

*“Enseigner la recherche en train de se faire”*



*Chaire de  
Physique de la Matière Condensée*

**PETITS SYSTEMES THERMOELECTRIQUES:  
*CONDUCTEURS MESOSCOPIQUES  
ET GAZ D'ATOMES FROIDS***

Antoine Georges

Cycle « Thermoélectricité »  
2012 - 2014

# Séance du 18 décembre 2013

## Cours 6

### SUPERFLUIDS:

When entropy propagates as a wave  
(« First » and « second » sound  
and their possible coupling)

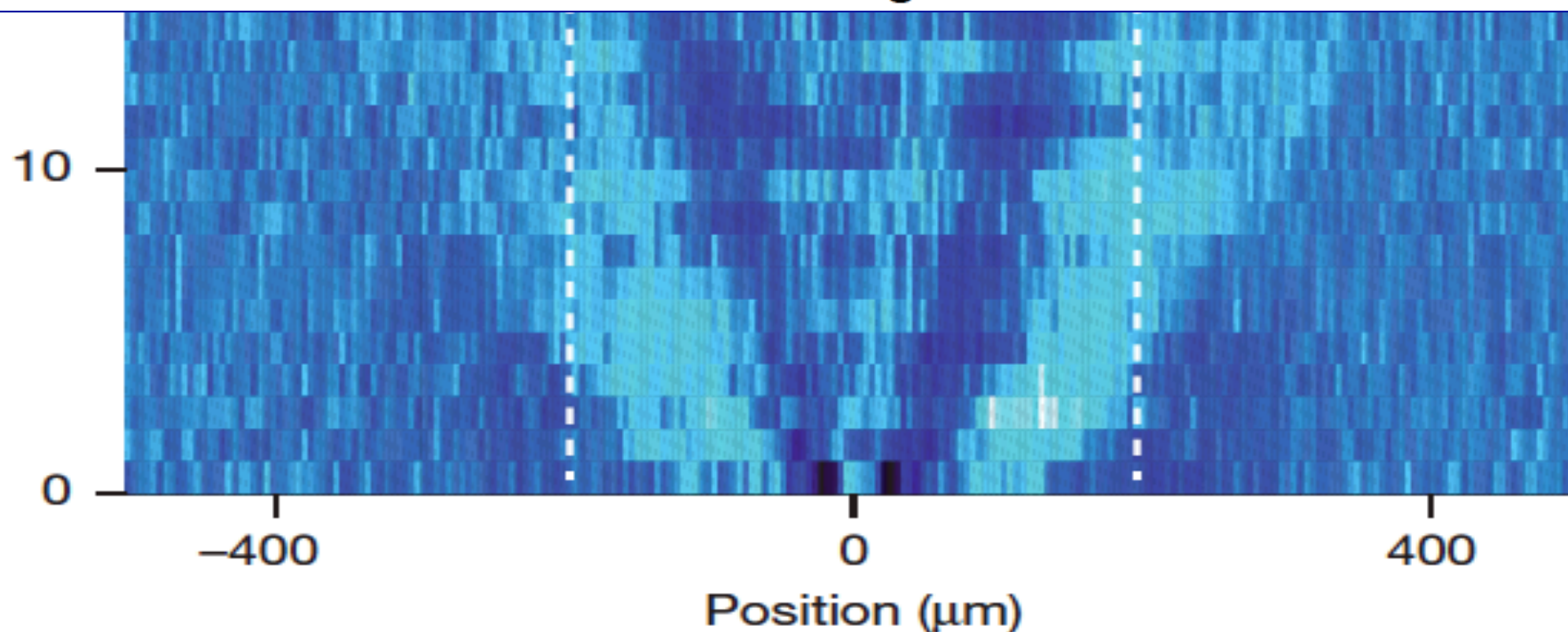
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# Séminaire - 18/12/12013

**Rudolf Grimm**

*(Institut für Experimental Physik, Universität Innsbruck)*

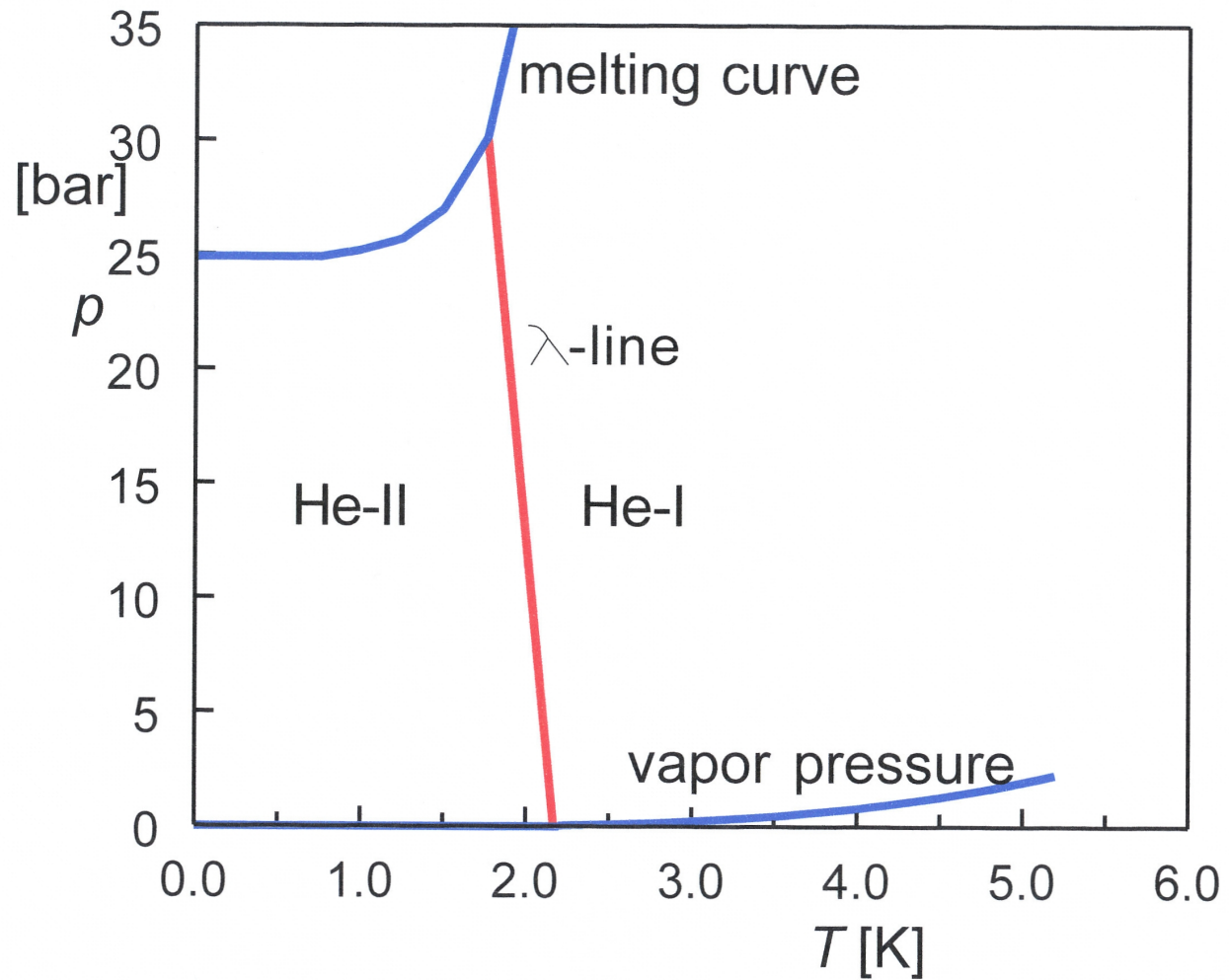
***Observation of second sound and more recent developments  
in ultracold Fermi gases***



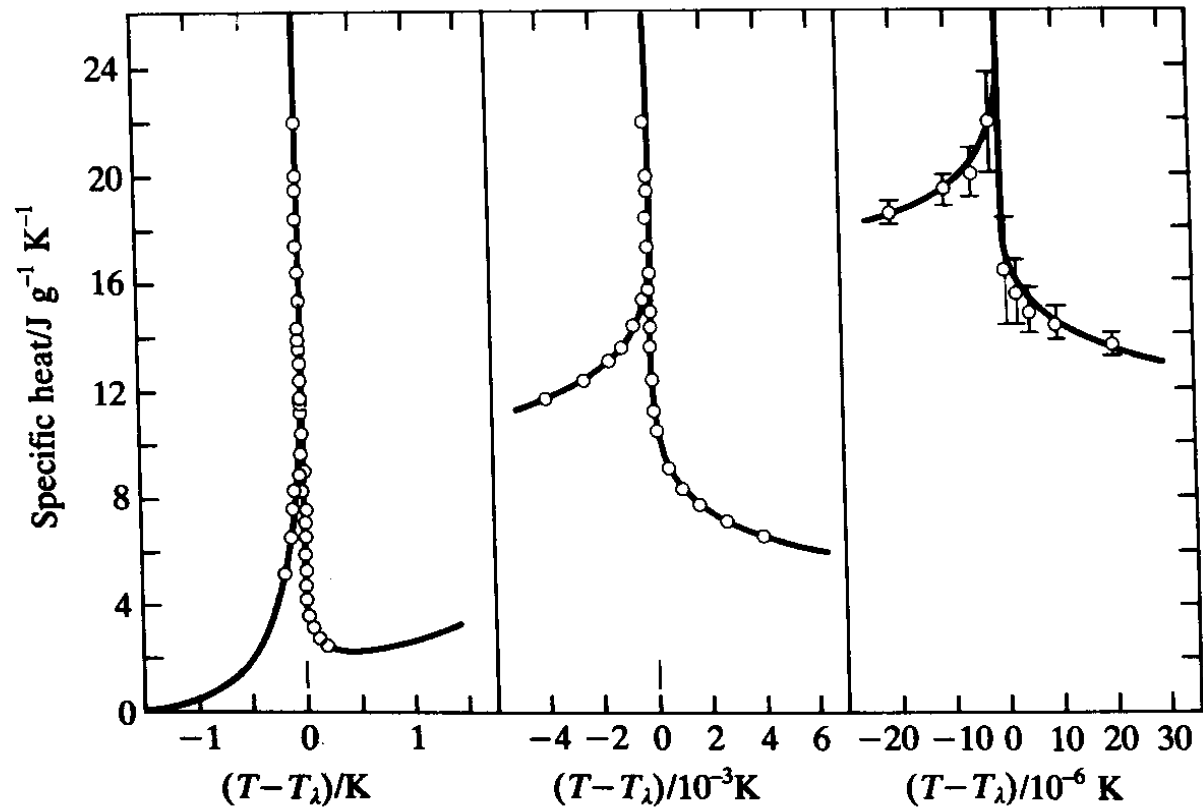
# SUPERFLUIDS

- For a long time, only known example: Helium 4 below 2.17 K (1937)
- Much later (1970's) superfluidity of the fermionic isotope Helium 3 ( $\sim 2.5$  mK)
- The discovery and study of Bose-Einstein condensation in ultra-cold atomic gases, and of superfluidity in two-component Fermi gases, have rejuvenated the field since the mid-1990's

# Helium 4: Some reminders



# The “lambda”-transition



**Fig. 15.3.** The specific heat of liquid  $^4\text{He}$  under the saturated vapour pressure as a function of  $T - T_\lambda$ . The width of the small vertical line just above the origin indicates the portion of the diagram shown expanded (in width) in the curve directly to the right (after Buckingham and Fairbank [193].)

# The Fascinating History of the Discovery of Superfluidity

- *Scientific discovery as a collective process*
- See:
- Sébastien Balibar:
- J. Low Temp Phys 146, 441 (2007)
- La pomme et l'atome (Odile Jacob, 2005)
- Slides on website @ENS
- Also: A.Griffin, J.Phys. Cond. Mat. 21, 164220 (2009)

# The experimental discovery of superfluidity ...1927-32

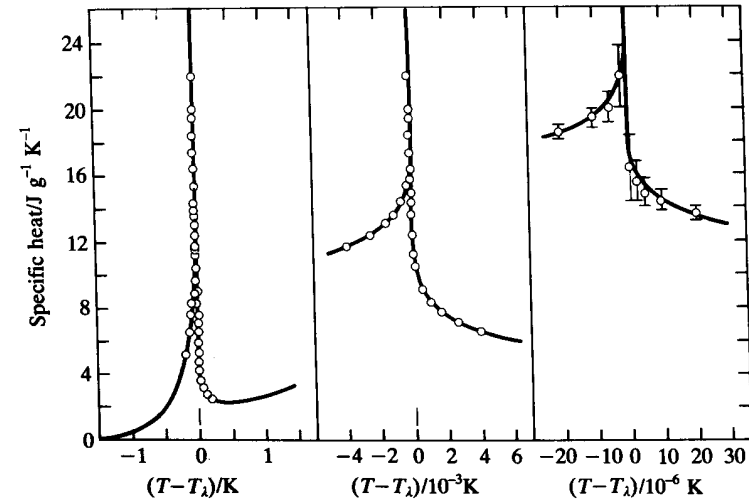
(slide: courtesy S.Balibar)

## Several steps :

1927-32: W.H. Keesom, M. Wolfke and K. Clusius (Leiden): liquid He has two different states which they call « helium I » above 2.2K and « helium II » below 2.2K. A singularity in the specific heat at 2.2 K (the « lambda point »)

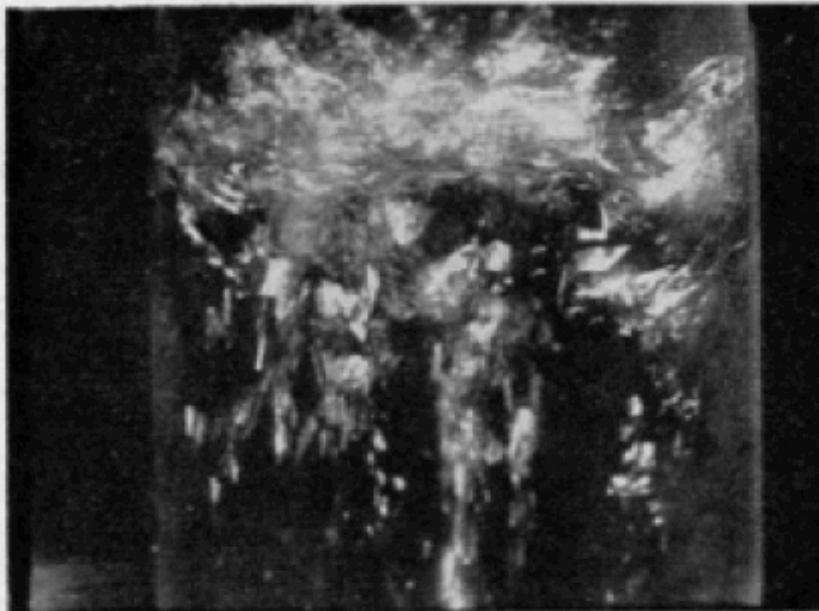
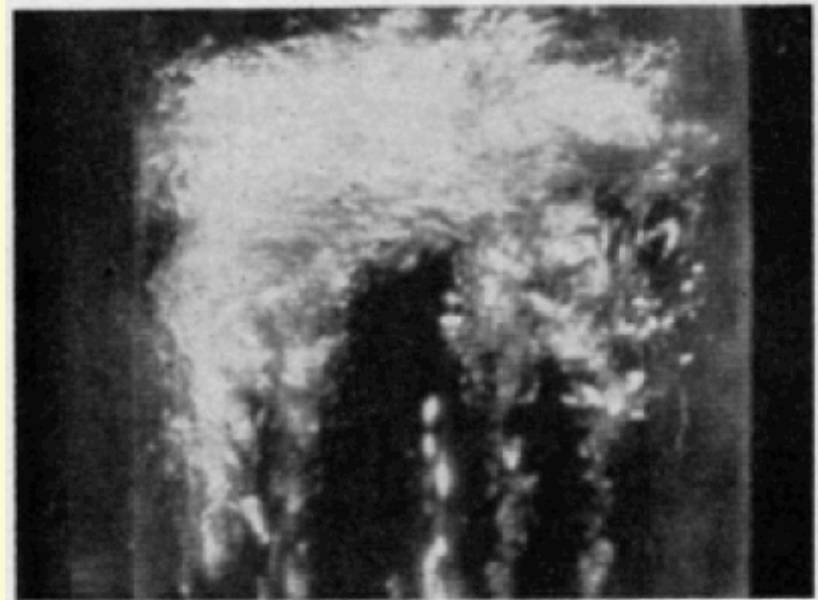
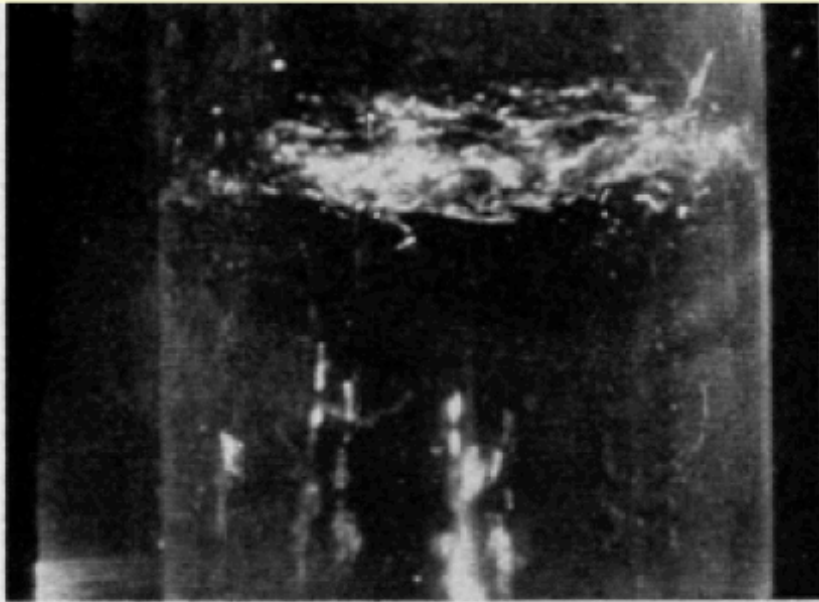
1930: W.H. Keesom and J.N. van der Ende (Leiden): liquid He flows very easily through narrow slits (superleaks)

1932: McLennan et al. (Toronto): liquid He stops boiling below 2.2K



**Fig. 15.3.** The specific heat of liquid <sup>4</sup>He under the saturated vapour pressure as a function of  $T - T_\lambda$ . The width of the small vertical line just above the origin indicates the portion of the diagram shown expanded (in width) in the curve directly to the right (after Buckingham and Fairbank [193].)





From Khalatnikov, Int.Sci.Tech, 1964

# The experimental discovery of superfluidity ...1935-37

(slide: courtesy S.Balibar)

*J.O. Wilhem, A.D. Misener and A.R. Clark (Toronto, 1935):  
the viscosity of liquid He drops down below 2.2 K*

*B.V. Rollin (Oxford, 1935)*

*W.H. Keesom and A. Keesom (Leiden, 1936)*

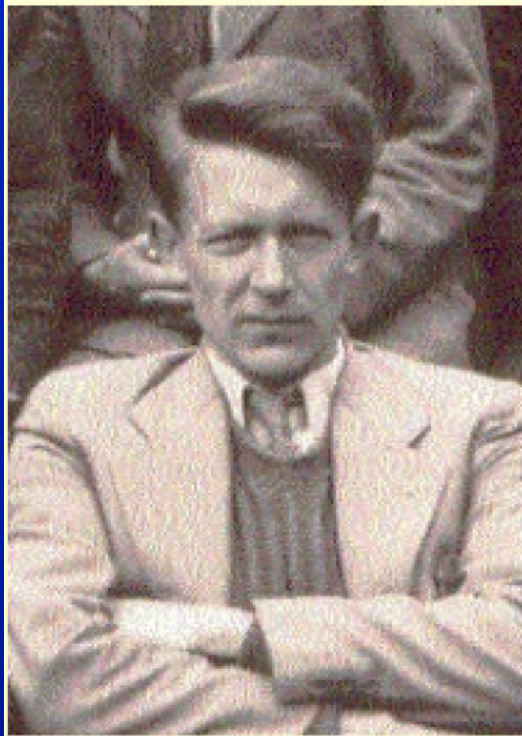
*J.F. Allen R. Peierls and M.Z. Uddin (Cambridge, 1937) :  
the thermal conductivity of liquid He increases below 2.2 K*

*December 1937: two articles side by side in Nature*

*(« THE DISCOVERY »)*

*P. Kapitza (Moscow)*

*J.F. Allen and A.D. Misener (Cambridge, UK)*



Pictures from:  
A.Griffin's article (op. cit.)

# 1938: Annus Mirabilis, Annus Horribilis

- Dec 3, 1937: Letter by Kapitza sent to Nature
- Dec 22, 1937: Letter by Allen and Misener sent to Nature
- Nature 141, 243 (1938): Allen and Jones report the fountain effect (see lecture 5, Dec 10)
- Nature 141, 643 (1938): Fritz London proposes that superfluidity is related to BEC
- Nature 141, 913 (1938) and Comptes Rendus Académie des Sciences 207; 1035, 1186 (1938): Laszlo Tisza proposes 2-fluid model

# Fritz London and Laszlo Tisza find shelter in Paris

(Institut Henri Poincaré  
and Collège de France)

Paul Langevin, Jean Perrin, Frédéric Joliot,  
Edmond Bauer take action to host scientists  
fleeing nazism, fascism and antisemitism

Finally I wish to thank M. Paul Langevin who has kindly accepted me in his laboratory.

I thank M. E. Bauer very much for the constant interest he has shown in my work and for his many critical remarks.

I thank MM. F. London and F. Simon for the fruitful discussions I have had with them.

I thank the « French Committee for the Reception and Organization of Foreign Scientists » which has helped me to carry out my research in Paris.

Manuscript received on 23 October 1939.

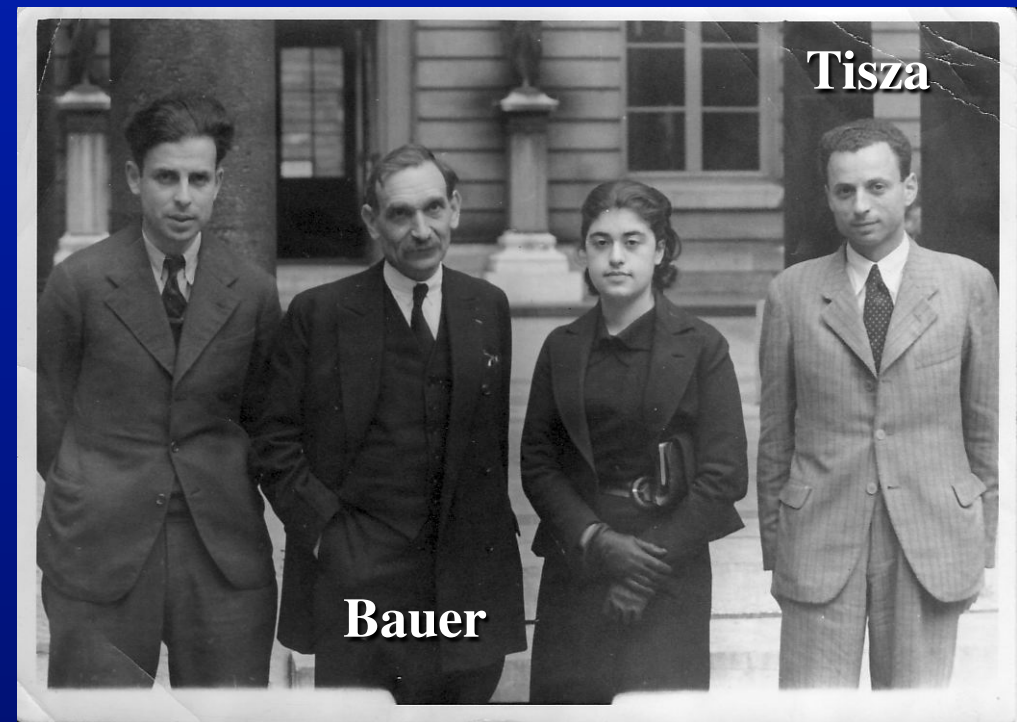
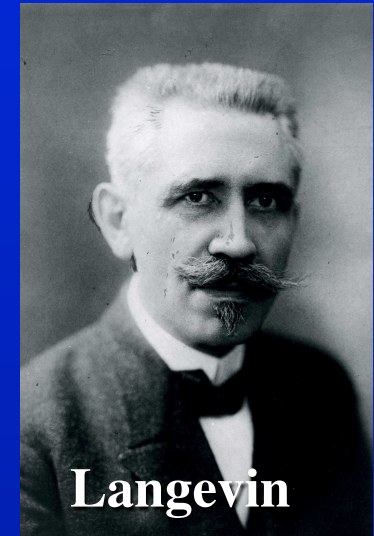
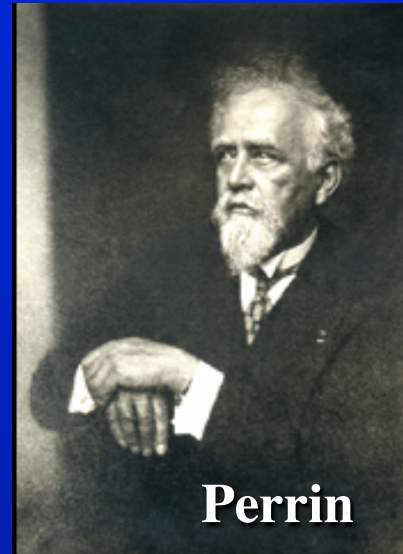
Laszlo Tisza, Journal de Physique et Le Radium, 1, 164 et 350 (1939)  
In these two articles as well as Nature 141, 913 (1938), Laszlo Tisza sets the basis of the two-fluid description !

# Tisza meets London in Paris

*1937 : a group of French intellectuals (Paul Langevin, Jean Perrin (Nobel 1926, and secretary of Research in the Front Populaire), Frederic Joliot-Curie (Nobel 1935) and Edmond Bauer welcome foreign scientists escaping from antisemitism in Eastern Europe*

*This is where Tisza (Collège de France) meets Fritz London (Institut Henri Poincaré, 500 m distance) who had recommended him to Langevin*

*Slide: courtesy S.Balibar*

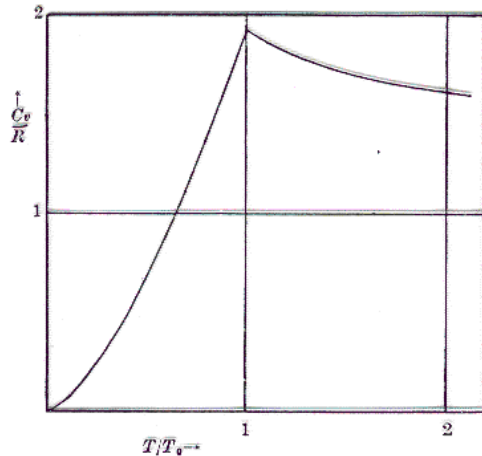




5 mars 1938, Institut Henri Poincaré :

Fritz London:  
Bose-Einstein Condensation  
may explain superfluidity !

Landau apparently always rejected this idea...  
But it is now clearly validated  
by the discoveries on cold gases



SPECIFIC HEAT OF AN IDEAL BOSE-EINSTEIN GAS.

(4) Though actually the  $\lambda$ -point of helium resembles rather a phase transition of second order, it seems difficult not to imagine a connexion with the condensation phenomenon of the Bose-Einstein statistics. The experimental values of the temperature of the  $\lambda$ -point ( $2.19^\circ$ ) and of its entropy ( $\sim 0.8 R$ ) seem to be in favour of this conception. On the other hand, it is obvious that a model which is so far away from reality that it simplifies liquid helium to an ideal gas, cannot, for high temperatures, yield but the value  $C_p = 3/2 R$ , and also for low temperatures the ideal Bose-Einstein gas must, of course, give too great a specific heat, since it does not account for

the effective mass  $m^*$  being of the order of magnitude of the mass of the atoms. But in the present case we are obliged to apply Bose-Einstein statistics instead of Fermi statistics.

(3) In his well-known papers, Einstein has already discussed a peculiar condensation phenomenon of the 'Bose-Einstein' gas; but in the course of time the degeneracy of the Bose-Einstein gas has rather got the reputation of having only a purely imaginary existence. Thus it is perhaps not generally known that this condensation phenomenon actually represents a discontinuity of the derivative of the specific heat (phase transition of third order). In the accompanying figure the specific heat ( $C_p$ ) of an ideal Bose-Einstein gas is represented as a function of  $T/T_0$ , where

$$T_0 = \frac{h^2}{2\pi m^* k} \left( \frac{n}{2,615} \right)^{2/3}.$$

With  $m^*$  = the mass of a He atom and with the mol. volume  $\frac{N_l}{n} = 27.6 \text{ cm.}^3$  one obtains  $T_0 = 3.09^\circ$ . For  $T < T_0$  the specific heat is given by

expected to furnish quantitative insight into the properties of liquid helium.

The conception here proposed might also throw a light on the peculiar transport phenomena observed with He II (enormous conductivity of heat<sup>5</sup>, extremely small viscosity<sup>6</sup> and also the strange fountain phenomenon recently discovered by Allen and Jones<sup>7</sup>).

A detailed discussion of these questions will be published in the *Journal de Physique*.

F. LONDON.

Institut Henri Poincaré,  
Paris.  
March 5.

<sup>1</sup> Fröhlich, H., *Physica*, 4, 639 (1937).

<sup>2</sup> Allen, J. F., and Jones, H., *NATURE*, 141, 243 (1938).

<sup>3</sup> Simon, F., *NATURE*, 133, 529 (1934).

<sup>4</sup> London, F., *Proc. Roy. Soc., A*, 153, 576 (1936).

<sup>5</sup> Rollin, *Physica*, 2, 557 (1935); Keesom, W. H., and Keesom, H. P., *Physica*, 3, 359 (1936); Allen, J. F., Peierls, R., and Zaki Uddin, M., *NATURE*, 140, 62 (1937).

<sup>6</sup> Burton, E. F., *NATURE*, 135, 265 (1935); Kapitza, P., *NATURE*, 141, 74 (1938); Allen, J. F. and Misener, A. D., *NATURE*, 141, 75 (1938).

Slide: courtesy S. Balibar

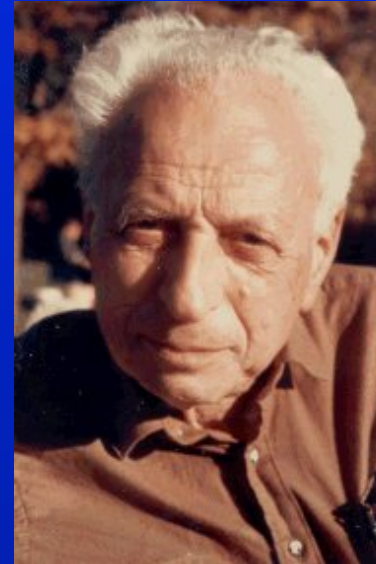


## ***Laszlo Tisza 1938 : le « modèle à deux fluides »***

A detailed discussion of the problem will be given in the *Journal de Physique*. I am greatly indebted to Dr. F. London for the opportunity of seeing his paper before publication.

L. TISZA.

Laboratoire de Physique Expérimentale,  
Collège de France,  
Paris. April 16.



Slide:  
courtesy  
S.Balibar

***deux fluides: le condensat et les atomes non-condensés***

***le condensat est à  $T=0$  , ne transporte pas d'entropie et ne peut participer à la dissipation (viscosité nulle)***

***les atomes non-condensés constituent un « fluide normal » qui transporte de l'entropie et peut échanger de l'énergie (viscosité non-nulle)***

***il existe deux champs de vitesse indépendants:  $v_s$  et  $v_n$***

***la température détermine le rapport entre les densités des deux fluides  
la dissipation dépend de la géométrie de l'expérience***

***si le superfluide seul s'écoule (à travers un poreux),  $T$  diminue  
un gradient de  $T$  produit un effet thermomécanique inverse, un écoulement du superfluide vers la région chaude (effet fontaine)***

# Walking together...

*L. Tisza (ENS-Paris, June 14, 2001 + e-mail to SB Sept. 4, 2001):*

*January 1938. On a Sunday we took a walk in the Bois de Verrières*

*The novelty of the effect became strikingly apparent in the Allen-Jones*

*fountain effect that started London and myself on our speculative spree... He jumped on me the BEC. I was at once delighted...*

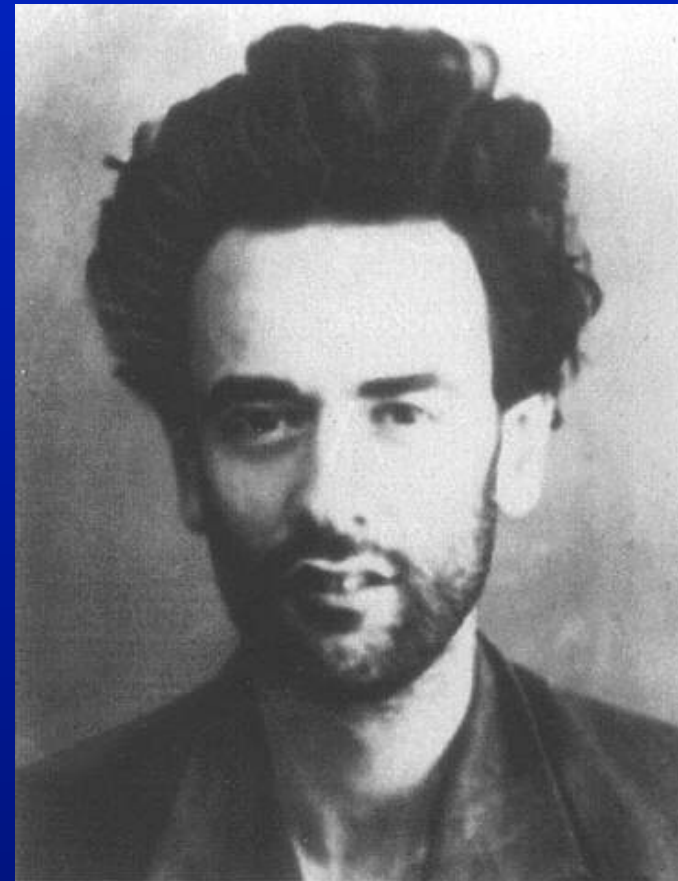
*There followed a sleepless night and by morning a rough outline of two fluid idea was in place. My idée fixe was that no value of viscosity however small could reconcile the capillary flow experiments with those on oscillating disks of Toronto... There had to be two fluids and to my mind this became evident in the fountain effect. One of the fluids had to be superfluid, the other viscous...*

*The next morning, I proudly reported my contribution to Fritz. He was outraged. I assigned to the two Bose Einstein components their own velocity field. Here London demurred...*



*Slide:  
courtesy  
S. Balibar*

Kapitza rescues Landau out of Stalin's jails  
(where he spent ~ 1 year from mid-1938 till mid-1939...)  
Landau builds his famous theory of superfluidity (1941)



Kapitza, Landau, ca. 1938

# Bose-Einstein Condensation: Long-Range Order and Condensate Wave-Function

(Bogoliubov 1947→, Penrose and Onsager, 1951,1956)

One-body density matrix:

$$n_1(\mathbf{r}, \mathbf{r}') \equiv \langle \Psi^\dagger(\mathbf{r}) \Psi(\mathbf{r}') \rangle$$

(Off-Diagonal) Long-Range Order: As  $\mathbf{r}$  and  $\mathbf{r}'$  are separated,  $n_1$  tends to a non-zero value " $\langle \psi \rangle$ "

$$n(\mathbf{p}) = N_0 \delta(\mathbf{p}) + \bar{n}(\mathbf{p})$$

Condensed fraction:  $n_0 \equiv \frac{N_0}{N}$

Eigenfunctions of 1-body density matrix:

$$\hat{n}_1 |\phi_\alpha\rangle = n_\alpha |\phi_\alpha\rangle$$

$$\hat{\Psi}(\mathbf{r}) = \phi_0(\mathbf{r})a_0 + \sum_{\alpha \neq 0} \phi_\alpha(\mathbf{r})a_\alpha \simeq \Psi_0(\mathbf{r}) + \delta\hat{\Psi}$$

Wave-function of the condensate (macroscopic part of field operator):

$$\Psi_0(\mathbf{r}) = \sqrt{N_0} \phi_0(\mathbf{r})$$

Amplitude and Phase:

$$\Psi_0(\mathbf{r}) = |\Psi_0(\mathbf{r})| e^{i\Phi(\mathbf{r})}$$

Evolution in the Heisenberg representation  $\rightarrow$  chemical potential

$$\Psi_0(\mathbf{r}, t) = \Psi_0(\mathbf{r}, 0) e^{-i\mu t/\hbar}$$

# The gradient and time-derivative of the phase

From the Heisenberg equation of motions, one can deduce a relation between the condensate wave-function in the frame where the fluid is at rest and the laboratory frame where the superfluid component has velocity  $v_s$ :

$$\Psi_{lab}(\mathbf{r}, t) = |\Psi_0(\mathbf{r} - \mathbf{v}_s t)| e^{i\Phi(\mathbf{r}, t)}$$

$$\Phi(\mathbf{r}, t) = \frac{1}{\hbar} \left[ m \mathbf{v}_s \cdot \mathbf{r} - \left( \frac{1}{2} m v_s^2 + \mu \right) t \right]$$

Irrotational  
Flow  
 $\Phi$ : potential

$$\mathbf{v}_s = \frac{\hbar}{m} \nabla \Phi, \quad \hbar \frac{\partial \Phi}{\partial t} = -\mu - \frac{1}{2} m v_s^2$$

First (~Josephson) relation also follows  
from superfluid current

$$\frac{\hbar}{m} \text{Im} \Psi_0^* \nabla \Psi_0$$

# The Two Sounds of Superfluids

SUR LA THÉORIE DES LIQUIDES QUANTIQUES. APPLICATION A L'HÉLIUM LIQUIDE <sup>(1)</sup>. II

Par L. TISZA.

Laboratoire de physique expérimentale du Collège de France.

**Sommaire.** — La théorie du liquide de Bose-Einstein est étendue aux cas où les deux « phases » (définies dans I) ont des vitesses différentes. En général, ces courants se compensent en ce qui concerne le flux de matière, mais ils sont toujours accompagnés d'un flux de chaleur (effet thermomécanique). Quant à la transition des atomes d'une phase à l'autre, elle comporte un échange de chaleur avec l'entourage. Ce mécanisme peut rendre compte de la supraconductibilité thermique observée dans l'He II. La théorie prévoit sous certaines conditions une vitesse de propagation des inégalités de température (ondes de température). La compensation du flux de matière n'est pas complète au voisinage des parois solides. *Cet effet thermomécanique proprement dit* permet de comprendre les phénomènes capillaires observés dans l'He II (le phénomène de la fontaine d'hélium, l'écoulement non hydrodynamique, etc.).

Laszlo Tisza, Journal de Physique et Le Radium, 1, 164 et 350 (1939)

5° Les différences de température à pression constante provoquent des courants opposés des deux phases, accompagnés d'un transport important de chaleur, mais se compensant en général quant au transport de matière. (Effet thermomécanique.) La théorie prévoit sous certaines conditions la propagation d'ondes de température. Une transition entre les deux phases n'a lieu en premier approximation qu'aux lieux d'échange de chaleur avec l'entourage.



# The two-fluid model and its hydrodynamics

(Tisza 1938-39, Landau 1941)

Assumption: the motion of the fluid can be described as the coupled motion of two components: a superfluid component with velocity  $v_s$  and a 'normal' component with velocity  $v_n$

In the following:  $\rho_s$  and  $\rho_n$  designate respectively the mass density of the superfluid and normal component ( $\rho_s$  not to be confused with the condensed fraction  $n_0$ )

Total mass density and mass current (= momentum density)

$$\rho = \rho_n + \rho_s \quad , \quad m\mathbf{j} = \rho_s \mathbf{v}_s + \rho_n \mathbf{v}_n$$

## Coupled equations for mass density and pressure

$$\frac{\partial \rho}{\partial t} + \text{div}(mj) = 0 \quad \text{Conservation of particle number}$$

$$m \frac{\partial j}{\partial t} = -\nabla p \quad \text{Equation of motion under pressure force}$$

$$\frac{\partial^2 \rho}{\partial t^2} = \nabla^2 p$$

Together with equation of state  $p=p(\rho)$  this leads to usual sound-waves in a conventional fluid

## Coupled equations for entropy and temperature

All irreversible processes and dissipation are assumed to be negligible here (viscosity of normal part = 0). We assume that the hydrodynamic regime applies, and linearize all equations.

$$\frac{\partial s}{\partial t} + \text{div}(s_V v_n) = 0$$

Conservation of entropy ( $s_V$ : per unit volume)  
The superfluid component does not carry any entropy !

Change variable to entropy per unit mass  $s=s_V/\rho$  in the following.  
After simple algebra:

$$\frac{\partial s}{\partial t} + \frac{s\rho_s}{\rho} \text{div}(v_n - v_s) = 0$$

We see here that the RELATIVE velocity between superfluid and normal components appears.

Using the dynamical equations for the phase (above):

$$\mathbf{v}_s = \frac{\hbar}{m} \nabla \Phi, \quad \hbar \frac{\partial \Phi}{\partial t} = -\mu - \frac{1}{2} m v_s^2$$
$$\rightarrow m \frac{\partial v_s}{\partial t} + m (v_s \cdot \nabla) v_s \simeq m \frac{\partial v_s}{\partial t} = -\nabla \mu$$

Thermodynamics allows to relate the gradient of chemical potential to that of pressure and temperature:

$$\frac{\rho}{m} \nabla \mu = -s \nabla T + \nabla p$$

So that one obtains the second equation of motion as:

$$\rho_n \frac{\partial}{\partial t} (v_n - v_s) + \rho s \nabla T = 0$$

**Temperature gradient controls the RELATIVE flow !**

5° Les différences de température à pression constante provoquent des courants opposés des deux phases, accompagnés d'un transport important de chaleur, mais se compensant en général quant au transport de matière. (Effet thermomécanique.)

Finally, combining:

$$\frac{\partial s}{\partial t} + \frac{s\rho_s}{\rho} \operatorname{div}(v_n - v_s) = 0$$
$$\rho_n \frac{\partial}{\partial t}(v_n - v_s) + \rho s \nabla T = 0$$

One obtains the second evolution equation linking entropy (per unit mass) and temperature:

$$\frac{\partial^2 s}{\partial t^2} = \frac{\rho_s}{\rho_n} s^2 \nabla^2 T$$

In summary:

$$\frac{\partial^2 \rho}{\partial t^2} = \nabla^2 p \quad , \quad \frac{\partial^2 s}{\partial t^2} = \frac{s^2 \rho_s}{\rho_n} \nabla^2 T$$

These two equations have to be supplemented by the thermodynamic equation of state, leading to wave-like solutions

$$\text{A priori, } \rho = \rho(p, T) \quad , \quad s = s(p, T)$$

So that the (pressure, temperature) and (density, entropy) variations are coupled: these waves are a priori not pure pressure and temperature waves.

In practice however, this depends on the superfluid we consider

## Matrix of thermodynamic coefficients:

$$\begin{pmatrix} \delta\rho \\ \delta s \end{pmatrix} = \begin{pmatrix} \left. \frac{\partial\rho}{\partial p} \right|_T & \left. \frac{\partial\rho}{\partial T} \right|_p \\ \left. \frac{\partial s}{\partial p} \right|_T & \left. \frac{\partial s}{\partial T} \right|_p \end{pmatrix} \begin{pmatrix} \delta p \\ \delta T \end{pmatrix} \equiv M \begin{pmatrix} \delta p \\ \delta T \end{pmatrix}$$

## Wave-like solutions:

Also define:

$$N \equiv \begin{pmatrix} 1 & 0 \\ 0 & \frac{s^2 \rho_s}{\rho_n} \end{pmatrix}$$

Look for wave-like solutions of the form  $e^{i(kx-\omega t)} = e^{-i\omega(t-x/c)}$ :

$$\det [c^2 M - N] = 0$$

$$c^4 \left[ \left. \frac{\partial\rho}{\partial p} \right|_T \left. \frac{\partial s}{\partial T} \right|_p - \left. \frac{\partial s}{\partial p} \right|_T \left. \frac{\partial\rho}{\partial T} \right|_p \right] - c^2 \left[ \left. \frac{\partial s}{\partial T} \right|_p + \frac{s^2 \rho_s}{\rho_n} \left. \frac{\partial\rho}{\partial p} \right|_T \right] + \frac{s^2 \rho_s}{\rho_n} = 0$$

Using thermodynamic identities:

$$c^4 - \left[ \left. \frac{\partial p}{\partial\rho} \right|_s + \frac{kT \rho_s s^2}{\rho_n c_v} \right] c^2 + \frac{kT \rho_s s^2}{\rho_n c_v} \left. \frac{\partial p}{\partial\rho} \right|_T = 0$$

## More on the thermodynamics of the reservoirs: including the non-diagonal terms

$$\begin{pmatrix} \Delta N \\ \Delta S \end{pmatrix} = \underline{K} \begin{pmatrix} \Delta \mu \\ \Delta T \end{pmatrix} = \begin{pmatrix} \kappa & \kappa \alpha_r \\ \kappa \alpha_r & C_\mu / T \end{pmatrix} \begin{pmatrix} \Delta \mu \\ \Delta T \end{pmatrix}$$

$$K_{11} \equiv \kappa = \left. \frac{\partial N}{\partial \mu} \right|_T = - \frac{\partial^2 \Omega}{\partial \mu^2}$$

$$K_{12} = K_{21} \equiv \alpha_r \kappa = \left. \frac{\partial N}{\partial T} \right|_\mu = - \frac{\partial^2 \Omega}{\partial \mu \partial T} = \left. \frac{\partial S}{\partial \mu} \right|_T$$

$$K_{22} \equiv \frac{C_\mu}{T} = \left. \frac{\partial S}{\partial T} \right|_\mu = - \frac{\partial^2 \Omega}{\partial T^2}$$

Grand-potential:  $\Omega \equiv -k_B T \ln Z_{gc}$  ,  $S = - \left. \frac{\partial \Omega}{\partial T} \right|_\mu$  ,  $N = - \left. \frac{\partial \Omega}{\partial \mu} \right|_T$



## (Almost-) uncoupled case: Helium

dilatation coefficient  $\left. \frac{\partial s}{\partial p} \right|_T \simeq 0$ :

$$\left[ c^2 \left. \frac{\partial \rho}{\partial p} \right|_T - 1 \right] \left[ c^2 \left. \frac{\partial s}{\partial T} \right|_p - \frac{s^2 \rho_s}{\rho_n} \right] = 0$$

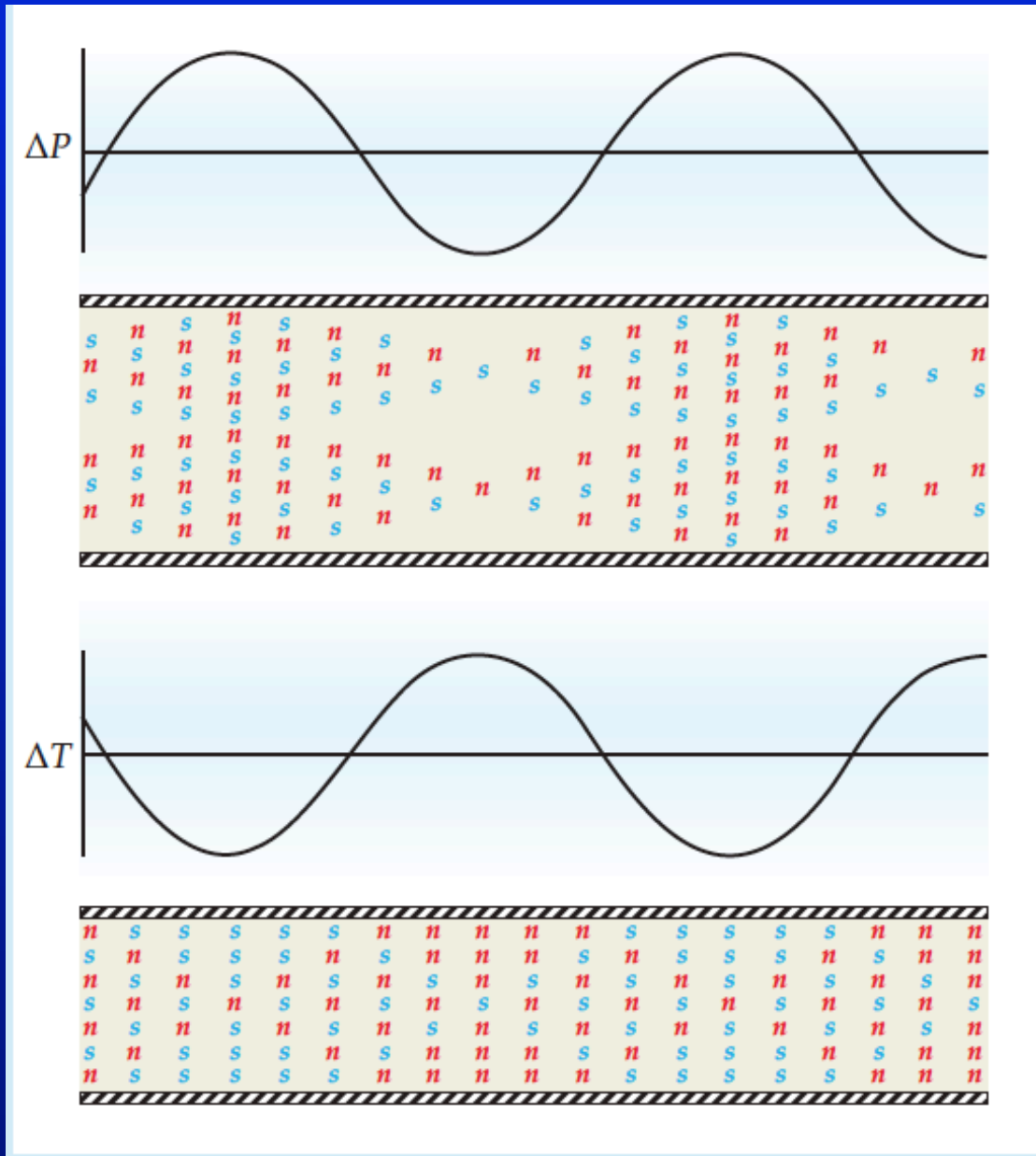
$$c_1^2 = \left. \frac{\partial p}{\partial \rho} \right|_T \simeq \left. \frac{\partial p}{\partial \rho} \right|_s \propto \frac{1}{\kappa}, \quad c_2^2 = \frac{s^2 \rho_s}{\rho_n} \frac{T}{c_p}$$

Contains superfluid fraction !

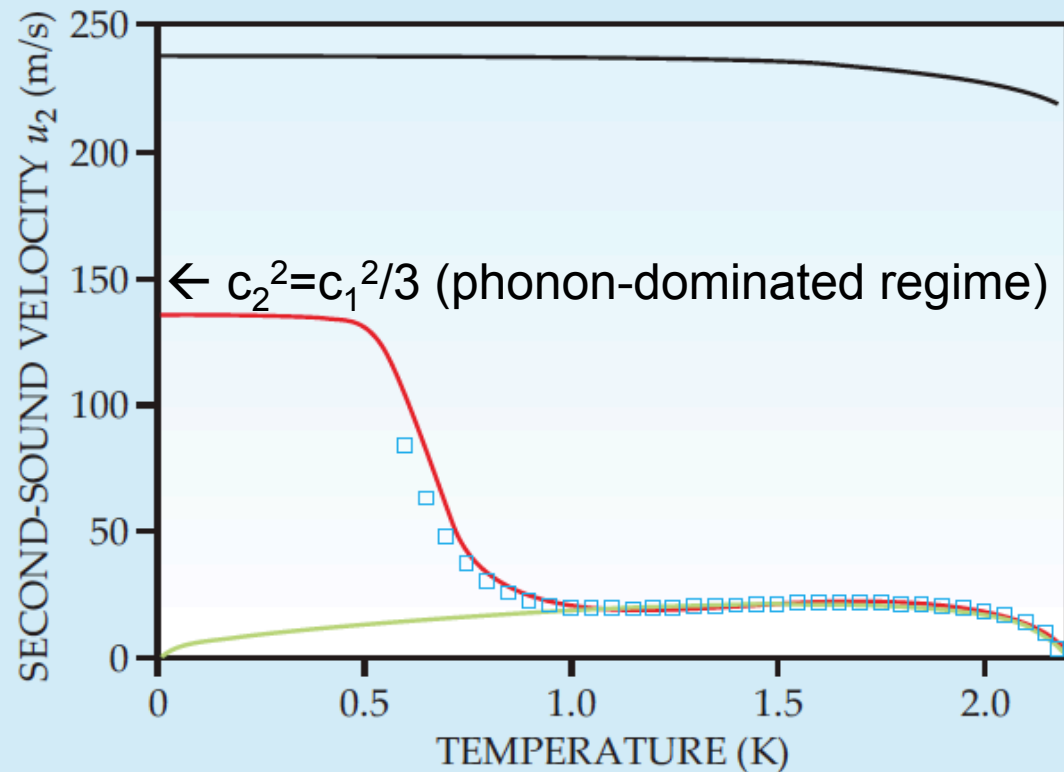
In this case:

- 'First sound': conventional pressure/density wave
- Second sound: pure temperature/entropy wave

Relative motion ( $v_n - v_s$ ) of superfluid vs. normal component, out of phase

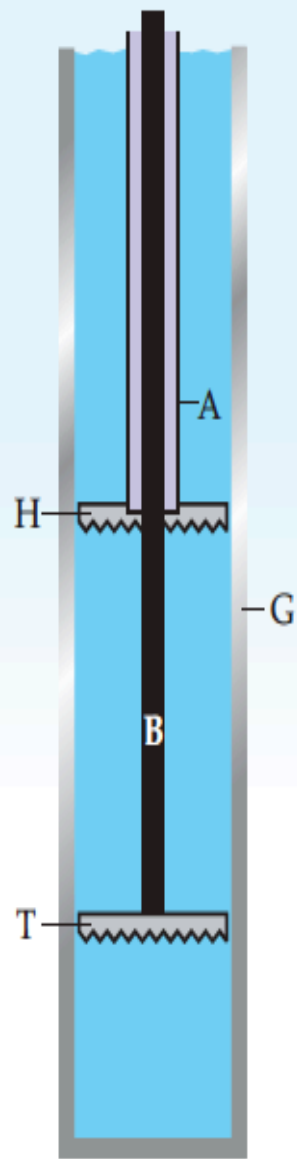


Donnelly, Physics Today, 10/2009

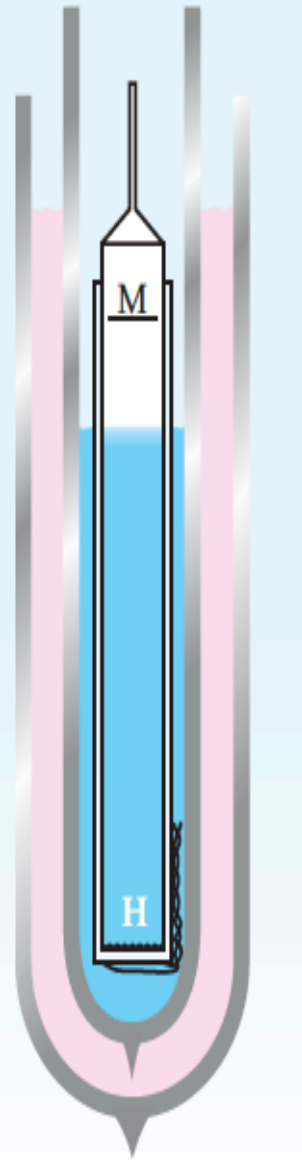


**Figure 2. The velocity of second sound** as computed by Evgeny Lifshitz (red curve) and by Laszlo Tisza (green curve); for comparison, the upper, black curve shows the velocity of first sound. The blue points are today's accepted values of second-sound velocity. Box 2 describes why Tisza's formula did not work.

a

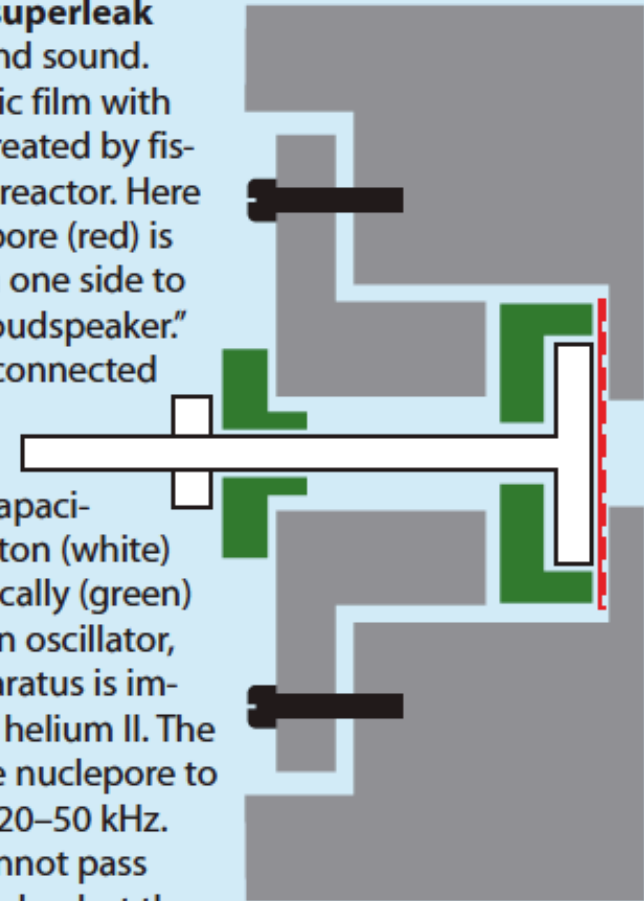


b



### Figure 4. Modern superleak transducer for second sound.

Nuclepore is a plastic film with micron-size holes created by fission fragments in a reactor. Here a thin film of nuclepore (red) is plated with gold on one side to make a capacitor "loudspeaker." The frame (gray) is connected to a bias voltage and connected to ground by a large capacitor, the rod and button (white) are insulated electrically (green) and connected to an oscillator, and the whole apparatus is immersed in a bath of helium II. The oscillator causes the nuclepore to vibrate, typically at 20–50 kHz. The normal fluid cannot pass through the film's holes, but the superfluid can, and the transducer can act as a generator or detector for second sound.



tuned to 2 kHz and its signal rectified. As the liquid helium bath evaporated, the free surface fell and resonant peaks appeared from the microphone output, from which the velocity of second sound could be deduced. (Adapted from ref. 7.)

# Dilute Bose Gases

(dilatation not negligible  $\rightarrow$  coupling)

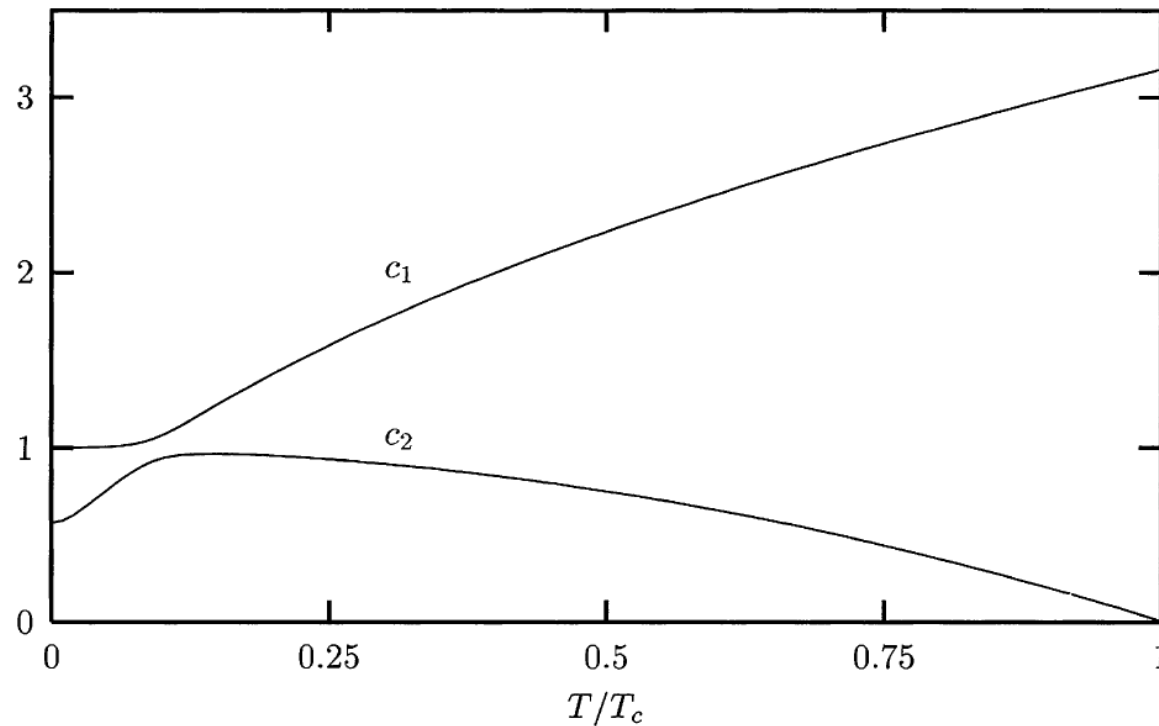


FIG. 6.1. Schematic representation of the velocity of first and second sound in a dilute Bose gas as a function of temperature. The figure points out the effect of hybridization between the two modes taking place at low temperature. Above  $T_c$  only the first sound survives

From: Pitaevskii and Stringari, Bose-Einstein Condensation (Oxford)

# Second-sound in solids

(Suggestion: Ward and Wilks, 1951)

Schematically:

$$c_v \frac{\partial T}{\partial t} + \text{div} j_Q = 0 \quad , \quad \frac{\partial}{\partial t} j_Q + \frac{1}{\tau} j_Q + \frac{c^2}{3} c_v \nabla T \simeq 0$$

When all anharmonic phonon coupling and scattering can be neglected: wave-like propagation of temperature fluctuation (rather than Fourier diffusive law)

For an early review, see:

Ackerman and Guyer, *Annals of Physics* 50, 128 (1968)

Observation in solid He:

Ackerman et al. *Phys Rev Lett* 16, 789 (1966)

c

# Séminaire - 18/12/12013

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**Observation of second sound and more recent developments  
in ultracold Fermi gases**

