Reading in the brain

3. Symbol grounding: How the acquisition of symbols affects numerical cognition

Stanislas Dehaene

Collège de France,
and
INSERM-CEA
Cognitive Neuroimaging Unit
NeuroSpin Center, Saclay, France
www.unicog.org

Summary of the two previous talks

1. Reading acquisition leads to the specialization of a left ventral occipito-temporal region for letter strings (**Visual word form area**)
2. The VWFA provides a **quick non-conscious access** to left temporal language areas.

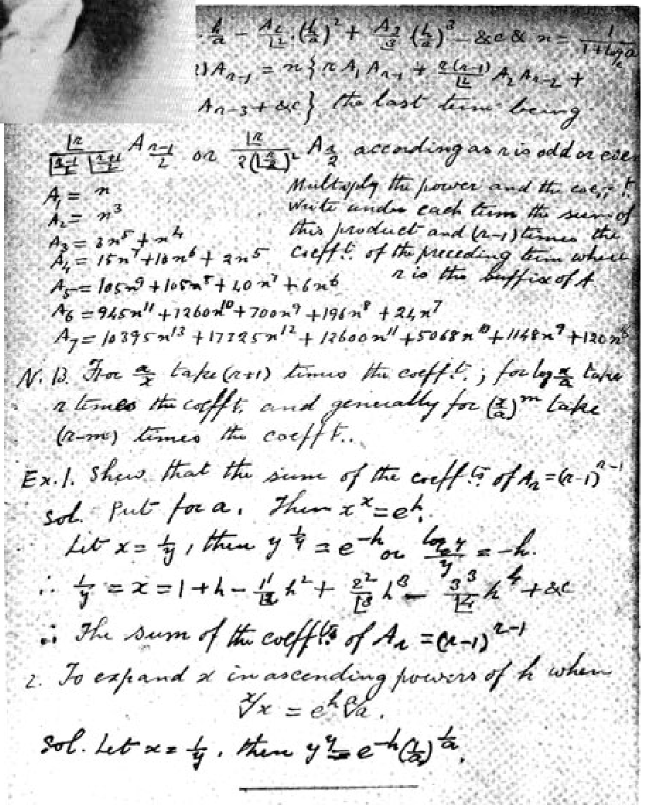
Today: **How is cognition affected by symbol acquisition?**

- Literacy improves **phonological awareness** and memory for meaningless **linguistic** material (Morais et al.)
- Are non-verbal **semantic** representations also altered as they become attached to a symbol? **The case of numbers**

Two mathematicians

Srinivasa Ramanujan
(1887-1920)

$$\frac{1}{\pi} = \frac{2\sqrt{2}}{9801} \sum_{k=0}^{\infty} \frac{(4k)!(1103+26390k)}{(k!)^4 396^{4k}}$$


according as n is odd or even

Multiply the power and the coeff.; Write under each term the sum of this product and (n-1) times the coeff. of the preceding term where n is the suffix of A.

$A_1 = n$
$A_2 = n^3$
$A_3 = 3n^5 + n^4$
$A_4 = 15n^7 + 10n^6 + 2n^5$
$A_5 = 105n^9 + 105n^8 + 40n^7 + 6n^6$
$A_6 = 945n^{11} + 1260n^{10} + 700n^9 + 196n^8 + 24n^7$
$A_7 = 10395n^{13} + 17325n^{12} + 12600n^{11} + 5068n^{10} + 1148n^9 + 120n^8$

N.B. For $\frac{a}{x}$ take (n+1) times the coeff.; for $\log\frac{x}{a}$ take n times the coeff. and generally for $(\frac{x}{a})^m$ take (n-m) times the coeff.

Ex. 1. Show that the sum of the coeff.s of $A_n = (n-1)^{n-1}$

Sol. Put for a. Then $x^x = e^h$.

Let $x = \frac{1}{y}$, then $y^{\frac{1}{y}} = e^{-h}$ or $\frac{\log y}{y} = -h$.

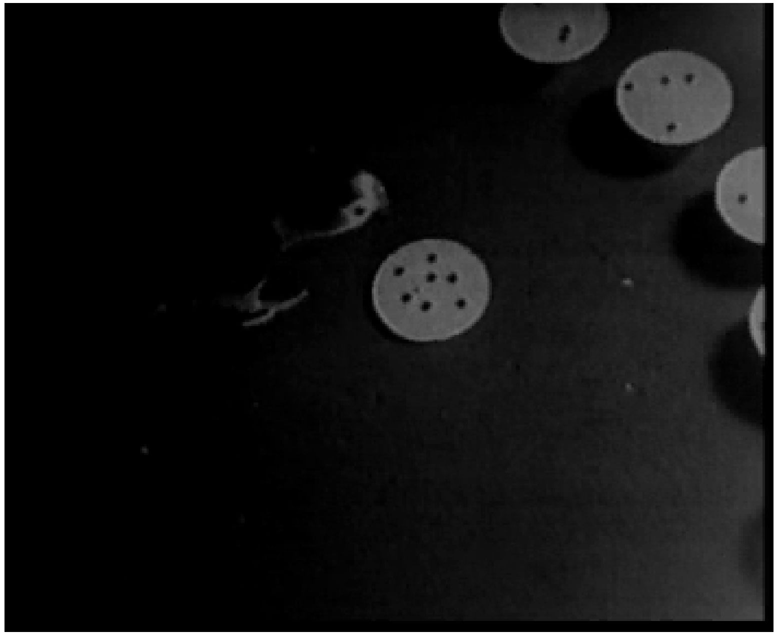
$\therefore \frac{1}{y} = x = 1 + h - \frac{1}{\lfloor 2} h^2 + \frac{2^2}{\lfloor 3} h^3 - \frac{3^3}{\lfloor 4} h^4 + \&c$

$\therefore$ The sum of the coeff.s of $A_n = (n-1)^{n-1}$

2. To expand x in ascending powers of h when $\sqrt[x]{x} = e^h \sqrt[a]{a}$.

Sol. Let $x = \frac{1}{y}$, then $y^y = e^{-h}(\frac{1}{a})^{\frac{1}{a}}$.

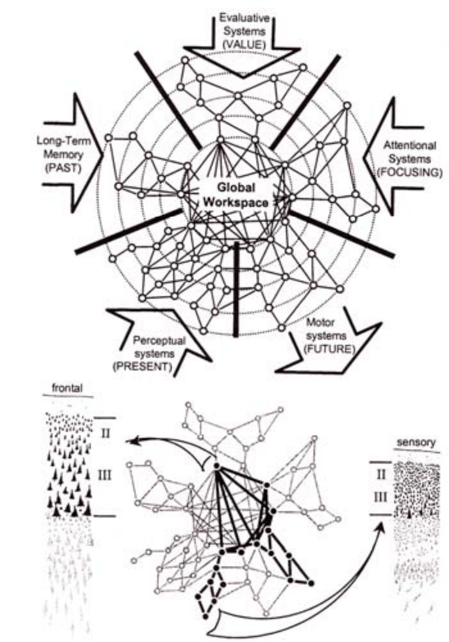
Otto Köhler's parrott (ca. 1955)



A cognitive neuroscience perspective on mathematics

- During its evolution, our primate brain has been endowed with elementary representations that are adequate to certain aspects of the external world.
- These internalized representations of time, space, and number, shared with many animal species, provide the **foundations of mathematics**.

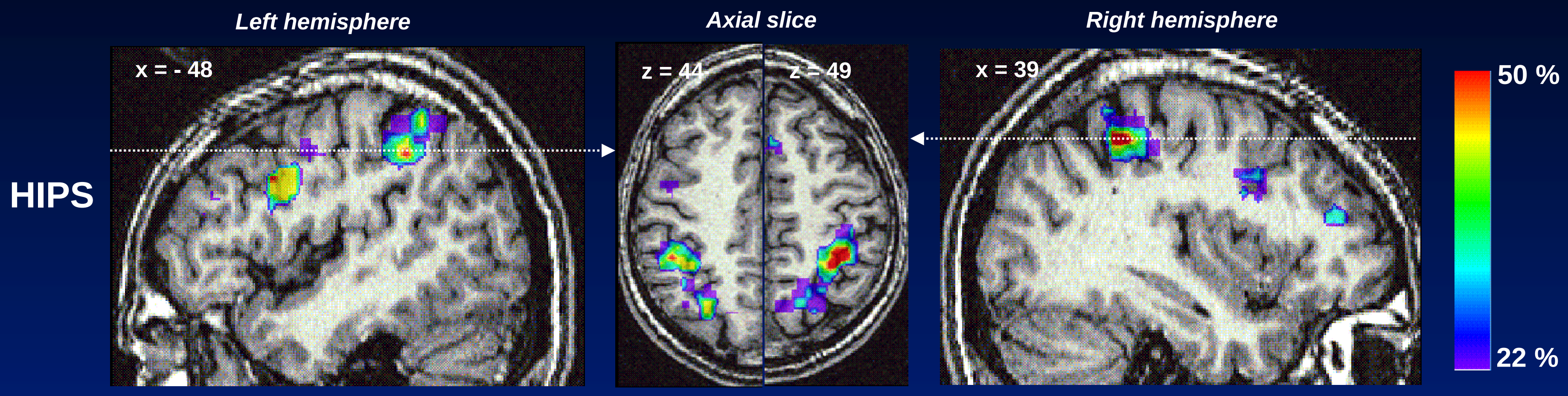
- Unique to humans, however, is the capacity to achieve integration of those internal senses
- The cultural construction of mathematics can be seen as a search for coherence amongst internal representations – which is reproduced at a faster pace during education
- What role does the acquisition of symbols play in this internal synthesis process? How does exposure to a system of Arabic numerals and number words change the organization of the number sense?



Plan of the talk

- Neural coding of numerosity
  - A quantity code is present in the intraparietal area of human adults and babies
  - Optimal decision mechanisms based on this code can explain human psychophysics
- Understanding of number symbols
  - Symbols are mapped onto numerosities
  - However, the numerosity code may be changed by learning symbols
  - Linear mapping of number onto space
- Brain mechanisms of number-space mappings

Previous studies of number sense and the horizontal segment of the intraparietal sulcus (HIPS)



- All numerical tasks activate this region
  (e.g. addition, subtraction, comparison, approximation, digit detection…)

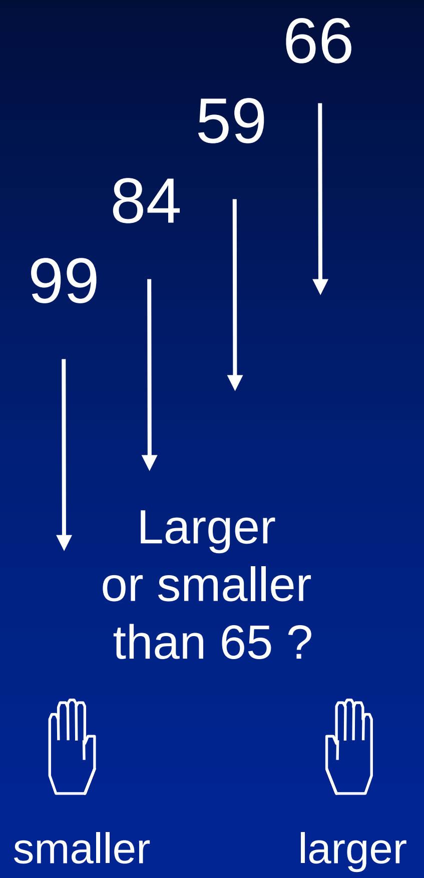
- This region fulfils two criteria for a semantic-level representation:

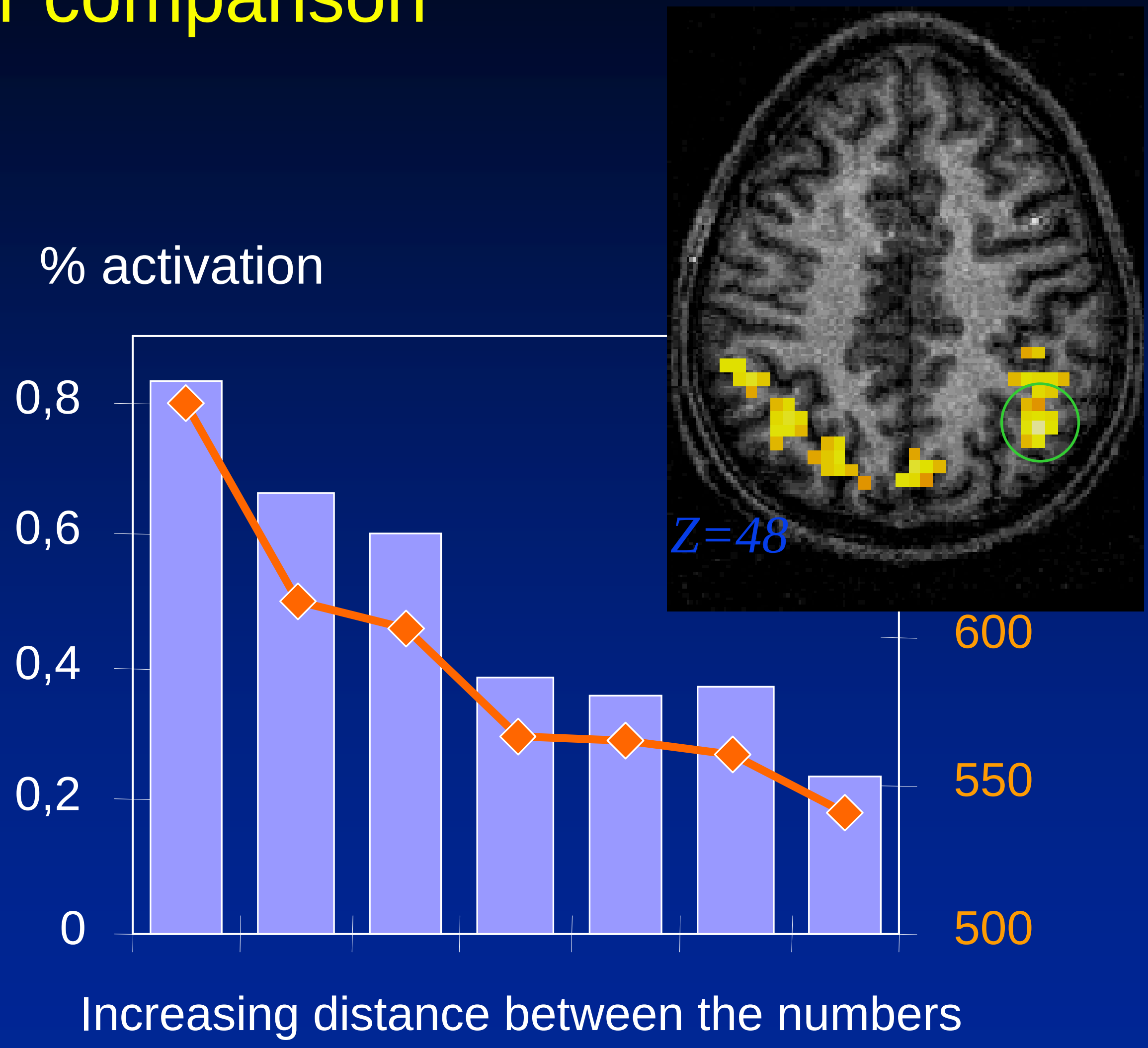
-It responds to number in various formats (Arabic digits, written or spoken words), more than to other categories of objects (e.g. letters, colors, animals…)

-Its activation varies according to a semantic metric (numerical distance, number size)

Dehaene, S., Piazza, M., Pinel, P., & Cohen, L. (2003). *Cognitive Neuropsychology*

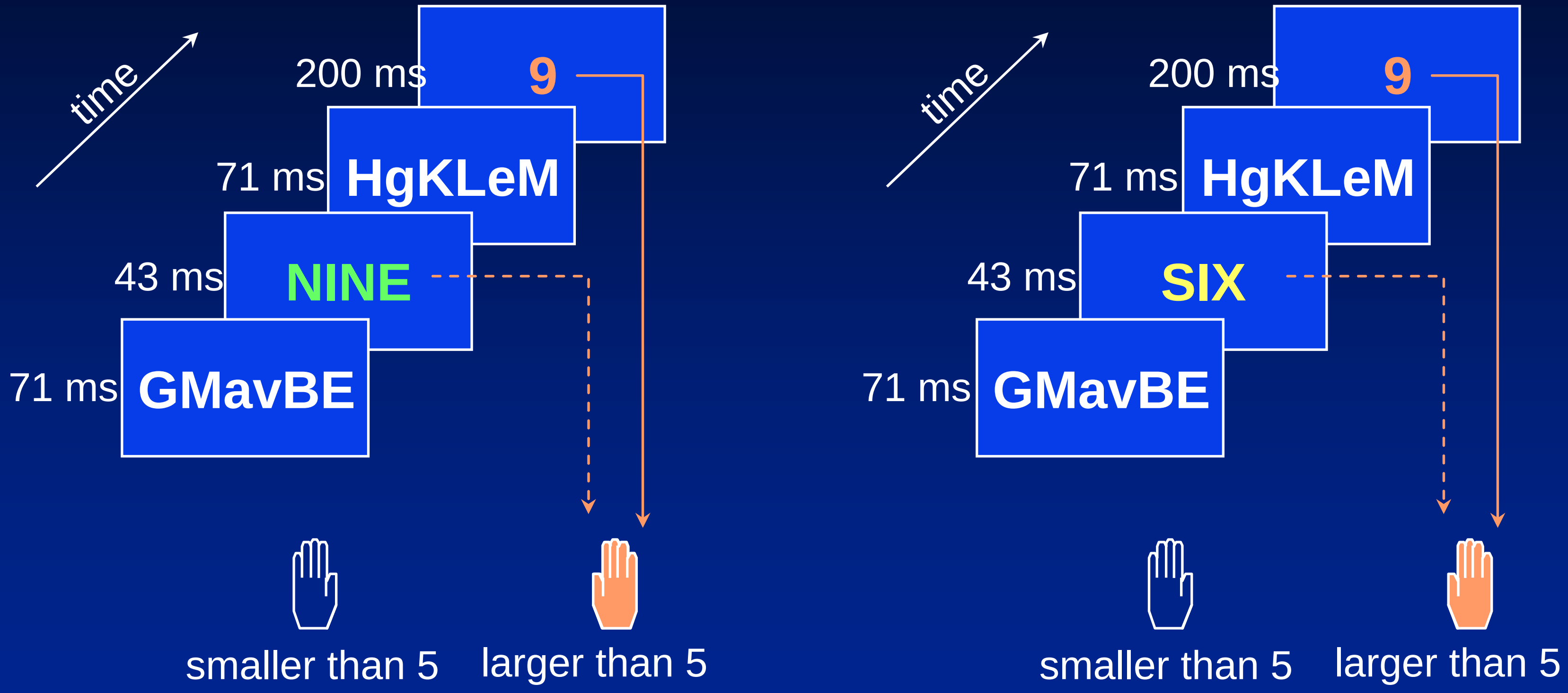
Neural bases of the distance effect in symbolic number comparison





Pinel, P., Dehaene, S., Riviere, D., & LeBihan, D. (2001). *Neuroimage, 14*(5), 1013-1026.

QUANTITY REPETITION PRIMING: A MARKER OF UNCONSCIOUS SEMANTIC PROCESSING?

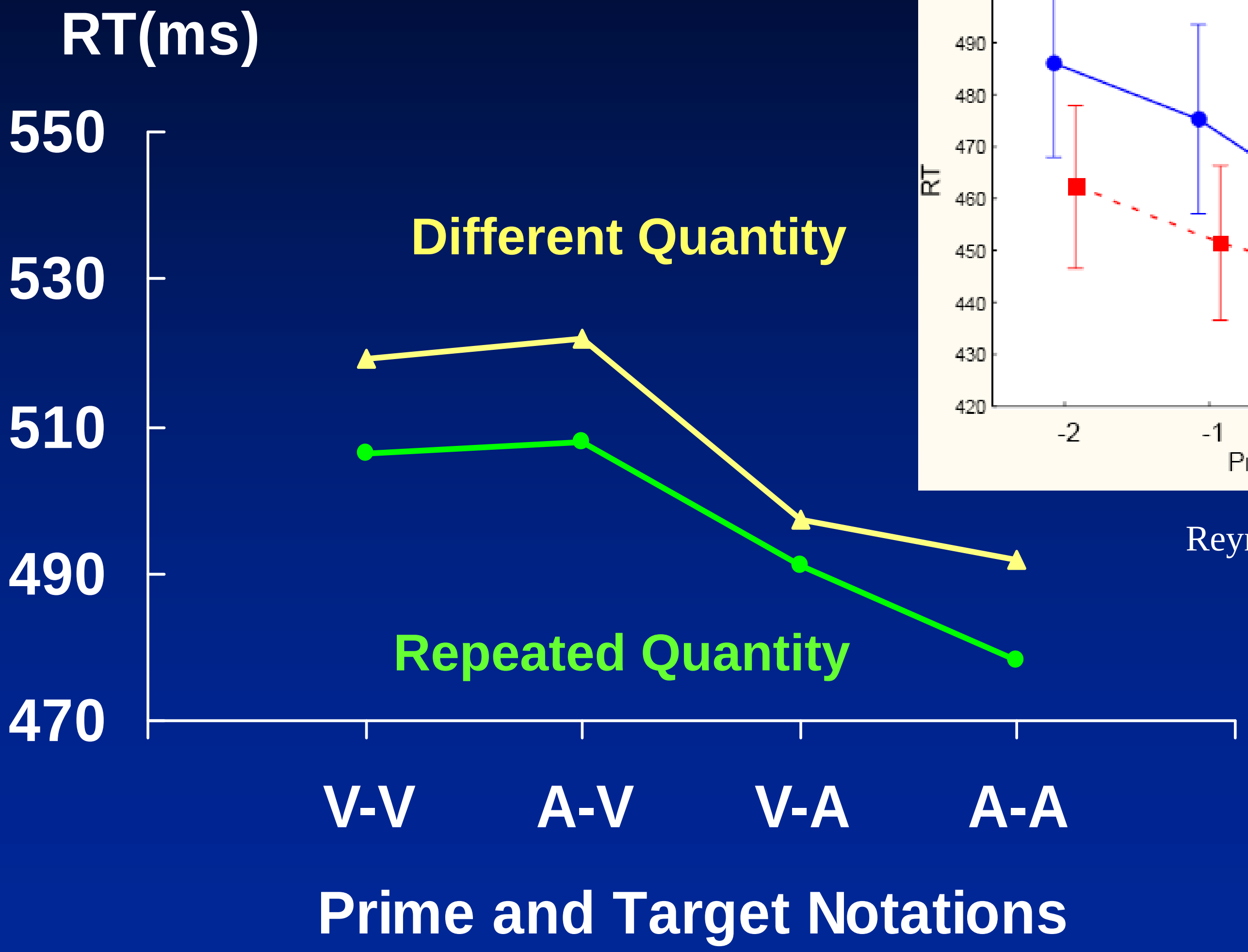


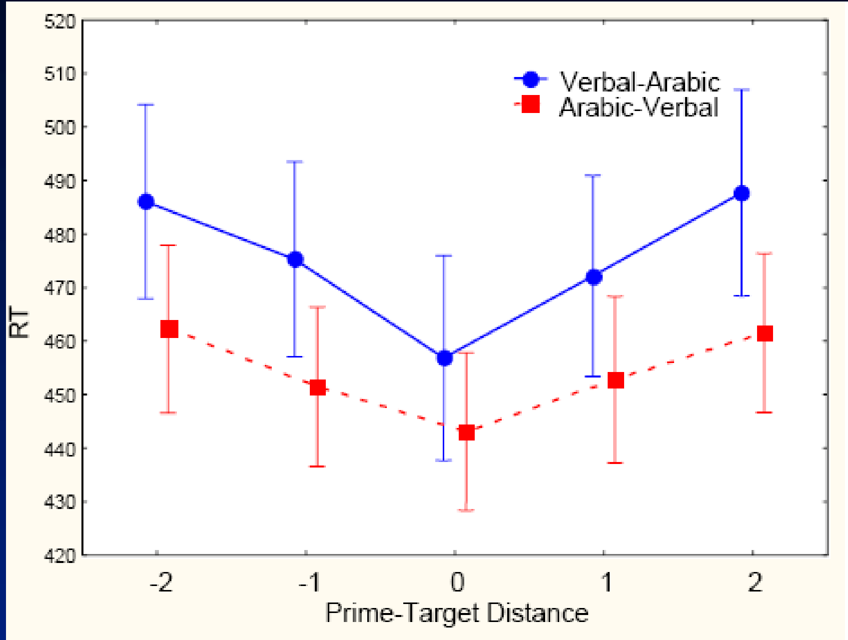
Same Response
Repeated Quantity

Same Response
Different Quantity

Lionel Naccache

Behavioral Quantity Priming
irrespective of number notation



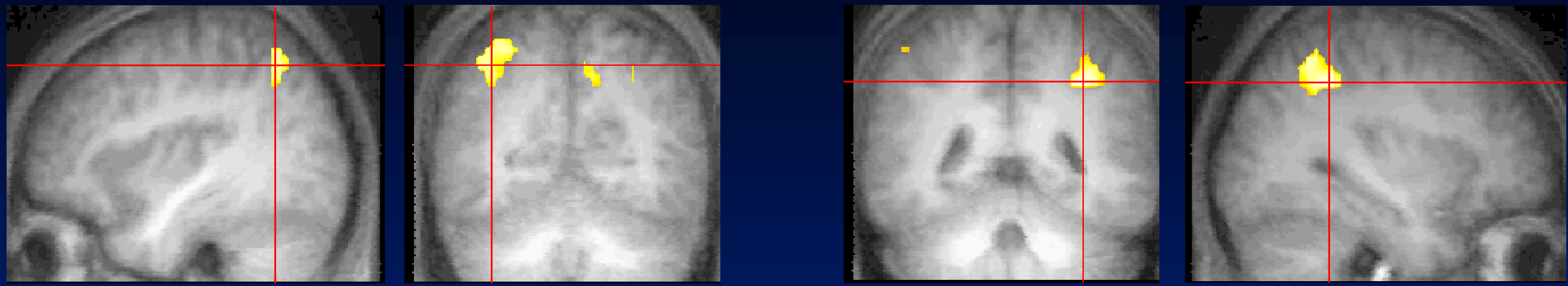


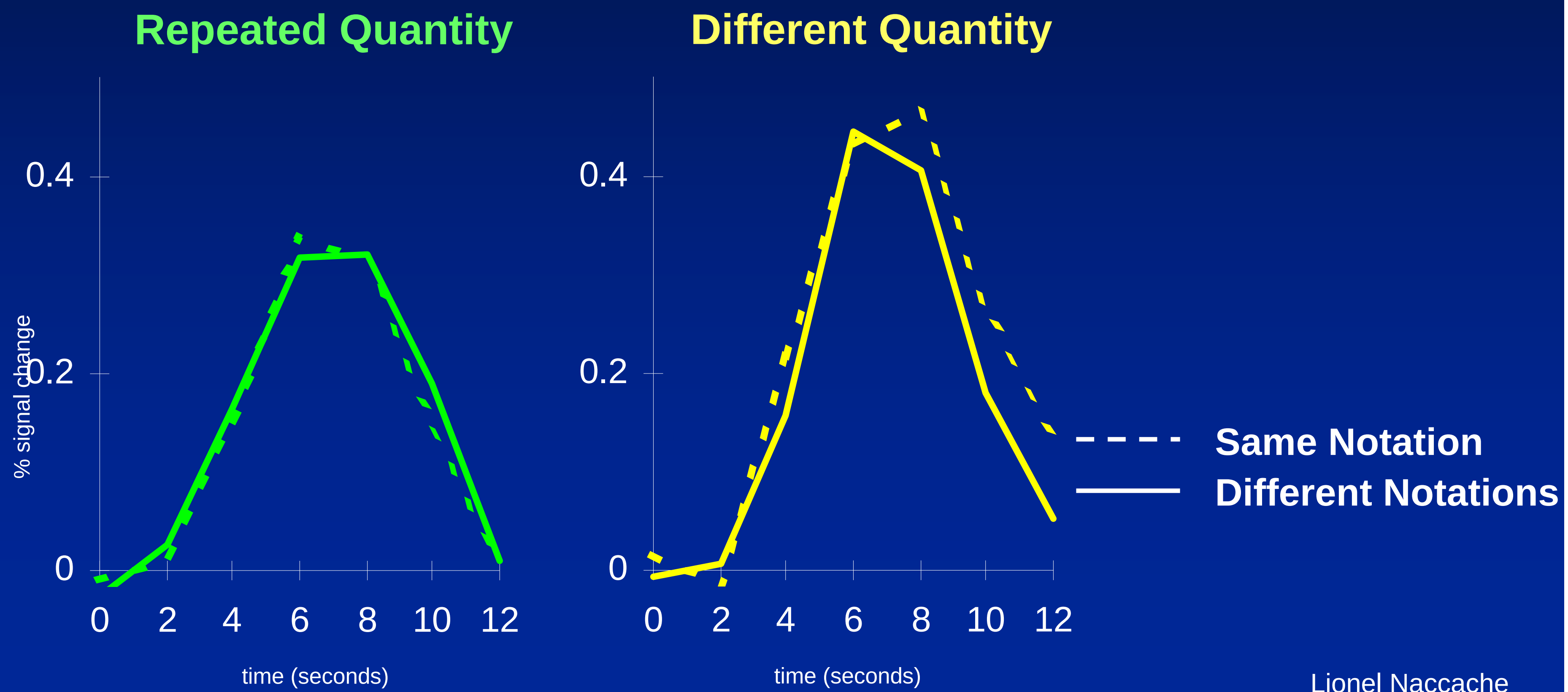
Reynvoet & Brysbaert (2004)

Lionel Naccache

UNCONSCIOUS PARIETAL PROCESSING OF QUANTITY

The intraparietal quantity system shows a notation-independent repetition effect



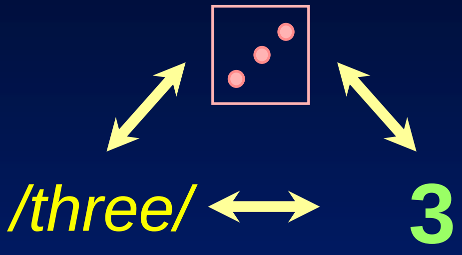


Lionel Naccache

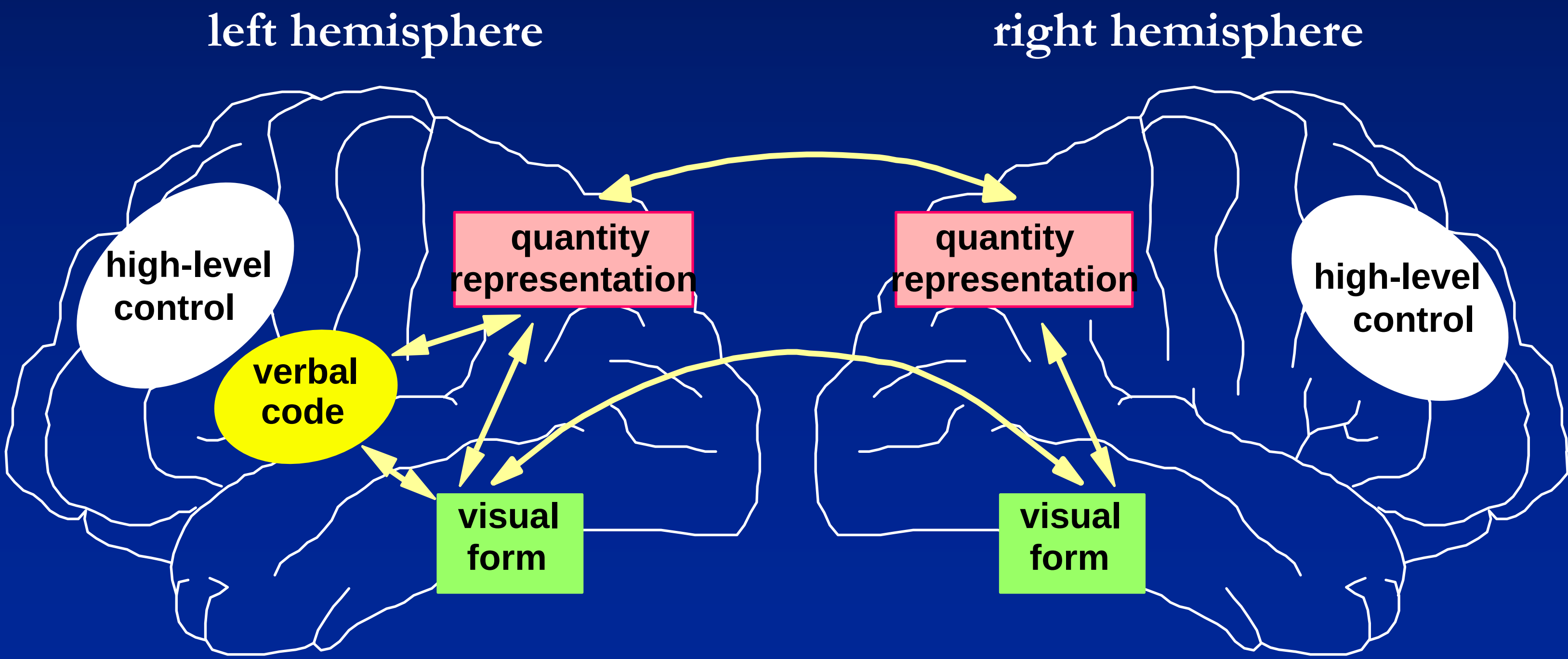
Attaching symbols to quantities:
The triple-code model of number processing

Quantity code
Number comparison; Proximity judgment; Approximation
Quantity manipulations (e.g. subtraction)

Verbal code
Spoken comprehension and production
rote tables (e.g. multiplication)



Arabic code
Arabic reading and writing
multidigit calculations



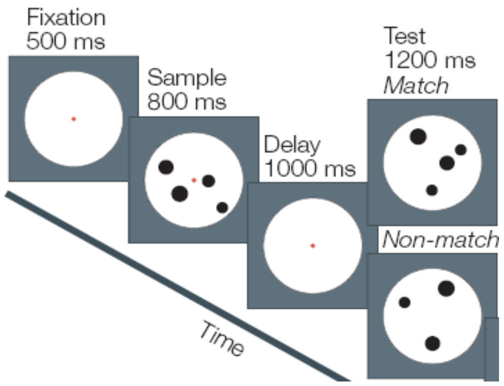
Plan of the talk

- Neural coding of numerosity
  - A quantity code is present in the intraparietal area of human adults and babies
  - Optimal decision mechanisms based on this code can explain human psychophysics
- Understanding of number symbols
  - Symbols are mapped onto numerosities
  - However, the numerosity code may be changed by learning symbols
  - Linear mapping of number onto space
- Brain mechanisms of number-space mappings

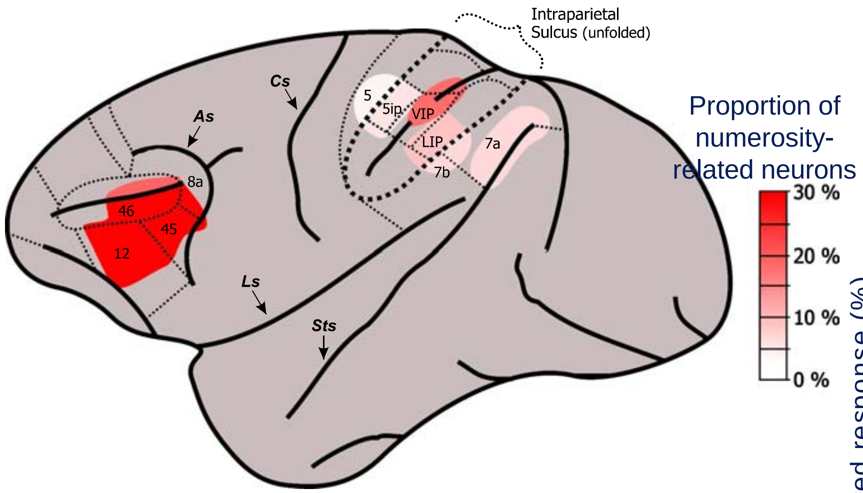
Number neurons in the monkey

(Nieder, Freedman & Miller, 2002; Nieder & Miller, 2003, 2004, 2005)

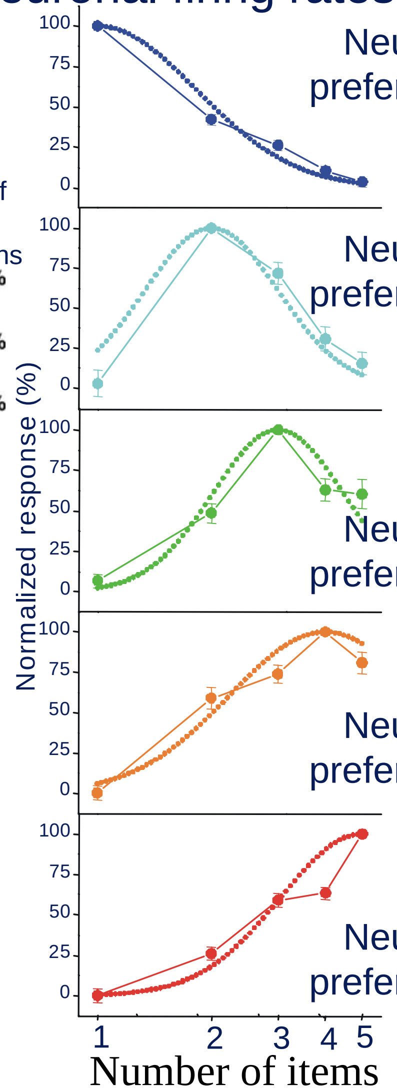
Task:
same-different judgement with small numerosities



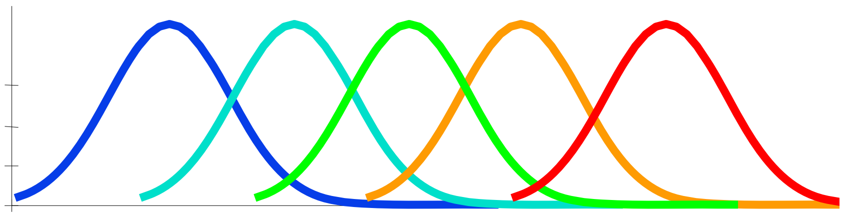
Anatomy



Neuronal firing rates



The Dehaene-Changeux (1993) model:
Coding by Log-Gaussian numerosity detectors



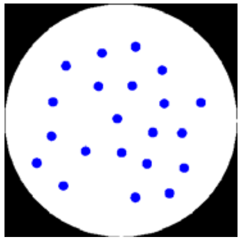
Internal logarithmic scale : log(*n*)

Nieder, A., Freedman, D. J., & Miller, E. K. (2002). Representation of the quantity of visual items in the primate prefrontal cortex. *Science, 297*(5587), 1708-1711.
Nieder, A., & Miller, E. K. (2003). Coding of cognitive magnitude. Compressed scaling of numerical information in the primate prefrontal cortex. *Neuron, 37*(1), 149-157.

From numerosity detectors to numerical decisions: Elements of a mathematical theory

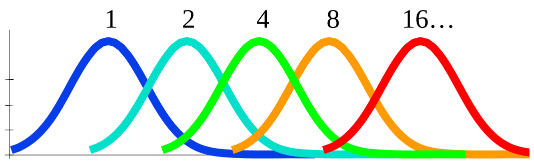
(S. Dehaene, *Attention & Performance* chapter, 2007)

Stimulus of numerosity *n*

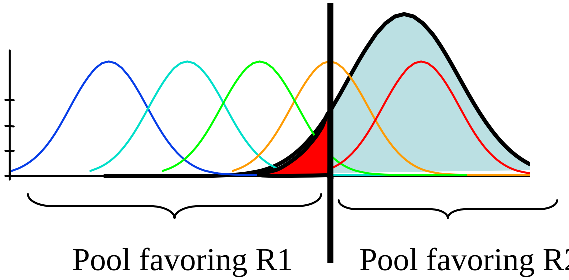


Response in simple arithmetic tasks:
-Larger or smaller than x?
-Equal to x?

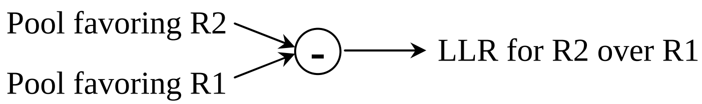
1. Coding by Log-Gaussian numerosity detectors



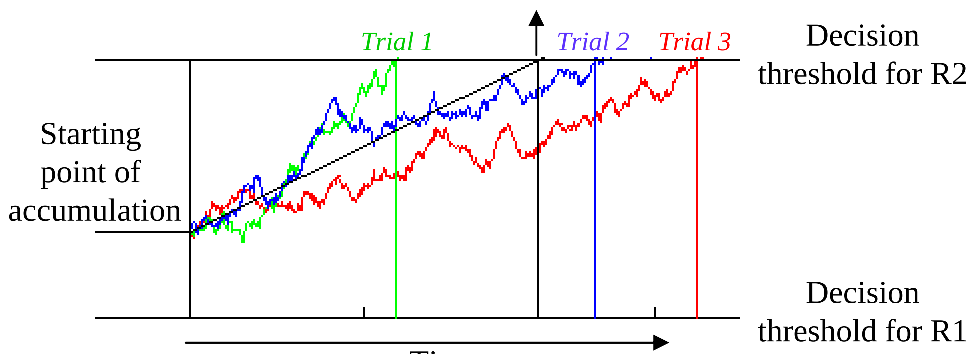
2. Application of a criterion and formation of two pools of units



3. Computation of log-likelihood ratio by differencing

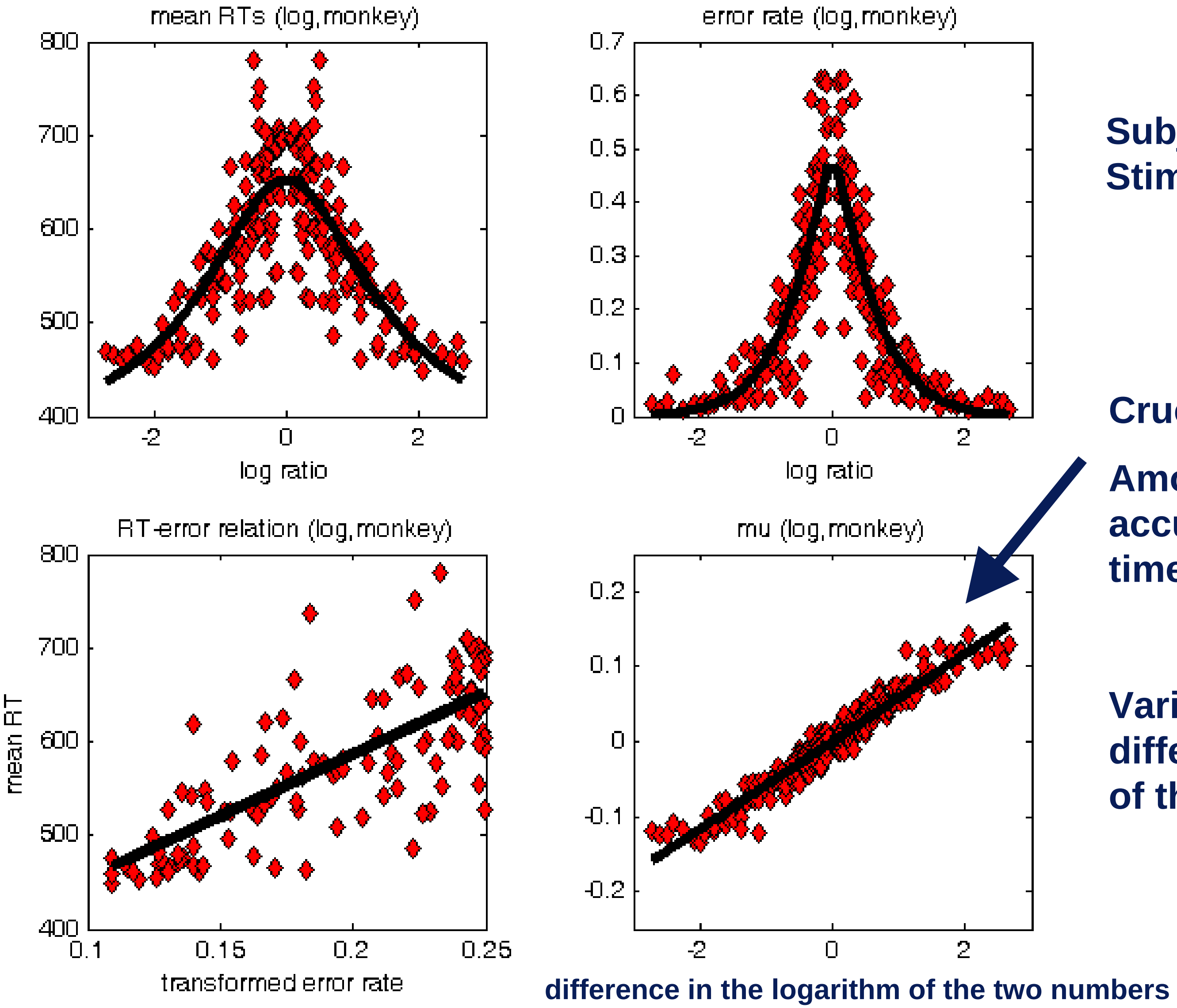


4. Accumulation of LLR, forming a random-walk process



Example: Which of two numerosities is the larger?

Data from Cantlon & Brannon (2006)



Subjects = monkeys
Stimuli = sets of dots

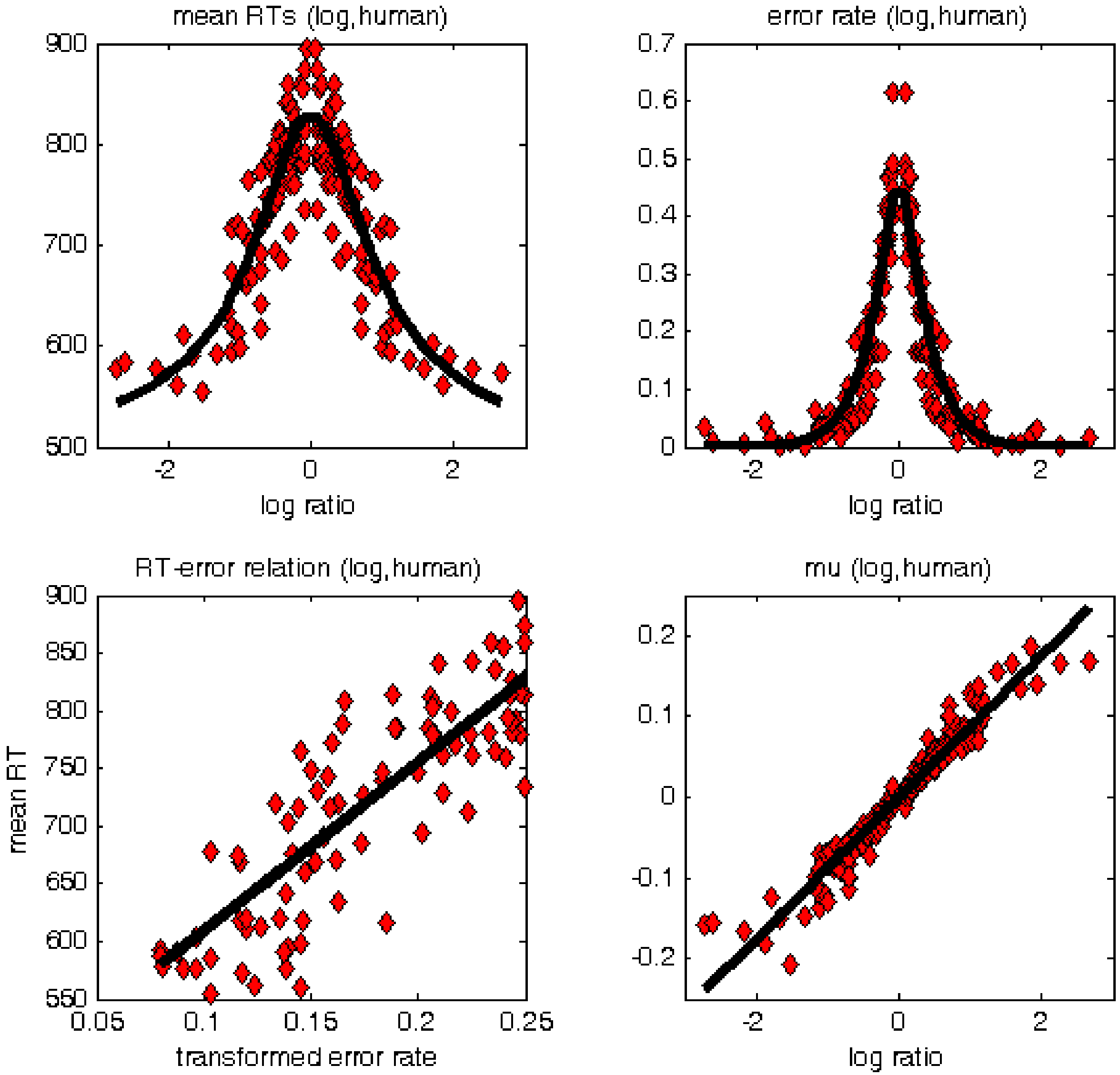
Crucial hidden variable:

Amount of information accumulated per unit of time

Varies linearly with the difference in the logarithm of the two numbers

Example: Which of two numerosities is the larger?

Cantlon, J. F., & Brannon, E. M. (2006). Shared system for ordering small and large numbers in monkeys and humans. *Psychol Sci, 17*(5), 401-406.

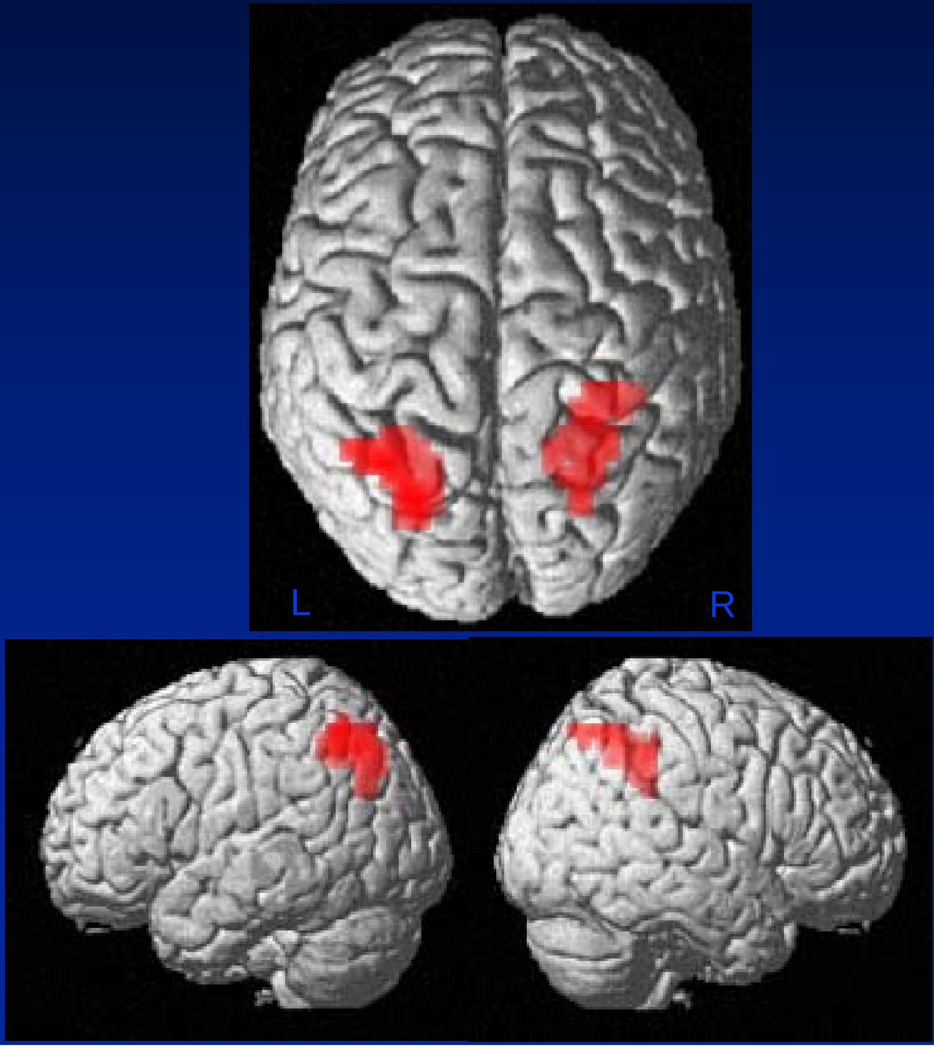


- **Humans and monkeys have very similar competence for numerosity decision (though humans are slighly slower and more precise)**
- **RTs and errors have exactly the shape predicted by the theory**
- **Performance depends on numerosity ratio (or equivalently, difference of logarithms)**

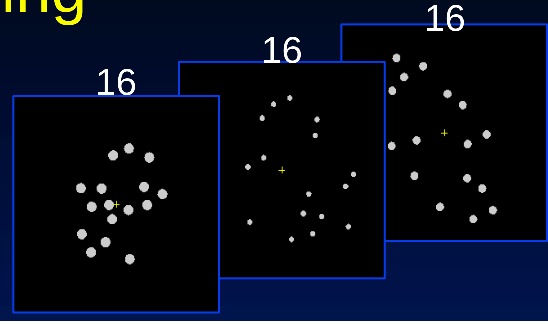
Does human IPS contain number neurons? fMRI adaptation reveals Log-Gaussian turning in the human intraparietal sulcus

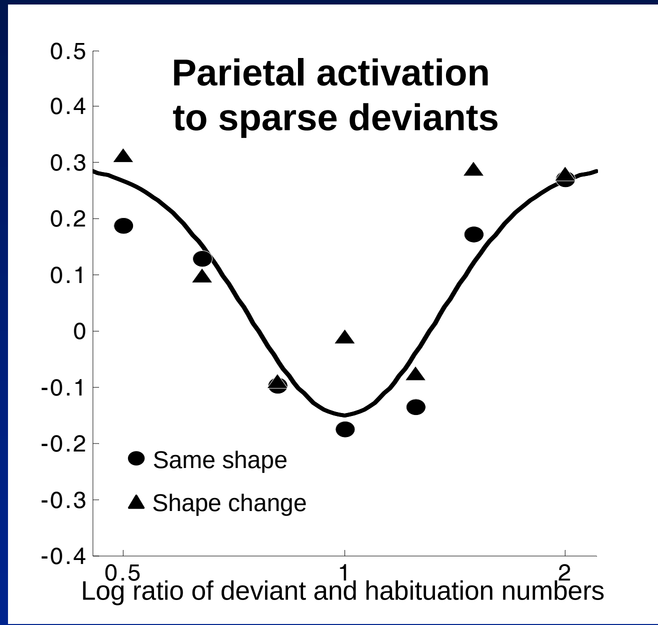
Piazza, Izard, Pinel, Le Bihan & Dehaene, Neuron 2004

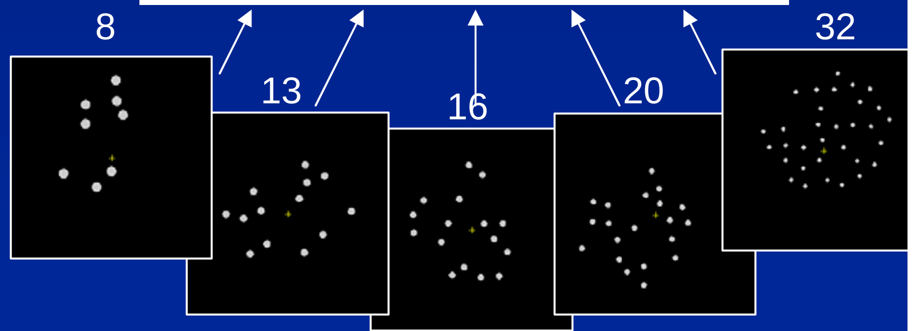
Regions responding to a change in number



Adaptation to a fixed number

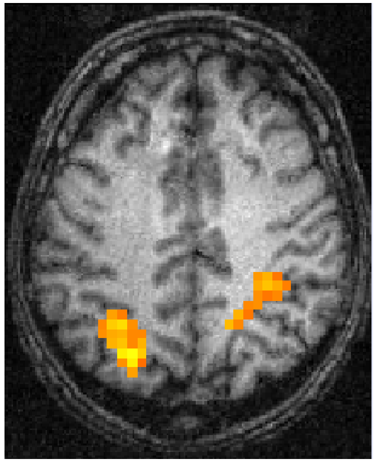






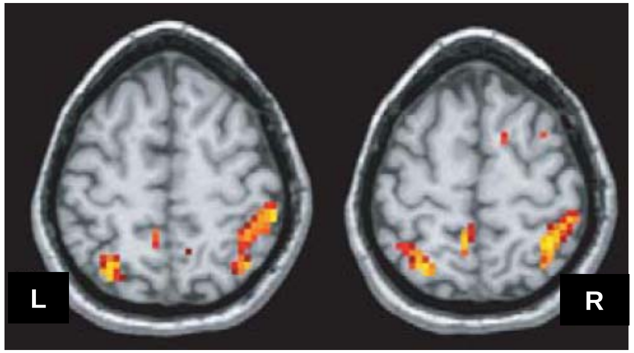
A basic dorsal-ventral organization for shape vs number

Initial study: effect of number change

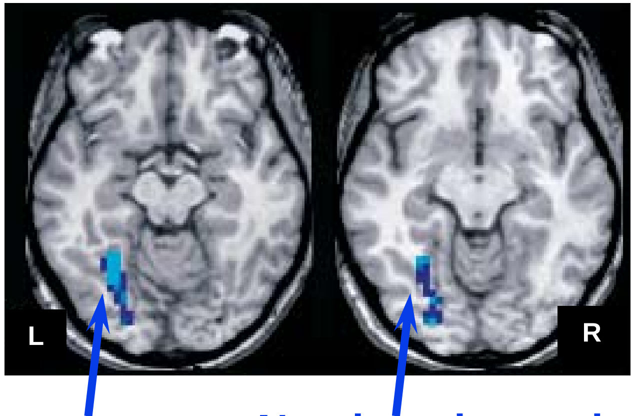


Number change in intraparietal cortex

Improved design by Cantlon, Brannon et al. (PLOS, 2006):

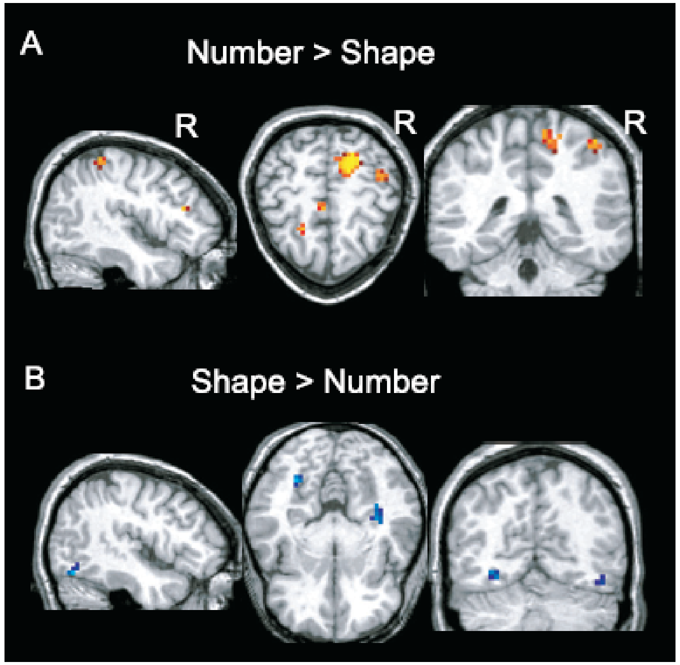


Number change > Shape change in bilateral intraparietal sulci

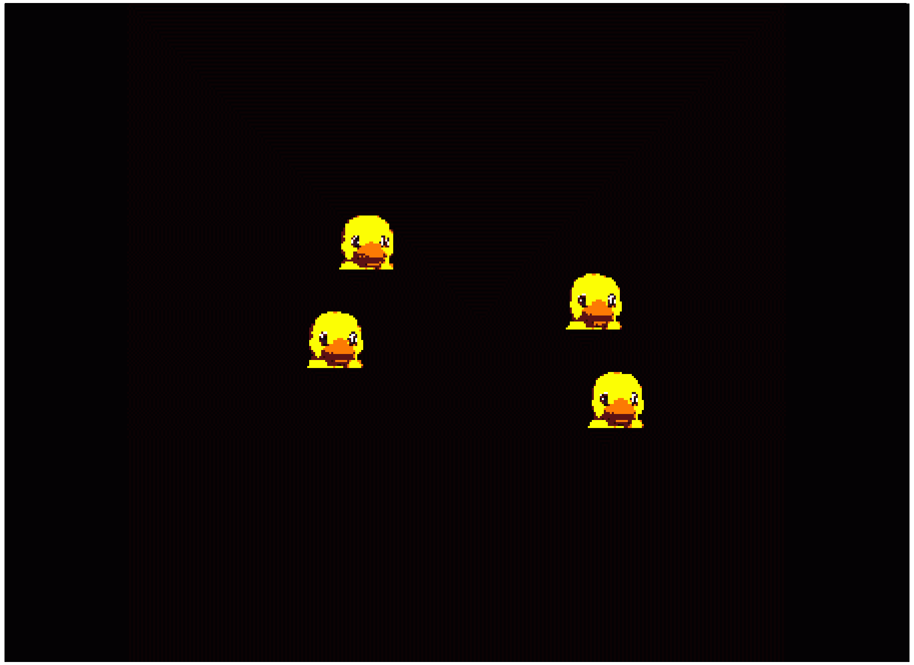


Shape change > Number change in left inferior temporal cortex

Number and shape in four-year-olds



Do infants show numerosity adaptation and recovery?
An ERP experiment



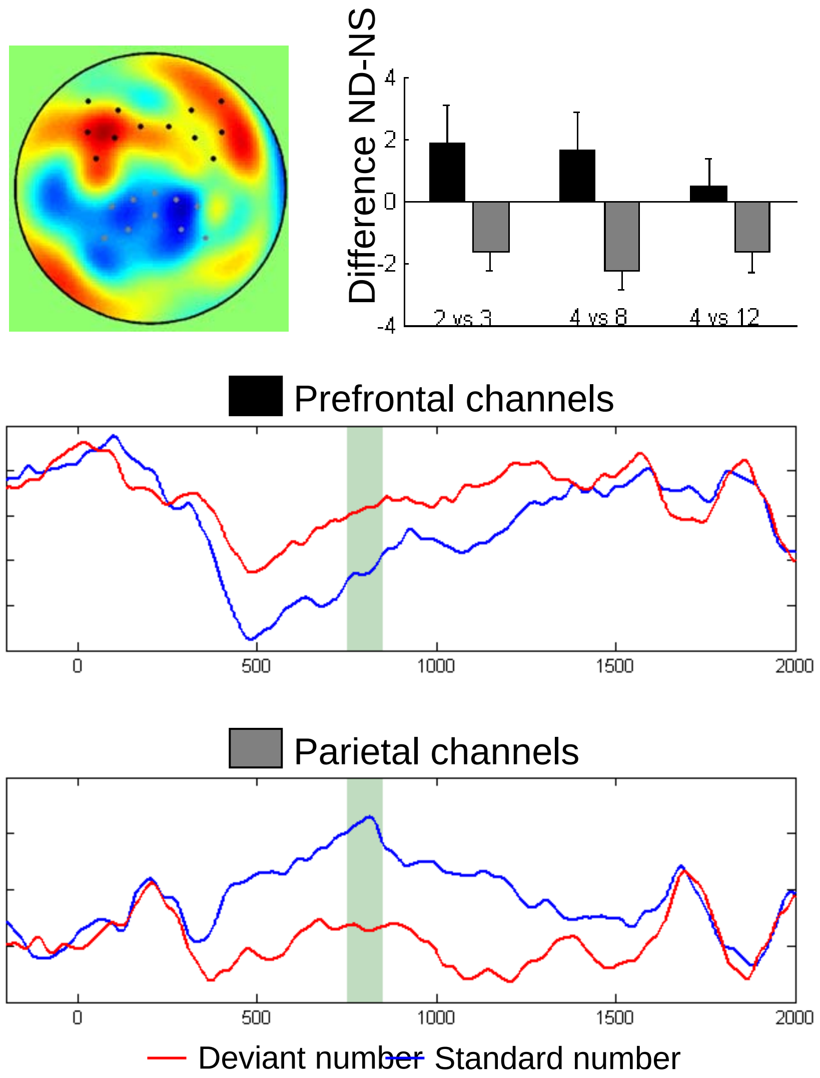
2 x 2 design : numerosity and/or object change

3 pairs of numerosities:
4 vs 8 ; 4 vs 12 ; 2 vs 3

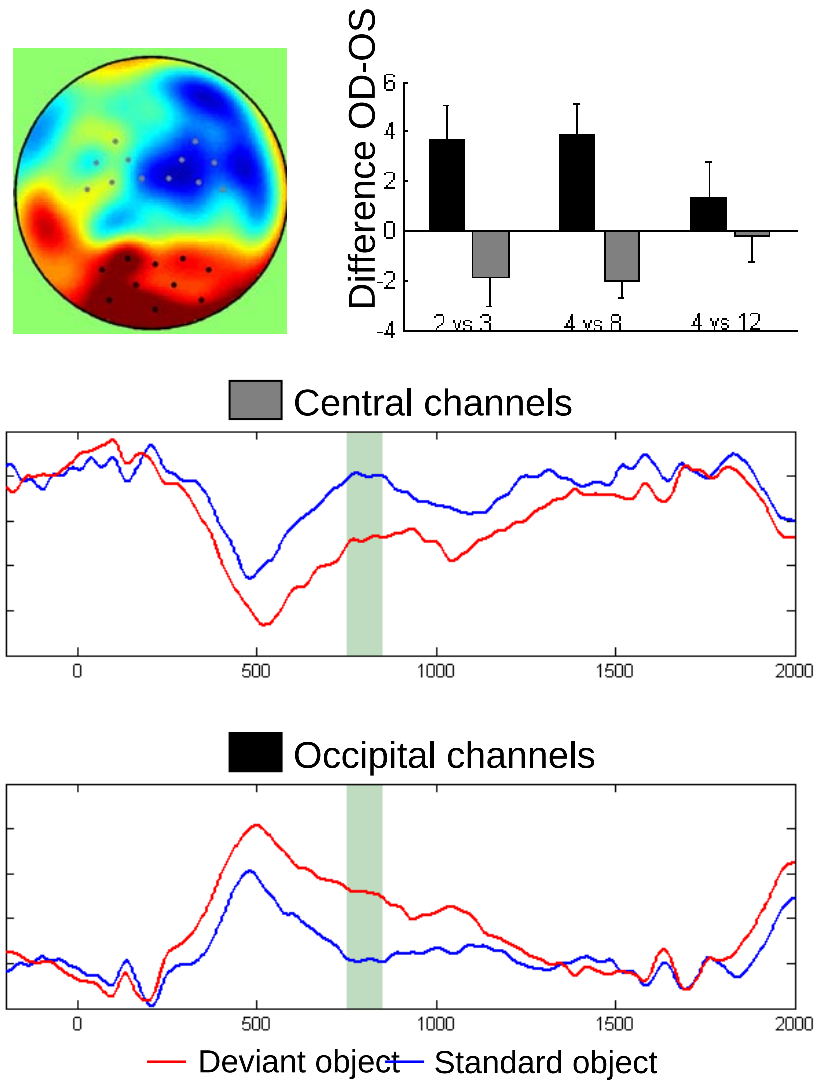
Twelve 3-4 month-old infants in each group

Véronique Izard
Ghislaine Dehaene-Lambertz

Number Change



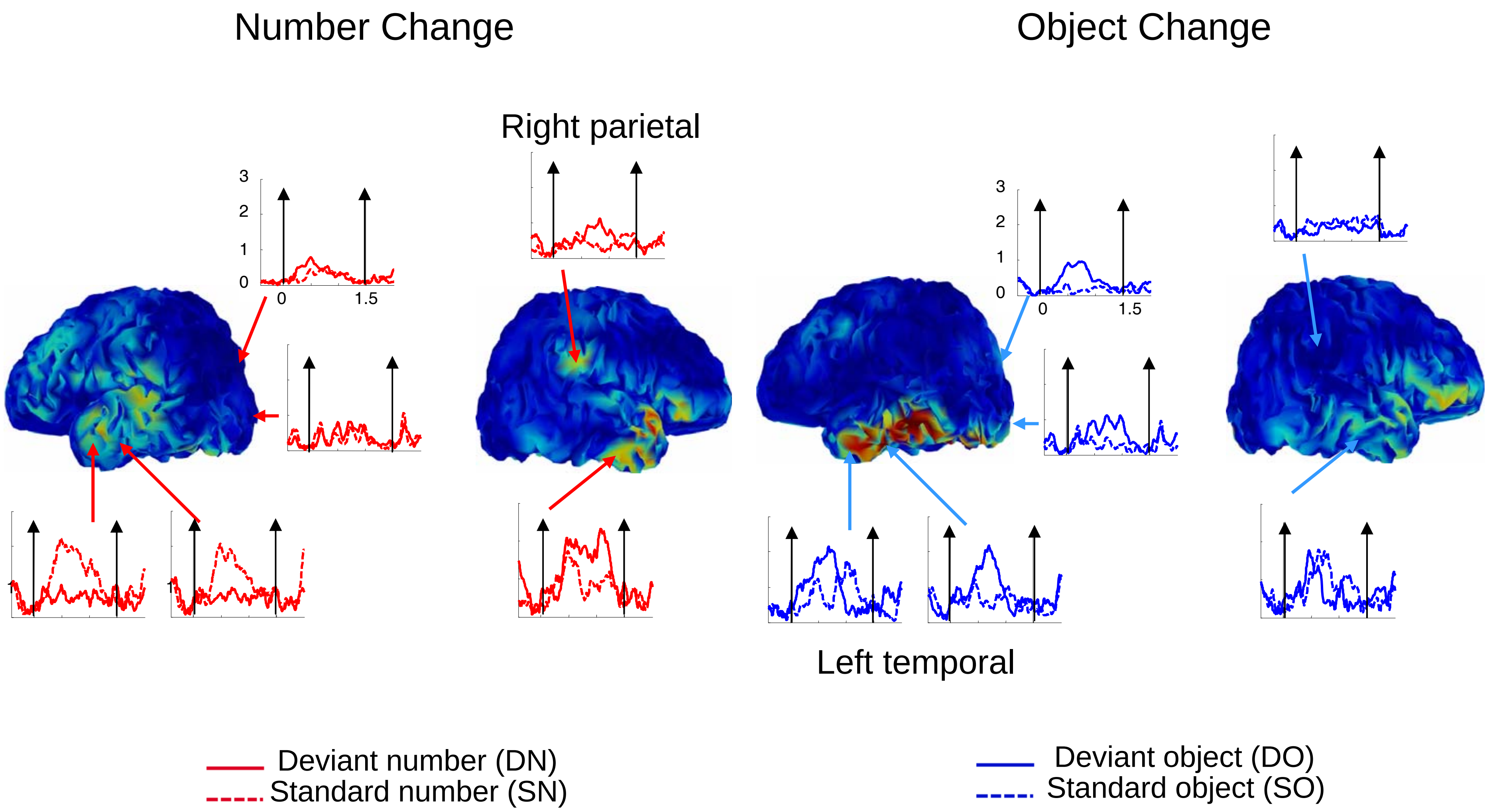
Object Change



Véronique Izard
Ghislaine Dehaene-Lambertz

A basic dorsal / ventral organization in 3-4 month old infants:

Right parietal response to number, left temporal response to objects



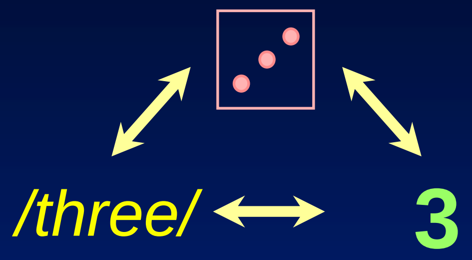
Plan of the talk

- Neural coding of numerosity
  - A quantity code is present in the intraparietal area of human adults and babies
  - Optimal decision mechanisms based on this code can explain human psychophysics
- Understanding of number symbols
  - Symbols are mapped onto numerosities
  - However, the numerosity code may be changed by learning symbols
  - Linear mapping of number onto space
- Brain mechanisms of number-space mappings

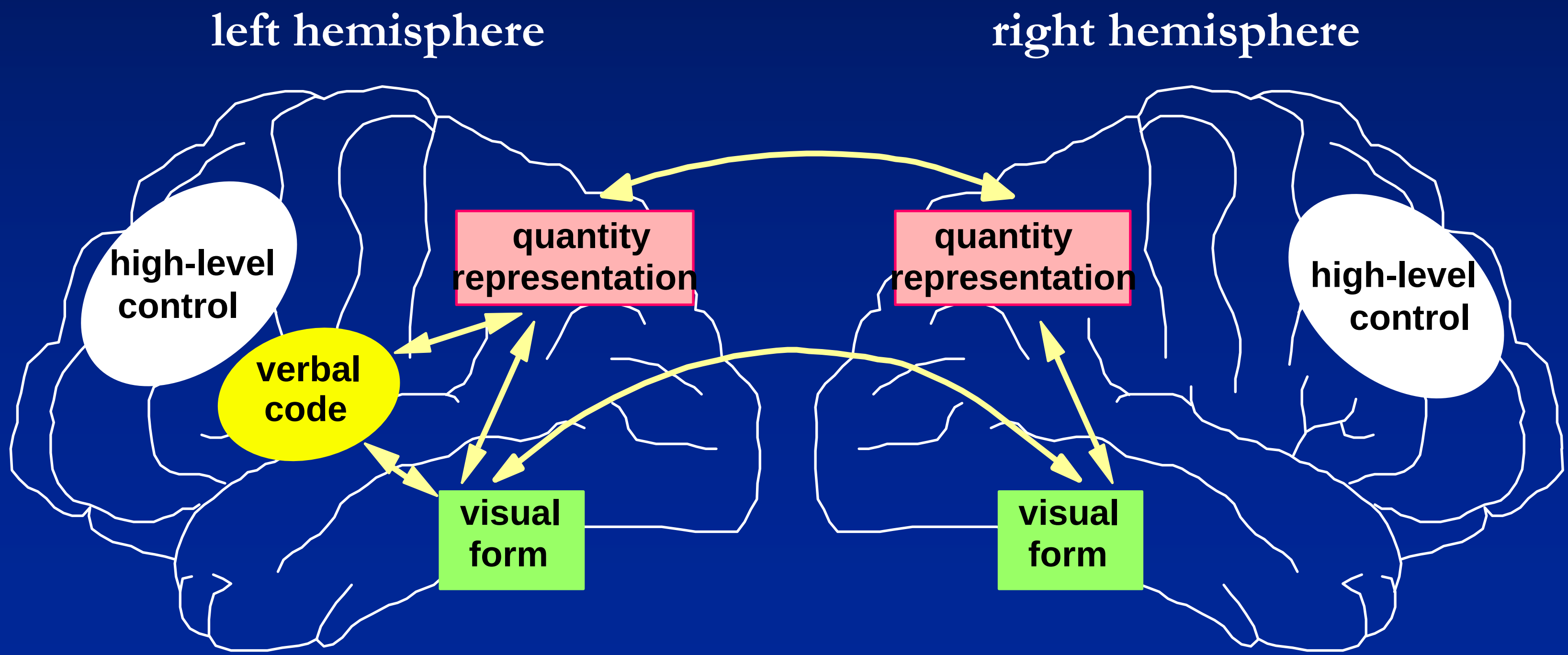
Attaching symbols to quantities:
The triple-code model of number processing

Quantity code
Number comparison; Proximity judgment; Approximation
Quantity manipulations (e.g. subtraction)

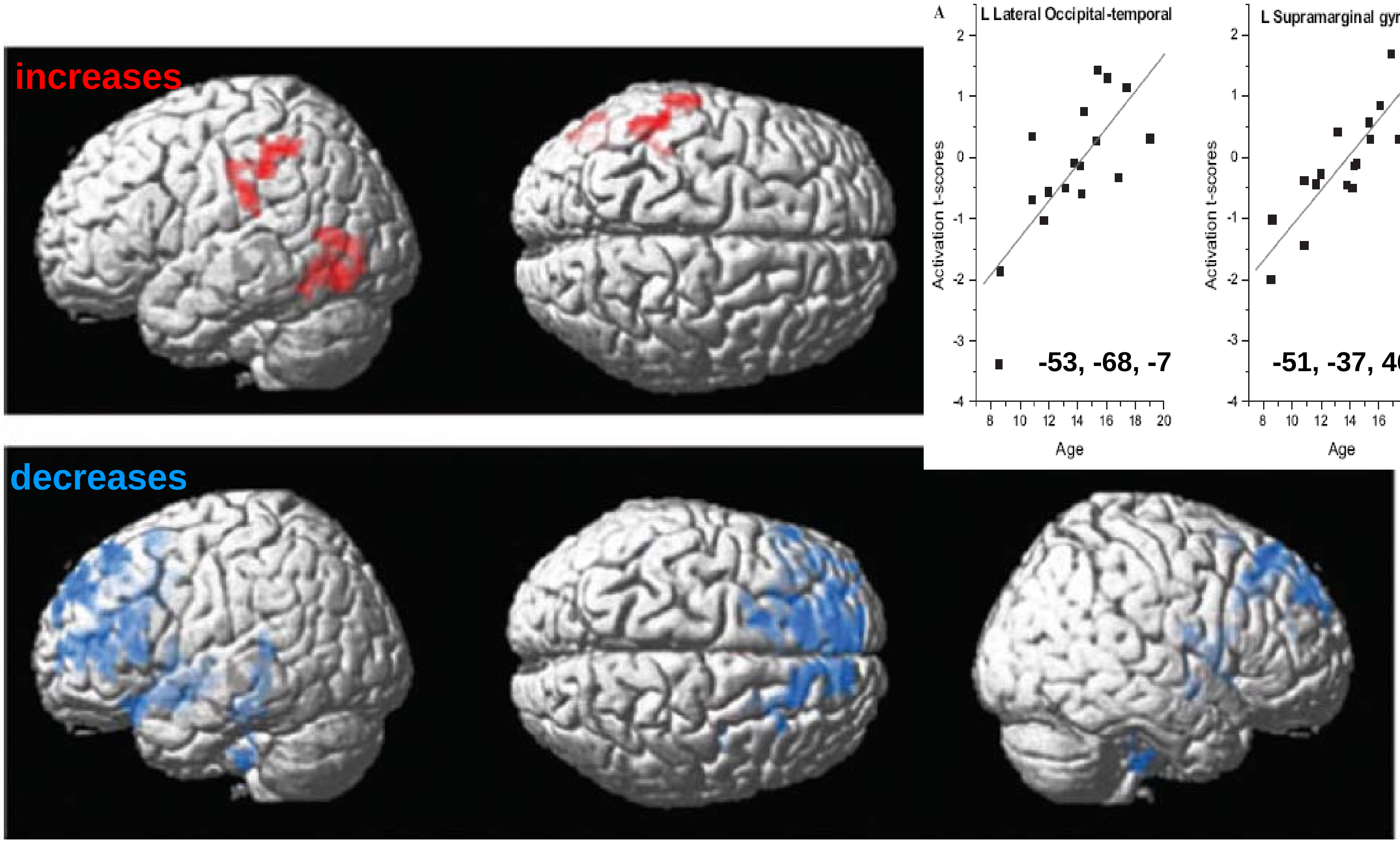
Verbal code
Spoken comprehension and production
rote tables (e.g. multiplication)



Arabic code
Arabic reading and writing
multidigit calculations



Changes in activation with age during mental arithmetic

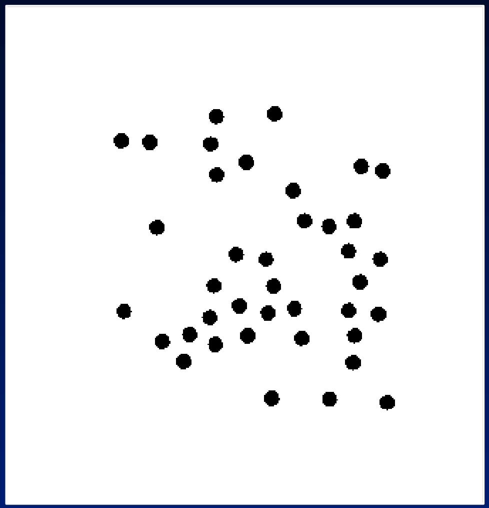


Rivera, S. M., Reiss, A. L., Eckert, M. A., & Menon, V. (2005). Developmental Changes in Mental Arithmetic: Evidence for Increased Functional Specialization in the Left Inferior Parietal Cortex. *Cereb Cortex, 15*(11), 1779-1790.

For a related result (increase in left parietal distance effect from 10 years-old to adults), see:
Ansari, D., & Dhital, B. (2006). Age-related changes in the activation of the intraparietal sulcus during nonsymbolic magnitude processing: an event-related functional magnetic resonance imaging study. *J Cogn Neurosci, 18*(11), 1820-1828.

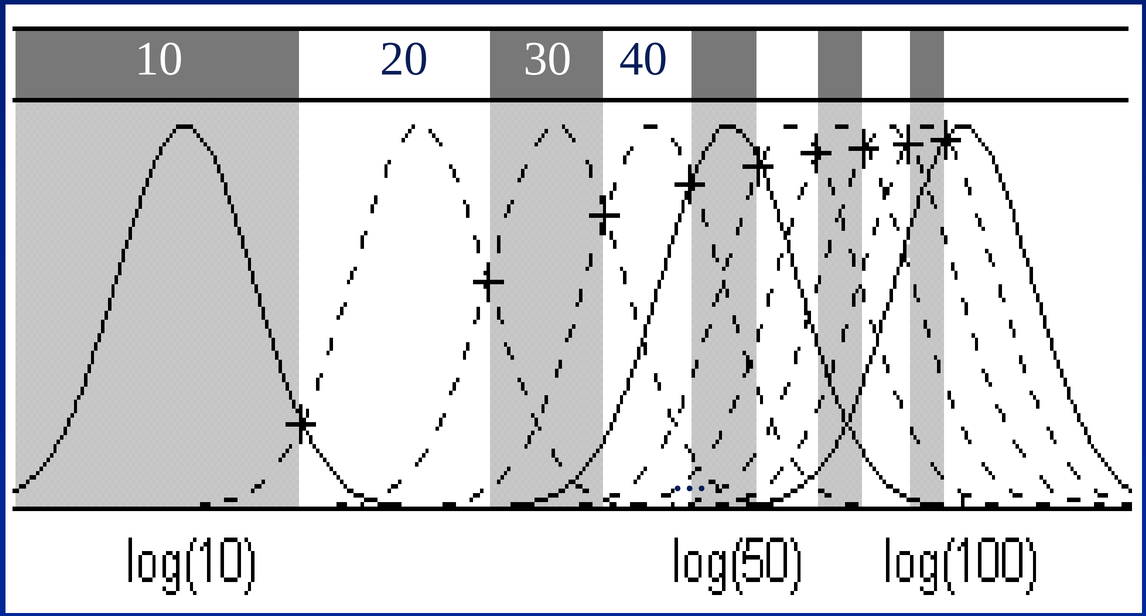
Model of numerosity naming

(Izard & Dehaene, *Cognition* 2007)



How many dots?

Theory:
Each number word corresponds to a range on the analog number line

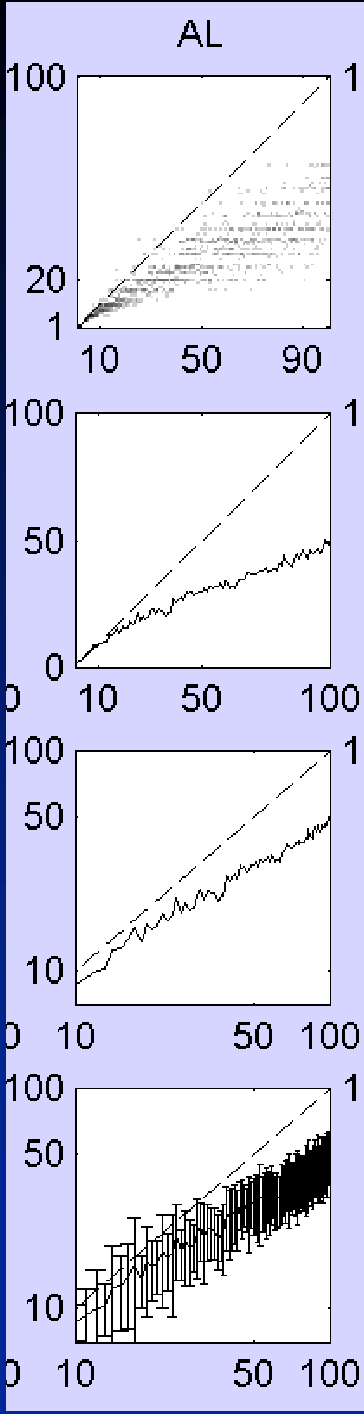


Distribution of the responses (linear plot)

Mean response (linear plot)

Mean response (loglog plot)

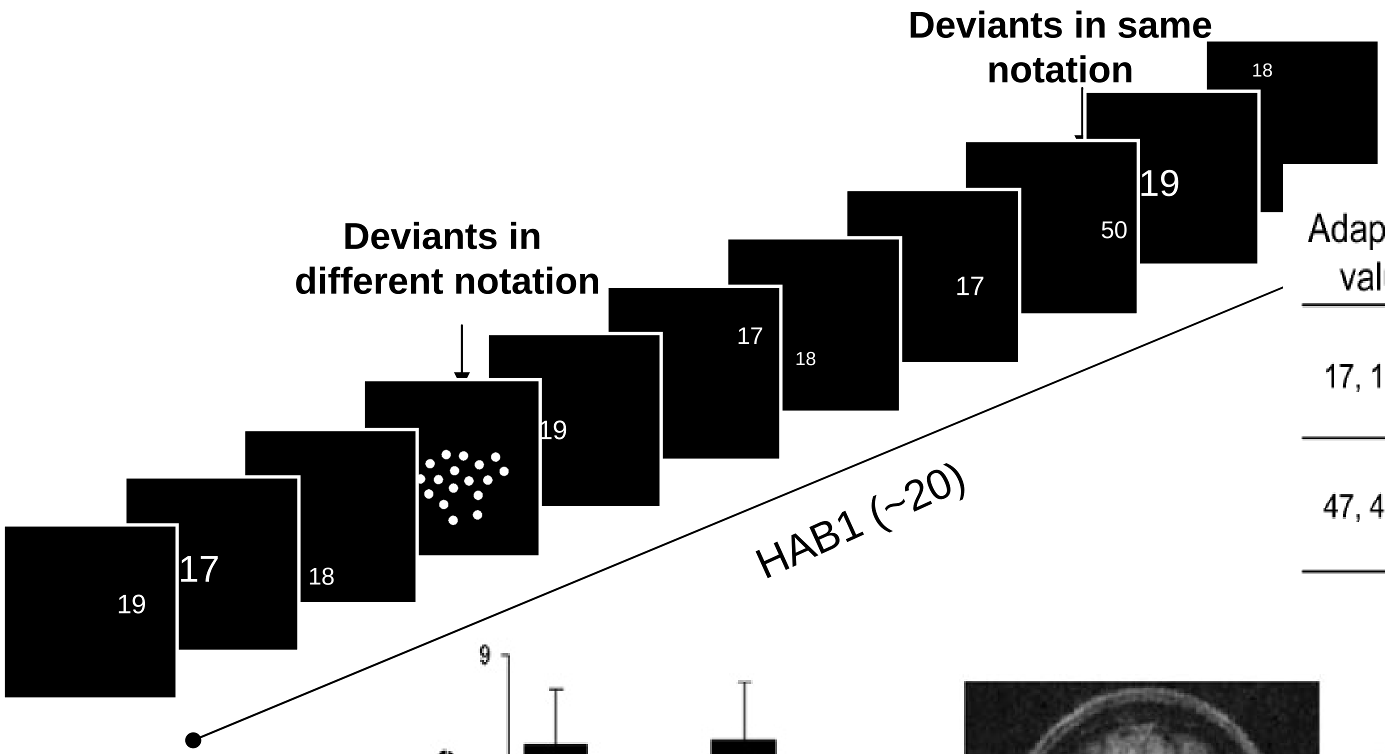
Standard deviation of the responses (loglog plot)



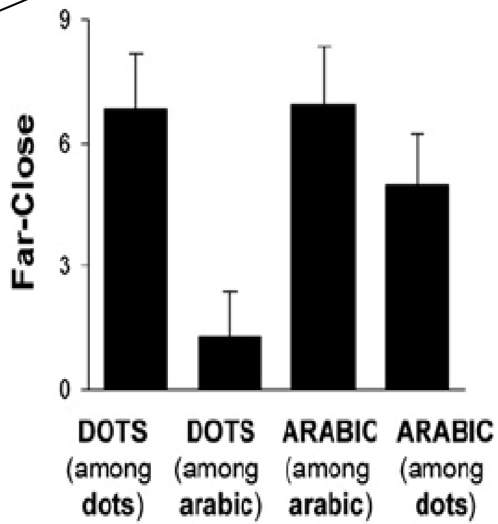
An fMRI study of cross-notation adaptation

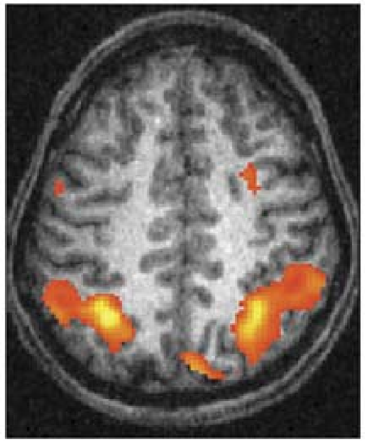
Piazza, Pinel and Dehaene, Neuron 2007

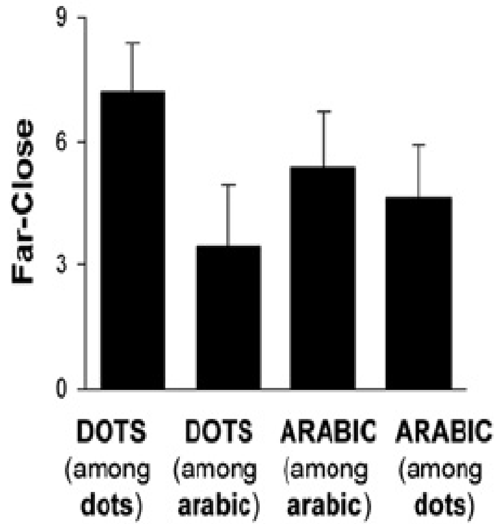
- Do the same neurons code for the symbol 20 and for twenty dots?



Adaptation values	Deviant values: 20	Deviant values: 50
17, 18, or 19	close	far
47, 48, or 49	far	close

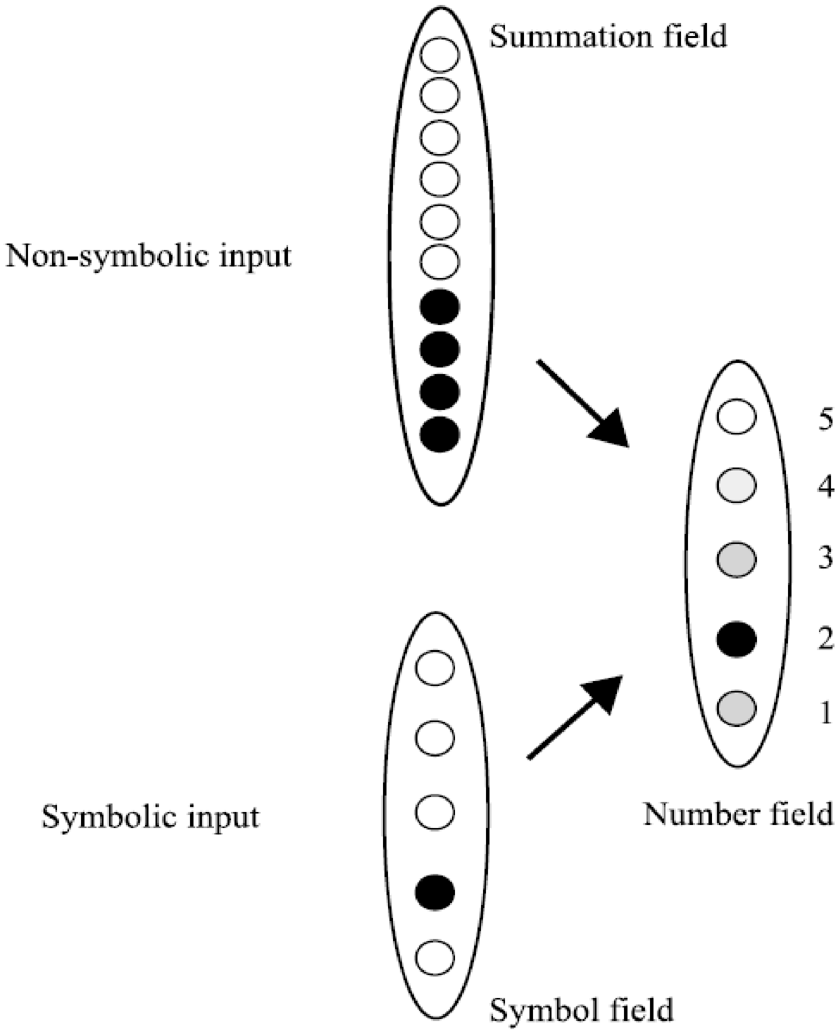






How is the numerosity representation changed by learning symbols?

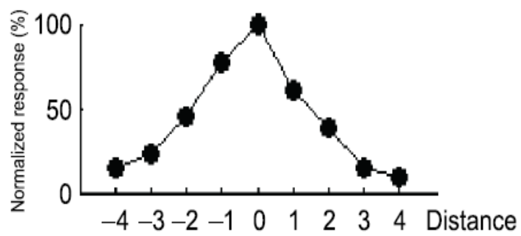
Verguts, T., & Fias, W. (2004). Representation of number in animals and humans: a neural model.
J Cogn Neurosci, 16(9), 1493-1504.



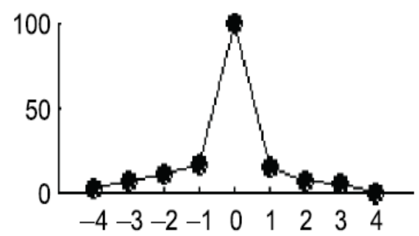
After learning, in the ***same*** neurons…

The numerosity tuning curves become narrower

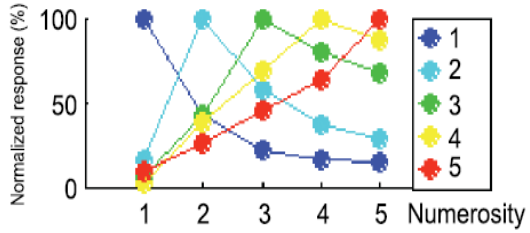
Non-symbolic input

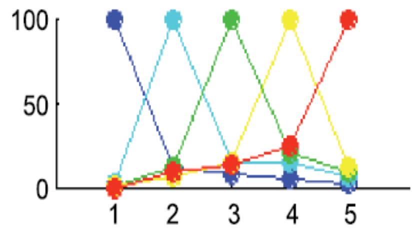


Symbolic input



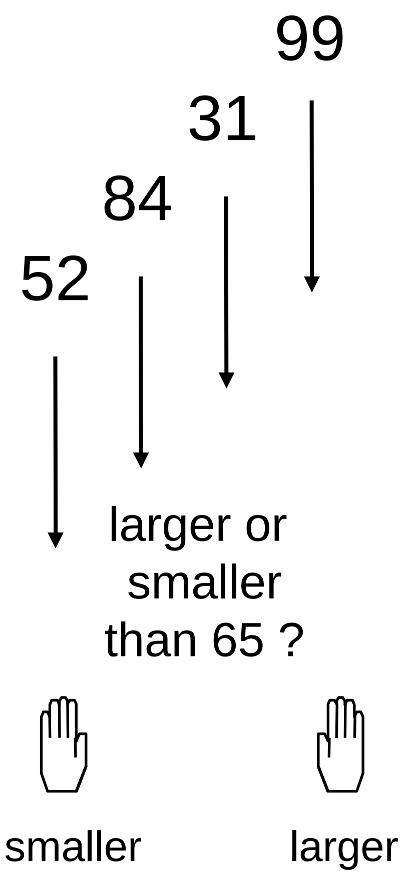
They cease to increase in width with number

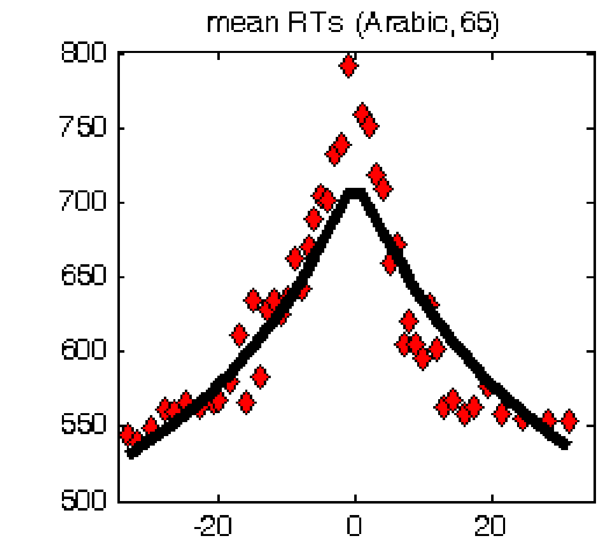




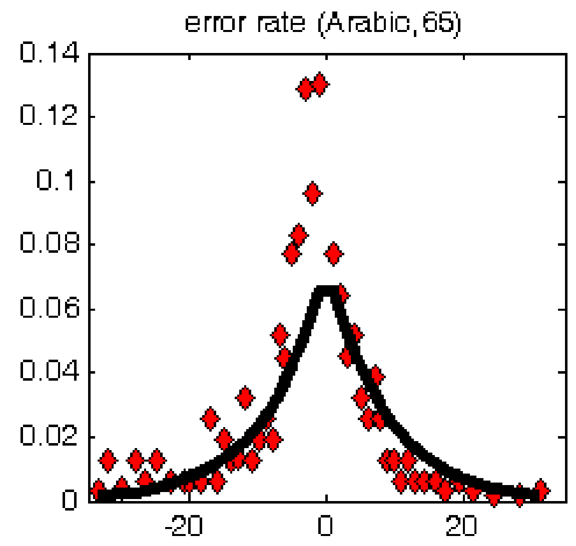
Which of two *Arabic numerals* is the larger?

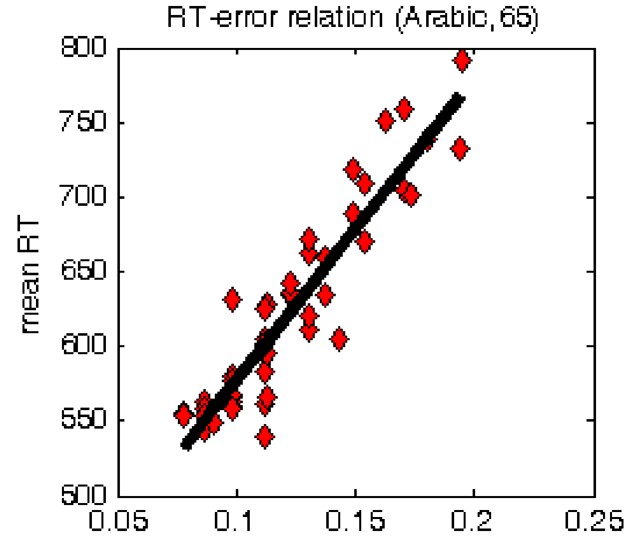
Subjects = humans
Stimuli = Arabic numerals



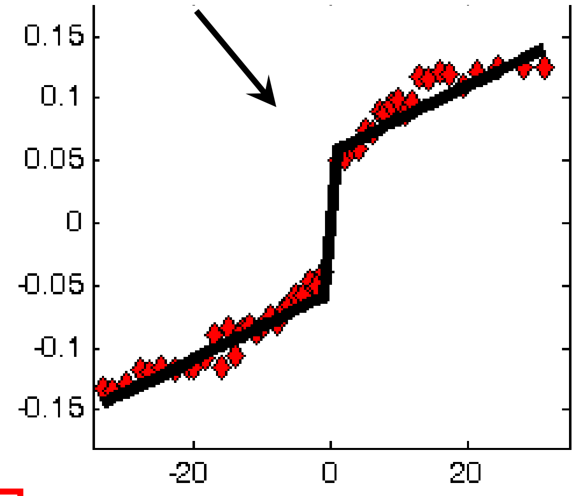


linear difference between the two numbers





Amount of information accumulated per unit of time

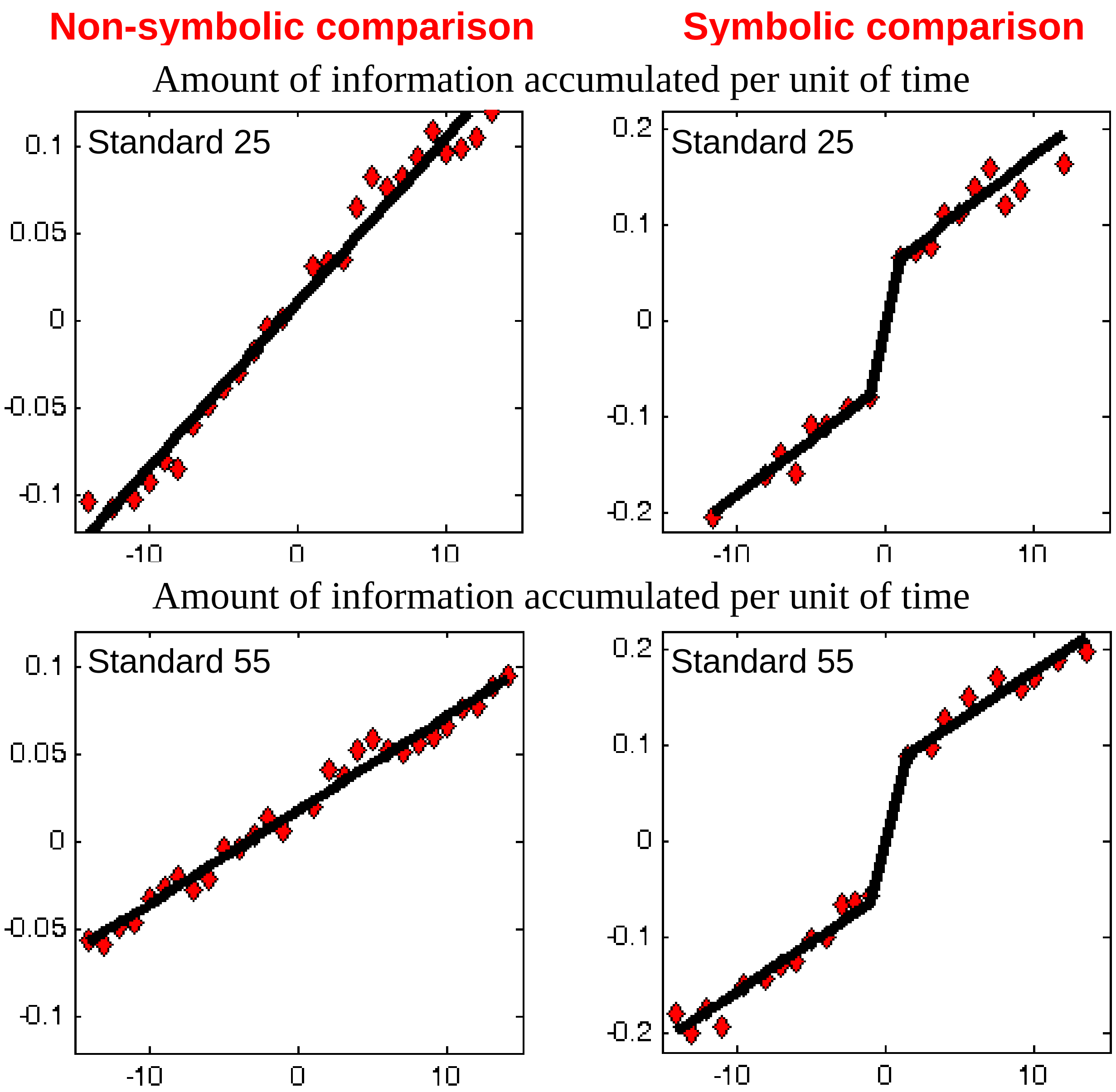


linear difference between the two numbers

Performance depends on the linear difference of the two numbers

Non-symbolic and symbolic comparison within the same subjects

10 human adults compared sets of dots or Arabic numerals to a fixed reference, either 25 or 55



Conclusion

The number representation is profoundly different for symbolic and non-symbolic numbers:

-Exact, not approximate representation

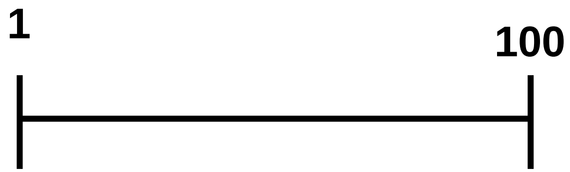
-Linear, not logarithmic representation

Development of the linear understanding of number

(Siegler & Opfer, 2003; Siegler & Booth, 2004)

Number-Space mapping task:

« Please point to where number x should fall »



A major change occurs during mathematical education : switch from a logarithmic to a linear understanding of number

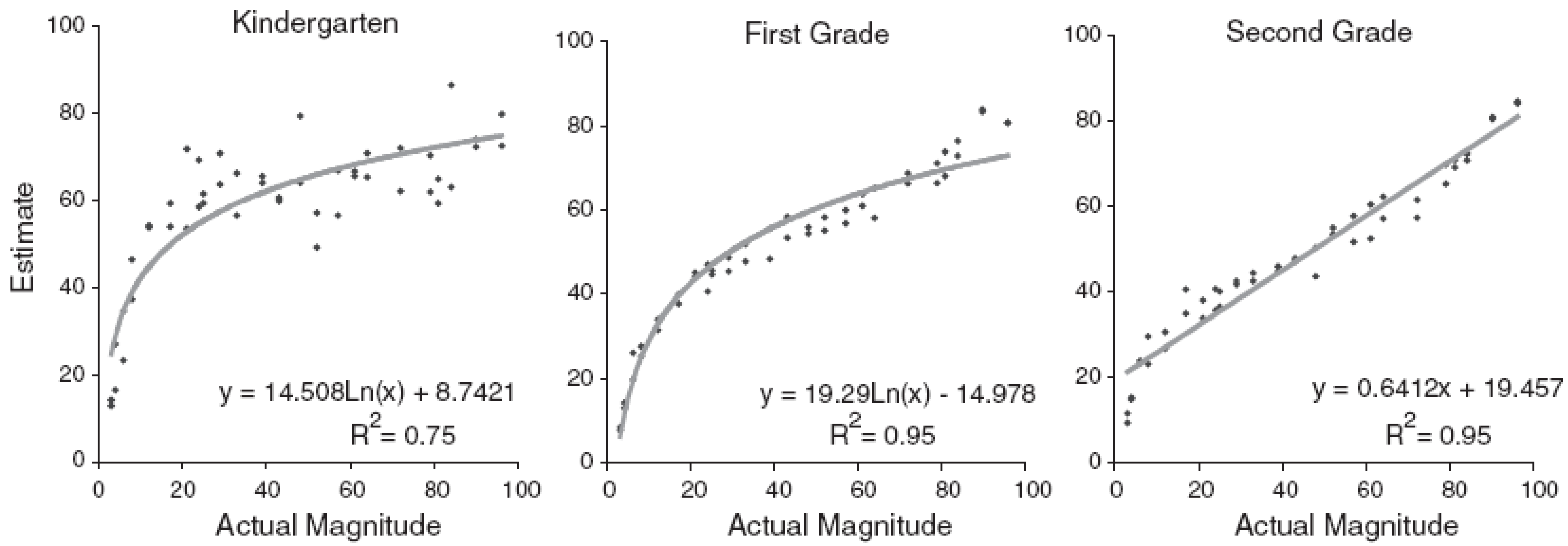


Figure 2. Progression from logarithmic pattern of median estimates among kindergartners (left panel) to linear pattern of estimates among second graders (right panel) in Experiment.

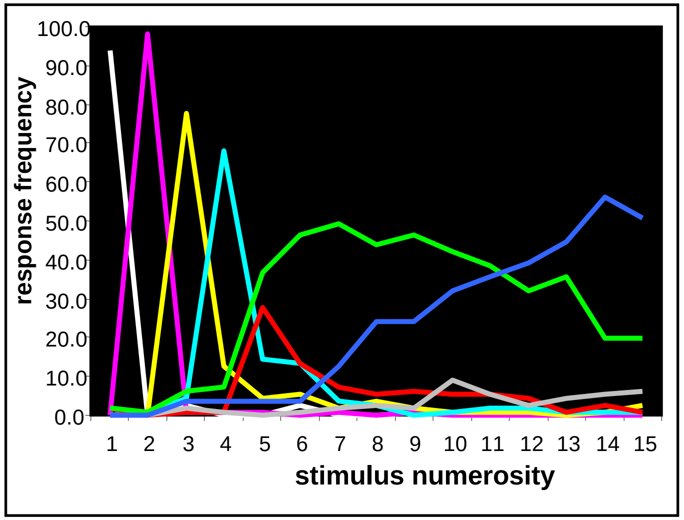
Numerical cognition without words in the Munduruku

Pica, Lemer, Izard, & Dehaene, Science, 2004

- pug ma = one
- xep xep = two
- ebapug = three
- ebadipdip = four
- pug põgbi = one hand
- xep xep põgbi = two hands
- adesu/ade gu = some, not many
- ade/ade ma = many, really many

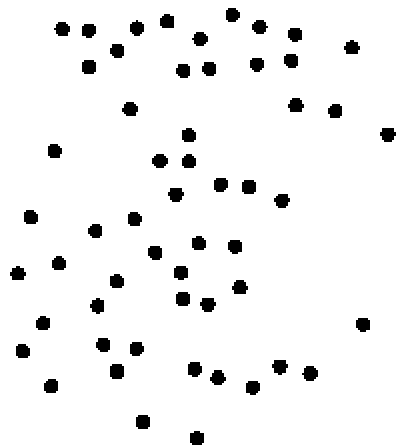
A reduced lexicon of number words

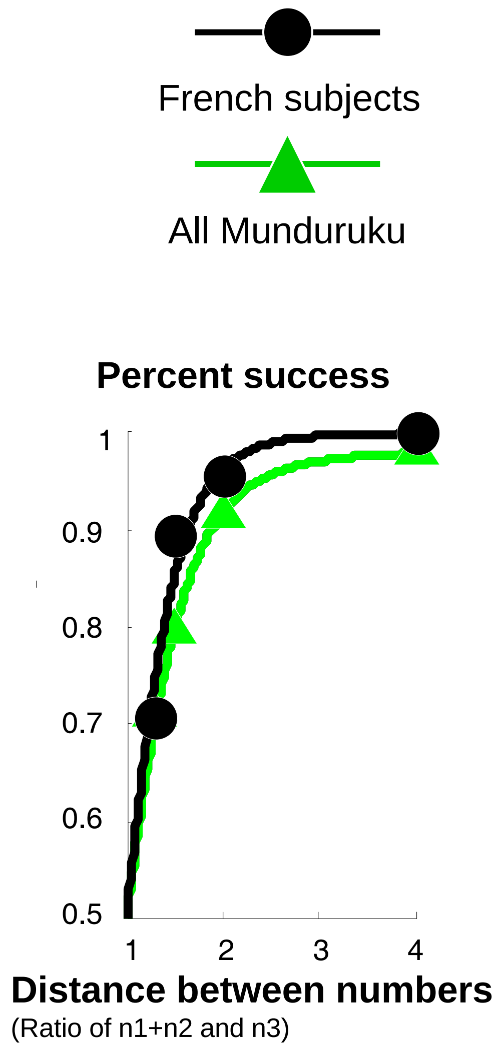
Munduruku number words refer to approximate numerosity



Munduruku adults and children can do approximate arithmetic with non-verbal numerosities (e.g. 40+30 is larger than 50) but not exact arithmetic (e.g. 7-6=1)

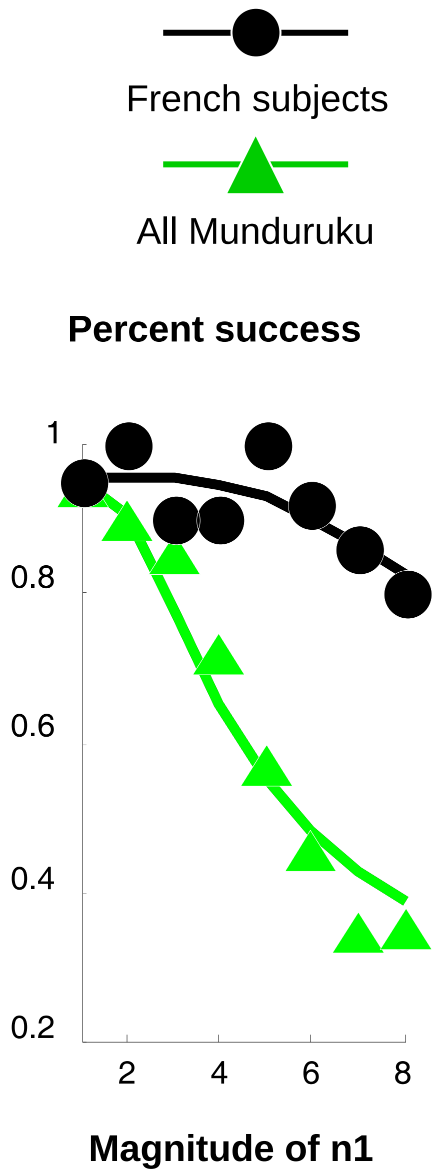
Success in approximate addition and comparison





Pica, Lemer, Izard, & Dehaene, Science, 2004

Failure in exact subtraction of small quantities



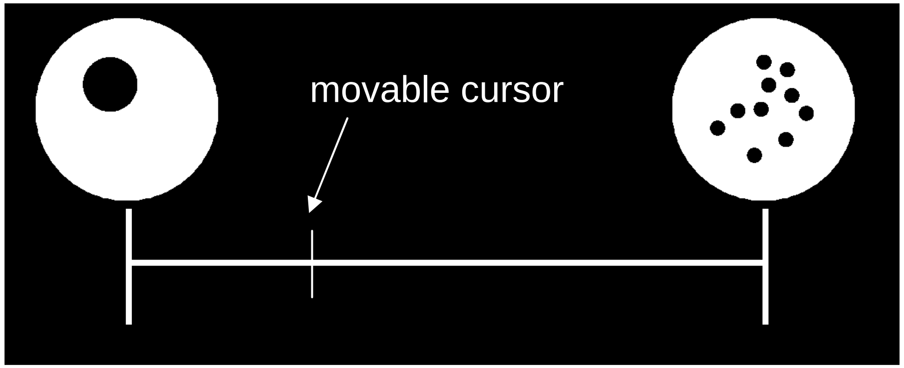
Pica, Lemer, Izard, & Dehaene, Science, 2004

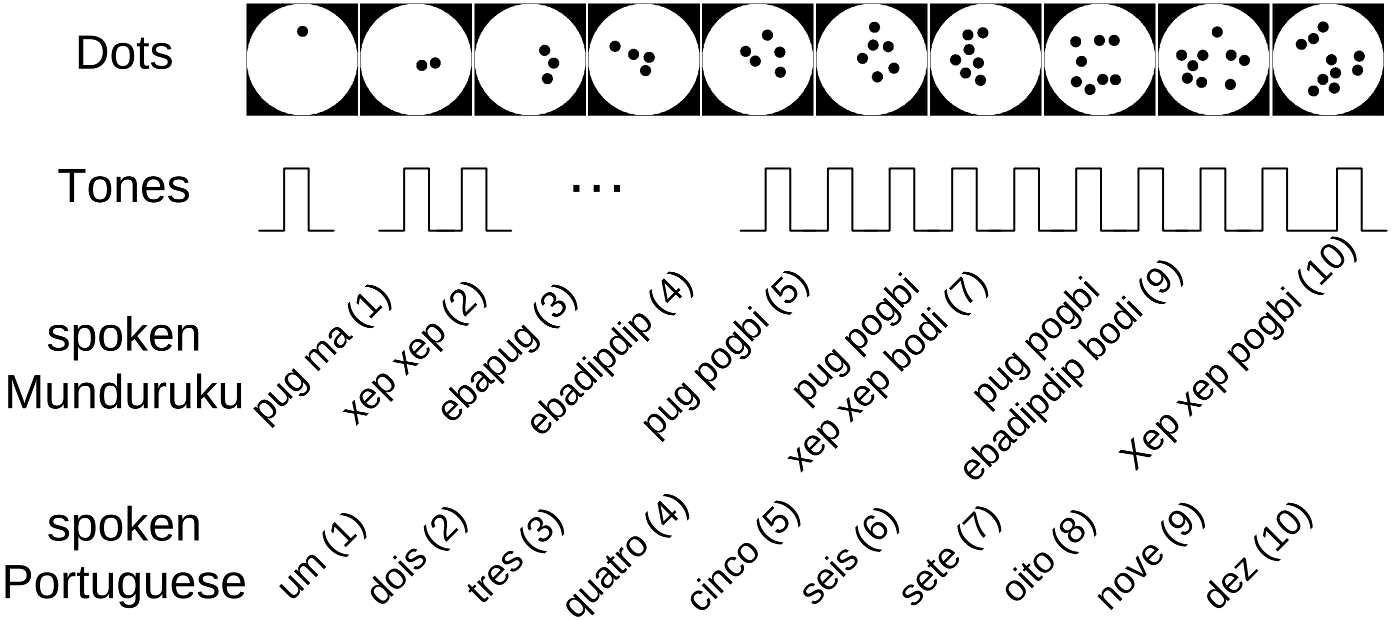
A trace of approximate number sense in Munduruku sculpture?

In this Munduruku necklace, two miniature hands (out of 35) only have three fingers!

Number-Space mapping in the Munduruku

Munduruku children and adults were asked to point to the location corresponding to a certain number.
Would they show a compressive mapping even in adults? And for numbers as small as 1-10?





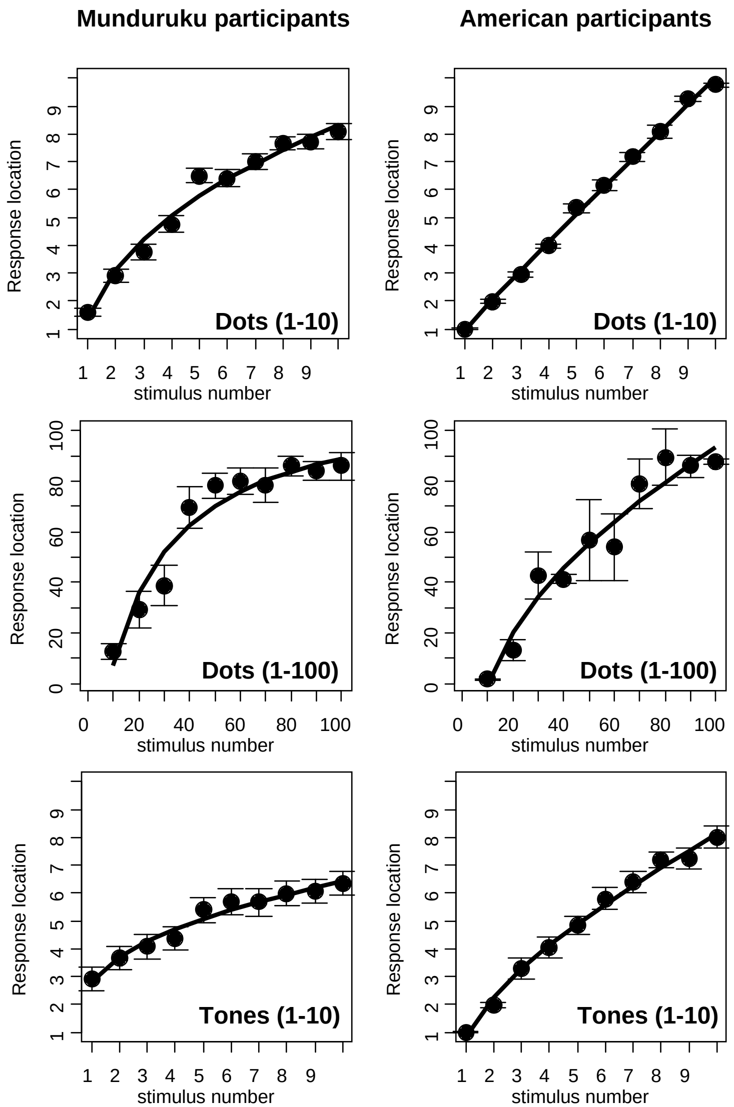
Dehaene, Izard, Spelke and Pica, in preparation

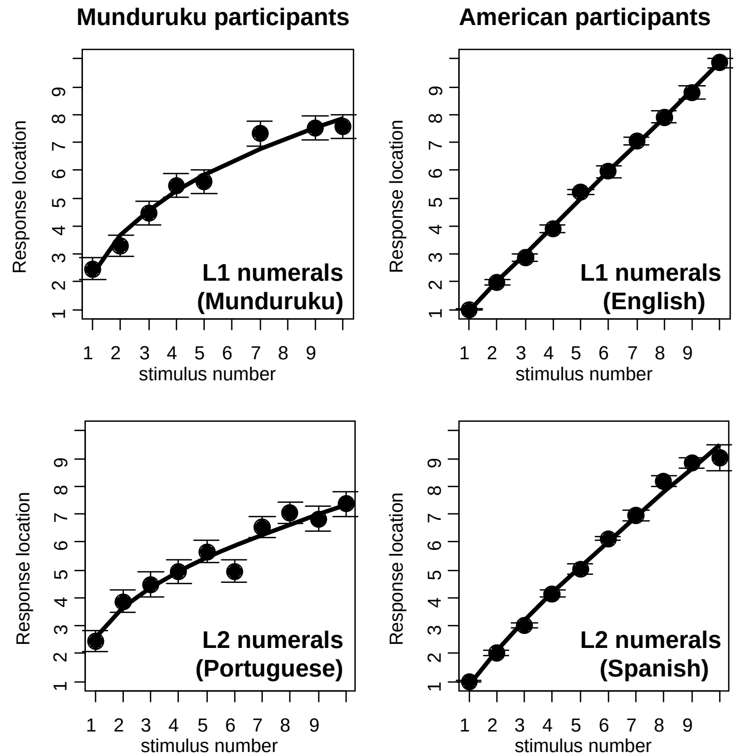
Logarithmic Number-Space mapping in the Munduruku

Munduruku children and adults show a compressive mapping

- For dot patterns or series of 1-10 tones

- For Munduruku words and even for Portuguese numerals



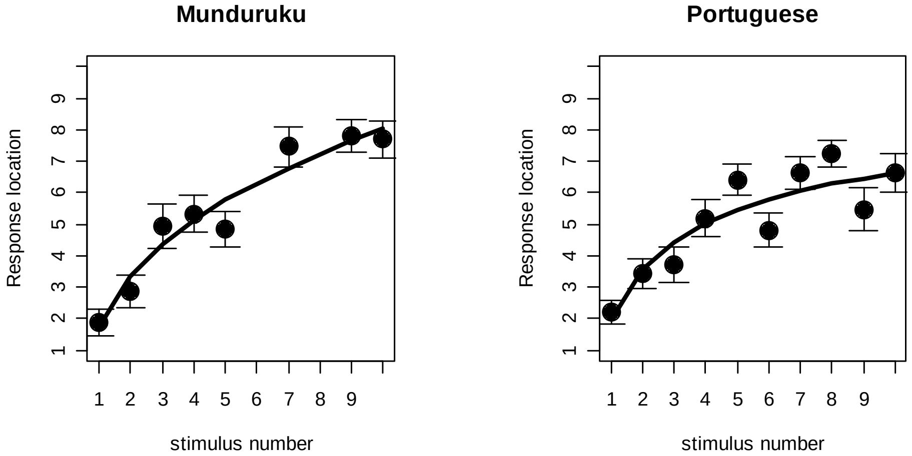


Dehaene, Izard, Spelke and Pica, in preparation

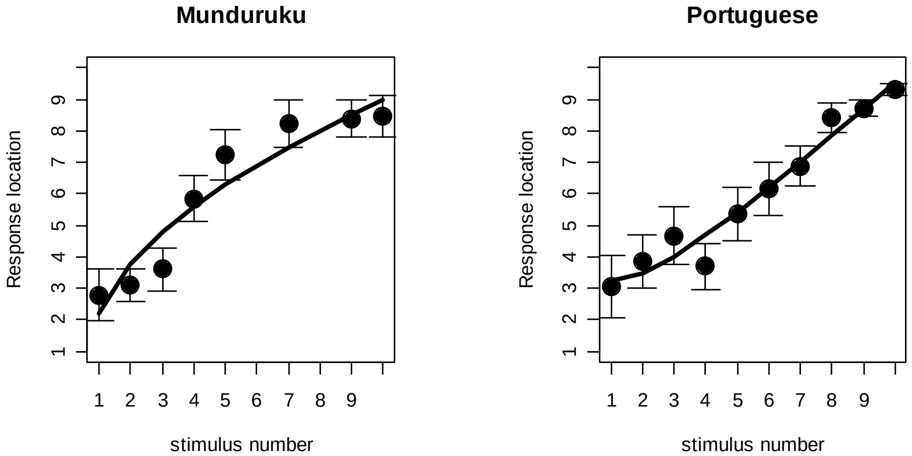
Effect of education on log versus linear responding in Portuguese, but not in Munduruku

Munduruku participants only

Very little education

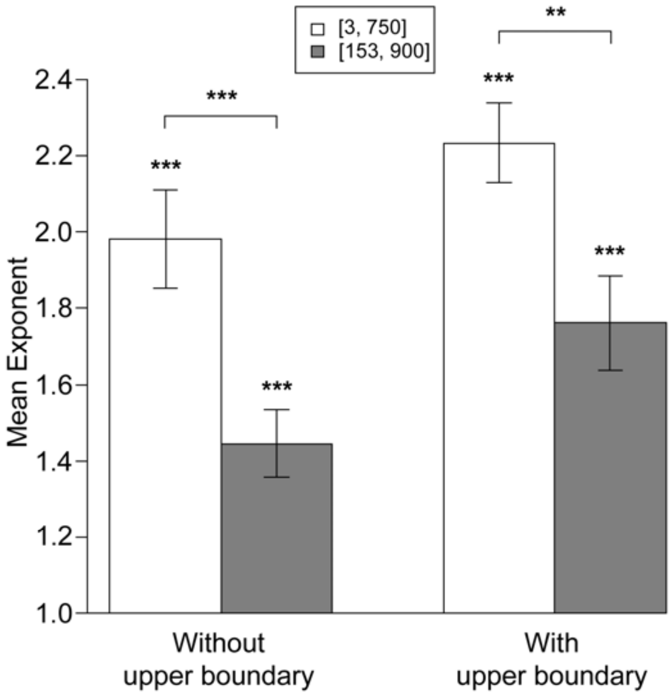


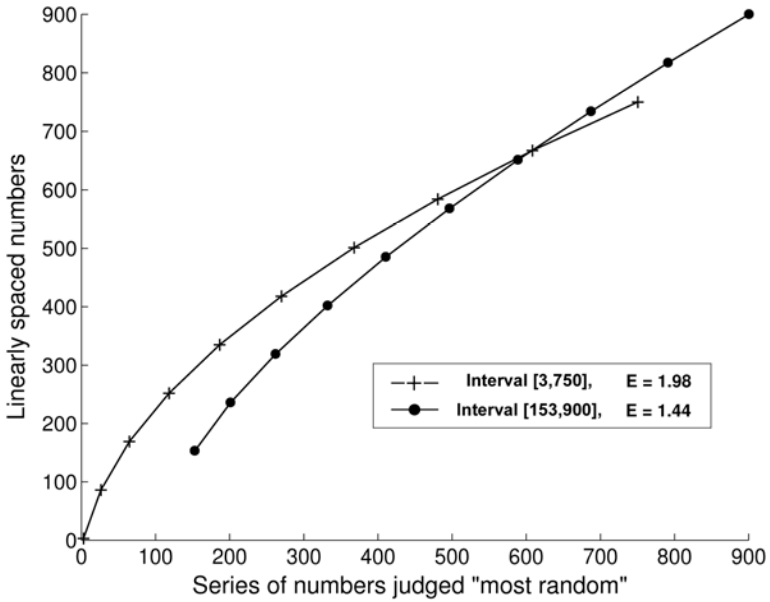
More education



A logarithmic representation of number words remains dormant in educated adults

- For very large numbers: what's in the middle of « one thousand » and « one billion »?
- For smaller numbers : bias in judgement of random sequences (Banks & Coleman, 1981):
    - We asked subjects to listen to random sequences of number words and rate whether each sequence comprised « more small numbers » or « more large numbers »
    - In fact, sequences were generated by a power law whose exponent varied according to a staircase procedure
    - The indifference point does not correspond to a linear sequence, but to a compressed sequence with (objectively) more small numbers, as if sampling uniformly from a compressed scale.
    - This result is very robust and resists number-space training or exposure to linear sequences





Viarouge, Hubbard, Sackur & Dehaene, submitted

Plan of the talk

- Neural coding of numerosity
  - A quantity code is present in the intraparietal area of human adults and babies
  - Optimal decision mechanisms based on this code can explain human psychophysics
- Understanding of number symbols
  - Symbols are mapped onto numerosities
  - However, the numerosity code may be changed by learning symbols
  - Linear mapping of number onto space
- Brain mechanisms of number-space mappings

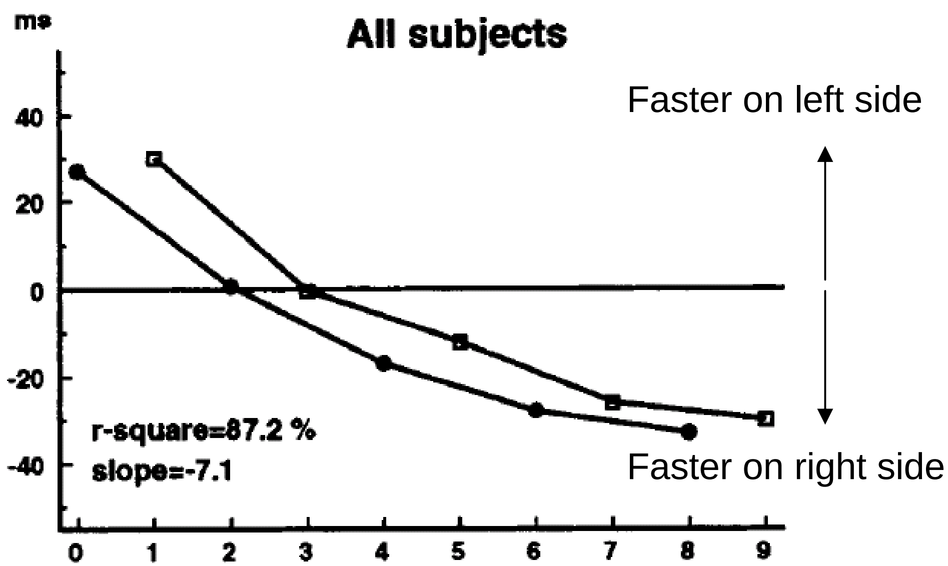
Behavioral evidence for interactions between Number and Space

Spatial-**N**umerical **A**ssociation of **R**esponse **C**odes

= **SNARC effect**

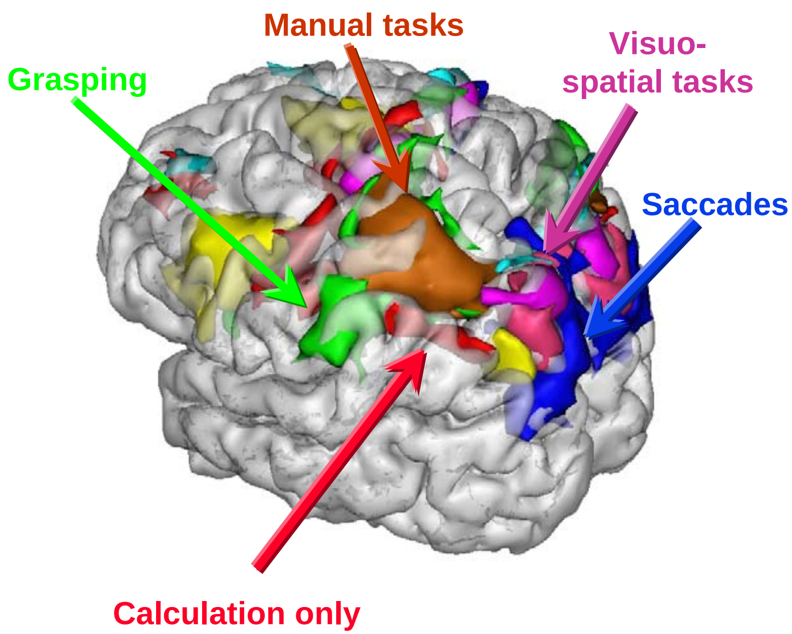
(Dehaene et al., 1993)

RT(right key) minus RT(left key)



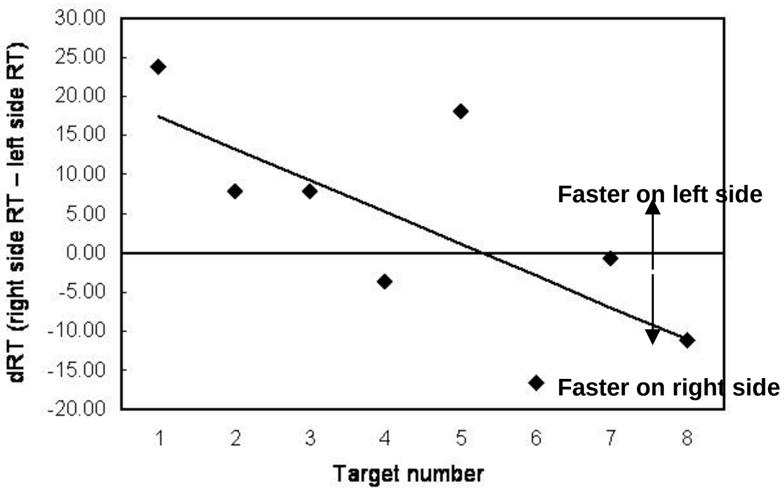
Hemispatial neglect
in numerical bisection task
(Zorzi et al., 2002)

Might these effects arise from parietal cross-activation?



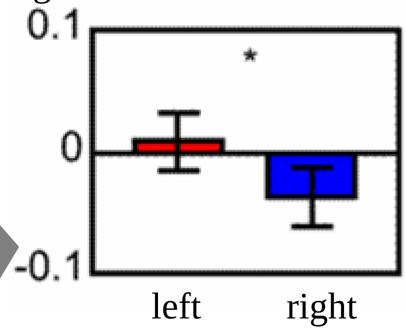
Brain mechanisms of Spatial-Numerical Assocation (SNARC): an fMRI experiment

Parity judgement
with variable mappings
+ calculation
+ saccades

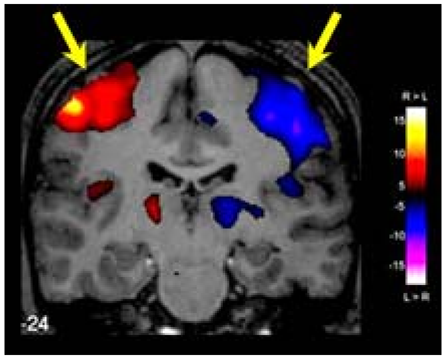


Cross-talk from the number code in the intraparietal sulcus to the spatial code in area LIP may explain the SNARC effect

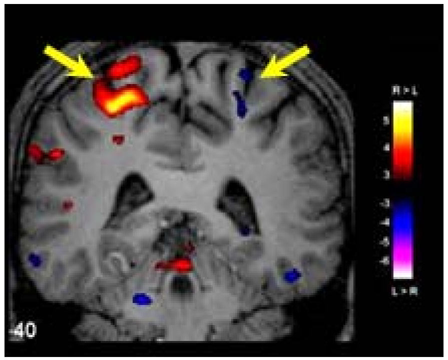
Large – small numbers contrast



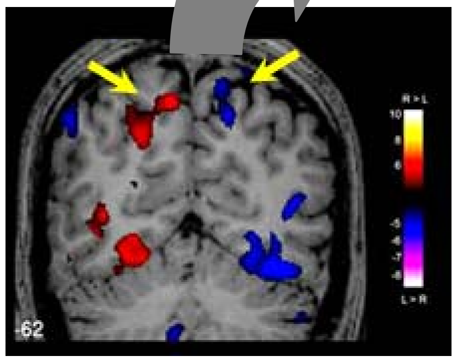
Three different intraparietal regions for spatial direction of



Hands (anterior)

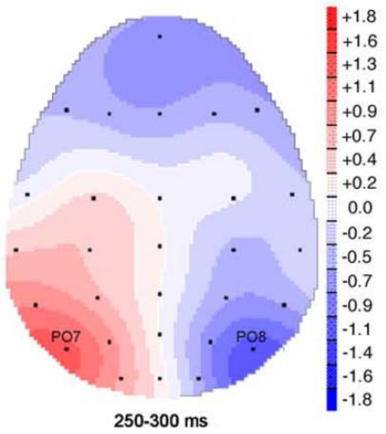


Space (middle)



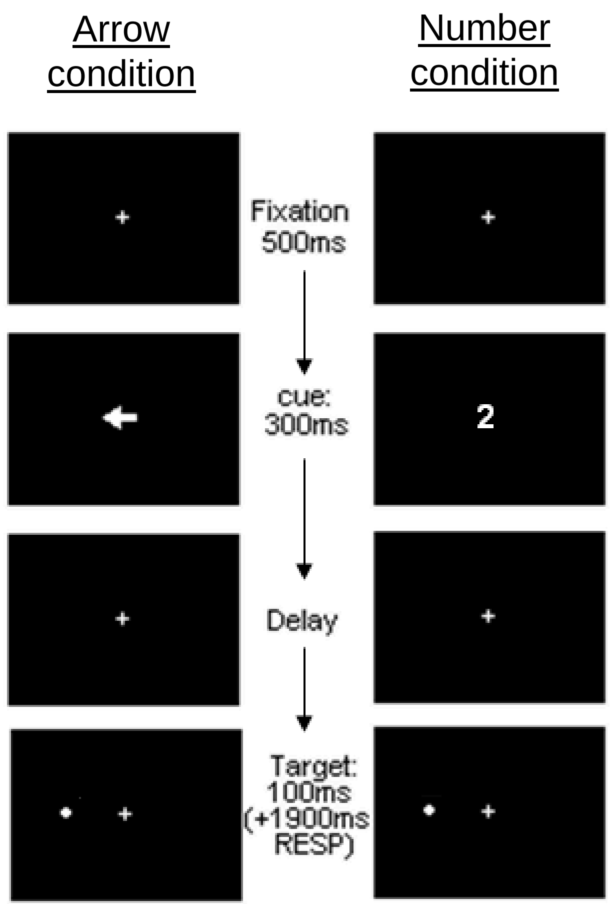
Saccades (posterior)

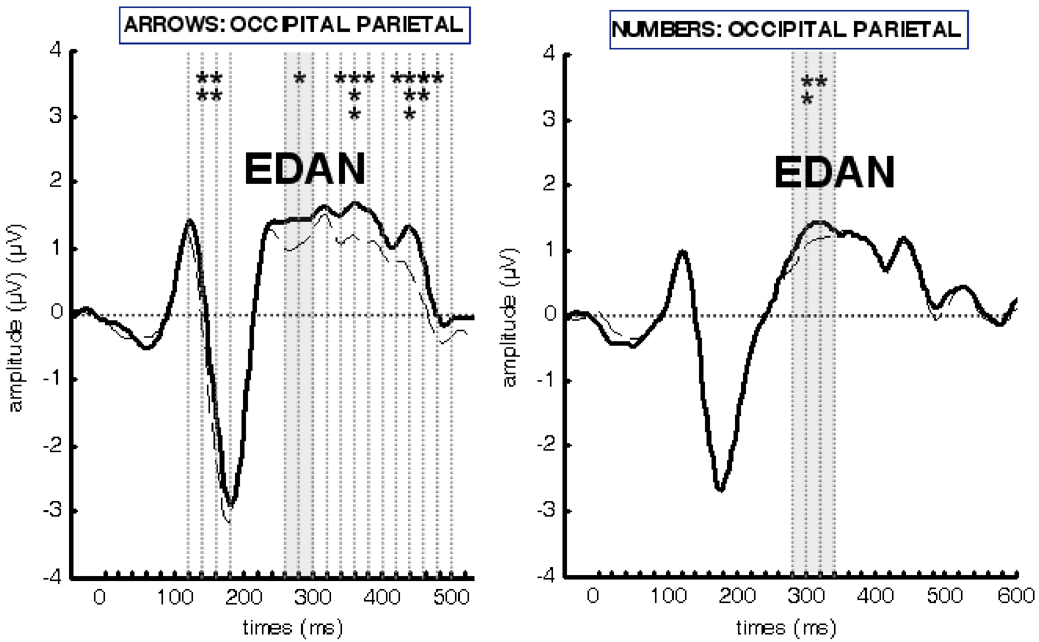
Hubbard, Pinel & Dehaene, submitted



Do numbers elicit automatic shifts of spatial attention? An ERP study

- The EDAN waveform indexes a lateralized orientation of spatial attention (~300 ms after a cue)
- Would an EDAN be elicited by irrelevant numbers?
- Task: detect a small dot (left or right of fixation) preceded by a non-informative arrow or number





« Ipsilateral » arrows or numbers
induce a greater positivity than
« Contralateral » arrows or numbers

Ranzini, Hubbard & Dehaene, in preparation

Conclusions: from animal number sense to human arithmetic

- All humans start in life with an elementary number sense based upon populations of number neurons in the intraparietal sulcus.
- Number symbols probably acquire their meaning by linking neural populations coding symbol shapes to those coding for nonsymbolic numerical quantities (a physical solution to the « grounding problem »)
- The acquisition of number symbols profoundly transforms the number system

- We develop an exact representation of large numbers
- We move from a logarithmic to a linear representation of numbers
- These two changes may happen in left parietal cortex
- Cross-talk with posterior parietal cortex may explain our intuition that numbers map onto space, a metaphor that plays an essential in higher mathematics.