



Perceptual inference and learning

Collège de France 2008

Abstract

We start with a statistical formulation of Helmholtz's ideas about neural energy to furnish a model of perceptual inference and learning that can explain a remarkable range of neurobiological facts. Using constructs from statistical physics it can be shown that the problems of inferring what cause our sensory inputs and learning causal regularities in the sensorium can be resolved using exactly the same principles. Furthermore, inference and learning can proceed in a biologically plausible fashion. The ensuing scheme rests on Empirical Bayes and hierarchical models of how sensory information is generated. The use of hierarchical models enables the brain to construct prior expectations in a dynamic and context-sensitive fashion. This scheme provides a principled way to understand many aspects of the brain's organization and responses.



Overview

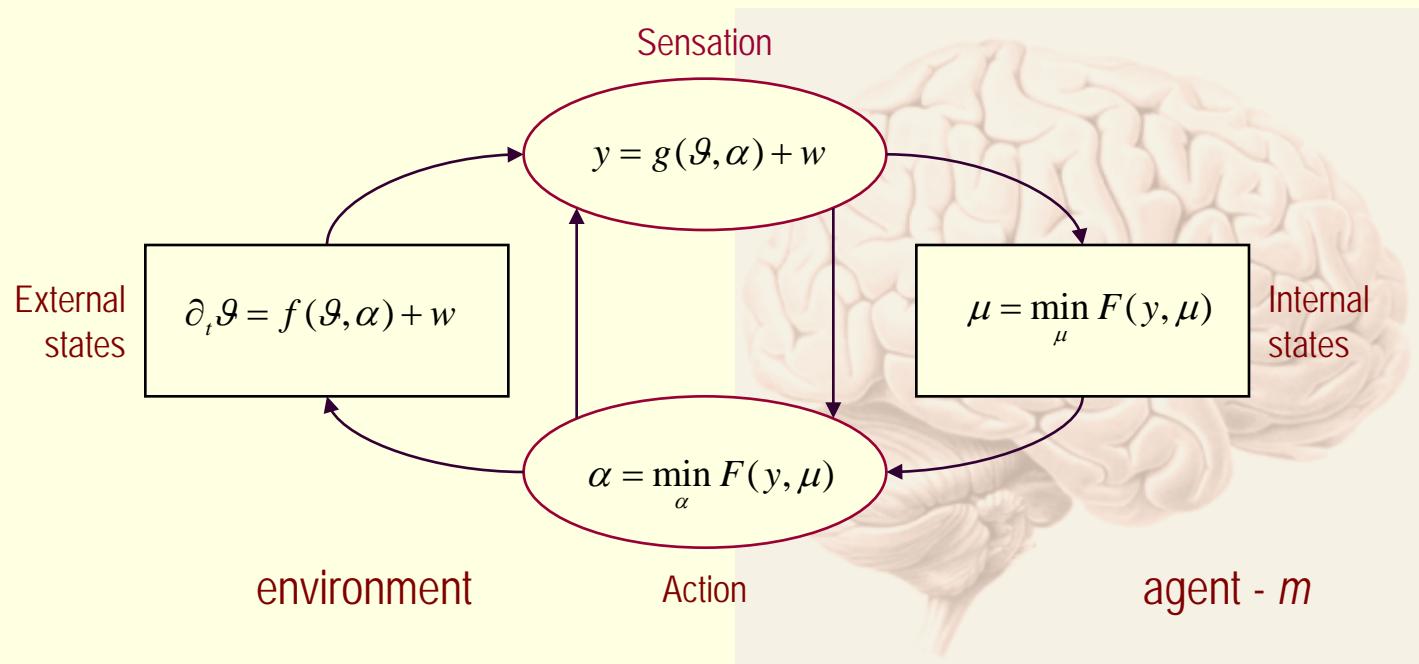
Inference and learning under the free energy principle
Hierarchical Bayesian inference

A simple experiment

Bird songs (inference)
Structural and dynamic priors
Prediction and omission
Perceptual categorisation

Bird songs (learning)
Repetition suppression
The mismatch negativity

Exchange with the environment



Separated by a Markov blanket

The free-energy principle

$$F = -\langle \ln p(y|\alpha, \vartheta | m) \rangle_q + \langle \ln q(\vartheta) \rangle_q \geq -\ln p(y | m)$$

Action to minimise a bound on surprise

$$\begin{aligned} F &= -\langle \ln p(y|\alpha) | \vartheta, m \rangle_q + D(q \| p(\vartheta)) \\ \alpha &= \min_{\alpha} F \\ &= \max_{\alpha} \langle \ln p(y|\alpha) | \vartheta, m \rangle_q \end{aligned}$$



Perception to optimise the bound

$$\begin{aligned} F &= -\ln p(y | m) + D(q(\vartheta; \mu) \| p(\vartheta | y)) \\ \mu &= \min_{\mu} F \Rightarrow \\ q(\vartheta; \mu) &\rightarrow p(\vartheta | y) \end{aligned}$$

The ensemble density and its parameters

$$q(\vartheta; \mu) = q(u; \mu_u)q(\theta; \mu_\theta)q(\gamma; \mu_\gamma)$$

Perceptual inference

$$\mu_u = \min_{\mu} F$$

Perceptual learning

$$\mu_\theta = \min_{\mu} F$$

Perceptual uncertainty

$$\mu_\gamma = \min_{\mu} F$$

Hierarchical models and message passing

Top-down messages

Bottom-up messages

$$v^{(i)} = g(x^{(i)}, v^{(i+1)}) + \varepsilon_v^{(i)}$$

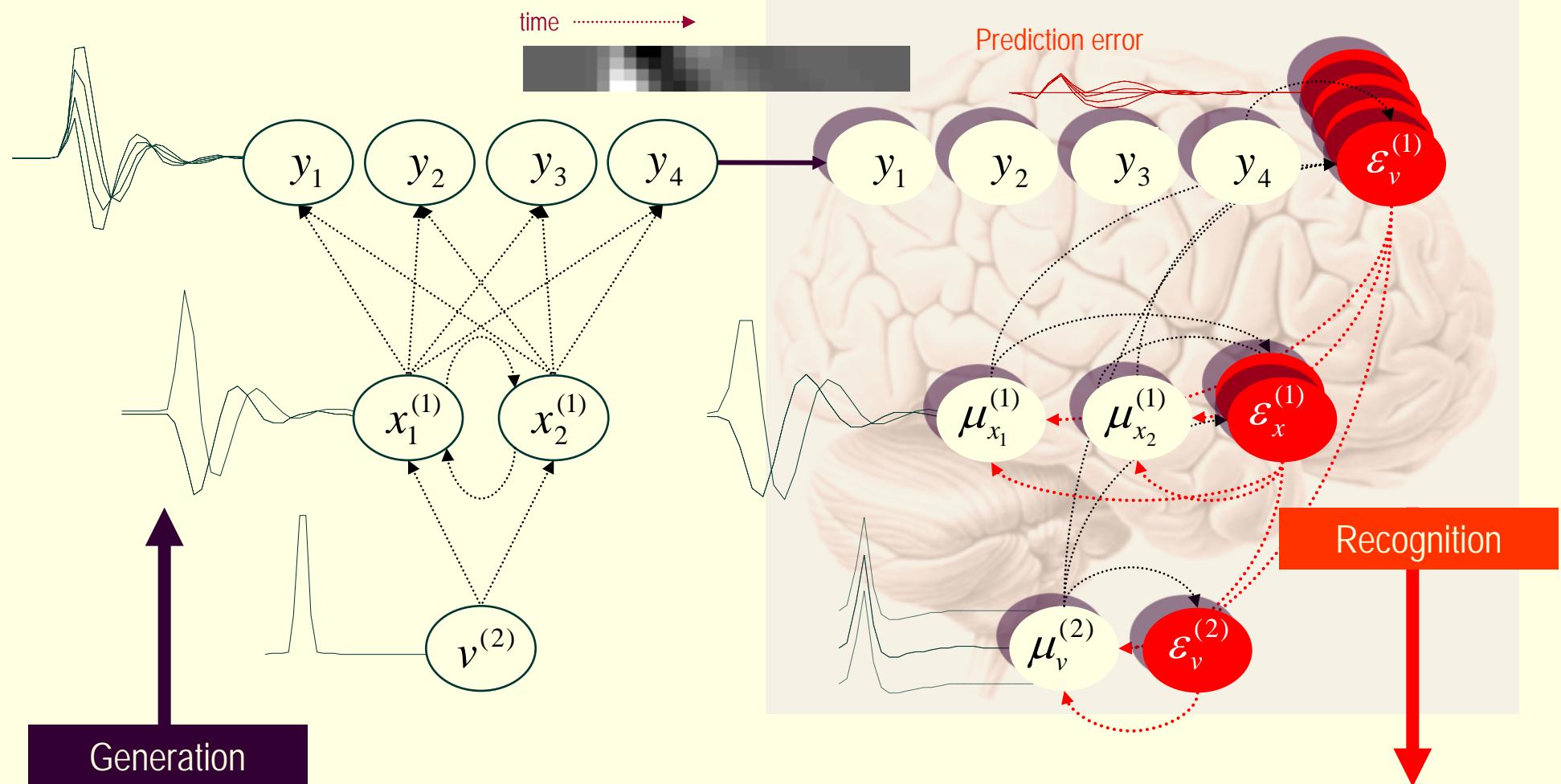
$$\partial_t x^{(i)} = f(x^{(i)}, v^{(i+1)}) + \varepsilon_x^{(i)}$$

$$\varepsilon_v^{(i)} = \mu_v^{(i)} - g(\mu_x^{(i)}, \mu_v^{(i+1)})$$

$$\varepsilon_x^{(i)} = \partial_t \mu_x^{(i)} - f(\mu_x^{(i)}, \mu_v^{(i+1)})$$

$$\partial_t \mu_v^{(i)} = -\partial_{v\varepsilon^{(i-1)}} F \varepsilon^{(i-1)} - \partial_{v\varepsilon^{(i)}} F \varepsilon^{(i)}$$

$$\partial_t \mu_x^{(i)} = -\partial_{x\varepsilon^{(i)}} F \varepsilon^{(i)}$$



Empirical Bayes and hierarchical models

D-Step
Perceptual inference

$$\begin{aligned}\partial_t \mu_v^{(i)} &= -\partial_v \varepsilon_u^{(i-1)T} \mu_\gamma^{(i-1)} \varepsilon_u^{(i-1)} - \mu_\gamma^{(i)} \varepsilon_u^{(i)} \\ \varepsilon_v^{(i)} &= \mu_v^{(i)} - g(\mu_x^{(i)}, \mu_v^{(i+1)})\end{aligned}$$

Bottom-up

Lateral

Top-down

Recurrent message passing among neuronal populations, with top-down predictions changing to suppress bottom-up prediction error

E-Step
Perceptual learning

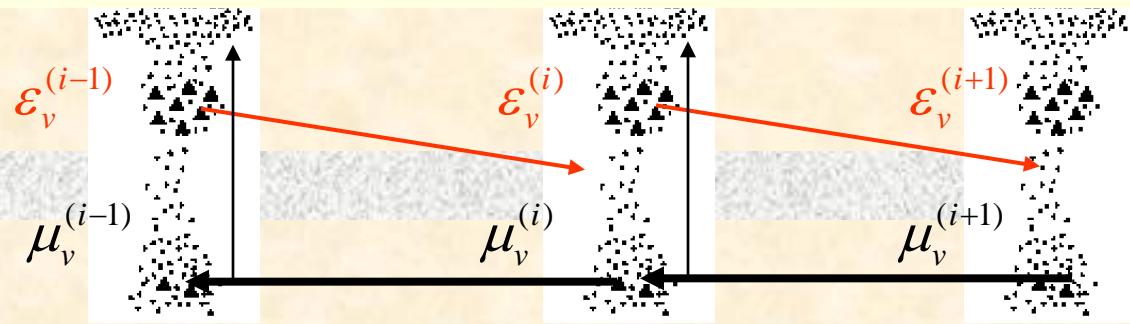
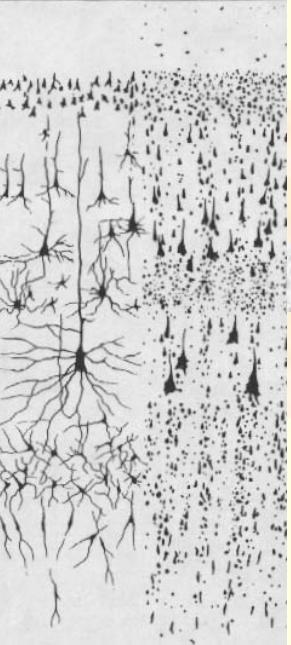
$$\partial_t \mu_\theta^{(i)} = -\langle \partial_\theta \varepsilon_u^{(i)T} \mu_\gamma^{(i-1)} \varepsilon_u^{(i)} \rangle$$

Associative plasticity, modulated by precision

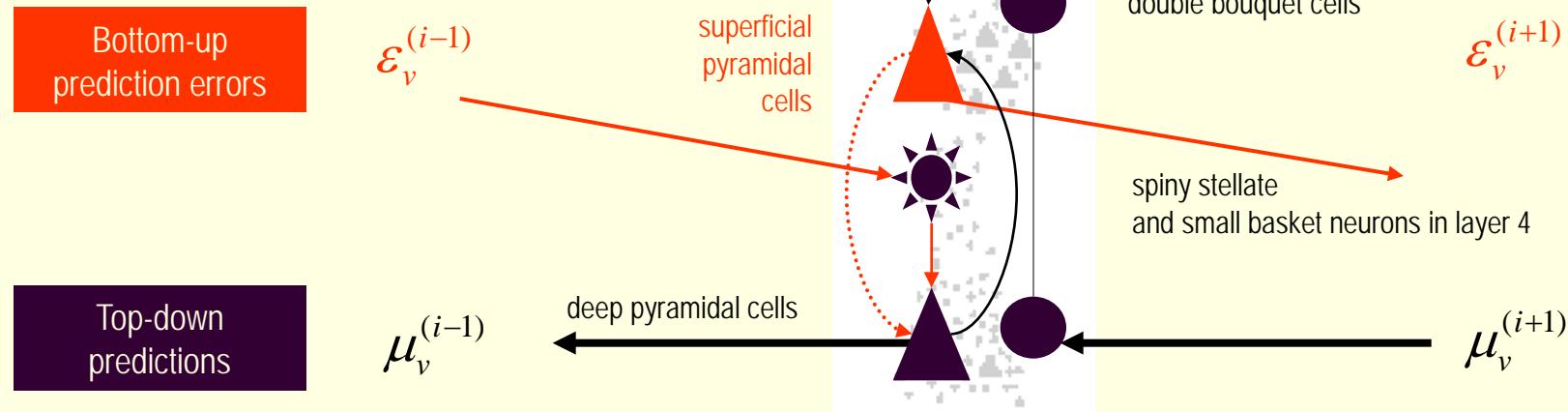
M-Step
Perceptual uncertainty

$$\partial_t \mu_\gamma^{(i)} = \mu_\gamma^{(i)-1} - \varepsilon_u^{(i)T} \varepsilon_u^{(i)}$$

Encoding of precision through classical neuromodulation or plasticity in lateral connections



$$\epsilon_v^{(i)} = \mu_v^{(i)} - g(\mu_x^{(i)}, \mu_v^{(i+1)})$$



$$\partial_t \mu_v^{(i)} = -\partial_v \epsilon_u^{(i-1)T} \mu_\gamma^{(i-1)} \epsilon_u^{(i-1)} - \mu_\gamma^{(i)} \epsilon_u^{(i)}$$

Neural implementation in cortical hierarchies (c.f. evidence accumulation models)



Overview

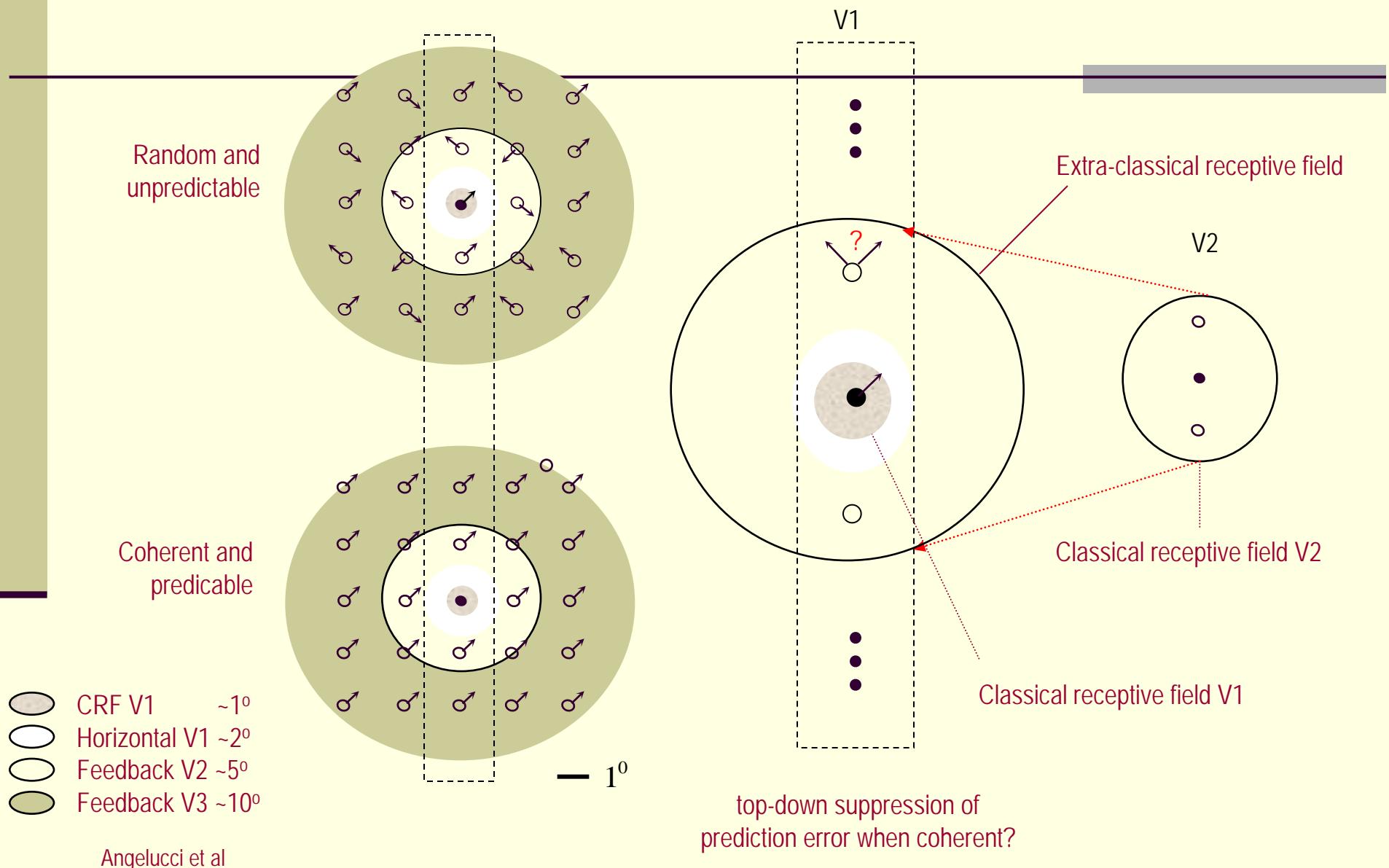
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A simple experiment

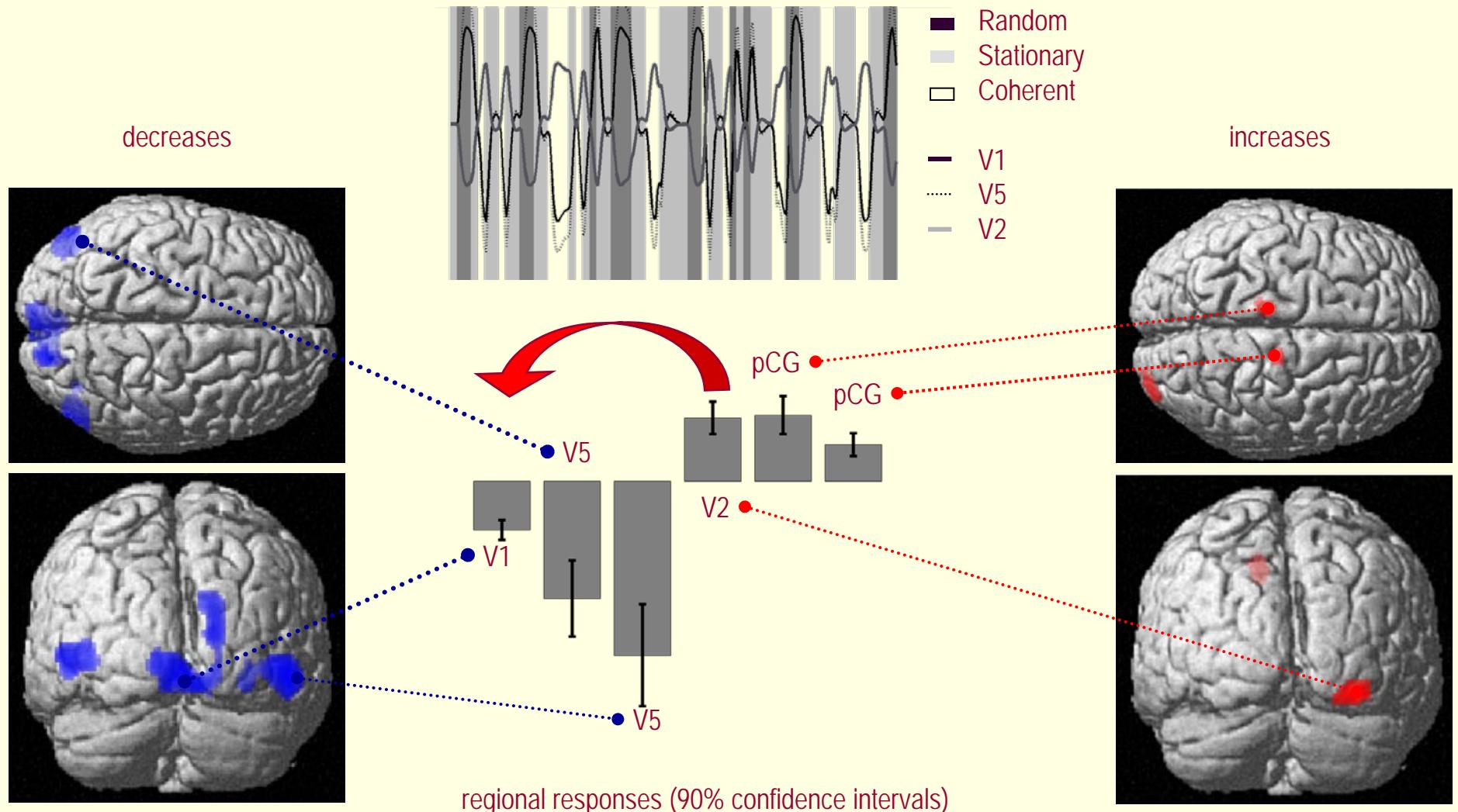
Bird songs (inference)
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Bird songs (learning)
Repetition suppression
The mismatch negativity

A brain imaging experiment with sparse visual stimuli



Suppression of prediction error with coherent stimuli





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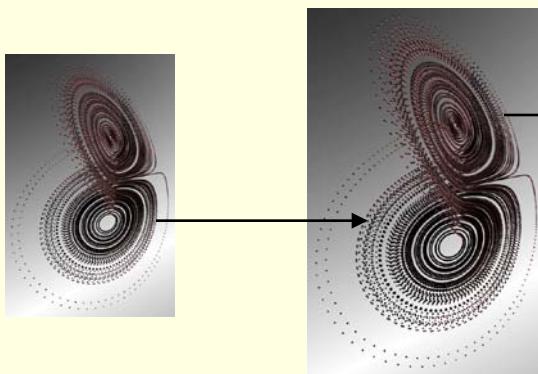
Perceptual categorisation

Bird songs (learning)

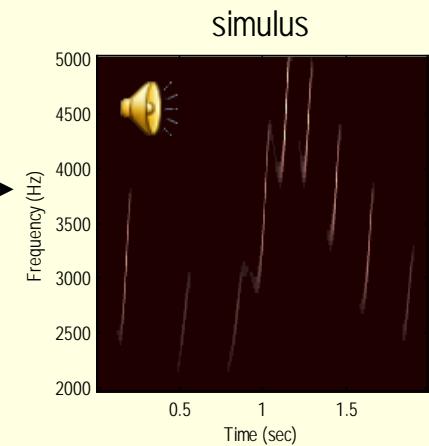
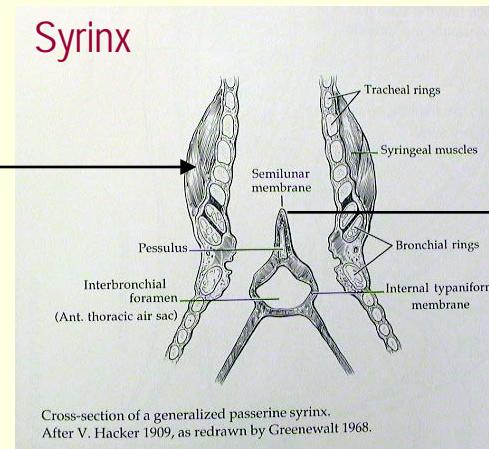
Repetition suppression

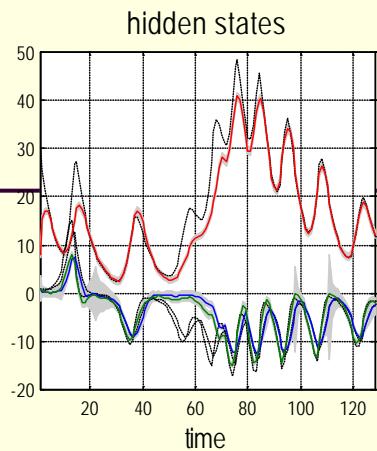
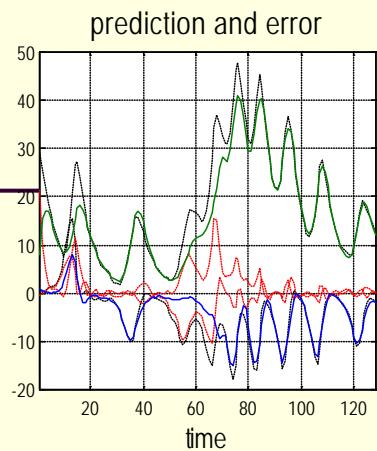
The mismatch negativity

Synthetic song-birds

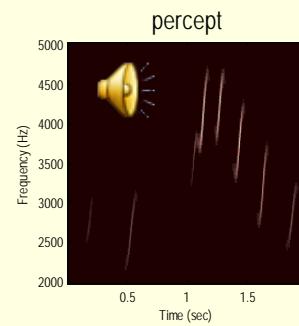
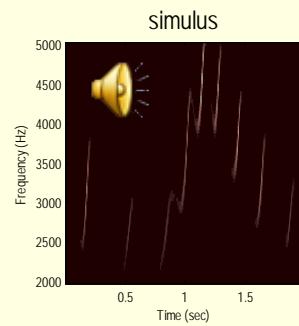
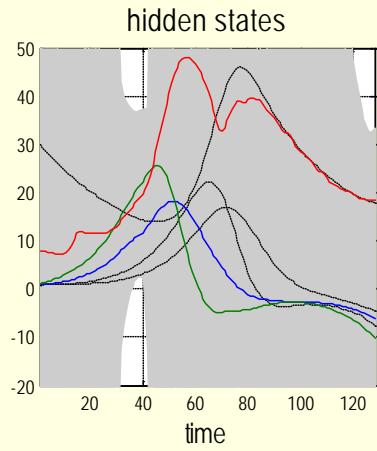
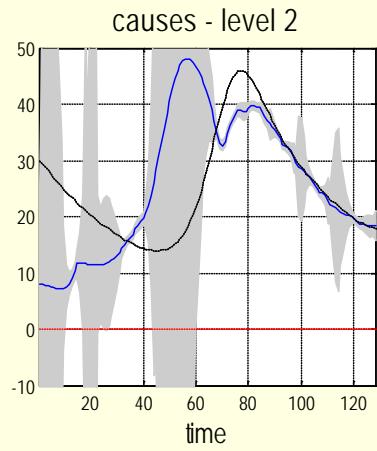


Neuronal hierarchy



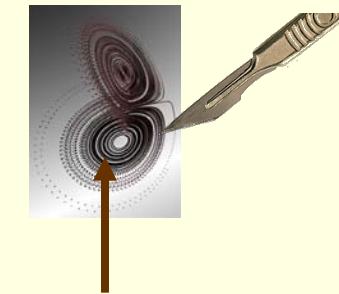
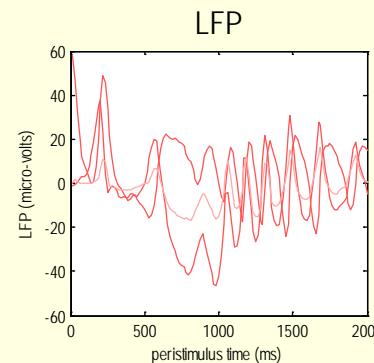
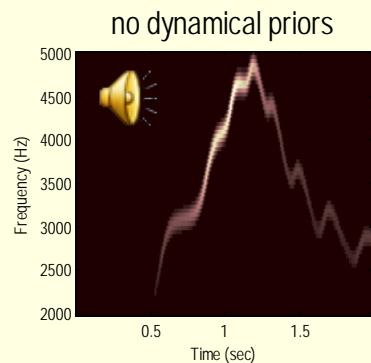
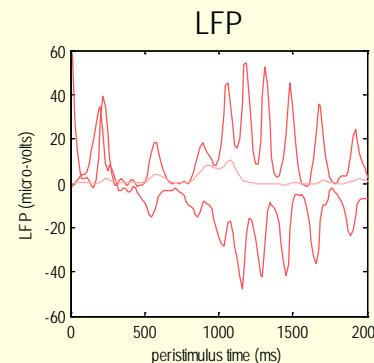
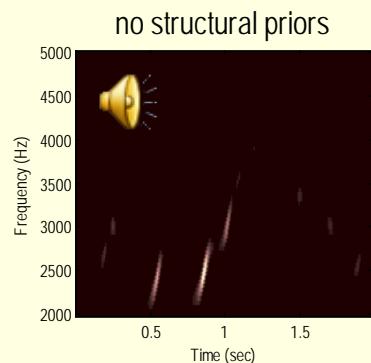
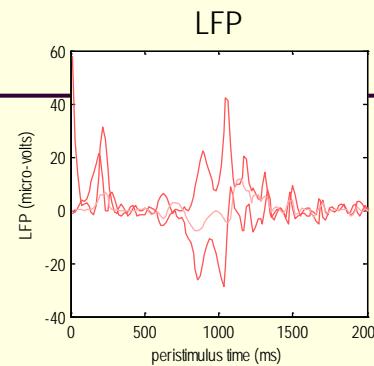
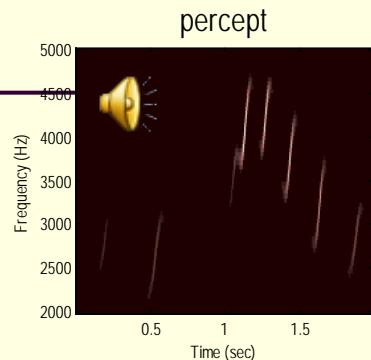


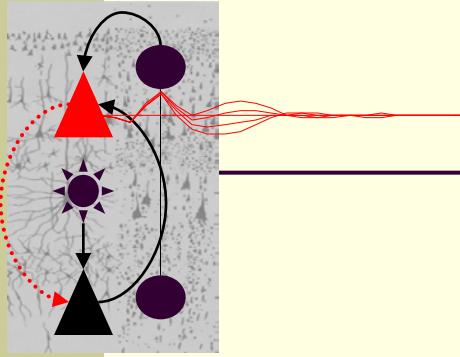
Song recognition with DEM



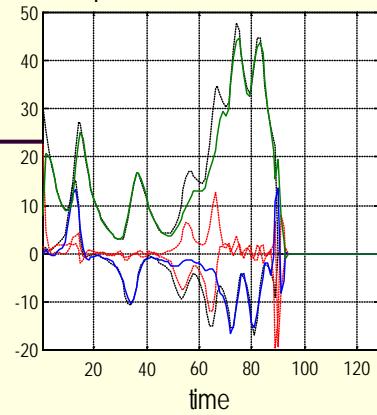


... and broken birds

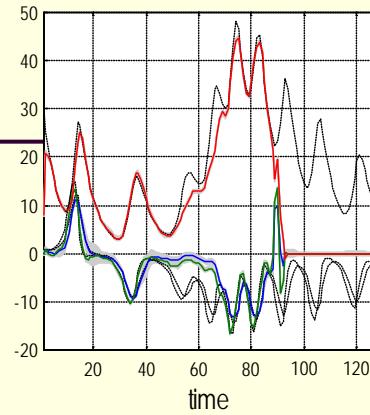




prediction and error

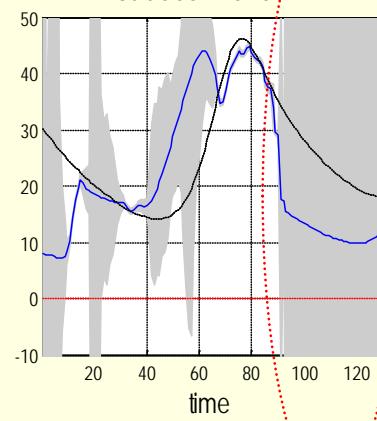


hidden states

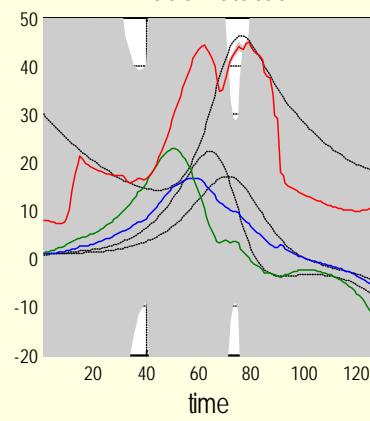


... omitting the last chirps

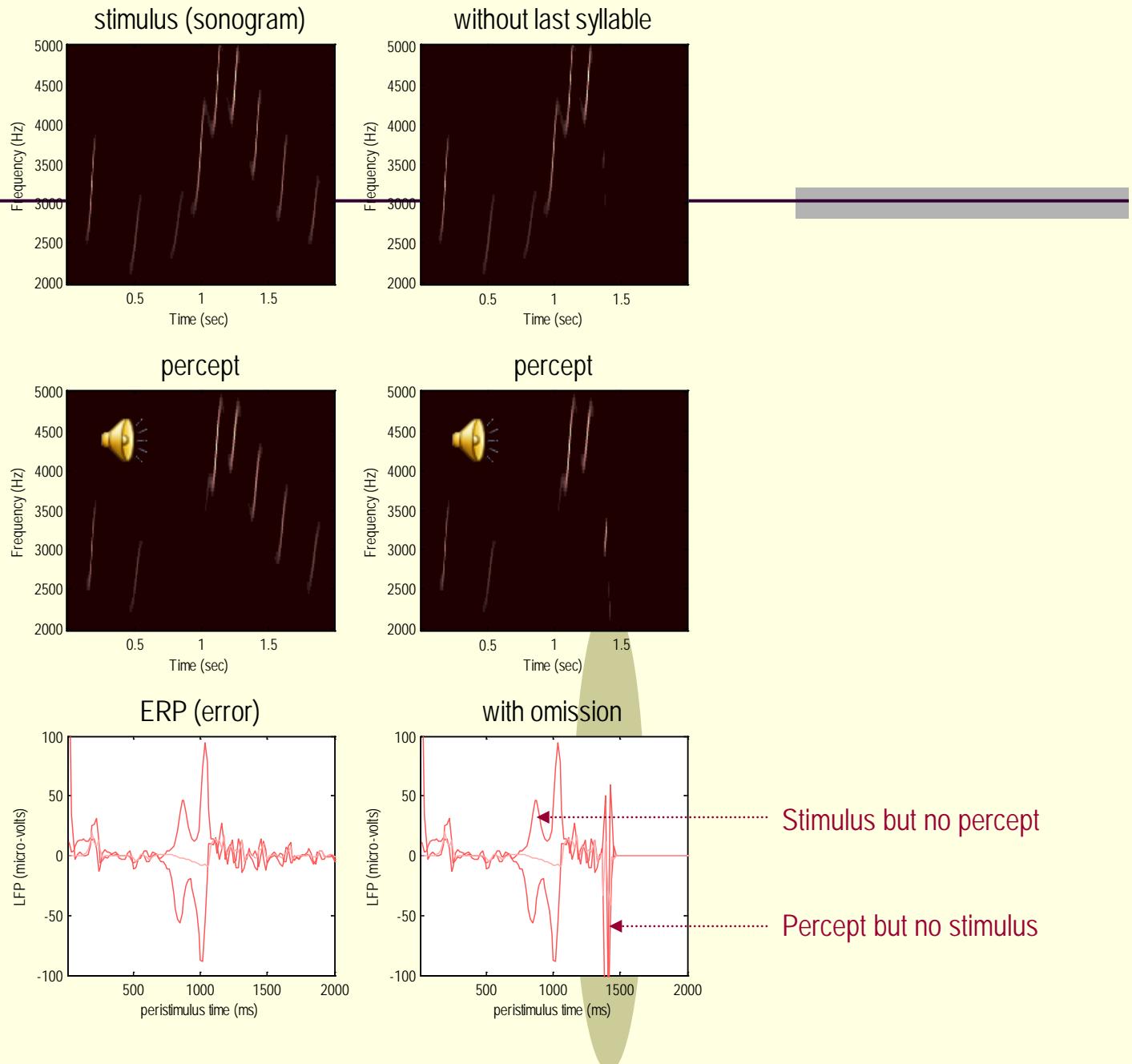
causes - level 2



hidden states



omission and violation of predictions





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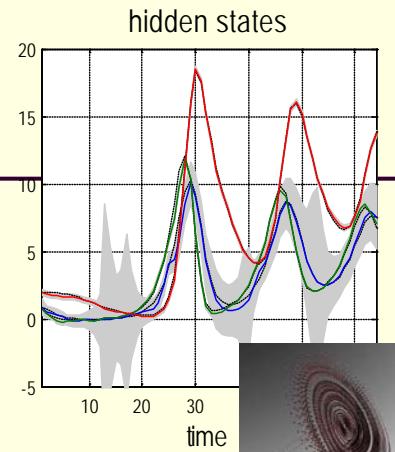
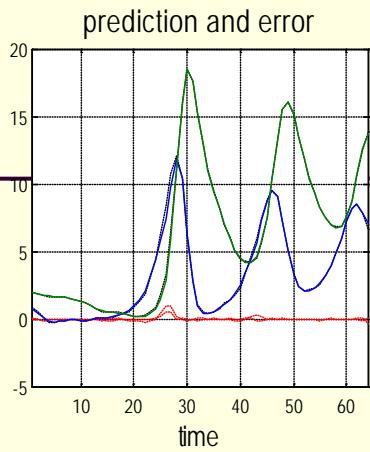
Prediction and omission

Perceptual categorisation

Bird songs (learning)

Repetition suppression

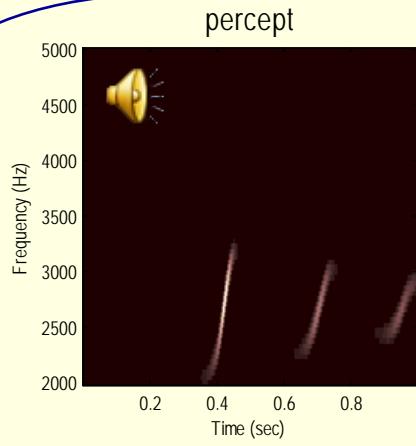
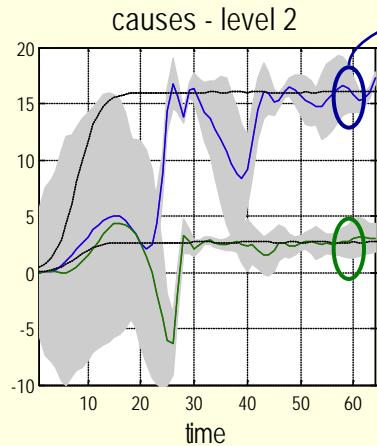
The mismatch negativity



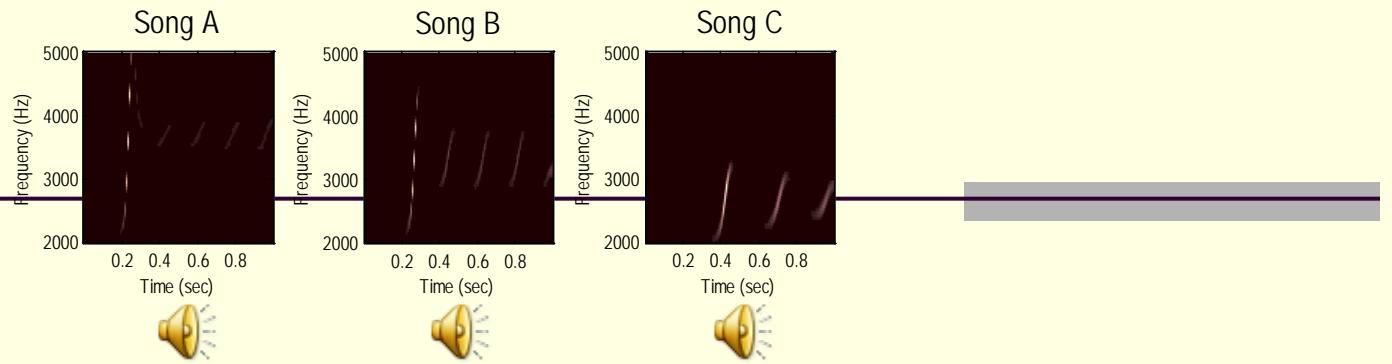
A simple song

Encoding sequences in terms of attractor manifolds

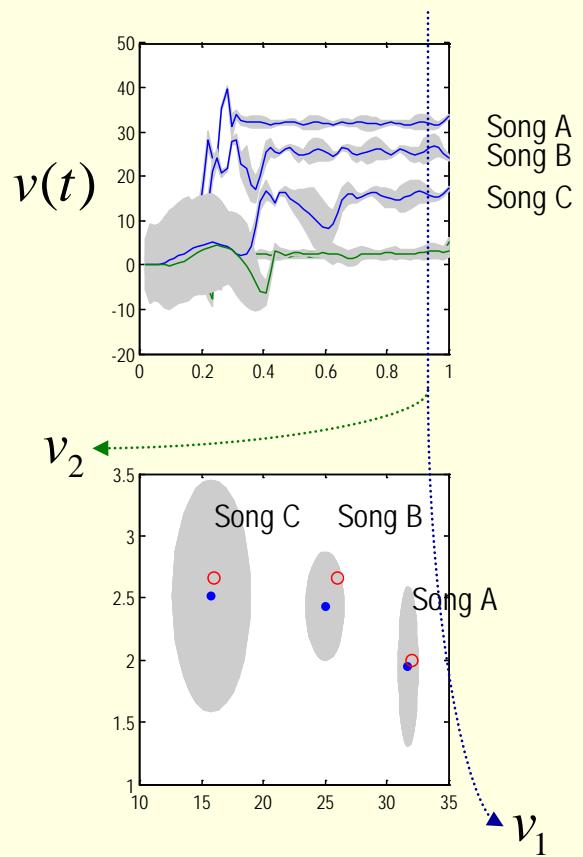
$$v(t)$$



$$f(x) = \begin{bmatrix} 18x_2 - 18x_1 \\ v_1x_1 - 2x_3x_1 - x_2 \\ 2x_1x_2 - v_2x_3 \end{bmatrix}$$



Categorizing
sequences
90% confidence
regions





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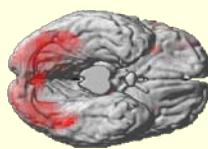
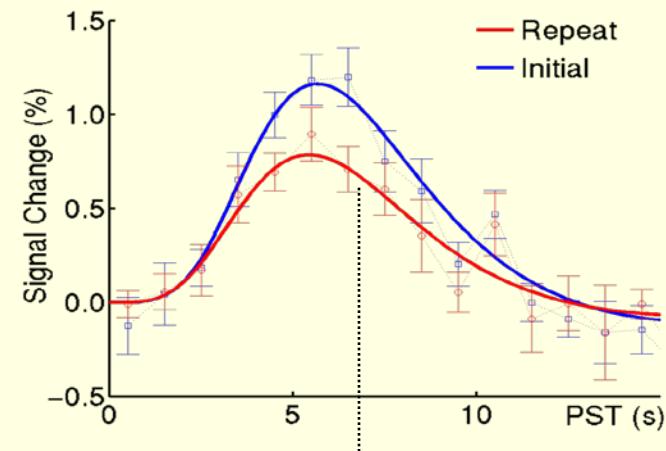
Perceptual categorisation

Bird songs (learning)

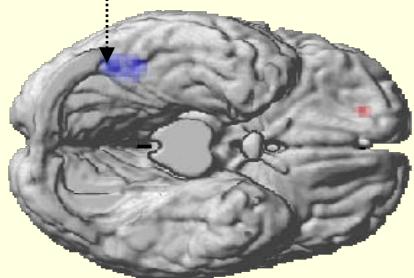
Repetition suppression

The mismatch negativity

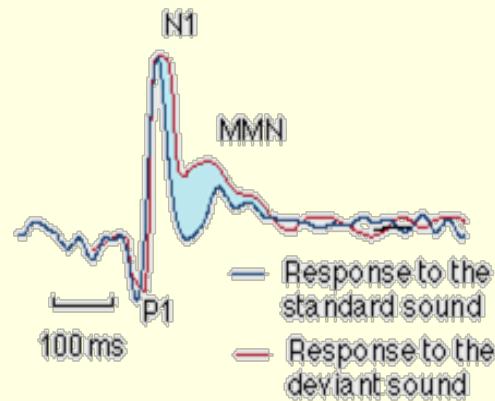
Repetition suppression and the MMN



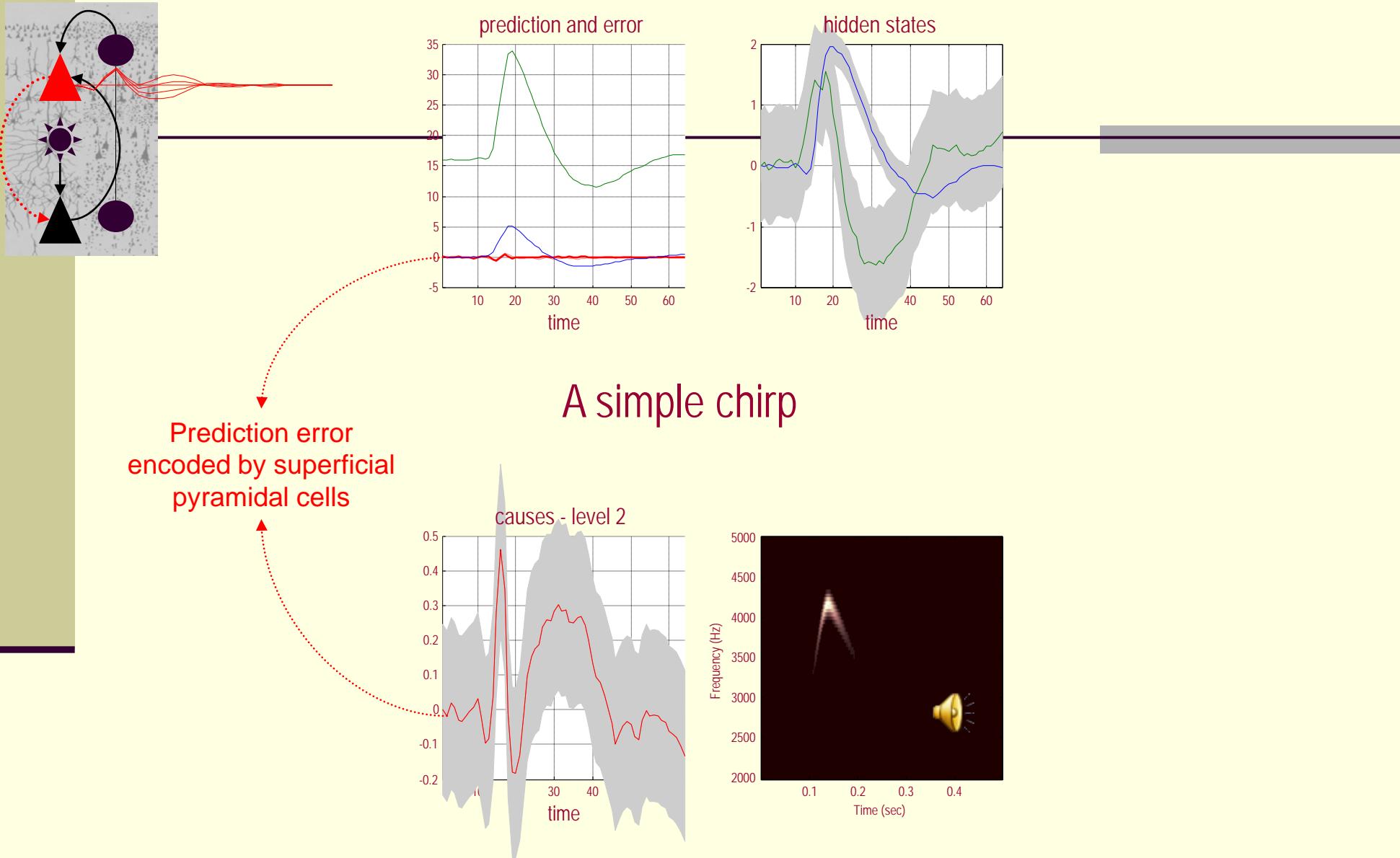
Main effect of faces



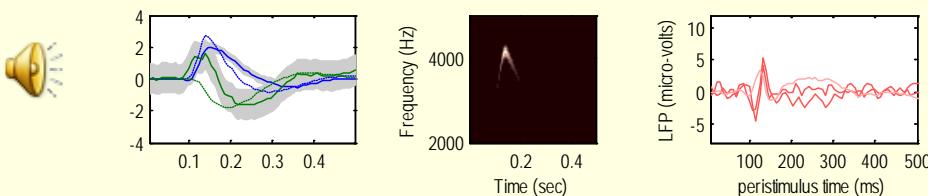
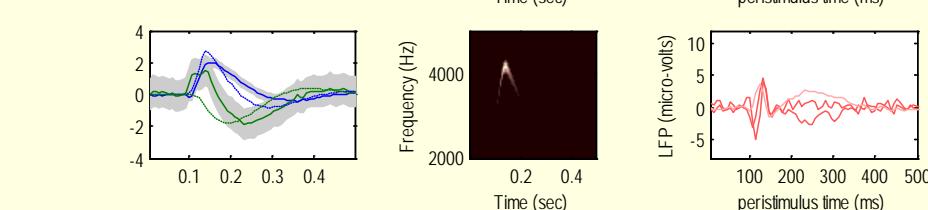
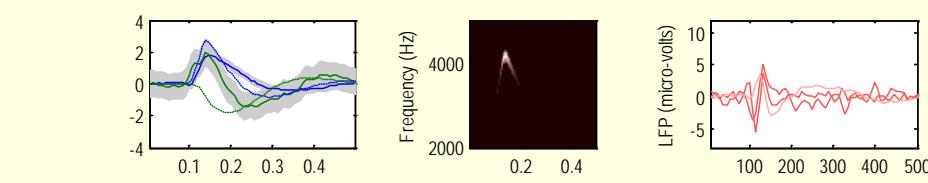
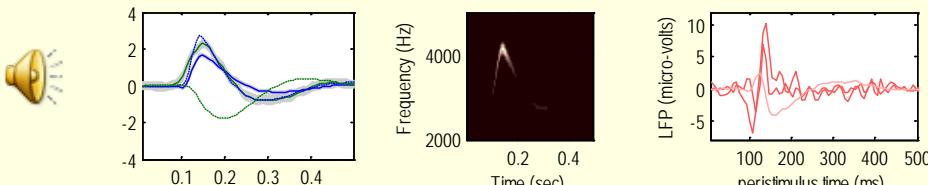
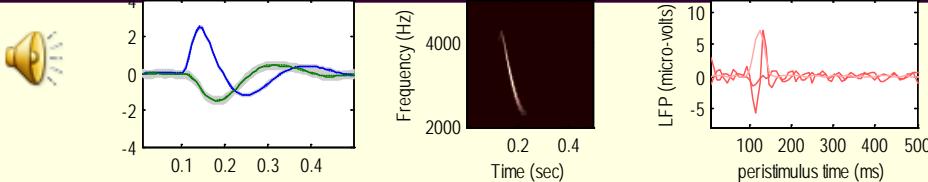
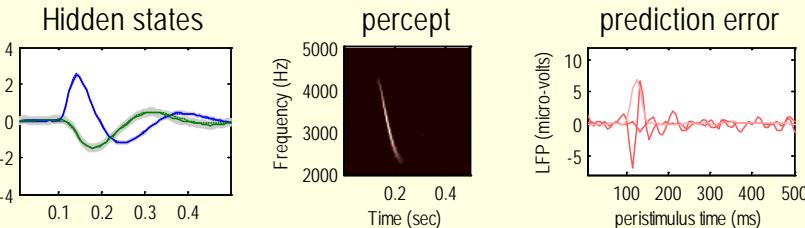
Suppression of inferotemporal responses to repeated faces



The MMN is an enhanced negativity seen in response to any change (deviant) compared to the standard response.



Simulating ERPs to repeated chirps



$$\mu_u = \min_{\mu} F$$

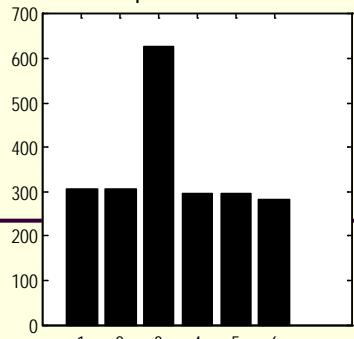
Perceptual inference:
suppressing error over
peristimulus time

Perceptual learning:
suppression over
repetitions

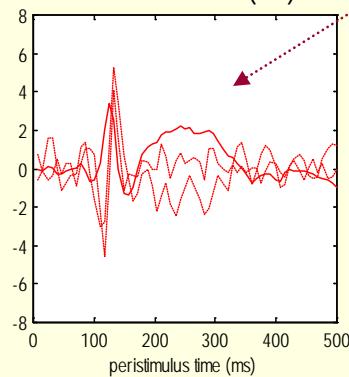
$$\mu_\theta = \min_{\mu} F$$

The MMN

SSQ prediction error

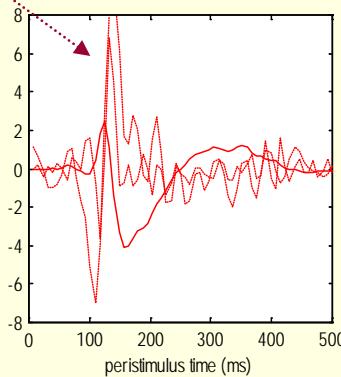


LFP: Standard (P1)



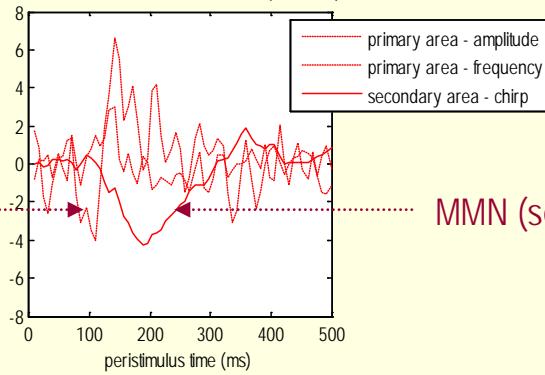
Last presentation
(after learning)

LFP: Oddball



First presentation
(before learning)

Difference waveform (MMN)



Enhanced N1 (primary area)

MMN (secondary area)

P300 (tertiary area)?



Summary

- A free energy principle can account for several aspects of action and perception
- The architecture of cortical systems speak to hierarchical generative models
- Estimation of hierarchical dynamic models corresponds to a generalised deconvolution of inputs to disclose their causes
- This deconvolution can be implemented in a neuronally plausible fashion by constructing a dynamic system that self-organises when exposed to inputs to suppress its free energy
- Minimisation of free energy proceeds over many spaces, including the state of a model (perception), its parameters (learning), its hyperparameters (salience and attention) and the model itself (selection in somatic or evolutionary time).