College de France, June 4, 2013

SUPERFLUIDTY IN
ULTRACOLD ATOMIC GASES
(SUPERCURRENTS AND SUPERSTRIPES)

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PLAN OF THE LECTURES

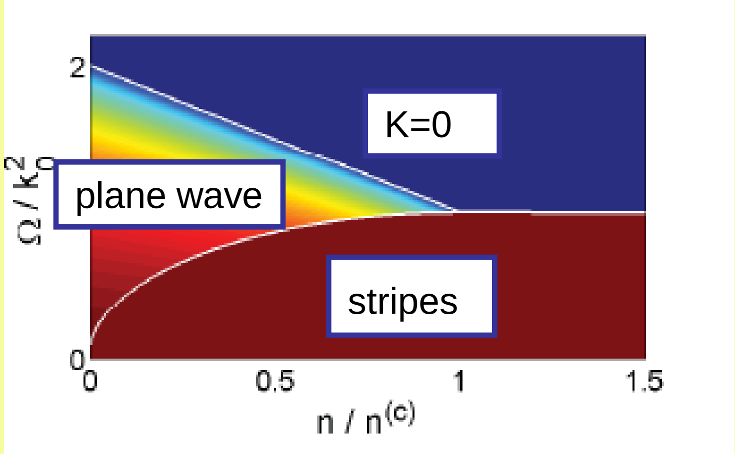
Lecture 1. **Brief summary of superfluidity** in ultracold gases.
Some open questions

Lecture 2. A tale of two sounds (**first and second sound**)

Lecture 3. Spin-orbit (SO) coupled Bose-Einstein condensed gas:
new quantum phases and **anisotropic superfluidity**

Lecture 4. **Superstripes and supercurrents** in SO coupled BECs

In lecture 3 we have discussed some properties of the quantum phases predicted by the 1D spin-orbit coupled Bose-Einstein condensates. We have shown that spin-orbit coupling deeply affects the dynamic behavior in the plane wave and single minimum phases.



This talk:

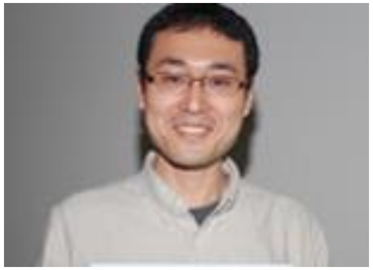
- **Dynamic instability of supercurrents in plane wave phase**
  Role of Galilean invariance

- **Dynamic behavior in stripe phase**
  and effects of supersolidity

Yun Li

Giovanni Martone

Lev Pitaevskii



Tomoki Ozawa

Supercurrent and dynamic instability

Tomoki Ozawa, Lev Pitaevskii and S.S.
(arXiv: 1305.0645, PRA 2013)

In uniform systems **Galilean** invariance implies that critical velocity of a supercurrent state coincides with Landau critical velocity

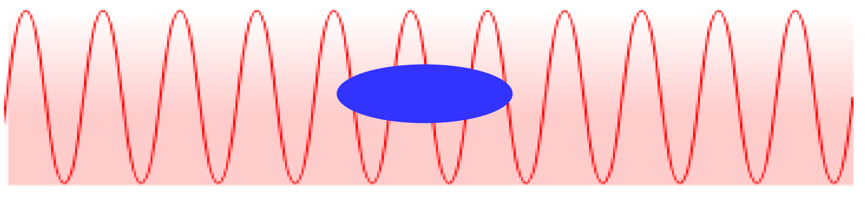
Frame at rest
(moving superfluid)

Superfluid at rest
(moving frame or moving impurity

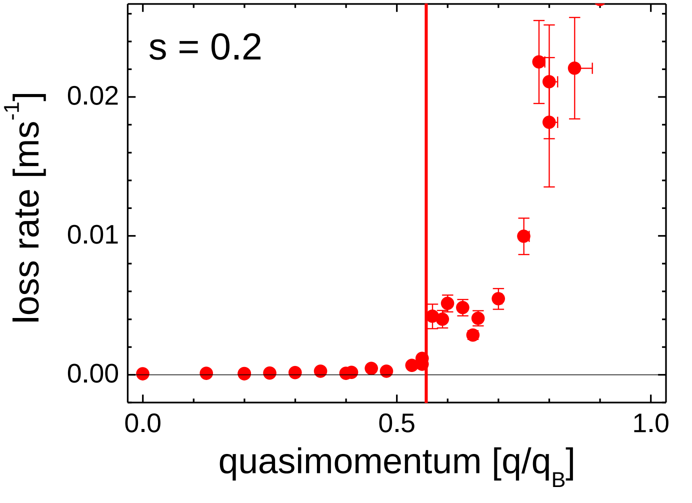
$$v_c = \min_p \frac{\varepsilon(p)}{p}$$

An external periodic potential (optical lattice) breaks **translational** and **Galilean invariance** giving rise to density modulations.

Critical velocity of supercurrent differs from Landau critical velocity. Supercurrent can become dynamically unstable



(Fallani et al., 2004)



Spin-orbit coupled Hamiltonian **breaks Galilean** invariance without breaking **translational** invariance

⇓

Critical velocity for supercurrents differs from Landau critical velocity!

(Zhu et al. EPL 2012, Zheng et al. arXiv:1212.6832)

Furthermore: supercurrents can exhibit **dynamic instability** even in **uniform** configurations

From equation of continuity

$$\partial\rho_q/\partial t = i[H,\rho_q] = -q/2\sum_i [(p_{x,i}-k_0\sigma_{z,i})e^{iqx_i} + e^{iqx_i}(p_{x,i}-k_0\sigma_{z,i})]$$

one identifies q=0 current operator $J_x = \sum_i (p_{x,i}-k_0\sigma_{z,i}) \neq P_x$

- **Current differs from momentum** operator $P_x = \sum_i p_{x,i}$
  At equilibrium current is zero
  but average momentum is not zero
  (in plane wave phase all atoms have momentum $k_1 \neq 0$)

$$H = \sum_i h_0(i) + V_{\text{int}}$$

$$h_0 = \frac{1}{2}[(p_x - k_0\sigma_z)^2 + p_\perp^2] + \frac{1}{2}\Omega\sigma_x + \frac{1}{2}\delta\sigma_z$$

- **Momentum** operator **commutes** with H (translational invariance) $[H,\vec{P}]=0$

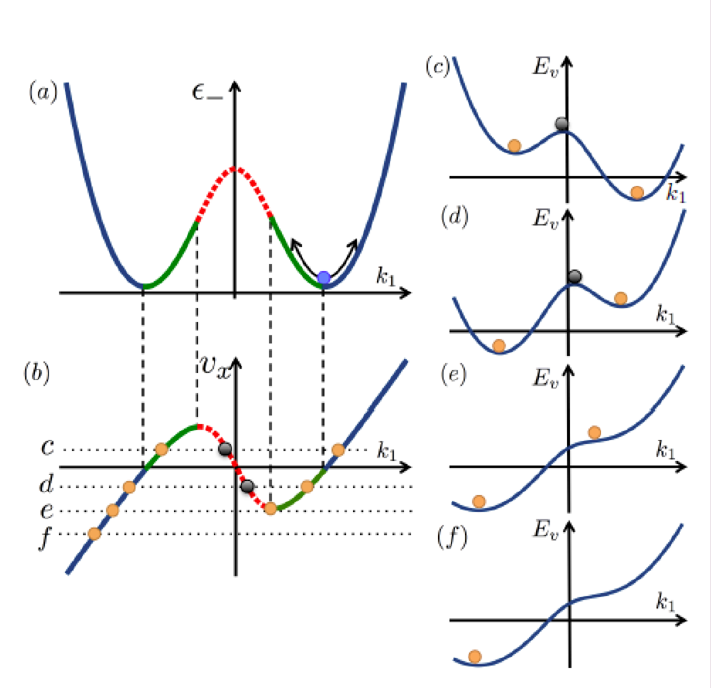
- **Current** operator does **NOT commute** (non commutativity caused by spin term in the current, breaking of Galilean invariance) $[H,\vec{J}]\neq 0$

How to construct the super current state

To generate the **supercurrent state** we calculate the stationary states of the constrained Hamiltonian

$$H_v = H - vP_x$$

We consider **plane wave** phase and minimize energy with the ansatz



$$\Psi = \sqrt{\frac{N}{V}} \begin{pmatrix} \cos\theta \\ -\sin\theta \end{pmatrix} e^{ik_1 x}$$

New value of momentum k_1 depends on velocity

$$v = k_1 (1 - k_0 / \sqrt{k_1^2 + (\Omega / 2k_0)^2})$$

fixing the value of the current $< J_x >= Nv$

Number of stationary solutions depends on velocity.
If initially $k_1 > 0$ and **v>0**, the system will continously evolve into the **global minimum**. If instead **v<0** the system continuosly evolves into a **metastable state**, the global minimum being at negative k_1.
For large negative velocity the metastable state disappears.

We have calculated the dispersion relation of the elementary excitations (Bogoliubov modes) on top of the supercurrent state.

Result for phonon dispersion in the limit

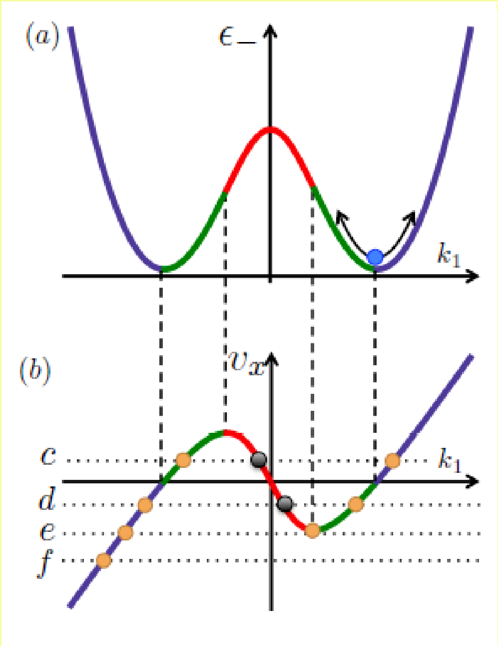
$G_2 << G_1 \;\; (G_1 = n(g + g_{\uparrow\downarrow})/4, G_2 = n(g - g_{\uparrow\downarrow})/4$

$$c_{\pm} = \sqrt{gn \frac{\partial^2 \epsilon_-(k_1)}{\partial k_1^2}} \pm v,$$

$1/m^*$

where

$$\epsilon_-(k_1) = \frac{k_1^2 + k_0^2}{2m} - \sqrt{\left(\frac{k_1 k_0}{m}\right)^2 + \left(\frac{\Omega}{2}\right)^2}$$



- Phonon velocity fixed by effective mass and shift $\pm v$
- Value of effective mass depends on velocity

$$v = k_1(1 - k_0 / \sqrt{k_1^2 + (\Omega / 2k_0)^2})$$

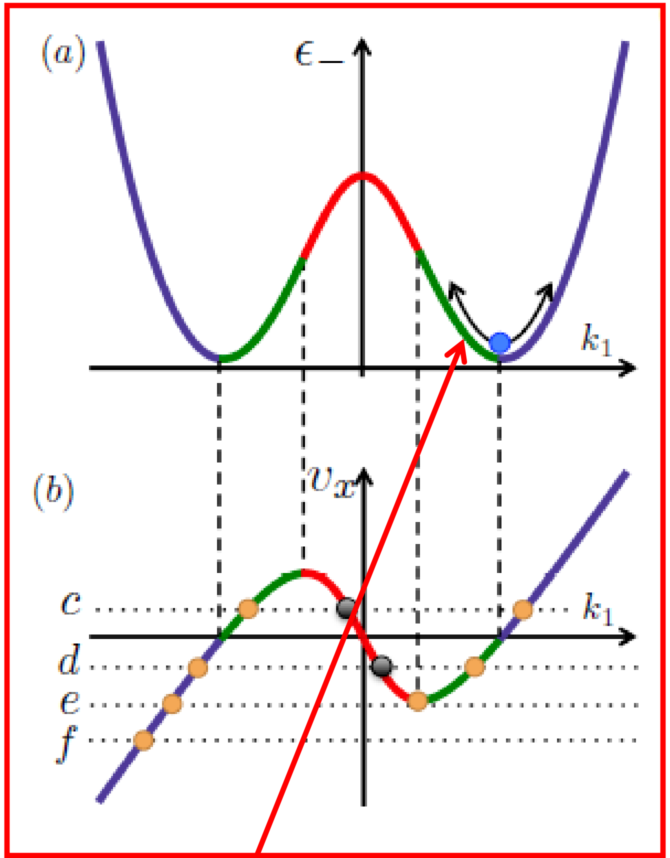
Dynamic instability emerges when effective mass becomes negative (**red region**) .

Similar behavior happens in BECs in optical lattices (dynamic instability measured in Florence experiments).

In **optical lattices** negative effective mass is caused by **band strucure** (Smerzi et al. 2002)

$$c = \sqrt{\frac{\tilde{g}n}{m^*}\cos\frac{mv_0 d}{\hbar} \pm \frac{\hbar}{m^* d}\sin\frac{mv_0 d}{\hbar}}$$

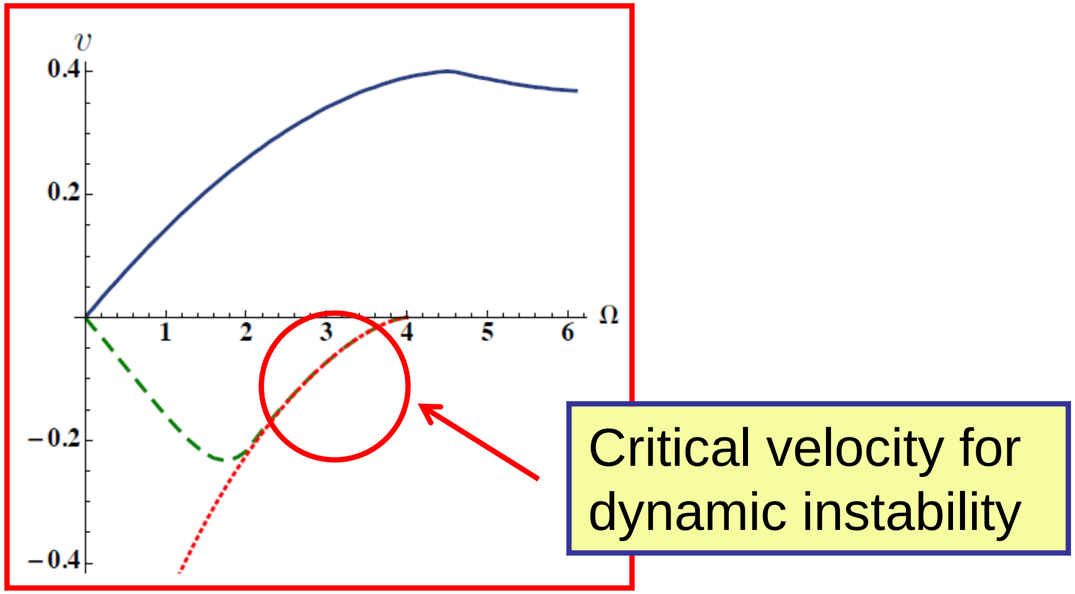
In **spin-orbit** gases (plane wave phase) negative effective mass is caused by **double minimum** structure



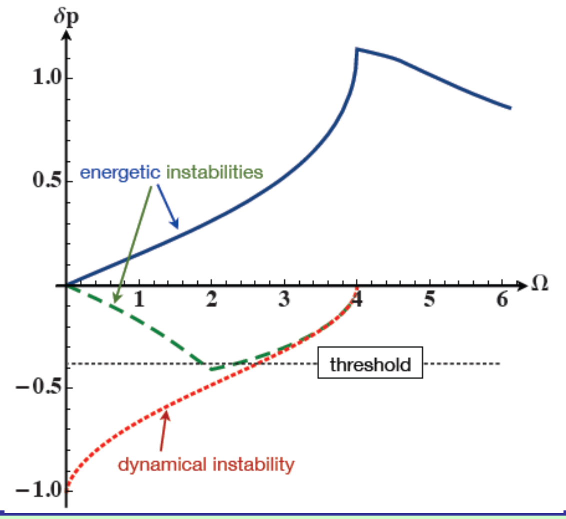
In harmonic trap a kick to the gas will gives rise to an oscillation around the minimum (dipole oscillation). If the kick is too large (even if positive) the system will enter the **red** region of dynamic instability

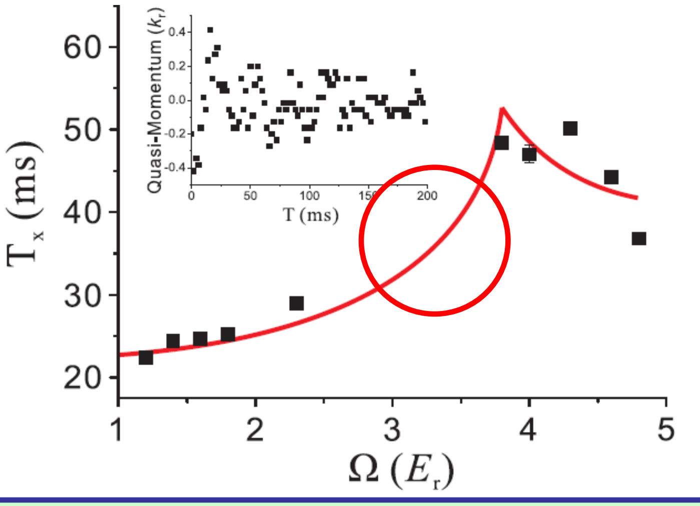
Critical value of velocity (and consequently of momentum kick) generating dynamic instability depends crucially on the value of Raman coupling Ω. It becomes smaller and smaller as Ω approaches the phase transition to the single minimum phase.

In the single minimum phase dynamical instability is absent



Recent experiments on **dipole oscillation** reveal strong dependence of collective frequency on spin-orbit coupling (Zhang et al. PRL 2012)



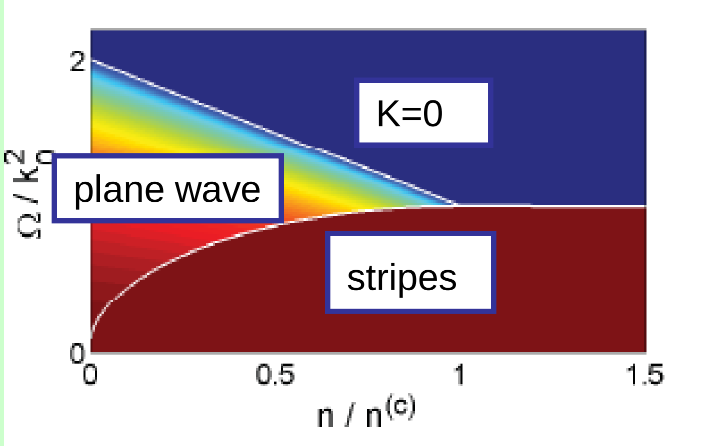


In the region below phase transition between plane wave and k=0 momentum phase, system exhibits instability. We argue that **noisy signal** in this region is due to the emergence of **dynamic instability** (too large momentum kick used to excite center of mass mode)

Dynamic instability can be also generated by a **quench** of the **Raman** coupling (exps not yet available)

Elementary excitations
and the **stripe phase**

Reminder of the stripe phase



Hamiltonian:

$$H = \sum_i h_0(i) + \frac{1}{2}\int d\vec{r}\left[(g+g_{\uparrow\downarrow})n^2 + (g-g_{\uparrow\downarrow})(n_+ - n_-)^2\right]$$

$$h_0 = \frac{1}{2}[\left(p_x - k_0\sigma_z\right)^2 + p_\perp^2] + \frac{1}{2}\Omega\sigma_x + \frac{1}{2}\delta\sigma_z$$

We make simplifying choice $g_{\uparrow\uparrow} = g_{\downarrow\downarrow} \equiv g, \ \delta = 0$ (if $g_{\uparrow\uparrow} \neq g_{\downarrow\downarrow}$ one can choose an effective magnetic field to compensate the asymmetry effect)

Order parameter:

$$\Psi = \sqrt{\frac{N}{2V}}\left[\begin{pmatrix}\cos\theta \\ -\sin\theta\end{pmatrix}e^{ik_1x} + \begin{pmatrix}\sin\theta \\ -\cos\theta\end{pmatrix}e^{-ik_1x} + higher\ harmonics\right]$$

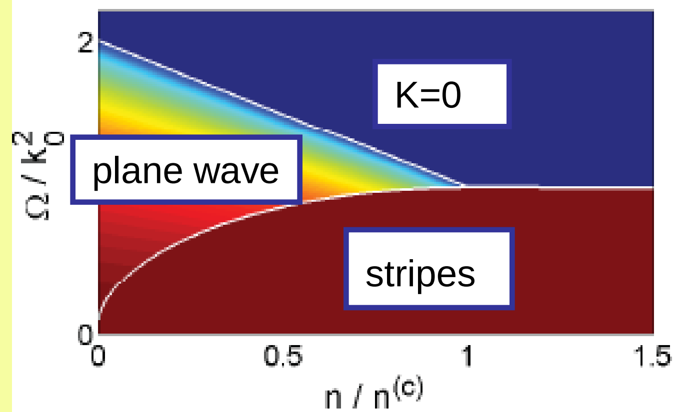
Contrast of fringes

Density modulations

$$n(x) = n\left(1 + \frac{\Omega}{2k_0^2}\cos 2k_1x\right) + higher\ harmonics$$

Absence of spin polarization (cfr plane wave phase: $<\sigma_z> = k_1/k_0$)

Stripe phase results from energetic competition between density and spin dependent terms in interaction term



$$H_{\text{int}} = \frac{1}{2}\int d\vec{r}\left[(g+g_{\uparrow\downarrow})n^2 + (g-g_{\uparrow\downarrow})(n_\uparrow - n_\downarrow)^2\right]$$

Cost due to density modulations

Cost due to spin polarization

$$\gamma = \frac{g - g_{\uparrow\downarrow}}{g + g_{\uparrow\downarrow}}$$

Stripe phase exists only if $g > g_{\uparrow\downarrow}$

Energetically favourable for values of the Raman coupling smaller than critical value

$$\Omega_{cr} = 2k_0^2 \sqrt{\frac{2\gamma}{1+2\gamma}}$$

(formula holds in small coupling limit $gn\,,\, g_{\uparrow\downarrow}n << k_0^2$)

For values $\Omega > \Omega_{cr}$ one enters the plane wave phase

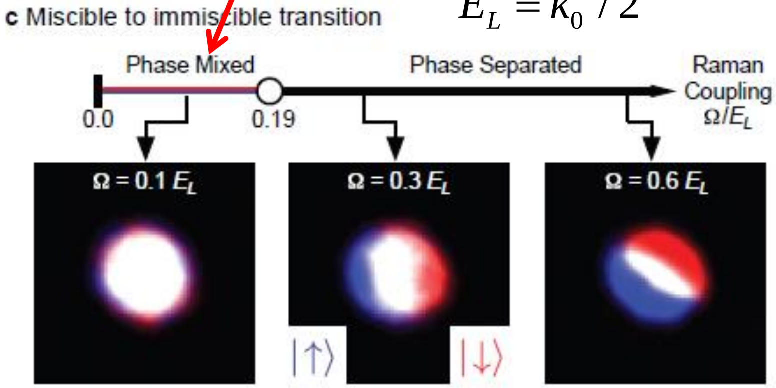
Stripe phase already realized experimentally (but stripes not yet observed !)
Phase transition observed at the predicted value of Raman coupling Ω . In Rb coupling constants are very close each other and phase transition to plane wave occurs at small values of Ω .

Fringes contrast $\Omega / 2k_0^2$ and fringes separation π / k_1 are too small to be observed in situ.

Effects of stripes more easily revealed in **excitation spectrum**

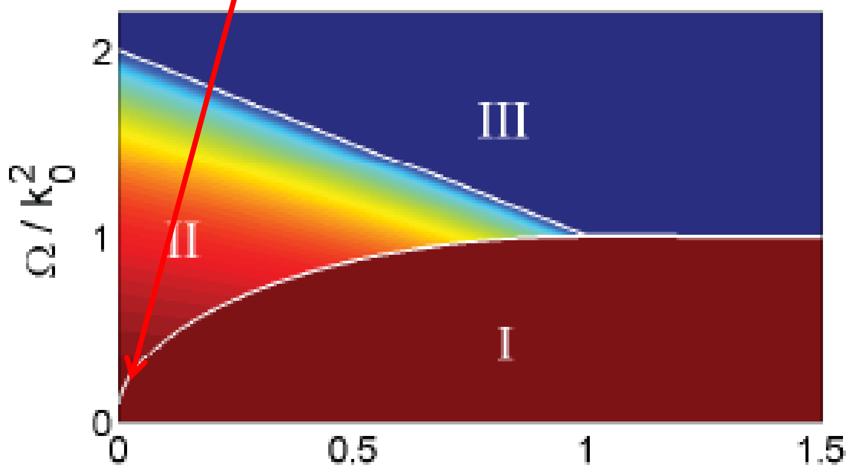
Stripe phase

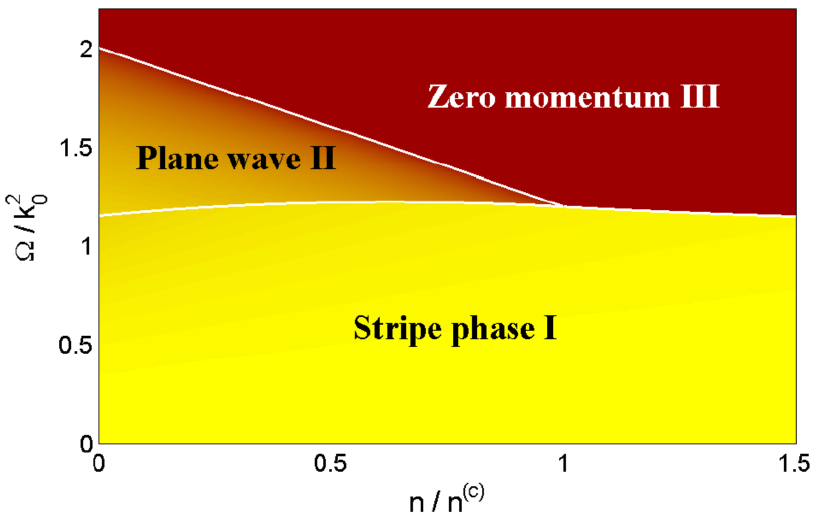
$E_L = k_0^2 / 2$



Lin et al., Nature 2011

In Rb Ω_{cr} is small. To increase the effects of the contrast (fixed b Ω) choose larger values of $\gamma = (g_{\uparrow\uparrow} - g_{\uparrow\downarrow})/(g_{\uparrow\uparrow} + g_{\uparrow\downarrow})$ (different atomic species, different trapping conditions)

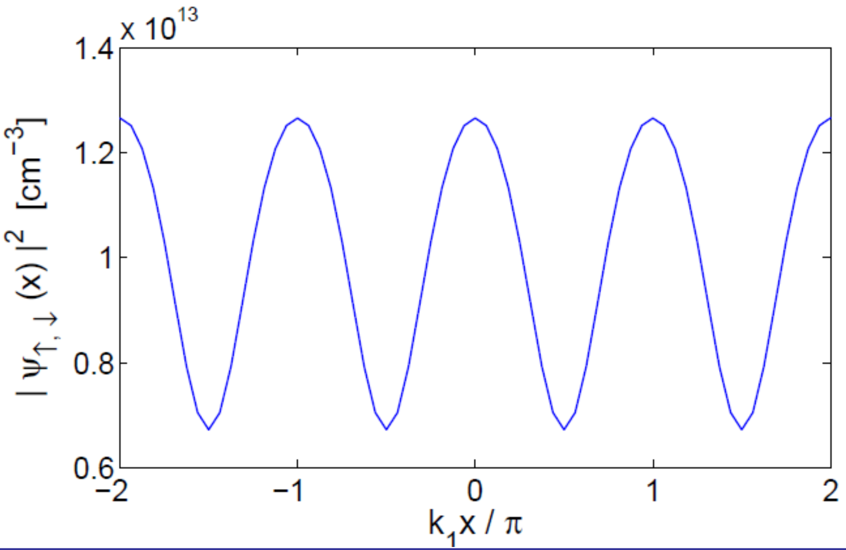




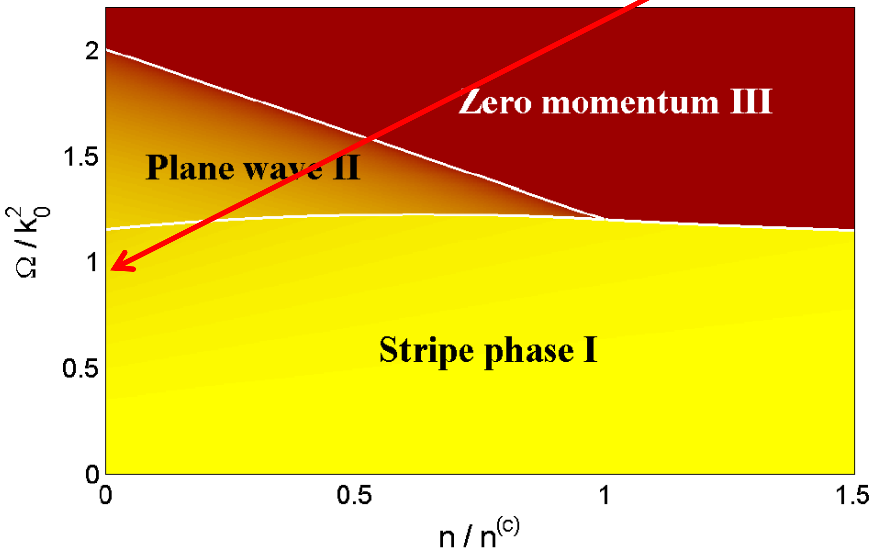
Rb parameters in Spielman exp

$$\Omega_{cr} = 2k_0^2 \sqrt{\frac{2\gamma}{1+2\gamma}}$$

Optimized choice with larger γ



Sizable density modulations at $\Omega = k_0^2$



Spectrum of elementary excitations in the **stripe** phase

Excitation spectrum in the **plane wave** phase

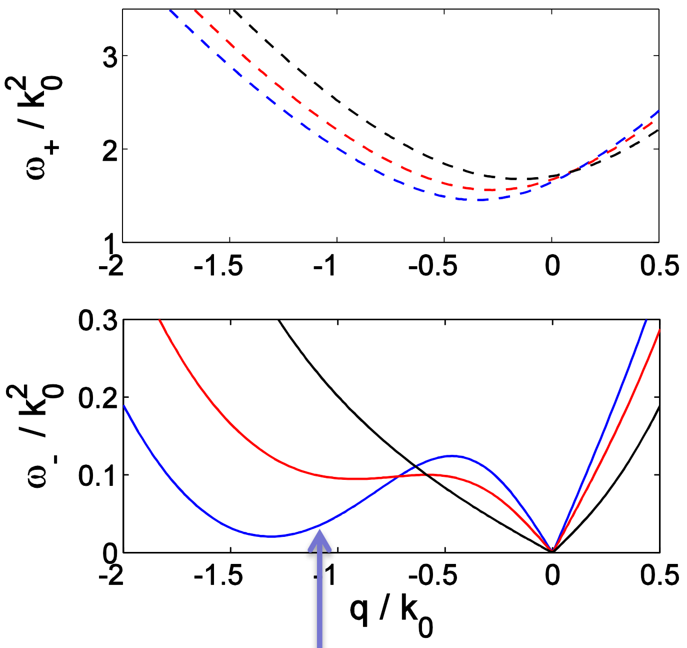
Giovanni Martone, Yun li, Lev Pitaevskii and S.S.. PRA 2012

Despite of spinor nature of the gas the occurrence of Raman coupling gives rise to a **single gapless** branch

Occupation of sp state with finite momentum yields **rotonic structure.**

Evidence for **anisotropy + parity breaking** in excitation spectrum

When Raman coupling approaches the transition to the stripe phase **roton minimum becomes lower** and lower (onset of crystallization)



Raman coupling close to transition to stripe phase

$$G_1/k_0^2 = 0.5,\ G_2/k_0^2 = 0.12$$

$$\Omega/k_0^2 = 1.22,\ 1.33,\ 1.46$$

Excitation spectrum in the **stripe** phase

Yun Li, Giovanni Martone, Lev Pitaevskii and S.S.
arXiv: 1303.6903, PRL 2013

Stripe phase exhibits spontaneous breaking of both gauge (**BEC**) and continous **translational** symmetries

Spontaneous breaking of continuous **translational** symmetry is expected to give rise to **second gapless** (Goldstone) mode, characterized by band structure. Typical behavior of supersolids.

We solve the linearized Gross-Pitaevskii equations in stripe phase using **Bloch formalism**

$$i\partial_t \Psi = \left[h_0 + \frac{1}{2}(g + g_{\uparrow\downarrow})\Psi^+\Psi + \frac{1}{2}(g - g_{\uparrow\downarrow})\Psi^+\sigma_z\Psi\sigma_z \right]\Psi$$

$$\begin{pmatrix} \psi_\uparrow \\ \psi_\downarrow \end{pmatrix} = e^{-i\mu t}\left[\begin{pmatrix} \psi_{0\uparrow} \\ \psi_{0\downarrow} \end{pmatrix} + \begin{pmatrix} u_\uparrow(\mathbf{r}) \\ u_\downarrow(\mathbf{r}) \end{pmatrix} e^{-i\omega t} + \begin{pmatrix} v^*_\uparrow(\mathbf{r}) \\ v^*_\downarrow(\mathbf{r}) \end{pmatrix} e^{i\omega t} \right]$$

$$u_{\mathbf{q}\,\uparrow,\downarrow}(\mathbf{r}) \;=\; e^{-ik_1 x}\sum_{\bar{K}} U_{\mathbf{q}\,\uparrow,\downarrow\,\bar{K}}\, e^{i\mathbf{q}\cdot\mathbf{r}+i\bar{K}x}$$

$$v_{\mathbf{q}\,\uparrow,\downarrow}(\mathbf{r}) \;=\; e^{ik_1 x}\sum_{\bar{K}} V_{\mathbf{q}\,\uparrow,\downarrow\,\bar{K}}\, e^{i\mathbf{q}\cdot\mathbf{r}-i\bar{K}x}$$

Inclusion of **many bands** in the ground state and in the Bogoliubov amplitudes is crucial to ensure correct gapless (phononic) behavior at small q and good convergency

Nature of two gapless branches:

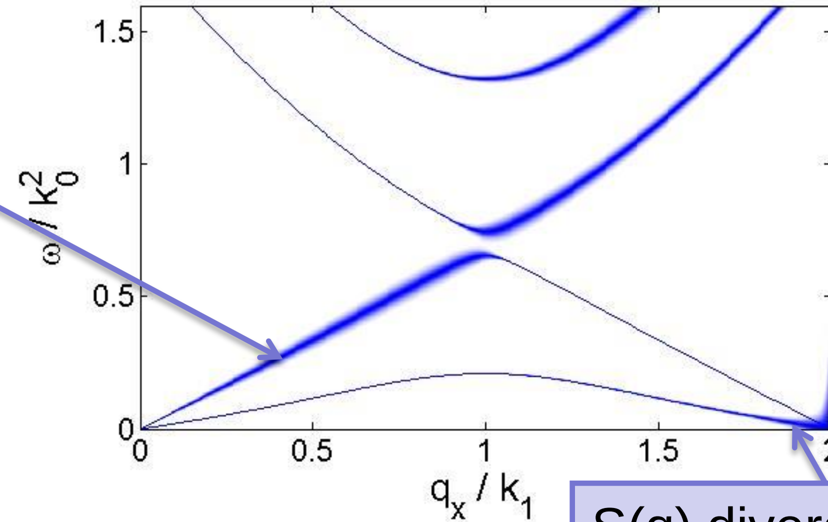
Upper branch is **density** wave. At small q exhausts static structure factor

$$S(q) = \int d\omega S(q,\omega)$$

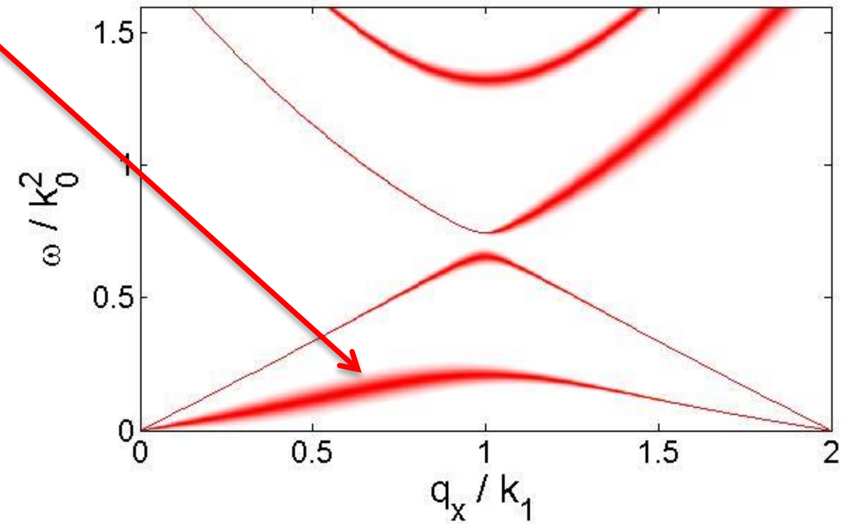
Lower branch is **spin** wave at small q. Responsible for **divergent** behavior of S(q) at the Brillouin vector

Dynamic structure factor measurable with 2-photon Bragg spectroscopy

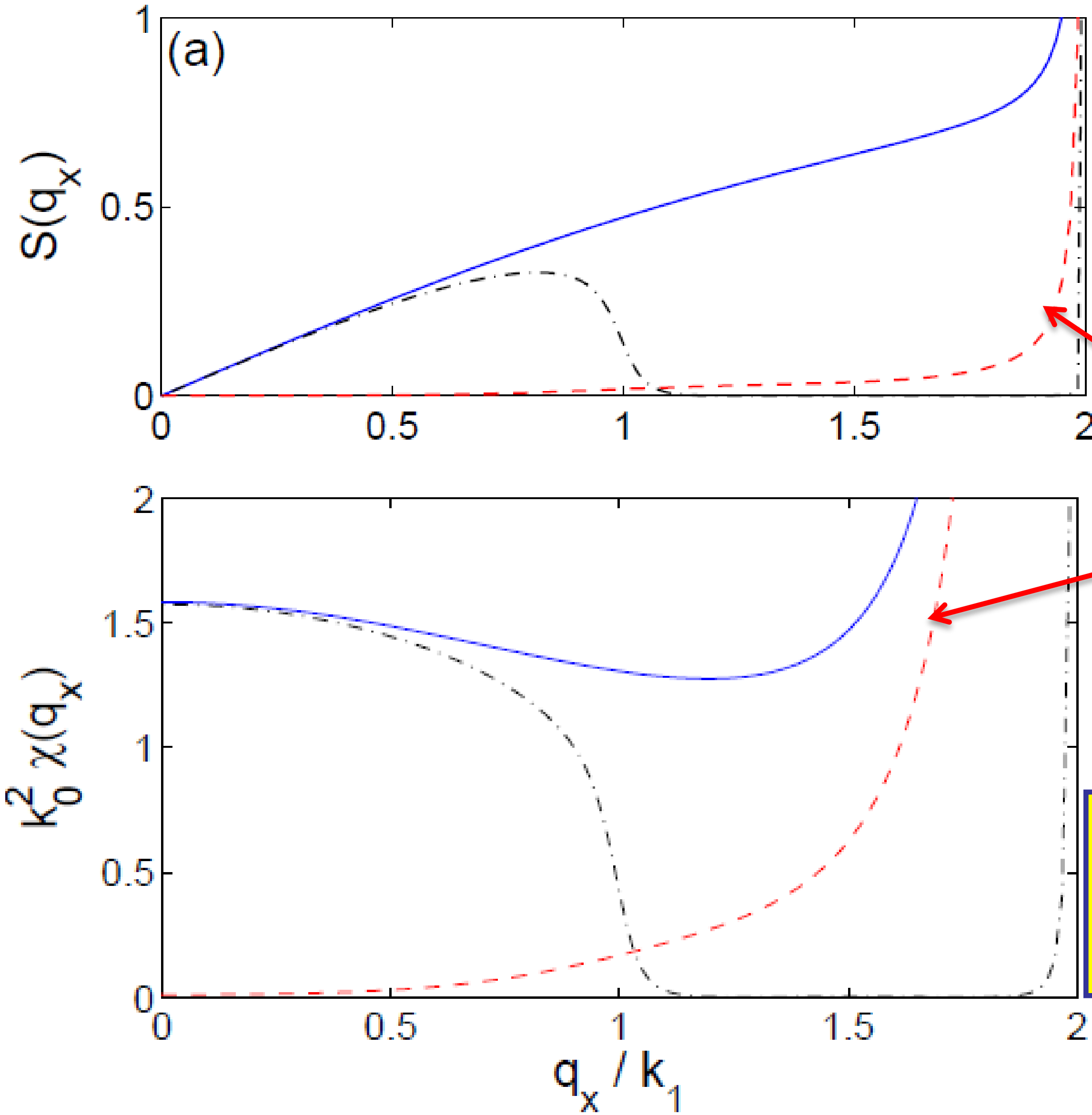
Density structure factor



S(q) diverges !!



Spin structure factor



Static structure factor

$$S(q) = \int d\omega S(q,\omega)$$

Divergent behavior caused by new low frequency branch

Static response function

$$\chi(q) = \int d\omega \omega^{-1} S(q,\omega)$$

Proof of divergent behavior of static structure factor $S(q)$ and static response function $\chi(q)$ near Brillouin vector

Proof is based on **rigorous inequalities** of statistical mechanics (m_k are moments of dynamic structure factor relative to F and G)

$$m_{-1}(F)m_1(G) \ge |<[F^+,G]>|^2$$

Bogoliubov inequality

$$m_0(F)m_0(G) \ge |<[F^+,G]>|^2$$

Uncertainty principle inequality

Inequalities hold for **any choice** of the operators **F and G**.

Useful choice ($q_B = 2k_1$)

$$F = \sum_j e^{iqx_j}$$

$$G = \sum_j (p_{xj} e^{i(q-q_B)x_j} + e^{i(q-q_B)x_j} p_{xj})/2$$

$$<[F^+,G]> = Nq < e^{-iq_B x} >$$

$$m_{-1}(F) = \chi(q)$$

$$m_0(F) = S(q)$$

$$m_1(G) = <[G^+[H,G]]>$$

$$m_0(G) \le \sqrt{m_1(G)m_{-1}(G)}$$

Inequalities provide rigorous bounds to $S(q)$ and $\chi(q)$

Near the Brillouin point static structure factor and static response exhibit divergent behavior

$$S(q) \propto \frac{|< e^{iq_B x} >|^2}{\sqrt{<[P_x(q-q_B),[H,P_x(q-q_B)]]>}} \approx \frac{1}{q-q_B}$$

$$\chi(q) \propto \frac{|< e^{iq_B x} >|^2}{<[P_x(q-q_B),[H,P_x(q-q_B)]]>} \approx \frac{1}{(q-q_B)^2}$$

$$P_x(q-q_B) = \sum_j (p_{xj} e^{i(q-q_B)x_j} + e^{i(q-q_B)x_j} p_{xj})/2$$

Divergent behavior requires:

- existence of crystalline order parameter $< e^{iq_B x} > \neq 0$

- translational invariance of the Hamiltonian $<[P_x(q-q_B),[H,P_x(q-q_B)]]>_{q\to q_B} \propto (q-q_B)^2$

Need for spontaneous breaking of translational symmetry !

Divergent behavior of static structure factor belongs to a general class of infrared divergencies exhibited by systems with **spontaneous breaking of continuous symmetries**

Other examples:

- momentum disribution in the presence of Bose-Einstein condensation: $n(p) \propto \frac{n_0}{p}$ (Gavoret-Nozieres)

- Spin structure factor in FFLO configurations of spin polarized Fermi superfluids

Smoking gun for superstripes (and supersolidity ?)

- **Double gapless band** structure
- **Divergent** behavior of S(q) at the Brillouin point

Conclusions

- **Supercurrent** in plane wave phase (uniform density) can exhibit **dynamic instability** as a consequence of breaking of Galiean invariance. Critical value of velocity differs from Landau criterion

- Stripe phase exhibits typical features of **supersolidity** (simultaneous spontaneous breaking of two continous symmetries) characterized by (smoking gun):

  - **two gapless bands**,
  - **divergent behavior of S(q)** at the Brillouin border

Collective oscillations and Bragg scattering experiments can probe the new superfluid features

Open issues (emerging from lectures 1 to 4) related to superfluidity in ultracold atomic gases

- Are the effects of quantum fluctuations on the structure of the **core of dark solitons** in the unitary Fermi gas crucial to explain major discrepancies between Mit exp and BdG predictions?

- Need for many body calculations of the **superfluid density** in 3D unitary gas (first data now available from measurement of second sound)

- Need for theory and exps on **second sound** and superfluid density in **2D BKT** Bose gases

- How to realize a **superleak** with cold atomic gases ? (control superfluid flow, cfr recent thermoelectric effect exps at ETH)

- Exp evidence for **rotons** in **spin orbit** coupled Bose gases still missing (Bragg spectroscopy)

- Exp evidence for **superstripes** in spin orbit coupled Bose gases (**double gapless band** structure)

The Trento BEC team
http://bec.science.unitn.it/