# Thermoelectric transport of ultracold fermions : theory

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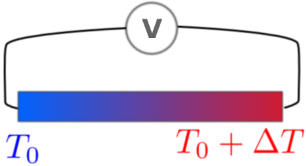
# Introduction to Thermoelectricity

- **Seebeck effect :** A difference of temperature $\Delta T$ creates a voltage $\Delta V$

$$\Delta V = \alpha \Delta T$$




- **Peltier effect :** An electric current $I$ generates a heat current $I_Q = \Pi \cdot I = (T \cdot \alpha) \cdot I$

i. **Seebeck coefficient $\alpha$** : Entropy per carrier $I_S = \alpha I_N - \lambda \Delta T$

i. **Stationary effects** : permanent currents/differences

L. Onsager, Phys. Rev. 38, 2265 (1931)& Phys. Rev. 37, 405 (1931)

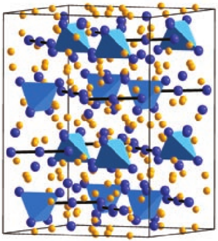
H.B. Callen, Phys. Rev. 73, 1349-1358 (1948)

# Thermoelectricity and materials

**Good thermoelectrics** : a recurrent interest in material physics

- Good Peltier cooling
- Efficient wasted heat recovery : energy saving purposes

**Idea** : increase figure of merit $ZT = \frac{\sigma_{el} T \alpha^2}{\sigma_{th}}$



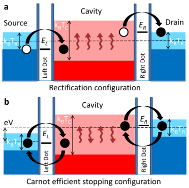
**Fundamental purposes** :
- High temperature transport properties
- Understanding of electron/phonon coupling ...

G. Snyder & E. Toberer, Nat. Mat. **7** 105 (2008)

# Thermoelectricity and mesoscopic physics

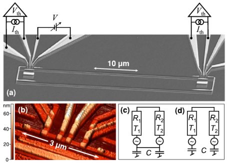
**GOALS :**

- Extract energy from fluctuating environment
- Optimize energy to electricity conversion (cooling purposes)



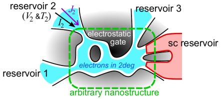

## Heat engine with quantum dots

A. Jordan *et al.* Phys. Rev. B 87, 075312 (2013)



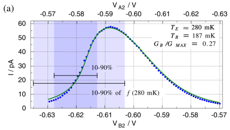

## Quantum limited refrigerator

Timofeev *et al* PRL (2009)




## Nonlinear thermoelectric transport

R. Whitney Phys. Rev. B, 88, 064302 (2013)




## Quantum dot refrigerator

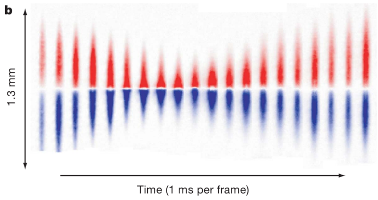
Prance *et al* PRL (2009)

**What about cold atoms?**

# Introduction - Transport and cold atoms

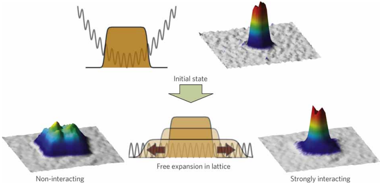
## Spin transport (MIT, 2011)

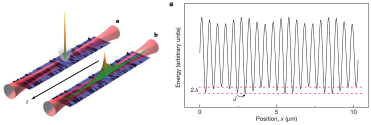
A. Sommer *et al.*, Nature




## Interactions (LMU, 2012)

U. Schneider *et al.*, Nat. Phys.



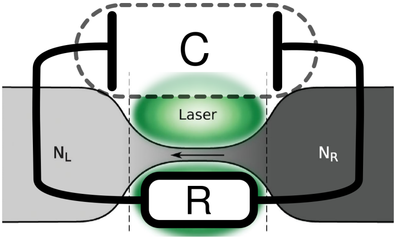



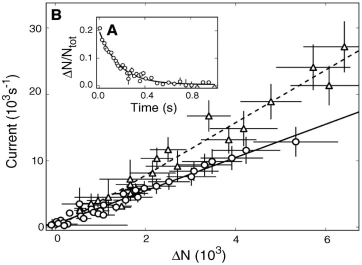

## Disorder (Inst. d'optique - LENS, 2008)

J. Billy *et al.*-G. Roati *et al.*, Nature

Also:
H. Ott *et al*, Phys. Rev. Lett. 92, 160601 (2004)
S. Palzer *et al*, Phys. Rev. Lett. 103, 150601 (2009)
J. Catani *et al*, Phys. Rev. A 85, 023623 (2012)
K.K. Das *et al*, Phys. Rev. Lett. 103, 123007 (2009)
And many others ...

# Introduction - Experimental motivation




**ETH, 2012**

J.P. Brantut *et al.*, Science

→ Realization of a two terminal transport setup

Discharge of a mesoscopic capacitor (reservoirs) in a resistor (the channel)

⇒ Simulation of mesoscopic physics with cold atoms

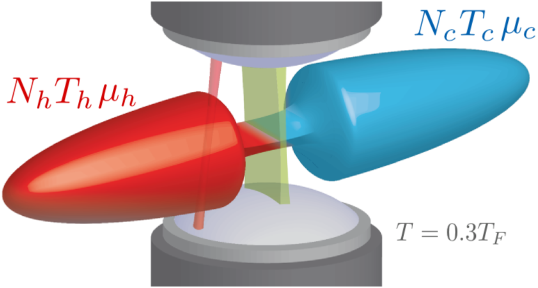
**Question** : Can this setup demonstrate offdiagonal transport ?

# Outline

# Outline

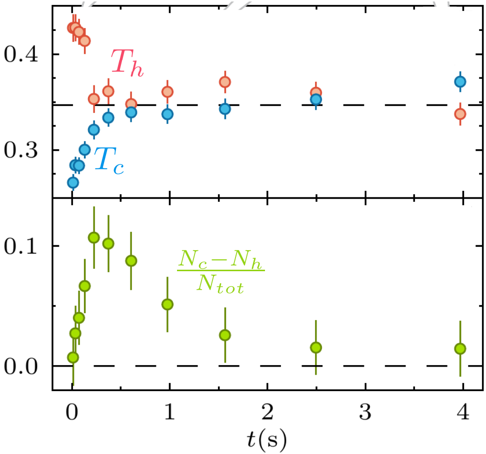
1. Experimental setup and theory framework
2. Thermoelectricity with cold atoms
3. A cold atom based heat engine

# Experimental procedure




i. Heating of one reservoir at constant particle number (closed channel)
ii. Reopen the channel
iii. Monitor temperature difference and particle number imbalance

# Typical results




- Temperatures : ≃ exp. relaxation
- Particle number imbalance :
  - i. Transient evolution
  - ii. Current from hot to cold

⇒**Thermoelectricity !**

# Transport in linear response

<u>**Our approach**</u> : Constriction $\leftrightarrow$ Black box responding linearly

$\equiv$ **Linear circuit picture**

**Linear response :** $$\begin{pmatrix} I_N \\ I_{\mathscr{S}} \end{pmatrix} = -\underline{\mathscr{L}} \begin{pmatrix} \Delta\mu \\ \Delta T \end{pmatrix}, \quad \underline{\mathscr{L}} = \begin{pmatrix} \mathscr{L}_{11} & \mathscr{L}_{12} \\ \mathscr{L}_{12} & \mathscr{L}_{22} \end{pmatrix}$$

**And thermodynamics :** $$\begin{pmatrix} \Delta N \\ \Delta S \end{pmatrix} = \underline{\mathscr{M}} \begin{pmatrix} \Delta\mu \\ \Delta T \end{pmatrix}, \quad \underline{\mathscr{M}} = \begin{pmatrix} \kappa & \gamma \\ \gamma & \frac{C_\mu}{T} \end{pmatrix}$$

$\Rightarrow$ Equations for chemical potential and temperature difference

$$\frac{d}{dt} \begin{pmatrix} \Delta\mu \\ \Delta T \end{pmatrix} = -\underline{\mathscr{M}}^{-1} \underline{\mathscr{L}} \begin{pmatrix} \Delta\mu \\ \Delta T \end{pmatrix}$$

# Transport equations and coefficients

Equations for particle number and temperature difference :

$$\tau_0 \frac{d}{dt}\begin{pmatrix} \Delta N/\kappa \\ \Delta T \end{pmatrix} = -\underline{\Lambda}\begin{pmatrix} \Delta N/\kappa \\ \Delta T \end{pmatrix}, \underline{\Lambda} = \begin{pmatrix} 1 & -\alpha \\ -\frac{\alpha}{\ell} & \frac{L+\alpha^2}{\ell} \end{pmatrix}.$$

$\Rightarrow$ Discharge of a capacitor through a resistor, including thermal properties

Global timescale $\tau_0 = \frac{\kappa}{\mathscr{L}_{11}} \sim RC$

**Effective transport coefficients :**

$L \equiv \mathscr{L}_{22}/\mathscr{L}_{11} - (\mathscr{L}_{12}/\mathscr{L}_{11})^2 \sim R/TR_T \rightarrow$ Lorenz number

$\ell \equiv C_N/\kappa T \rightarrow$ Reservoir analogue to $L$

$\alpha \equiv \alpha_r - \alpha_{ch} \equiv \gamma/\kappa - \mathscr{L}_{12}/\mathscr{L}_{11} \rightarrow$ Total Seebeck coefficient

**Both thermodynamic and linear response coeffs participate to transport**

**Competition for offdiagonal contributions**

# How to compute transport coefficients ?

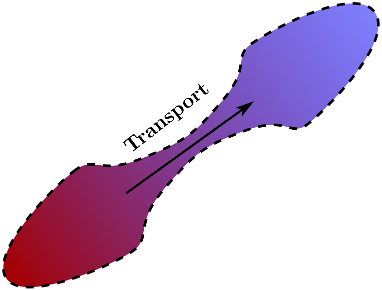
Need to take care of constriction **and** reservoirs
→ Reservoirs and constriction are treated separately

### Reservoirs

- Trapped Fermi gas
- Noninteracting fermions

### Constriction

- Geometry: Transverse harmonic trap
  1-2-3 D
- Conduction regime :
  Ballistic or diffusive
- Response coefficients :
  Landauer-Büttiker formalism



# Computing coefficients

- **Thermo. coefficients** $\Rightarrow$ Proportional to moments of $DOS \cdot \partial f / \partial \varepsilon$

$$\mathscr{R}_n = \int_0^{+\infty} d\varepsilon\, g_r(\varepsilon)\left(-\frac{\partial f}{\partial \varepsilon}\right)(\varepsilon-\mu)^n$$

$$\kappa \rightarrow n=0,\ \gamma \rightarrow n=1\ \frac{C_\mu}{T} \rightarrow n=2$$

- **Transport coefficients** $\Rightarrow$ Proportional to moments of $\Phi \cdot \partial f / \partial \varepsilon$

$$\mathscr{T}_n = \int_0^{+\infty} d\varepsilon\, \Phi(\varepsilon)\left(-\frac{\partial f}{\partial \varepsilon}\right)(\varepsilon-\mu)^n$$

$$\mathscr{L}_{11} \rightarrow n=0,\ \mathscr{L}_{12} \rightarrow n=1\ \mathscr{L}_{22} \rightarrow n=2$$

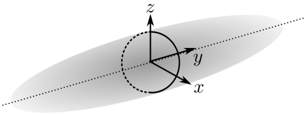
$\Phi$ : transport function $\simeq$ DOS $\cdot$ velocity $\cdot$ transmission
$\propto$ Differential conductance

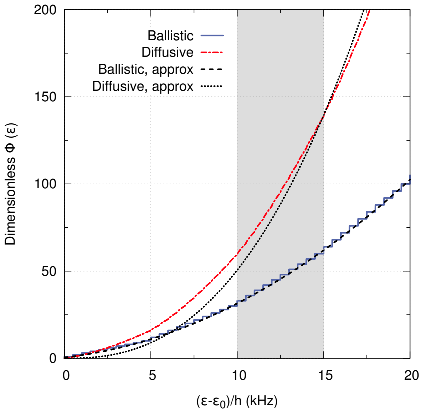

R. Kim *et al* Appl. Phys. Lett. **105**, 034506 (2009)

G. D. Mahan & J. O. Sofo, PNAS **93**, 7436 (1996)

# Transport function

$$\Phi(\varepsilon) = \sum_{n_x,n_z} \int dk_y \frac{\hbar k_y}{M} T(k)\delta\left(\varepsilon - \frac{\hbar^2 k_y^2}{2M} - n_z h\nu_z - n_x h\nu_x\right)$$




**Ballistic** : $T(k) = 1,$

$$\Phi(\varepsilon) \simeq \frac{1}{2}\left(1 + \left\lfloor \frac{\varepsilon-\varepsilon_0}{h\nu_x} \right\rfloor\right)\left(1 + \left\lfloor \frac{\varepsilon-\varepsilon_0}{h\nu_z} \right\rfloor\right)$$

**Diffusive** : $T(k) \simeq \frac{l(k)}{\mathscr{L}},$

$$\Phi(\varepsilon) \simeq \frac{4}{15}\frac{\tau_s}{\mathscr{L}}\sqrt{\frac{2}{M}}\frac{(\varepsilon-\varepsilon_0)^{5/2}}{h\nu_x h\nu_z}$$

For $\nu_z = 5\ kHz,\ \nu_x = 0.5\ kHz$

# Simple estimates for Seebeck(s)

Two contributions to thermopower :

$$\textbf{Channel} : \alpha_{ch} = \frac{\mathcal{L}_{12}}{\mathcal{L}_{11}}$$

$$\textbf{Reservoirs} : \alpha_r = \frac{\gamma}{\kappa}$$

At low $T$ $\Rightarrow$ Mott-Cutler formula :

$$\boxed{\alpha_{ch} \simeq \frac{\pi^2 k_B^2 T}{3} \frac{\Phi'(\mu)}{\Phi(\mu)}} \quad \text{and} \quad \boxed{\alpha_r \simeq \frac{\pi^2 k_B^2 T}{3} \frac{g_r'(\mu)}{g_r(\mu)}}$$

**Reservoirs ≡ 3D harmonic traps** : $g_r \propto \varepsilon^2 \Rightarrow \alpha_r \simeq \frac{2\pi^2 k_B^2 T}{3\mu}$

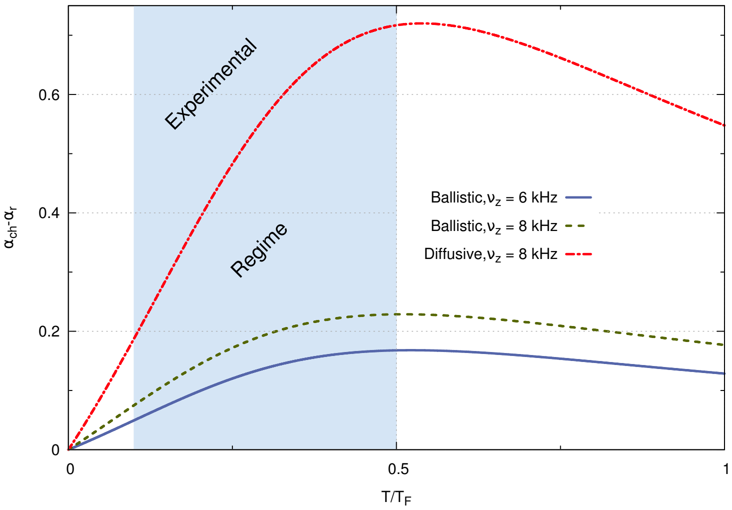
Two cases for the channel:

I. **Diffusive:** $\Phi \propto \varepsilon^{5/2}$ -> $\alpha_{ch} \simeq \frac{5\pi^2 k_B^2 T}{6\mu}$ : **Channel dominates**

II. **Ballistic:** $\Phi \propto \varepsilon^2$ -> $\alpha_{ch} \simeq \frac{2\pi^2 k_B^2 T}{3\mu}$ : **Need corrections, small effect expected**

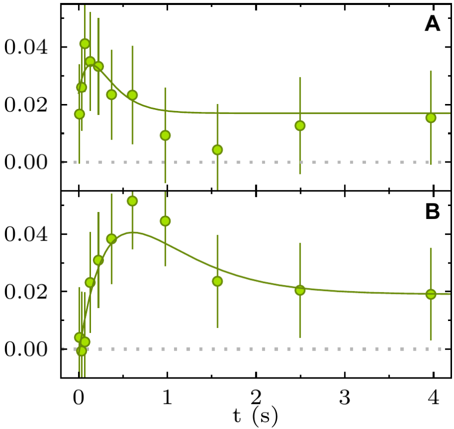
M. Cutler and N. F. Mott, Phys. Rev. 181, 1336 (1969)

# Resulting Seebeck

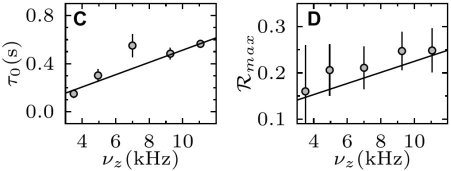


# Outline

# Comparison data-theory in the ballistic case




Particle imbalance vs. time for :
**A** 3.5 *kHz* and **B** 9.3 *kHz*

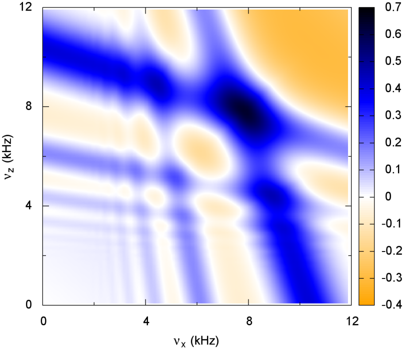



Characteristics vs confinement
(ab-initio predictions):

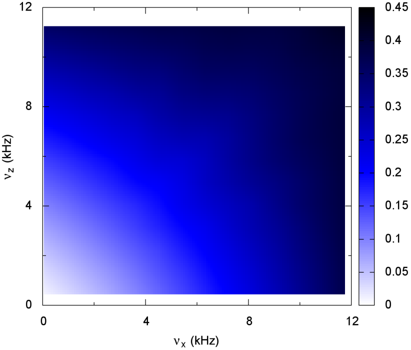
- $\tau_0$: Good agreement with theory vs $\nu_z$
- Response $\mathscr{R}_{max} = \frac{T_F \Delta N(max)}{N_{tot} \Delta T_0}$

# Nanostructuration with harmonic confinement

$T/T_F = 0.1$




$T/T_F = 0.3$





Hicks and Dresselhaus, Phys. Rev. B **47** 12727 (1993)

Hicks and Dresselhaus, Phys. Rev. B **47** 16631 (1993)

# *En route* for diffusive conduction

Unified description :

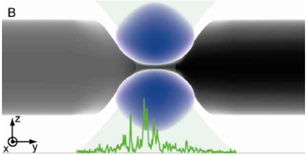
$$T(\varepsilon) = \frac{l(\varepsilon)}{\mathscr{L} + l(\varepsilon)}$$

$l(\varepsilon)$ : mean free path, $l(\varepsilon) = \tau_S v(\varepsilon)$

- **Low disorder** $l \gg \mathscr{L}$ : $T \to 1$ Ballistic transport
- **Strong disorder** $l \ll \mathscr{L}$ : $T \simeq \frac{l}{\mathscr{L}}$ Diffusive transport

On the experimental side :

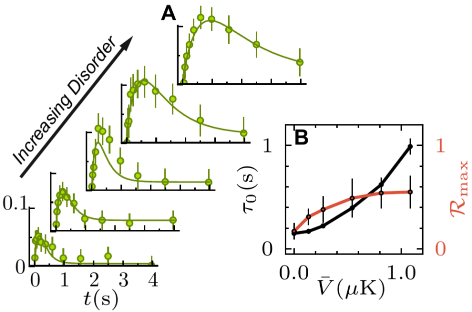
**Speckle potential**




Energy-independent $\tau_S \Rightarrow$ **Universal regime at strong speckle**

Seebeck coefficient is a ratio of conduction coefficients

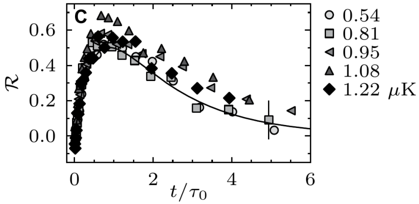
# Enhancing thermoelectric response



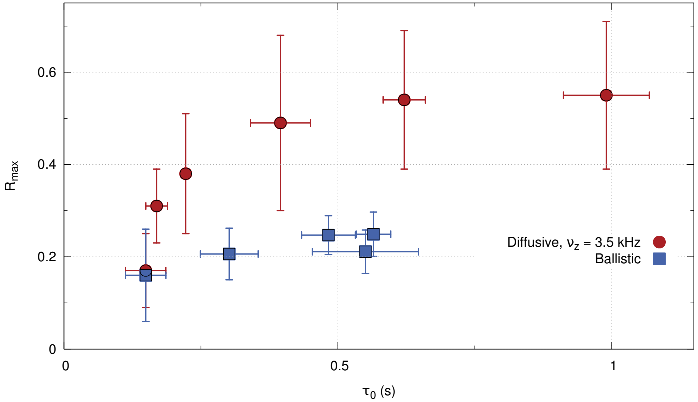

- Thermoelectric effect grows with disorder
- **At strong disorder**
  **The effect saturates : Constant $\tau_S$ ✓**
- Seebeck coefficient ≠ Conductivity

Rescaled evolution of particle imbalance :

**Universal regime**



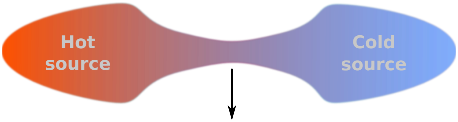
# Systematic comparison between ballistic and diffusive conduction

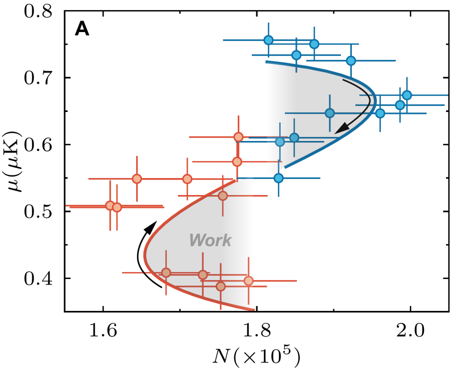



- Comparison of **ballistic** and **diffusive** channel
- Disorder more efficient than geometry
- Resistance ≠ thermopower

# Outline

# The setup as a heat engine




- Reservoirs ≡ Hot and Cold sources
- Channel : converts heat into (chemical) work

- Evolution in the $\mu - N$ plane
- Access to thermodynamic evolution
  ⇒ Extraction of work

**QUESTION** : Efficiency of the process ?

# Efficiency

**No DC regime** ⇒ compare work, not power
**Expression for the efficiency** : compare output chemical work to heat

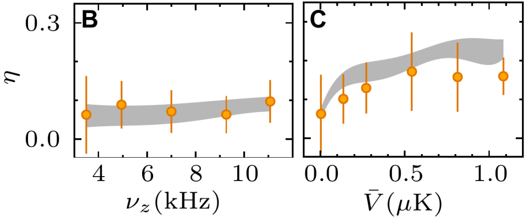
$$\eta \equiv \frac{Work}{Heat} = \frac{\int_{evolution} \Delta\mu \cdot d\Delta N}{\int_{evolution} \Delta T \cdot d\Delta S} = \frac{\int_0^\infty dt \Delta\mu \cdot I_N}{\int_0^\infty dt \Delta T \cdot I_S}$$

Solution to transport equations ⇒ $\eta$ **in terms of transport coefficients :** $\ell$, $L$, $\alpha$

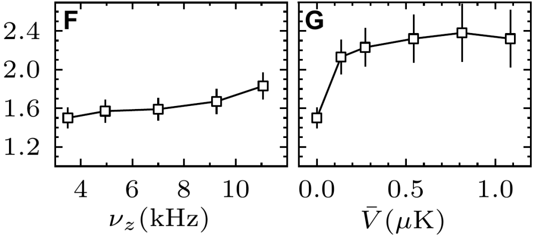
$$\boxed{\eta = \frac{-\alpha\alpha_r}{\ell + L + \alpha^2 - \alpha\alpha_r}}$$

i. Comparison to data ?
ii. Relation to channel properties ?
iii. Output power ?

# Results : Efficiency and ZT




- $\eta$ grows with confinement and speckle
- Slow dynamics ⇒ most efficient



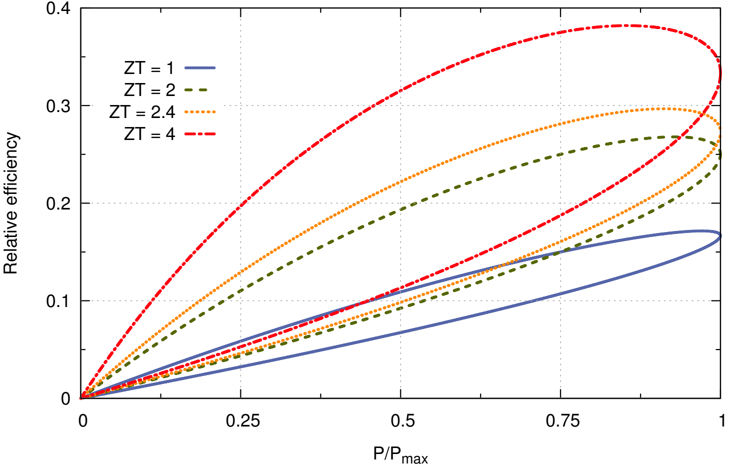

**For the channel only** :

Thermoelectric figure of merit $ZT = \frac{\alpha_{ch}^2}{L}$

$ZT \rightarrow 2.4$ : > than any material

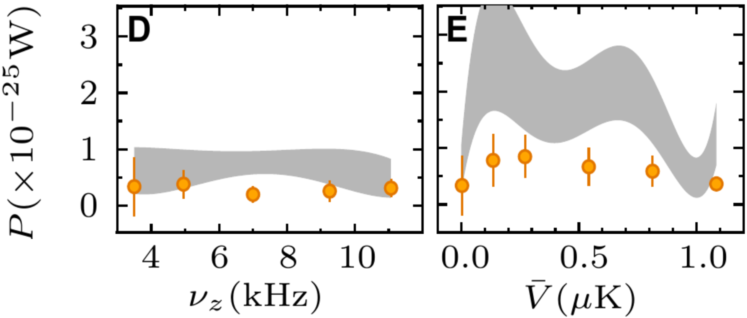
## Power and efficiency

For the channel only :




**Optimize power ≠ optimize efficiency**

# Results : Output Power




- Cycle averaged power $\frac{W}{\tau_0}$
- Optimize power $\neq$ Optimizing efficiency
- Strong confinement/speckle : **Slow dynamics $\Rightarrow$ Low power**

# Conclusions-Outlook

- Thermoelectricity with cold atoms !
- Transport : combination of reservoir and channel properties
- Control on the effect via geometry ($\simeq$ nanostructuration) or transport regime (ballistic -> diffusive)
- High figure of merit
- High-T transport without phonons

**What's next ?**

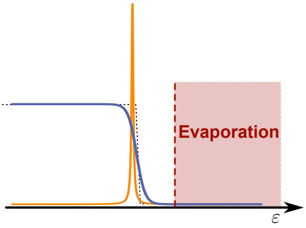
- Superfluid : Thermomechanical (fountain) effects
- Interactions : improvement of thermopower
- Lattice ?


J.P- Brantut, CG, J. Meineke, S. Krinner, D. Stadler, C. Kollath, T. Esslinger & A. Georges
**Science 342, 713-715 (2013)**

CG, C. Kollath & A. Georges **arxiv:1209.3942**

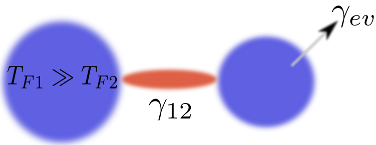
# Outlook - Cooling by transport : Peltier effect

**Peltier effect** ≡ injection of electron and holes around $\mu$ :
⇒ Rectification of the Fermi distribution

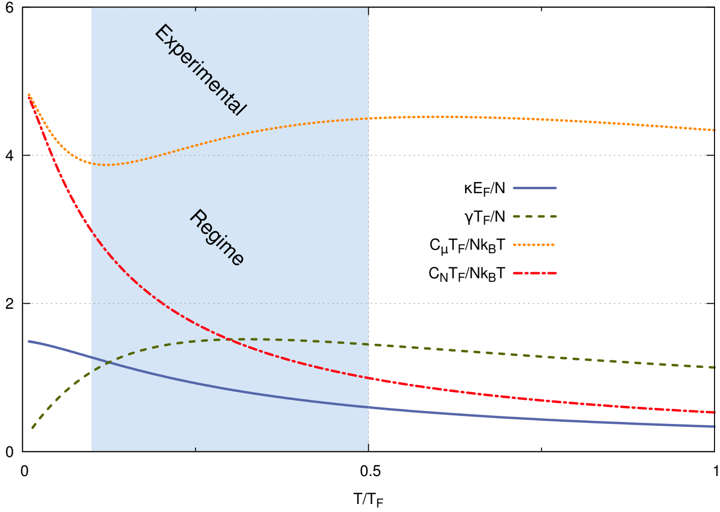



Here : Design transport properties

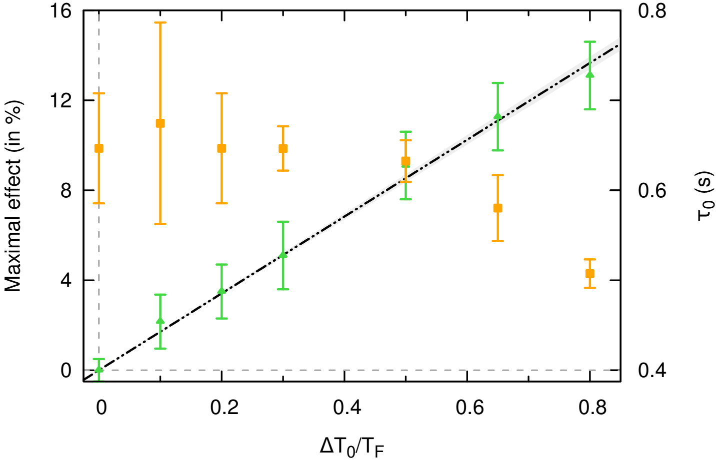
⇒ Injection in a chosen energy window
**Goal** : improve evaporative cooling



# Thermodynamic coefficients



# Linear response



# Lorentz et al