

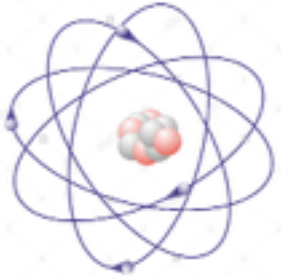
# Excitonic condensation of strongly correlated electrons

*Jan Kuneš*



# Magnetism of 'non-magnetic ions'

single atom



energy spectrum:

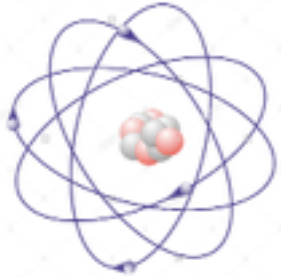
magnetic multiplet 

ground state (S=0) 

Examples:  $\text{Co}^{3+}$ ,  $\text{Fe}^{2+}$ ,  $\text{Ir}^{5+}$ ,  $\text{Os}^{4+}$ ,  $\text{Ru}^{4+}$  in cubic crystal field

# Magnetism of 'non-magnetic ions'

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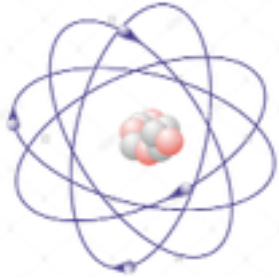
Examples:  $\text{Co}^{3+}$ ,  $\text{Fe}^{2+}$ ,  $\text{Ir}^{5+}$ ,  $\text{Os}^{4+}$ ,  $\text{Ru}^{4+}$  in cubic crystal field



combing a bald head

# Magnetism of 'non-magnetic ions'

single atom

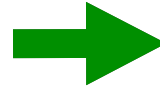


energy spectrum:

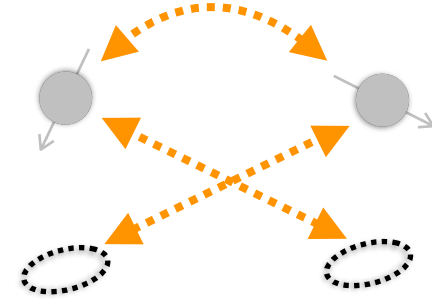
magnetic multiplet 

ground state (S=0) 

crystal



inter-atomic interaction:



Examples:  $\text{Co}^{3+}$ ,  $\text{Fe}^{2+}$ ,  $\text{Ir}^{5+}$ ,  $\text{Os}^{4+}$ ,  $\text{Ru}^{4+}$  in cubic crystal field

*L. Balents, PRB 62, 2346 (2000)*

*G. Khaliullin, PRL 111, 197201 (2013)*

*JK and P. Augustinský, PRB 90, 235112 (2014)*

*T. Kaneko, Y. Ohta and S. Yunoki, PRB 97, 155131 (2018)*

*J. Nasu et al., PRB 93, 205136 (2016)*

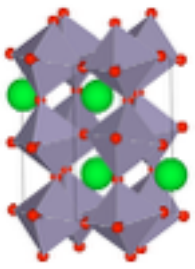
*T. Tatsuno et al., JPSJ 85, 083706 (2016)*

*T. Yamaguchi, K. Sugimoto and Y. Ohta, JPSJ 86, 043701 (2017)*

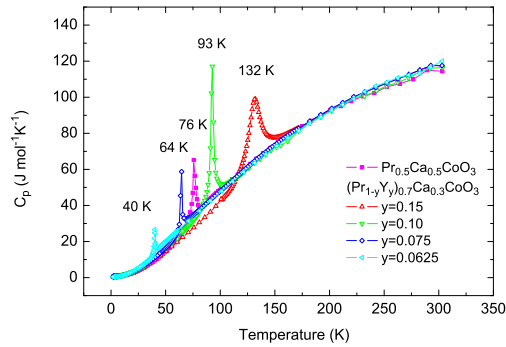
⋮



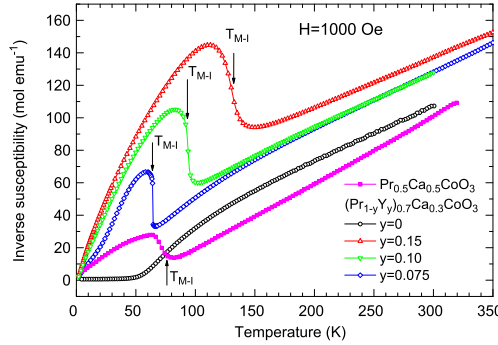
# $(\text{Pr}_{1-y}\text{Y}_y)_{1-x}\text{Ca}_x\text{CoO}_3$



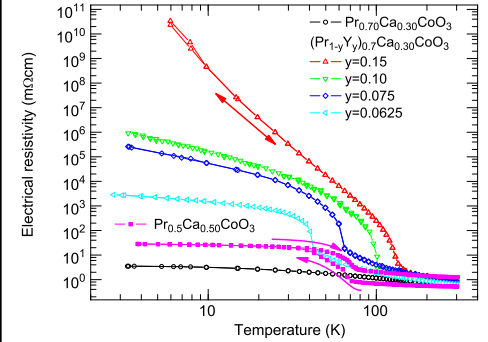
## specific heat



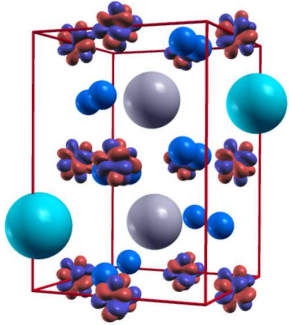
## spin susceptibility



## transport

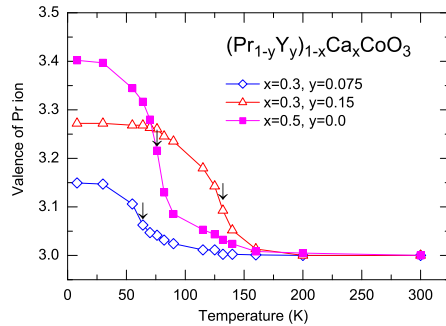


## theory

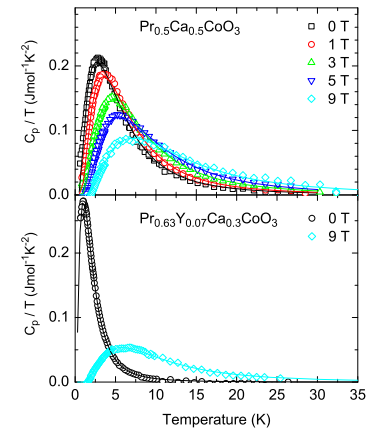


## Crystal field

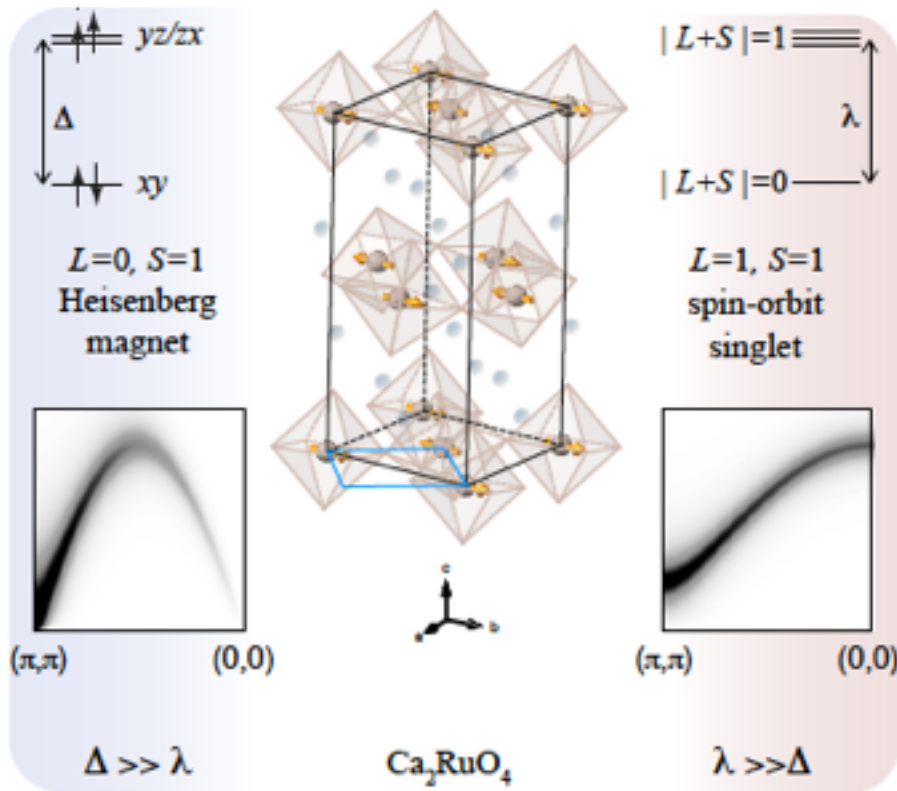
## XAS->valence transition



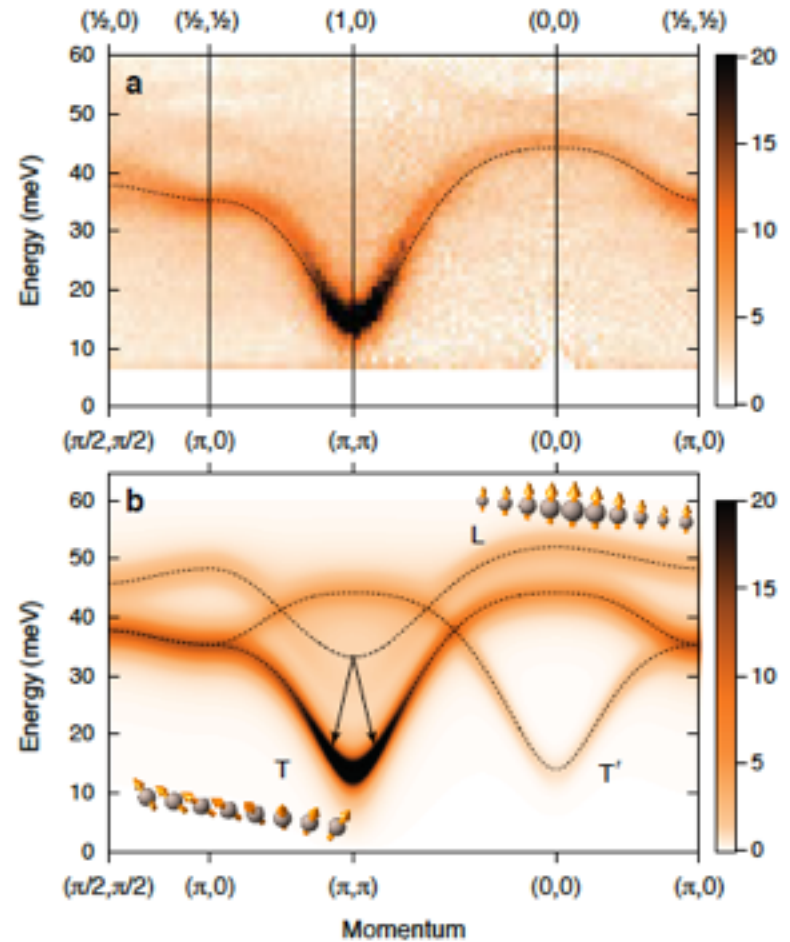
## low-T specific heat



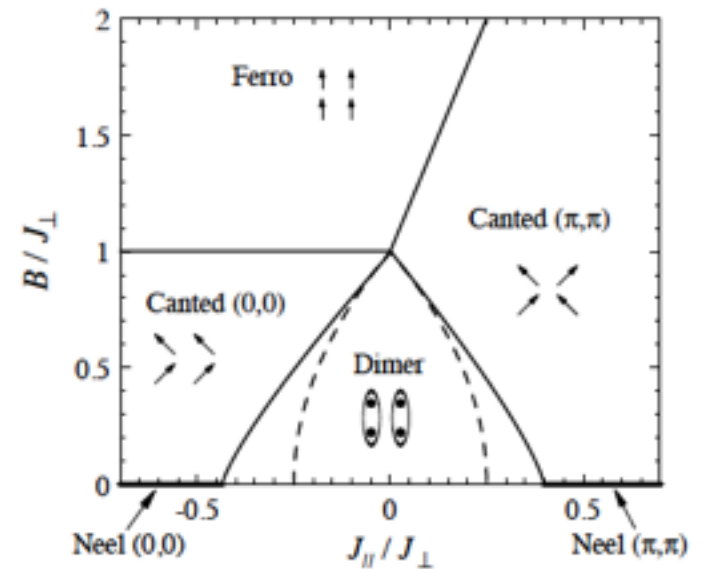
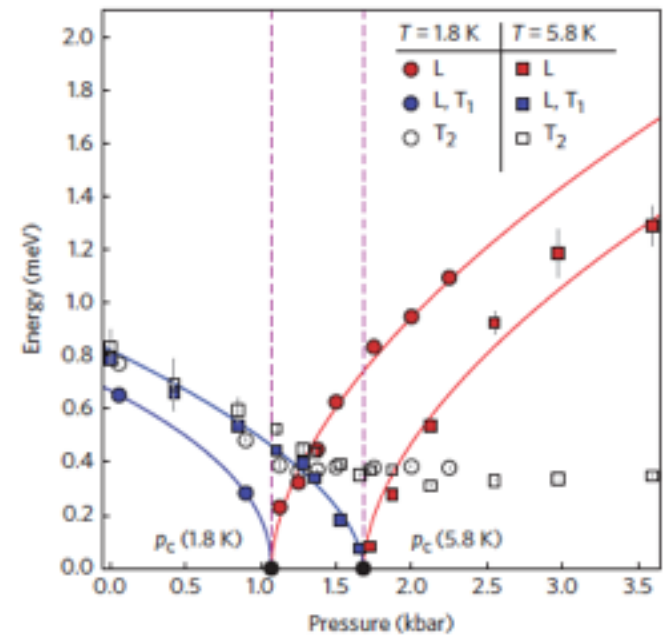
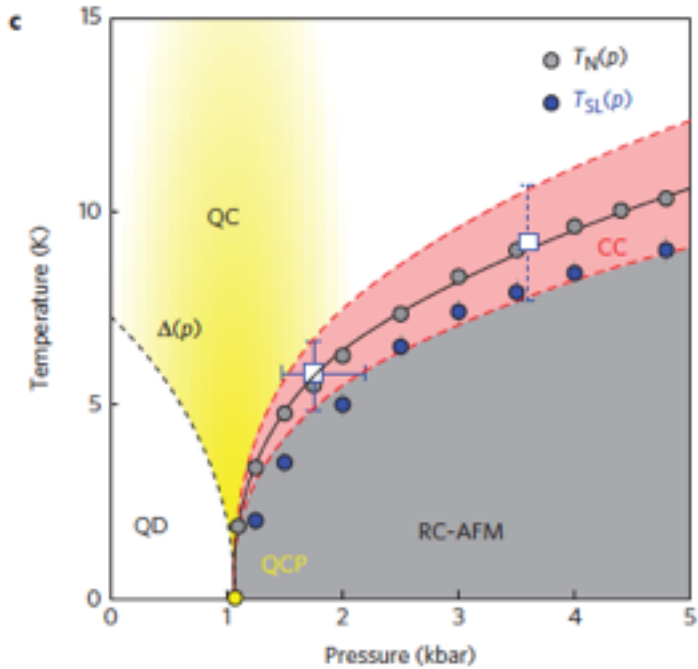
# Ca<sub>2</sub>RuO<sub>4</sub>



## Spin-orbit coupling



# TlCuCl<sub>3</sub>



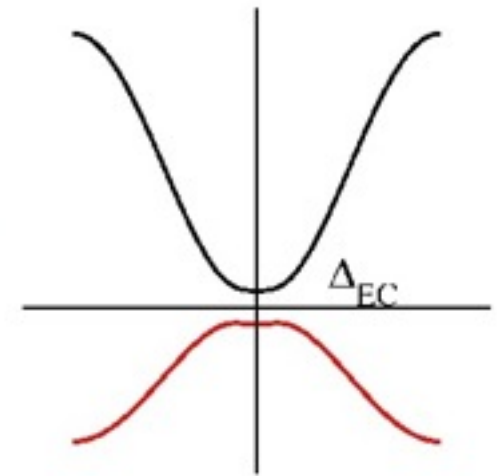
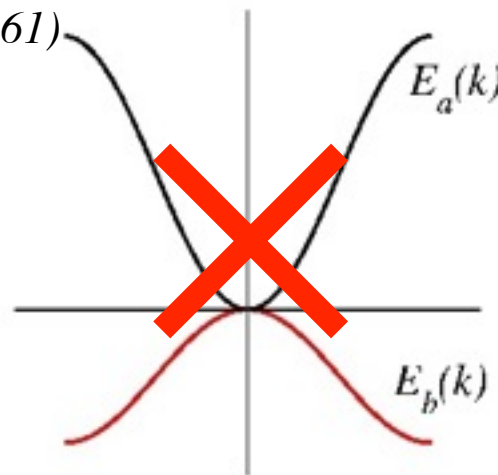
bi-layer Heisenberg model

# Excitonic insulator



“The first point that we must make about this model is that the predicted continuous increase in the number of free electrons and holes from the value zero is not possible. An electron and a positive hole will attract each other ... the electron and hole will always, when in the state of lowest energy, form pairs (excitons) ...”

*N. F. Mott, Philos. Mag. 6, 287 (1961)*

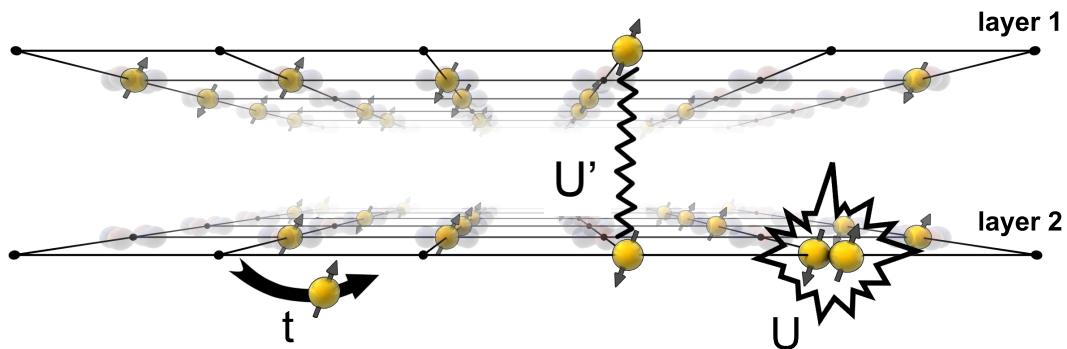


} BCS theories

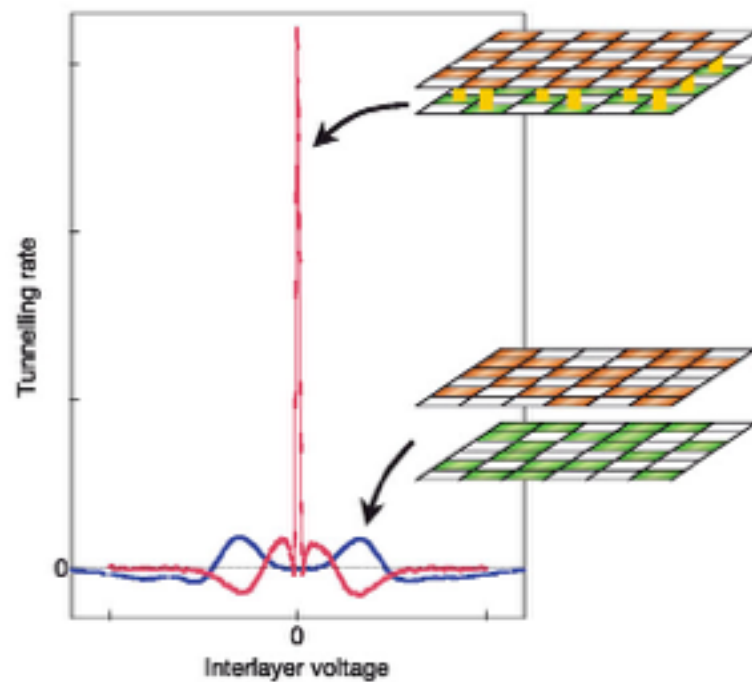
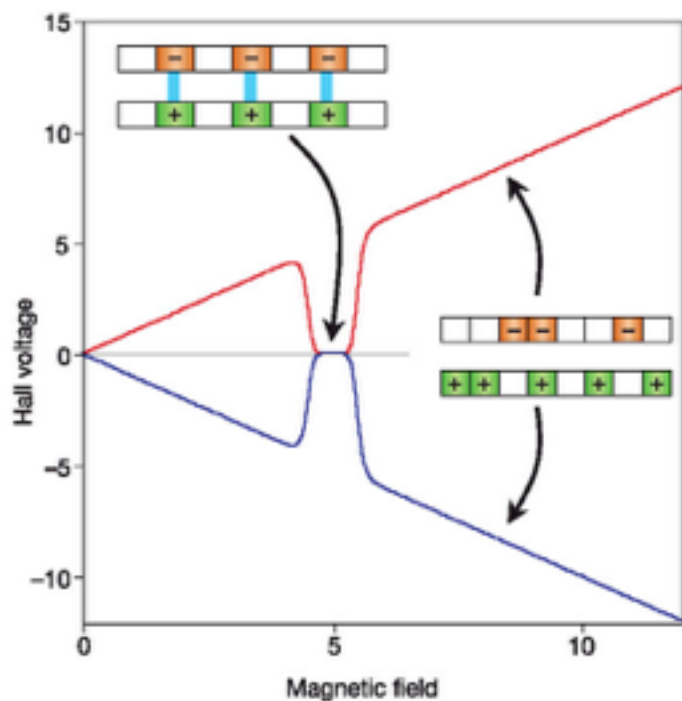
*R. S. Knox, Solid State Phys. Sup*  
*L. V. Keldysh and Y. V. Kopaev, Sov. Phys. Solid State 6, 2219 (1965)*  
*J. des Cloizeaux, J. Phys. Chem. Solids 26, 259 (1965)*  
*B. I. Halperin and T. M. Rice, Solid state physics, (1968)*

*H. Cercellier et al., PRL 99, 146403 (2007); C. Monney et al. PRL 106, 106404 (2011)*

# Experimental proof of exciton condensation: bilayers

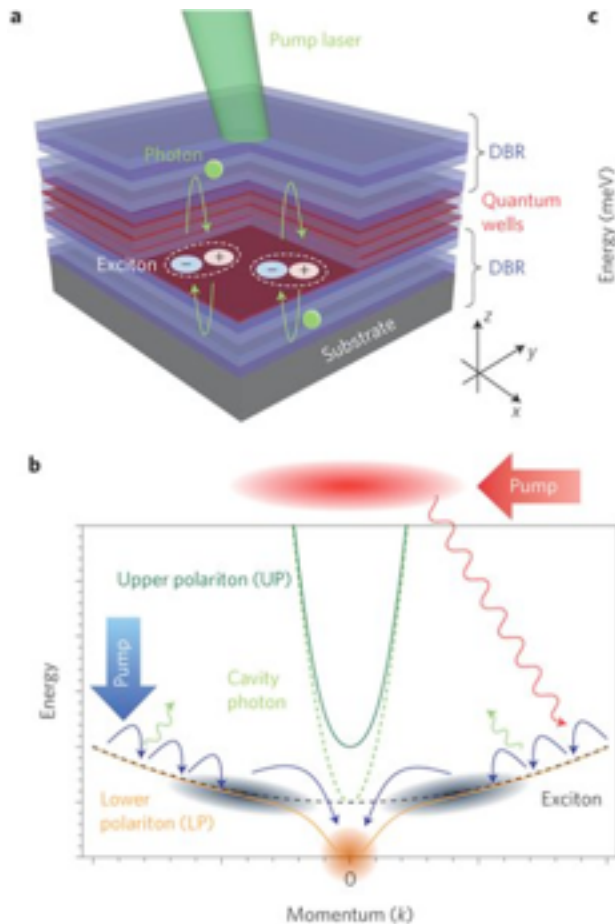


from Rademaker, 2014



*J. P. Eisenstein and A. H. MacDonald, Nature 432, 691 (2004)*

# Exciton-polariton condensates



PHYSICAL REVIEW LETTERS **122**, 017401 (2019)

## Superradiant Quantum Materials

Giacomo Mazza<sup>1,2,\*</sup> and Antoine Georges<sup>2,3,1,4</sup>

*T. Byrnes, N. Young Kim and Y. Yamamoto, Nat. Phys. **10**, 803 (2014)*

# Outline

- Two-orbital Hubbard model: strong coupling  
DMFT results
- Real materials  $(\text{Pr}_x\text{R}_{1-x})_y\text{Ca}_{1-y}\text{CoO}_3$
- How to detect exciton condensate?

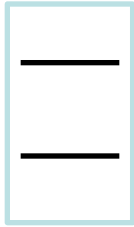
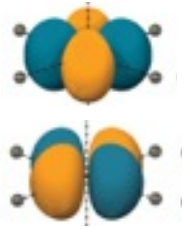


# Model set-up



# 2-orbital atom with 2 electrons

orbitals:



$E_0 + \Delta$

$E_0$

e-e interaction:



$U$



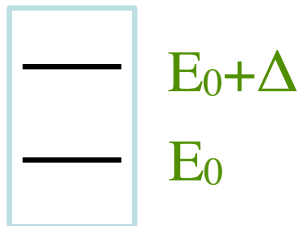
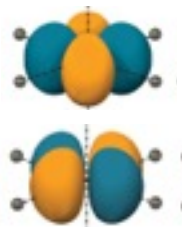
$-J$



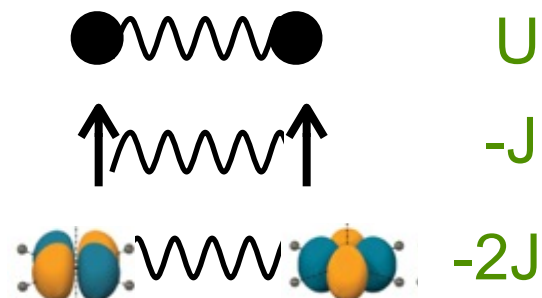
$-2J$

# 2-orbital atom with 2 electrons

orbitals:

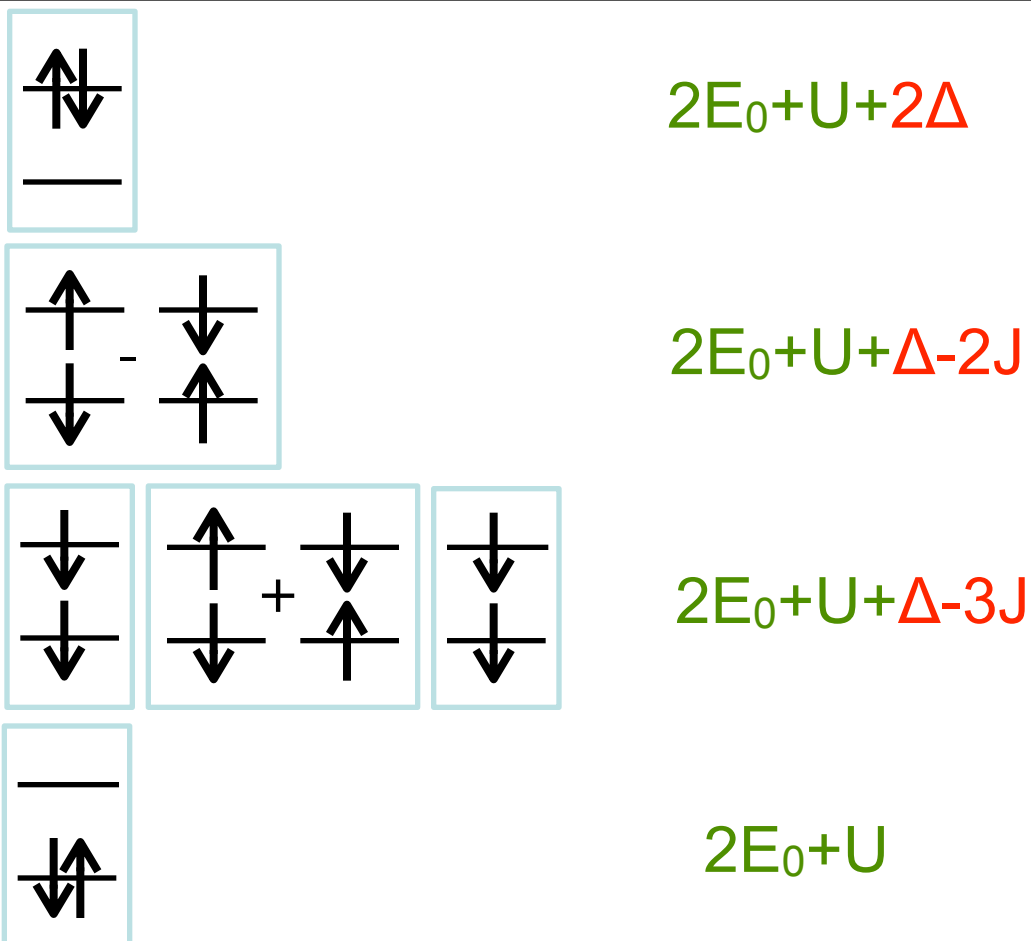


e-e interaction:



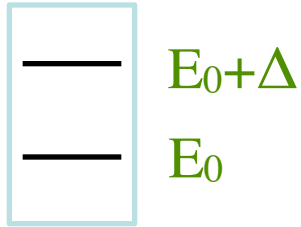
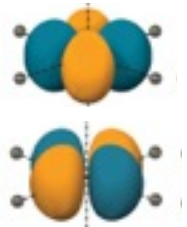
2-electron configurations:

$$\Delta \geq 3J$$

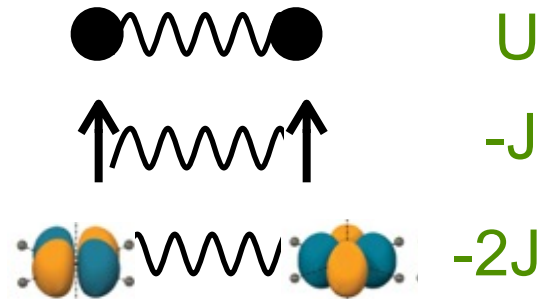


# 2-orbital atom with 2 electrons

orbitals:



e-e interaction:

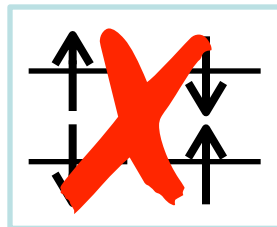


2-electron configurations:

$$\Delta \geq 3J$$

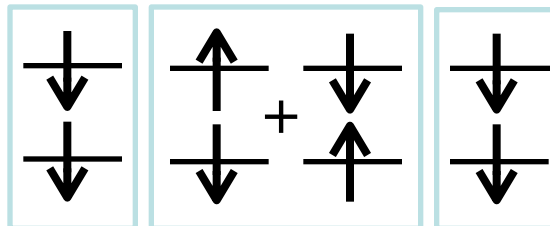


$$2E_0 + U + 2\Delta$$



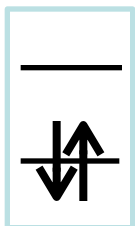
$$2E_0 + U + \Delta - 2J$$

High-spin state ( $S=1$ )



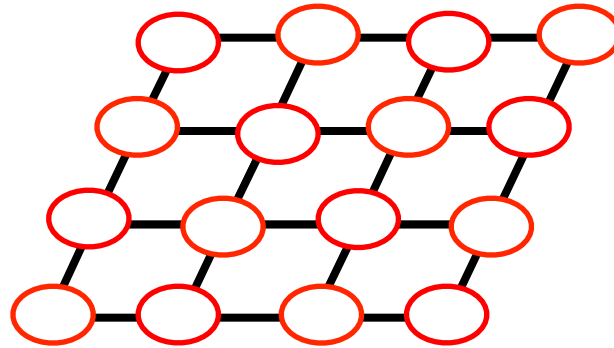
$$2E_0 + U + \Delta - 3J$$

Low-spin state ( $S=0$ )



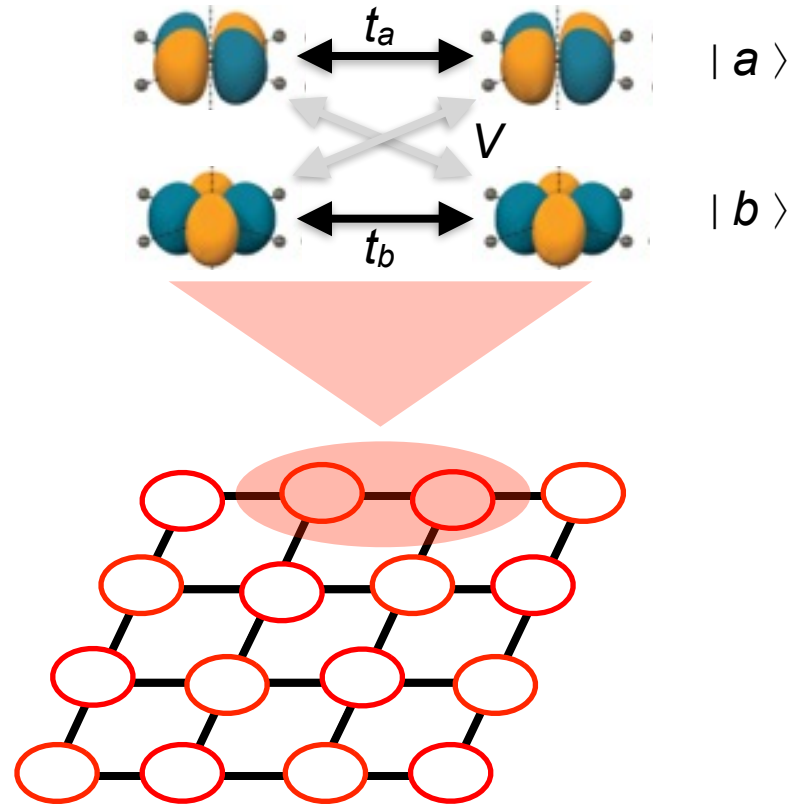
$$2E_0 + U$$

# Crystal of 2-orbital atoms



# Crystal of 2-orbital atoms

Hopping between neighbors



No Coulomb interaction between neighbors  $\Rightarrow$  Hubbard model

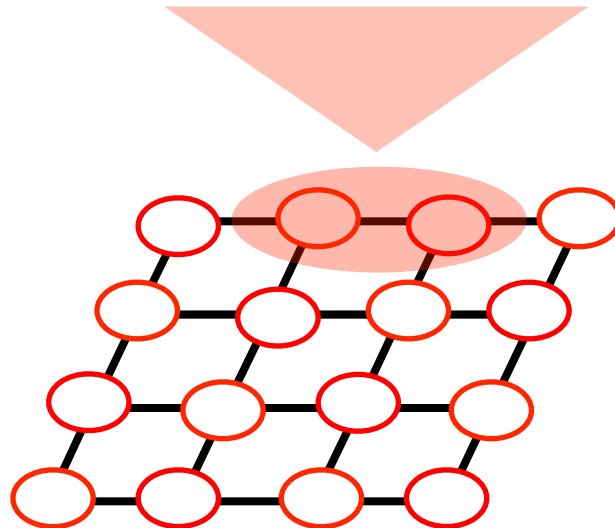
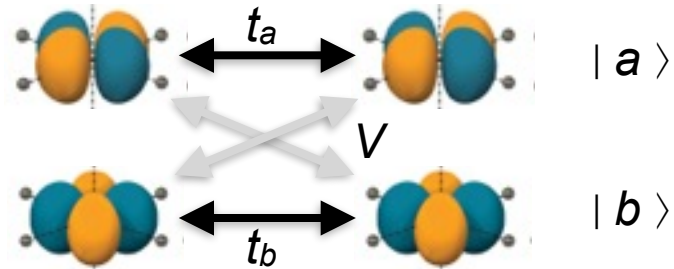
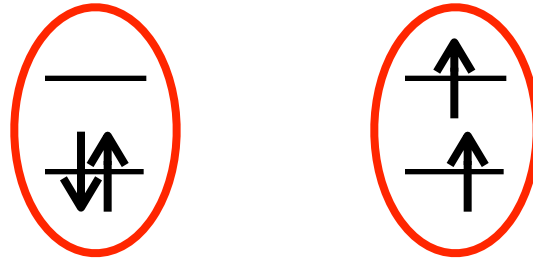
# Strong-coupling limit (hard-core bosons)



$$t \ll U$$

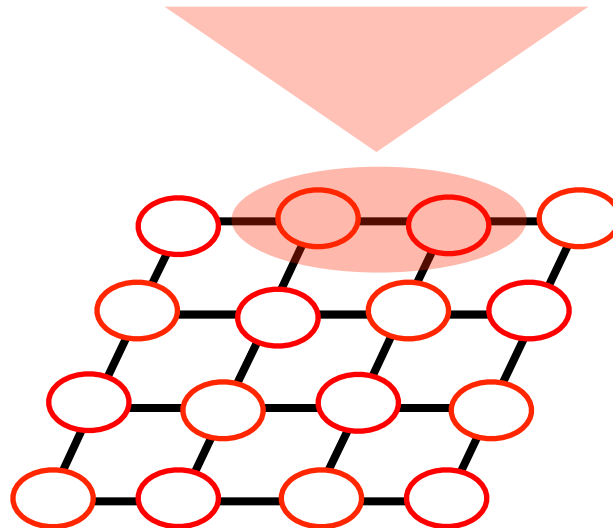
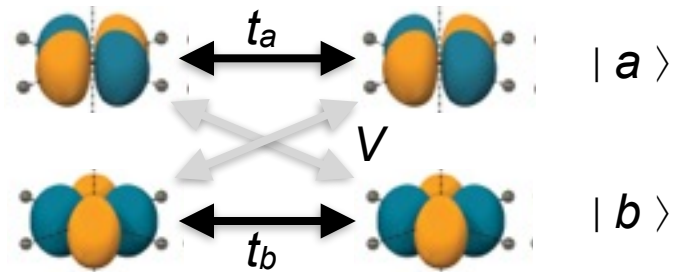
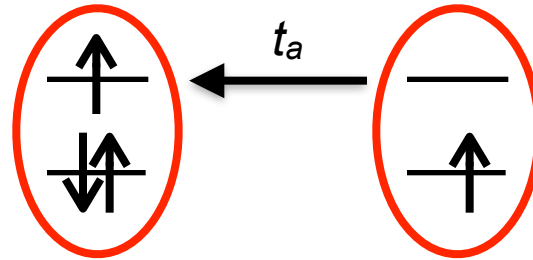


# Crystal of 2-orbital atoms

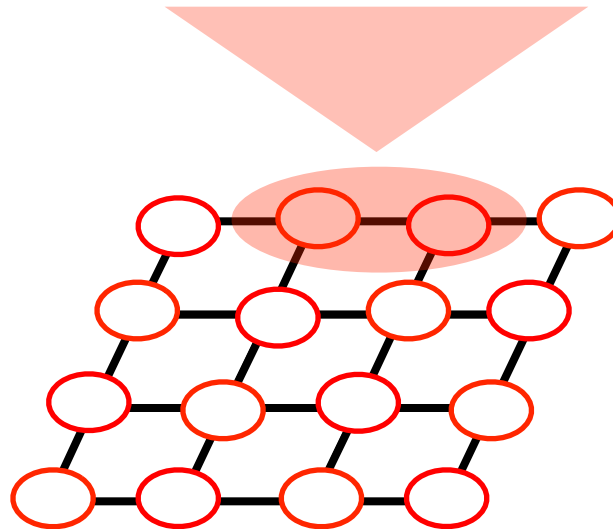
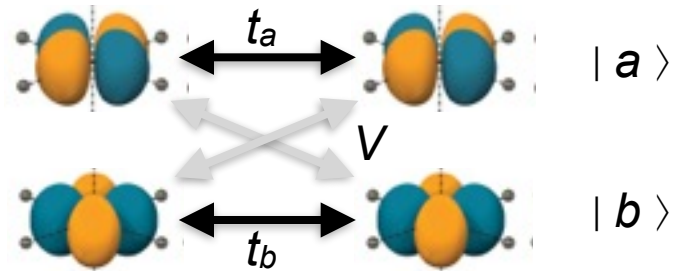
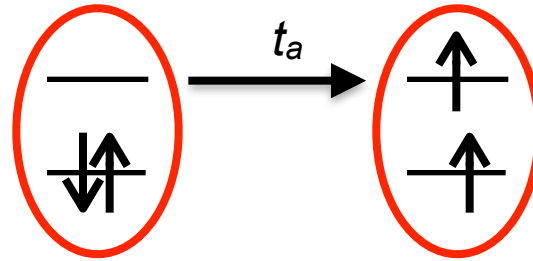


# Crystal of 2-orbital atoms

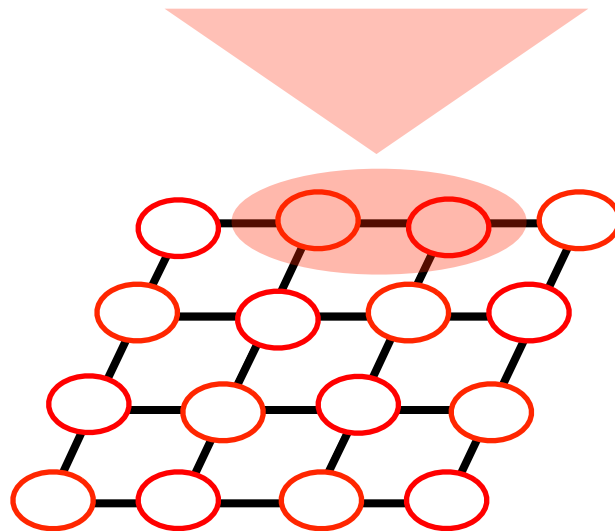
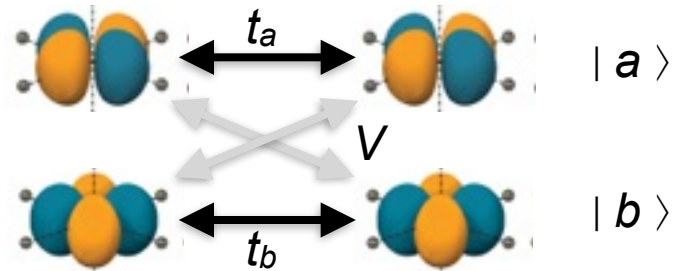
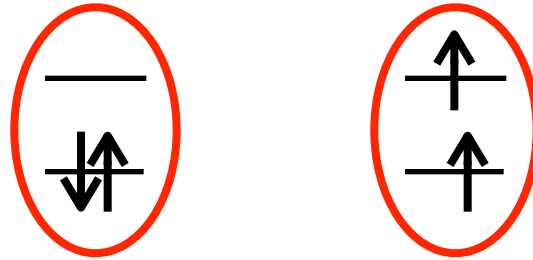
**+U**



# Crystal of 2-orbital atoms

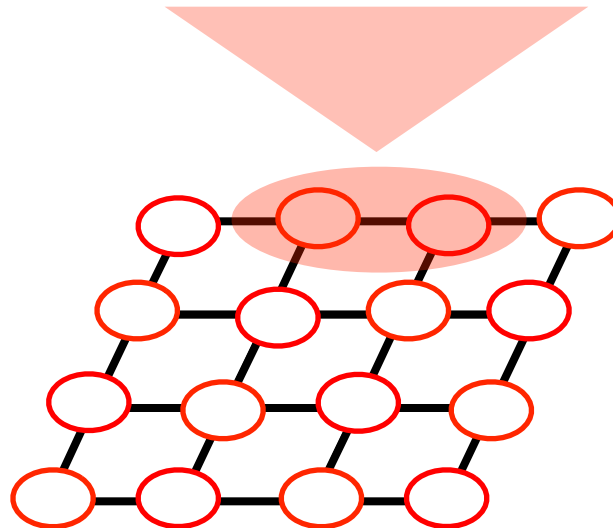
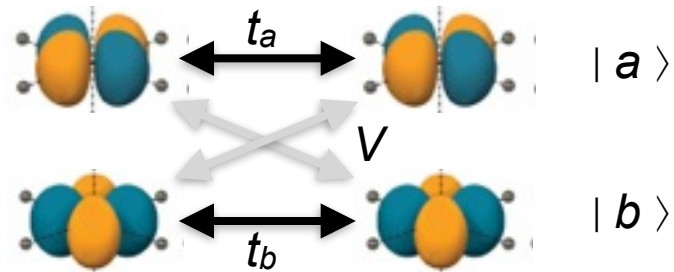
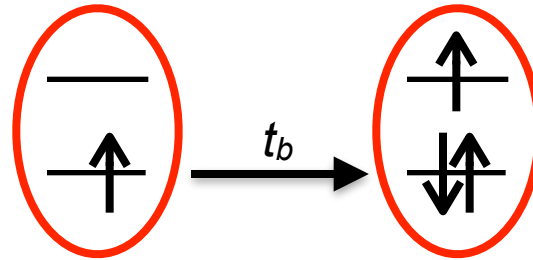


# Crystal of 2-orbital atoms

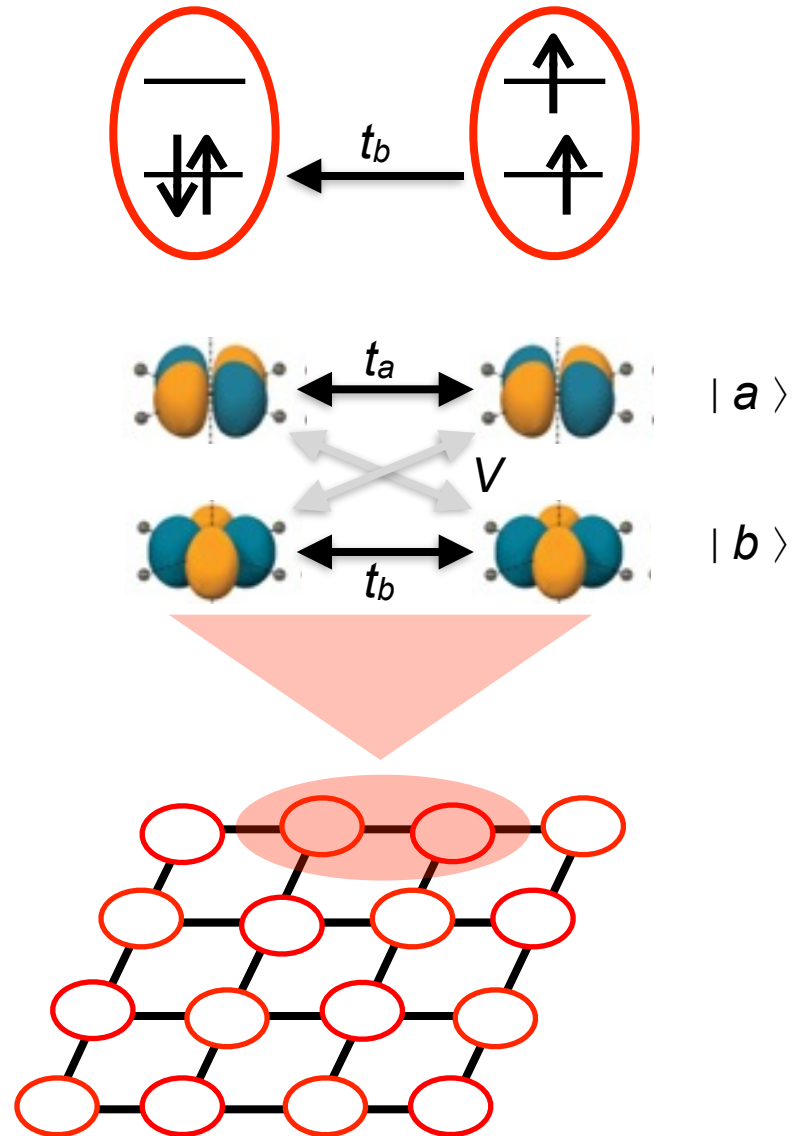


# Crystal of 2-orbital atoms

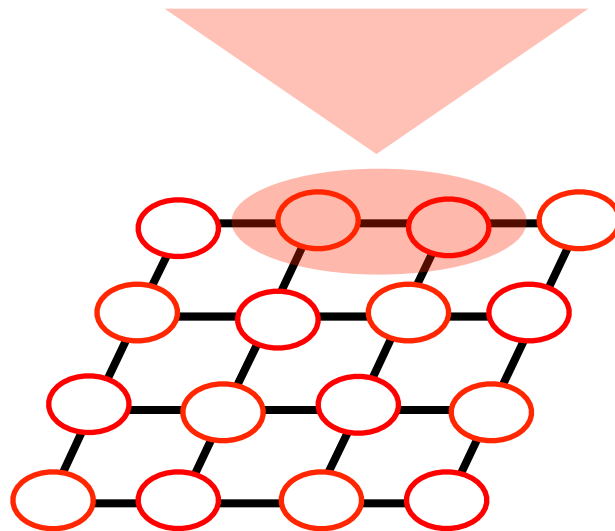
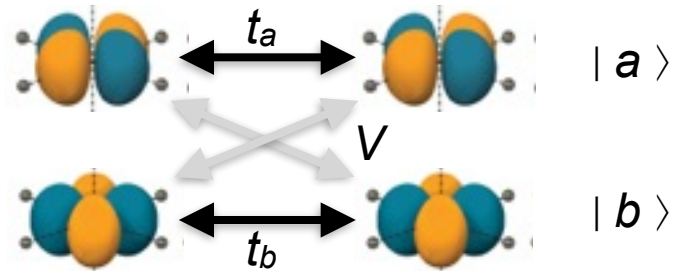
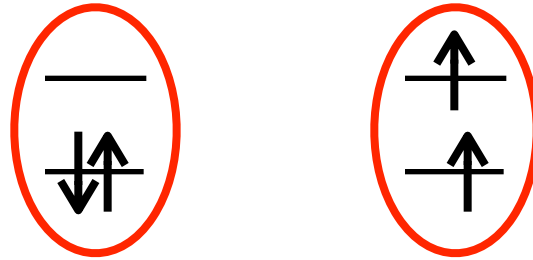
**+U**



# Crystal of 2-orbital atoms



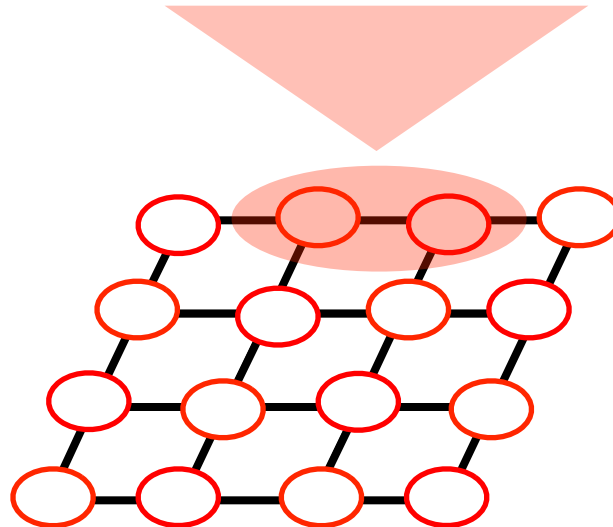
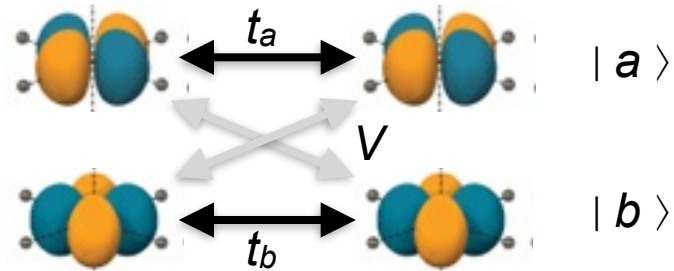
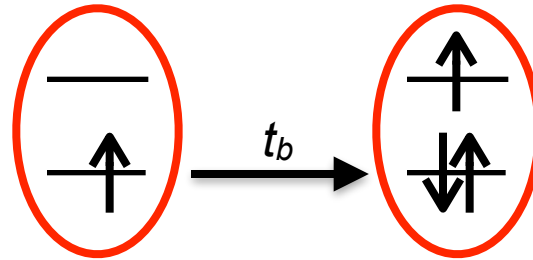
# Crystal of 2-orbital atoms



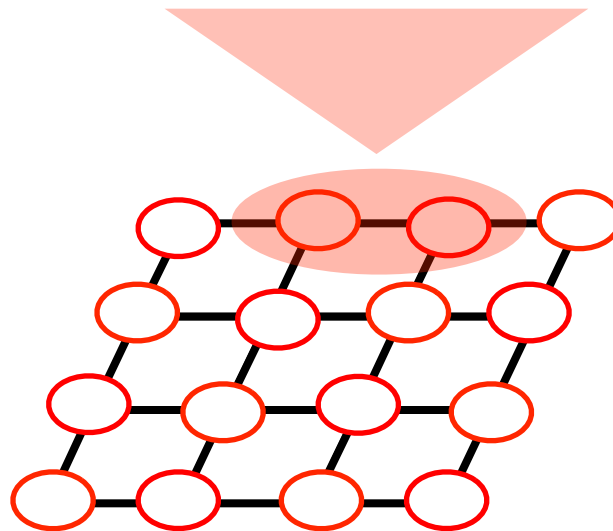
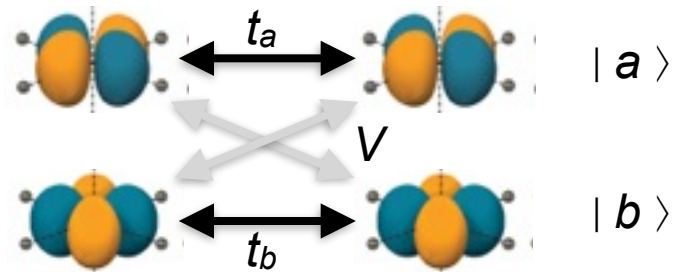
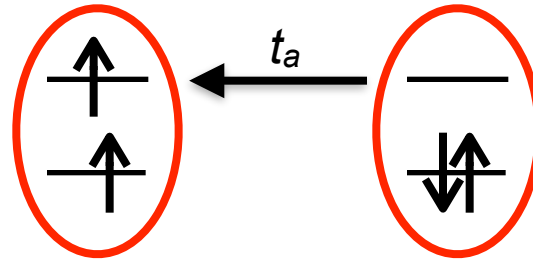


# Crystal of 2-orbital atoms

+U

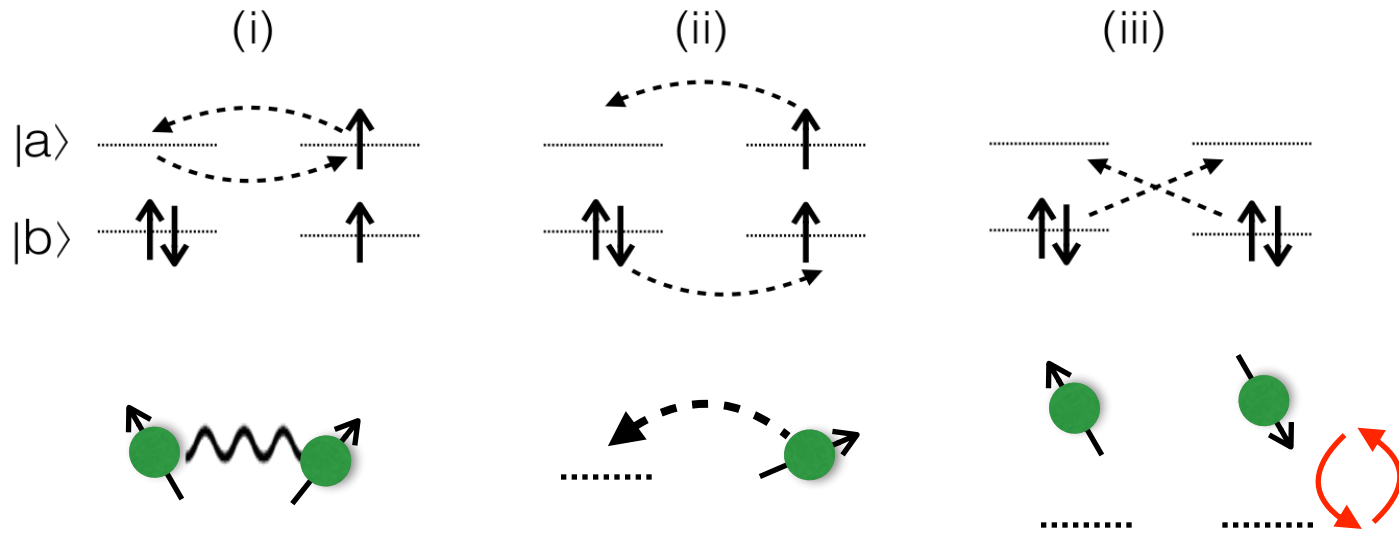


# Crystal of 2-orbital atoms



# Strong coupling theory

Typical 2nd order hopping processes:

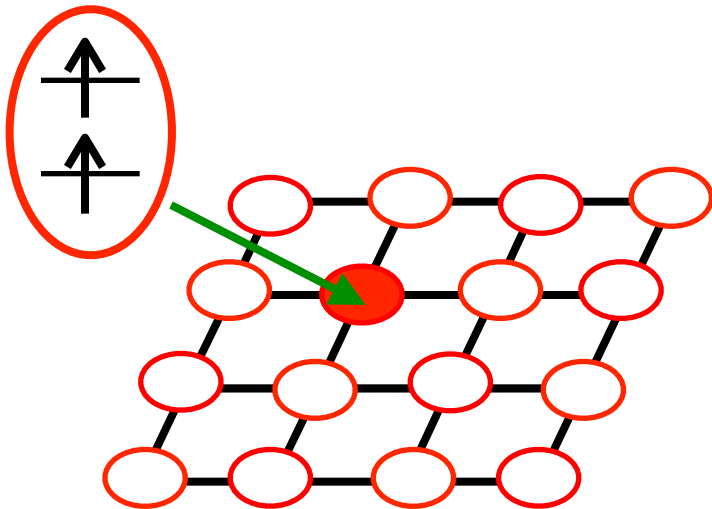
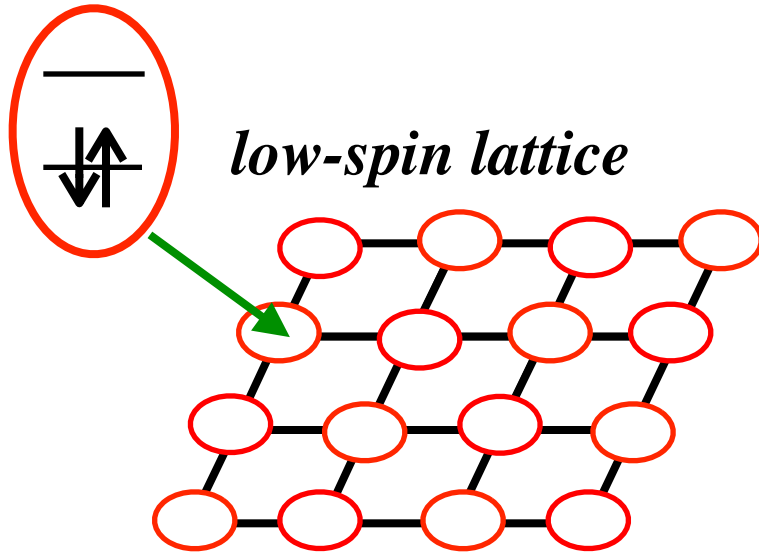


Effective Hamiltonian:

The new  particles are **bosons** and carry **spin  $S=1$** .

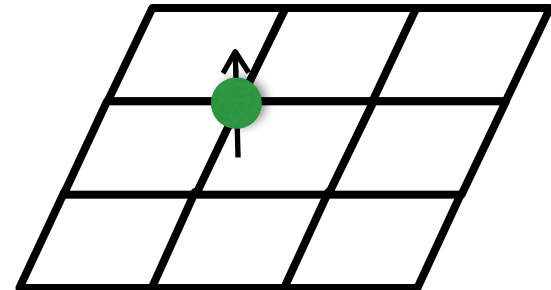
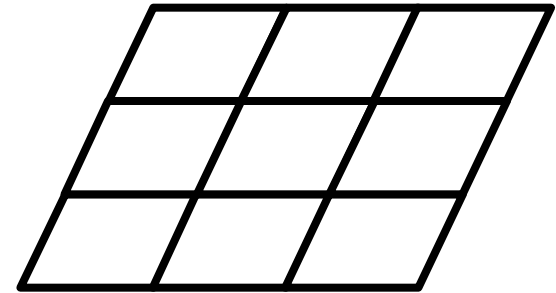
# Strong coupling theory

*Fermion (electron) picture*



*Boson (exciton) picture*

*vacuum state*



# Strong coupling theory

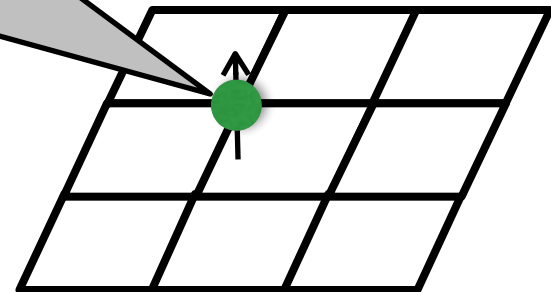
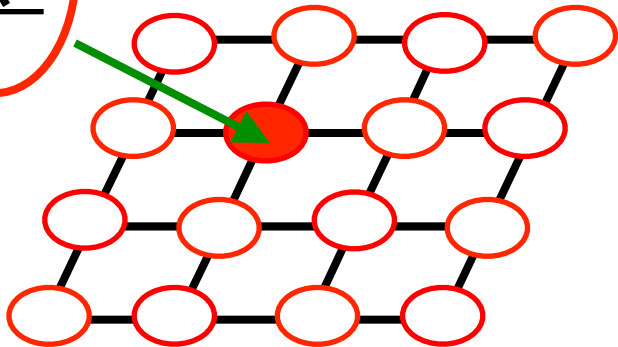
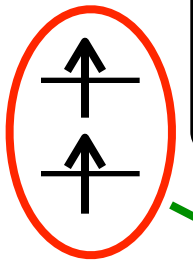
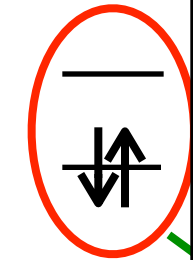
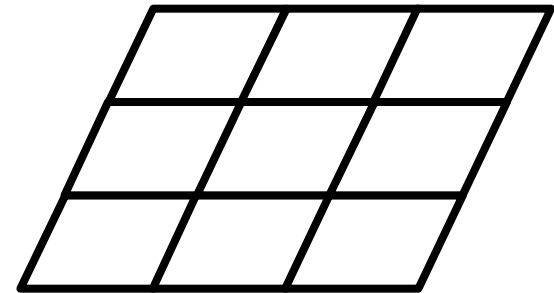
*Fermion (electron) picture*

**excitons are mobile !**



*Boson (exciton) picture*

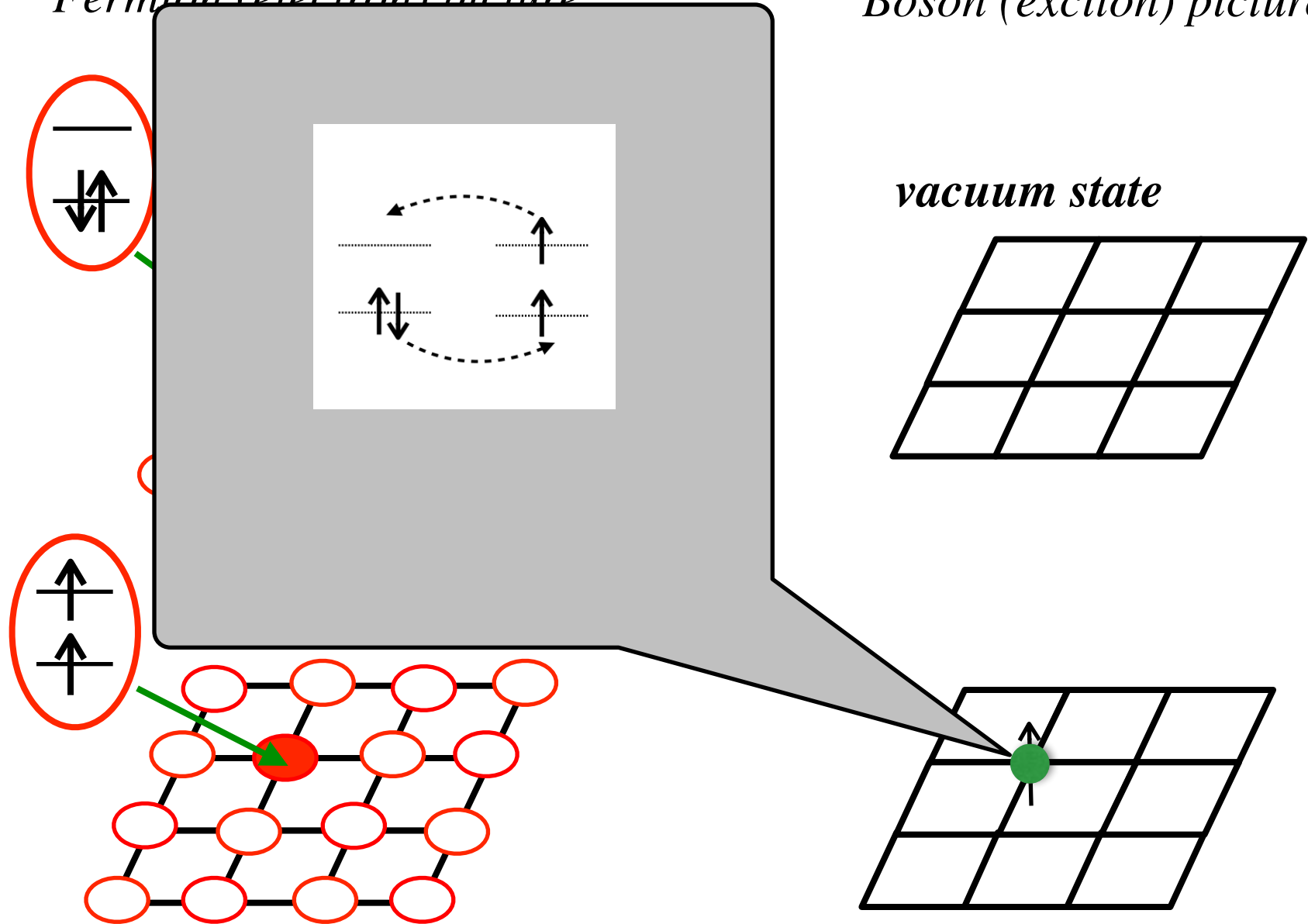
*vacuum state*



# Strong coupling theory

*Fermion (electron) picture*

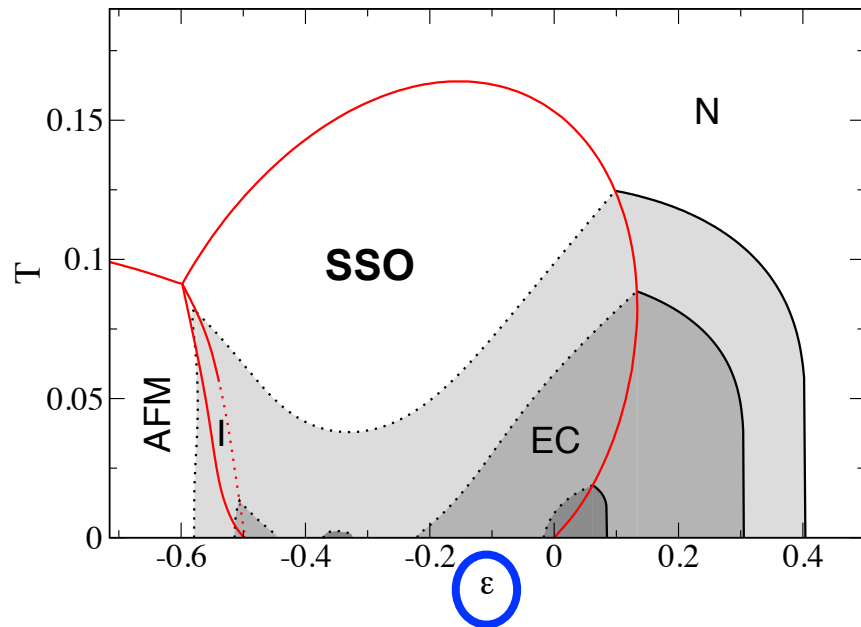
*Boson (exciton) picture*



# Mean-field theory for excitons

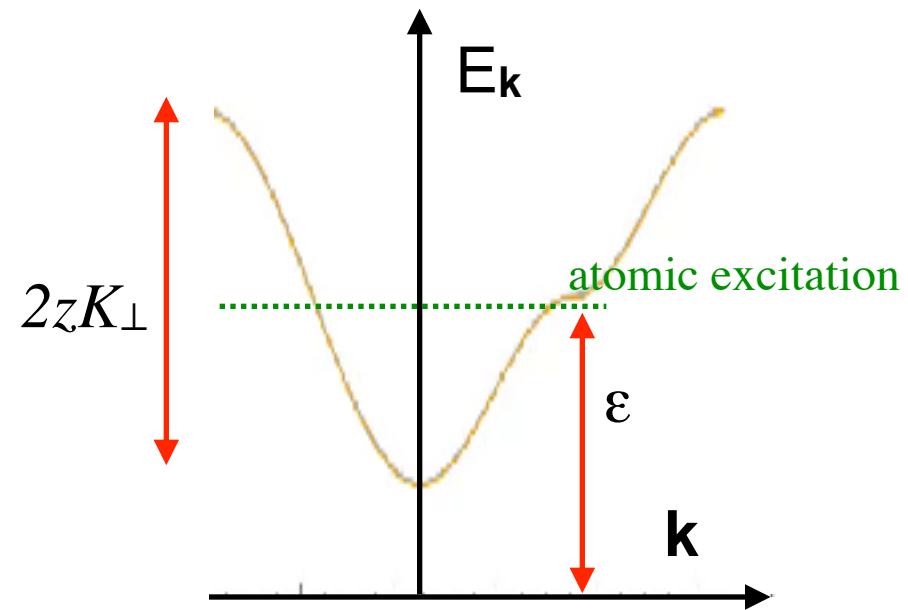
$$H_{\text{eff}} = \epsilon \sum_i n_i + K_{\perp} \sum_{\langle ij \rangle} (\mathbf{d}_i^{\dagger} \cdot \mathbf{d}_j + H.c.) + K_{\parallel} \sum_{\langle ij \rangle} n_i n_j + K_0 \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j$$

MF phase diagram:



*JK, JPCM 27, 333201 (2015)*

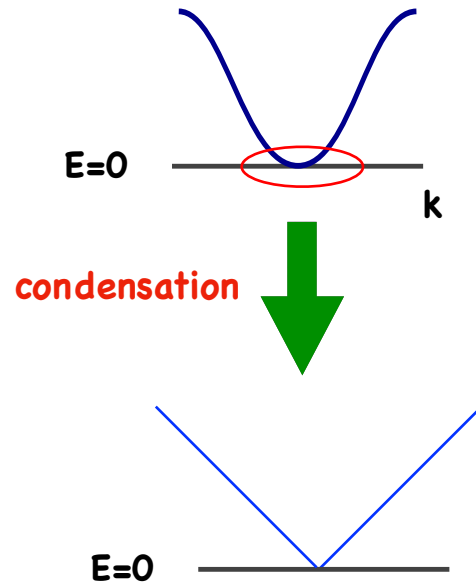
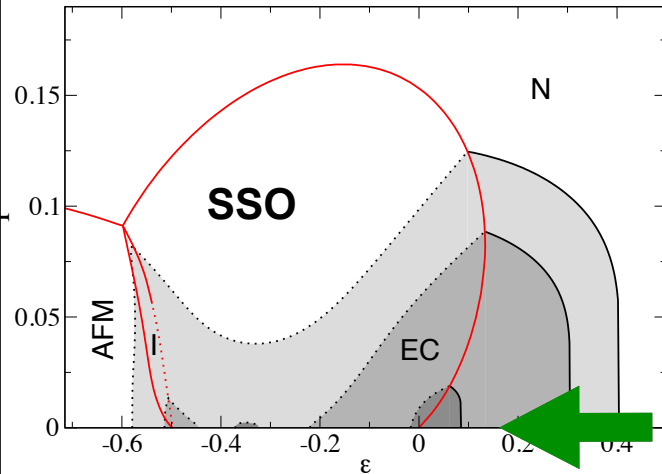
elementary excitations of normal phase



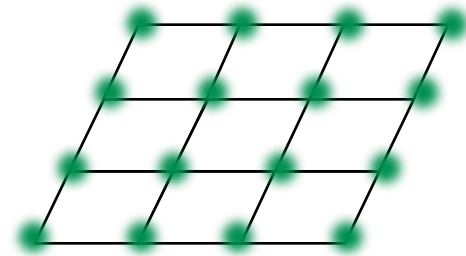


# Mean-field theory for excitons

MF phase diagram:



**Exciton condensate (superfluid):**



## Normal state ( $T > 0$ )

**statistical mixture** of vacuum  $|\cdots\rangle$  and boson  $|\downarrow\rangle$  states

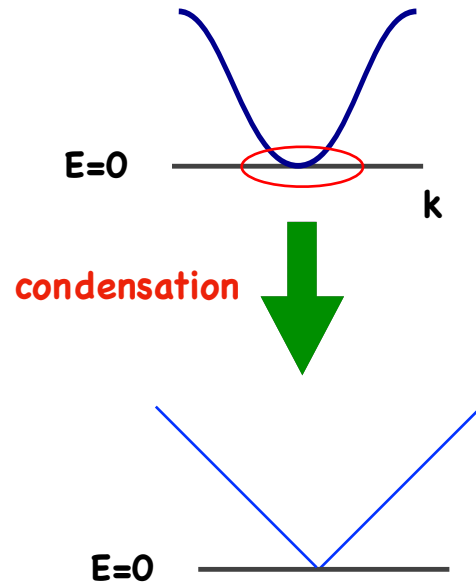
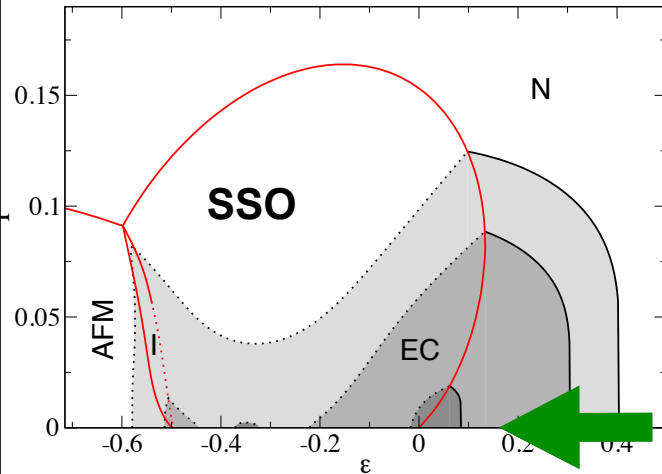
$$\langle n \rangle > 0, \quad \langle d_s \rangle = 0$$

low-spin ground state + thermally populated high-spin states

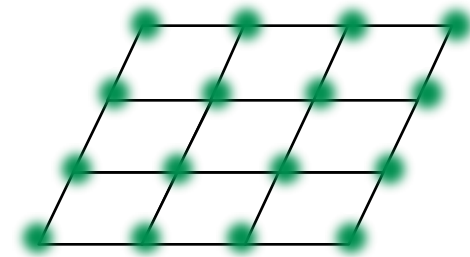
$$\chi \sim \frac{1}{T}, \quad \langle a_{\sigma}^{\dagger} b_{-\sigma} \rangle = 0$$

# Mean-field theory for excitons

MF phase diagram:



**Exciton condensate (superfluid):**



## Condensate

superposition  $\alpha \left| \cdots \right\rangle + \beta \left| \downarrow \right\rangle$  **ferromagnetic**

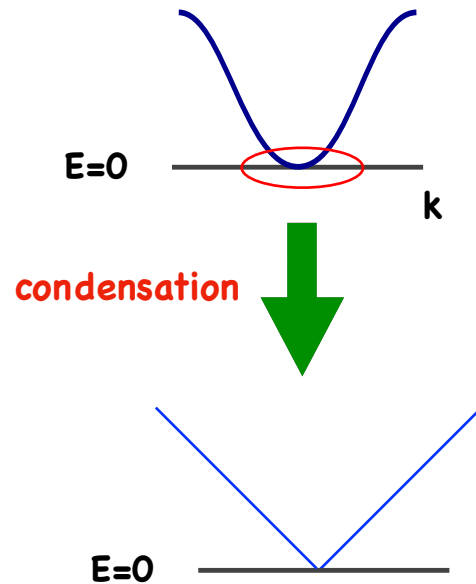
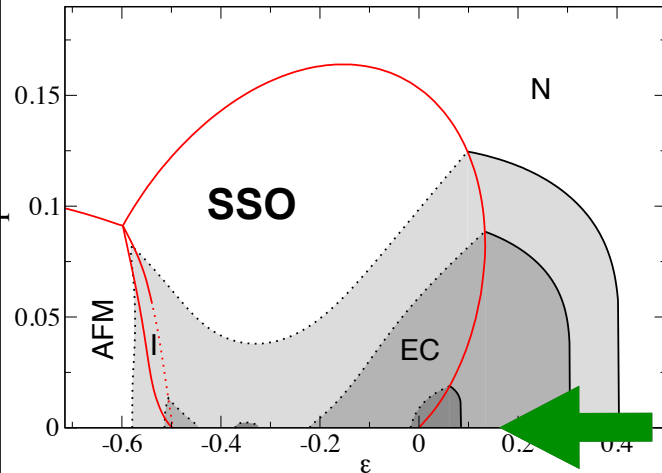
$$\langle n \rangle > 0, \quad \langle d_s \rangle \neq 0$$

hybridized state low-spin and high-spin states  $\alpha \left| \begin{smallmatrix} \cdots \\ \uparrow\downarrow \end{smallmatrix} \right\rangle + \beta \left| \begin{smallmatrix} \cdots \\ \downarrow\downarrow \end{smallmatrix} \right\rangle$

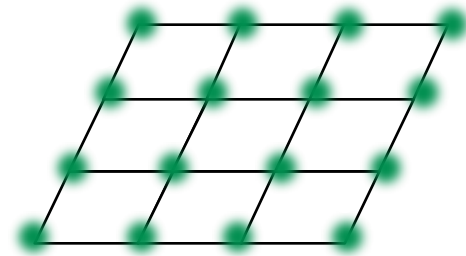
$$\langle a_\sigma^\dagger b_{-\sigma} \rangle \neq 0$$

# Mean-field theory for excitons

MF phase diagram:



**Exciton condensate (superfluid):**



## Condensate

superposition  $\alpha |\dots\rangle + \beta |\downarrow\rangle + \beta |\uparrow\rangle$  **polar**

$$\langle n \rangle > 0, \quad \langle d_s \rangle \neq 0$$

hybridized state low-spin and high-spin states  $\alpha |\dots\rangle + \beta |\dots\rangle$

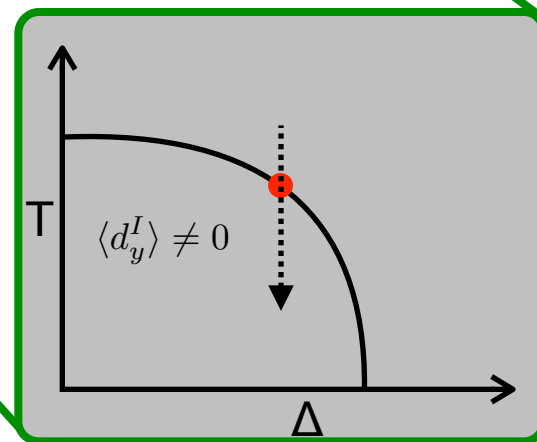
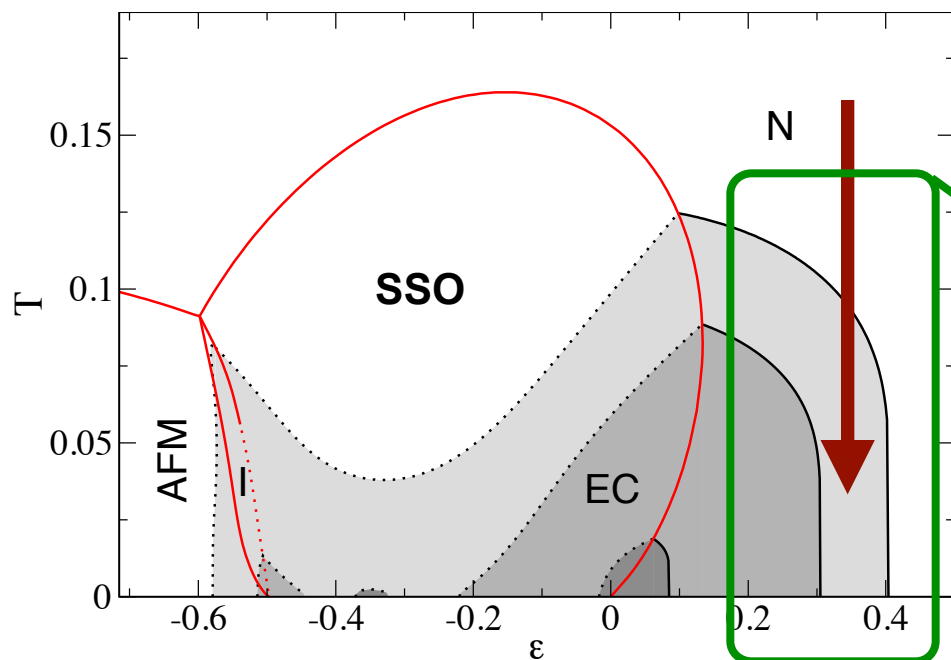
$$\langle a_\sigma^\dagger b_{-\sigma} \rangle \neq 0$$

# Back to fermions (Dynamical mean-field theory)



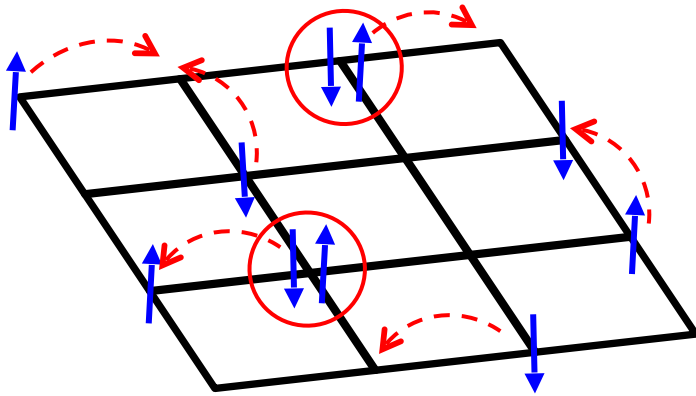
$$\phi = \sum_{\alpha\beta} \sigma_{\alpha\beta} \langle a_{\alpha}^{\dagger} b_{\beta} \rangle$$

# Numerical results (DMFT) excitonic instability:



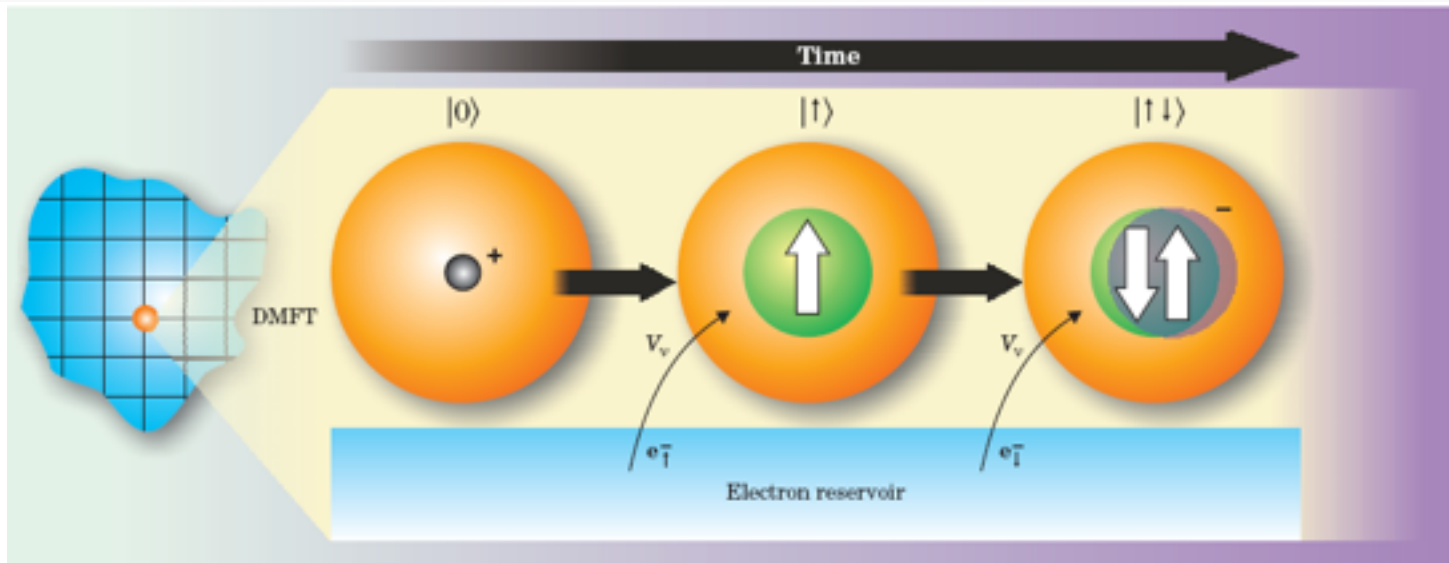
# Hubbard model

Two-band Hubbard model at  $n=2$  (half filling)



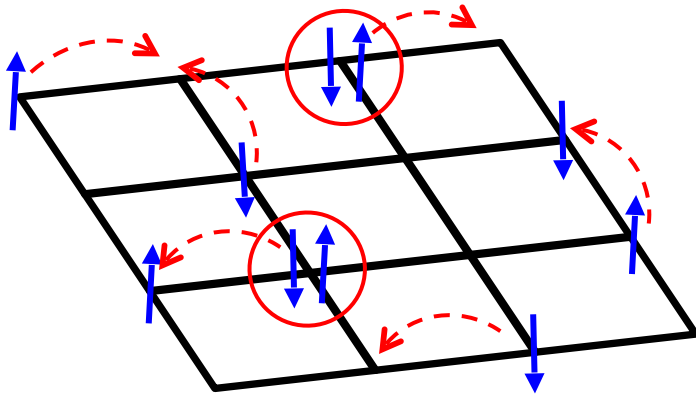
$$\begin{aligned}
 H_t &= \frac{\Delta}{2} \sum_{i,\sigma} (n_{i\sigma}^a - n_{i\sigma}^b) + \sum_{i,j,\sigma} (t_a a_{i\sigma}^\dagger a_{j\sigma} + t_b b_{i\sigma}^\dagger b_{j\sigma}) \\
 &\quad + \sum_{\langle ij \rangle, \sigma} (V_1 a_{i\sigma}^\dagger b_{j\sigma} + V_2 b_{i\sigma}^\dagger a_{j\sigma} + c.c.) \\
 H_{\text{int}}^{\text{dd}} &= U \sum_i (n_{i\uparrow}^a n_{i\downarrow}^a + n_{i\uparrow}^b n_{i\downarrow}^b) + (U - 2J) \sum_{i,\sigma} n_{i\sigma}^a n_{i-\sigma}^b \\
 &\quad + (U - 3J) \sum_{i\sigma} n_{i\sigma}^a n_{i\sigma}^b \\
 H'_{\text{int}} &= J \sum_{i\sigma} a_{i\sigma}^\dagger b_{i-\sigma}^\dagger a_{i-\sigma} b_{i\sigma} + J' \sum_i (a_{i\uparrow}^\dagger a_{i\downarrow}^\dagger b_{i\downarrow} b_{i\uparrow} + c.c.).
 \end{aligned}$$

## Dynamical mean-field theory



# Hubbard model

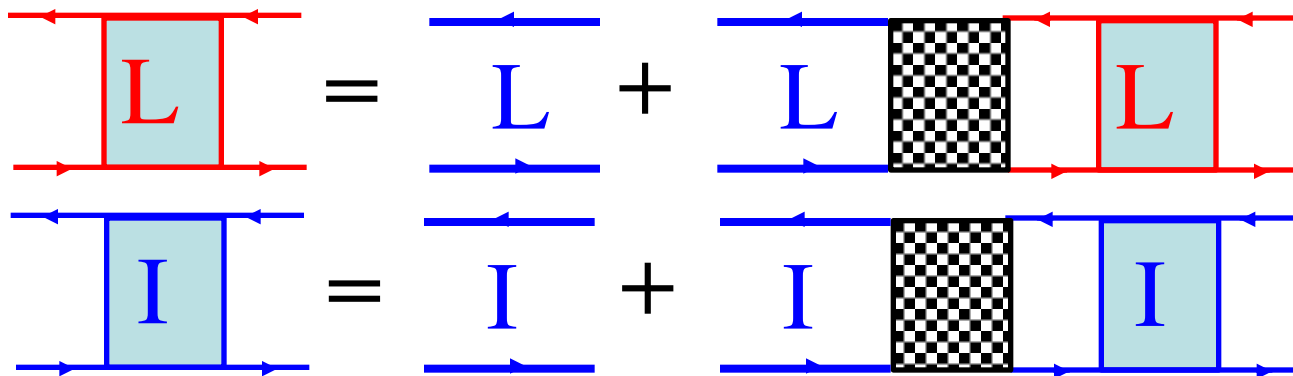
Two-band Hubbard model at  $n=2$  (half filling)



$$\begin{aligned}
 H_t &= \frac{\Delta}{2} \sum_{i,\sigma} (n_{i\sigma}^a - n_{i\sigma}^b) + \sum_{i,j,\sigma} (t_a a_{i\sigma}^\dagger a_{j\sigma} + t_b b_{i\sigma}^\dagger b_{j\sigma}) \\
 &\quad + \sum_{\langle ij \rangle, \sigma} (V_1 a_{i\sigma}^\dagger b_{j\sigma} + V_2 b_{i\sigma}^\dagger a_{j\sigma} + c.c.) \\
 H_{\text{int}}^{\text{dd}} &= U \sum_i (n_{i\uparrow}^a n_{i\downarrow}^a + n_{i\uparrow}^b n_{i\downarrow}^b) + (U - 2J) \sum_{i,\sigma} n_{i\sigma}^a n_{i-\sigma}^b \\
 &\quad + (U - 3J) \sum_{i\sigma} n_{i\sigma}^a n_{i\sigma}^b \\
 H'_{\text{int}} &= J \sum_{i\sigma} a_{i\sigma}^\dagger b_{i-\sigma}^\dagger a_{i-\sigma} b_{i\sigma} + J' \sum_i (a_{i\uparrow}^\dagger a_{i\downarrow}^\dagger b_{i\downarrow} b_{i\uparrow} + c.c.).
 \end{aligned}$$

## Bethe-Salpeter equation

- full  $(\omega, \nu, \nu')$  frequency structure
- local ph-irreducible vertex
- multi-orbital; no symmetry constraints
- Maxent for analytic continuation

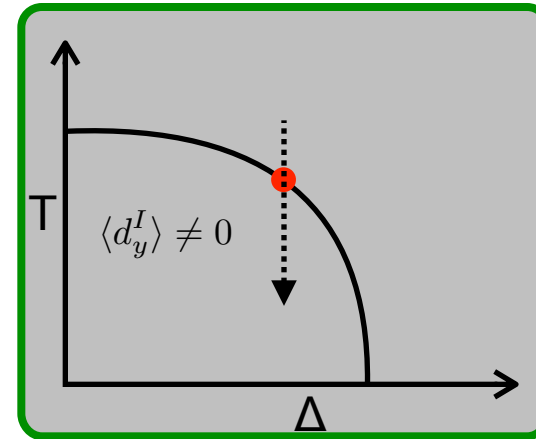
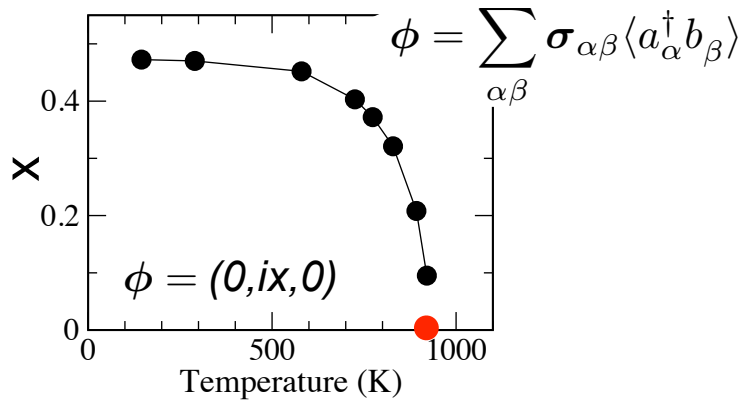


*lattice model*

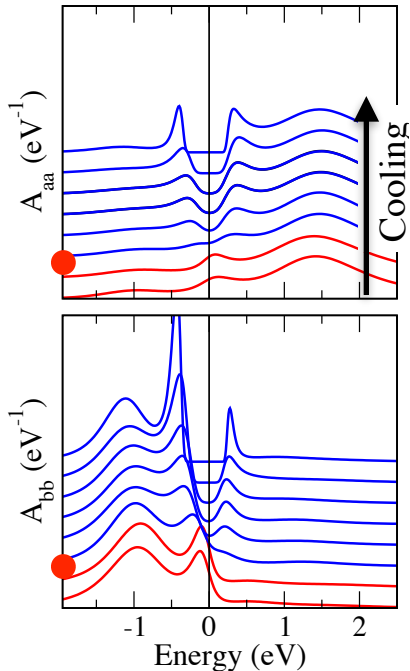
*impurity model*

# Polar excitonic condensate (half filling)

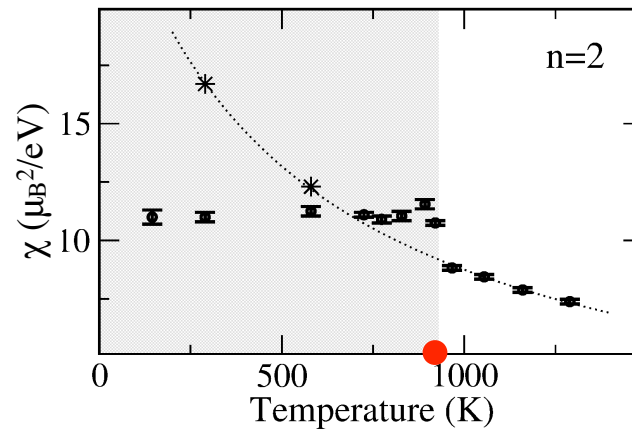
## Order parameter



## Spectral density



## Uniform spin susceptibility

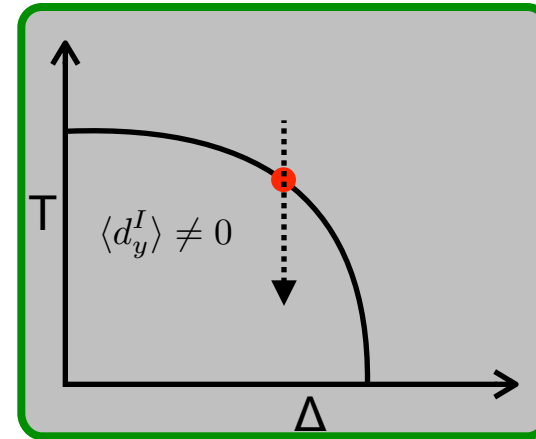
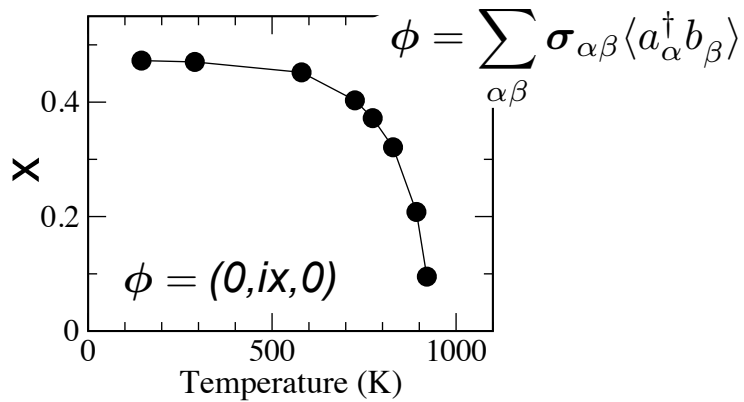


Model parameters (eV):  
 $U=4$ ,  $J=1$ ,  $\Delta_{CF}=3.40$   
 $t_a=0.4118$ ,  $t_b=-0.1882$   
 square lattice

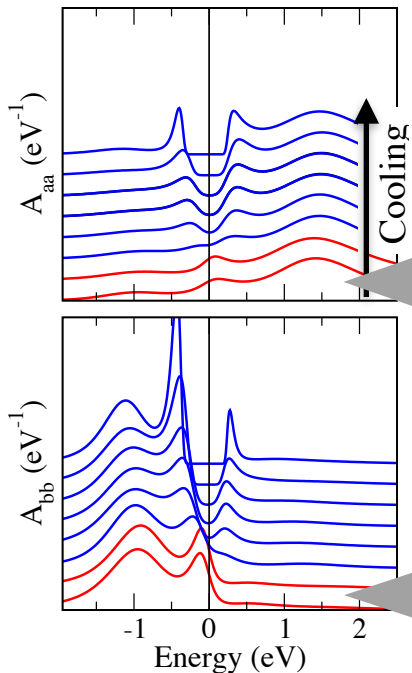


# Polar excitonic condensate (half filling)

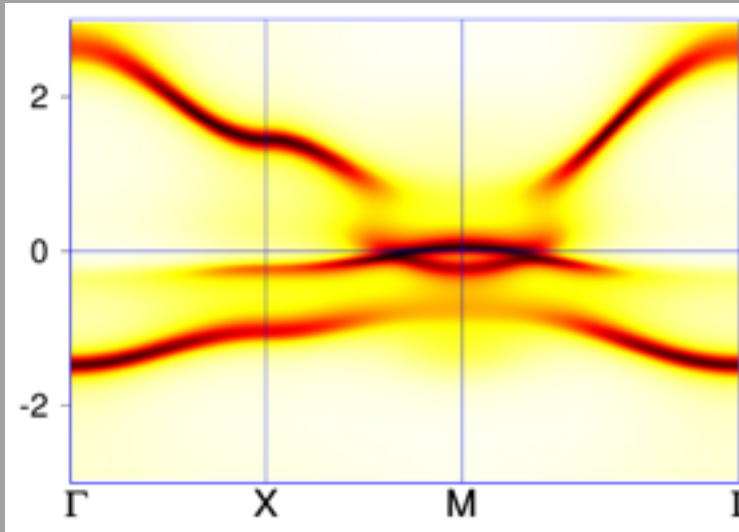
Order parameter



Spectral density



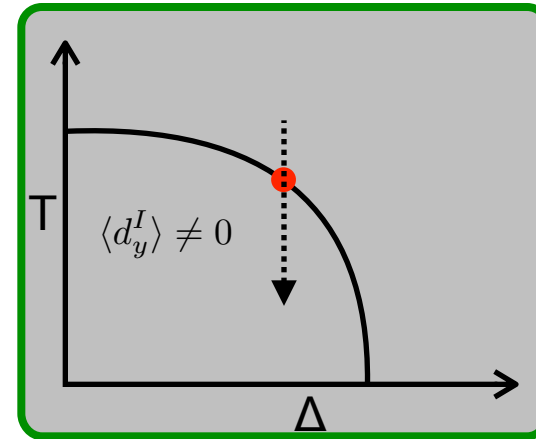
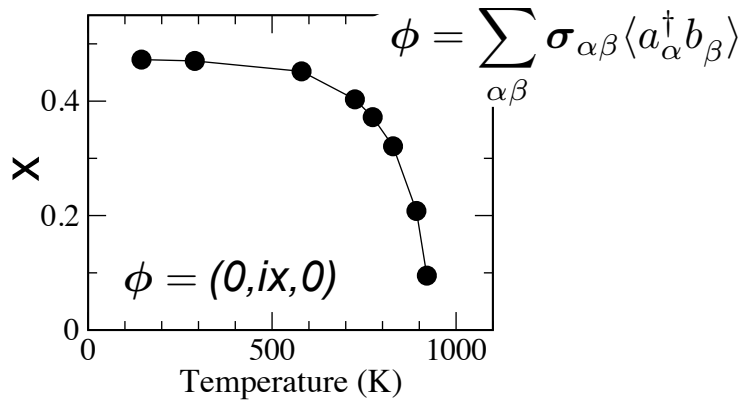
normal phase (high T)



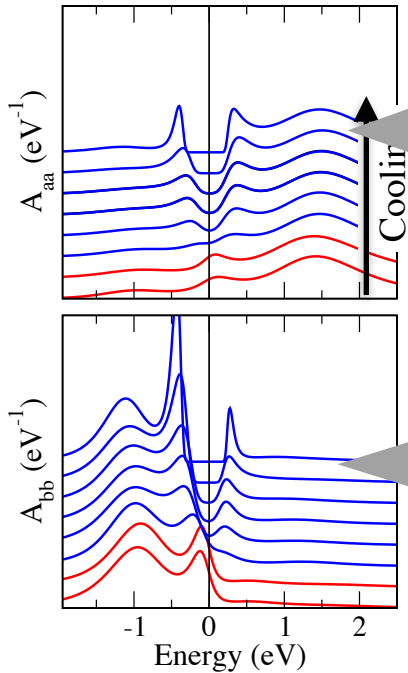
model parameters (eV):  
 $t_a = 0.4118$ ,  $t_b = -0.1882$   
 are lattice

# Polar excitonic condensate (half filling)

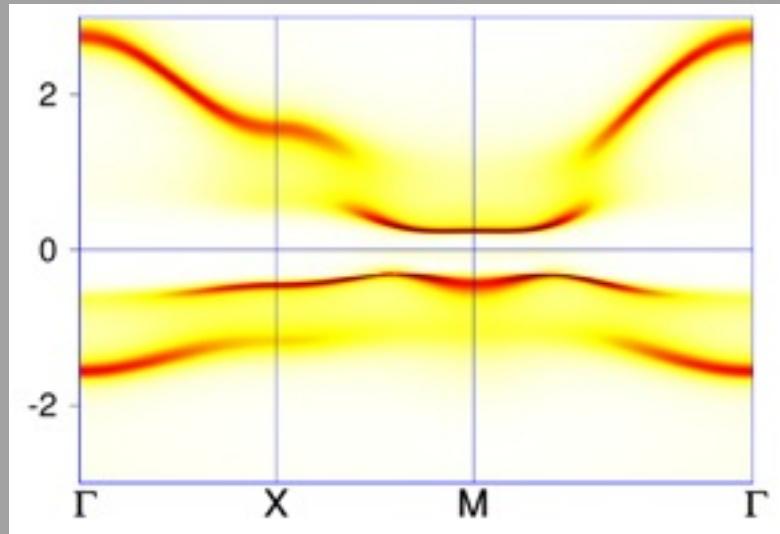
Order parameter



Spectral density

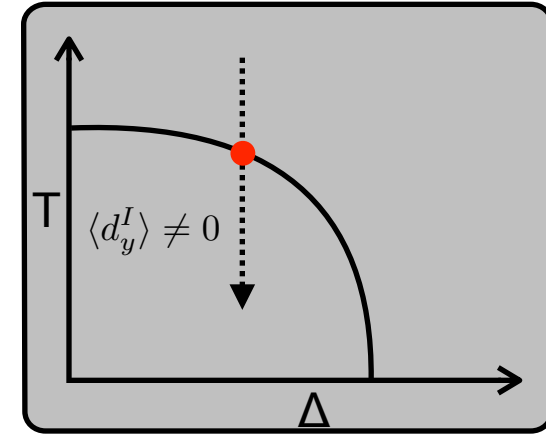
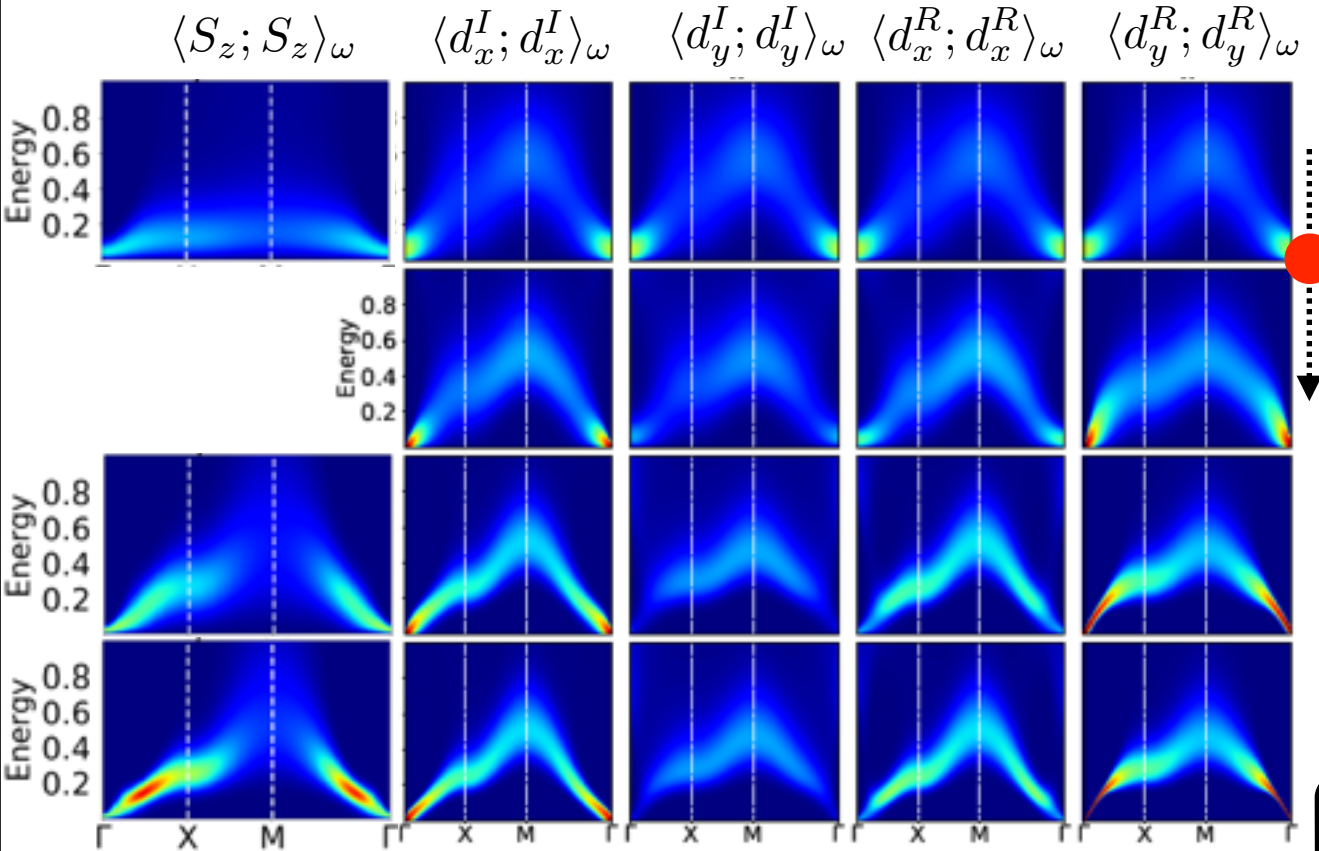


condensed phase (low T)



Model parameters (eV):  
 $t_a = 0.4118$ ,  $t_b = -0.1882$   
 are lattice

# Dynamical susceptibility



*D. Geffroy et al., PRL 122, 127601 (2019)*

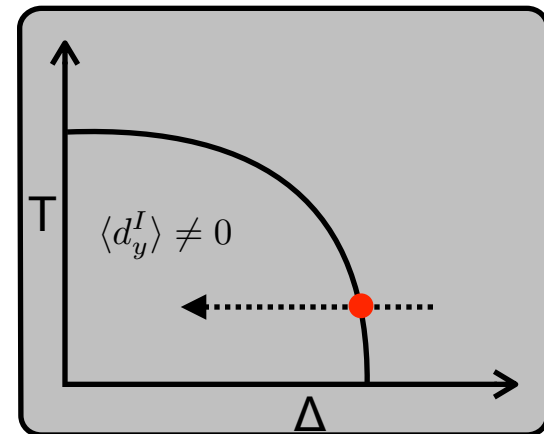
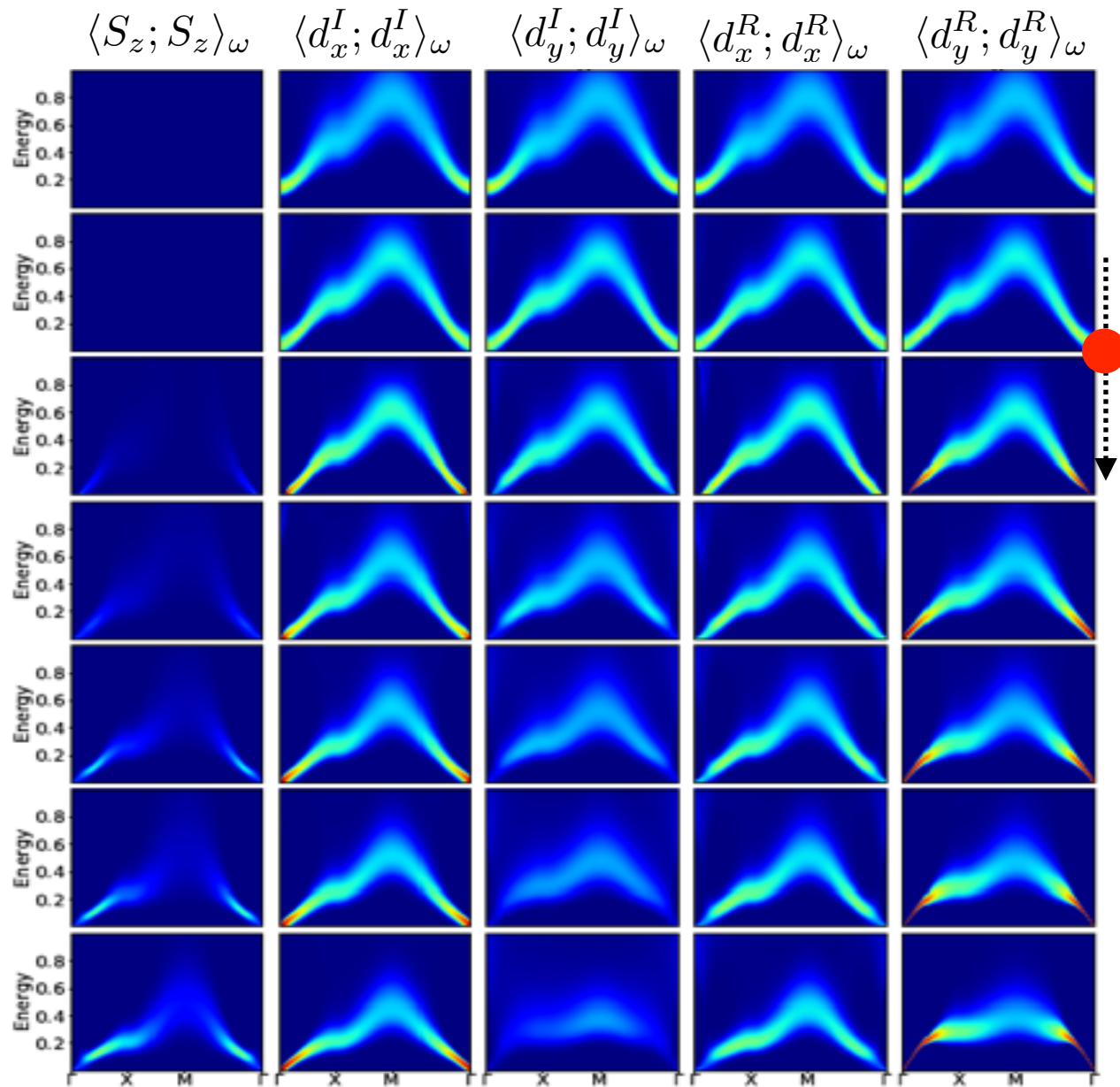
Formal defs:

$$d_{i\eta}^R = \sum_{\alpha\beta} \sigma_{\alpha\beta}^\eta (a_{i\alpha}^\dagger b_{i\beta} + b_{i\alpha}^\dagger a_{i\beta})$$

$$d_{i\eta}^I = i \sum_{\alpha\beta} \sigma_{\alpha\beta}^\eta (a_{i\alpha}^\dagger b_{i\beta} - b_{i\alpha}^\dagger a_{i\beta})$$

$$S_i^z = \sum_{\alpha\beta} \sigma_{\alpha\beta}^z (a_{i\alpha}^\dagger a_{i\beta} + b_{i\alpha}^\dagger b_{i\beta})$$

# Dynamical susceptibility



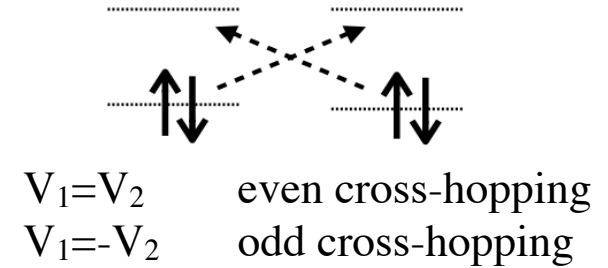
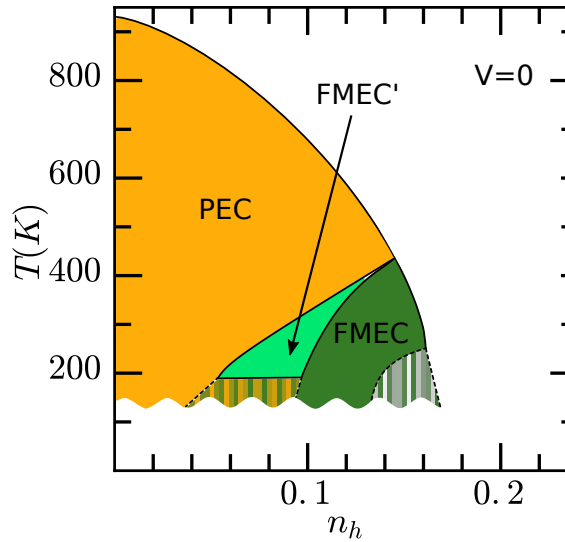
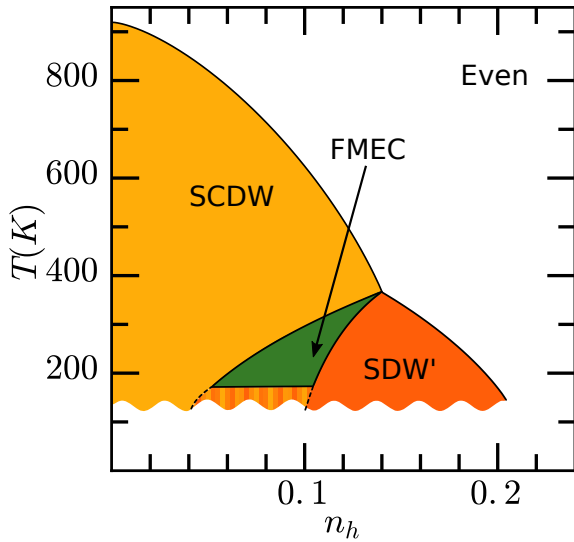
Formal defs:

$$d_{i\eta}^R = \sum_{\alpha\beta} \sigma_{\alpha\beta}^\eta (a_{i\alpha}^\dagger b_{i\beta} + b_{i\alpha}^\dagger a_{i\beta})$$

$$d_{i\eta}^I = i \sum_{\alpha\beta} \sigma_{\alpha\beta}^\eta (a_{i\alpha}^\dagger b_{i\beta} - b_{i\alpha}^\dagger a_{i\beta})$$

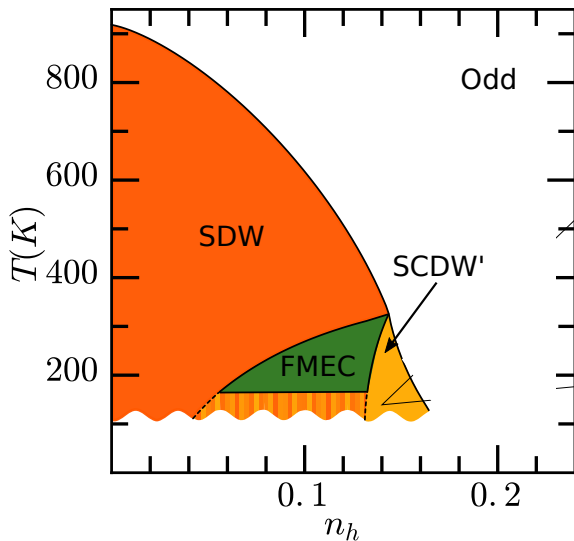
$$S_i^z = \sum_{\alpha\beta} \sigma_{\alpha\beta}^z (a_{i\alpha}^\dagger a_{i\beta} + b_{i\alpha}^\dagger b_{i\beta})$$

# Doping + cross-hopping



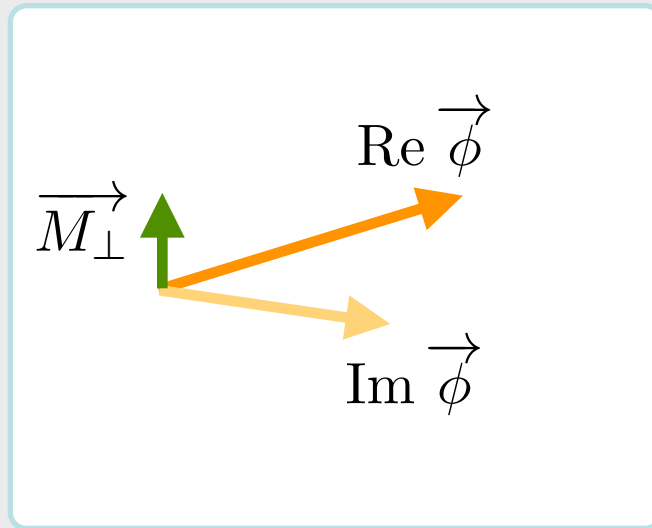
$$\phi = \sum_{\alpha\beta} \sigma_{\alpha\beta} \langle a_{\alpha}^{\dagger} b_{\beta} \rangle$$

$$\text{PEC} \begin{cases} \text{SDW} & \phi = \mathbf{x} \\ \text{SCDW} & \phi = i\mathbf{x} \end{cases}$$



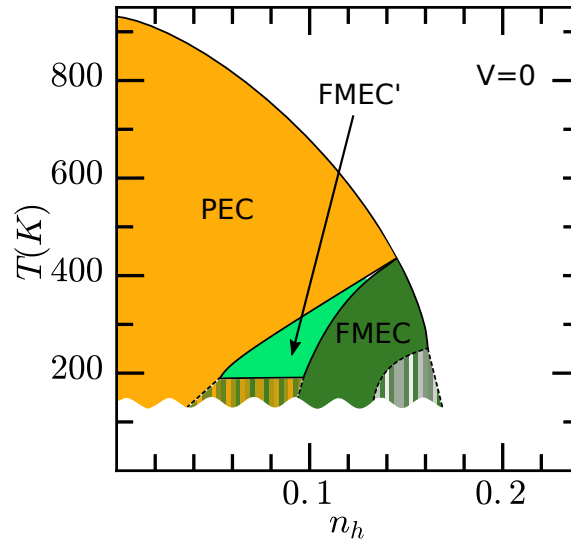
Condensate state	$M_{\perp}$	$M_{\parallel}$	$\mathbf{m}(\mathbf{r})$	$\mathbf{m}_{\mathbf{k}}$	$\text{Re } \phi$	$\text{Im } \phi$
FMEC	✓	✓, 0	✓	✓	✓	✓
SDW	0	0	✓	0	✓	0
SCDW	0	0	0	0	0	✓
SDW'	0	✓, 0	✓	✓	✓	0
SCDW'	0	0	0	✓	0	✓

$$\mathbf{m}_{\mathbf{k}} = \sum_{\alpha\beta} \sigma_{\alpha\beta} \langle a_{\mathbf{k}\alpha}^{\dagger} a_{\mathbf{k}\beta} + b_{\mathbf{k}\alpha}^{\dagger} b_{\mathbf{k}\beta} \rangle$$



$$\phi = \sum_{\alpha\beta} \sigma_{\alpha\beta} \langle a_\alpha^\dagger b_\beta \rangle$$

# Doping



$$H_{\text{eff}} = \varepsilon \sum_i n_i + K_{\perp} \sum_{\langle ij \rangle} \left( \mathbf{d}_i^{\dagger} \cdot \mathbf{d}_j + H.c. \right) + K_{\parallel} \sum_{\langle ij \rangle} n_i n_j + K_0 \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j$$

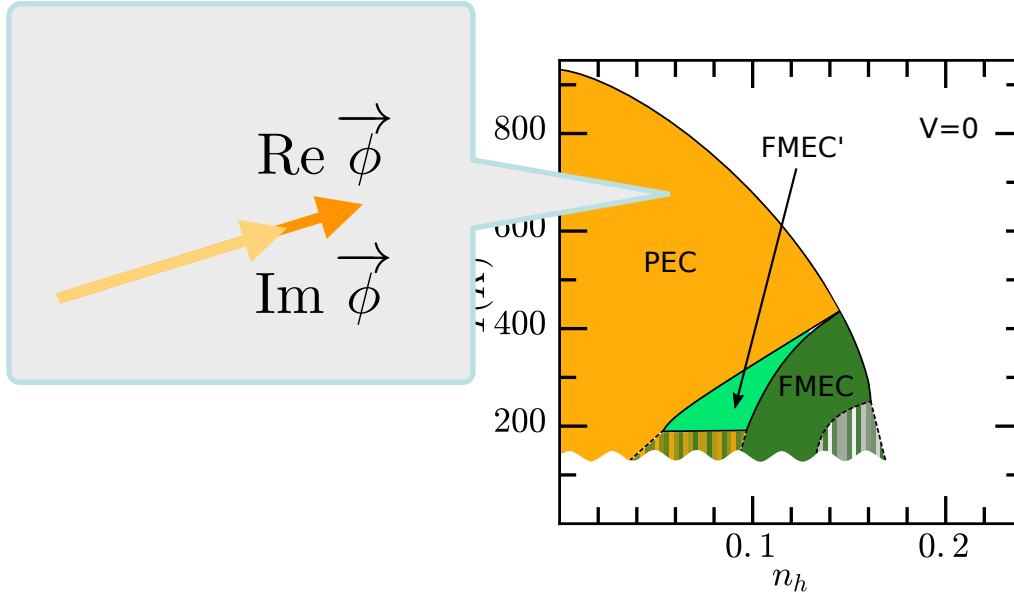
$$K_0 > 0$$

$$\text{PEC: } \psi = \prod_i (\alpha + \beta d_{i\mathbf{x}}^{\dagger}) |0\rangle$$

$$K_0 < 0$$

$$\text{FMEC: } \psi = \prod_i (\alpha + \beta d_{i\mathbf{1}}^{\dagger}) |0\rangle$$

# Doping



$$H_{\text{eff}} = \varepsilon \sum_i n_i + K_{\perp} \sum_{\langle ij \rangle} \left( \mathbf{d}_i^{\dagger} \cdot \mathbf{d}_j + H.c. \right) + K_{\parallel} \sum_{\langle ij \rangle} n_i n_j + K_0 \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j$$

$$K_0 > 0$$

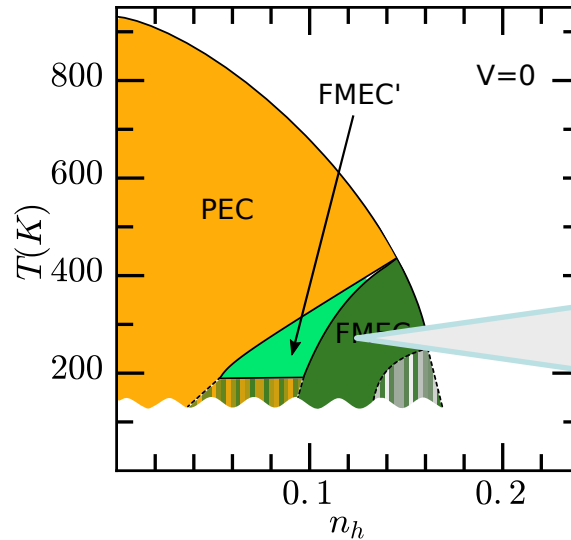
$$\text{PEC: } \psi = \prod_i (\alpha + \beta d_{i\mathbf{x}}^{\dagger}) |0\rangle$$

$$K_0 < 0$$

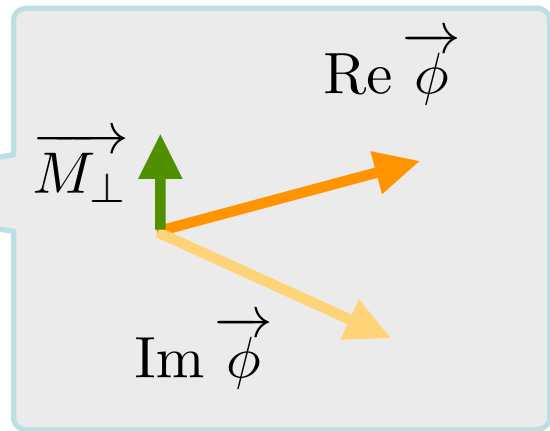
$$\text{FMEC: } \psi = \prod_i (\alpha + \beta d_{i\mathbf{1}}^{\dagger}) |0\rangle$$



# Doping



$$\mathbf{S}_i = i(\mathbf{d}_i^\dagger \times \mathbf{d}_i)$$



$$H_{\text{eff}} = \varepsilon \sum_i n_i + K_{\perp} \sum_{\langle ij \rangle} (\mathbf{d}_i^\dagger \cdot \mathbf{d}_j + H.c.) + K_{\parallel} \sum_{\langle ij \rangle} n_i n_j + K_0 \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j$$

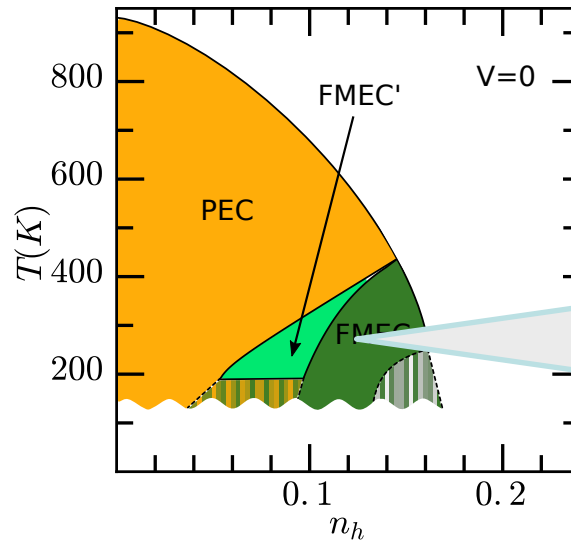
$$K_0 > 0$$

$$\text{PEC: } \psi = \prod_i (\alpha + \beta d_{i\mathbf{x}}^\dagger) |0\rangle$$

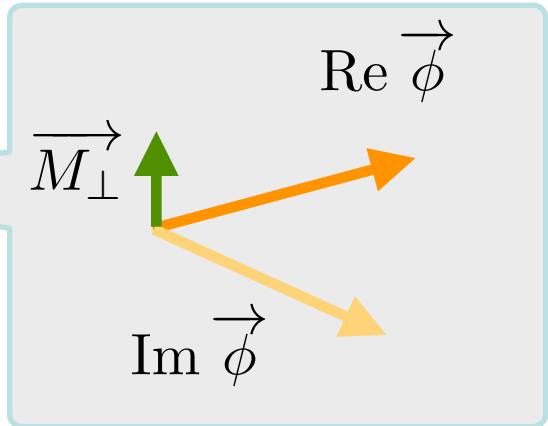
$$K_0 < 0$$

$$\text{FMEC: } \psi = \prod_i (\alpha + \beta d_{i\mathbf{1}}^\dagger) |0\rangle$$

# Doping



$$S_z = d_1^\dagger d_1 - d_{-1}^\dagger d_{-1} \\ = i(d_x^\dagger d_y - d_y^\dagger d_x)$$



$$H_{\text{eff}} = \varepsilon \sum_i n_i + K_\perp \sum_{\langle ij \rangle} (\mathbf{d}_i^\dagger \cdot \mathbf{d}_j + H.c.) + K_\parallel \sum_{\langle ij \rangle} n_i n_j + K_0 \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j$$

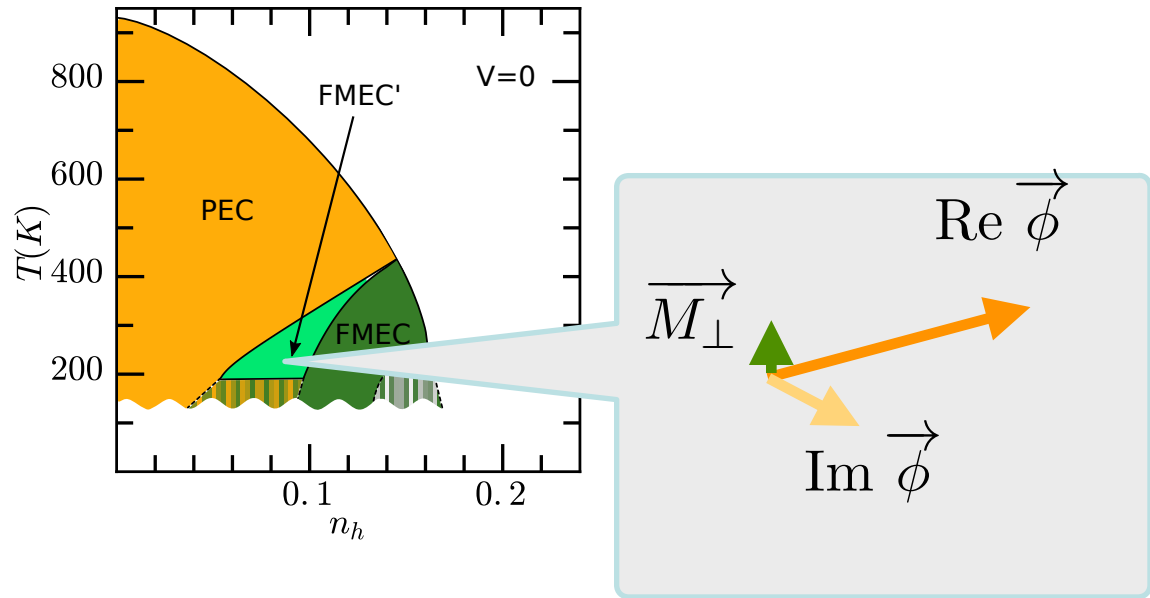
$$K_0 > 0$$

$$\text{PEC: } \psi = \prod_i (\alpha + \beta d_{i\mathbf{x}}^\dagger) |0\rangle$$

$$K_0 < 0$$

$$\text{FMEC: } \psi = \prod_i (\alpha + \beta d_{i\mathbf{1}}^\dagger) |0\rangle$$

# Doping



$$H_{\text{eff}} = \varepsilon \sum_i n_i + K_\perp \sum_{\langle ij \rangle} (\mathbf{d}_i^\dagger \cdot \mathbf{d}_j + H.c.) + K_\parallel \sum_{\langle ij \rangle} n_i n_j + K_0 \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j$$

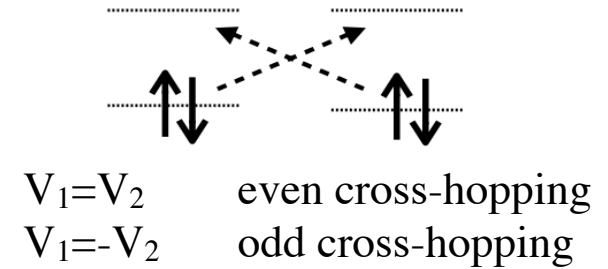
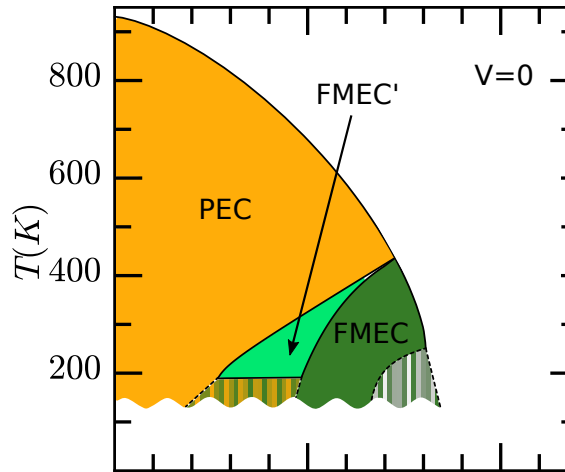
$$K_0 > 0$$

$$\text{PEC: } \psi = \prod_i (\alpha + \beta d_{i\mathbf{x}}^\dagger) |0\rangle$$

$$K_0 < 0$$

$$\text{FMEC: } \psi = \prod_i (\alpha + \beta d_{i\mathbf{1}}^\dagger) |0\rangle$$

# Cross-hopping

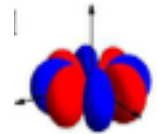


$$H_{\text{eff}} = \varepsilon \sum_i n_i + K_{\perp} \sum_{\langle ij \rangle} (\mathbf{d}_i^{\dagger} \cdot \mathbf{d}_j + H.c.) + K_{\parallel} \sum_{\langle ij \rangle} n_i n_j + K_0 \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j$$

$$+ K_1 \sum_{\langle ij \rangle} (\mathbf{d}_i^{\dagger} \cdot \mathbf{d}_j^{\dagger} + \mathbf{d}_i \cdot \mathbf{d}_j)$$

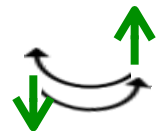
$K_{\perp} K_1 > 0$  SDW:

$$\psi = \prod_i (\alpha + \beta d_{ix}^{\dagger}) |0\rangle$$

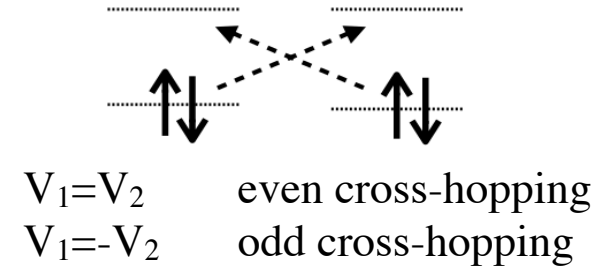
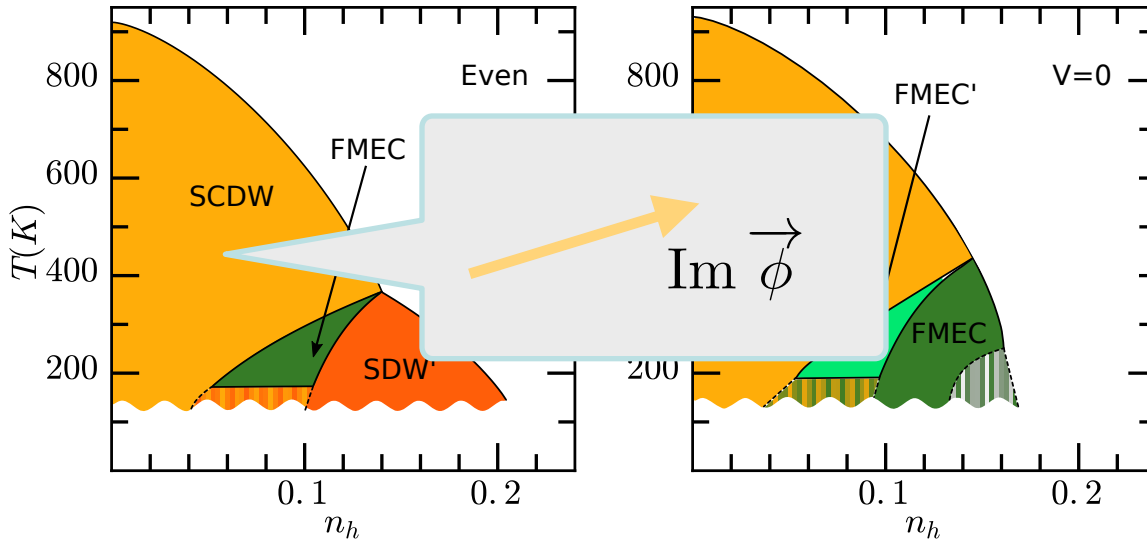


$K_{\perp} K_1 < 0$  SCDW:

$$\psi = \prod_i (\alpha + i\beta d_{ix}^{\dagger}) |0\rangle$$

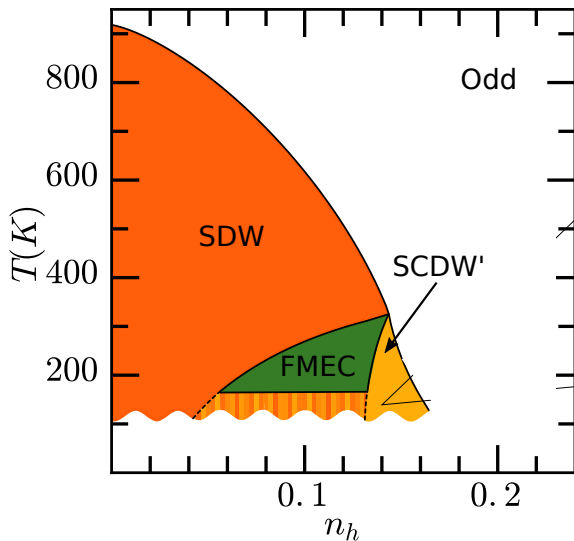


# Doping + cross-hopping



$$\phi = \sum_{\alpha\beta} \sigma_{\alpha\beta} \langle a_{\alpha}^{\dagger} b_{\beta} \rangle$$

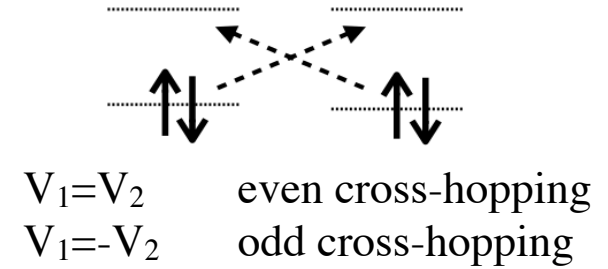
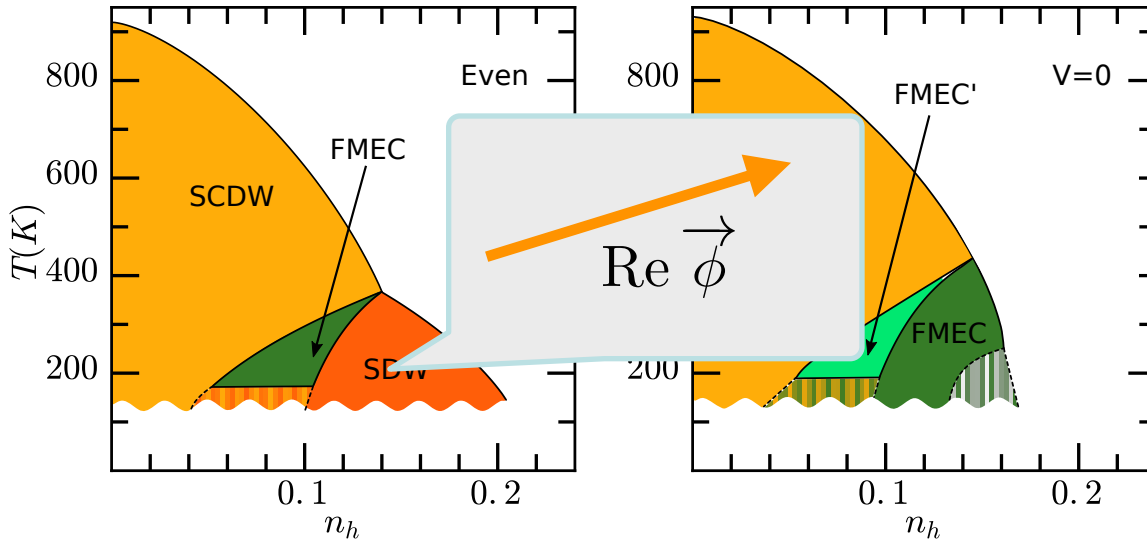
$$\text{PEC} \begin{cases} \text{SDW} & \phi = \mathbf{x} \\ \text{SCDW} & \phi = i\mathbf{x} \end{cases}$$



Condensate state	$M_{\perp}$	$M_{\parallel}$	$\mathbf{m}(\mathbf{r})$	$\mathbf{m}_{\mathbf{k}}$	$\text{Re } \phi$	$\text{Im } \phi$
FMEC	✓	✓, 0	✓	✓	✓	✓
SDW	0	0	✓	0	✓	0
SCDW	0	0	0	0	0	✓
SDW'	0	✓, 0	✓	✓	✓	0
SCDW'	0	0	0	✓	0	✓

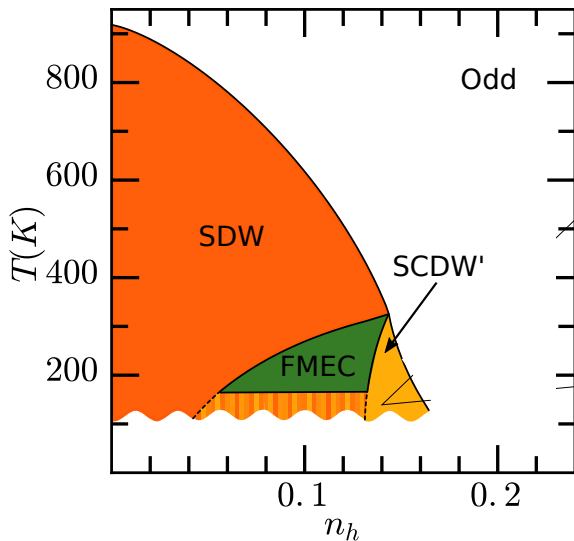
$$\mathbf{m}_{\mathbf{k}} = \sum_{\alpha\beta} \sigma_{\alpha\beta} \langle a_{\mathbf{k}\alpha}^{\dagger} a_{\mathbf{k}\beta} + b_{\mathbf{k}\alpha}^{\dagger} b_{\mathbf{k}\beta} \rangle$$

# Doping + cross-hopping



$$\phi = \sum_{\alpha\beta} \sigma_{\alpha\beta} \langle a_{\alpha}^{\dagger} b_{\beta} \rangle$$

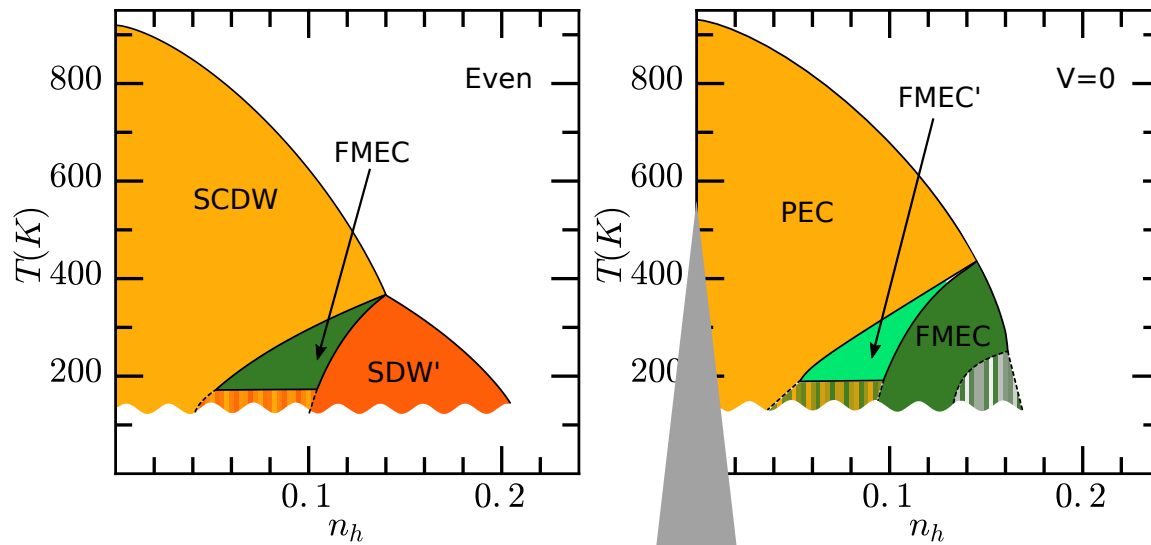
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FMEC	✓	✓, 0	✓	✓	✓	✓
SDW	0	0	✓	0	✓	0
SCDW	0	0	0	0	0	✓
SDW'	0	✓, 0	✓	✓	✓	0
SCDW'	0	0	0	✓	0	✓

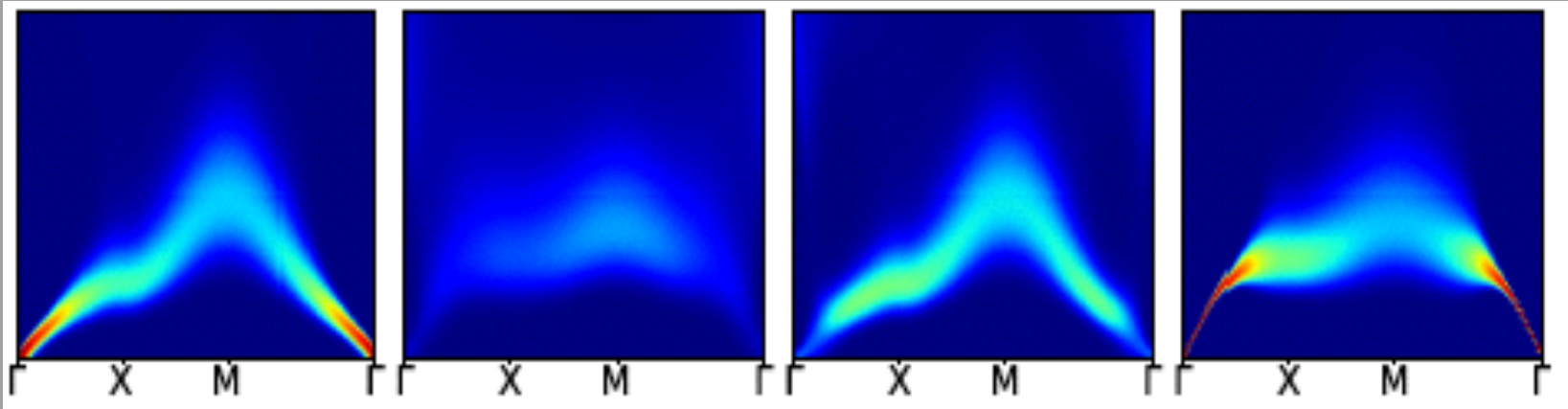
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# Doping + cross-hopping

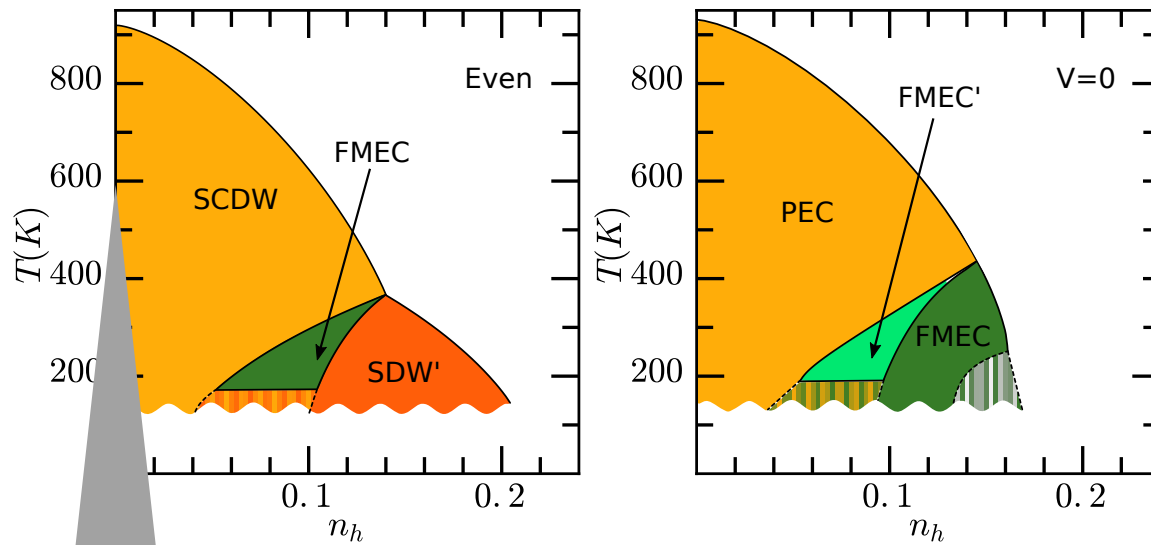


*spin GM*

*phase GM*

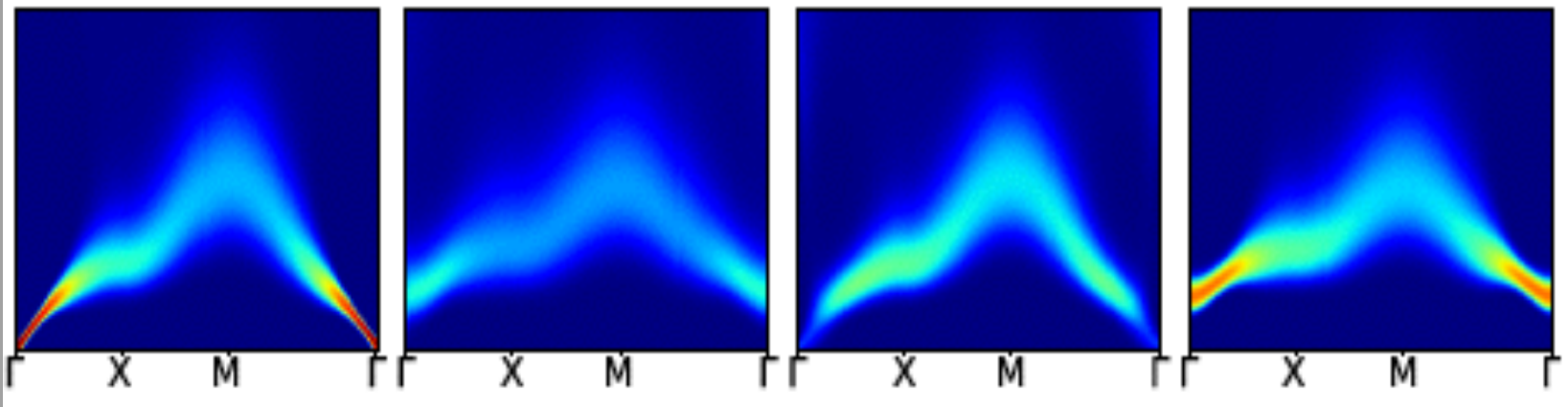


# Doping + cross-hopping



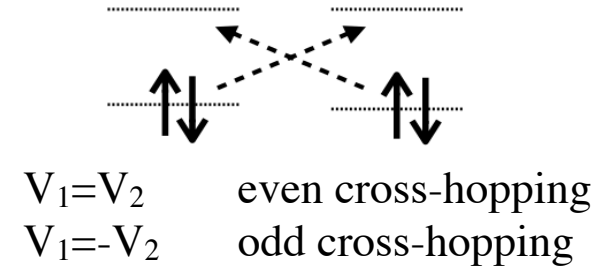
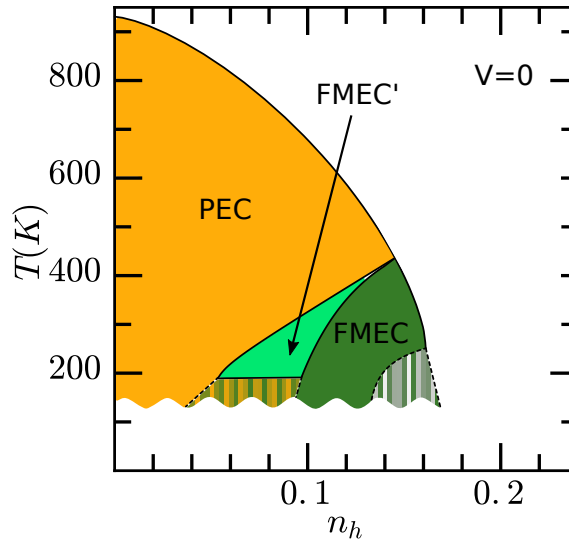
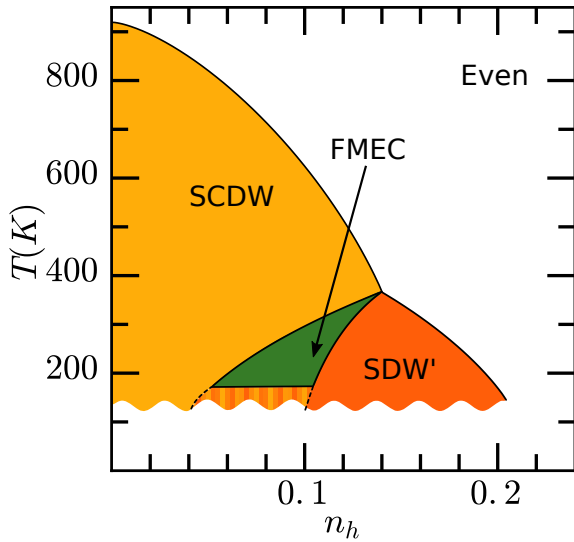
*spin GM*

*phase mode*



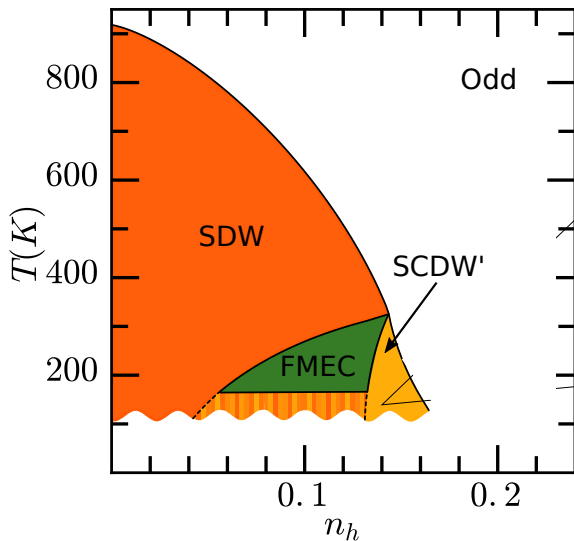


# Doping + cross-hopping



$$\phi = \sum_{\alpha\beta} \sigma_{\alpha\beta} \langle a_{\alpha}^{\dagger} b_{\beta} \rangle$$

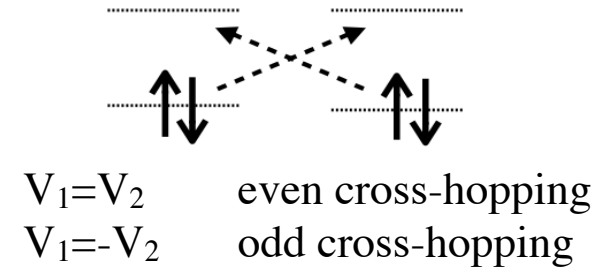
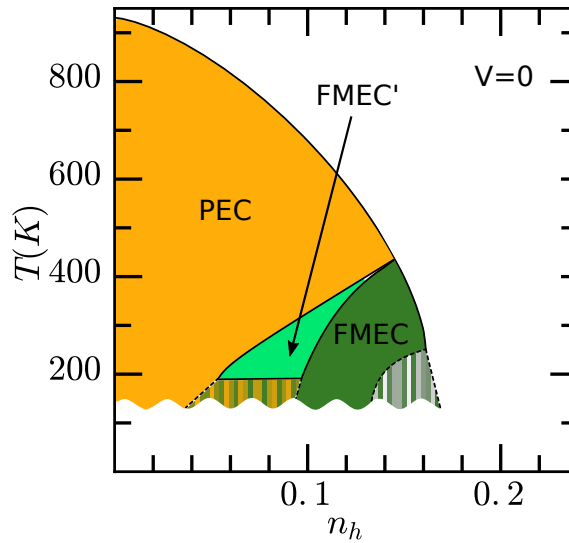
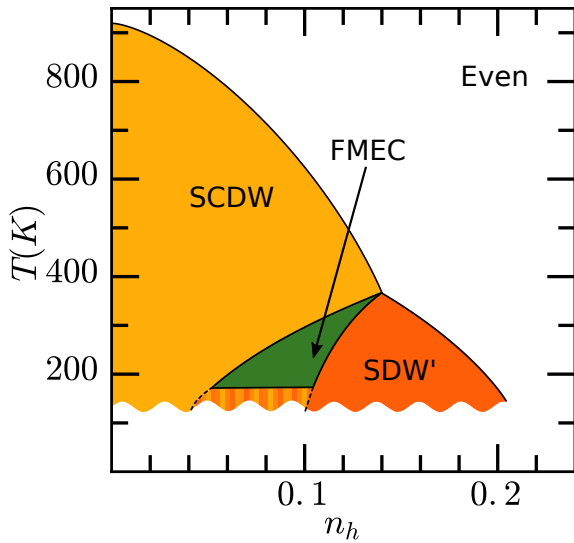
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SDW	0	0	✓	0	✓	0
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SDW'	0	✓, 0	✓	✓	✓	0
SCDW'	0	0	0	✓	0	✓

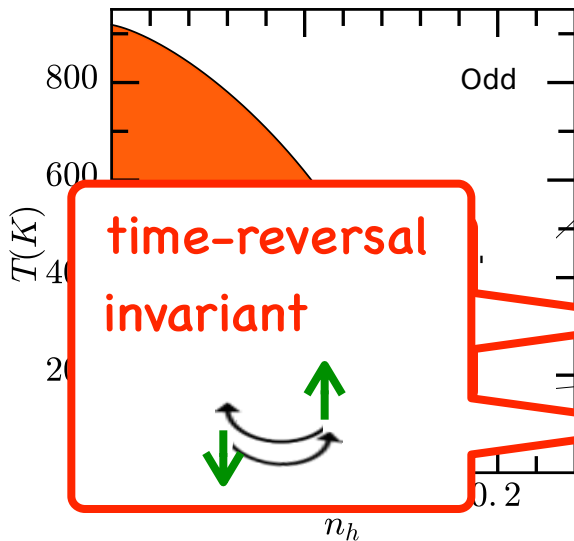
$$\mathbf{m}_{\mathbf{k}} = \sum_{\alpha\beta} \sigma_{\alpha\beta} \langle a_{\mathbf{k}\alpha}^{\dagger} a_{\mathbf{k}\beta} + b_{\mathbf{k}\alpha}^{\dagger} b_{\mathbf{k}\beta} \rangle$$

# Doping + cross-hopping



$$\phi = \sum_{\alpha\beta} \sigma_{\alpha\beta} \langle a_{\alpha}^{\dagger} b_{\beta} \rangle$$

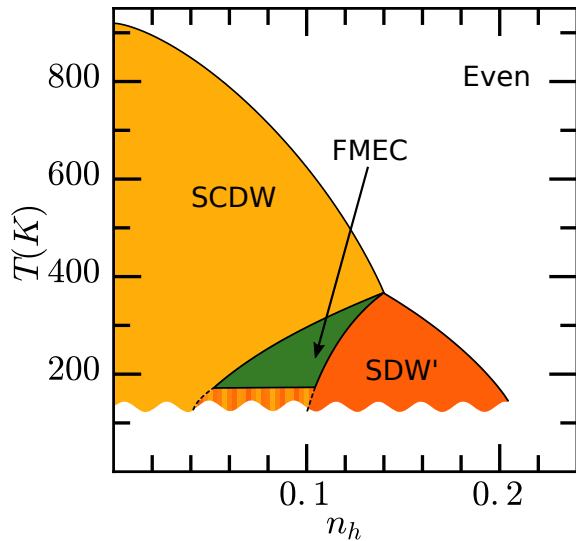
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SDW	0	0	✓	0	✓	0
SCDW	0	0	0	0	0	✓
SDW'	0	✓, 0	✓	✓	✓	0
SCDW'	0	0	0	✓	0	✓

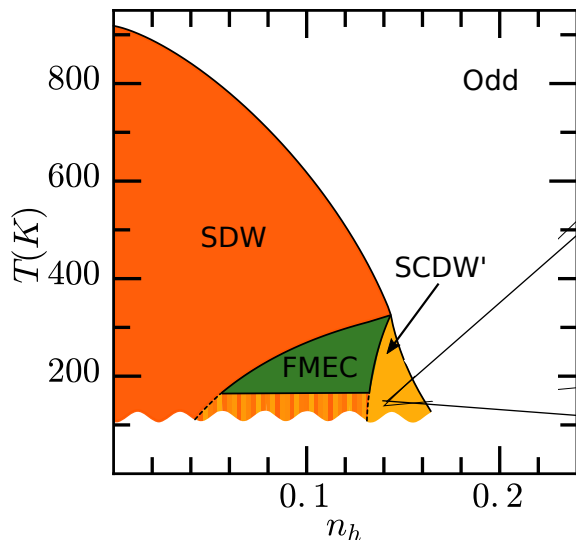
$$\mathbf{m}_{\mathbf{k}} = \sum_{\alpha\beta} \sigma_{\alpha\beta} \langle a_{\mathbf{k}\alpha}^{\dagger} a_{\mathbf{k}\beta} + b_{\mathbf{k}\alpha}^{\dagger} b_{\mathbf{k}\beta} \rangle$$

# Doping + cross-hopping

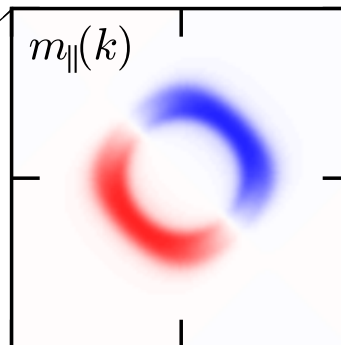


Condensate state	$M_{\perp}$	$M_{\parallel}$	$\mathbf{m}(\mathbf{r})$	$\mathbf{m}_{\mathbf{k}}$	$\text{Re } \phi$	$\text{Im } \phi$
FMEC	✓	✓, 0	✓	✓	✓	✓
SDW	0	0	✓	0	✓	0
SCDW	0	0	0	0	0	✓
SDW'	0	✓, 0	✓	✓	✓	0
SCDW'	0	0	0	✓	0	✓

$$\mathbf{m}_{\mathbf{k}} = \sum_{\alpha\beta} \sigma_{\alpha\beta} \langle a_{\mathbf{k}\alpha}^{\dagger} a_{\mathbf{k}\beta} + b_{\mathbf{k}\alpha}^{\dagger} b_{\mathbf{k}\beta} \rangle$$



Spin density in  $k$ -space (Brillouin zone)

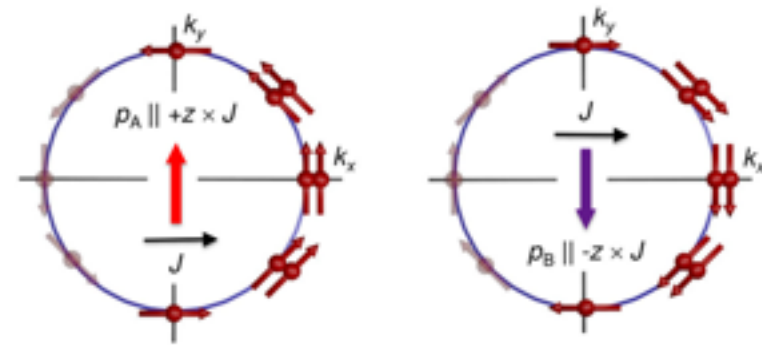


- $k$ -space spin texture
- bulk spin current ?
- spontaneous spin-orbit coupling

Cite as: P. Wadley *et al.*, *Science* 10.1126/science.aab1031 (2016).

# Electrical switching of an antiferromagnet

P. Wadley,<sup>1\*</sup> B. Howells,<sup>1\*</sup> J. Železný,<sup>2,3</sup> C. Andrews,<sup>1</sup> V. Hills,<sup>1</sup> R. P. Campion,<sup>1</sup> V. Novák,<sup>2</sup> K. Olejník,<sup>2</sup> F. Maccherozzi,<sup>4</sup> S. S. Dhesi,<sup>4</sup> S. Y. Martin,<sup>5</sup> T. Wagner,<sup>5,6</sup> J. Wunderlich,<sup>2,5</sup> F. Freimuth,<sup>7</sup> Y. Mokrousov,<sup>7</sup> J. Kuneš,<sup>8</sup> J. S. Chauhan,<sup>1</sup> M. J. Grzybowski,<sup>1,9</sup> A. W. Rushforth,<sup>1</sup> K. W. Edmonds,<sup>1</sup> B. L. Gallagher,<sup>1</sup> T. Jungwirth,<sup>2,1</sup>



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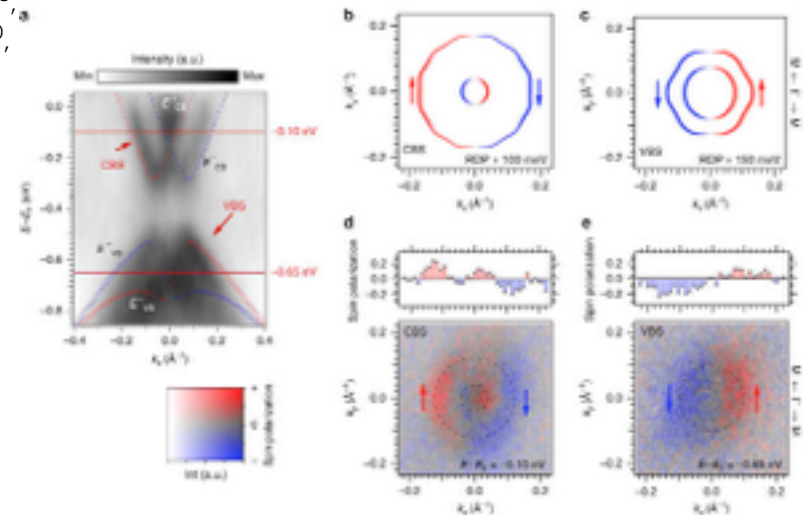
Received 30 Nov 2015 | Accepted 14 Apr 2016 | Published 18 May 2016

DOI: 10.1038/ncomms11621

OPEN

# Spin-texture inversion in the giant Rashba semiconductor BiTeI

Henriette Maaß<sup>1</sup>, Hendrik Bentmann<sup>1</sup>, Christoph Seibel<sup>1</sup>, Christian Tuschke<sup>2</sup>, Sergey V. Eremeev<sup>3,4,5</sup>, Thiago R.F. Peixoto<sup>1</sup>, Oleg E. Tereshchenko<sup>5,6,7</sup>, Konstantin A. Kokh<sup>5,7,8</sup>, Evgeni V. Chulkov<sup>4,5,9,10</sup>, Jürgen Kirschner<sup>2</sup> & Friedrich Reinert<sup>1</sup>



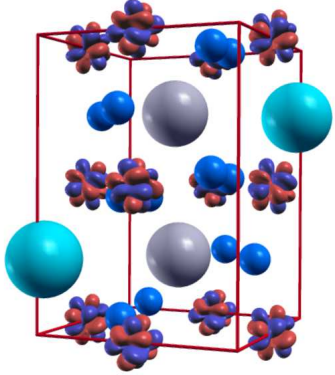
# Excitonic condensation in real materials



*Immanuel Kant*

“Experience without theory is blind, but theory without experience is mere intellectual play.”

### Pr<sub>0.5</sub>Ca<sub>0.5</sub>CoO<sub>3</sub>:

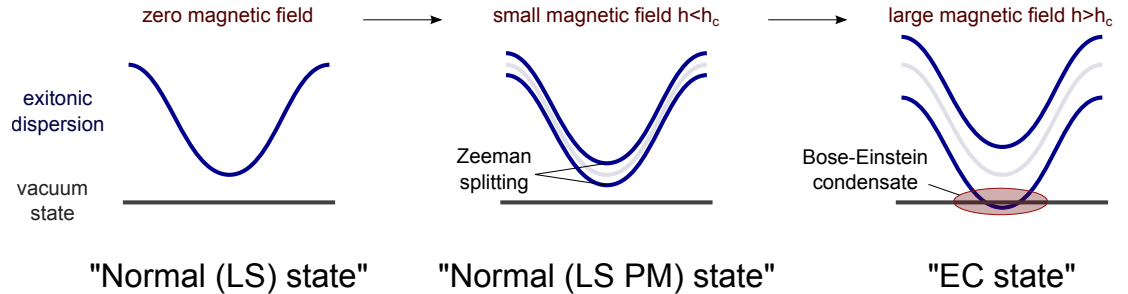


- metal-insulator transition
- disappearance of Co moments
- Pr<sup>3+</sup> → Pr<sup>4+</sup> valence transition
- TR breaking without ordered moments

Tsubouchi et al. 2002, 2004  
 Hejtmanek et al. 2010, 2013  
 Knizek et al., 2010, 2013  
 JK & Augustinsky, 2014

### LaCoO<sub>3</sub> in high field:

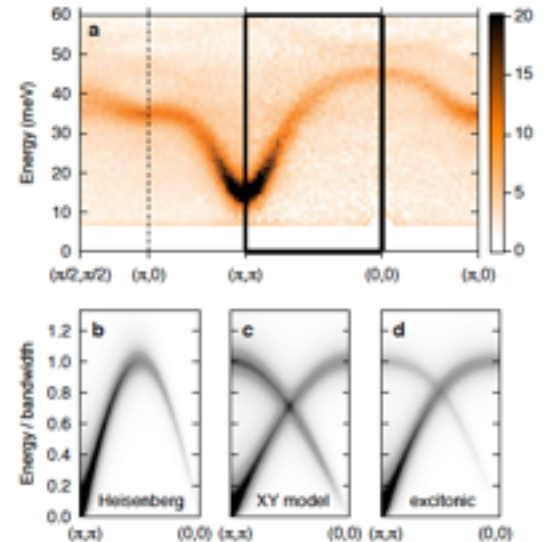
- positive  $dB_c/dT > 0$



Ikeda et al., 2016; Sotnikov & JK, 2016

### Ca<sub>2</sub>RuO<sub>4</sub>:

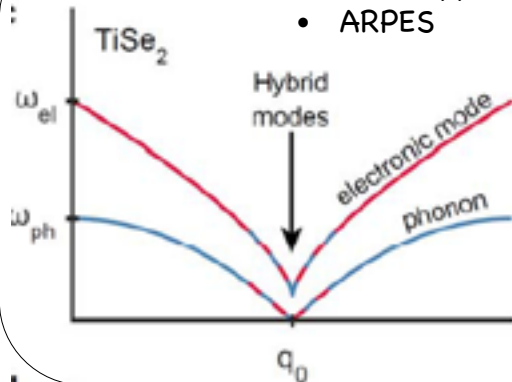
- INS in the ordered state



Jain et al., 2015

### 1T-TiSe<sub>2</sub>:

- EELS approaching instability
- ARPES

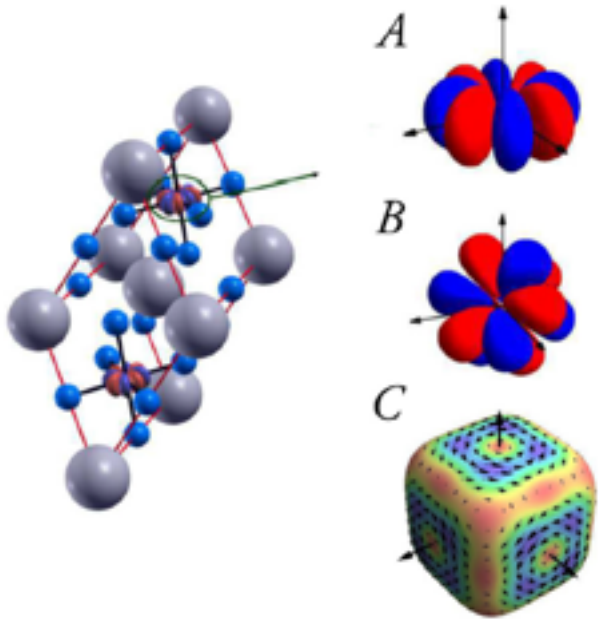


Kogar et al., 2013

# cubic d<sup>6</sup> cobaltite (LaCoO<sub>3</sub>): LDA+U results

Phase    Spin density    Order parameter

EC phase	$ \phi' $	$ \phi'' $	$E - E_0$ [meV/f.u.]
A-phase	0.151	0.000	-13.32
B-phase	0.191	0.000	-20.39
C-phase	0.202	0.009	-22.81



$$\phi_k^{(A)} = (-1)^k \begin{pmatrix} 0 & 0 & \lambda' \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\phi_k^{(B)} = (-1)^k \begin{pmatrix} 0 & 0 & \lambda' \\ 0 & 0 & \lambda' \\ 0 & 0 & \lambda' \end{pmatrix}$$

$$\phi_k^{(C)} = (-1)^k \begin{pmatrix} \lambda' & 0 & 0 \\ 0 & \lambda' & 0 \\ 0 & 0 & \lambda' \end{pmatrix} + i \begin{pmatrix} \lambda'' & 0 & 0 \\ 0 & \lambda'' & 0 \\ 0 & 0 & \lambda'' \end{pmatrix}$$

staggered

uniform

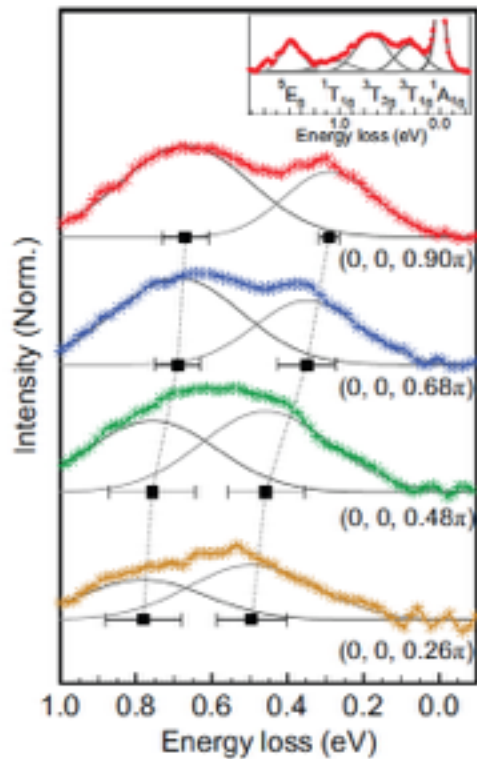
- Numerous excitonic phases are possible
- Rhombohedral distortion (real structure) suppresses excitonic order
- Spin-orbit coupling favors the excitonic order

# Real $\text{LaCoO}_3$

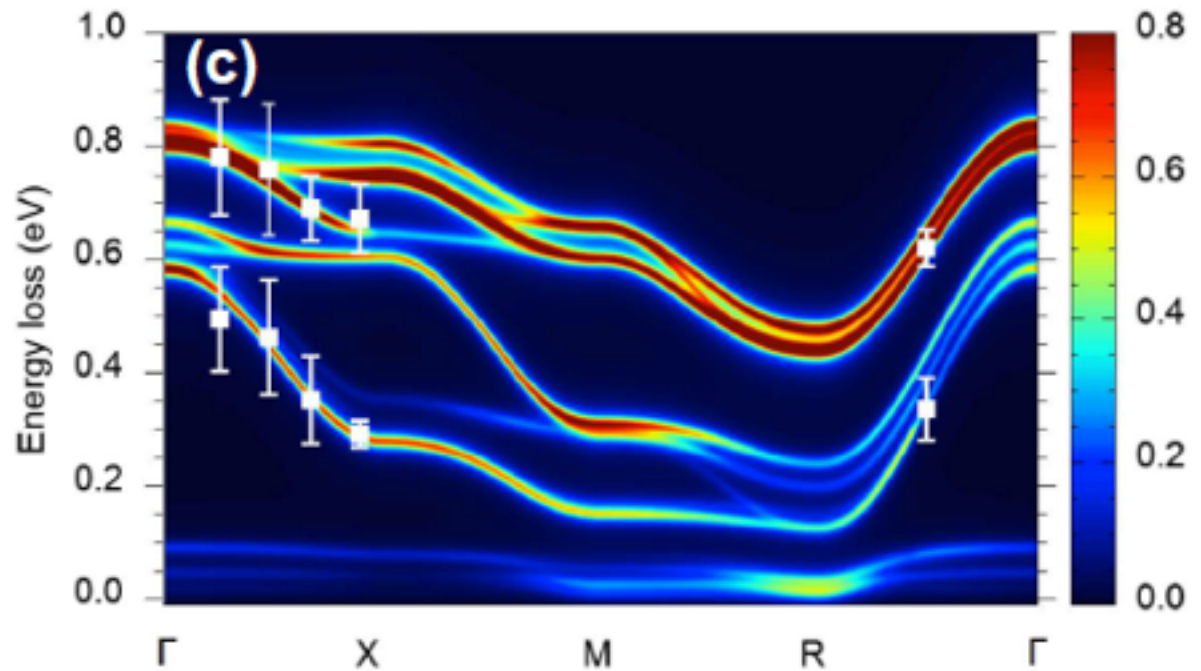
Are IS excitation mobile? **YES**

Do they condense? **NO**

Co L-edge RIXS



Exp + Theory





# Real $\text{LaCoO}_3$

Are IS excitation mobile? **YES**

Do they condense? **NO**

Why not? formation of immobile bi-excitons  $\text{IS} + \text{IS} \rightarrow \text{HS}$

Why not spin state order  $\text{HS-LS-HS-LS}$ ?

**important question**

$\text{HS} \rightleftharpoons \text{IS} + \text{IS} ???$

# Real $\text{LaCoO}_3$

Are IS excitation mobile? **YES**

Do they condense? **NO**

Why not? formation of immobile bi-excitons  $\text{IS} + \text{IS} \rightarrow \text{HS}$

Why not spin state order  $\text{HS-LS-HS-LS}$ ?

**important question**

$\text{HS} \rightleftharpoons \text{IS} + \text{IS} ???$

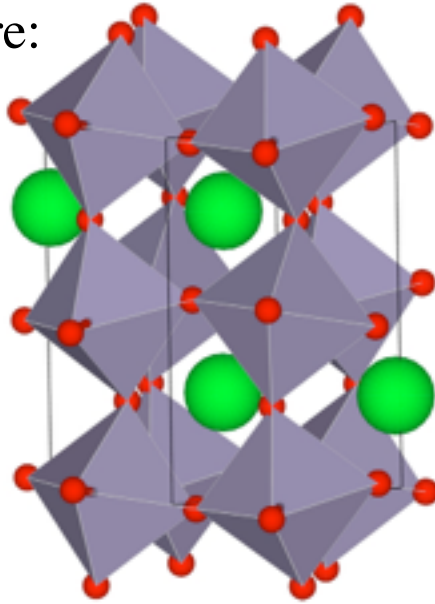
Possible route to exciton magnet in cobaltites:

2D structure, e.g.,  $(\text{SrLa})_2\text{CoO}_4$

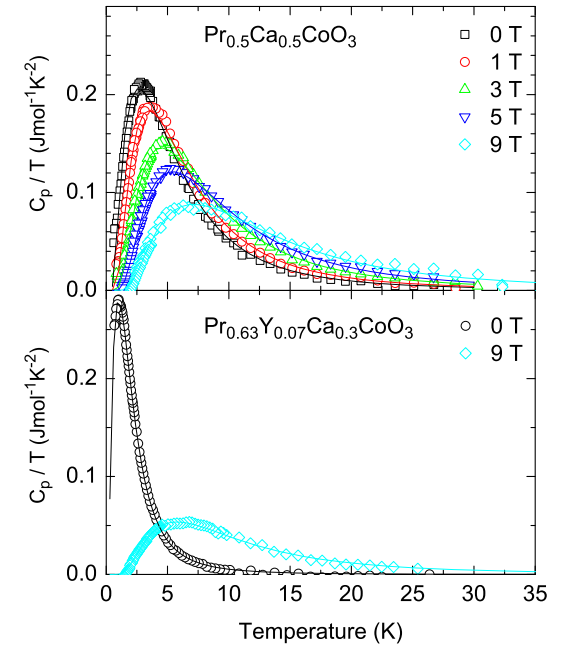
$\Rightarrow$  smaller  $E_{\text{IS}} - E_{\text{HS}}$

# Exchange splitting in $\text{Pr}_{0.5}\text{Ca}_{0.5}\text{CoO}_3$

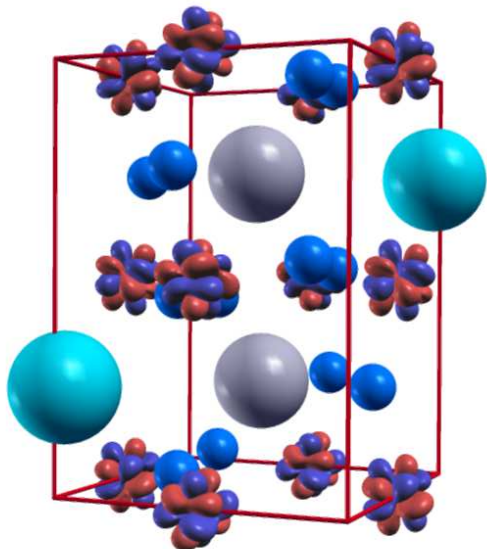
crystal structure:



$\text{Pr}^{4+}$  Schottky peak:



spin density in condensed phase (LDA+U)



*J. Hejtmanek et al., EPJB 86, 305 (2013)*

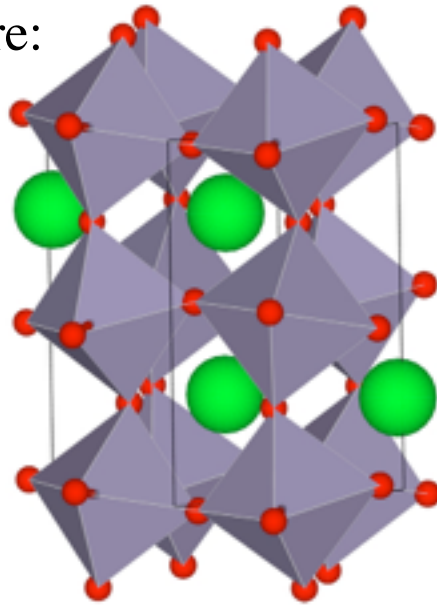
Kondo interaction of Pr moment with Co d-electrons:

$$H^{(n)} = \sum_{\alpha\alpha'} \sum_{mm'} \sum_i \mathbf{S} \cdot \boldsymbol{\sigma}_{\alpha\alpha'} J_{i,mm'}^{(n)} c_{im\alpha}^\dagger c_{im'\alpha'} + \text{c.c.}$$

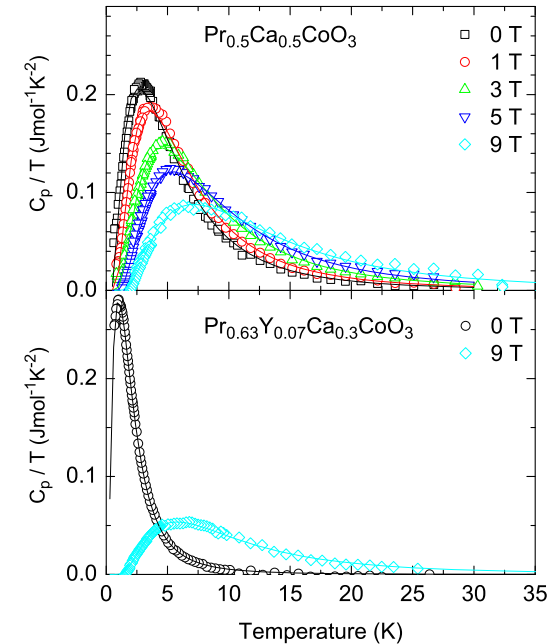
*JK and P. Augustinský, PRB 90, 235112 (2014)*

# Exchange splitting in $\text{Pr}_{0.5}\text{Ca}_{0.5}\text{CoO}_3$

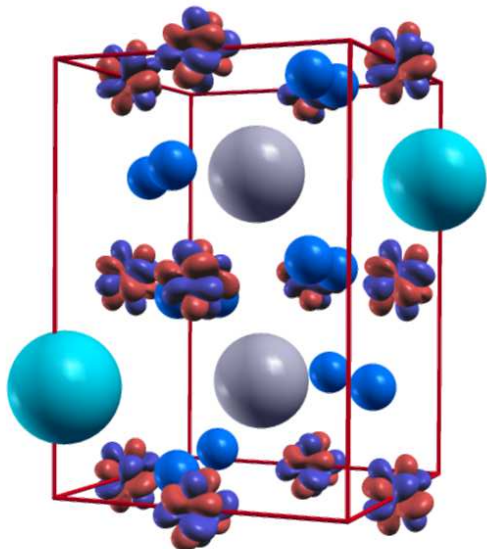
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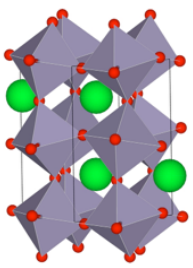
Kondo interaction of Pr moment with Co d-electrons:  
in the condensate

$$H^{(n)} = \sum_{\alpha\alpha'} \sum_{mm'} \sum_i \mathbf{S} \cdot h_{\gamma}^{(n)}$$

$$h_{\gamma}^{(n)} = \sum_{imm'} J_{i,mm'}^{(n)} \sum_{\alpha\alpha'} 2 \text{Re} \langle c_{im\alpha}^{\dagger} \sigma_{\alpha\alpha'}^{\gamma} c_{im'\alpha'} \rangle$$

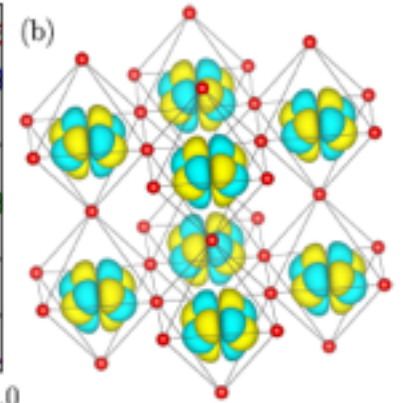
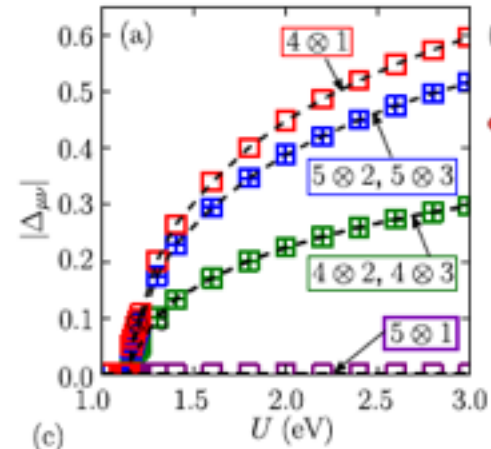
*JK and P. Augustinský, PRB 90, 235112 (2014)*

# Pr<sub>0.5</sub>Ca<sub>0.5</sub>CoO<sub>3</sub>

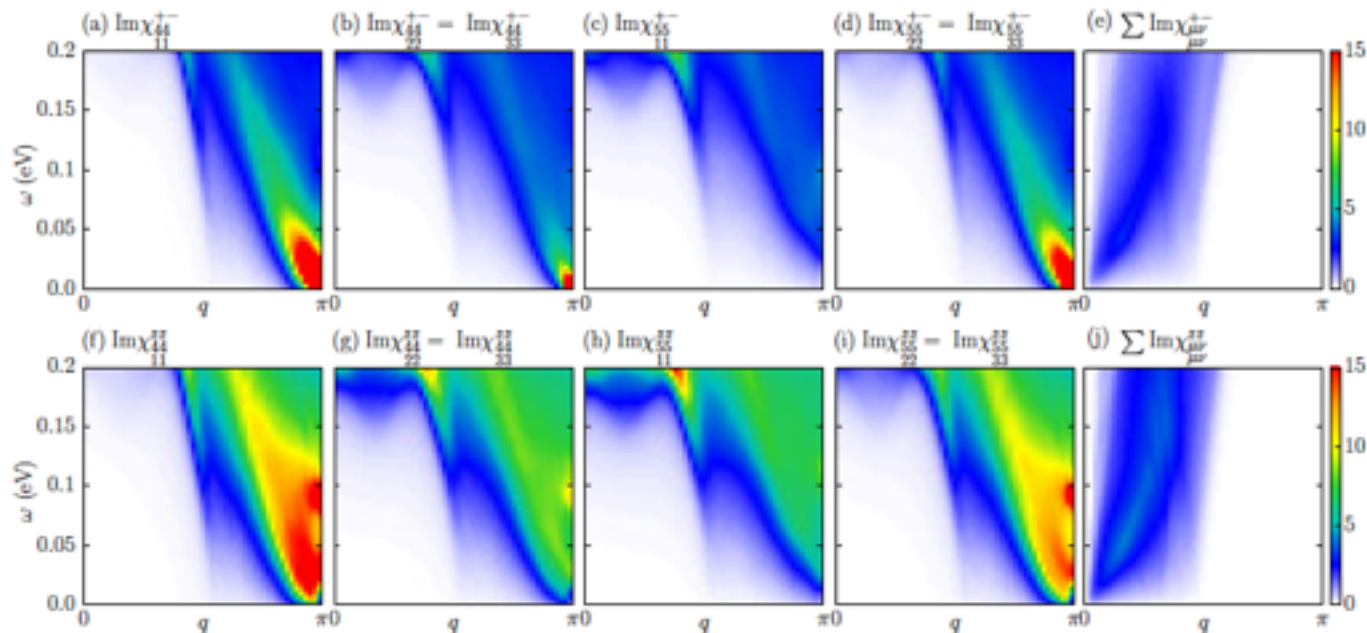


RPA calculation in ideal cubic structure

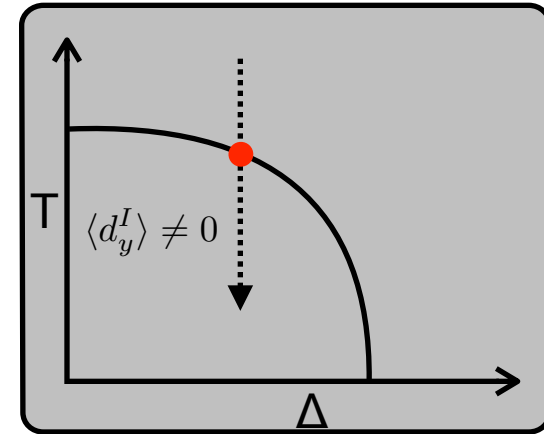
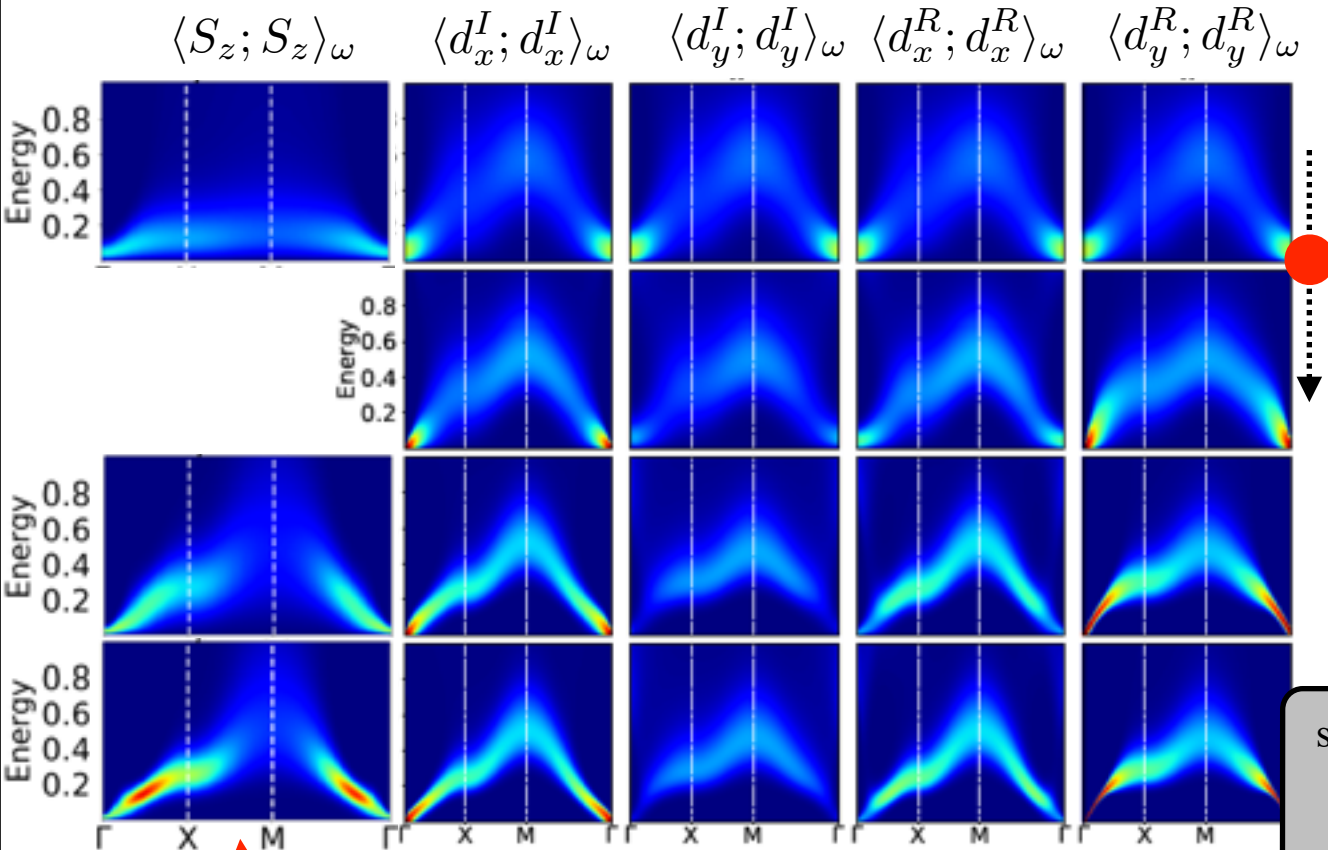
Order parameter:



Excitonic and spin susceptibility:



# How to detect excitonic condensate (PEC)?



Similar behavior recently observed in INS experiment  
on  $(\text{Pr}_{1-y}\text{Y}_y)_{1-x}\text{Ca}_x\text{CoO}_3$

*T. Mojoshi et al. PRB 98, 205105 (2018)*

strong-coupling picture:

$$\begin{aligned} S_z &= d_1^\dagger d_1 - d_{-1}^\dagger d_{-1} \\ &= i(d_x^\dagger d_y - d_y^\dagger d_x) \end{aligned}$$

in the PEC phase:

$$\begin{aligned} \langle d_y \rangle &= i\phi \quad \Rightarrow \\ S_z &\approx -\phi(d_x^\dagger + d_x) = -\phi d_x^R \end{aligned}$$

## Collaborators:



Atsushi Hariki



Andrii Sotnikov



Juan Fernandez Afonso



Dominique Geffroy

Ru-Pan Wang  
Frank de Groot  
Zdenek Jirak

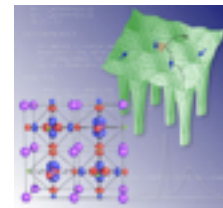


Pavel Augustinský

## Sponsors:



*ERC CoG EXMAG*



*DFG Research Unit FOR1346*



# Conclusions

- Excitonic magnetism provides a rich field of new physics with potentially interesting application
- Excitonic magnets have yet to be found (promising candidates exist)
- Experimental techniques for unambiguous identification of excitonic condensate have to be established





# Higgs mode

