Three-terminal quantum-dot thermoelectrics

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Outline

Introduction
- Quantum dots and Coulomb blockade
- Quantum dots and Thermoelectrics
- Three-terminal thermoelectrics

Coulomb-coupled conductors
- Coulomb-blockade regime
- Chaotic cavities
- Resonant tunneling

Harvesting bosons
- Phonons
- Magnons
- Microwave photons

Summary
Introduction
Quantum dots

Confine electrons in all three spatial directions, quasi 0-dimensional “Artificial atom”

Characteristic energy scales

- Charging energy $E_{\text{ch}} = E_C(N - C_g V_g/e)^2$, $E_C = \frac{e^2}{2C}$
- Level quantization $\Delta \varepsilon$
- Tunnel couplings $\Gamma = 2\pi|t|^2\rho$
Coulomb blockade

- Charging energy: \( E_{\text{ch}} = \frac{e^2}{2C} (N - C_g V_g/e)^2 \)
- Charge fixed away from degeneracy points
- Transport only at degeneracy points
- Coulomb oscillations
Energy quantization and level scheme

- Finite level spacing in small quantum dots: $E_N = E_{ch} + \sum_{i=1}^{N} \epsilon_i$
- Addition energy: $\Delta = 2E_C + \Delta \epsilon$
- $\Delta \epsilon \ll k_B T, eV$: many levels involved $\rightarrow$ metallic island
- $\Delta \epsilon \gg k_B T, eV$: only single level $\rightarrow$ quantum dot

Cobden and Nygård, PRL 2002
Transport regimes

- Sequential tunneling
  - Tunneling of single electron
  - First order in $\Gamma$
  - Real occupation of the dot
  - Energy conservation
  - Dominant on resonance $k_B T \gg \Gamma, |\epsilon|$ or $eV \gg \Gamma, |\epsilon|$
Transport regimes

- Cotunneling
  - Tunneling of two electrons
  - Second order in $\Gamma$
  - Virtual occupation of the dot
  - Energy conservation only for total process
  - Dominant off resonance $|\varepsilon| \gg k_B T, eV, \Gamma$
Transport regimes

- Resonant tunneling
  - Many electrons tunnel
  - Nonperturbative in $\Gamma$
  - Complicated many-body effects (Kondo)
  - Dominant for strong coupling $\Gamma \gg k_B T, eV$
Anderson model

Single-level quantum dot

\[ H = \sum_{\sigma} \varepsilon n_\sigma + U n_\uparrow n_\downarrow + \sum_{r k \sigma} \varepsilon_k a_{r k \sigma}^\dagger a_{r k \sigma} + \sum_{r k \sigma} t_{r} a_{r k \sigma}^\dagger c_{\sigma} + \text{H.c.} \]

\[
\begin{align*}
|0\rangle & \quad |\uparrow\rangle & \quad |\downarrow\rangle & \quad |d\rangle \\
E = 0 & \quad E = \varepsilon & \quad E = \varepsilon & \quad E = 2\varepsilon + U
\end{align*}
\]

- Level position \( \varepsilon \) (tunable)
- Coulomb energy \( U \)
- Tunnel coupling \( \Gamma_r = 2\pi |t_r|^2 \rho \) (often tunable)
- Temperature \( T \) (tunable)
- Voltage \( V \) (tunable)
Master equation approach

- Probability $P_\chi$ to find quantum dot in state $\chi \in \{0, \uparrow, \downarrow, d\}$
- Occupation probabilities obey master equation

\[ \dot{P}_\chi = \sum_{\chi'} W_{\chi \chi'} P_{\chi'} \]

- Transition rates $W_{\chi \chi'}$ from Fermi’s golden rule

\[ W_{\uparrow 0} = \sum_r \Gamma_r f_r(\varepsilon) \]
\[ W_{\uparrow d} = \sum_r \Gamma_r [1 - f_r(\varepsilon + U)] \]

- Current $I = \sum_{\chi \chi'} W^I_{\chi \chi'} P_{\chi'}$
Experimental realizations

- 2-dimensional electron gas
- Metallic nanoparticles
- Self-assembled quantum dots

Lu et al., Nature 2003
Kuemmeth et al., Nano Letters 2008
Hamaya et al., APL 2007
Experimental realizations

- Carbon nanotubes
- Graphene
- Nanowires
Thermoelectrics

- 1823 Seebeck effect: Heat $\rightarrow$ current
- 1834 Peltier effect: Current $\rightarrow$ Cooling

Advantages of thermoelectrics:

- No moving parts
- Scalable to the nanoscale
- Heat is ubiquitous

Disadvantage

- Low efficiency, small power
Mesoscopic physics and thermoelectrics

Fundamental research

- Experiments on quantum point contacts Molenkamp et al. PRL 1990

Can mesoscopic systems be useful for thermoelectric applications?

- Quantum wires and wells for thermoelectrics Hicks and Dresselhaus PRB 1993
- Sharp spectral features increase thermoelectric performance Mahan and Sofo PNAS 1996
Three-terminal thermoelectrics

- Connection to Coulomb-drag setups
- Crossed heat and charge currents
- Separation of heat source and rectifier
- Energy harvesting

Sánchez, Büttiker, PRB 2011

McClure et al., PRL 2007
Coulomb-coupled conductors
Coulomb-blockade regime

- **Coulomb-coupled** quantum dots
- Exchange **energy** but **no particles**
- Conductor dot: two cold reservoirs
- Gate dot: single hot reservoir
Coulomb-blockade regime

- Drive current by temperature bias
- Energy-dependent, asymmetric tunnel barriers
- Optimal heat to charge current conversion
- One energy quantum of the bath transfers one charge quantum
Coulomb-blockade regime

- Power $P = IV$
- Efficiency $\eta = P/J_g$
- Device reaches Carnot efficiency $\eta_C$ at stopping voltage
- Efficiency at maximum power $\eta_{\text{max}P} = \eta_C/2$
Capacitively coupled chaotic cavities

Open quantum dots: Large number $N$ of transport channels

How do current and power vary with $N$?

Asymmetric, energy-dependent transmissions $T_r = T_r^0 - eT'_r \delta U$
Chaotic cavities

- Current \( I = \frac{\Lambda}{\tau_{RC}} k_B (\Theta_1 - \Theta_2) \)

- Asymmetry parameter \( \Lambda = \frac{G_L G'_R - G_R G'_L}{(G_L + G_R)^2} \) where \( G_r = \frac{e^2}{h} T_r^0 \) and \( G'_r = \frac{e^3}{h} T'_r \)

- RC time \( \tau_{RC} \) determined by effective conductance and capacitance of double cavity

- Current independent of channel number, \( I \sim 0.1 \text{nA} \)

- Power scales as \( 1/N \), similar to Coulomb-blockade for a few open channels \( P \sim 1 \text{fW} \)

- Efficiency scales as \( 1/N^2 \), few percent of \( \eta_C \)
Power versus conductance

- Coulomb blockade regime: Power grows linear with conductance
- Open contacts: Power drops as inverse conductance
- Maximal power should be achieved for single channel
- ⇒ Consider resonant tunneling through quantum dots
Resonant tunneling

- Central cavity in thermal equilibrium with hot reservoir
- Cavity connected to two cold electronic reservoirs via quantum dots
- Quantum dots host single resonant level with width $\gamma$ and energy $E_{L,R}$
Resonant tunneling

Scattering matrix approach

- Charge current: \( I_j = \frac{2e}{\hbar} \int dE \ T_j(E) [f_j(E) - f_C(E)] \)
- Energy current: \( J_j = \frac{2}{\hbar} \int dE \ E T_j(E) [f_j(E) - f_C(E)] \)
- Transmission: \( T_j(E) = \frac{\gamma^2}{(E - E_j)^2 + \gamma^2} \)
- Heat current \( J \) from hot reservoir
- Conservation of charge and energy

\[
0 = I_L + I_R \\
0 = J + J_L + J_R
\]
Numerically optimize $\Delta E$, $\gamma$ and $V$ for maximal power

Optimal values: $\Delta E \approx 6k_B T$, $\gamma \approx k_B T$

Maximal power $P_{\text{max}} \sim 0.4(k_B \Delta T)^2/h$, about 0.1 pW at $\Delta T = 1$ K

Efficiency at maximum power $\eta_{\text{maxP}} \sim 0.2\eta_C$
Resonant tunneling

Scaling

- Swiss cheese sandwich with self-assembled quantum dots
- Dot positions do not have to match
- Dot size of $100 \, \text{nm}^2$ yields $10 \, \text{W/cm}^2$ at $\Delta T = 10 \, \text{K}$
- Robust with respect to fluctuations of level positions
Harvesting bosons
Phonons

- Quantum dot coupled to electronic reservoirs and phonon bath
- Linear-response thermoelectrics
- Left-right and particle-hole symmetry broken: $\Gamma_L(E) \neq \Gamma_R(E)$
- Flux-dependence of response coefficients in Aharonov-Bohm geometry
Magnons

- Quantum dot coupled to ferromagnetic electrodes and ferromagnetic insulator
- Bridge between energy harvesting and spin caloritronics
- Drive pure spin current or spin-polarized charge current by magnons
Microwave photons

- Double quantum dots connected via superconducting cavity
- Combines circuit QED and thermoelectrics
- Separate hot and cold part by macroscopic distance
- Reduce leakage heat currents
Summary
Summary

- Three-terminal quantum-dot thermoelectrics
- Coulomb-coupled conductors
  - Coulomb blockade: High efficiency, small power
  - Open dots: Small efficiency, small power
  - Resonant tunneling: Large power, good efficiency
- Boson-driven heat engines
  - Phonons (hard to control)
  - Magnons: spintronics
  - Microwave photons: circuit QED