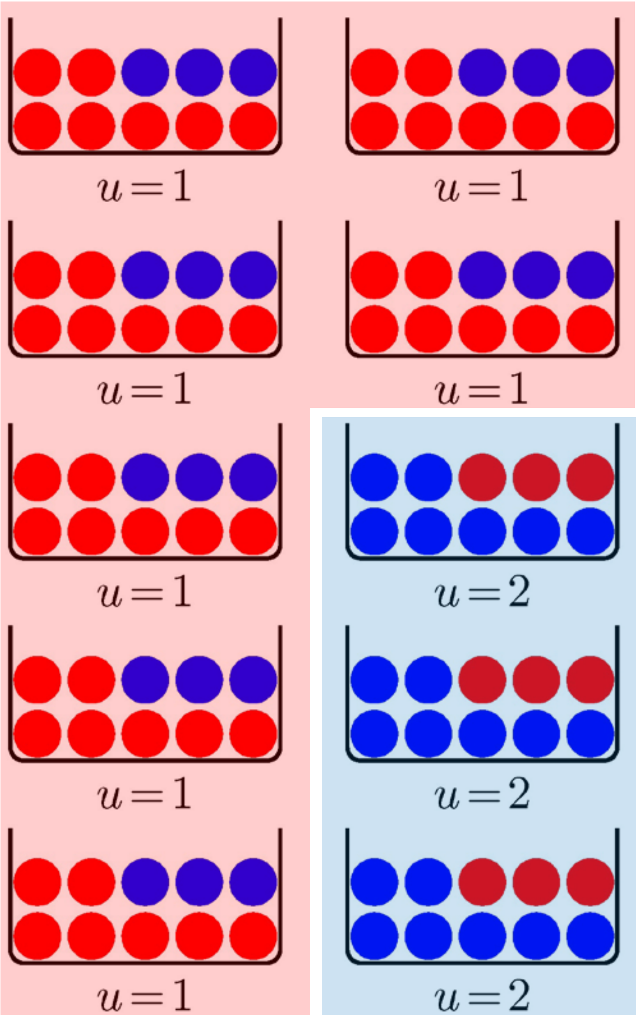
$\mathrm{P}(u)$ **Prior:** How probable was the hypothesis before the observation?
$\mathrm{P}(b|u)$ **Likelihood:** How probable is the observation given a hypothesis?
$\mathrm{P}(b)$ **Evidence:** How probable is the observation under all possible hypotheses?
$\mathrm{P}(u|b)$ **Posterior:** How probable is the hypothesis after the observation?

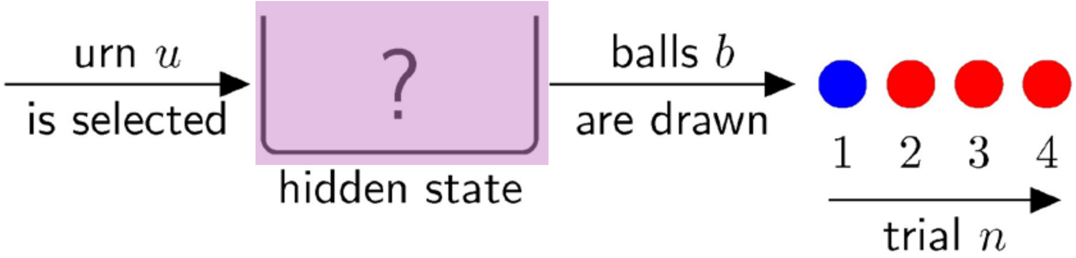
Prior distribution: $\mathrm{P}_{\mathcal{U}} = \{\mathrm{P}(u) \mid \forall\, u \in \mathcal{U}\}$

Posterior distribution: $\mathrm{P}_{\mathcal{U}|b} = \{\mathrm{P}(u|b) \mid \forall\, u \in \mathcal{U}\}$

Likelihood distribution: $\mathcal{L}_{\mathcal{B}|u} = \{\mathrm{P}(b|u) \mid \forall\, b \in \mathcal{B}\}$

Relation to Bayes‘ Theorem





Subjects inferred the type of the hidden urn based on an episode of sampling of four balls.

$u \in \mathcal{U} = \{1, 2\}$ **Hypothesis:** urn type

$b \in \mathcal{B} = \{$●, ●$\}$ **Observation:** ball type

$\mathrm{P}_{\mathcal{U}}$ **Prior & Posterior:** Urn distribution

$\mathcal{L}_{\mathcal{B}|u}$ **Likelihoods:** Ball distribution given *one* urn

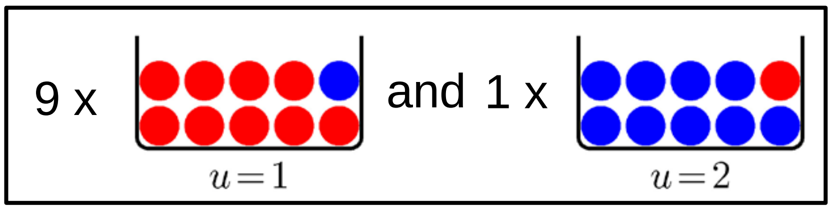
$\mathrm{P}_{\mathcal{B}}$ **Evidence:** Ball distribution over *all* urns

2 Urn-Ball Task
Experimental Conditions

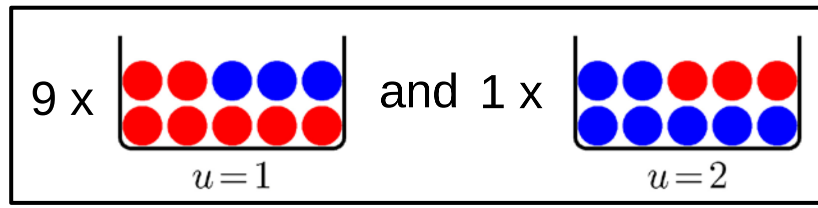
The four experimental conditions:
(50 episodes of ball sampling per condition)

certain prior:

certain likelihood

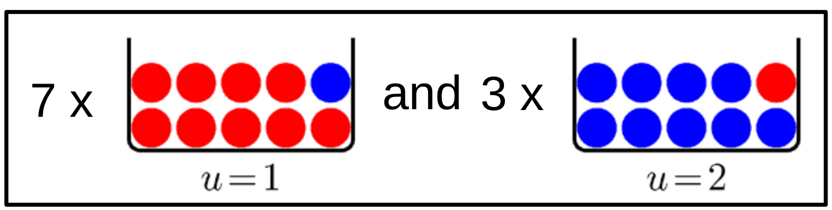


uncertain likelihood

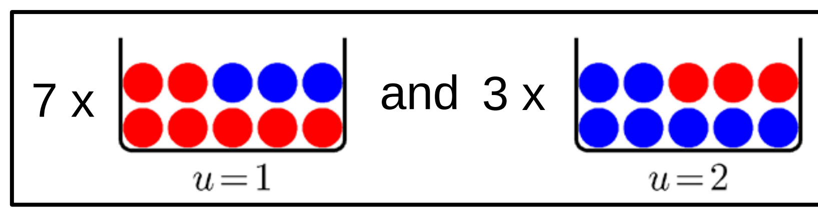


uncertain prior:

certain likelihood



uncertain likelihood



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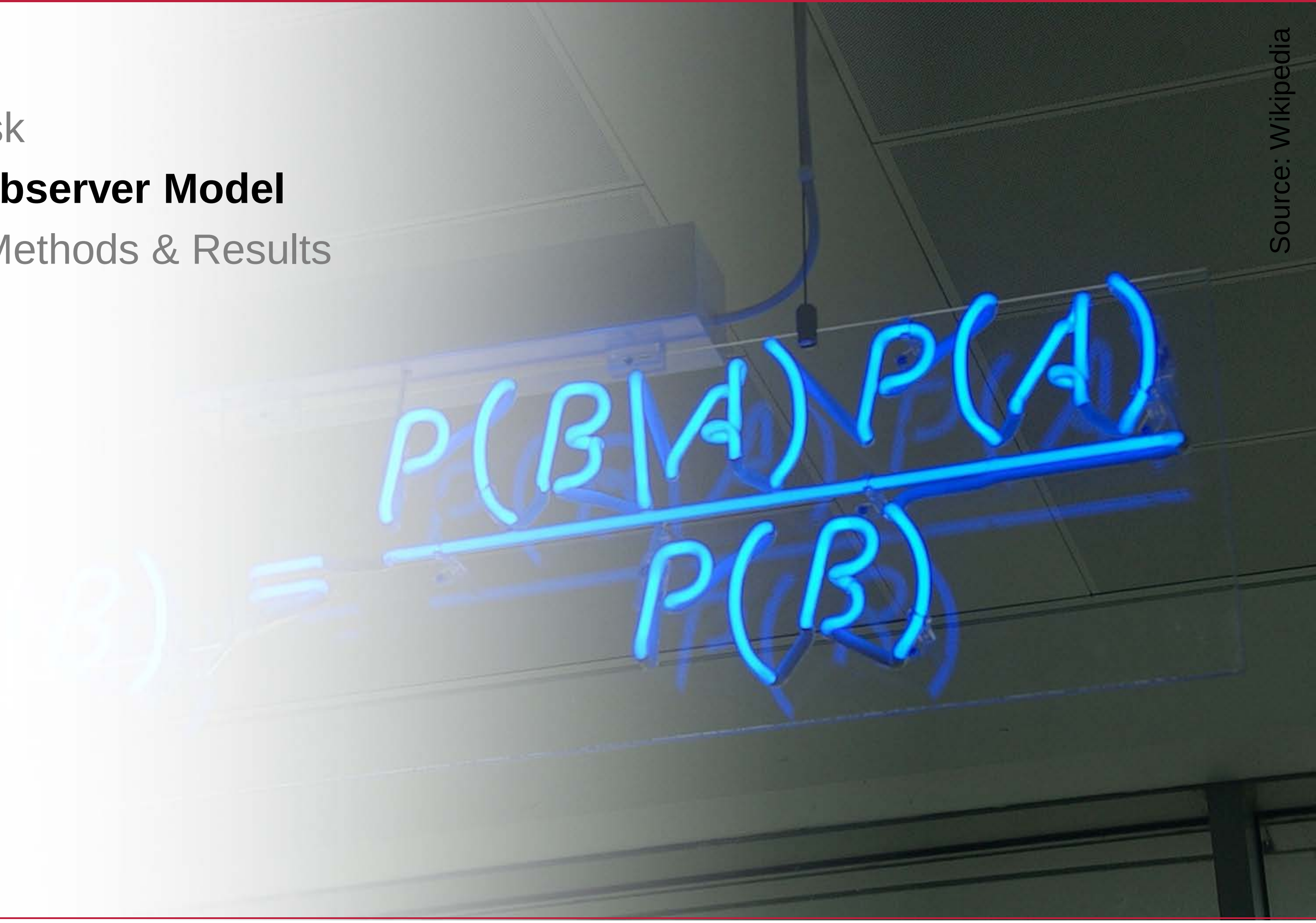

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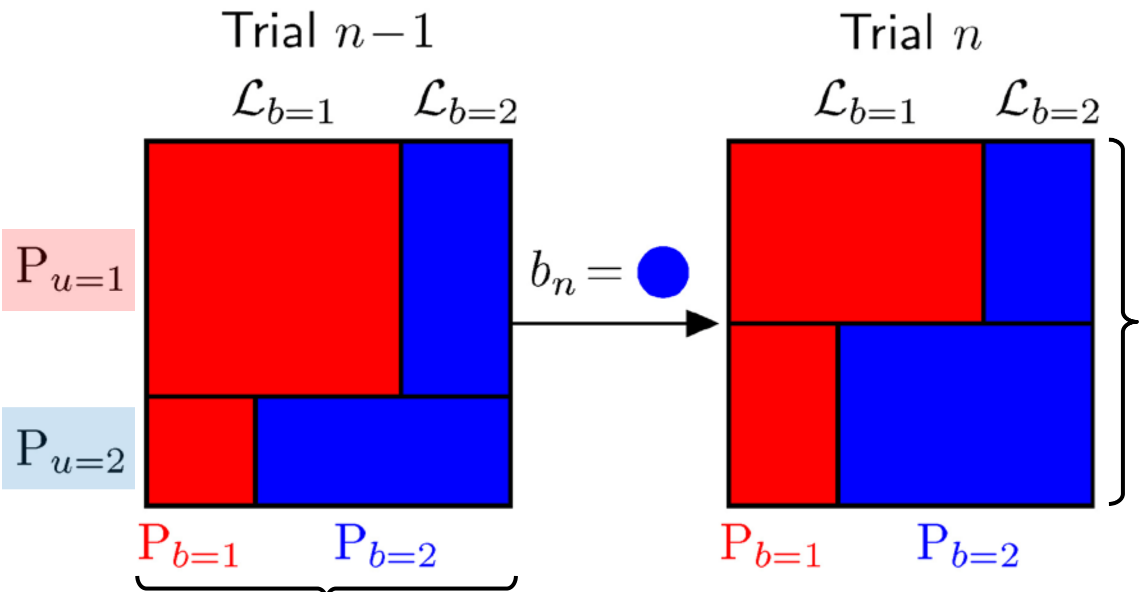
The BEL and PRE Distbributions



beliefs $\mathrm{P}_{\mathcal{U}}$ (belief updating)

predictions $\mathrm{P}_{\mathcal{B}}$ (prediction updating)

$\mathrm{P}_{\mathcal{U}}$ **Prior & Posterior**

$\mathcal{L}_{\mathcal{B}|u}$ **Likelihoods**

$\mathrm{P}_{\mathcal{B}}$ **Evidence**

$$\mathrm{P}_u(n) = \frac{\mathcal{L}_{b|u}\mathrm{P}_u(n-1)}{\mathrm{P}_b(n)}, \quad \forall u \in \mathcal{U}$$

$$\mathrm{P}_b(n+1) = \sum_{u\in\mathcal{U}} \mathcal{L}_{b|u}\mathrm{P}_u(n), \quad \forall b \in \mathcal{B}$$

Belief distribution (BEL): $\mathrm{P}_{\mathcal{U}}(n-1) \rightarrow \mathrm{P}_{\mathcal{U}}(n)$

Prediction distribution (PRE): $\mathrm{P}_{\mathcal{B}}(n) \rightarrow \mathrm{P}_{\mathcal{B}}(n+1)$

3 Bayesian Observer Model
Bayesian and Predictive Surprise

Surprise w.r.t *probability distribution updating:*

I_B **Bayesian surprise:** Kullback-Leibler divergence between two distributions [1]

Belief updating (BEL):

$$\mathrm{P}_{\mathcal{U}}(n-1) \rightarrow \mathrm{P}_{\mathcal{U}}(n): \quad I_B(n) = \mathrm{D}_{\mathrm{KL}}\left(\mathrm{P}_{\mathcal{U}}(n-1) \,||\, \mathrm{P}_{\mathcal{U}}(n)\right) = \sum_{u \in \mathcal{U}} \mathrm{P}_u(n-1) \log\left(\frac{\mathrm{P}_u(n-1)}{\mathrm{P}_u(n)}\right)$$

Prediction updating (PRE):

$$\mathrm{P}_{\mathcal{B}}(n) \rightarrow \mathrm{P}_{\mathcal{B}}(n+1): \quad I_B(n) = \mathrm{D}_{\mathrm{KL}}\left(\mathrm{P}_{\mathcal{B}}(n) \,||\, \mathrm{P}_{\mathcal{B}}(n+1)\right) = \sum_{b \in \mathcal{B}} \mathrm{P}_b(n) \log\left(\frac{\mathrm{P}_b(n)}{\mathrm{P}_b(n+1)}\right)$$

Surprise w.r.t the *probability* of the observation (PRE):

I_P **Predictive surprise:** Shannon surprise about the current observation [2]

$$I_P = -\log_2 \mathrm{P}(b)$$

[1] Itti L, Baldi P (2009) Bayesian surprise attracts human attention. *Vision Research* 49:1295-1306.

[2] Shannon CE, Weaver W (1948) The mathematical theory of communication. *Communication, Bell System Technical Journal* 27:379-423.

3 Bayesian Observer Model
Probability Weighting

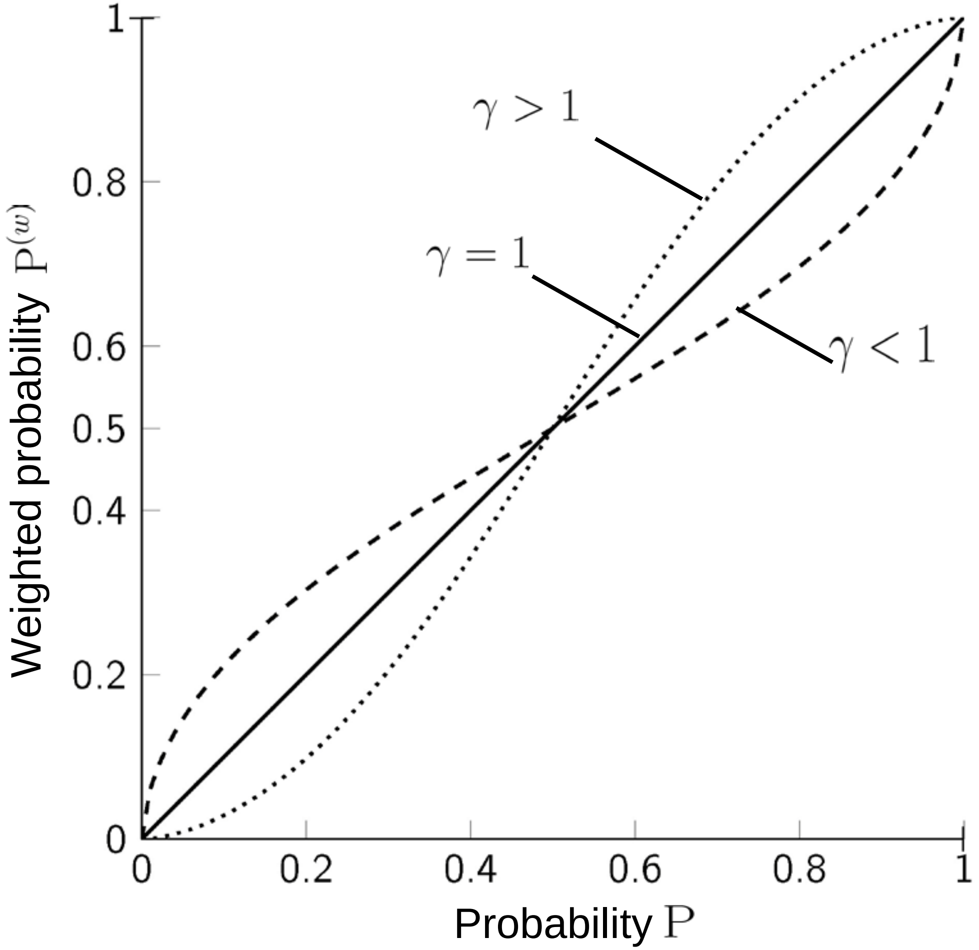
Estimates of probabilities are modulated by weighting functions [1]

Inverse S-shaped weighting function [2]

$$\mathrm{Lo}\left(w(\mathrm{P})\right) = \gamma \mathrm{Lo}(\mathrm{P}) + (1-\gamma)\mathrm{Lo}(\mathrm{P}_0)$$

$$\mathrm{Lo}\left(\mathrm{P}\right) = \log\left(\frac{\mathrm{P}}{1-\mathrm{P}}\right)$$

$\mathrm{P}^{(w)} := w(\mathrm{P})$ notation of the weighted probability



[1] Kahneman D, Tversky A (1979) Prospect theory: an analysis of decision under risk. *Econometrica* 47: 263-291.

[2] Zhang H, Maloney LT (2012) Ubiquitous log odds: a common representation of probability and frequency distortion in perception, action, and cognition. *Frontiers in Neuroscience* 6:1.

Effect of Probability Weighting

Applying probability weighting to the observer model:

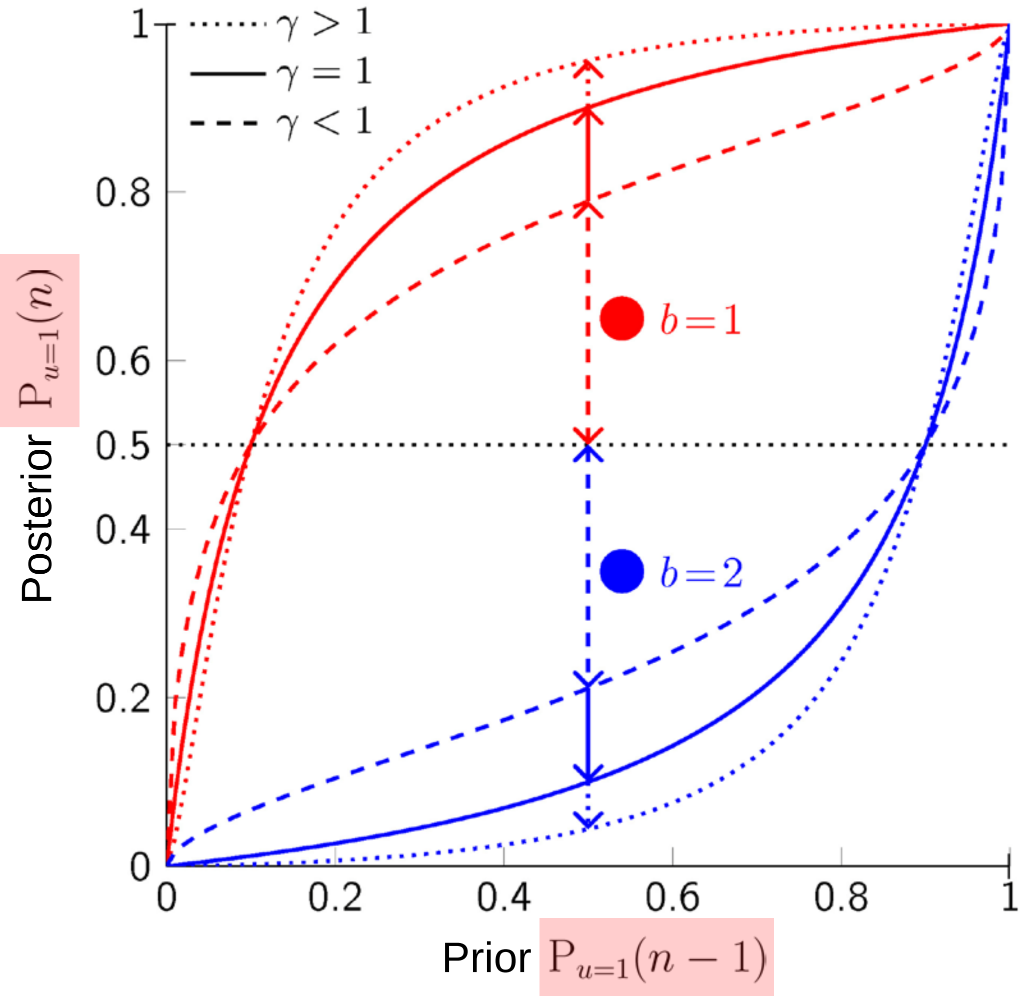
$\mathrm{BEL} \rightarrow \mathrm{BEL}^{(w)}$

$$\mathrm{P}_u(n) = \frac{\mathcal{L}_{b|u}^{(w)} \mathrm{P}_u^{(w)}(n-1)}{\mathrm{P}_b(n)}, \quad \forall u \in \mathcal{U}$$

with $\mathrm{P}_b(n) = \sum_{u \in \mathcal{U}} \mathcal{L}_{b|u}^{(w)} \mathrm{P}_u^{(w)}(n-1)$

$\mathrm{PRE} \rightarrow \mathrm{PRE}^{(w)}$

$$\mathrm{P}_b(n+1) = \sum_{u \in \mathcal{U}} \mathcal{L}_{b|u}^{(w)} \mathrm{P}_u^{(w)}(n), \quad \forall b \in \mathcal{B}$$



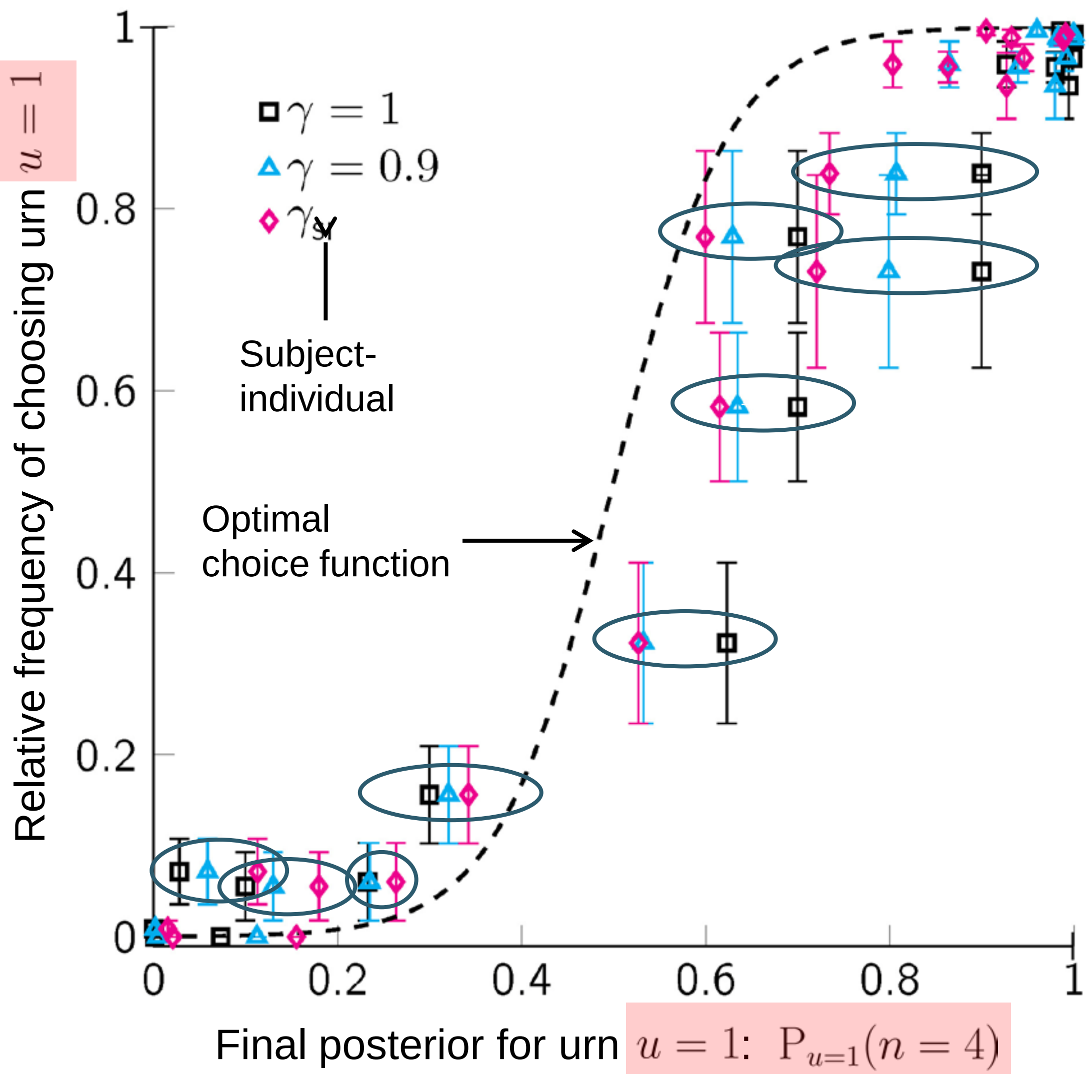
3 Bayesian Observer Model
Parameter Optimization Based on Behavioral Data

Data points (for display purpose):
- Four experimental conditions
- Five different ratios of ball colors

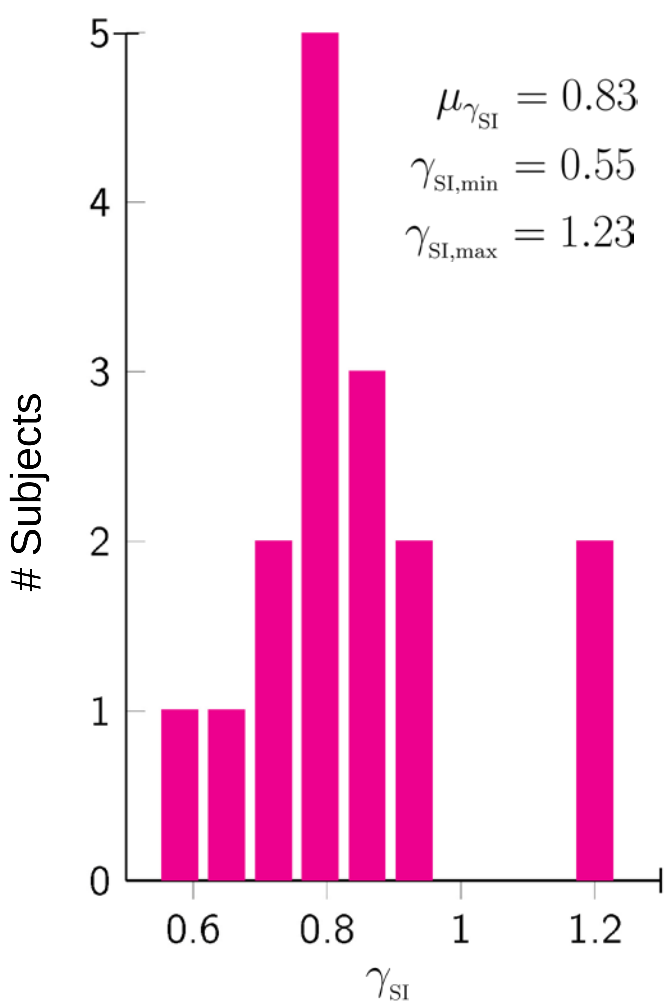
Optimization: Minimum mean squared error between data points and choice function

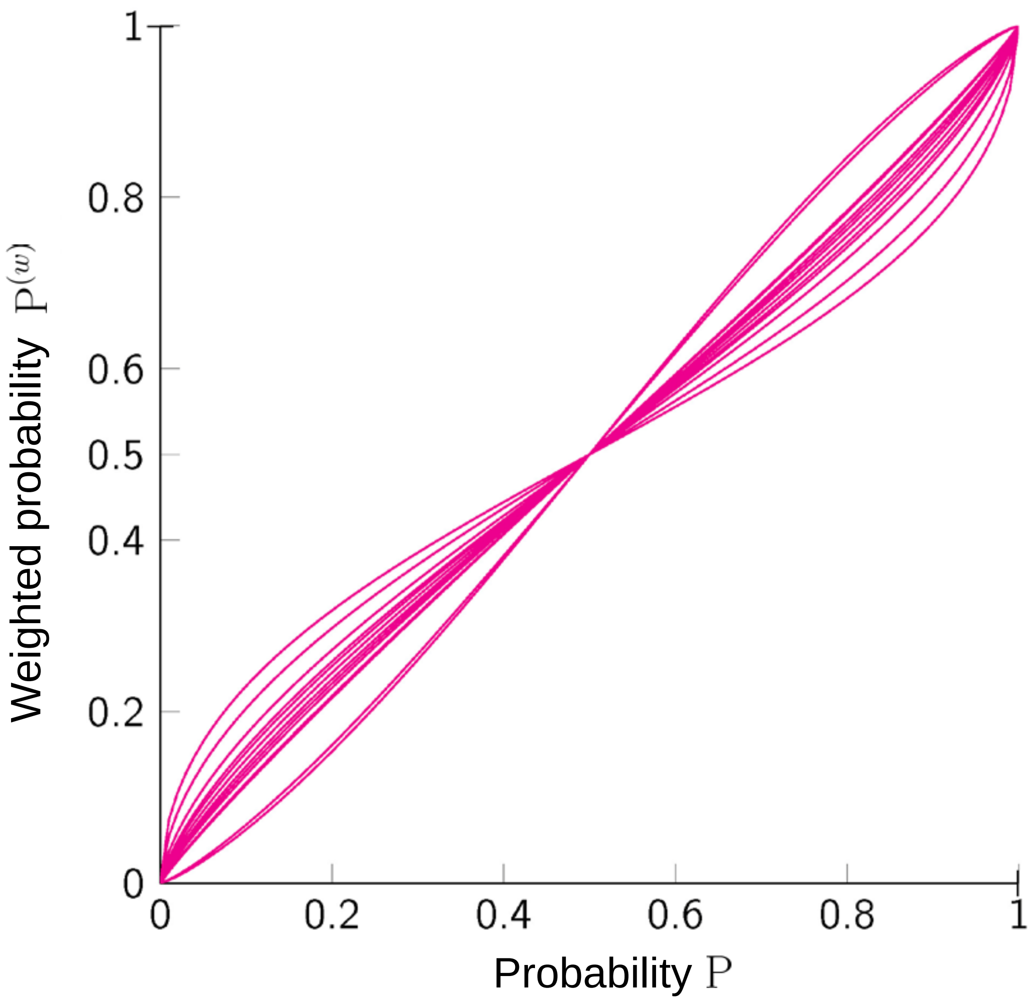
Clustering of data points around optimal choice function for $\gamma = 0.9$

Subject-individual γ_{SI} reduce error



3 Bayesian Observer Model
Individual Differences in Weighting Parameters





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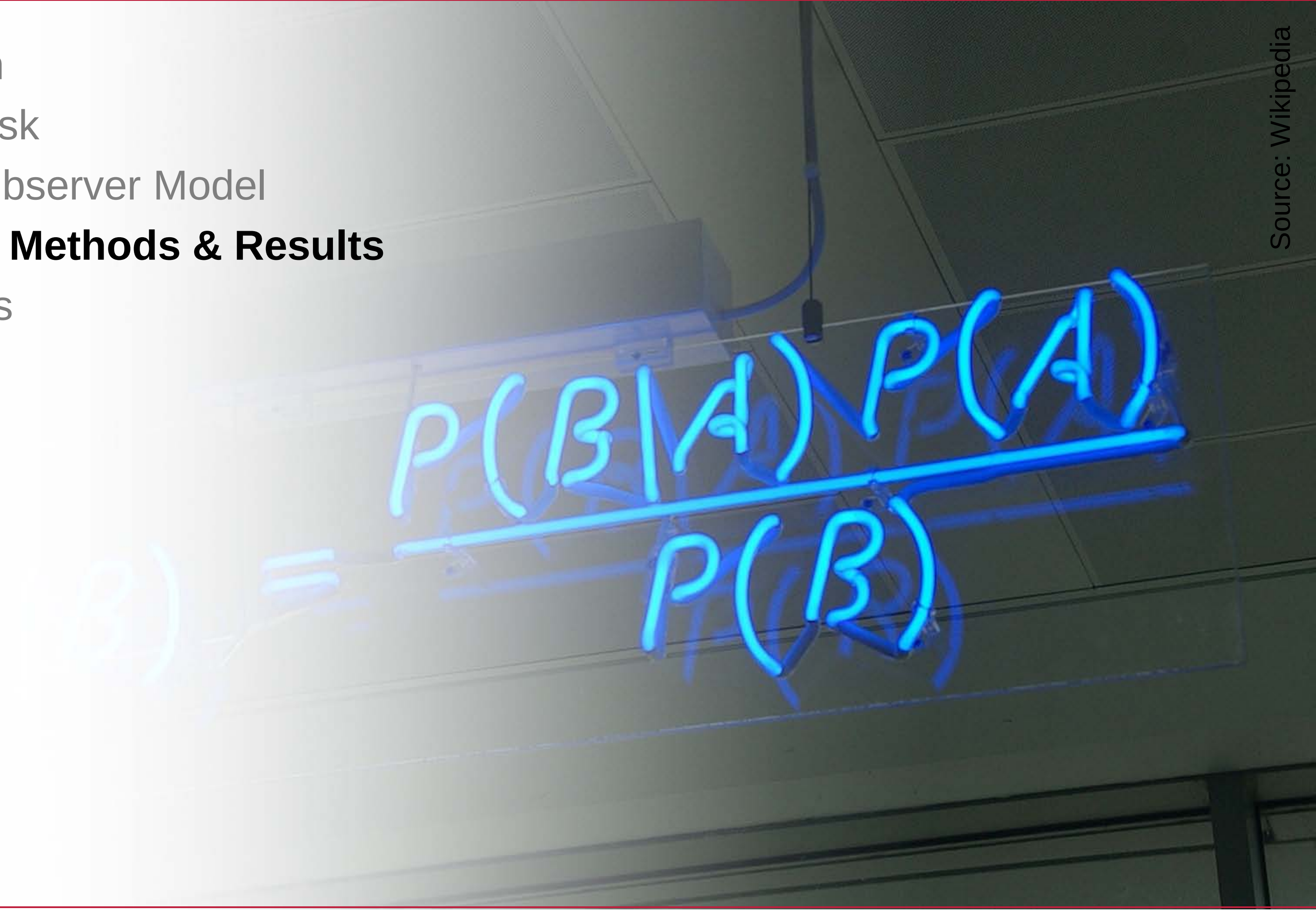

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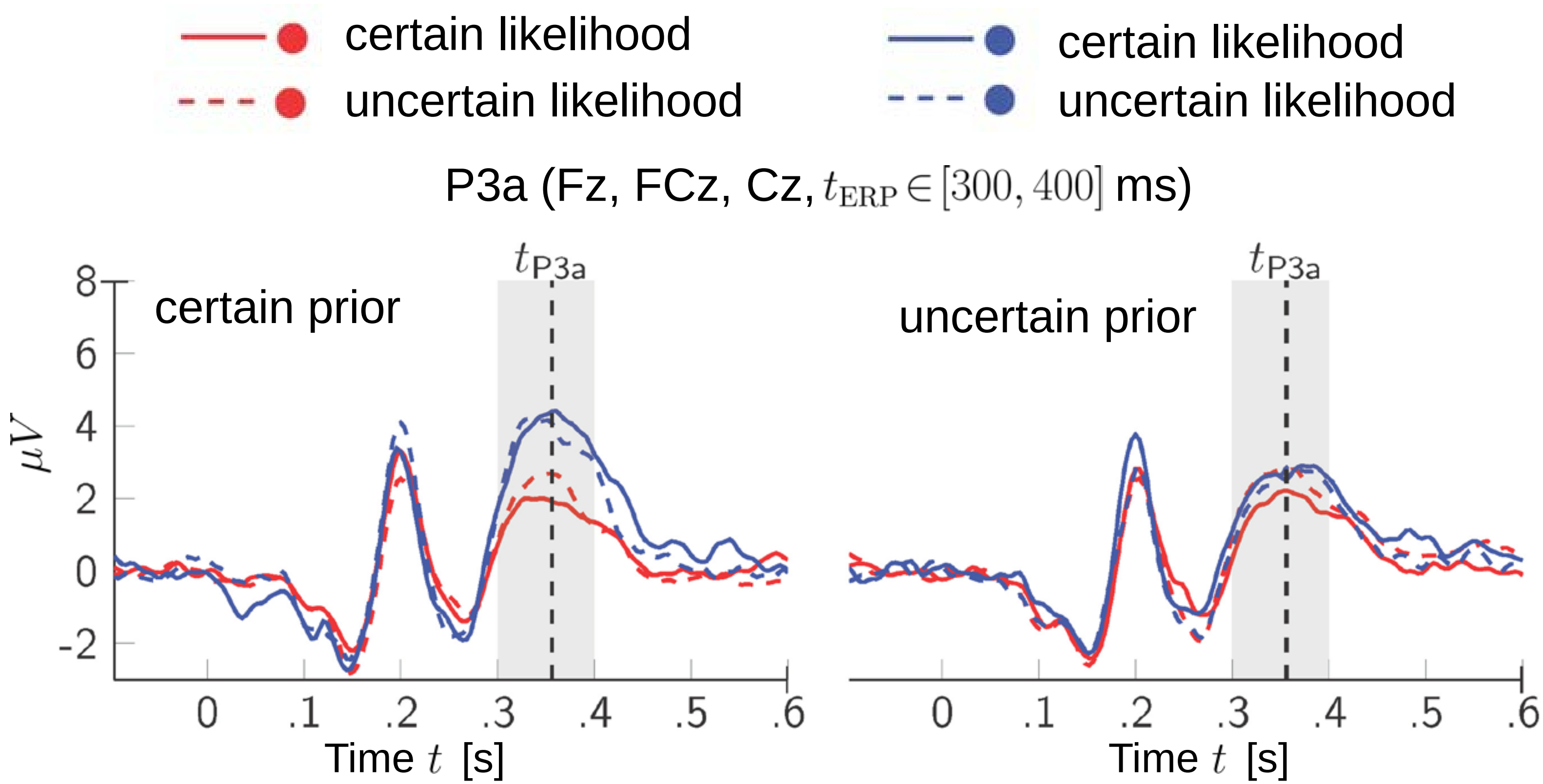
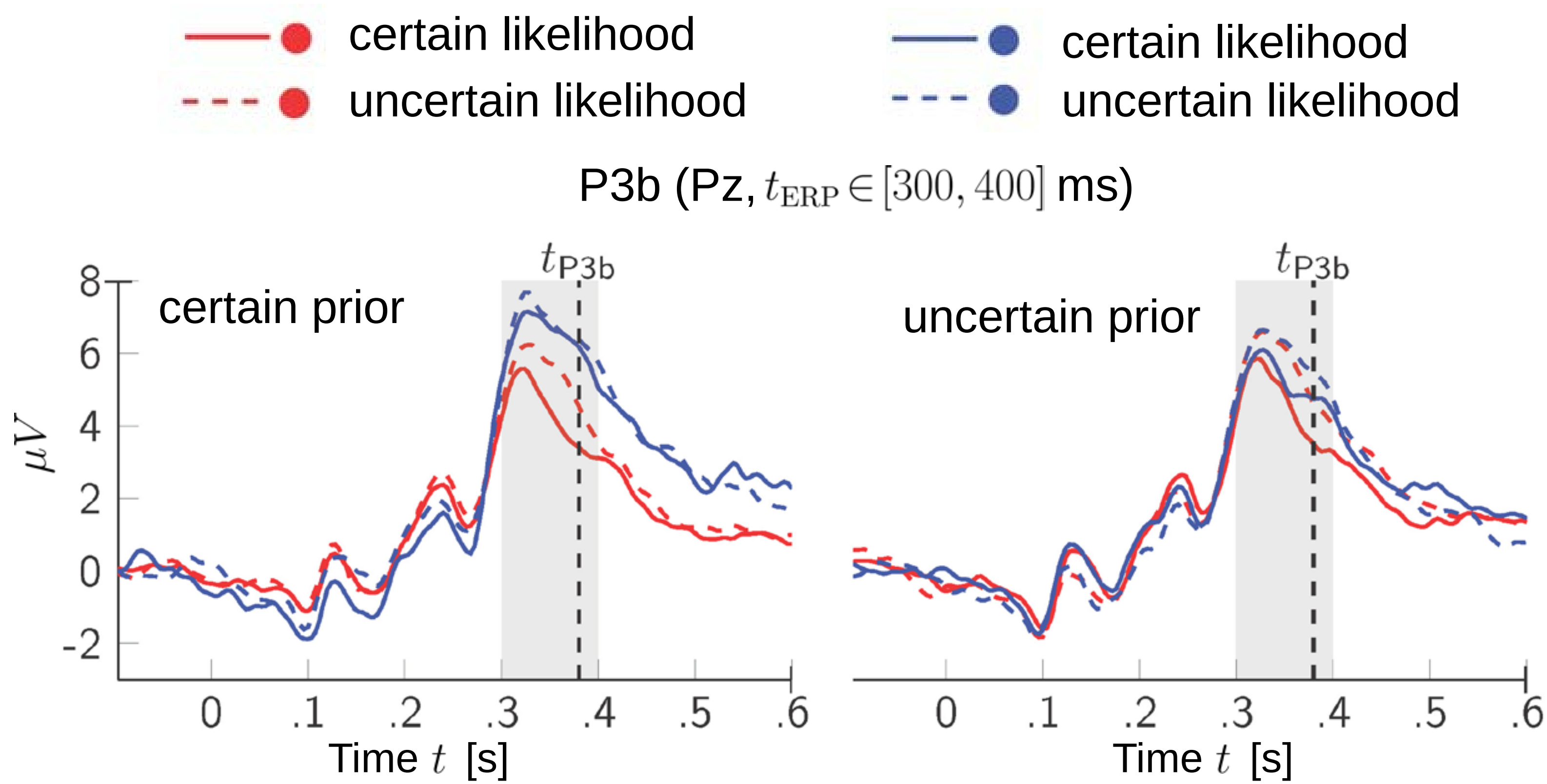
Grand-Average ERP waves



- Larger amplitudes in response to rare balls
- Central scalp topography
- Peak amplitudes and maximum variance at expected P3a latency

t_{ERP} latency of maximum variance

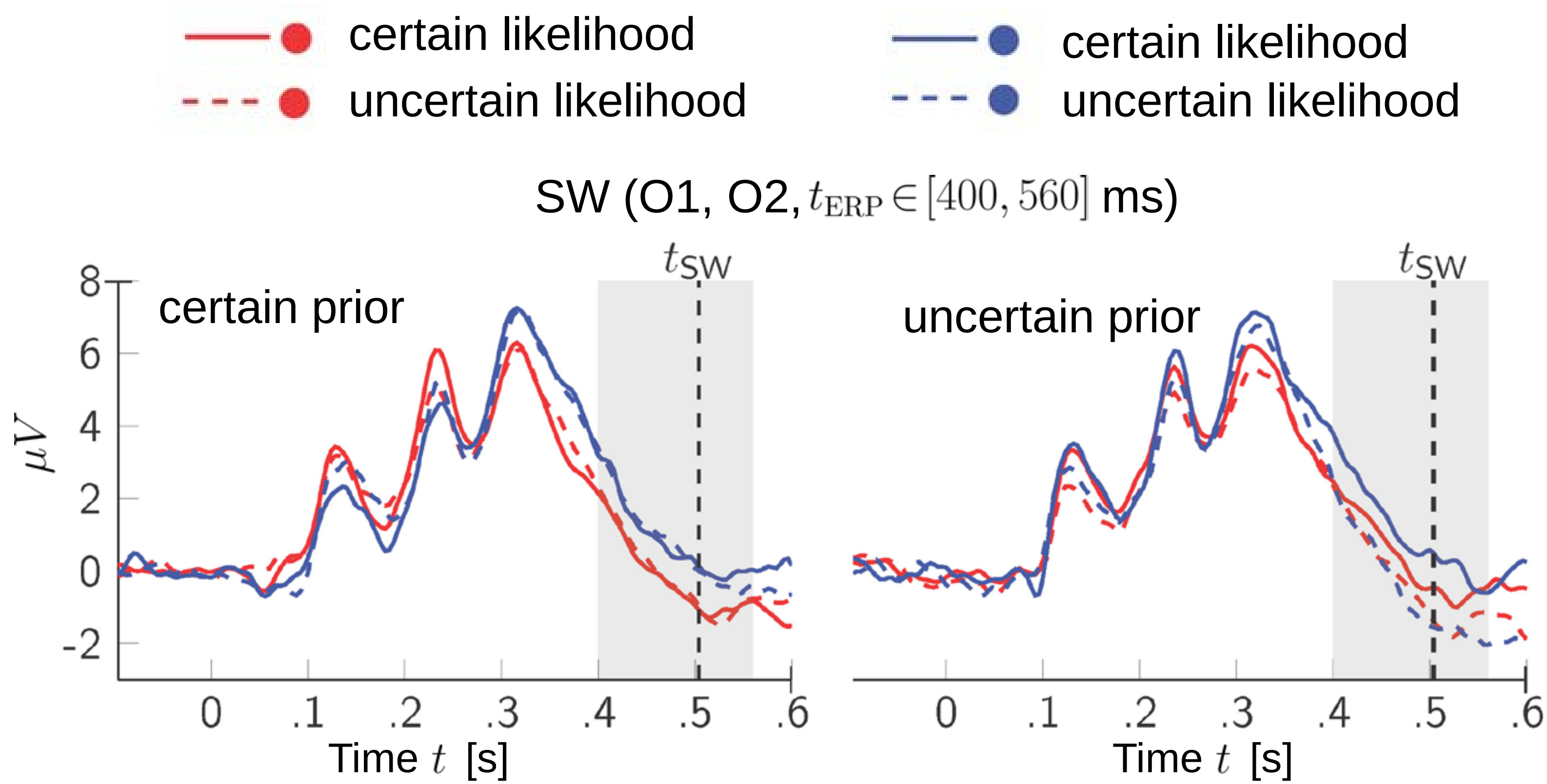
4 Methods
Grand-Average ERP waves



- Larger amplitudes in response to rare balls
- Parietal scalp topography
- Peak amplitudes at expected P3b latency, maximum variance slightly later

t_{ERP} latency of maximum variance

4 Methods
Grand-Average ERP waves



- Larger amplitudes in response to rare balls
- Parieto-occipital topography
- No distinct SW peaks, but distinct maximum variance latency

t_{ERP} latency of maximum variance

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Model Selection

Previous work suggests the following model space [1]:

ERP	model space $\mathcal{M}$
P3a	$\{I_B(\mathrm{BEL}),\, I_B(\mathrm{BEL}^{(w)}),\, I_B(\mathrm{BEL}_{\mathrm{SI}}^{(w)})\}$
P3b	$\{I_P(\mathrm{PRE}),\, I_P(\mathrm{PRE}^{(w)}),\, I_P(\mathrm{PRE}_{\mathrm{SI}}^{(w)})\}$
SW	$\{I_B(\mathrm{PRE}),\, I_B(\mathrm{PRE}^{(w)}),\, I_B(\mathrm{PRE}_{\mathrm{SI}}^{(w)})\}$

Model selection:

Exceedance probability φ_m [2]

Scalp maps:

Group log-Bayes factor [3]

$$\log\left(\mathrm{GBF}\right) = F_m - F_{m_0}$$

$m \in \mathcal{M}$	model
m_0	reference model
$\mathbf{y}$	measured data
$F_m \approx \log(\mathrm{p}(\mathbf{y}\|m))$	group log-evidence

[1] Kolossa A, et al. (2015) A computational analysis of the neural basis of Bayesian inference. *NeuroImage* 106:222–237.

[2] Stephan KE, et al. (2009) Bayesian model selection for group studies. *NeuroImage* 46:1004–1017.

[3] Stephan KE, et al. (2007) Comparing hemodynamic models with DCM. *NeuroImage* 38:387–401.

4 Results
Exceedance Probabilities and Scalp Maps

t_{P3a} = 356 ms

P3a

I_B	BEL	$\mathrm{BEL}^{(w)}$	$\mathrm{BEL}_{\mathrm{SI}}^{(w)}$
φ_m	<0.01	0.07	0.93

t_{P3b} = 380 ms

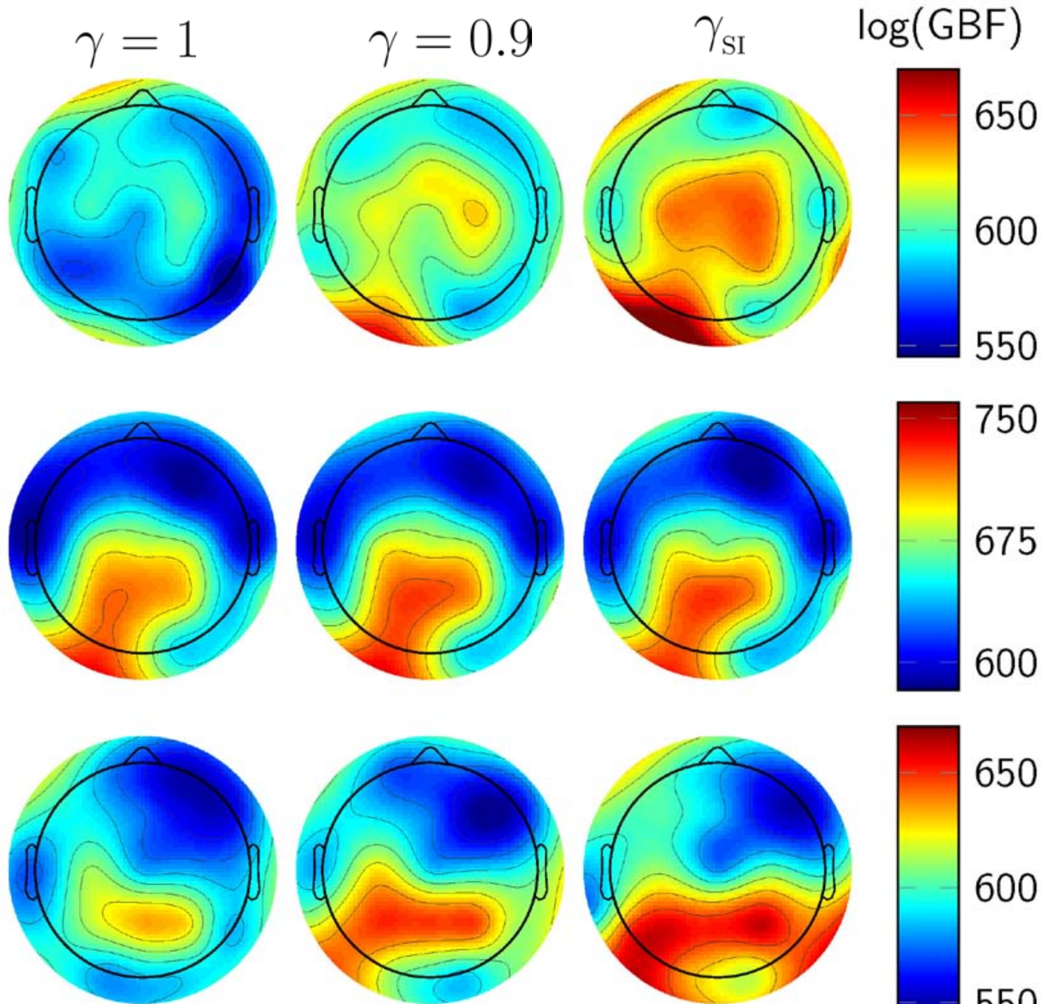
P3b

I_P	PRE	$\mathrm{PRE}^{(w)}$	$\mathrm{PRE}_{\mathrm{SI}}^{(w)}$
φ_m	0.01	0.07	0.92

t_{SW} = 504 ms

SW

I_B	PRE	$\mathrm{PRE}^{(w)}$	$\mathrm{PRE}_{\mathrm{SI}}^{(w)}$
φ_m	0.01	<0.01	0.99



Increasing φ_m due to probability weighting

Visibly increasing $\log(\mathrm{GBF})$ for all ERPs reflects φ_m

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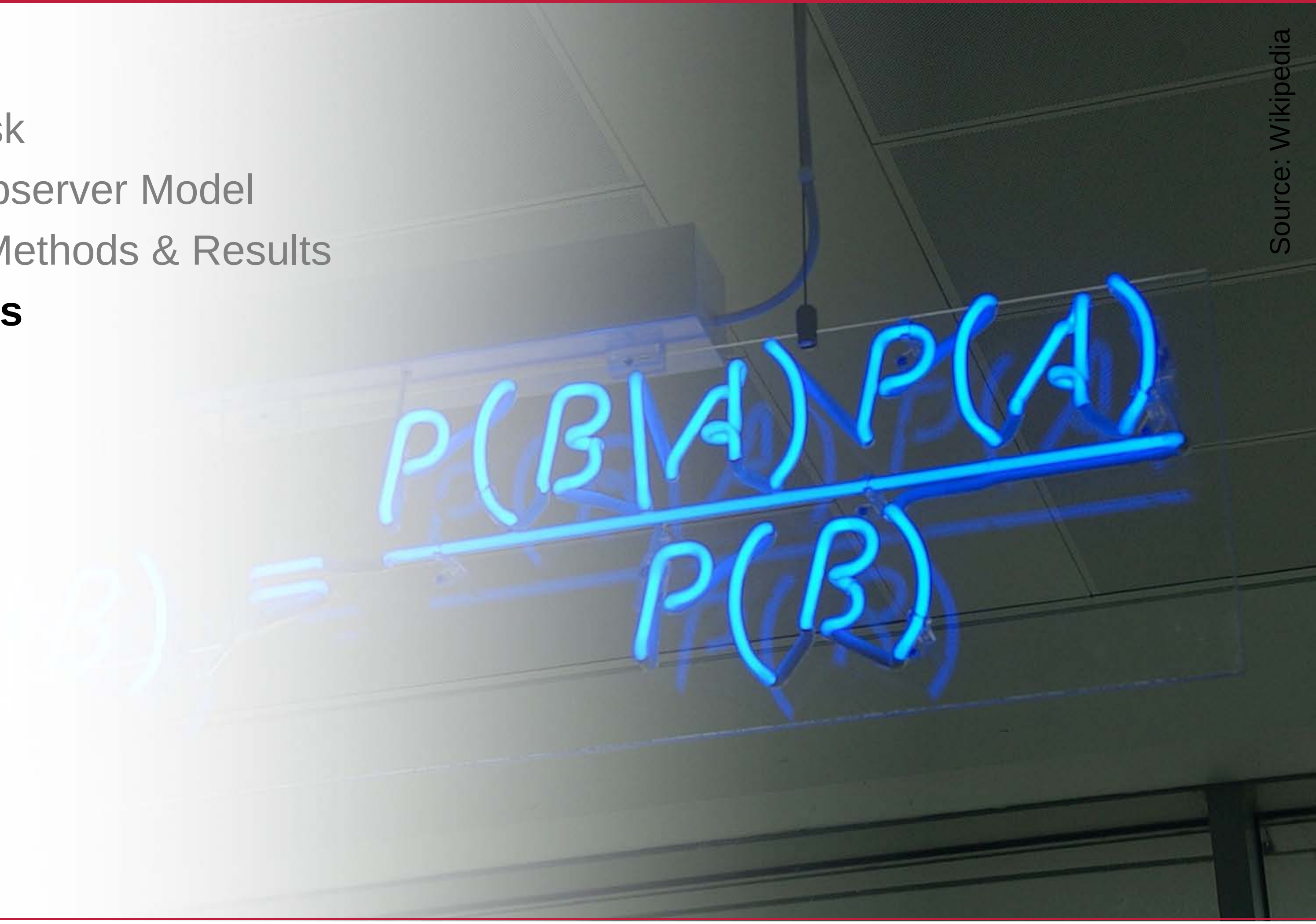
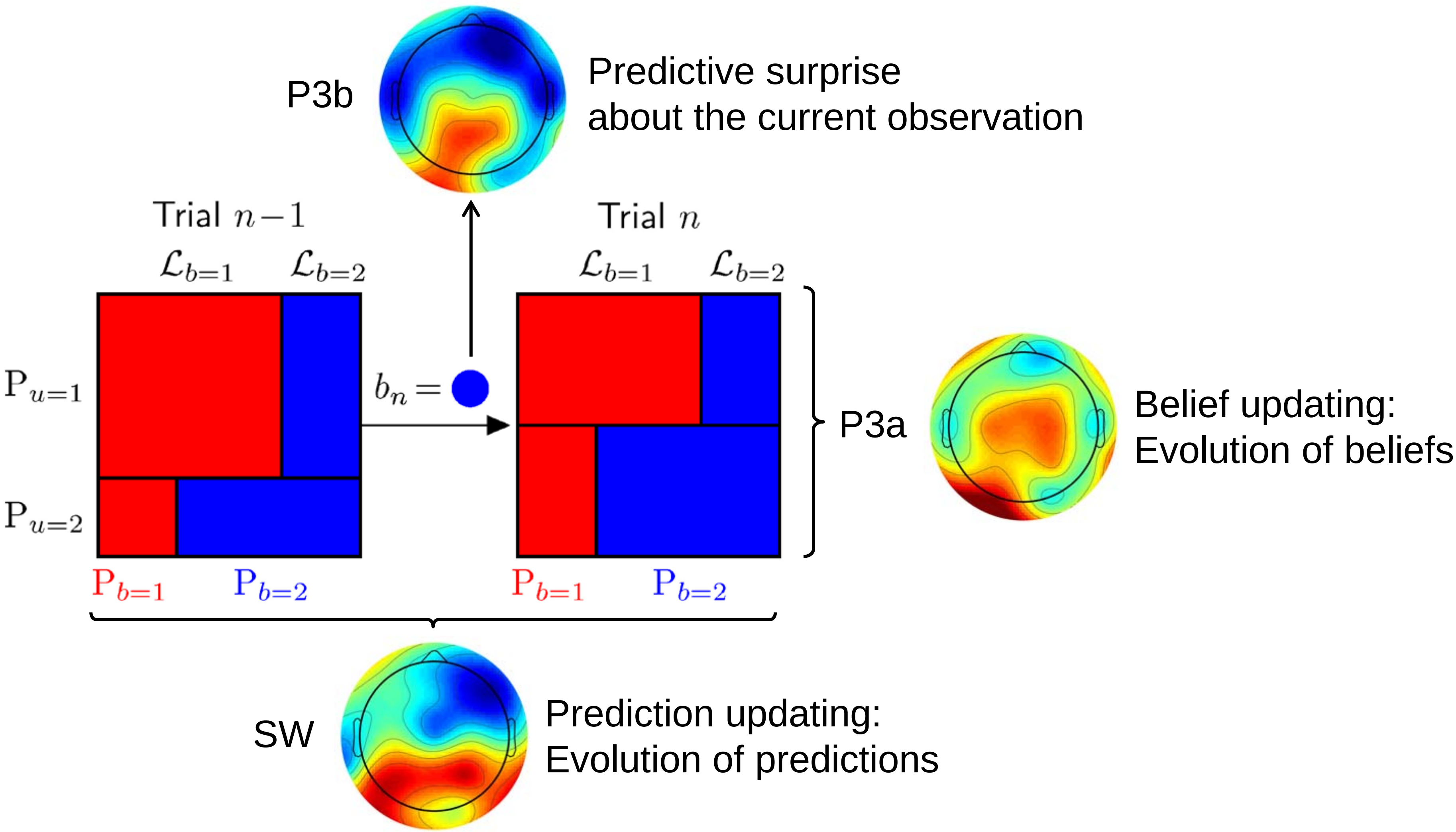


Source: Wikipedia

5 Conclusions
Bayesian Decomposition of Surprise and ERPs



5 Conclusions
ERP Data on the Bayesian Brain

The present study yielded ERP data on the neural bases of Bayesian inference:

- Dissociable cortical responses reflect formally defined aspects of surprise
- Future studies should target single-trial fluctuations of:
  - P3a: beliefs
  - SW: predictions
  - P3b: predictive surprise
- Probability weighting improves the fit of the Bayesian observer model
- The average Bayesian observer is biased towards uncertainty
- Subject-individual weighting parameters further improve the model fit
- Bayesian inference by the brain serves the minimization of surprise

Thank you for your attention.

Antonio Kolossa

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