

Quantum Condensed Matter Dynamics



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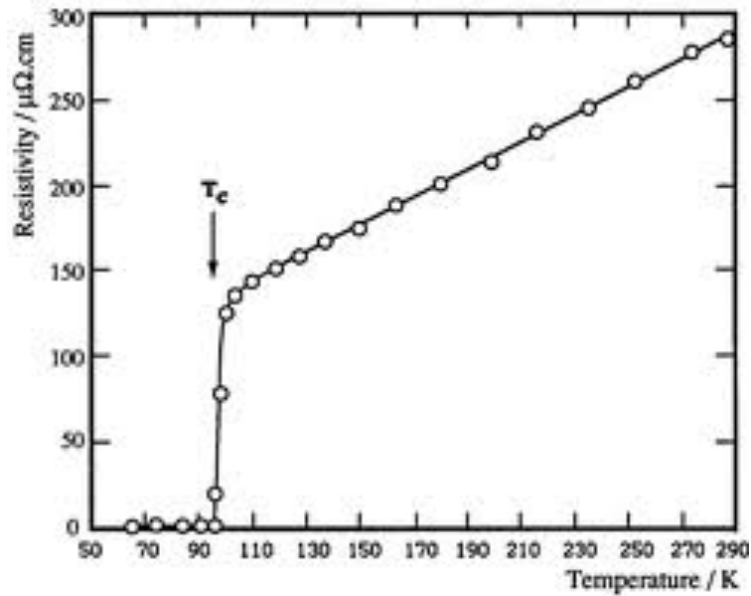
Lecture 2: Josephson Plasmonics

Quantum Materials are those solids which exhibit **macroscopic behavior** that cannot be understood, even qualitatively, without **quantum mechanics**.

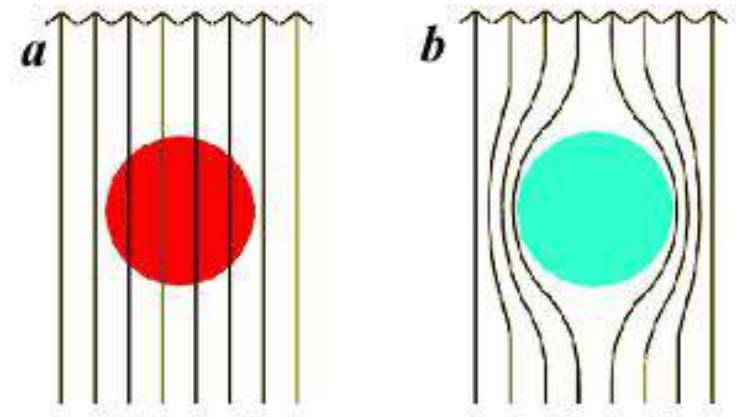
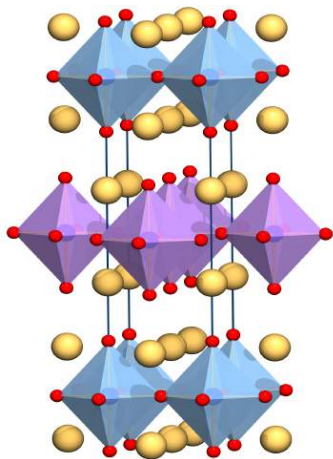
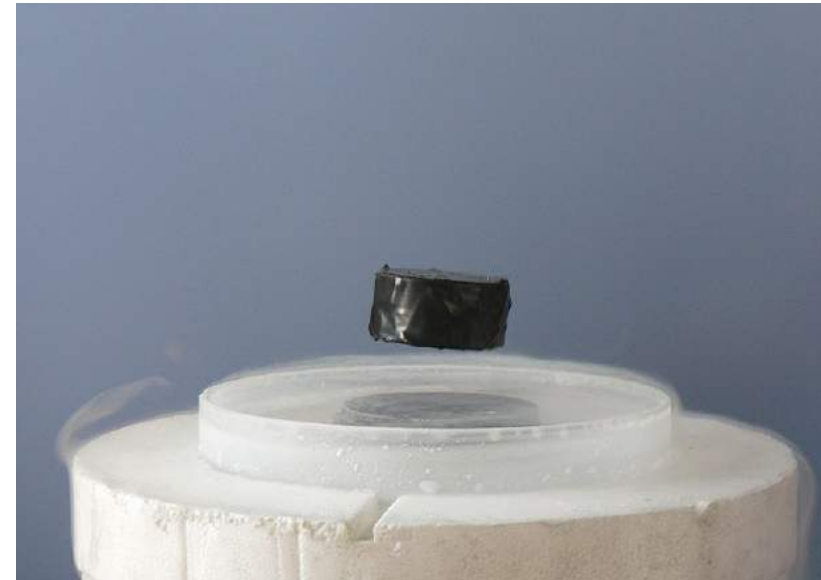
A prominent example of a Quantum Material is a **High T_c Superconductor**

High T_c Superconductivity

Zero DC resistance



Meissner effect



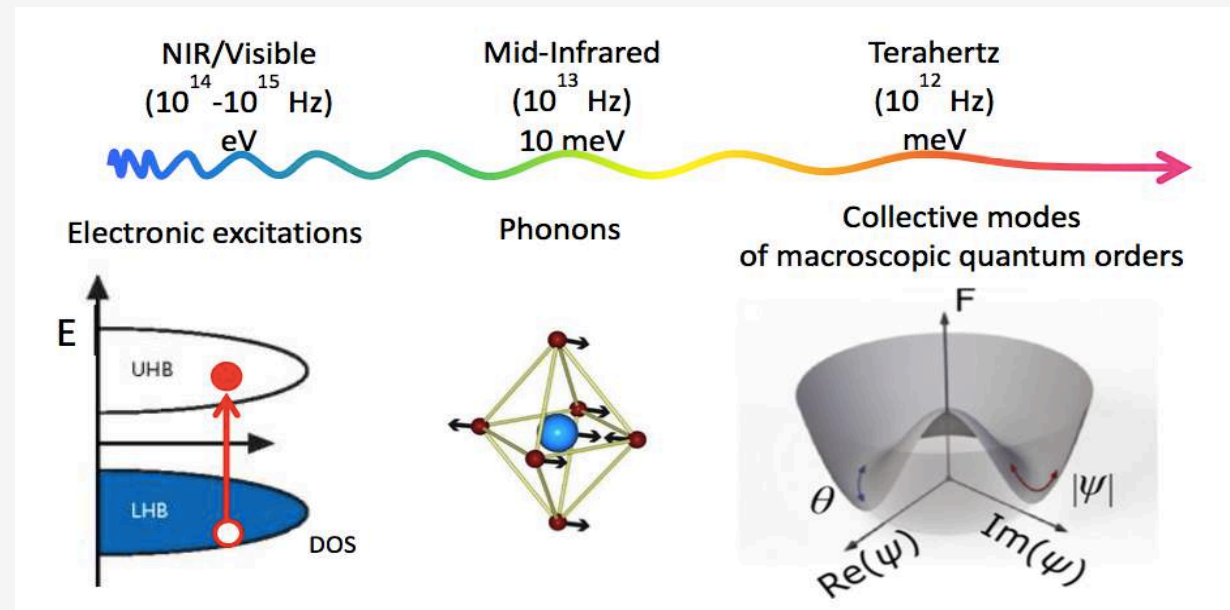
We study the **nonlinear excitation** of **collective modes** of quantum materials – generally using light

Goal 1: **reveal** and **create** functionalities that are **hidden** or **not present** at equilibrium.

Goal 2: Control **high speed** phenomena - explore new strategies for **device applications**.

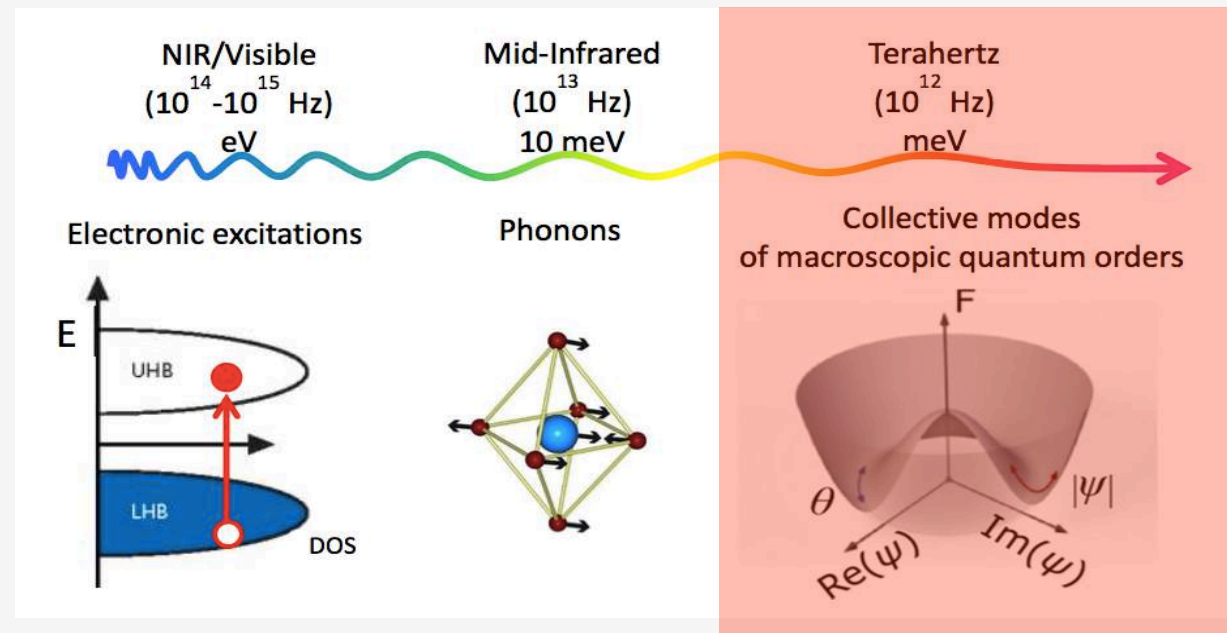
Driving frequency

We want to excite collective modes of quantum materials **beyond** their **linear response**

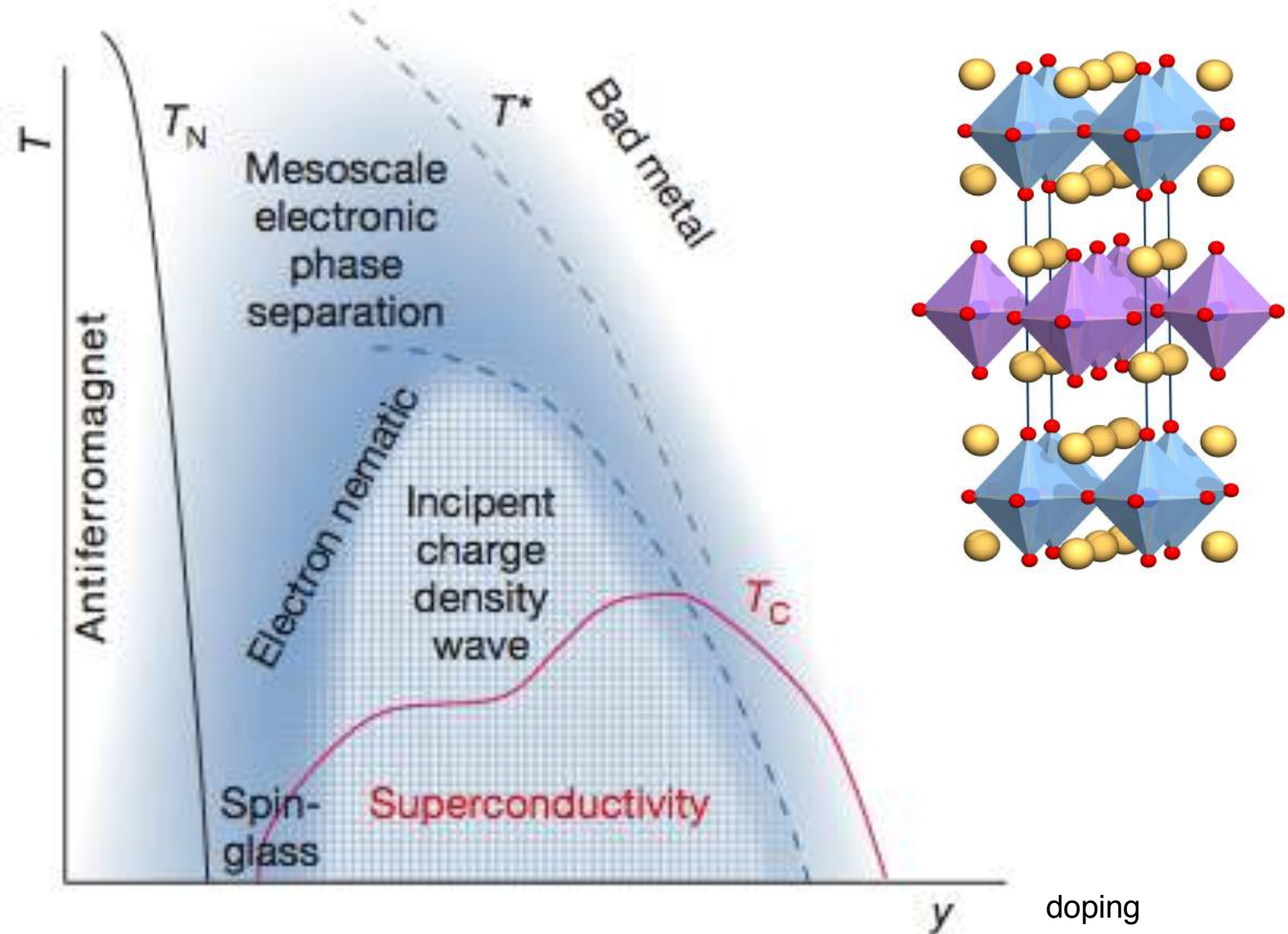


Driving frequency

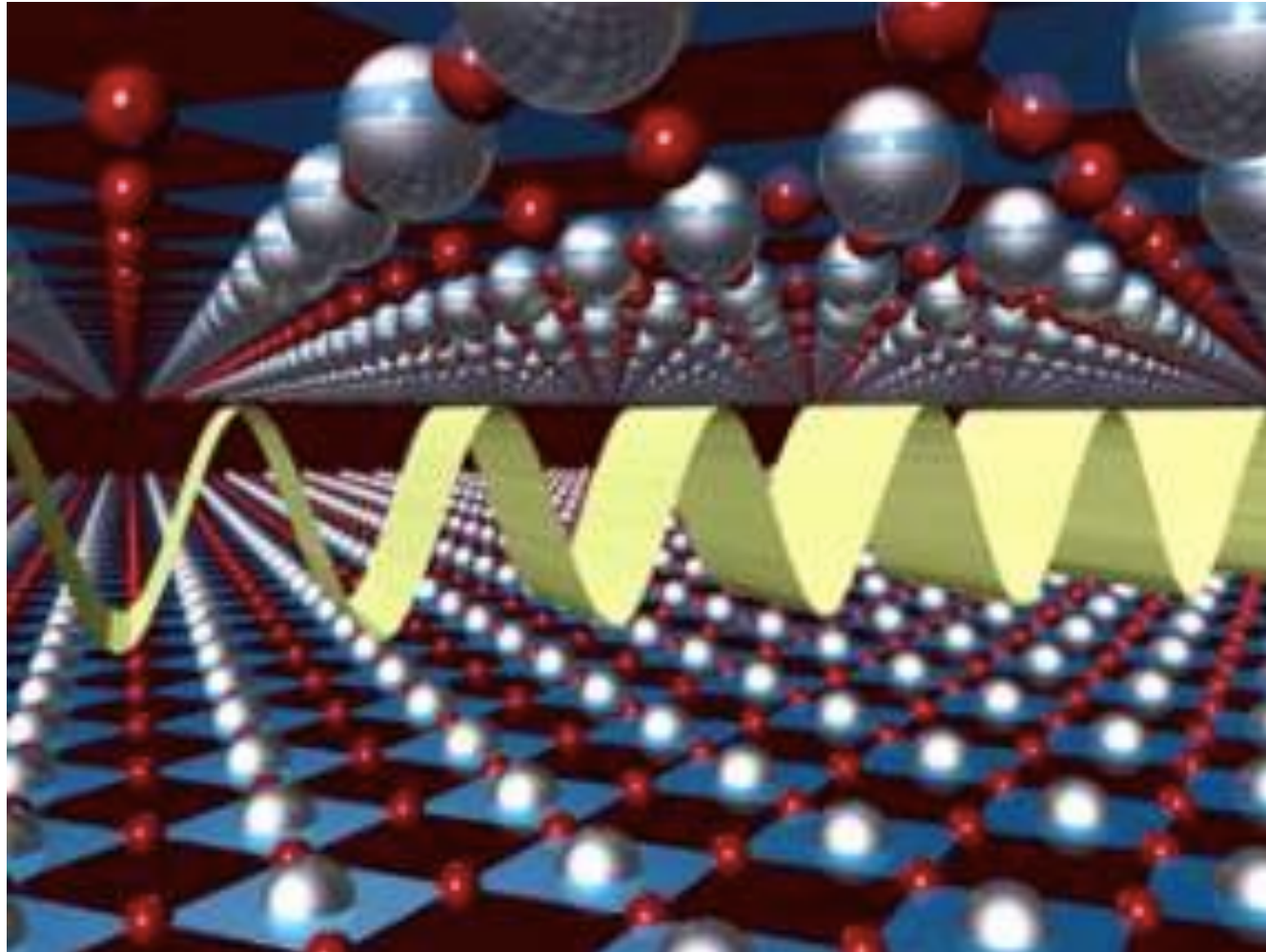
We want to excite collective modes of quantum materials **beyond** their **linear response**



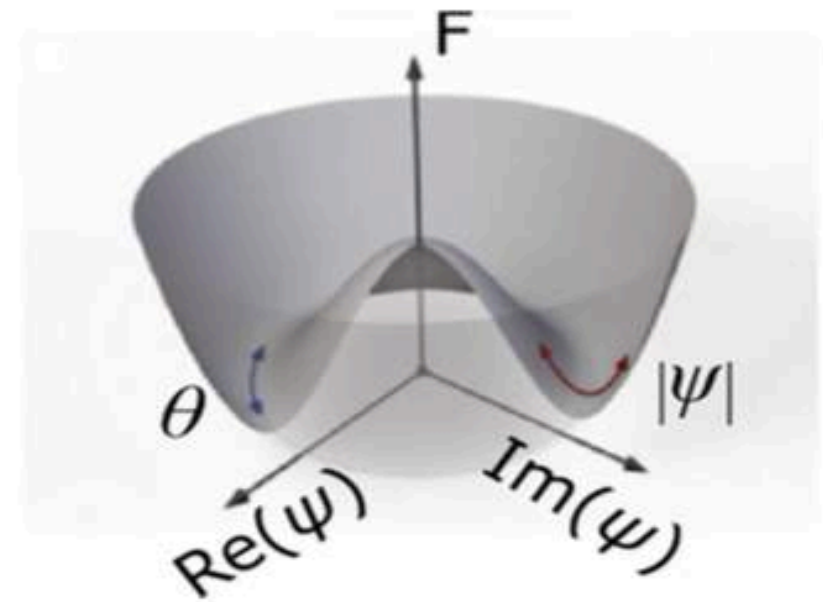
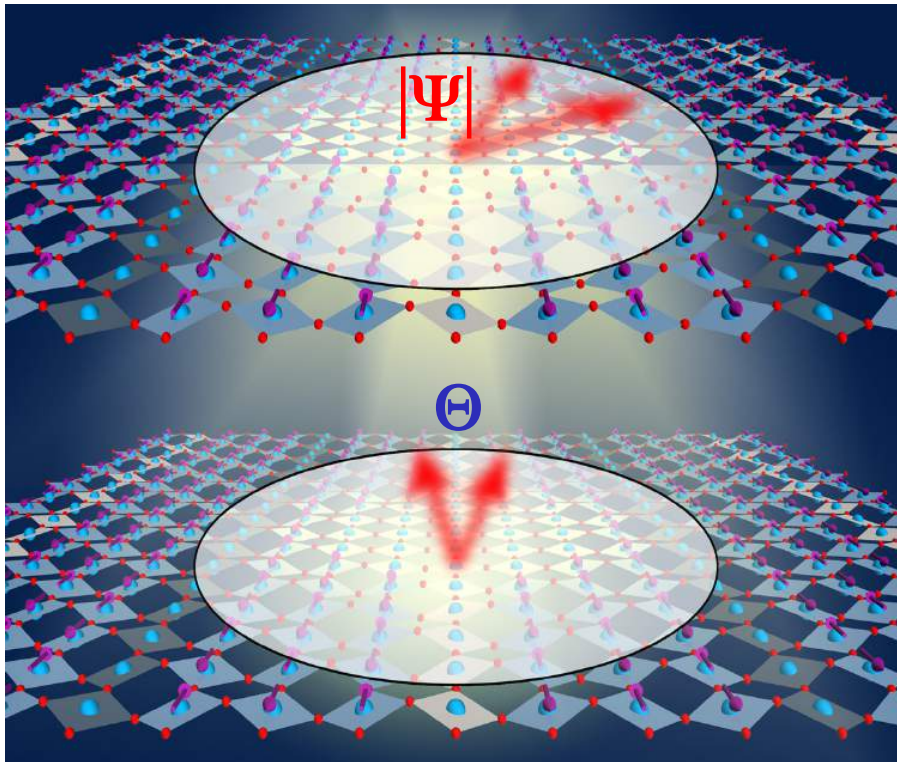
Cuprate superconductors: complexity at work



Superconducting Plasma Waves

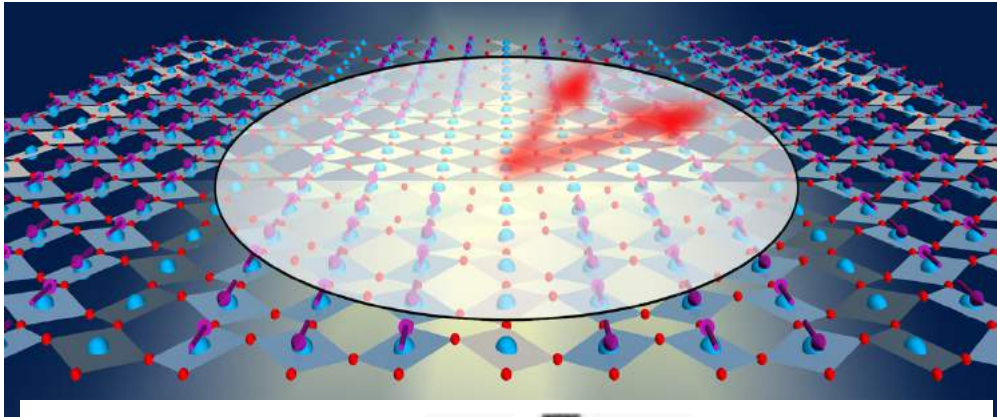


Superconducting order in cuprates

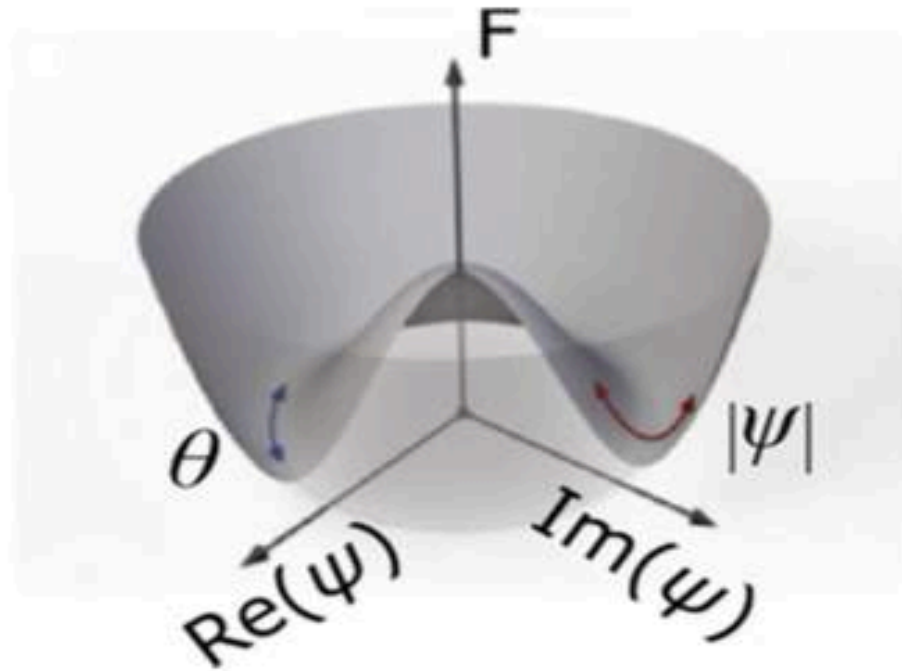


What do I see if $E(\omega)$ is polarized the plane

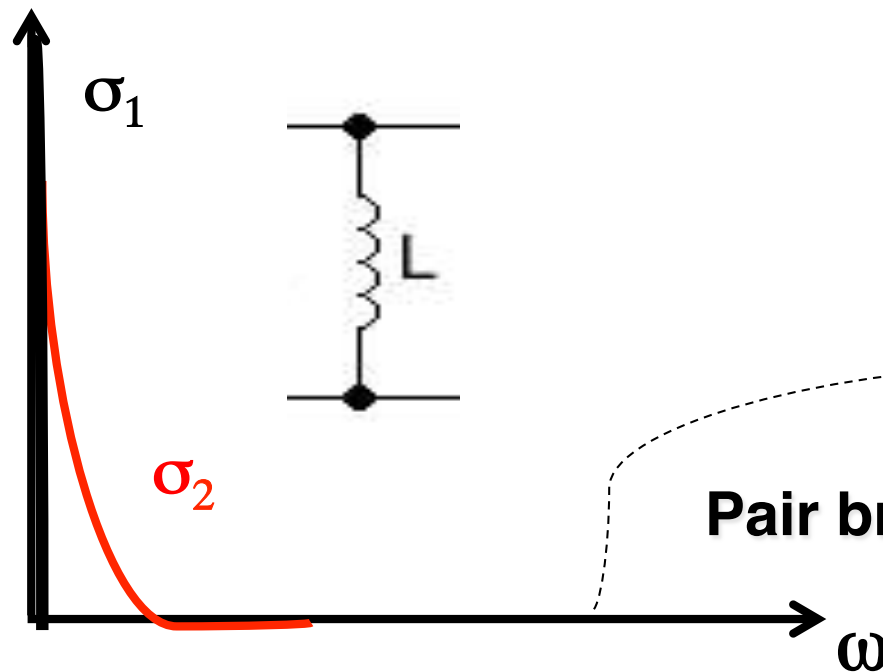
$E(\omega)$ 



◆ In plane



Reminder: Optics in a Superfluid



$$J(\omega) = [\sigma_1(\omega) + i\sigma_2(\omega)]E(\omega)$$

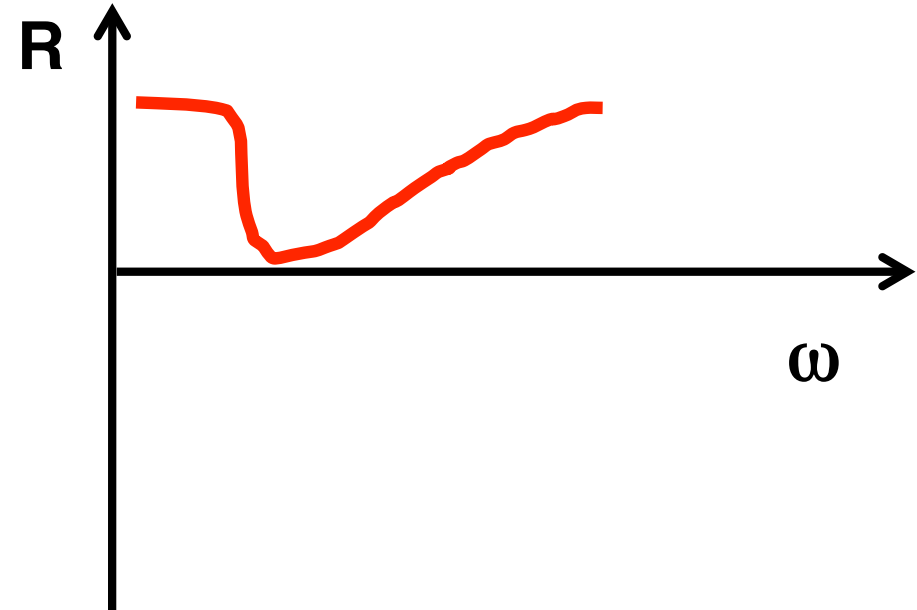
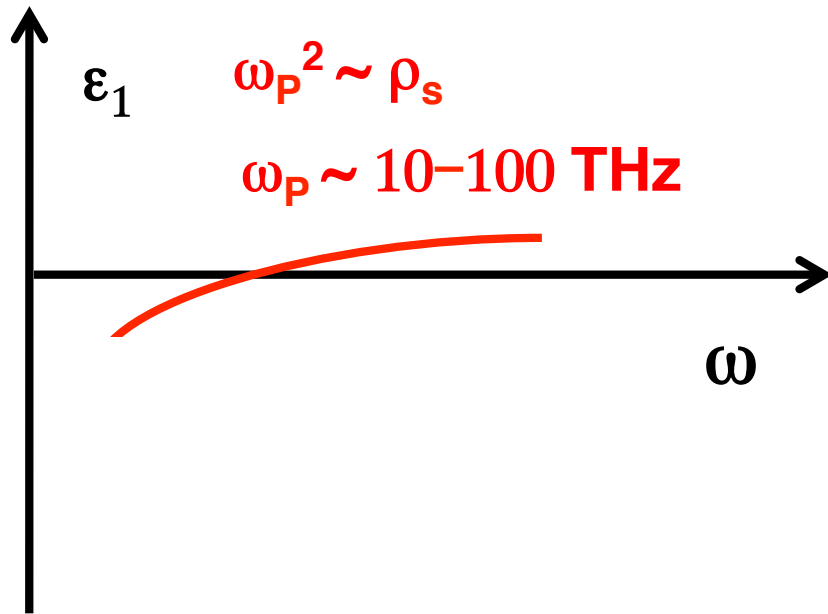
London superconductor

$$\sigma_1(\omega) = \pi/2(n_s e^2 / m^*)\delta(0)$$

$$\sigma_2(\omega) = n_s e^2 / m^* \omega$$



Optics in a Superfluid



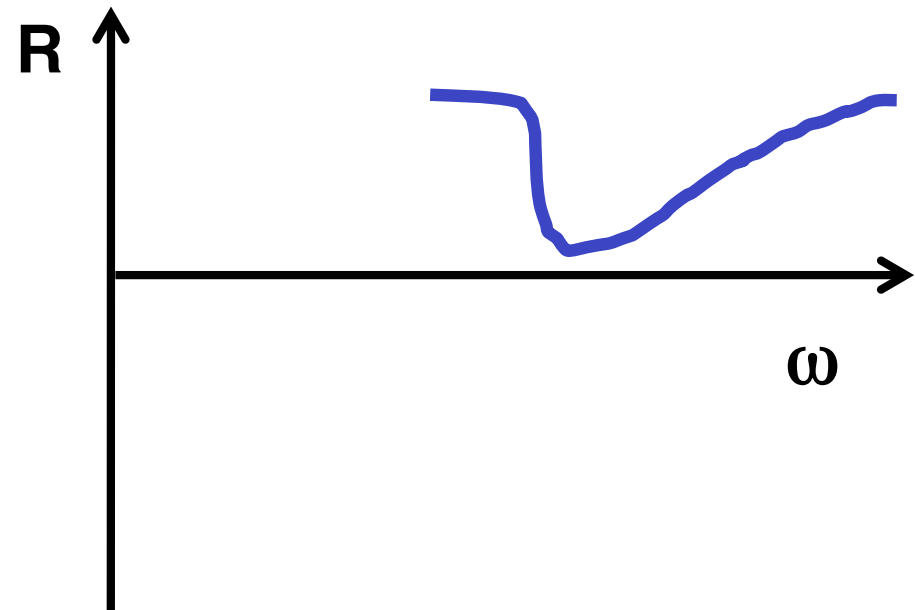
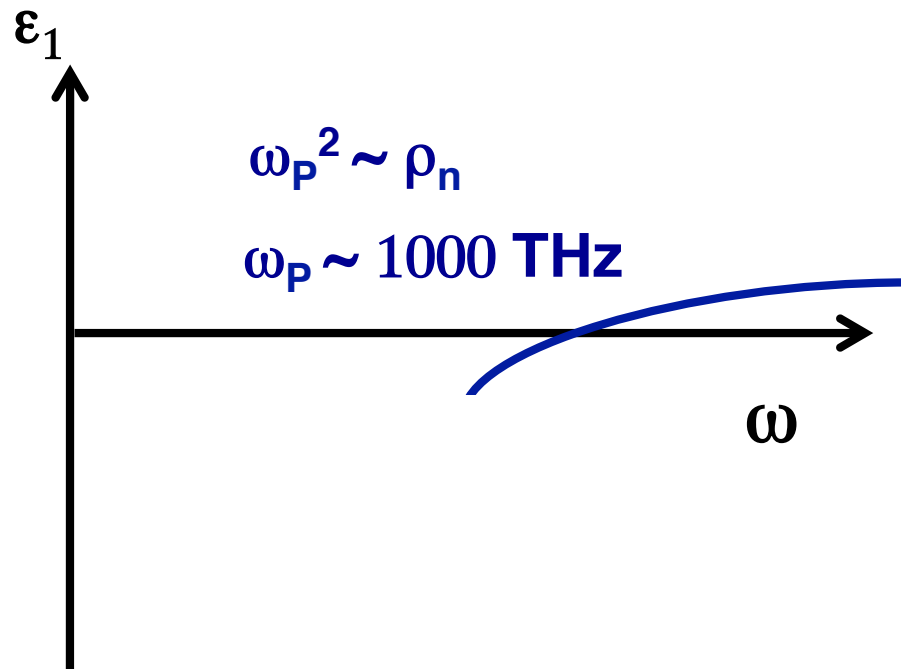
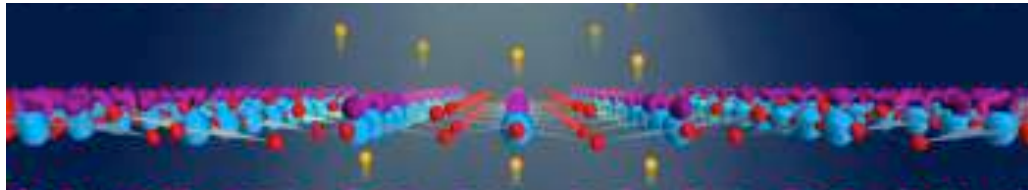
London superconductor

$$\varepsilon_1(\omega) = \varepsilon_0 - \frac{n_s e^2}{m^* \omega^2}$$

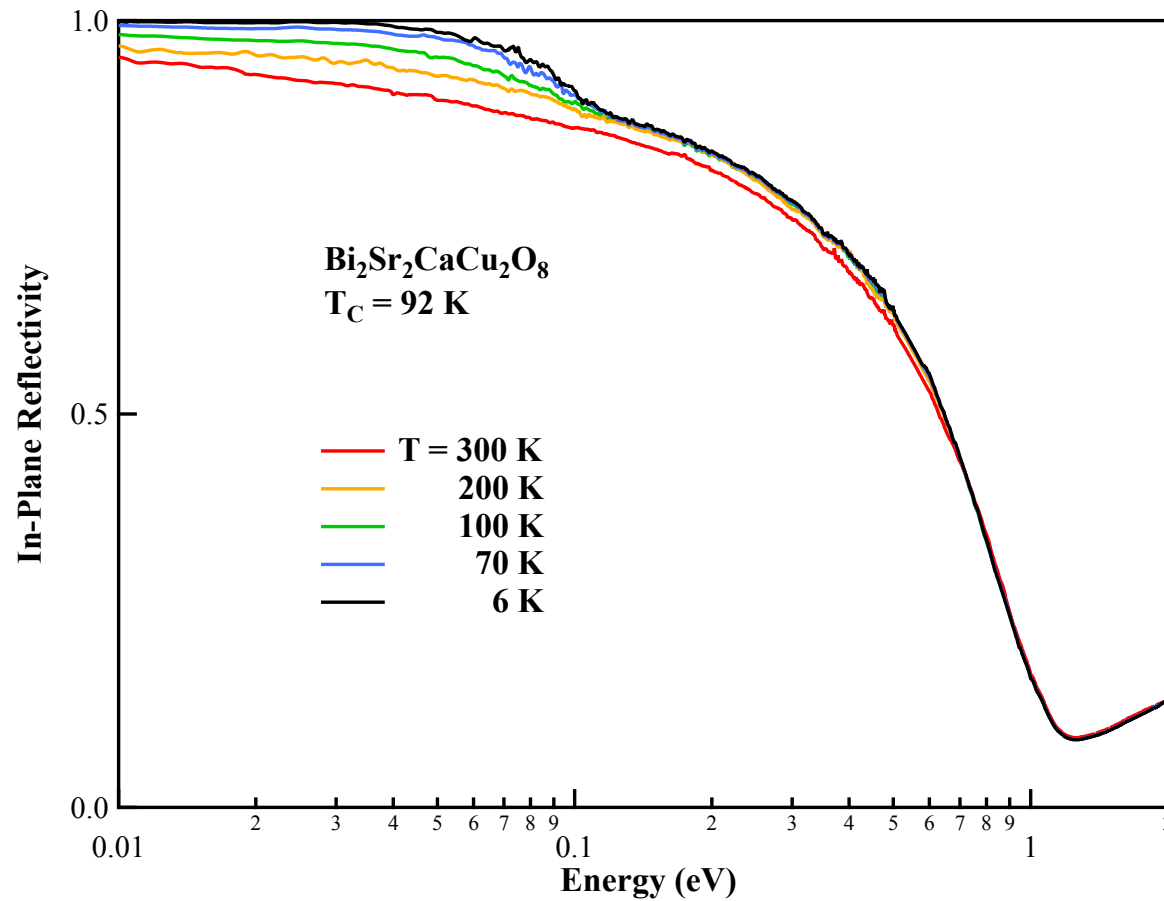
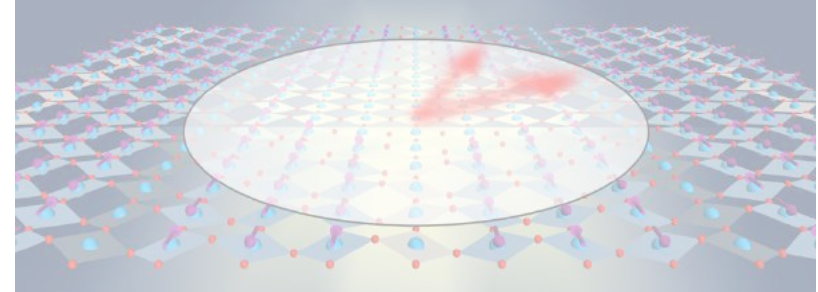
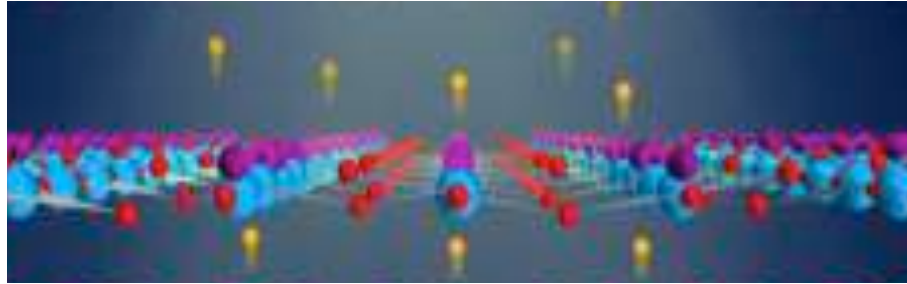
$$\varepsilon_2(\omega) = \frac{n_s e^2}{m^* \omega} \delta(0)$$

Also, response of quasi-particles

$E(\omega)$ 



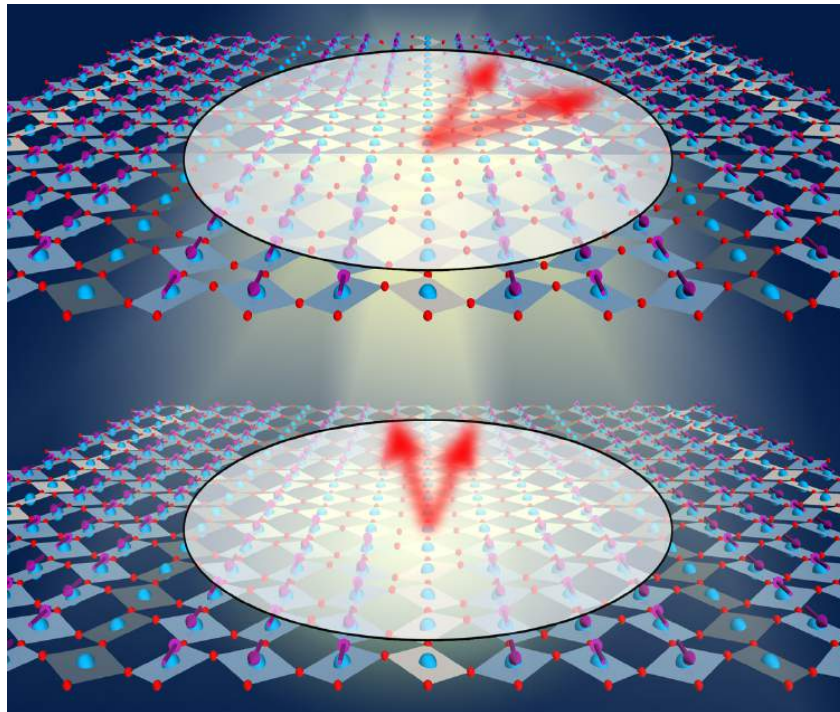
Quasi-particles dominate the in-plane response



Out of Plane

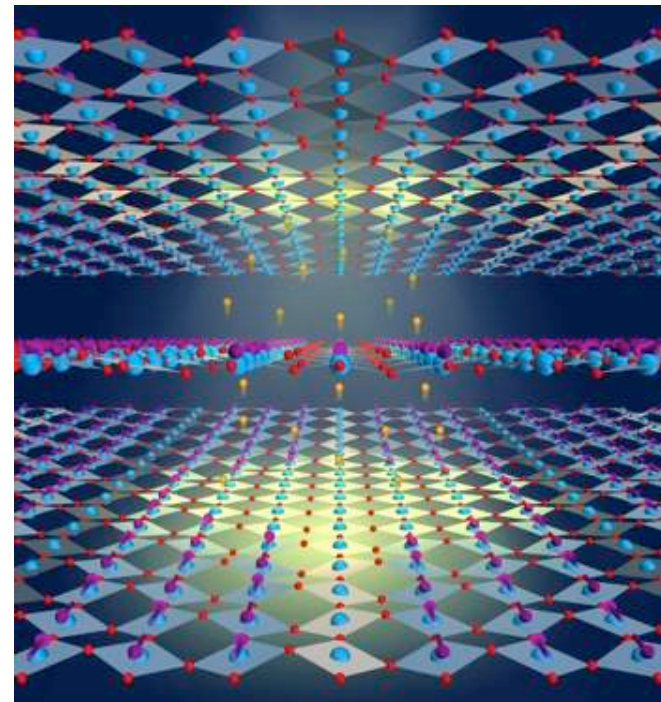
$$\omega_p^2 \sim \rho_s$$


$$\omega_p \sim 0.1 - 10 \text{ THz}$$



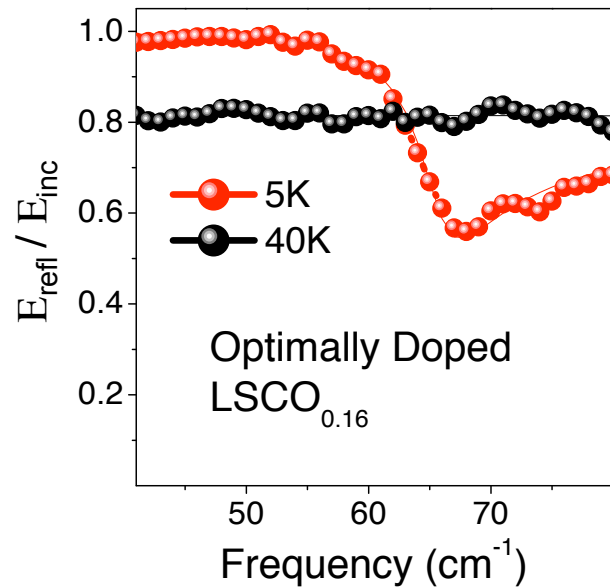
$$\omega_p^2 \sim \rho_{nz}$$

$$\omega_p \sim 0$$

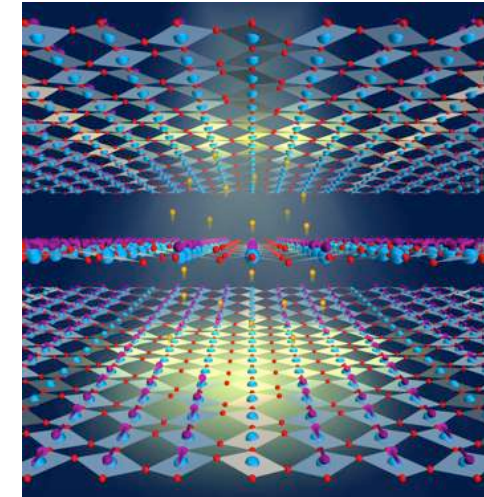
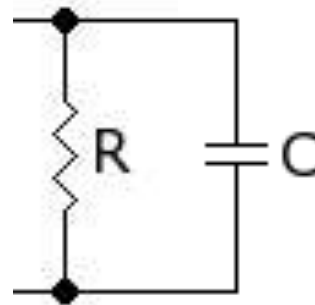



 $E(\omega)$

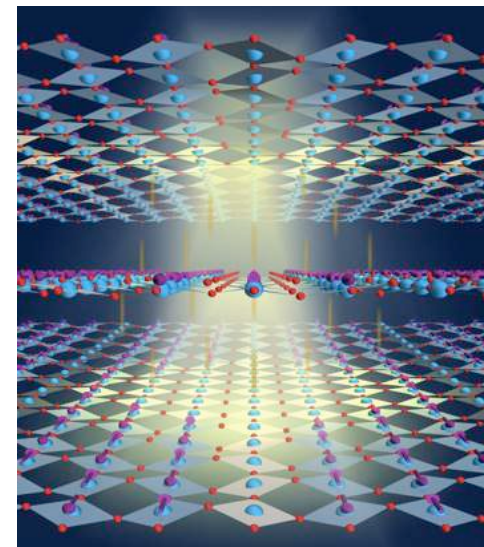
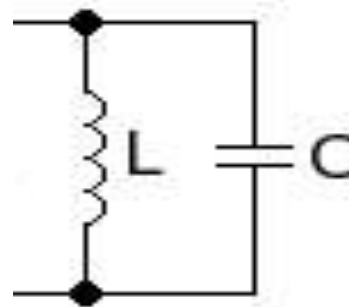
Josephson plasmons in cuprates



$T > T_c$



$T < T_c$



Kresin and Morawitz Phys. Rev. B (1988)

van der Marel and A. A. Tsvetkov Czech. J. Phys. (1996)

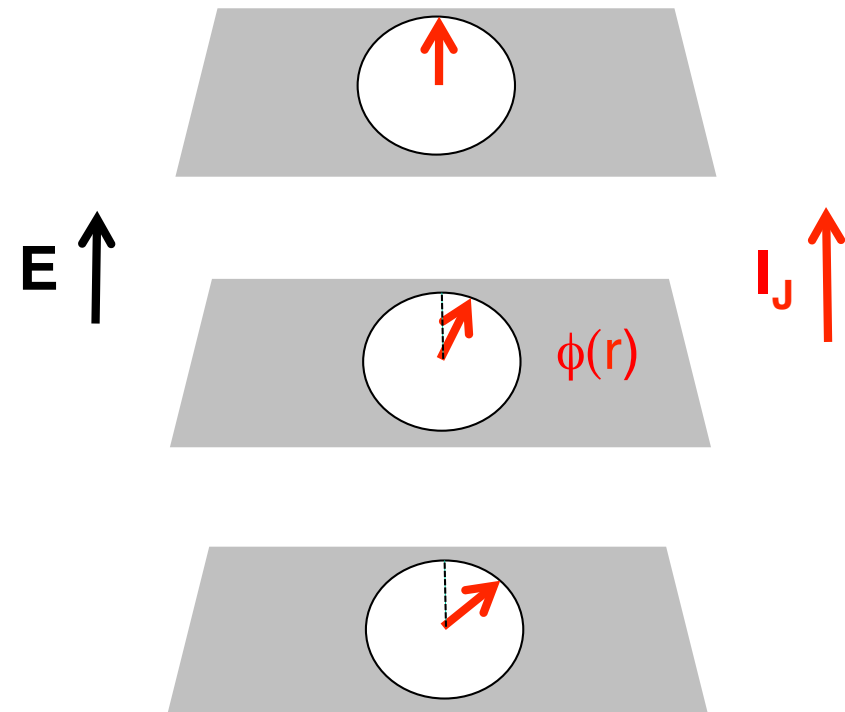
D.N. Basov et al. Science 283, 49 (1999)

First Josephson Equation

$$I_J = I_c \sin \phi$$

Second Josephson Equation

$$\frac{\partial \phi}{\partial t} = \frac{2eV}{\hbar}$$



A nonlinear inductor

First Josephson Equation

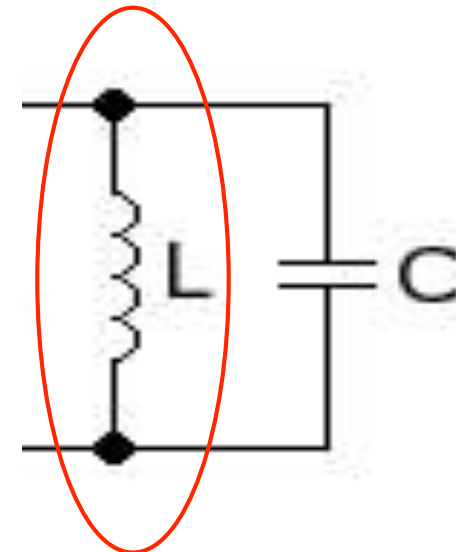
$$dI_J = I_c (\cos \phi) d\phi$$

$$dV = \frac{\hbar}{2e} \frac{1}{I_c \cos \phi} \frac{\partial}{\partial t} dI$$

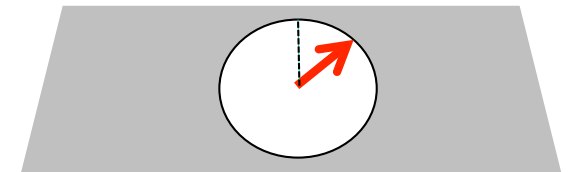
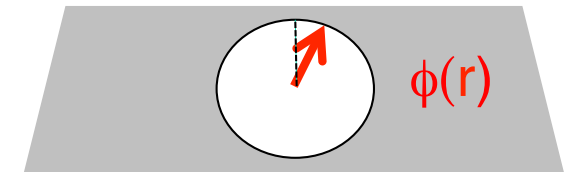
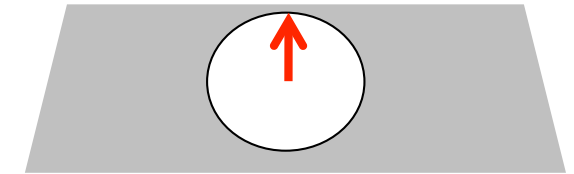
Second Josephson Equation

$$\frac{\partial}{\partial t} \phi = \frac{2eV}{\hbar}$$

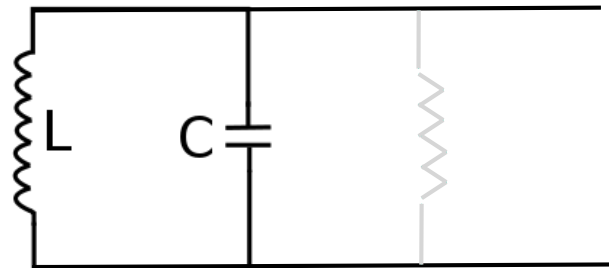
$$L = \frac{\hbar}{2e} \frac{1}{I_c \cos \phi}$$



Linear response: small phase gradients



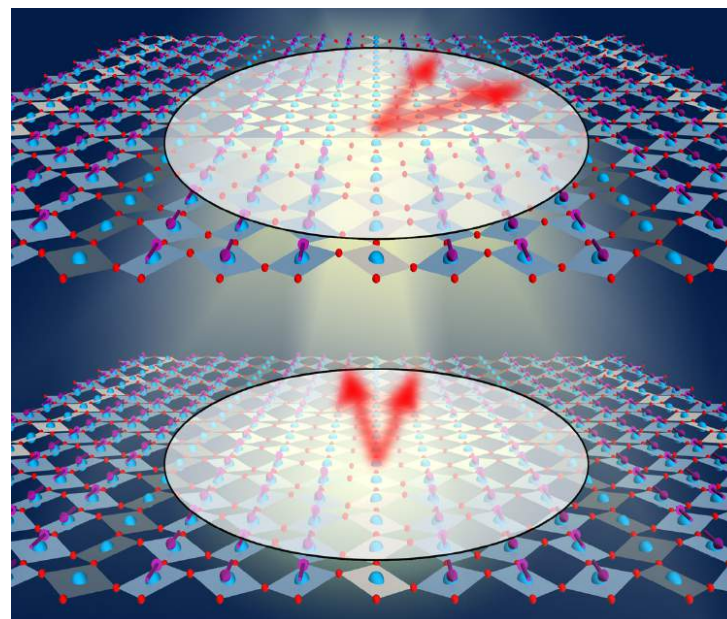
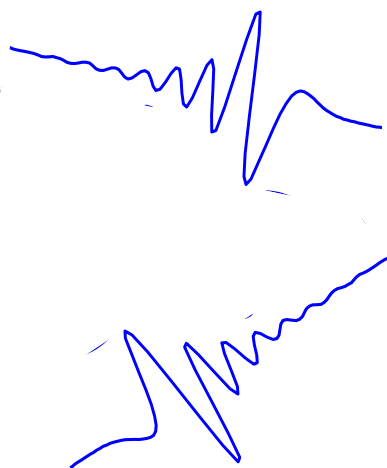
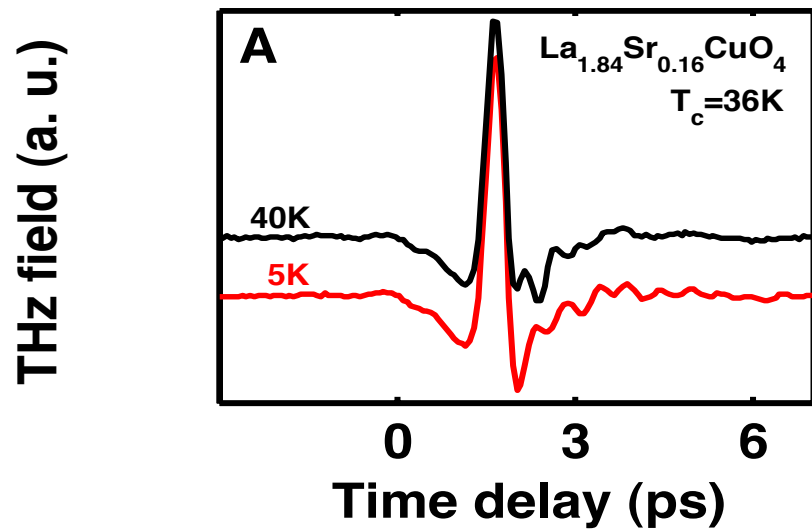
$$L = \frac{\hbar}{2e} \frac{1}{I_c \cos \phi} \approx \frac{\hbar}{2e I_c}$$



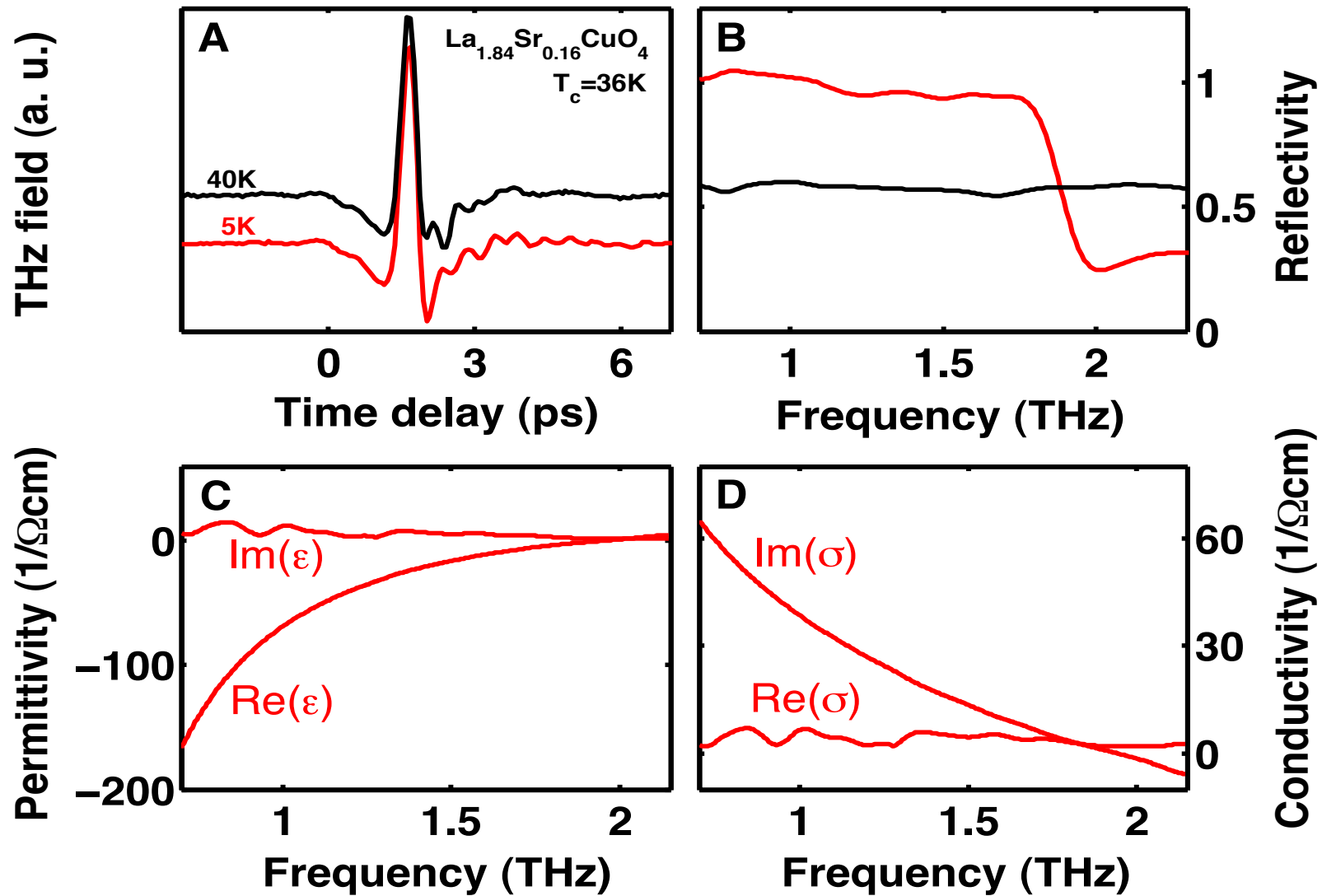
$$R \gg \omega L$$



La_{1.84}Sr_{0.16}CuO₄ (at 5K): THz reflection



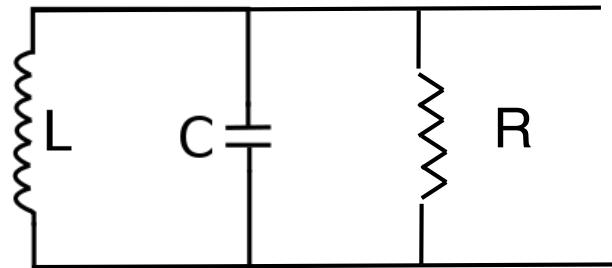
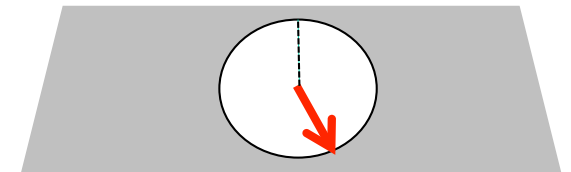
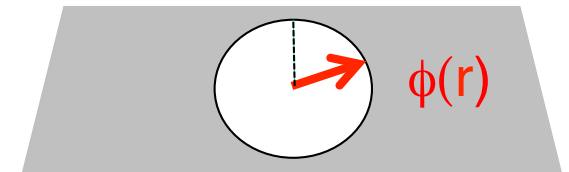
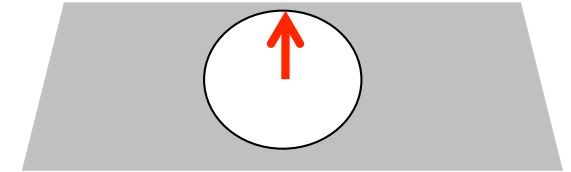
La_{1.84}Sr_{0.16}CuO₄ (at 5K)



Expected Response: Large Phase Differences

Resistive coupling is no longer shorted

$$L = \frac{h}{2e I_c \cos \phi} \gg \frac{h}{2e I_c} \approx R$$



Phase Differences: $\pi/2$

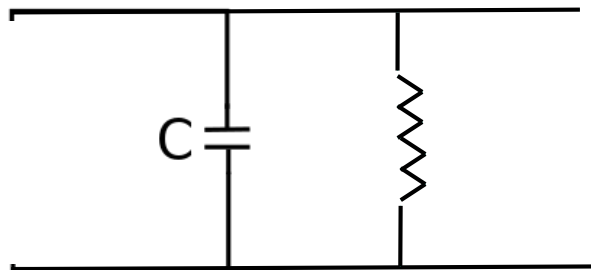
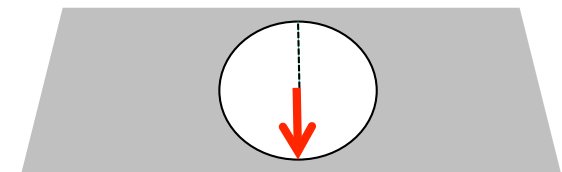
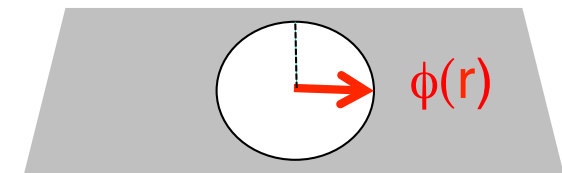
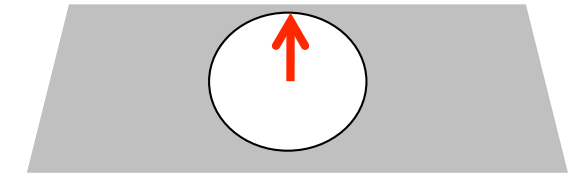
Second Josephson Equation

$$\frac{\partial \phi}{\partial t} = \frac{2eV}{\hbar}$$

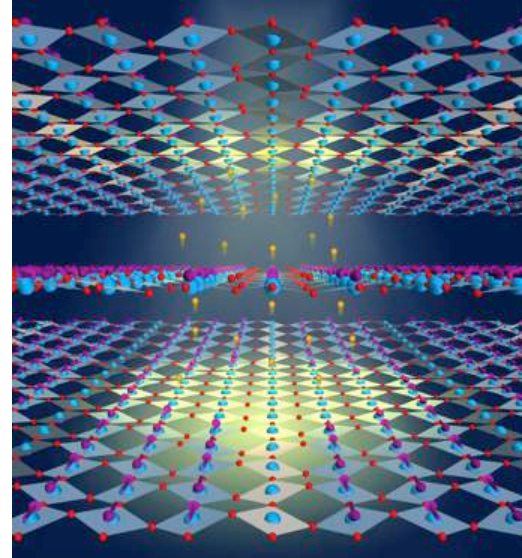
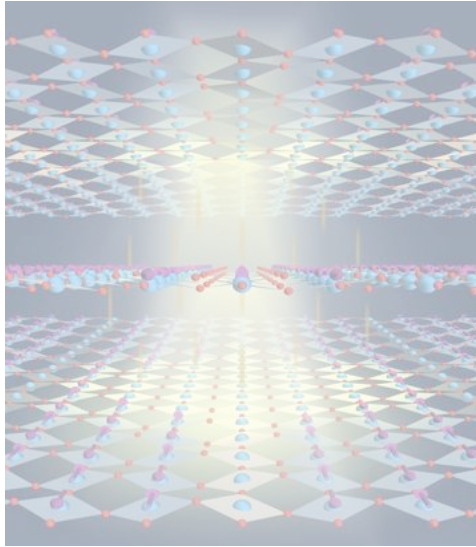
$$\phi(t) = \int_{-\infty}^t \frac{2eV(\tau)d\tau}{\hbar} = \frac{\pi}{2}$$

Inductive coupling is zero

$$L = \frac{\hbar}{2e I_c \cos \phi} = \infty$$

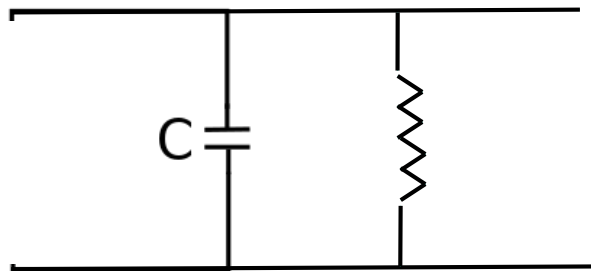


Phase Differences: $\pi/2$



Inductive coupling is zero

$$L = \frac{\hbar}{2e I_c \cos \phi} = \infty$$



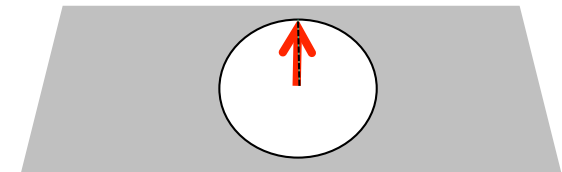
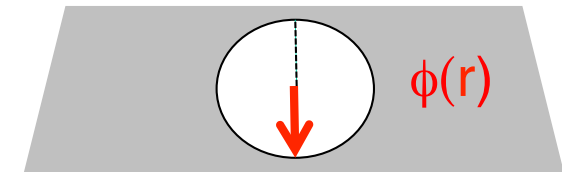
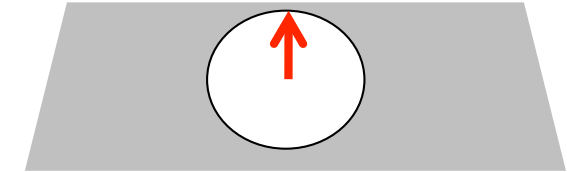
Phase Difference: π

Second Josephson Equation

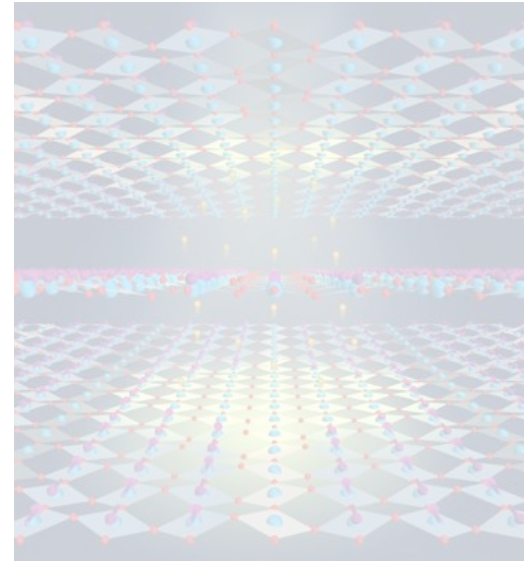
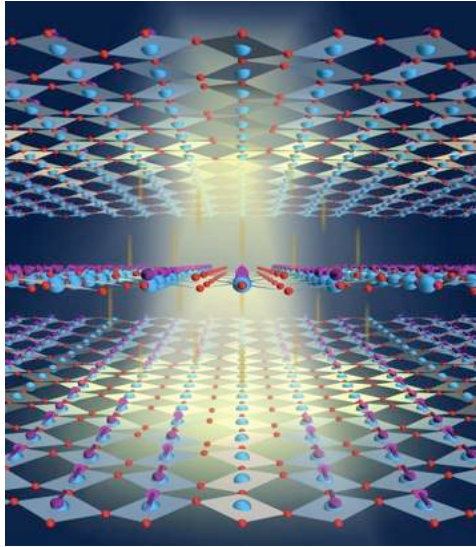
$$\frac{\partial \phi}{\partial t} = \frac{2eV}{\hbar}$$

$$\phi(t) = \int_{-\infty}^t \frac{2eV(\tau)d\tau}{\hbar}$$

$$L = \frac{\hbar}{2e} \frac{1}{I_c \cos \phi} \approx -\frac{\hbar}{2eI_c}$$

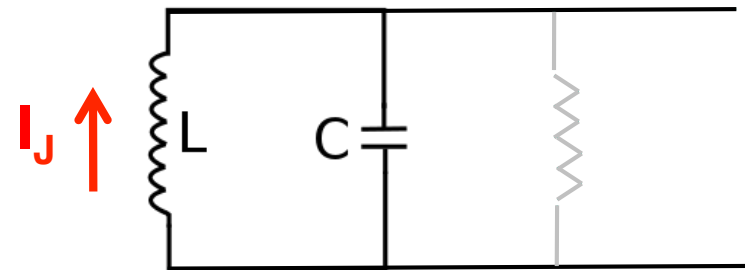


Phase Difference: π

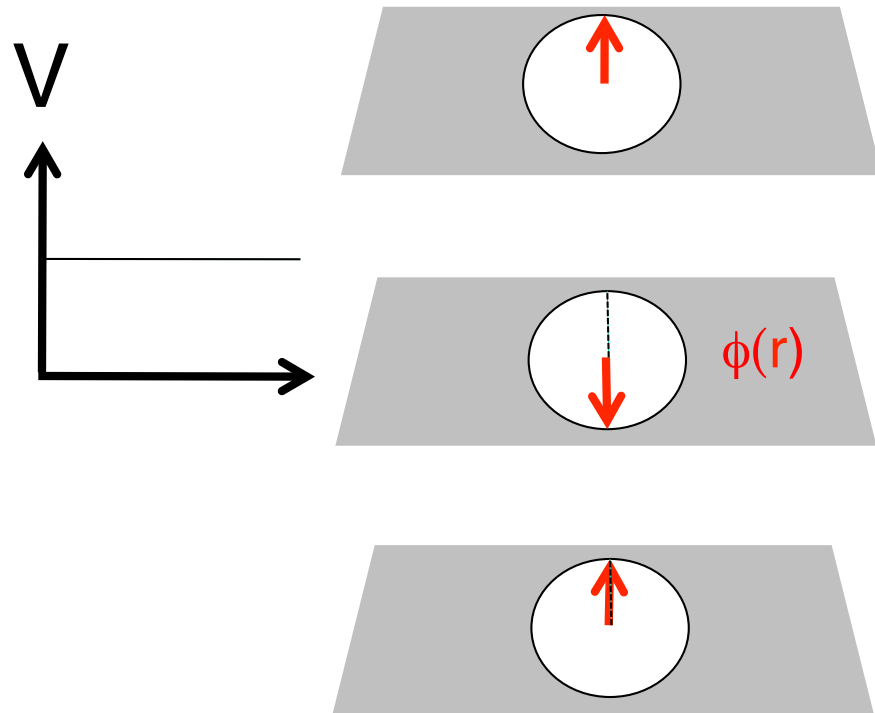


Inductive coupling is negative

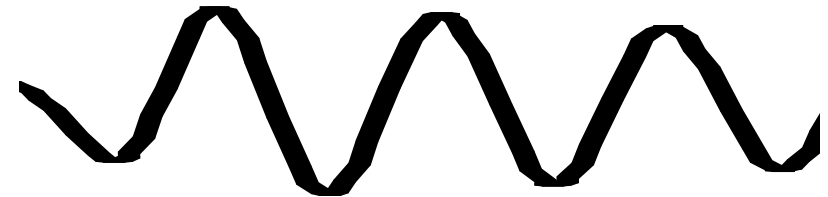
$$L = \frac{\hbar}{2e} \frac{1}{I_c \cos \phi} \approx -\frac{\hbar}{2eI_c}$$



The AC Josephson effect

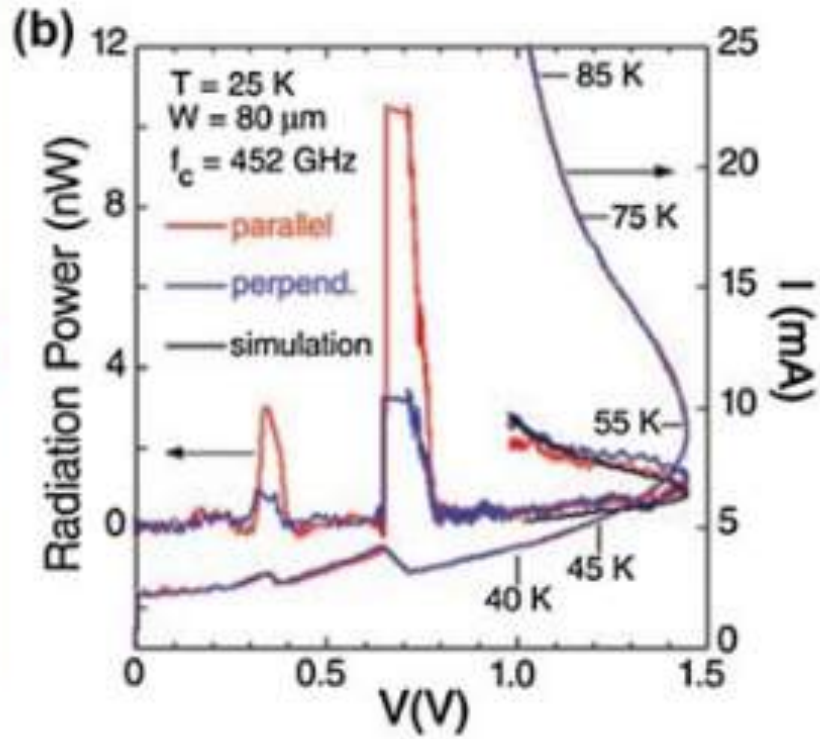
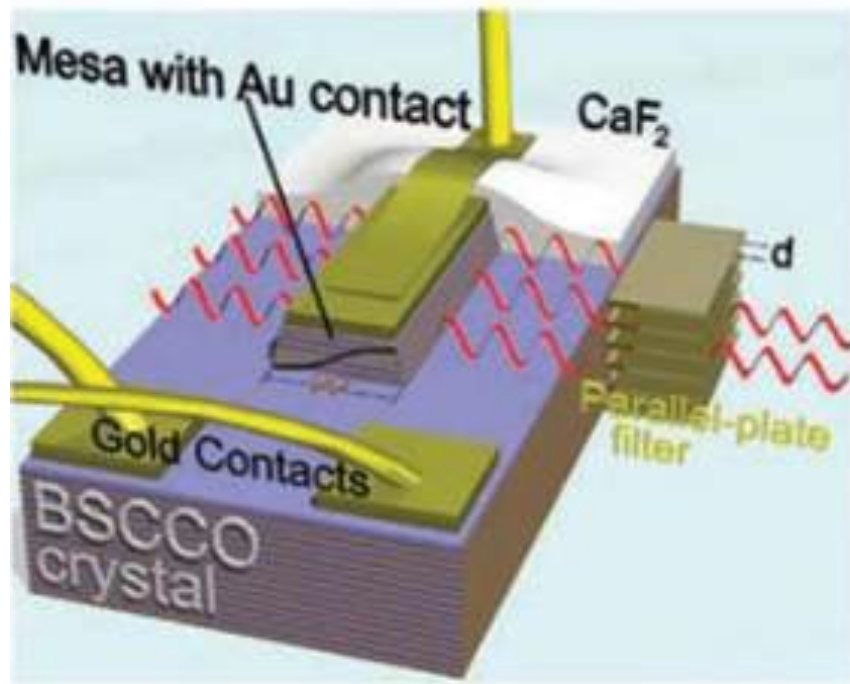


I_s



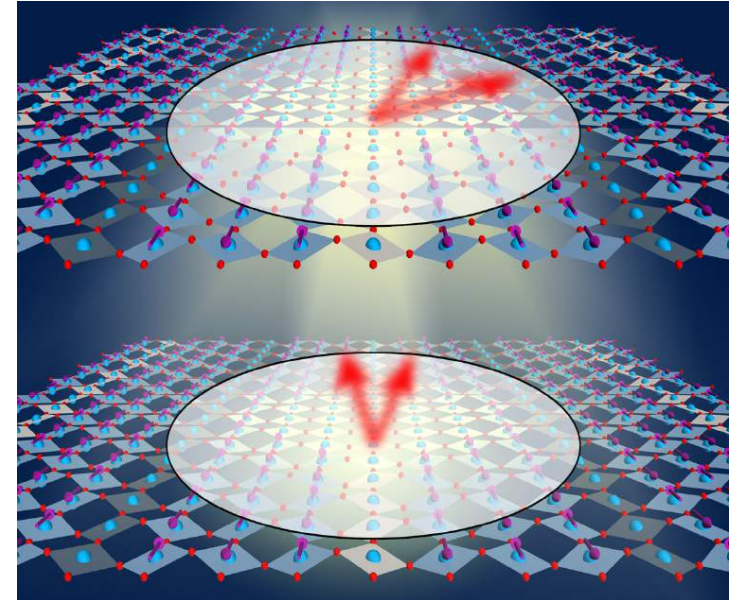
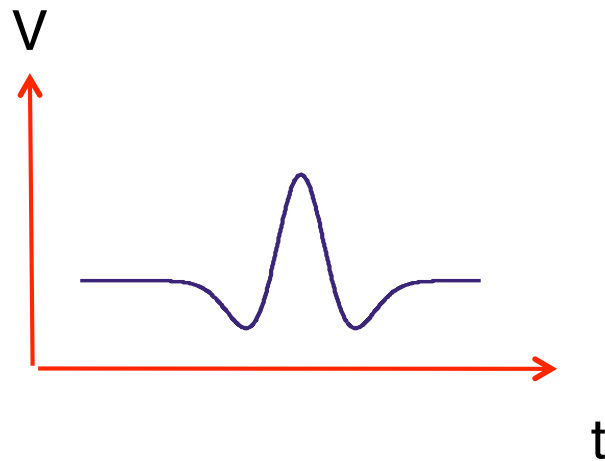
Voltage to frequency converter

In Cuprates: Coherent THz radiation with DC Bias



Can we flip phase at ultrafast rates ?

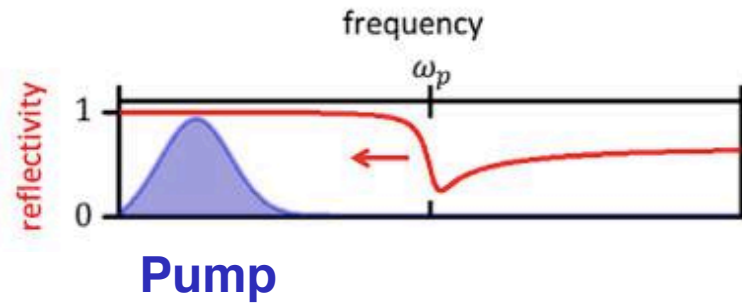
$$\phi(t) = \int_{-\infty}^t \frac{2eV(\tau)d\tau}{\hbar} = \frac{\pi}{2}$$



$$V = \frac{\pi}{4e} \frac{\hbar}{100 \text{ fs}} \approx mV$$

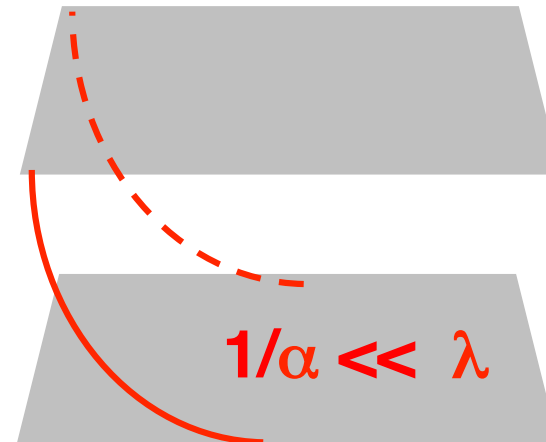
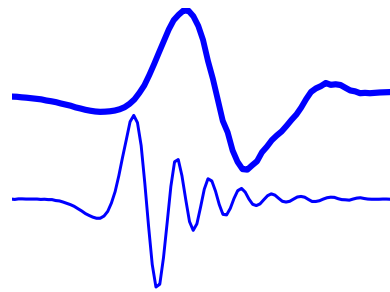
$$E \approx mV / (1nm) \approx \underline{100kV / cm}$$

Pump probe experiments

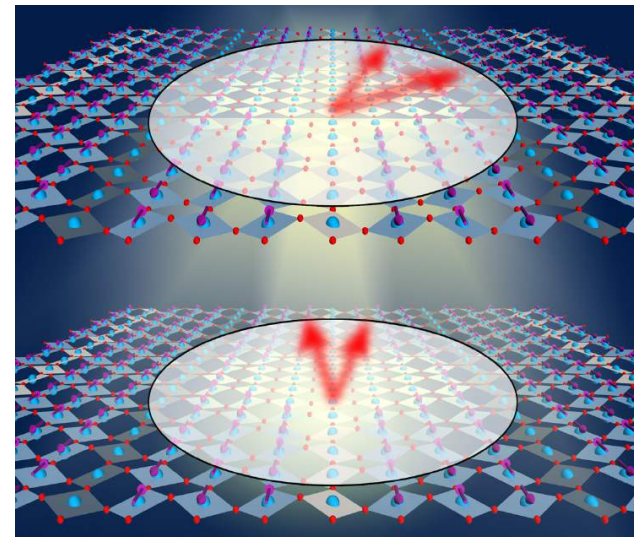
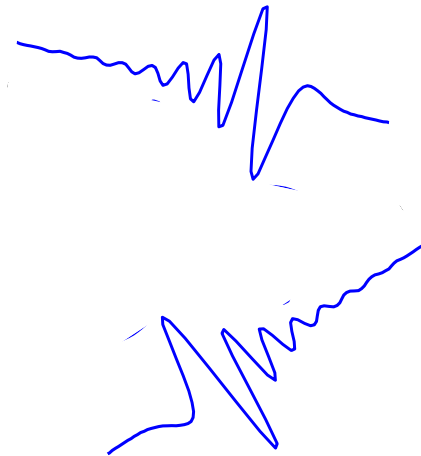
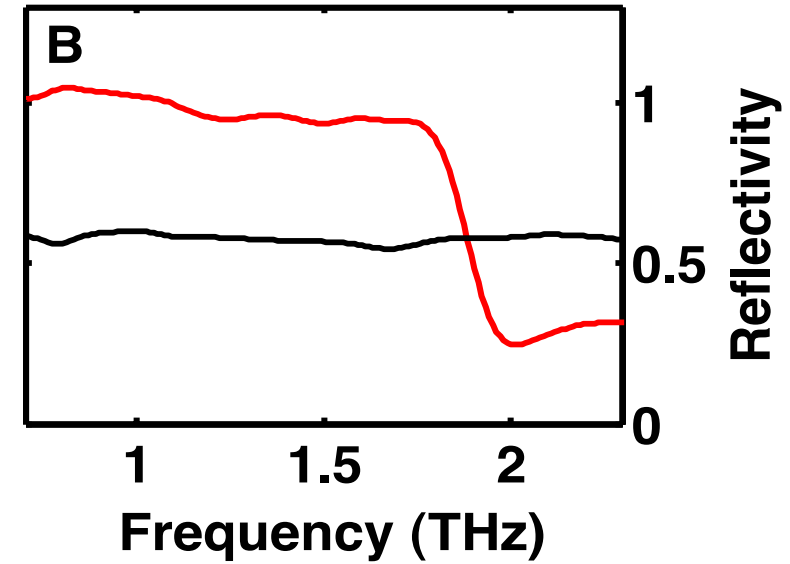
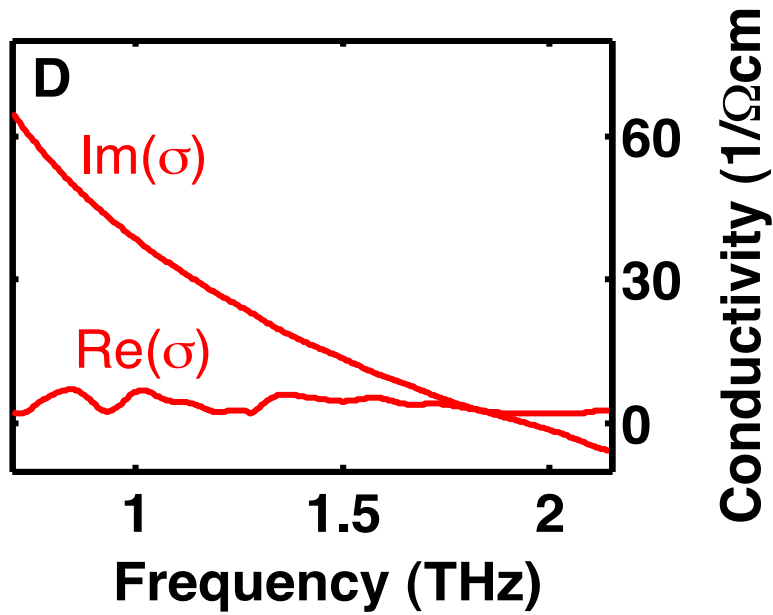


Pump

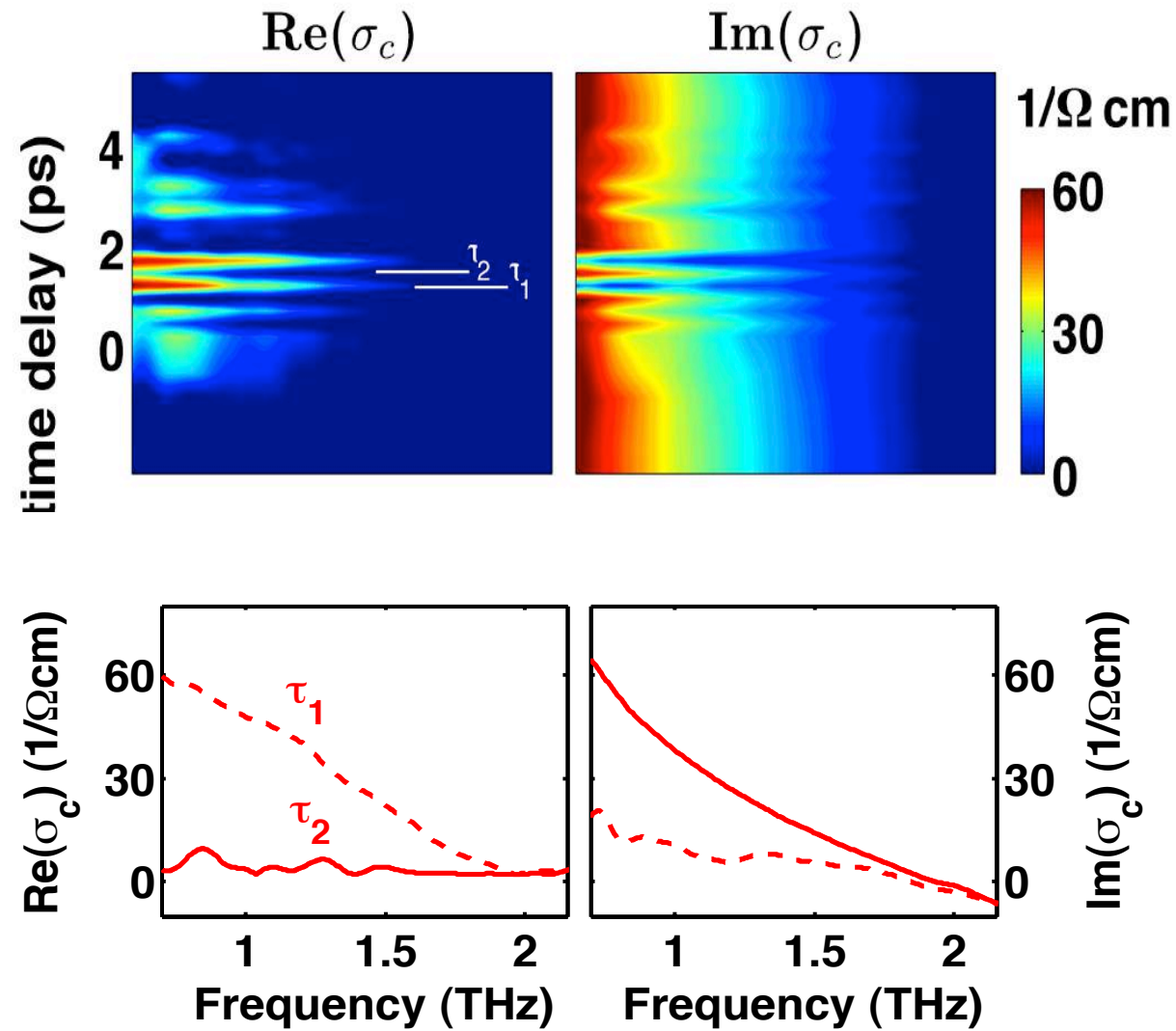
Probe



Probe alone: $\text{La}_{1.84}\text{Sr}_{0.16}\text{CuO}_4$ (at 5K)



Large Conductivity Oscillations



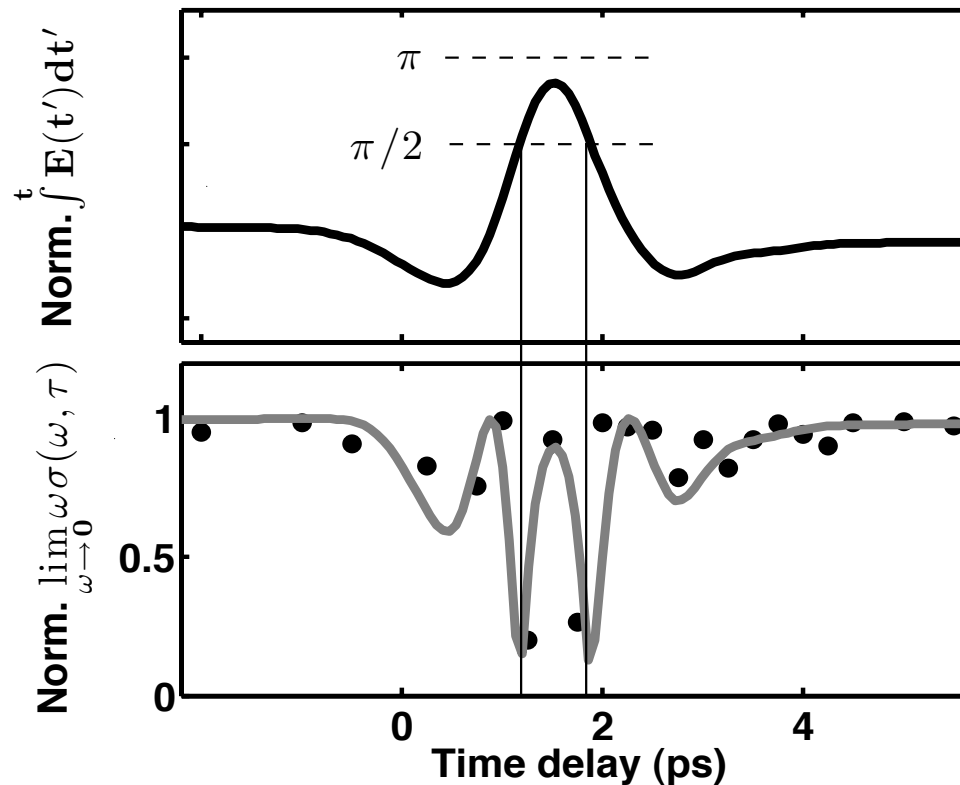
Oscillations in the interlayer coupling

$$\sigma_1(\omega) = \pi/2(n_s e^2 / m^*) \delta(0)$$

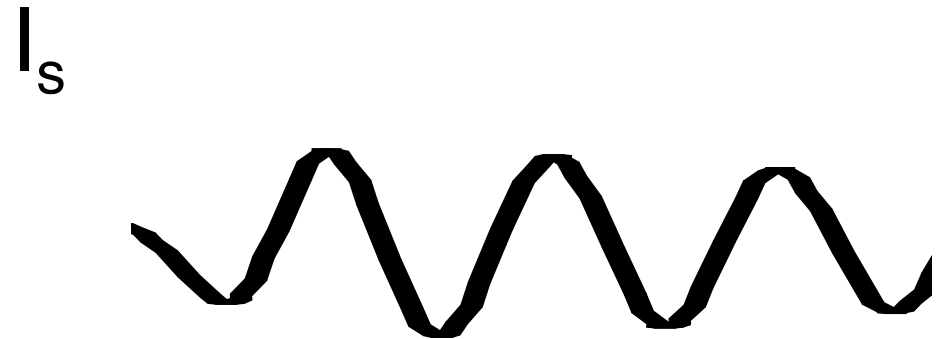
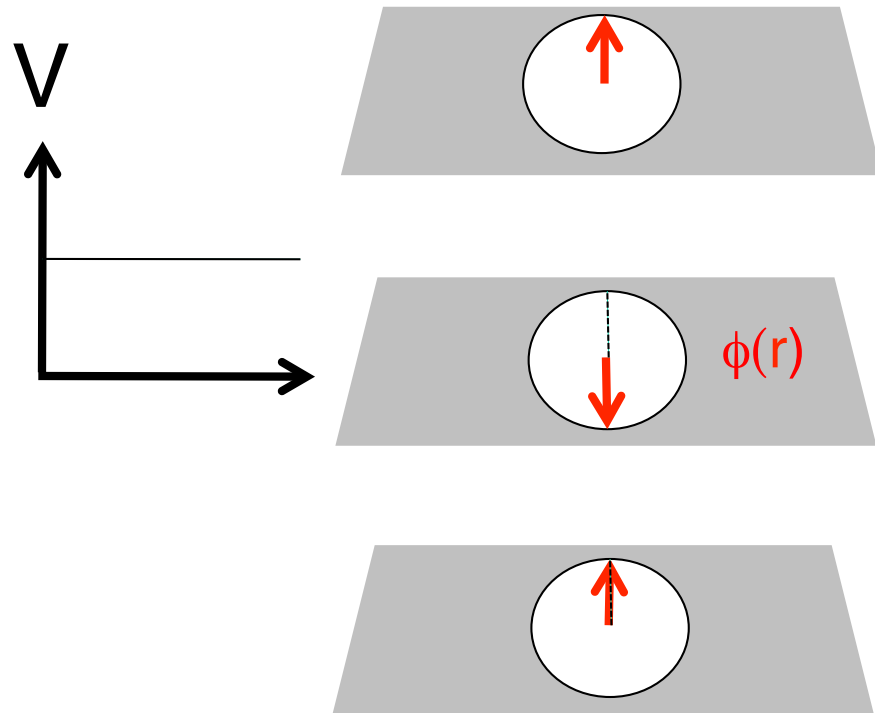
$$\sigma_2(\omega) = n_s e^2 / m^* \omega$$



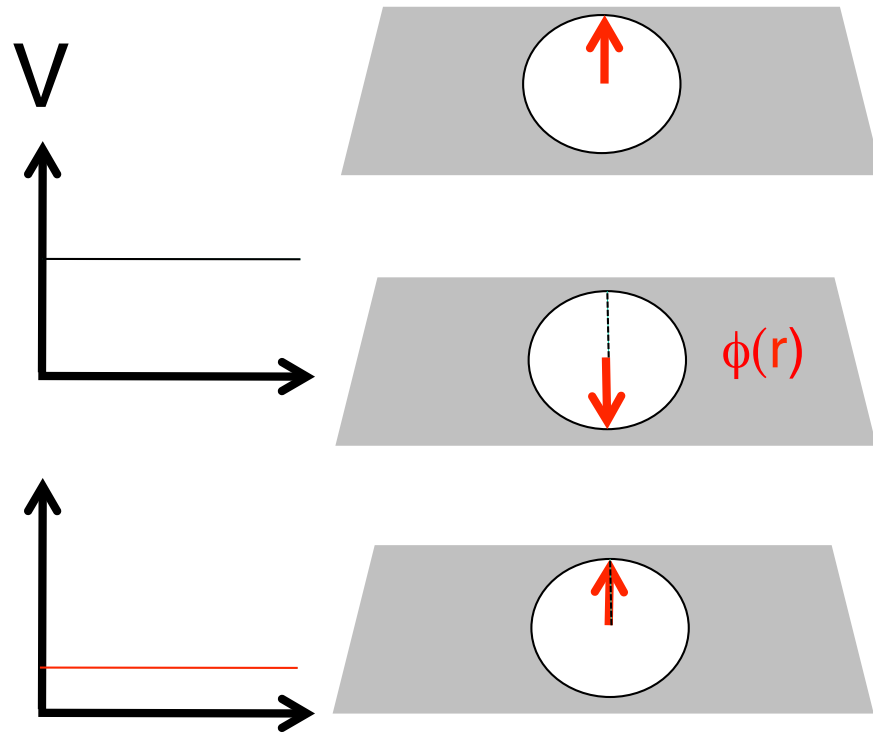
$$n_s \propto \lim_{\omega \rightarrow 0} \omega \sigma_2(\omega)$$



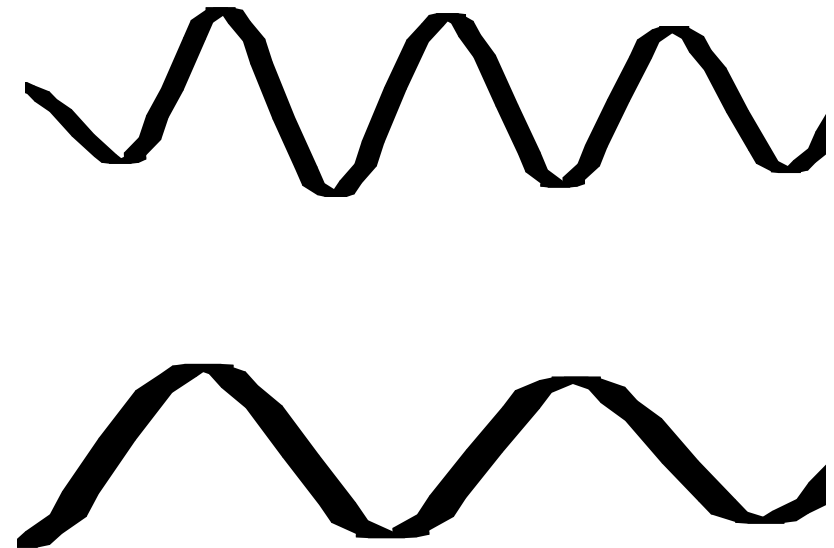
Quasi-DC field: Conductivity Oscillations



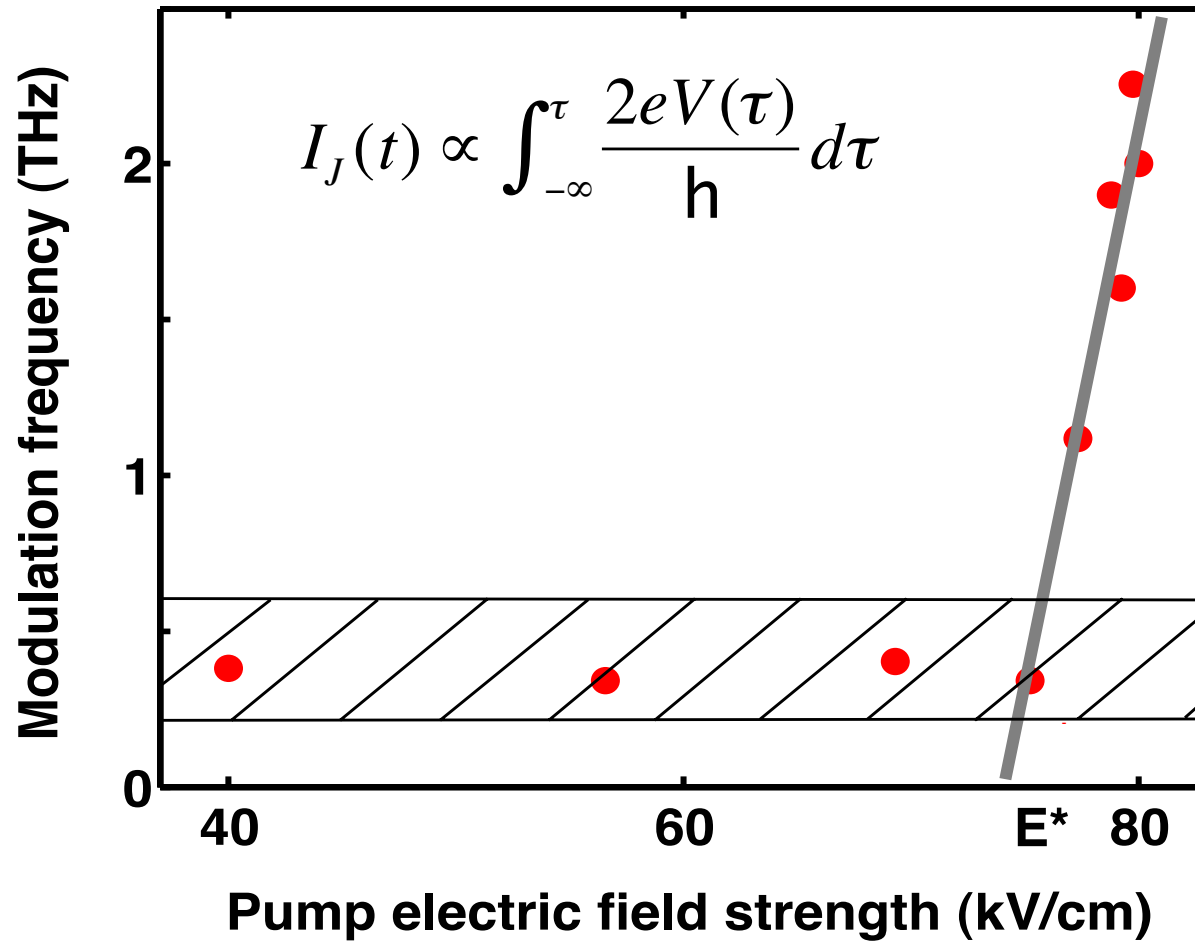
Expected: Voltage dependent frequency



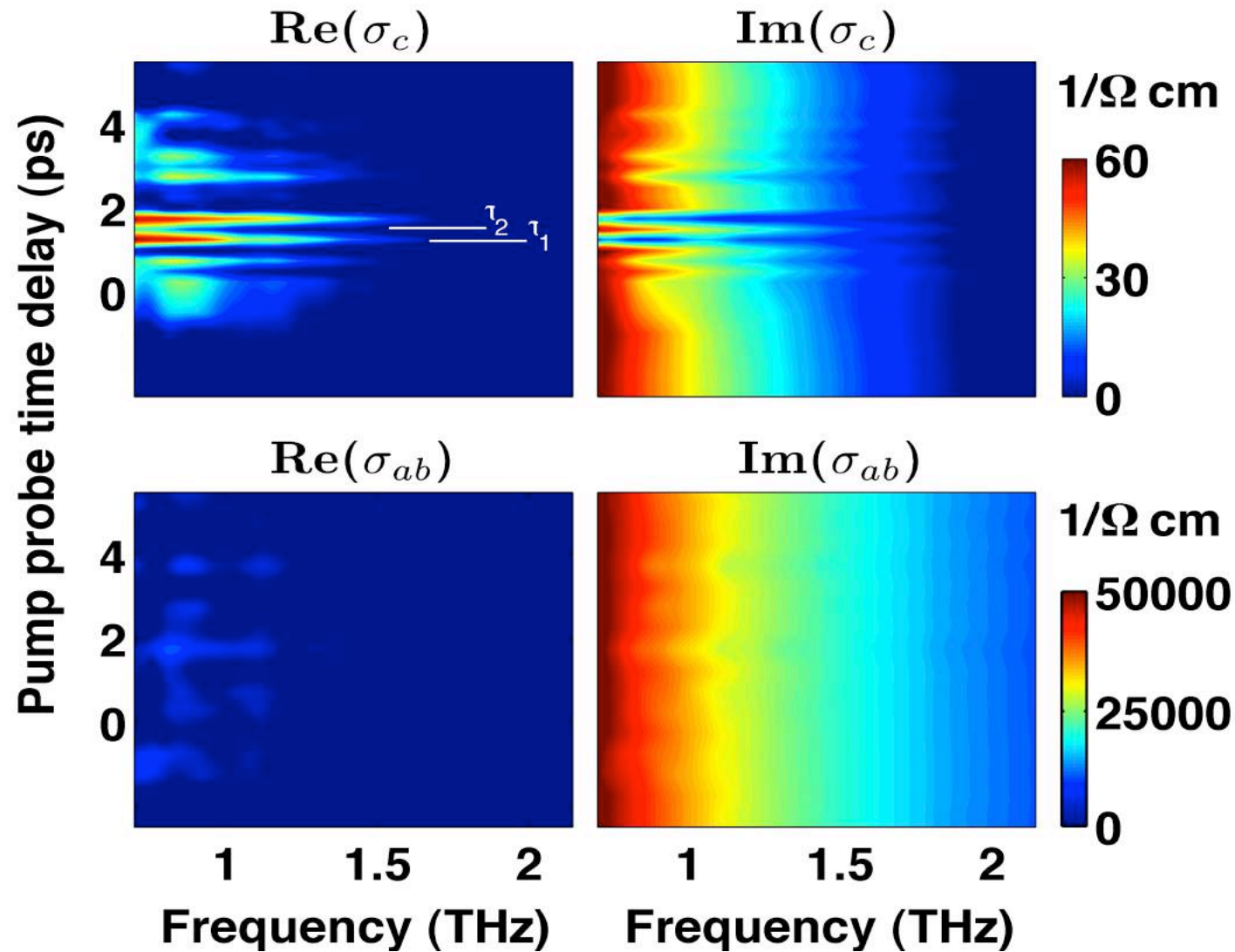
I_s



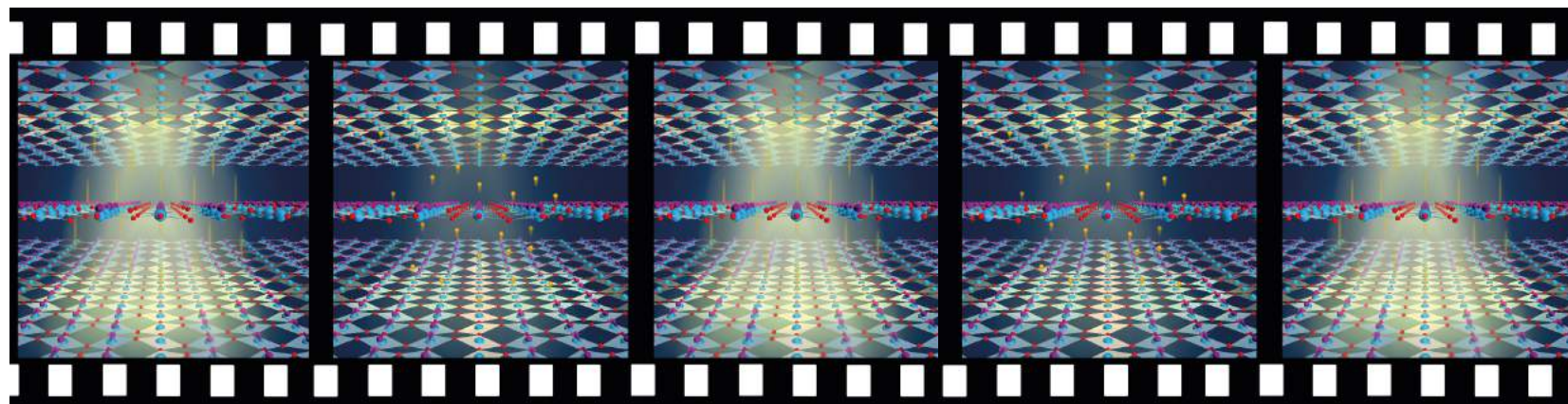
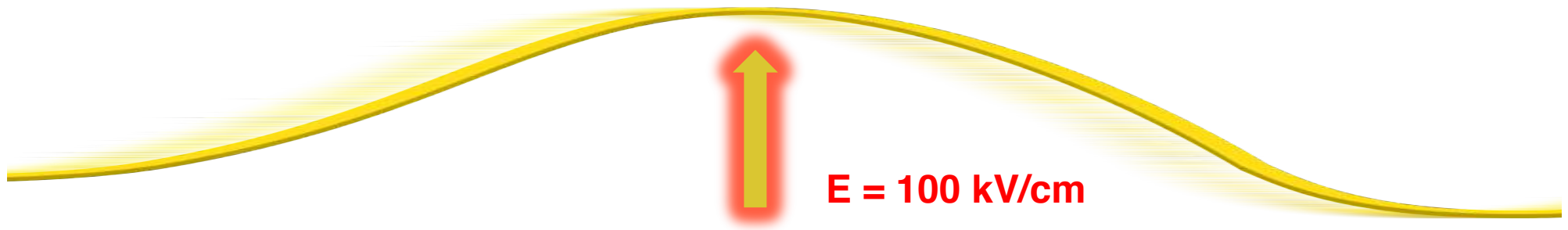
The oscillation frequency depends on field



Response in the Plane does not change



THz dimensionality oscillations



0

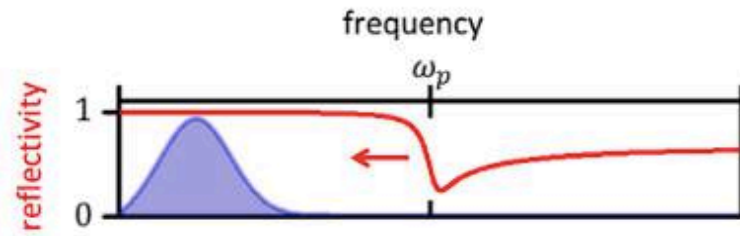
1.2 ps

1.5 ps

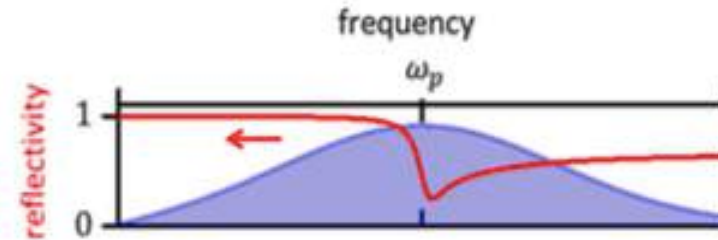
1.8 ps

2 ps

Tuning the pump wavelength to resonance

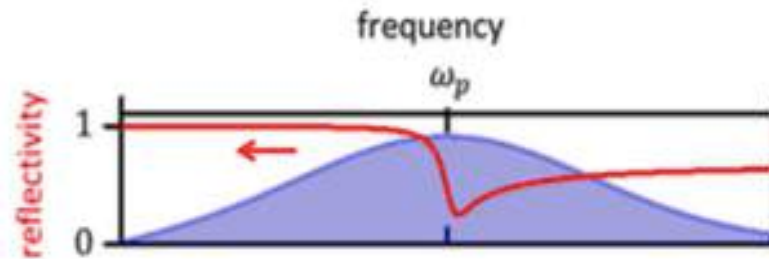


$$\omega_{\text{THz}} \ll \omega_p$$



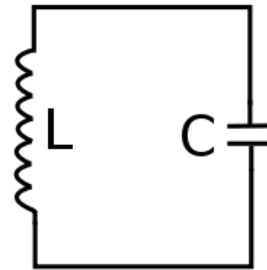
$$\omega_{\text{THz}} \sim \omega_p$$

Driven Plasma



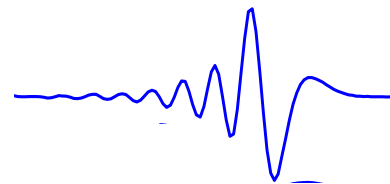
$$\omega_{\text{THz}} \sim \omega_p$$

$$L = \frac{\hbar}{2e} \frac{1}{I_c \cos \phi}$$

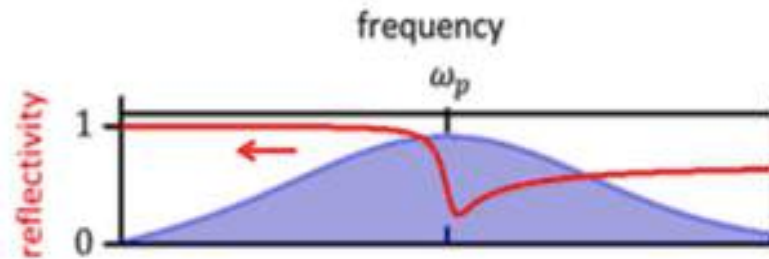


$$E(t) = E_0 \sin(\omega_{JP0} t)$$

$$\theta_{i,i+1}(t) = \theta_0 \cos(\omega_{JP0} t)$$



Driven Plasma



$$\omega_{\text{THz}} \sim \omega_p$$

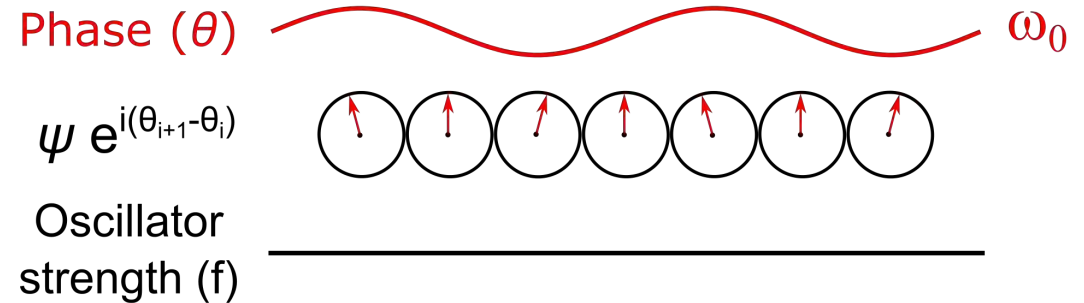
$$\omega_{P'}^2 = \omega_P^2 \cos[\theta_0 \cos(\omega_{JP_0} t)]$$

$$\omega_{P'}^2 = \omega_P^2 \left(1 - \frac{\theta_0^2 + \theta_0^2 \cos(2\omega_{JP_0} t)}{4} \right)$$

Strongly Driven Plasma

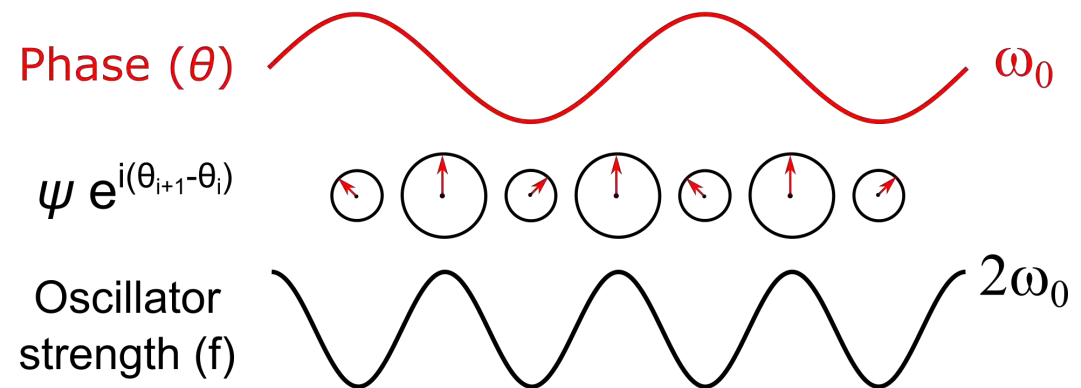
$$\theta_0 \ll \frac{\pi}{4}$$

Linear regime

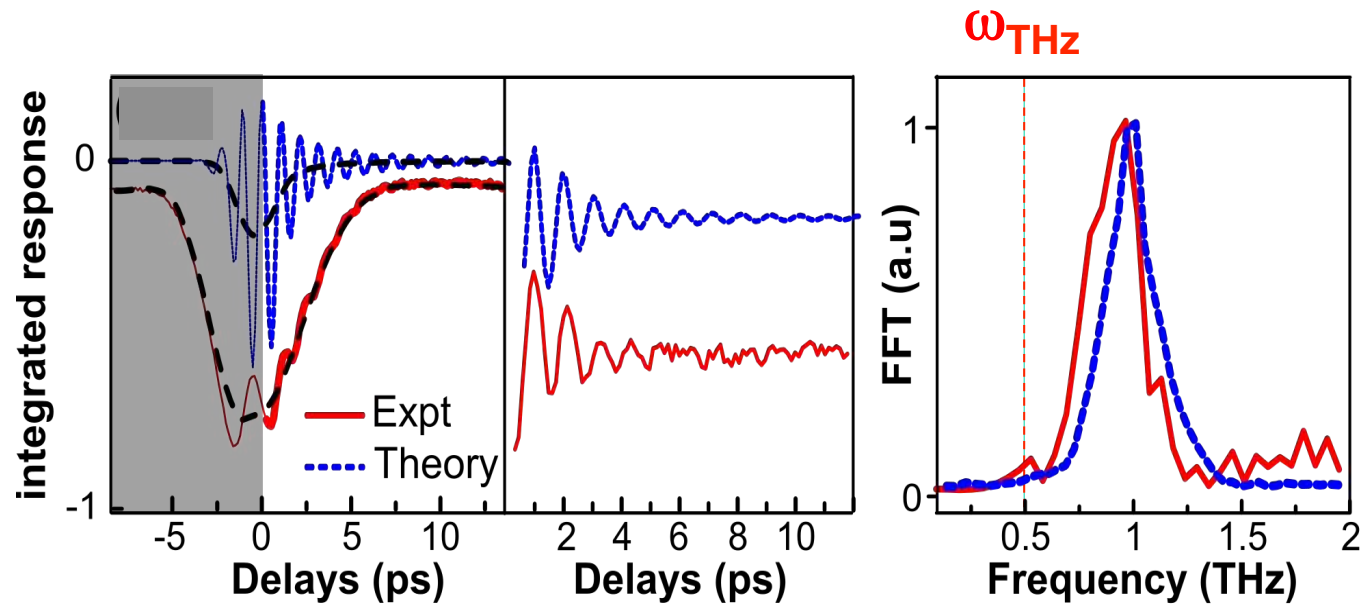
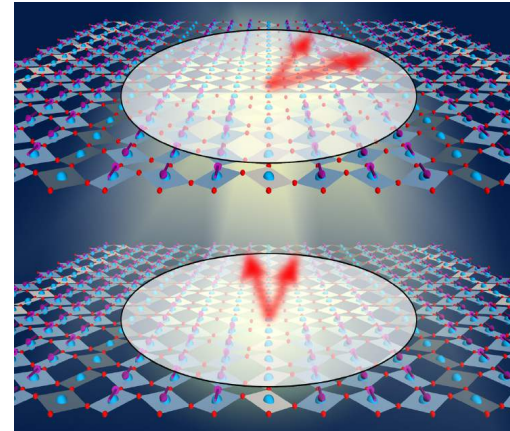
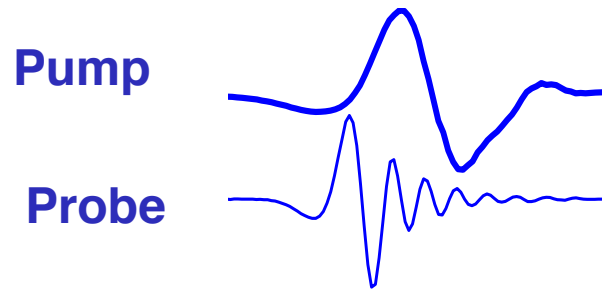


Nonlinear regime

$$\omega_{P'}^2 = \omega_P^2 \left(1 - \frac{\theta_0^2 + \theta_0^2 \cos(2\omega_{JP0}t)}{4} \right)$$



Superfluid stiffness driven at $2\omega_{\text{THz}}$

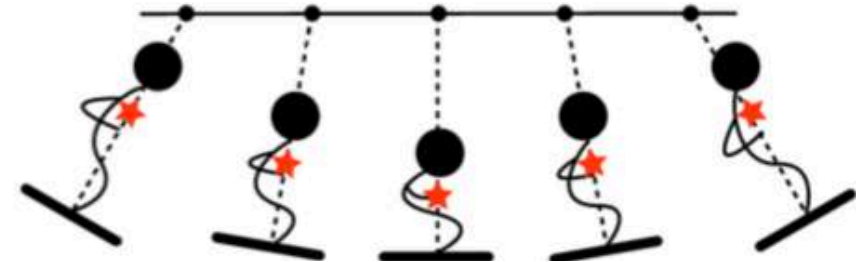


Superfluid stiffness driven at $2 \omega_{\text{THz}}$



Is this interesting ?

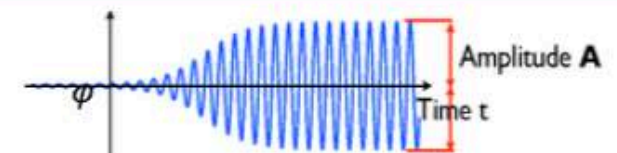
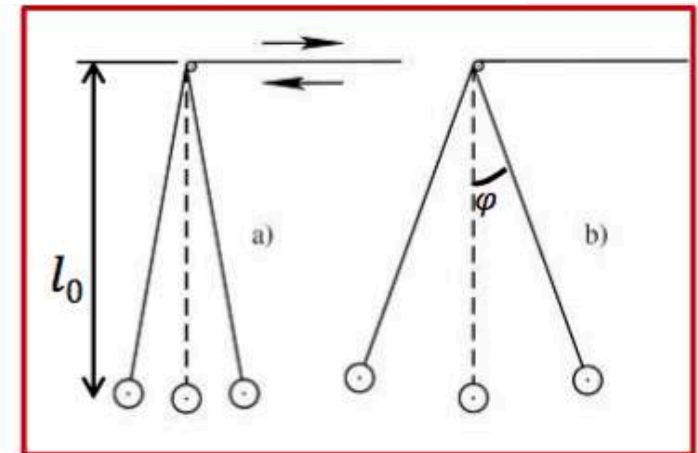
A parametrically driven pendulum



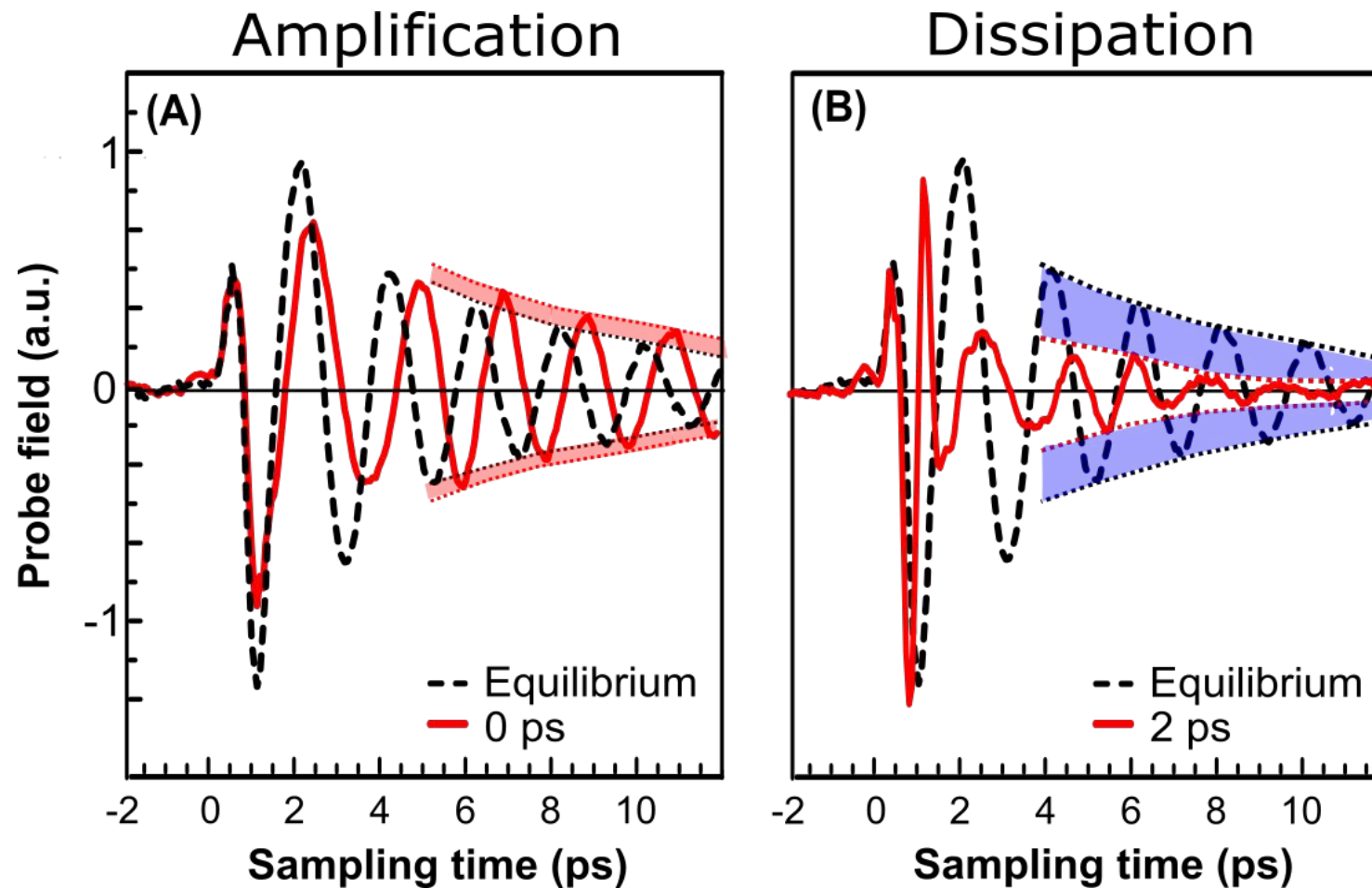
Oscillator strength is driven at 2ω

Mathieu's equation for parametric amplification is

$$\frac{\partial^2 \varphi}{\partial t^2} + \omega_p^2 \{1 + l_0(\cos(2\omega_p * t))\} \varphi = 0$$
$$F_{\text{driver}} = l_0(\cos(2\omega_p * t))$$

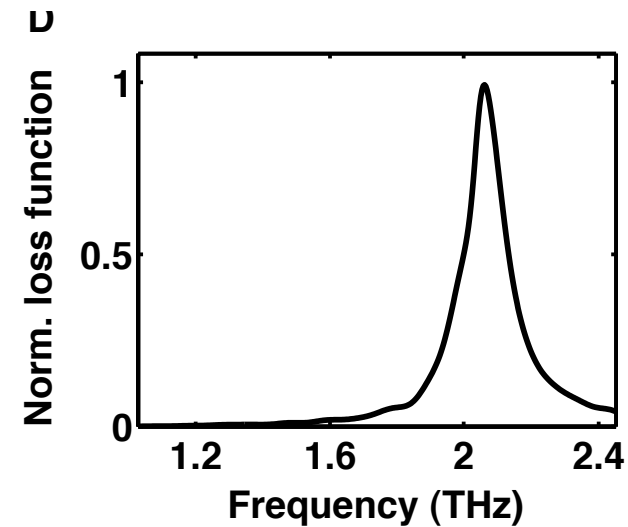
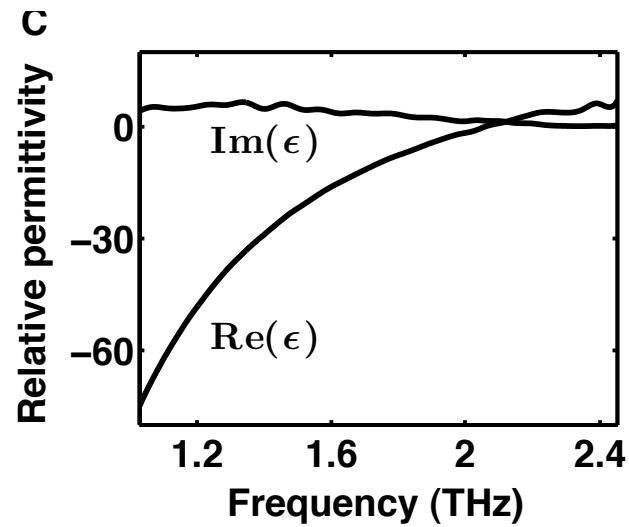


SC fluctuations are amplified or de-amplified



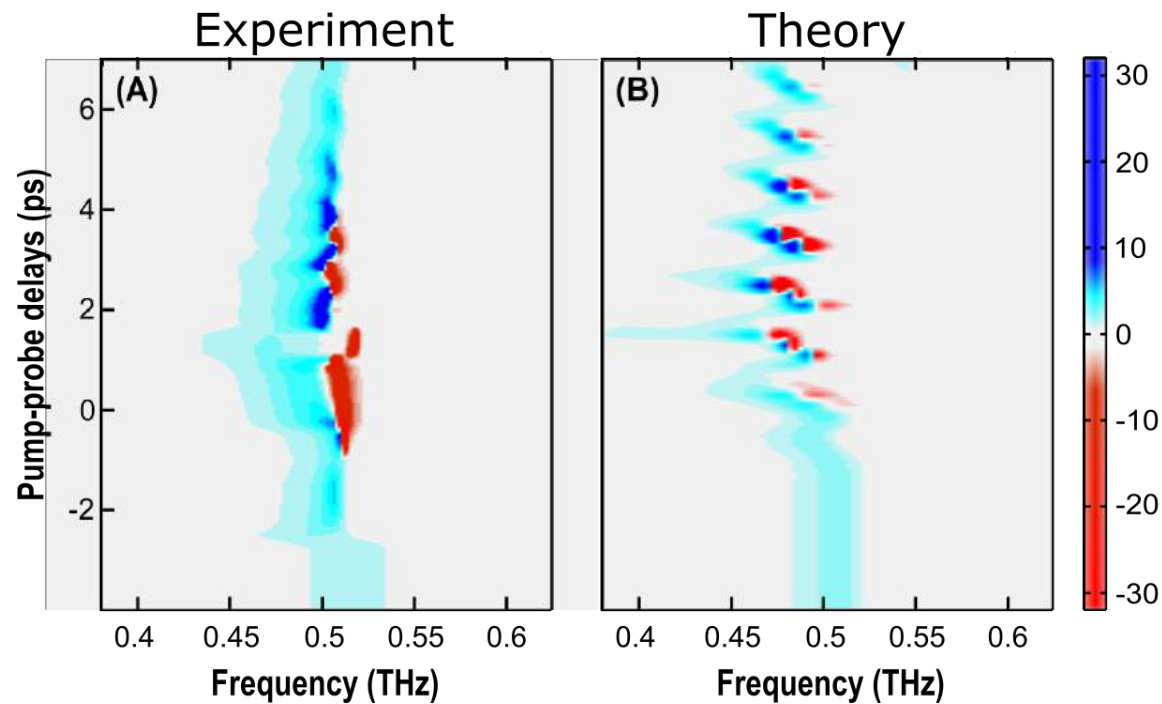
Loss function

$$-\text{Im}\left(\frac{1}{\epsilon(\omega)}\right)$$

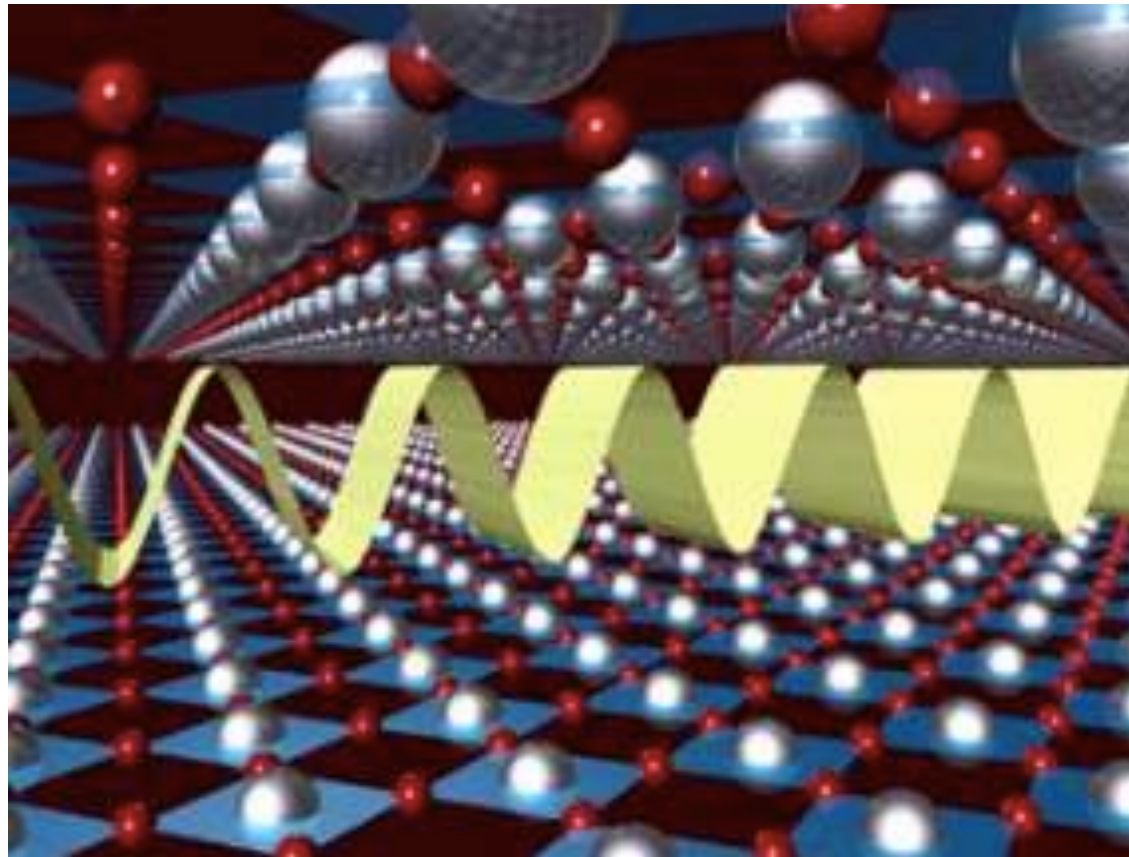


SC fluctuations are amplified or de-amplified

$L(\omega)$ oscillates between positive and negative values

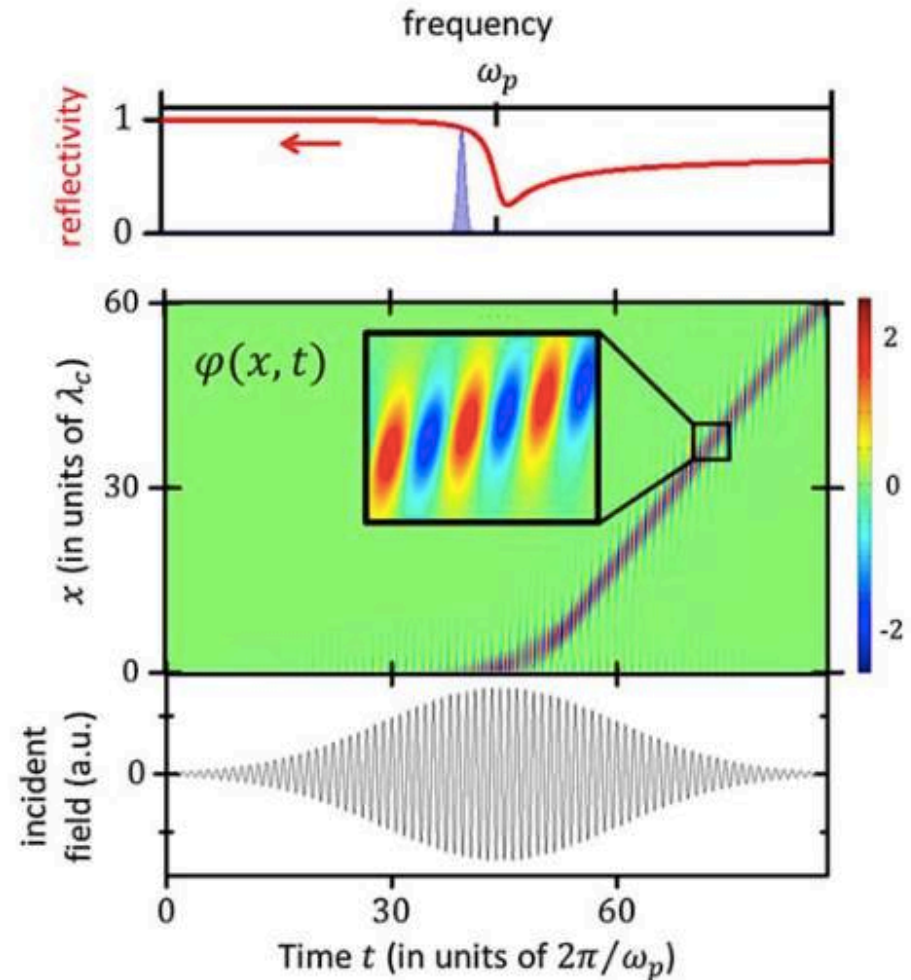
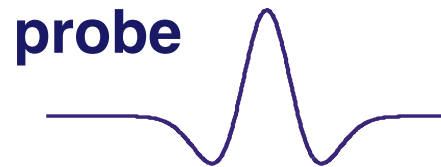
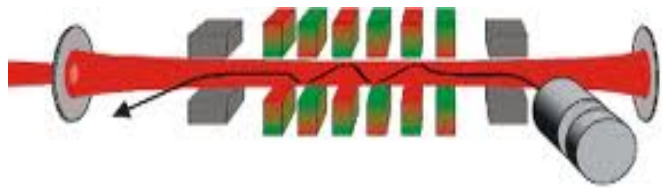


Amplification and de-amplification of SC fluctuations



Other work: Travelling Vortex-antivortex pairs

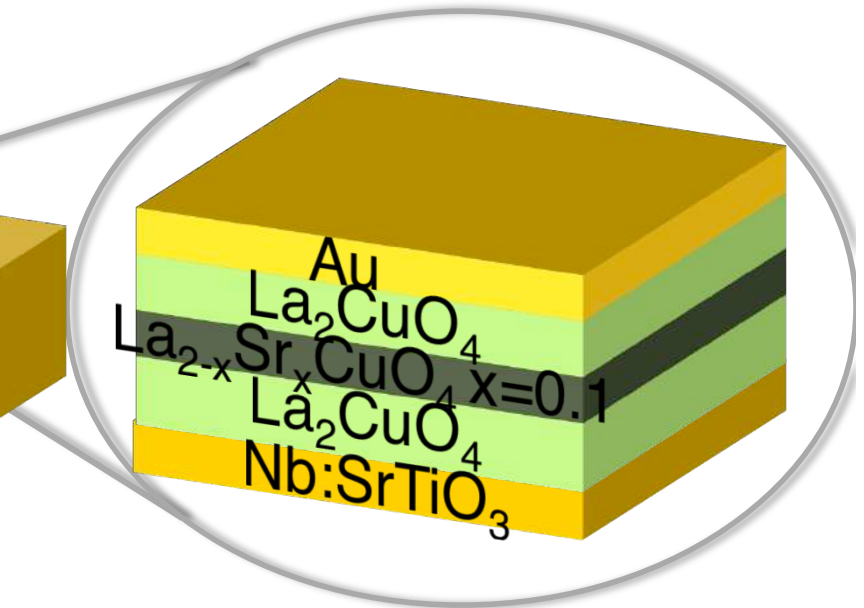
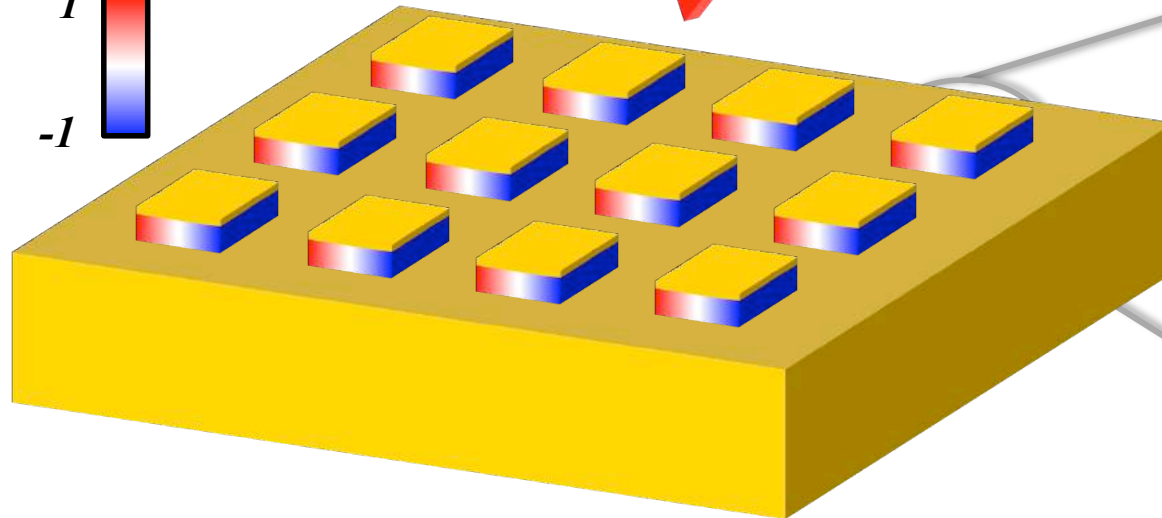
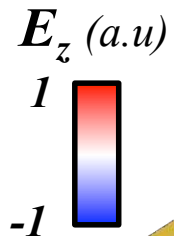
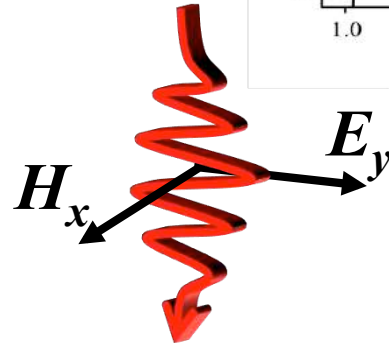
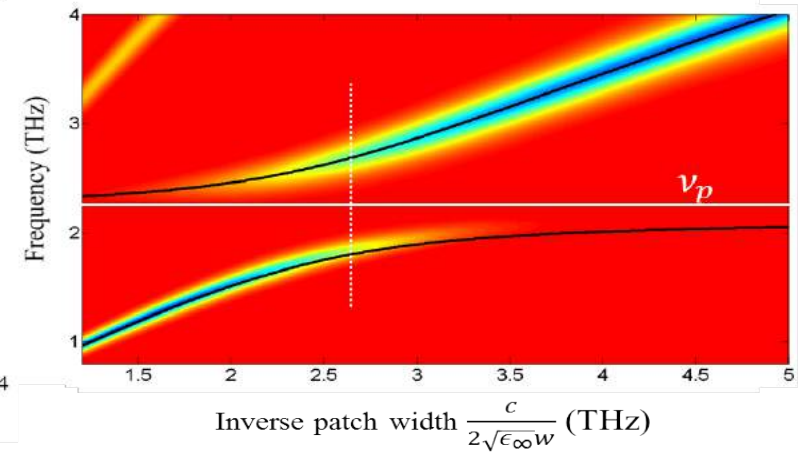
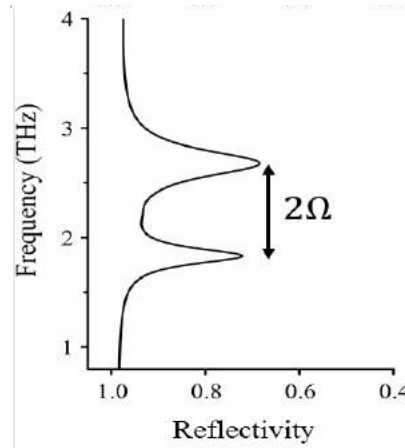
Free Electron Laser - pump



Ultrastrong coupling between light and superfluid

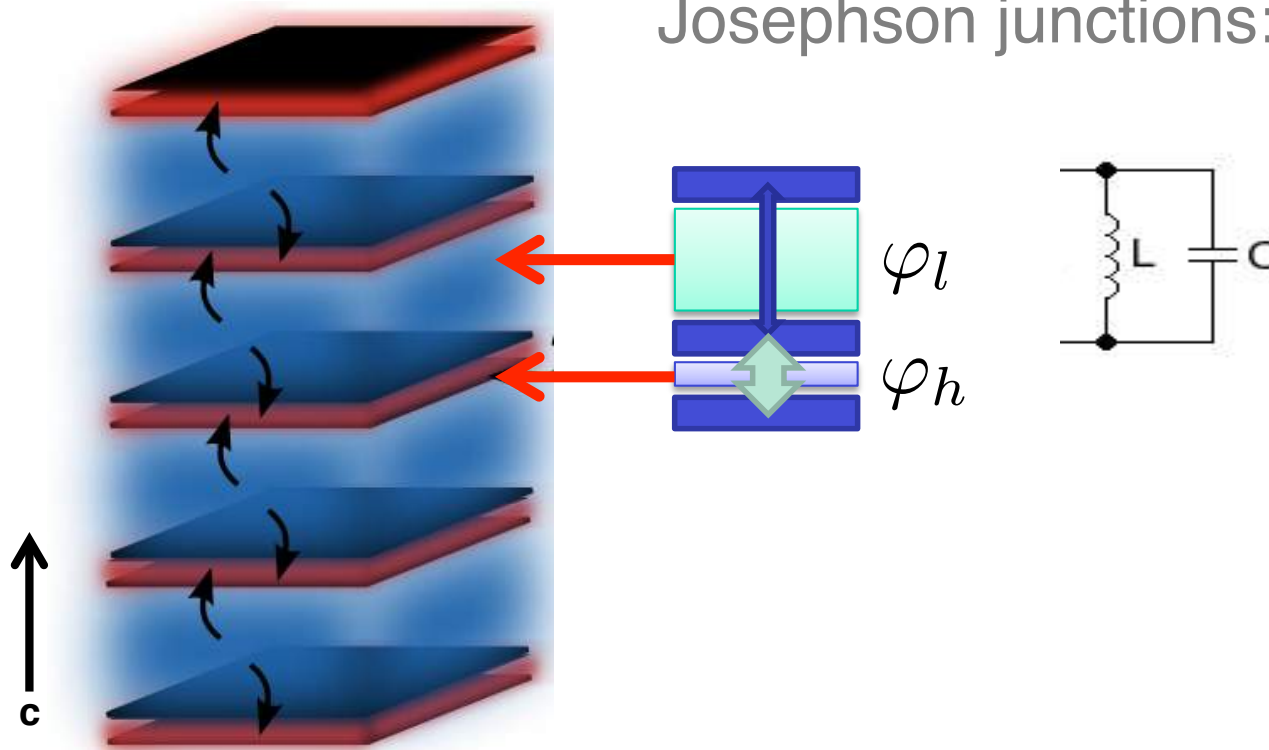


Y. Laplace

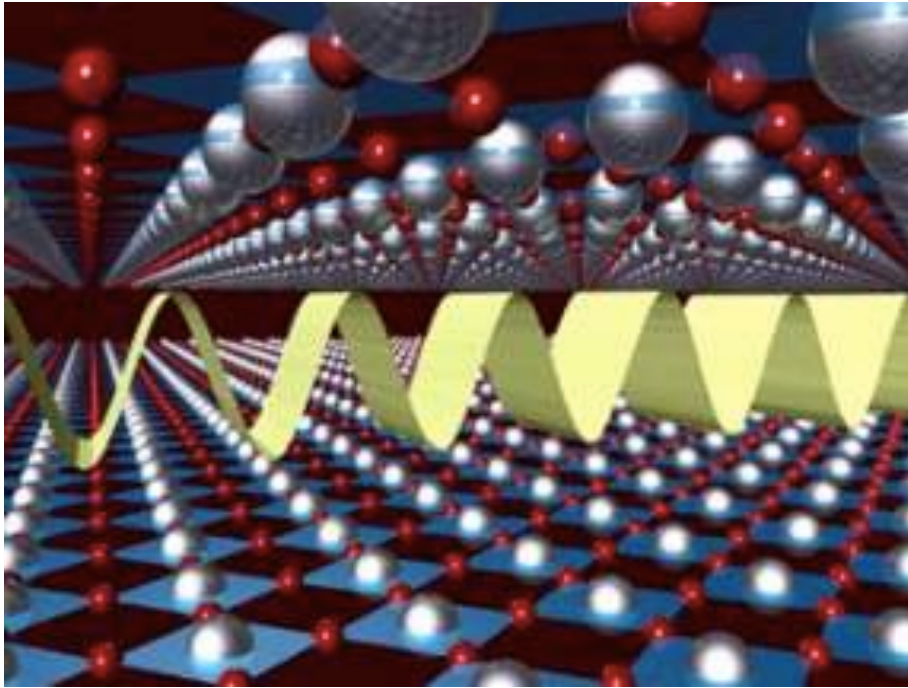


Parametric cooling

YBCO as a stack of Josephson junctions:



Manipulating Superconducting order in cuprates



A. Dienst et al., *Nature Photonics* 5, 485 (2011)

A. Dienst et al. *Nature Materials* 12, 535 (2013)

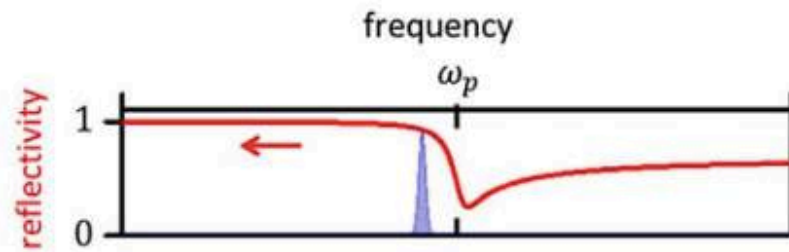
S. Denny, et al. *Phys. Rev. Lett.* 114, 137001 (2015)

J. Okamoto, et al. *Phys. Rev. Lett.* 117, 227001 (2016)

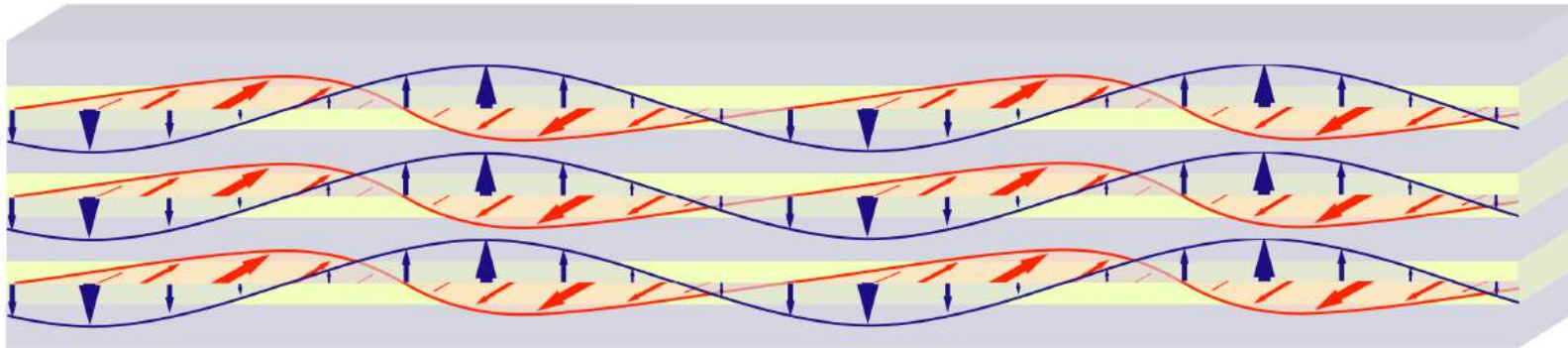
Rajasekaran et al., *Nature Physics* 12, 1012 (2016)

Y. Laplace & A. Cavalleri *Advances In Physics X* – 1, 387 (2016)

Theme 3: Understand Nonlinear Propagation

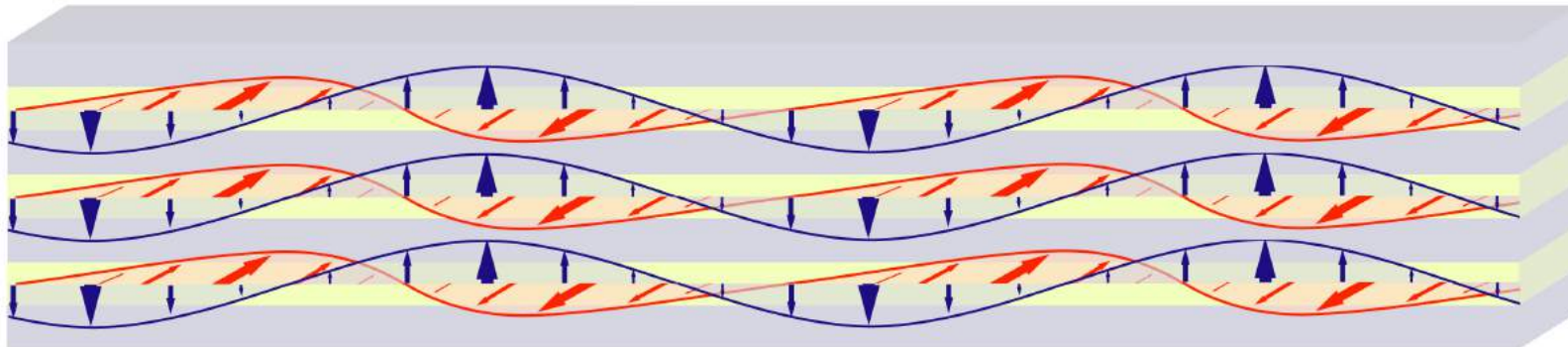


Spectrally pure nonlinear modes



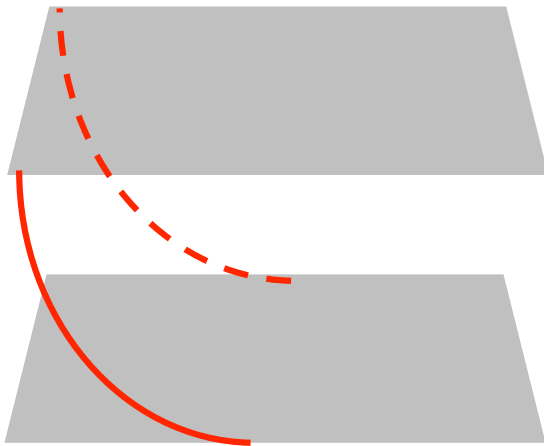
Sine Gordon Equation

$$\frac{\partial^2 \phi_z(x,t)}{\partial x^2} - \frac{\epsilon_r}{c^2} \frac{\partial^2 \phi_z(x,t)}{\partial t^2} = \frac{1}{\lambda_J^2} \sin \phi_z(x,t)$$



Small magnetic field - no time dependence

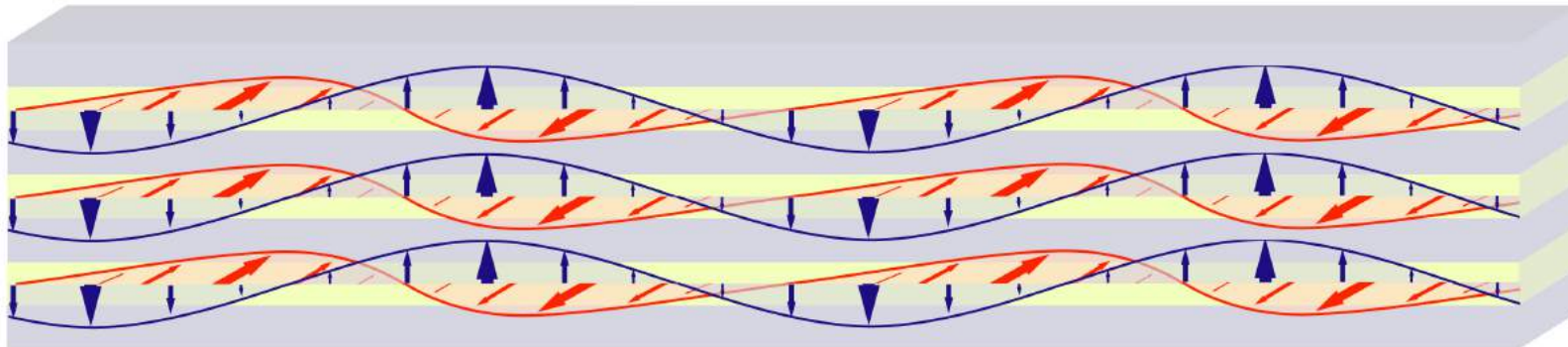
$$\frac{\partial^2 \phi_z(x,t)}{\partial x^2} = \frac{1}{\lambda_J^2} \phi_z(x,t)$$



Meissner effect

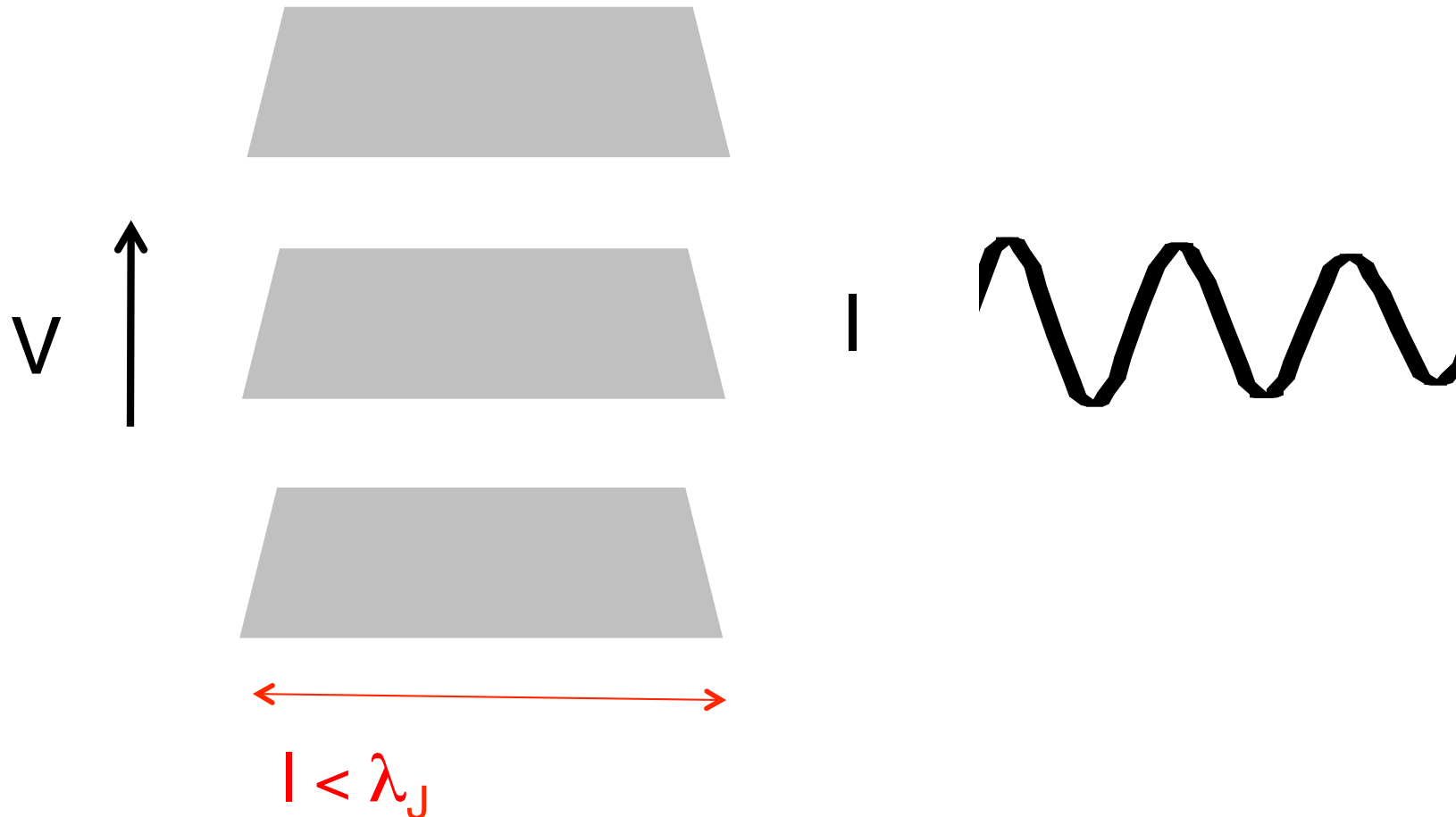
Linear Wave equation

$$\frac{\partial^2 \phi_z(x,t)}{\partial x^2} - \frac{\epsilon_r}{c^2} \frac{\partial^2 \phi_z(x,t)}{\partial t^2} = \frac{1}{\lambda_J^2} \phi_z(x,t)$$



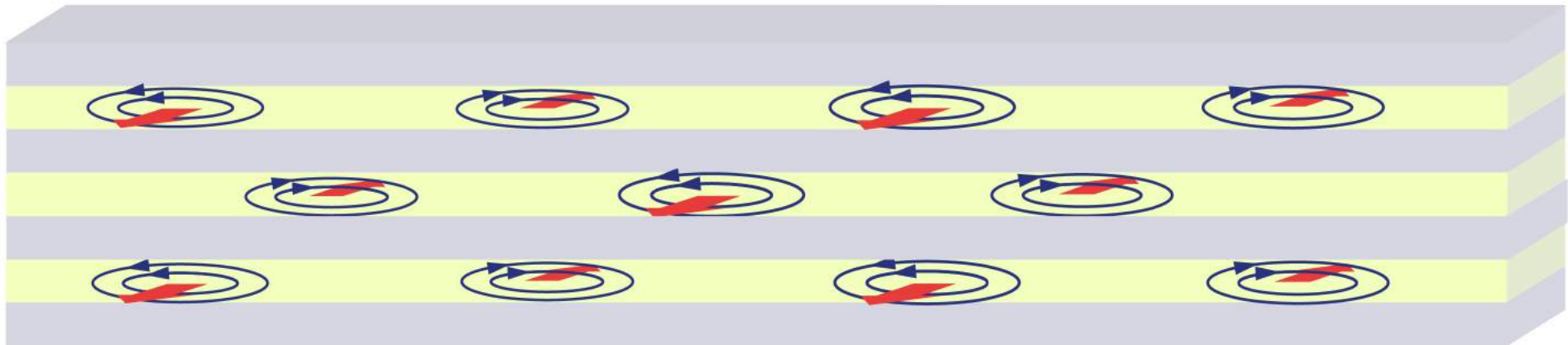
No space dependence – DC Voltage

$$\frac{\epsilon_r}{c^2} \frac{\partial^2 \phi_z(t)}{\partial t^2} = \frac{1}{\lambda_J^2} \sin \phi_z(t)$$



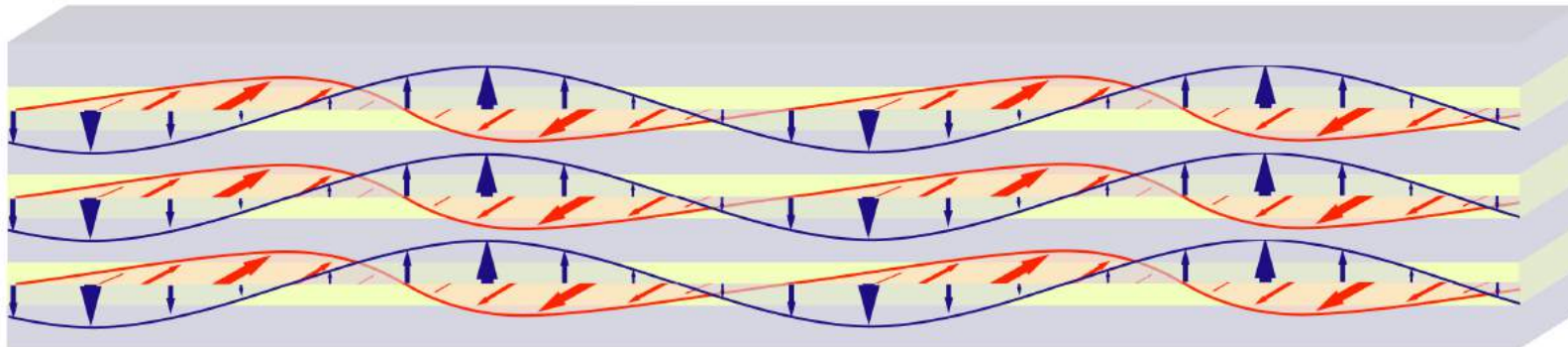
High Magnetic fields - vortex lattice

$$\frac{\partial^2 \phi_z(x, t)}{\partial x^2} = \frac{1}{\lambda_J^2} \sin \phi_z(x, t)$$

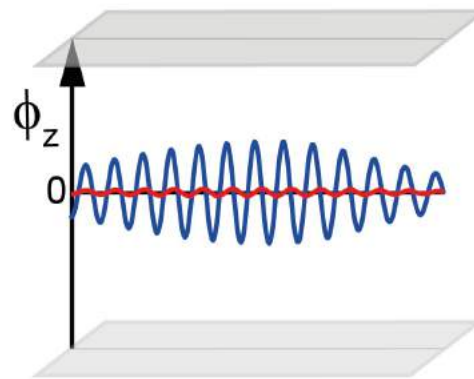
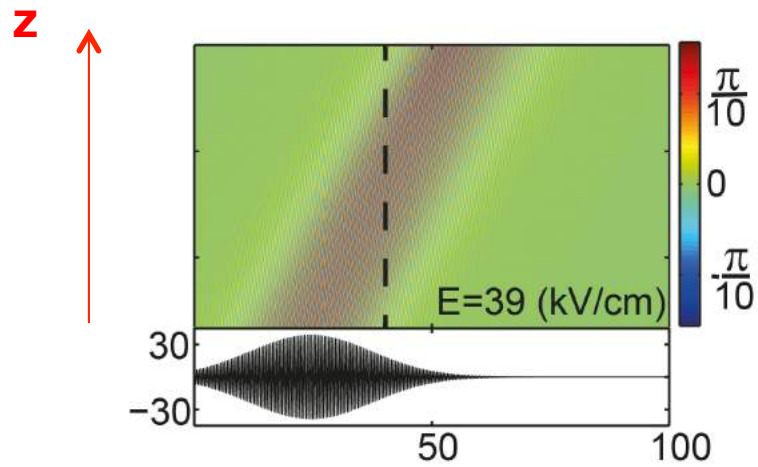
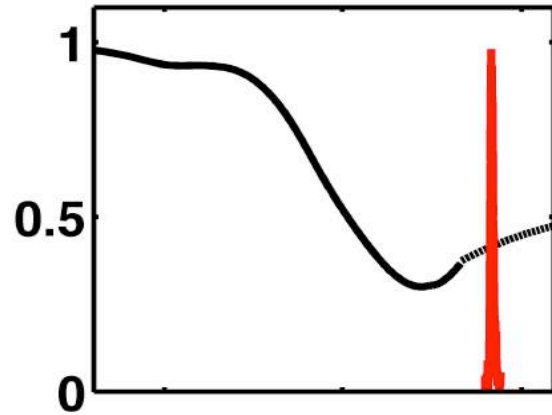


Sine Gordon Equation

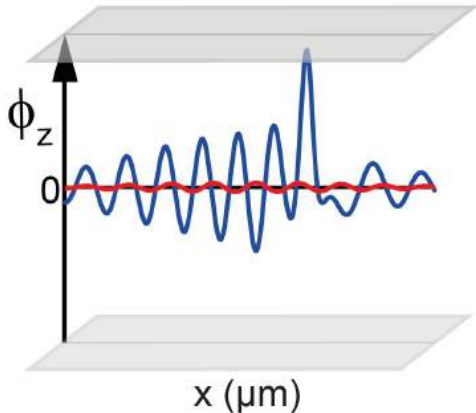
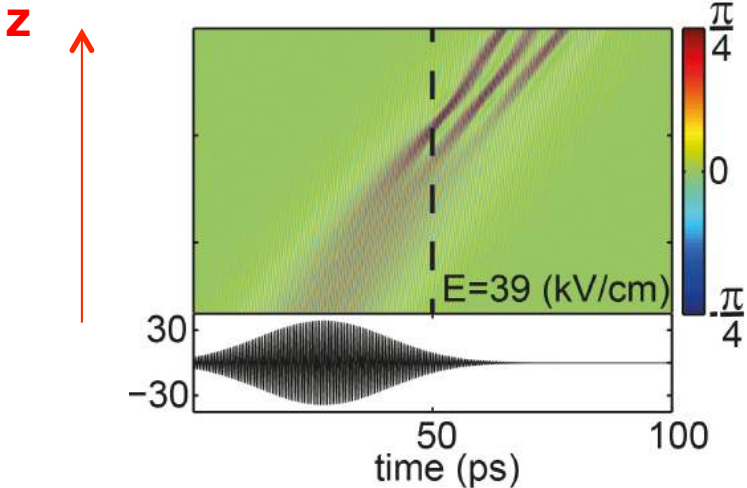
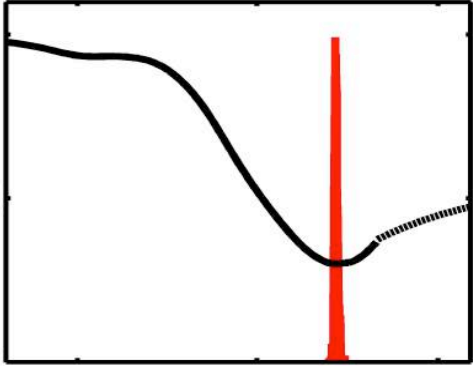
$$\frac{\partial^2 \phi_z(x,t)}{\partial x^2} - \frac{\epsilon_r}{c^2} \frac{\partial^2 \phi_z(x,t)}{\partial t^2} = \frac{1}{\lambda_J^2} \sin \phi_z(x,t)$$



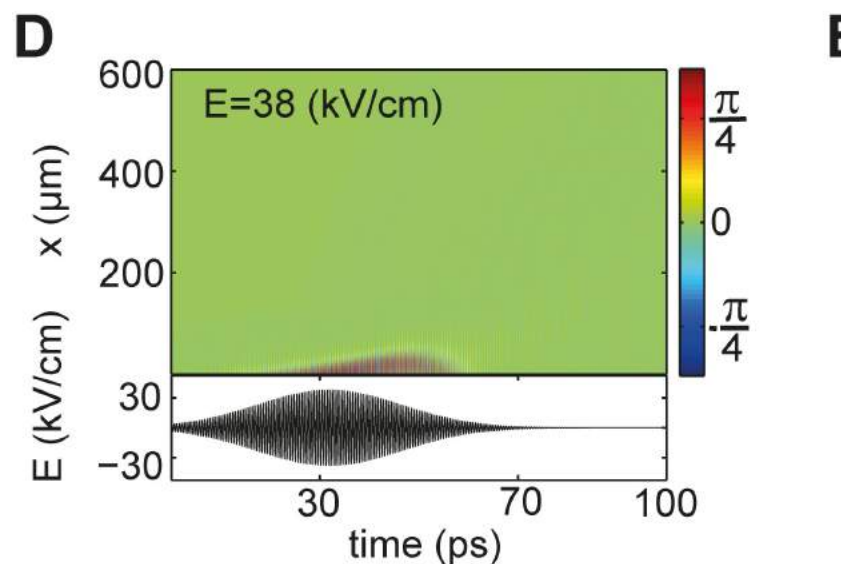
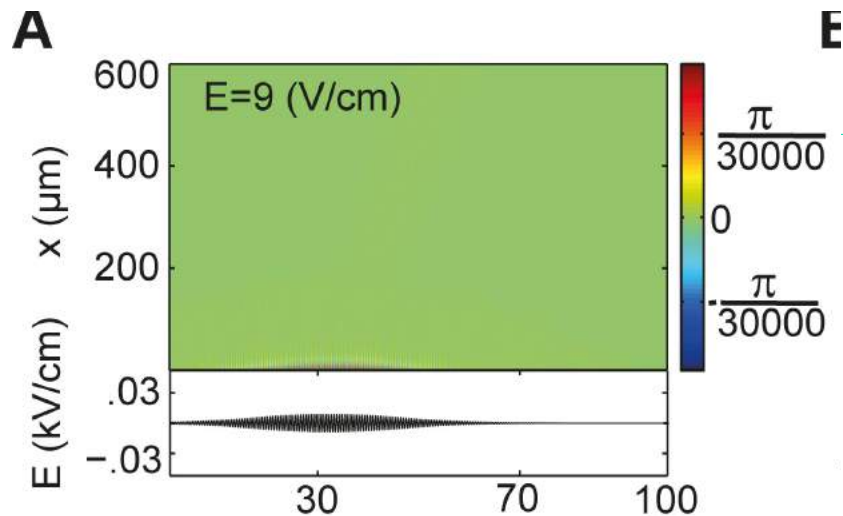
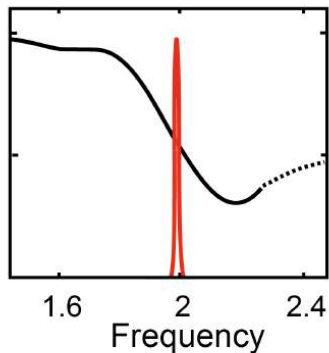
Simulations: 1.1 ω_{JPR}



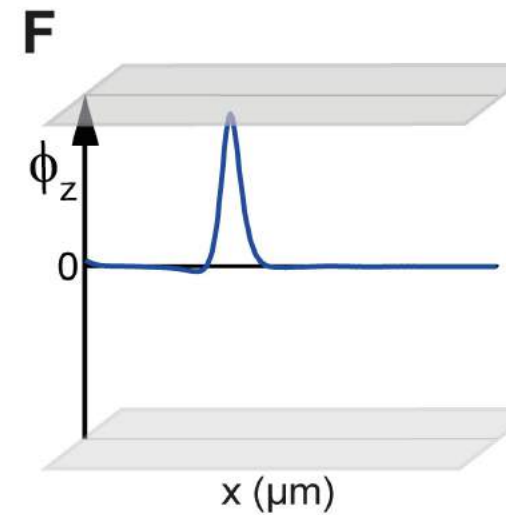
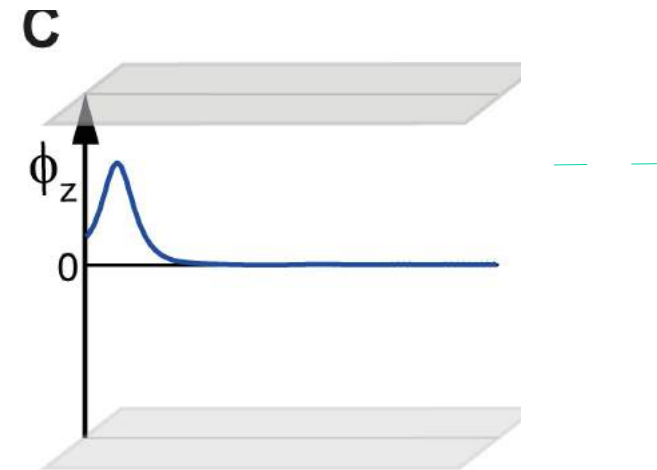
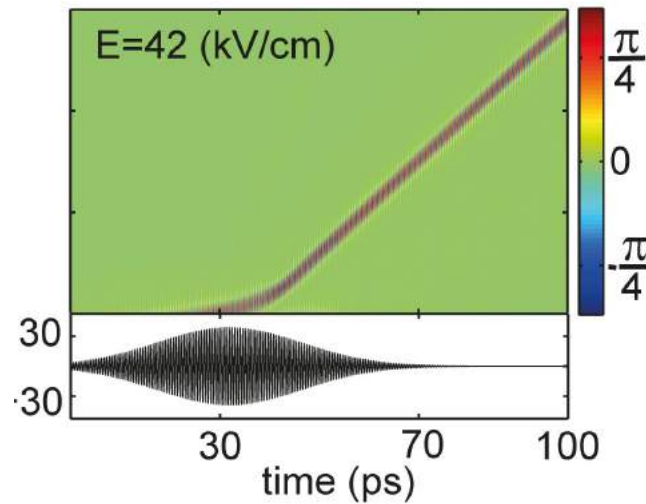
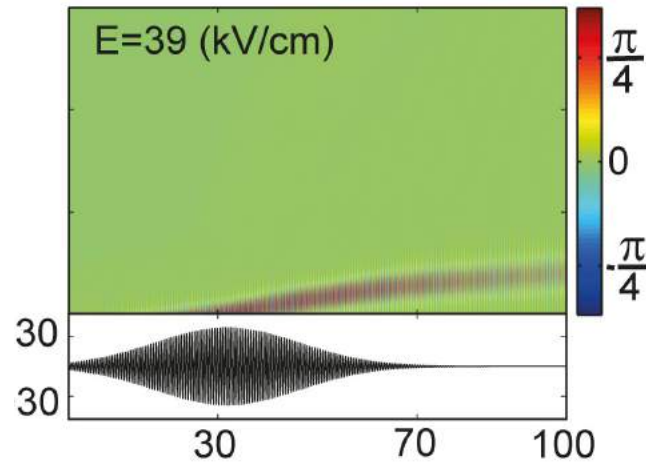
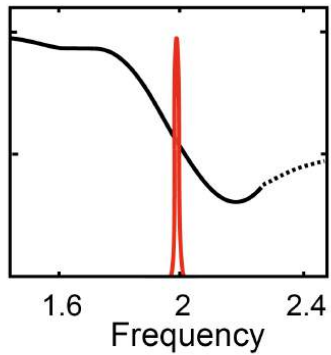
Simulations: $1.05 \omega_{JPR}$



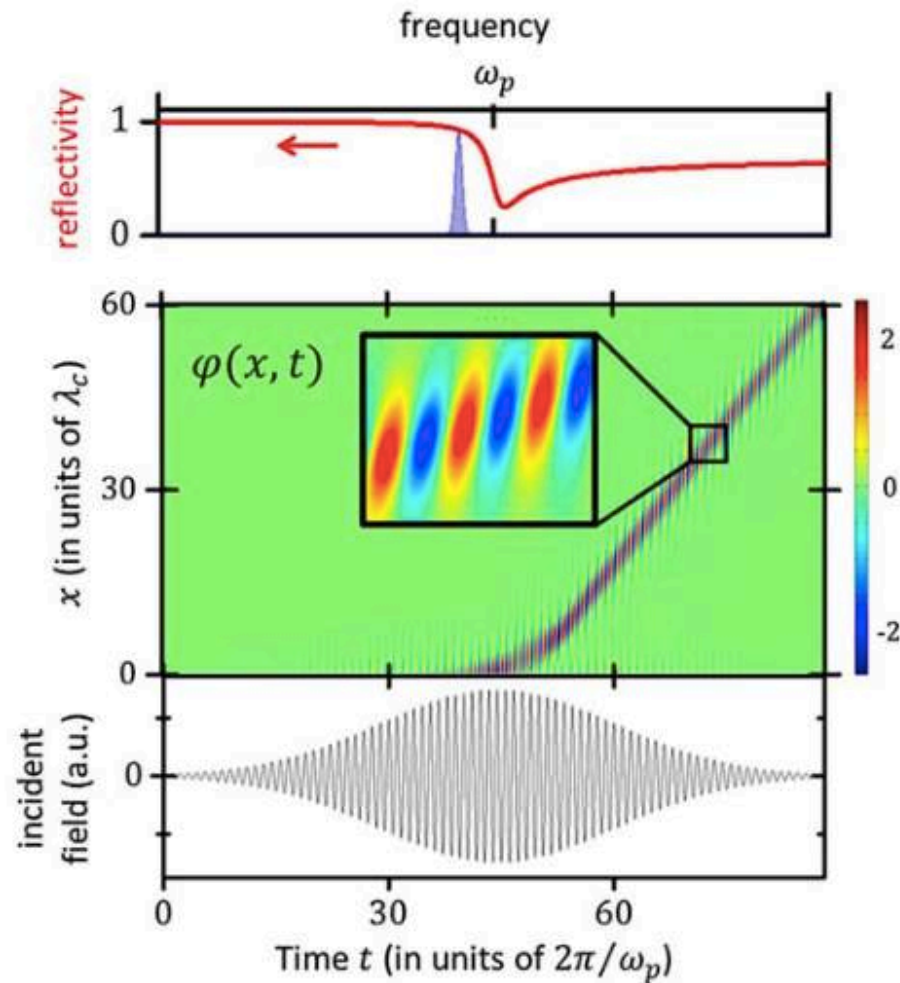
Simulations – below the edge: $0.97 \omega_{\text{JPRmpsd}}$



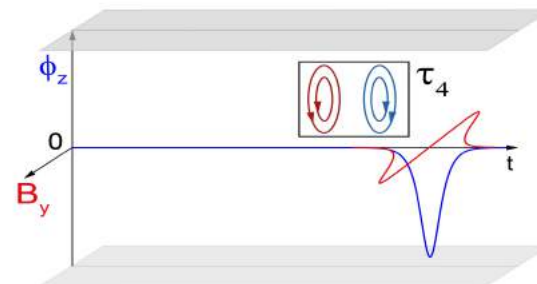
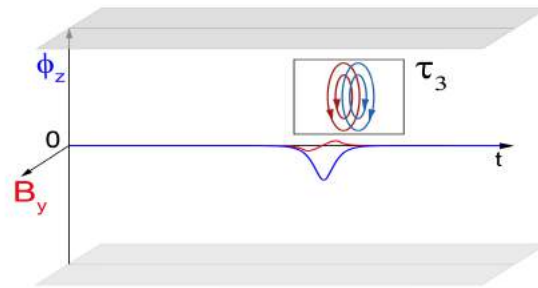
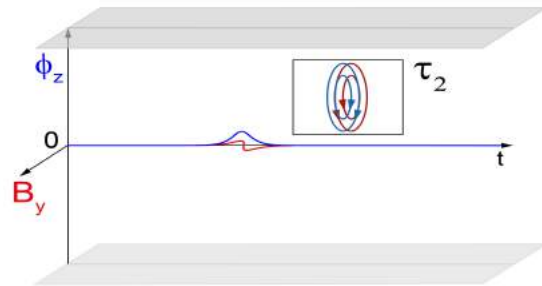
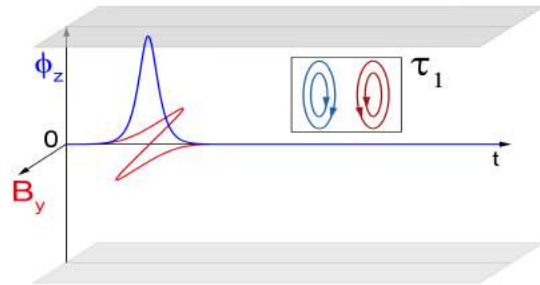
Simulations – high fields : $0.97 \omega_{JPR}$



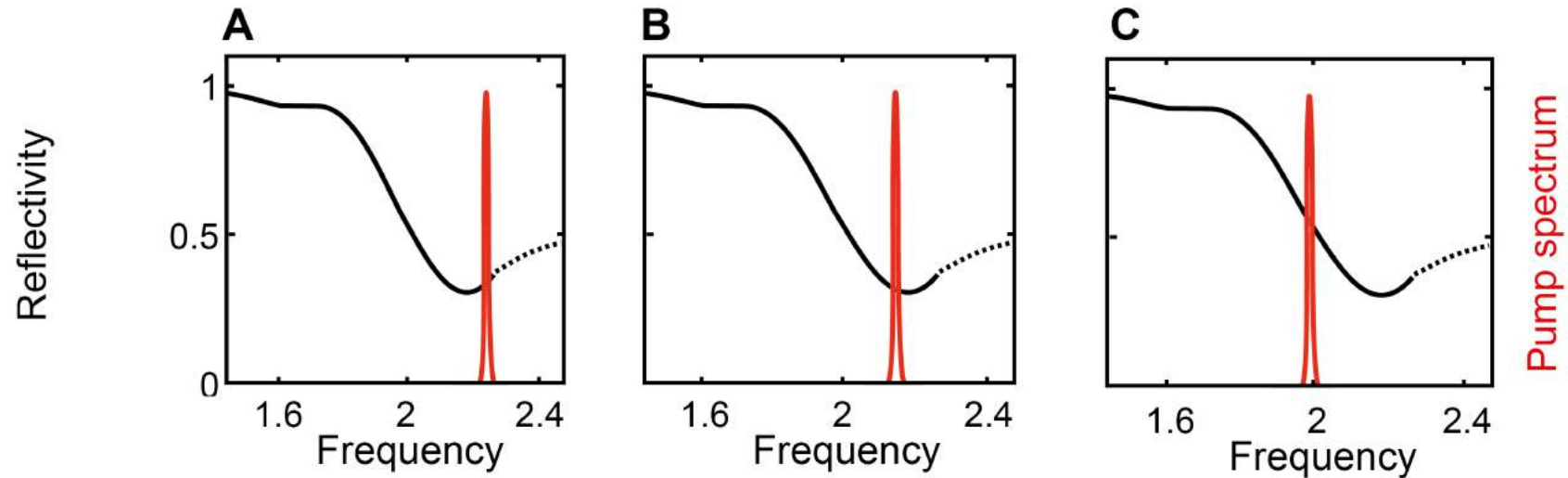
Nonlinear Propagation below the edge



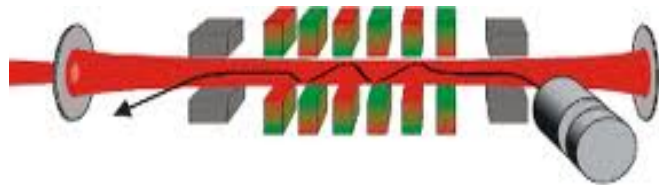
A travelling vortex-antivortex pair



Experimental realization: narrowband pump



Free Electron Laser - pump

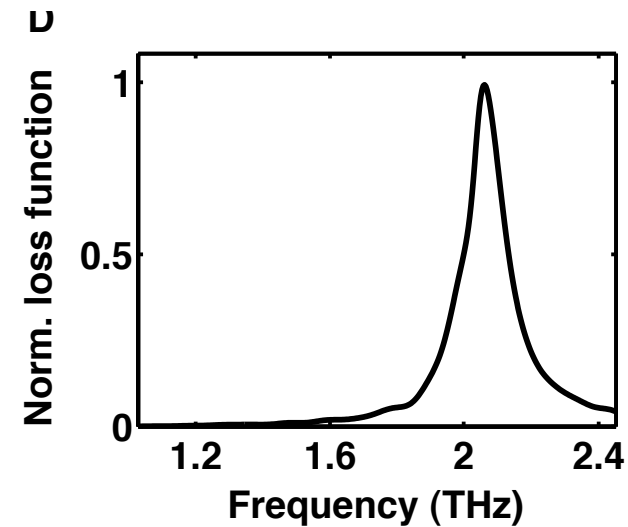
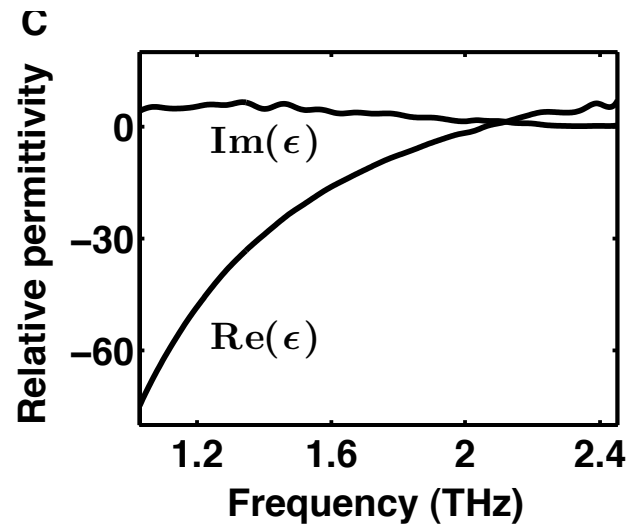


probe

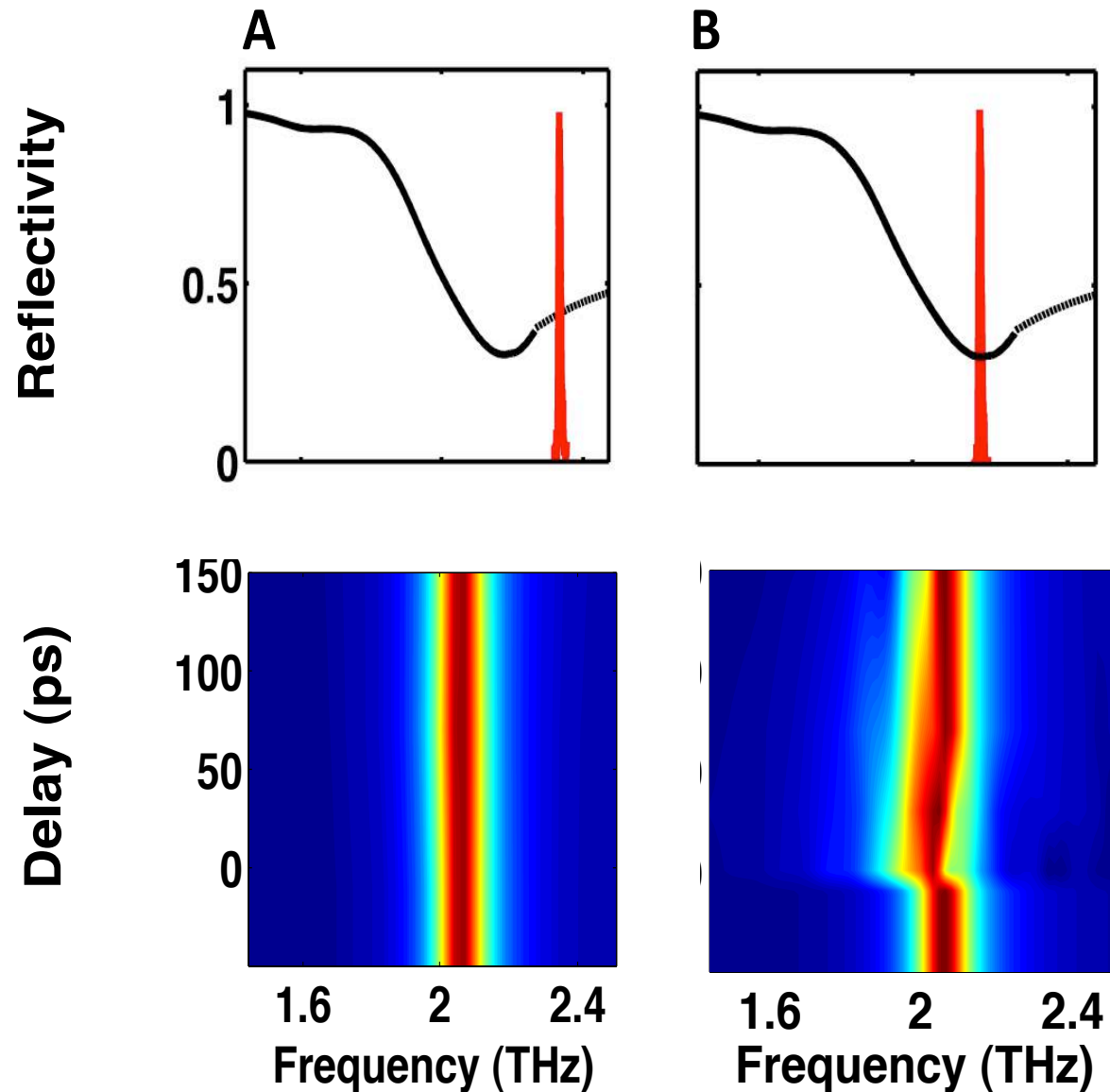


Loss function

$$-\text{Im}\left(\frac{1}{\epsilon(\omega)}\right)$$



Experiments: $1.1 \omega_{\text{JPR}}$ and $1.05 \omega_{\text{JPR}}$

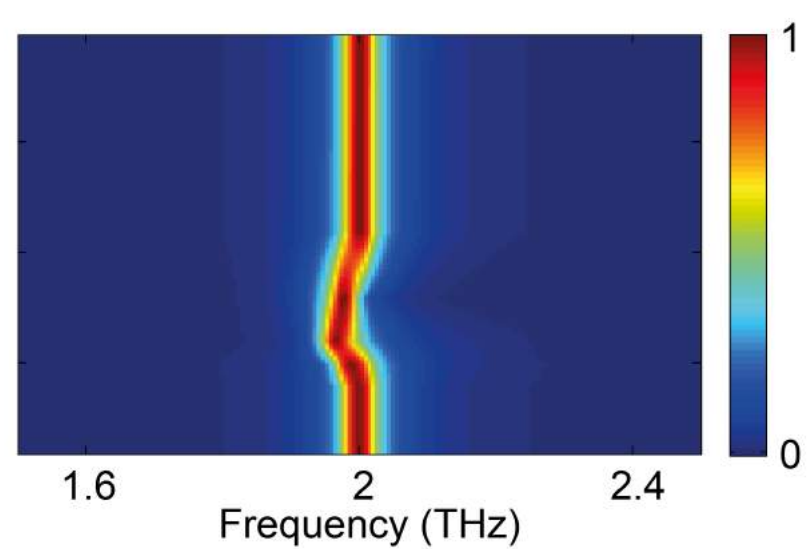
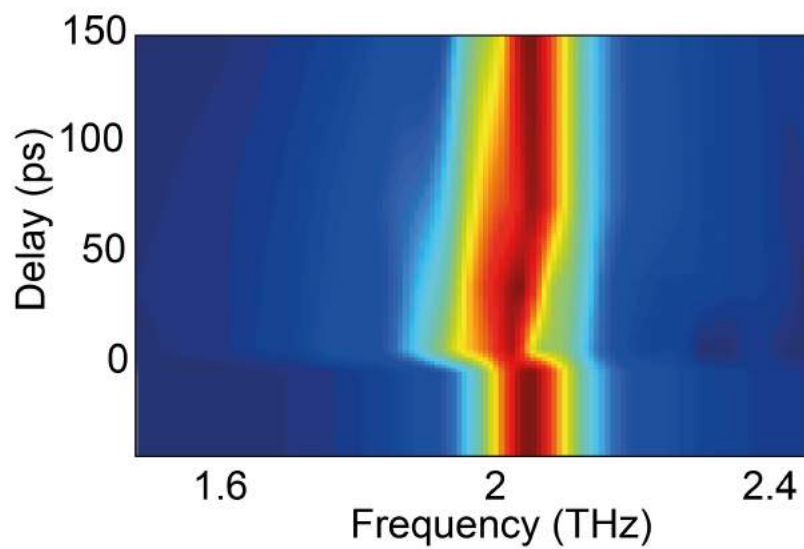
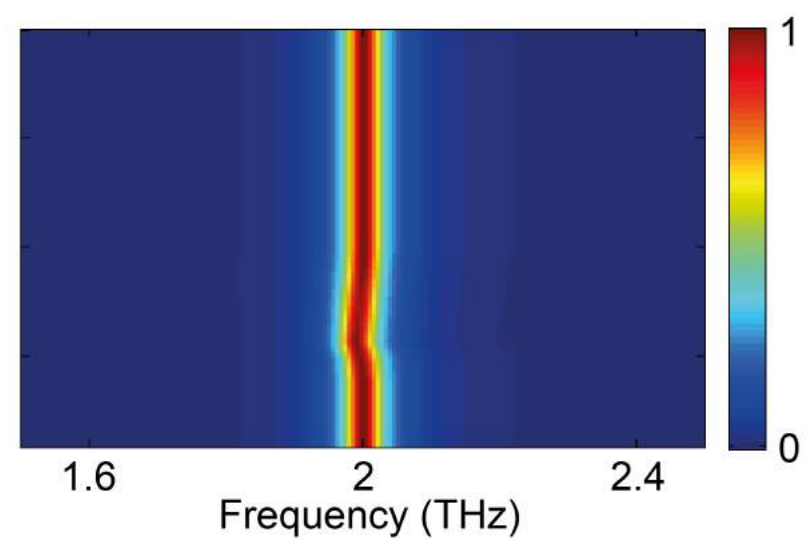
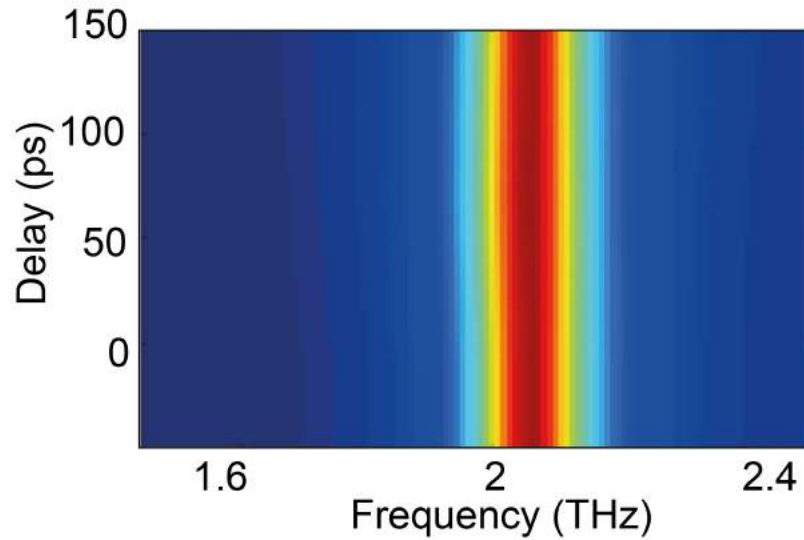


Nonlinear THz response: light induced transparency

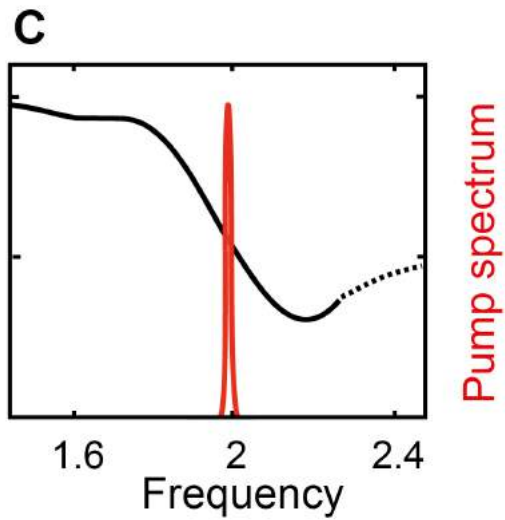


Experiment

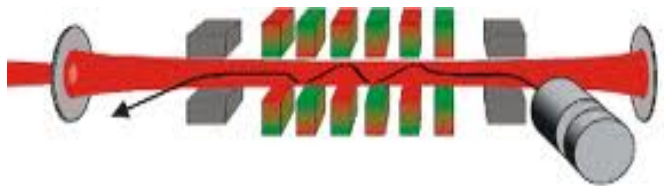
Theory



Experimental realization: narrowband pump



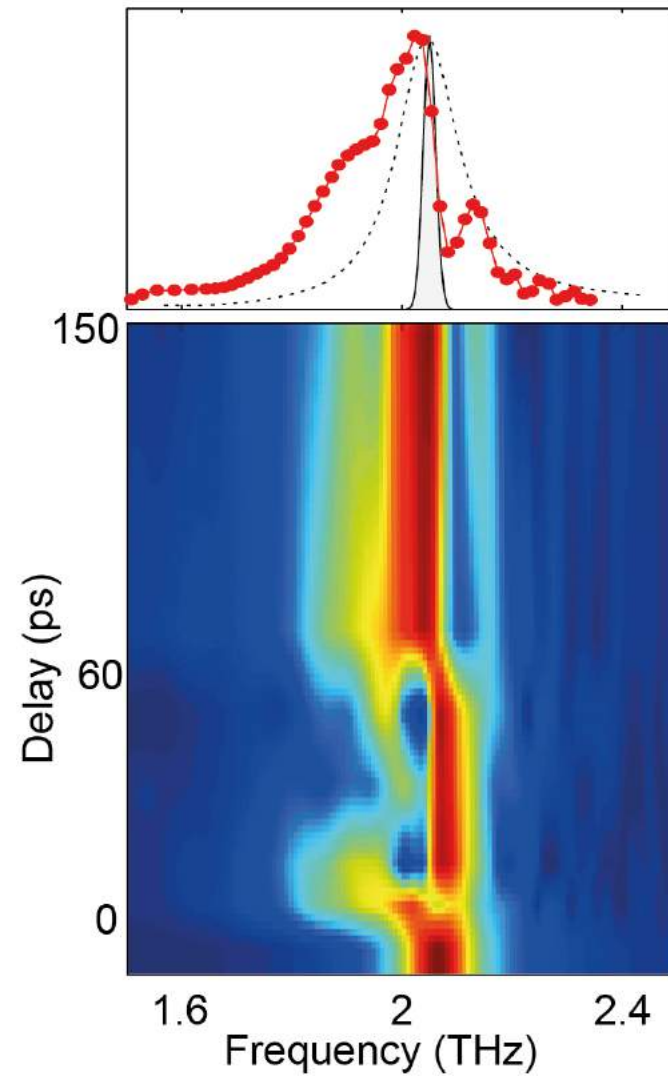
Free Electron Laser - pump



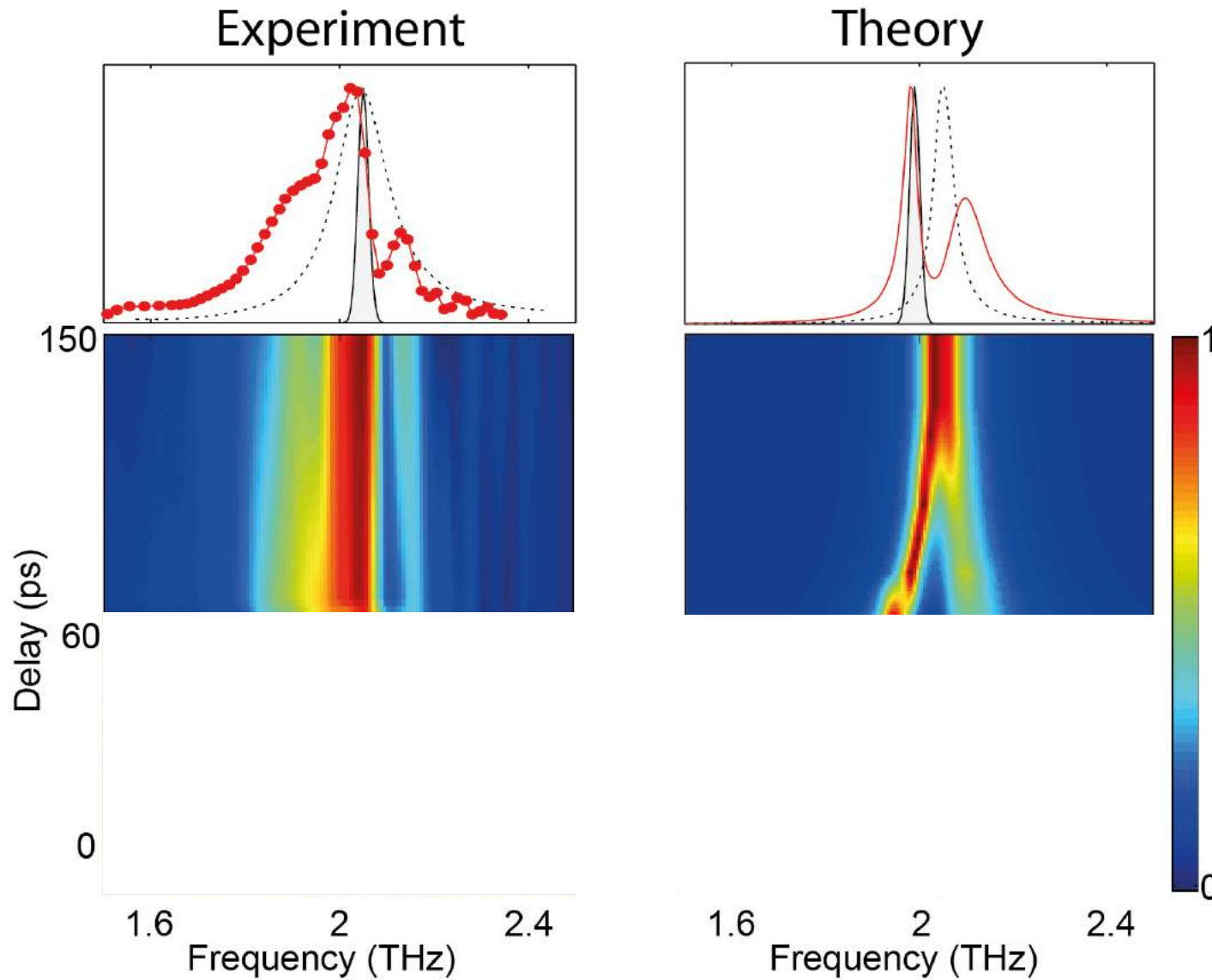
probe



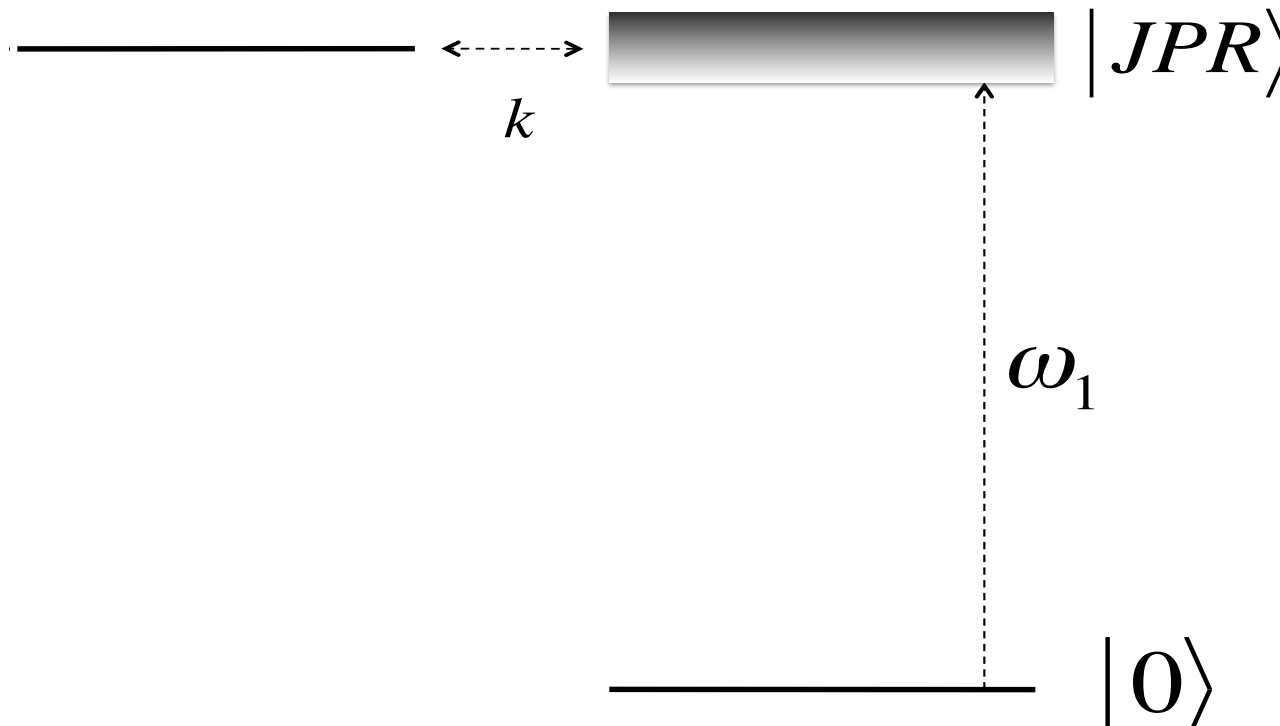
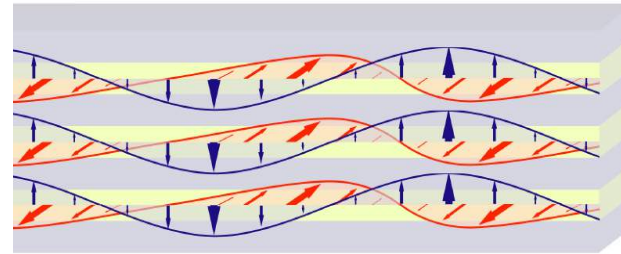
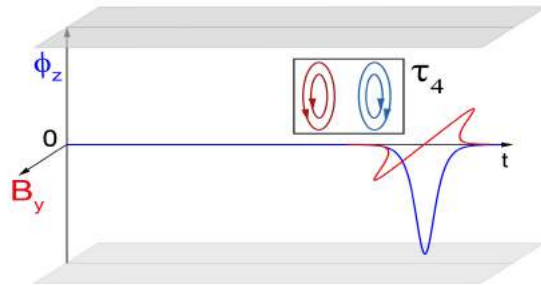
Experiment



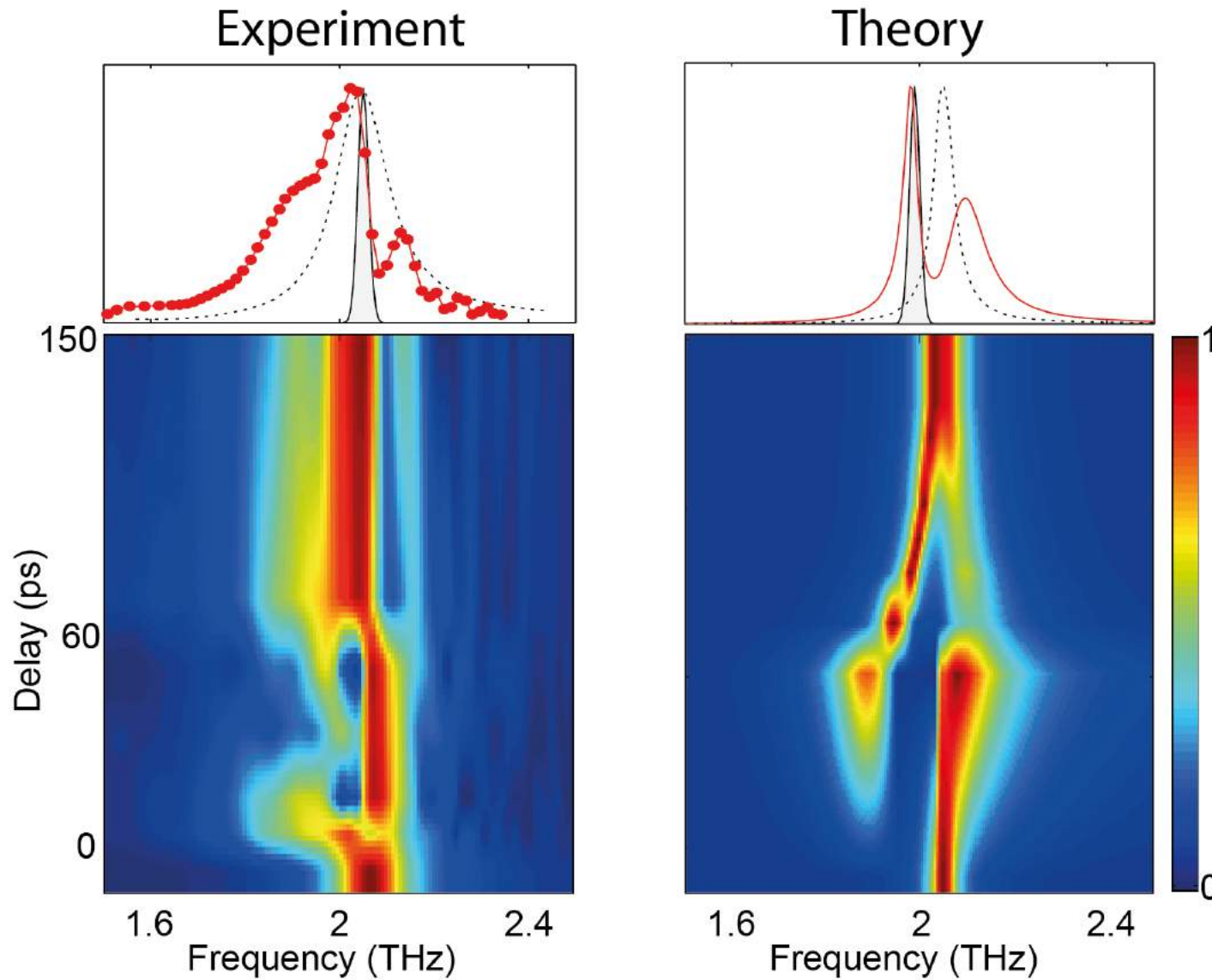
At resonance: Transparency window



Linear Waves interfere with Josephson Solitons



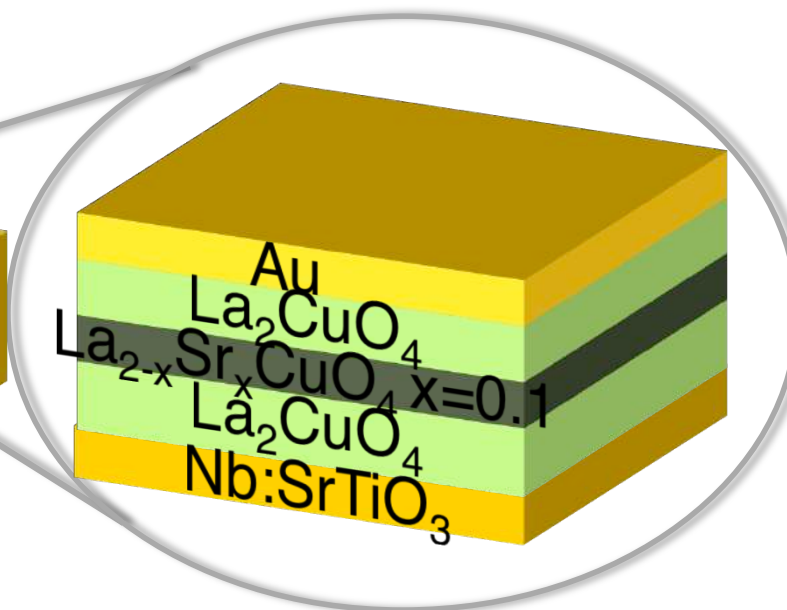
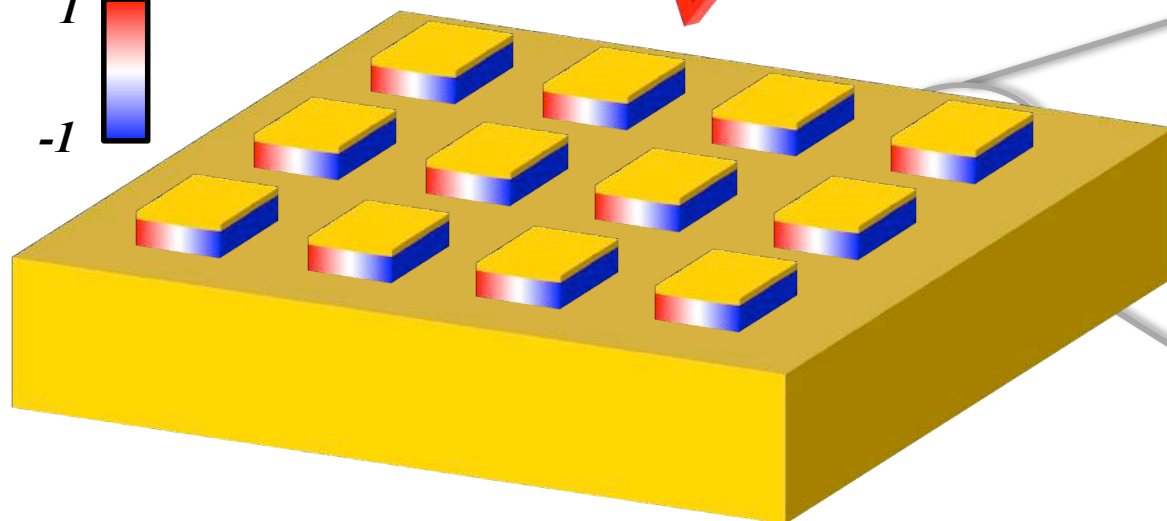
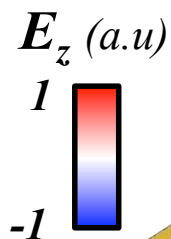
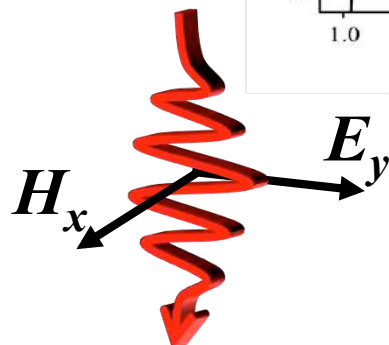
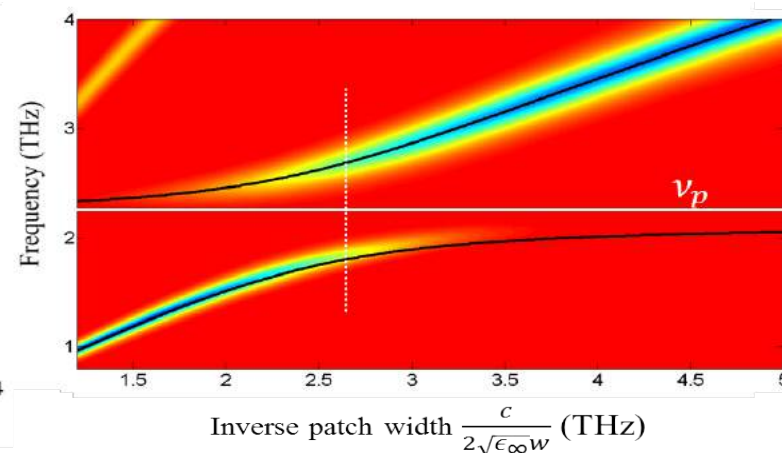
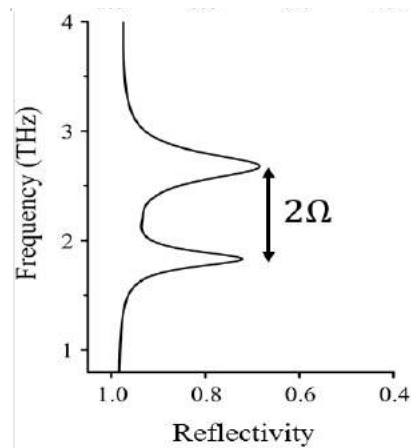
At resonance: Transparency window



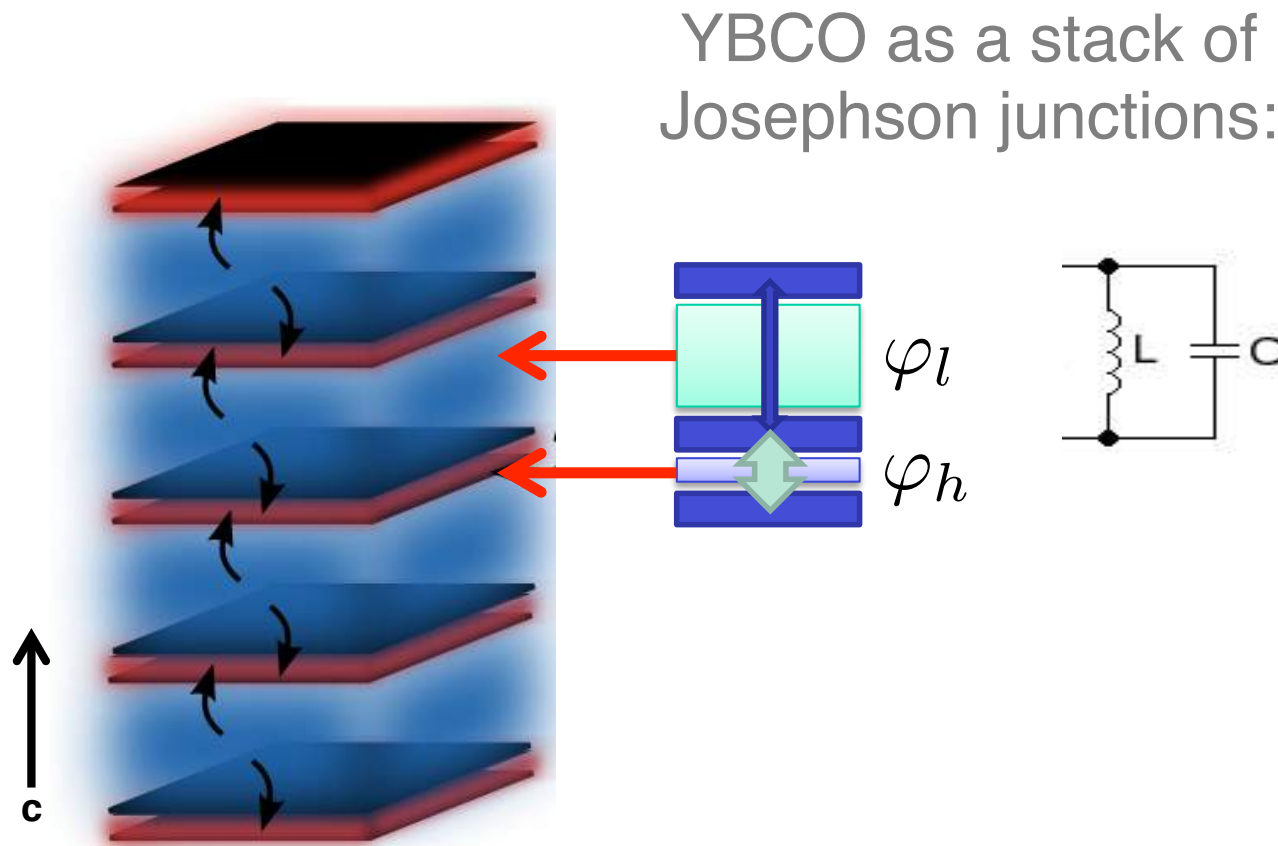
Ultrastrong coupling between light and superfluid



Y. Laplace



Parametric cooling



S. Denny, S. Clark, A. Cavalleri, D. Jaksch *Phys. Rev. Lett.* 114, 137001 (2015)

J. Okamoto, A. Cavalleri, L. Mathey *Phys. Rev. Lett.* 117, 227001 (2016)

Manipulating Superconducting order in cuprates

