SUPERFLUIDITY IN ULTRACOLD ATOMIC GASES

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PLAN OF THE LECTURES

Lecture 1. **Superfluidity** in ultra cold atomic gases: examples and open questions (May 14)

Lecture 2. A tale of two sounds (**first** and **second sound**) (May 21)

Lecture 3. **Spin-orbit** coupled Bose-Einstein condensed gases: quantum phases and **anisotropic dynamics** (May 28)

Lecture 4. **Superstripes** and **supercurrents** in spin-orbit coupled Bose-Einstein condensates (June 4)
Bose-Einstein condensation: first experiments

1995 (Jila+Mit)
(Macroscopic occupation of sp state)

1996 Mit (coherence + wave nature)

\[ N_0 / N \neq 0 \]

\[ \psi = \sqrt{n} e^{i\phi} \]
What was **new** with BEC in trapped atomic gases?

- Bose-Einstein condensation in both **momentum** and **coordinate** space

- **Diluteness** (Gross-Pitaevskii eq. for order parameter)

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New important knobs are now available. They permit to increase the effects of correlations.

- **Tuning** of **scattering length** (BEC-BCS crossover in Fermi gases)

- **Tuning** the **external conditions**, optical lattices and the superfluid-Mott insulator transition, 1D and 2D configurations, adding disorder …
Fermi Superfluidity: the BEC-BCS Crossover  
(Eagles, Leggett, Nozieres, Schmitt-Rink, Randeria)

BEC regime  
(molecules)

BCS regime  
(Cooper pairs)

Dilute Bose gas  
(size of molecules much smaller than interparticle distance)

At unitarity scattering length is much larger than interparticle distance: strongly interacting superfluid

Tuning the scattering length through a Feshbach resonance

BEC regime (molecules)

BCS regime (Cooper pairs)

unitary limit

At unitarity scattering length is much larger than interparticle distance: strongly interacting superfluid
Important (and old) questions

- Connections between BEC and superfluidity
- Can the condensate and superfluid densities be different?

Some answers

- Gross-Pitaevskii equation for the 3D BEC order parameter predicts important superfluid features (quantized vortices, irrotational hydrodynamic flow ...)
- In dilute 3D BECs condensate and superfluid densities are practically equivalent (quantum depletion is small). Crucial differences emerge in low D (BKT superfluidity) and in strongly interacting superfluids (liquid Helium, unitary Fermi gas). Measurement of superfluid density in strongly interacting Fermi gas is now available.
Superfluid He4

Superfluid fraction

Condensate fraction
Superfluid He4

Superfluid fraction

Condensate fraction

Unitary Fermi gas (lecture 2)

Sidorenkov et al., IBK-TN, 2013

Ku et al., MIT, 2012
Superfluid features in atomic gases (selection)

- Quantized vortices and solitons

- Absence of viscosity (critical velocity and supercurrents)

- Search for supersolidity

- Lambda transition and specific heat

- Hydrodynamic behavior (irrotationality, collective oscillations, first and second sound)
Quantized Vortices and solitons exhibit unique topological features

They are characterized by a peculiar behavior of the phase of the order parameter

\[ \Psi(\vec{r}) = e^{i\varphi} |\Psi(\vec{r})| \quad \text{quantized vortex} \]

\[ \Psi(z) = \text{sgn}(z) |\Psi(z)| \quad \text{dark soliton} \]

Due to the singularity of the phase at \( r=0 \) (vortex) and \( z=0 \) (dark soliton) the density of the superfluid is suppressed in the vicinity of the core. Gross-Pitaevskii theory predicts that the density exactly vanishes on the core. What happens in more correlated fluids (for example in the unitary Fermi gas?)
Recent **Mit experiment** on the oscillation of a soliton of superfluid Fermi gas in a harmonic trap raises dramatically the question of the structure of its core and of its effective mass

Yefsah et al., arXiv:1302.4736
Bogoliubov de Gennes theory predicts that at unitarity the core of the dark soliton is partially filled, differently from what happens in a dilute Bose gas.

Useful quantity characterizing the soliton core is the so called deficit of atoms:

\[ N_S = \int_{-\infty}^{+\infty} dx (n_1(x) - n_1(\infty)) \]

This quantity is crucial for the calculation of the effective mass of the soliton.

Antezza et al., PRA 2007
Effective mass of a dark soliton
(oscillation in harmonic trap)

The effective mass of a dark soliton is a crucial ingredient describing the oscillation of the soliton in a harmonic trap. Its value is the result of two different effects (Scott et al. PRL 2011):

- a dynamic effect accounted for by the derivative of the phase difference (left to right) with respect to the velocity of the soliton.

\[
\frac{M^*}{M} = \left( \frac{T_S}{T_Z} \right)^2 = 1 + \frac{\hbar n_1(+\infty)}{m_B N_S} \frac{d\Delta \varphi}{dv}
\]

- An equilibrium ingredient given by deficit of particles in the soliton region

\[
N_S = \int_{-\infty}^{+\infty} dx (n_1(x) - n_1(+\infty))
\]
In a **dilute Bose-Einstein condensed gas** both the gradient of the phase and the deficit of particles can be calculated analytically starting from GP equations.

The result is (Busch and Anglin, 2001, Konotop and Pitaevskii, 2004)

\[
\left( \frac{T_s}{T_z} \right)^2 = 2
\]

Frequency of oscillation is decreased by factor \( \sqrt{2} \)

Experiments in 1D harmonic traps confirm the prediction of theory with reasonably good accuracy

Becker et al. (2008), Weller et al. (2008)
In Scott et al. (PRL, 2011) we calculated the velocity gradient \( \frac{d(\Delta \phi)}{dv} \) of the phase difference and the deficit \( N_s \) of the soliton density along the BEC-BCS crossover, by solving the BdG equations. We find:

- The velocity gradient is practically constant along the crossover.
- The deficit \( N_s \) decreases (in modulus) as one moves from BEC to unitarity. The oscillation time accordingly increases.

![Graphs showing phase difference and soliton density](image-url)
Soliton oscillations in harmonic trap
(solution of time dependent Bogoliubov equations, Scott et al. 2011)

\[
\frac{1}{k_F a} = -0.5 \\
\text{(BCS)}
\]

\[
\frac{1}{k_F a} = 0 \\
\text{(Unitarity)}
\]

\[
\frac{1}{k_F a} = 0.5 \\
\text{(BEC)}
\]
Recent measurements of the **soliton oscillation** in harmonic trap reveals much stronger increase of effective mass when one moves from BEC to unitarity. Huge discrepancy with predictions of mean field theory (BdG eqs)

Mit exp: Yefsah et al., arXiv:1302.4736

BdG prediction
Scott et al., PRL 2011
Can we calculate quantum fluctuations inside soliton beyond mean field theory? Is the question relevant also for the effective mass of a quantized vortex? Can we envisage an experiment to measure the effective mass of a quantized vortex?
Landau’s critical velocity and supercurrents
Two different critical velocities:

- **Impurity moving with velocity v**
  Critical velocity fixed by Landau’s criterion
  (energetic instability, energy of excitation spectrum becomes negative)

- **Critical current.**
  Superfluid can support a metastable supercurrent up to a critical velocity

\[ v_c = \min_p \frac{\varepsilon(p)}{p} \neq 0 \]

In uniform superfluids the two criteria are equivalent (consequence of Galilean invariance)

In non uniform superfluids (eg. optical lattice) the two criteria are different and current can exhibit **dynamic instability**
(apperance of imaginary component in excitation spectrum)
Above critical velocity dissipative effect produced by moving optical lattice is observed

Example of Landau’s critical velocity: uniform Fermi superfluid at unitarity (energetic instability)

\[ v_c = \min_p \frac{\varepsilon(p)}{p} \]

(Mit, Miller et al, 2007)
Dispersion law along BCS-BEC crossover
(Cobescoat, M. Kagan, Stringari, 2006)

Landau’s critical velocity is highest near unitarity!!
Example of dynamic instability: BEC in moving periodic potential

\[ \omega + \delta \omega, \ k + \delta k \]

\[ \omega, \ -k \]

\[ w \pm dw, \ k \pm d k \]

(Fallani et al., 2004)

Example of dynamic instability: BEC in moving periodic potential

In tight binding limit dynamic instability starts when the effective mass associated with the presence of the optical lattice becomes negative.
Question addressed in Lecture 4

Are Landau’s criterion for critical velocity and stability criterion for persistent current always equivalent in uniform superfluids?

Recent possibility of generating uniform superfluids with Spin-Orbit Hamiltonians breaking Galilean invariance

Non trivial consequence on the stability of the supercurrent

RECENT WORK ON PERSISTENT CURRENTS

Measured critical velocity in 2D superfluids (ENS)
Persistent current in ring geometry with weak links (Nist)
Critical velocities in spinor mixtures (Cambridge)

.....
Search for **supersolidity** in ultracold atomic gases

Supersolidity is characterized by co-existence of two sponatenoeusly broken continuous symmetries:

- **Gauge symmetry** yielding BEC and superfluidity

- **Translational invariance** yielding crystalline structure
- First attempts to observe supersolidity in **solid helium** (Kim and Chan, Nature 2004) by observing quenching of moment of inertia

- **No conclusive proof** of supersolidity still available (Balibar, Nature 2012).

Recent theoretical proposals to realize supersolidity in ultracold atomic gases:

- **Rydberg atoms** with dipolar potentials softened at short distance

- Superstripe phase in **spin-orbit coupled BEC’s** (Lecture 4)
Superfluid

Supersolid

Normal solid

Excitation spectrum of a Bose gas with soft core repulsive potential
Saccani et al, PRL 2012

Double gapless band in the superstripe phase of a spin-orbit coupled BEC
Yun Li et al. PRL 2013
Thermodynamics of a strongly interacting Fermi superfluid gas:

The lambda transition
Thermodynamics and Universality of the Unitary Fermi gas (1/a=0)

Absence of interaction parameter implies that thermodynamics should obey universal law (Ho, 2004)

\[ \frac{P}{k_B T} \lambda_T^3 = f_p(\mu / k_B T) \]

\[ n \lambda_T^3 = f_p'(\mu / k_B T) \]

where \( \lambda_T = \sqrt{2\pi \hbar^2 / mk_B T} \) is thermal wave-length and \( f_p(x) \) is dimensionless, universal function.

All thermodynamic functions (entropy, compressibilities, specific heats etc.) can be expressed in terms of \( f_p(x) \) exact behavior known only at large negative \( x \) (classical regime) and at large positive \( x \) (phonon regime). Calculation of \( f_p(x) \) requires non trivial many-body approaches at finite \( T \).

Universal function \( f_p(x) \) and thermodynamic functions are now available experimentally.
Thermodynamics of interacting Fermi gas

Recent major contributions: ENS (Nascimbene et al., Nature 2010) and MIT (Ku et al., Science 2012)

MIT experiment has provided first direct evidence of lambda transition in specific heat. **Pressure** is measured by integrating radial density profile and using LDA result

\[
\begin{align*}
n_1(z) &= \int n(\vec{r}) \, dx \, dy = \frac{2\pi}{m\omega_{\perp}^2} P(x = y = 0) \\
V_{\text{ext}} &= (1/2m)[\omega_{\perp}^2(x^2 + y^2) + \omega_z^2 z^2]
\end{align*}
\]

holding in harmonic traps

In MIT exp measurement of T was replaced by measurement of **compressibility**

\[
\kappa = -\frac{1}{n^2} \left( \frac{dn}{dV_{\text{ext}}} \right)_T
\]
Experimental determination of critical temperature

\[ T_c / T_F = 0.167(13) \]

(determined by peak in specific heat and onset of BEC) in agreement with many-body predictions (Burowski et al. 2006; Haussmann et al. (2007); Goulko and Wingate 2010)
Universal function \( f_p(\mu/k_B T) \) gives access to all thermodynamic quantities, except superfluid density.

Question: how to measure the superfluid density?

(not available from equilibrium thermodynamics, needed transport phenomena)
Determination of superfluid density in strongly interacting Fermi gases through measurement of second sound (Innsbruck-Trento collaboration Nature, this week!)

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Meng Khoon Tey
Yan-Hua Hou
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Next Lecture