COLLÈGE DE FRANCE
1530

Chaire de Physique
de la Matière Condensée
Antoine Georges

Des métaux étranges aux trous noirs: autour des modèles SYK

SYK: Sachdev-Ye-Kitaev

Cours 1 – Introduction, Motivations, Vue d'ensemble

Cycle 2021-2022
10 mai 2022

COLLÈGE DE FRANCE
1530

Chaire de Physique
de la Matière Condensée
Antoine Georges

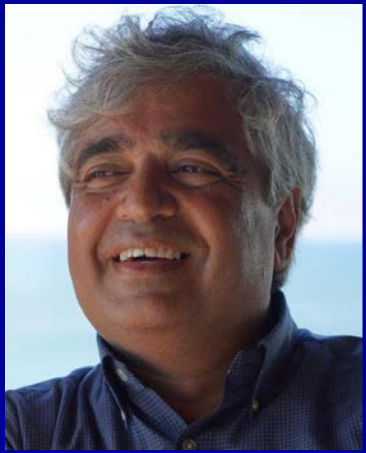
From Strange Metals to Black Holes: Sachdev-Ye-Kitaev (SYK) Models and related topics

Lecture 1 – Introduction, Motivations, Overview

2021-2022 Lectures
May 10, 2022

Organization of the Lectures

No Seminars this year!



- A cycle of invited lectures by Subir Sachdev on May 17,24,31 and June 7
- And a Workshop/Mini-Conference on June 2-3 *`Strange Metals, SYK Models and Beyond'* *(Participation open to all. Registration on website to open soon)*

Confirmed Speakers at the June 2-3 Workshop

- Laura Foini (IPhT Saclay)
- Blaise Goutéraux (CPHT, Ecole Polytechnique)
- Gaël Grissonnanche (Cornell)
- Nigel Hussey (Nijmegen)
- Olivier Parcollet (Flatiron Institute, New York)
- Silke Paschen (TU-Wien)
- Catherine Pépin (IPhT, Saclay)
- Lucile Savary (ENS Lyon, currently visiting CdF)
- Koenraad Schalm (Leiden)
- Marco Schiro (Collège de France)
- Julian Sonner (U. of Geneva)
- Jörg Schmalian (Karlsruhe)
- Evyatar Tulipman (Weizmann Institute)
- Dirk van der Marel (U. of Geneva)
- Pengfei Zhang (Caltech)

10 mai 2022 à 9h30 et 11h30
COURS (I+II) :
Métaux étranges, Dissipation Planckienne, modèles SYK : Introduction

17 mai 2022 à 9h30
COURS (III) : *Entropie et asymétrie spectrale des modèles SYK*
CONFÉRENCE INVITÉE — Subir Sachdev :
Beckenstein-Hawking Entropy of a Black Hole.

24 mai 2022 à 9h30
COURS (IV) : *Modèles t-J désordonnés : criticalité, dissipation Planckienne*
CONFÉRENCE INVITÉE — Subir Sachdev :
Schwarzian Theory of SYK Fluctuations and T-Linear Resistivity

Content of the lectures on this slide is <u>tentative</u> and will be adapted as the lectures go on…

31 mai 2022 à 9h30
COURS (V) : *Modèles t-J désordonnés : criticalité, dissipation Planckienne* (suite)
CONFÉRENCE INVITÉE — Subir Sachdev :
Fermi Surface Coupled to Gauge Fields (I)

2 et 3 juin 2022 — COLLOQUE/WORKSHOP :
Strange Metals, SYK Models and Beyond

7 juin 2022 à 9h30
COURS (VI) : *Perspectives et applications physiques*
CONFÉRENCE INVITÉE — Subir Sachdev :
Fermi Surface Coupled to Gauge Fields (II)

Mailing List

(Weekly announcement of lecture and seminar, etc.)

Send email to: listes-diffusion.cdf@college-de-france.fr
Subject line: subscribe chaire-pmc.ipcdf
…or: unsubscribe chaire-pmc.ipcdf

You can also just send me an email to be placed on the list

Website:

https://www.college-de-france.fr/site/antoine-georges/index.htm

Lectures are video recorded
and available on the CdF website

If you are interested in participating in the June 2-3 workshop:

- Please make sure you are on the mailing list of the lectures
- If you are not, please subscribe or send me an email
- Watch your inbox in the coming days for a mail containing registration information and please register immediately
- Registration is free but highly recommended to help w/ organisation

CCQ-New York

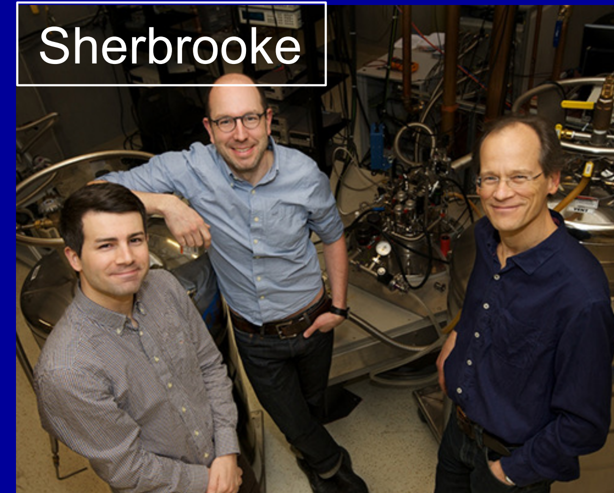
Philipp Dumitrescu

Olivier Parcollet

Nils Wentzell

Alex Wietek

Sherbrooke



Gaël Grissonnanche
Nicolas Doyron-Leyraud
Louis Taillefer

Adrien Gourgout

Harvard

Subir Sachdev

Henry Shackleton

THANK YOU!

Geneva

Dirk van der Marel

Christophe Berthod

Bastien Michon

Jernej Mravlje

Ljubljana

Cornell

Peter Cha

Eun-Ah Kim

Today: Lectures 1 and 2
(9:30-11:00 and 11:30-13:00)

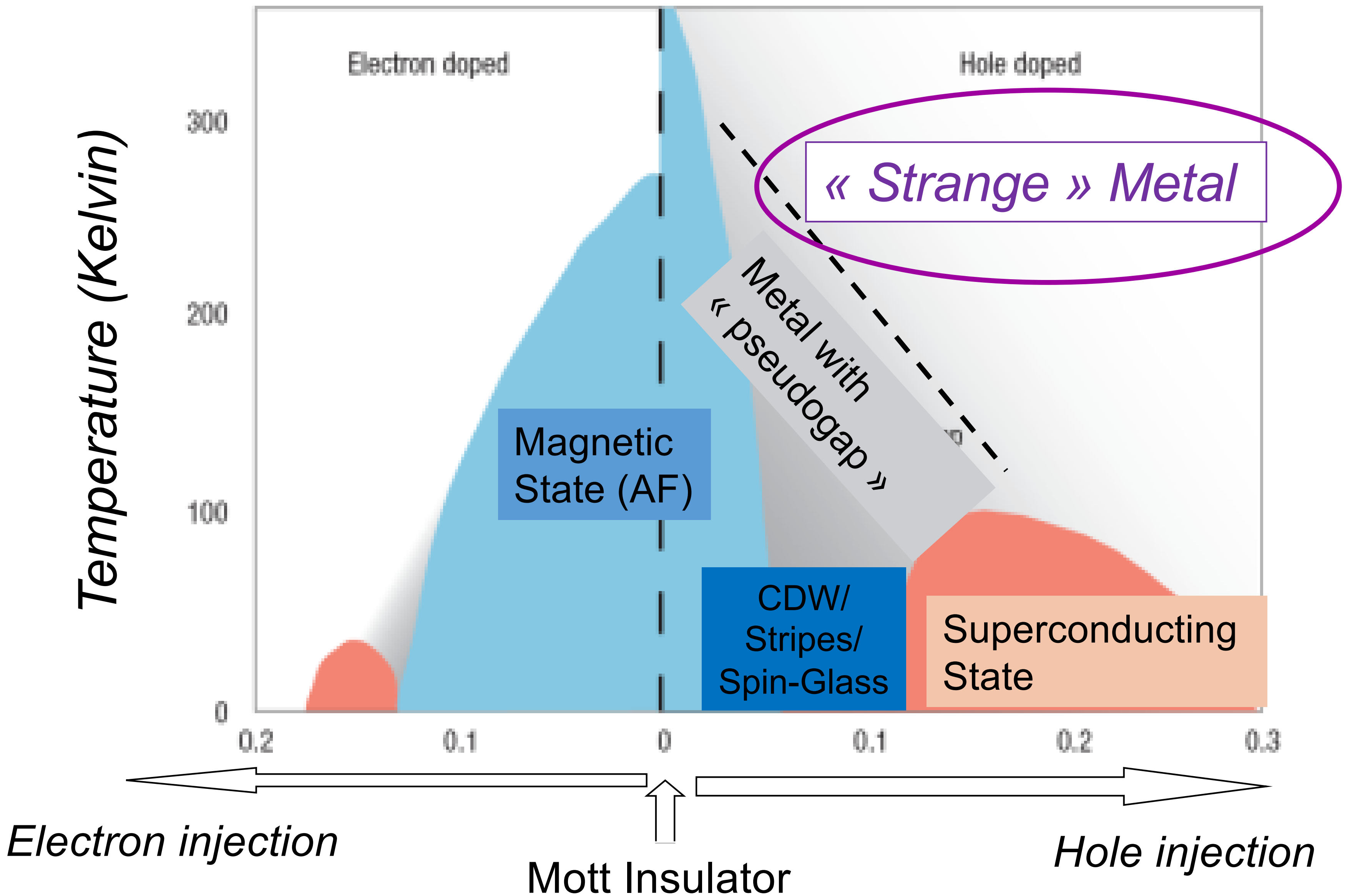
- Broad-band overview – Motivations (`Colloquium style’)
- Introduction to the SYK model and its solution

A central theme of these lectures: `Strange Metals' and the Breakdown of the Quasiparticle Concept!

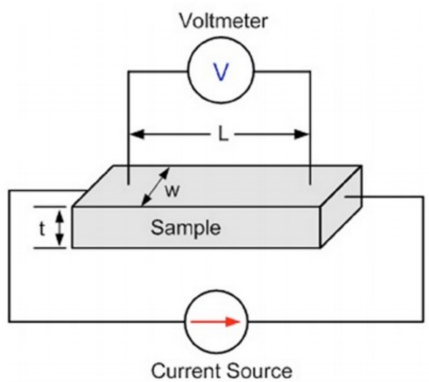
- An emerging theme in condensed matter physics since the discovery of the high-Tc copper-oxide (`cuprates') superconductors in 1986
- Experimental diagnosis: transport, optical (and other) spectroscopies
- Materials: High-Tc cuprates, Some Fe-based SCs,some Heavy-Fermion materials close to QCPs, Twisted bilayer graphene (TBLG), etc…

`Strange' Metals

Typical phase diagram of copper-oxide Superconductors



What is Strange About `Strange Metals'?
Linear Resistivity



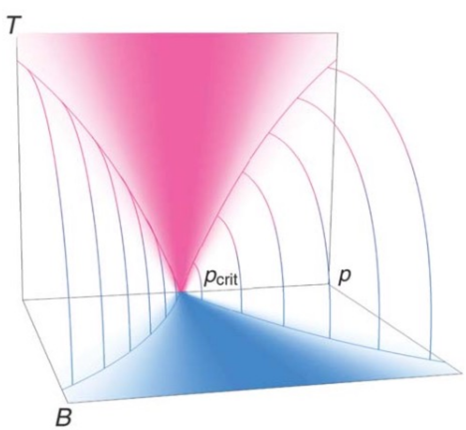
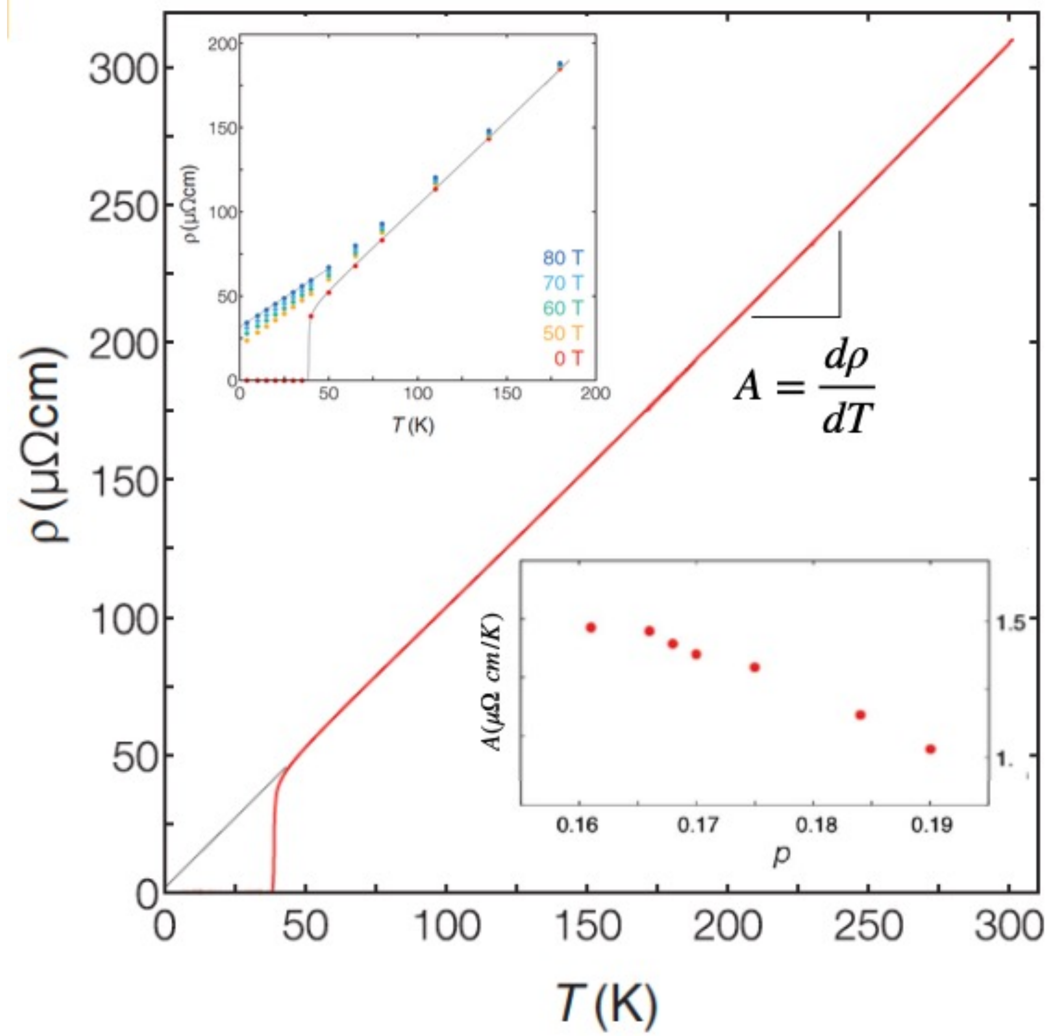
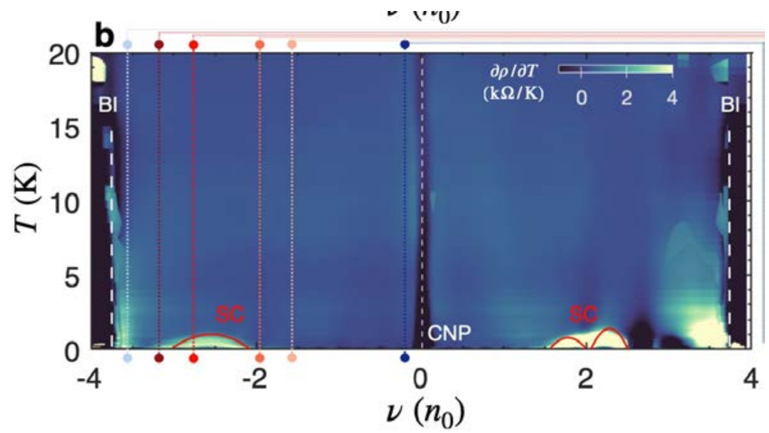


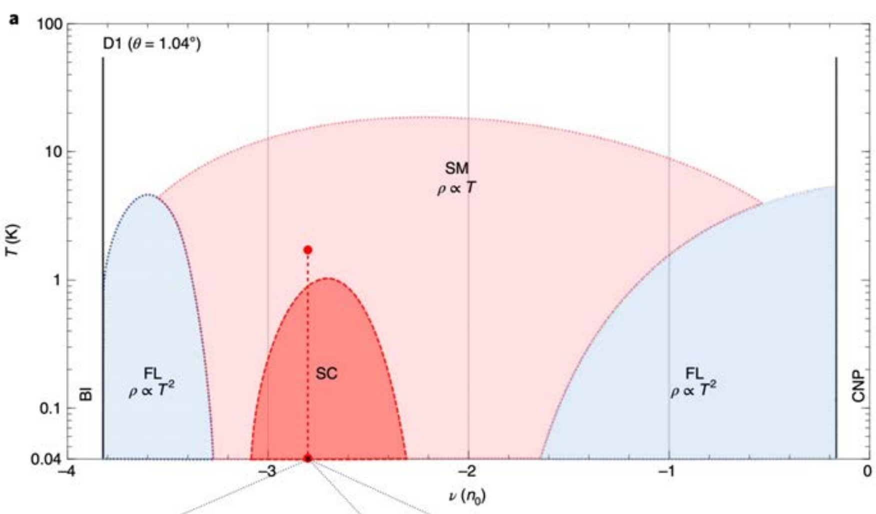
Fig. 3. Schematic doping-field-temperature (*p*-*B*-*T*) phase diagram in the vicinity of the critical doping p_{crit}. Note that the supercon-

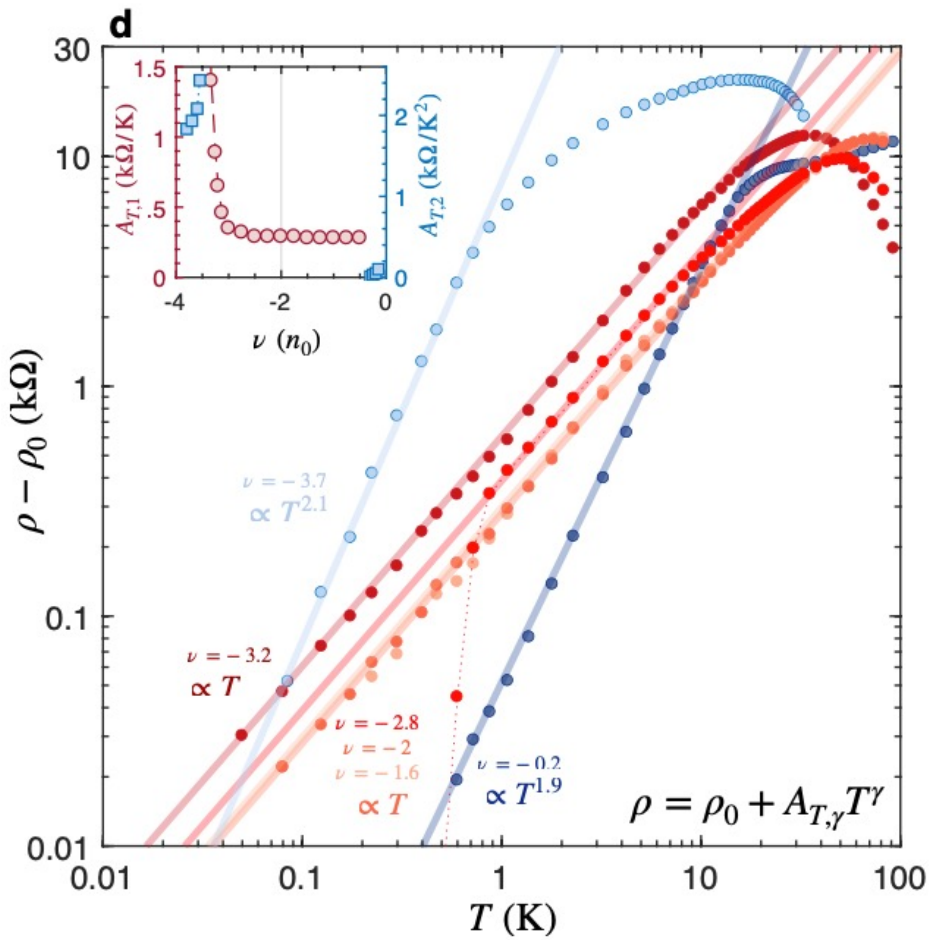


Giraldo-Gallo et al. Science, 2018. Resistivity of Sr-doped (19%) La_2CuO_4

Also in Twisted 2D Materials, e.g. Twisted Bilayer Graphene

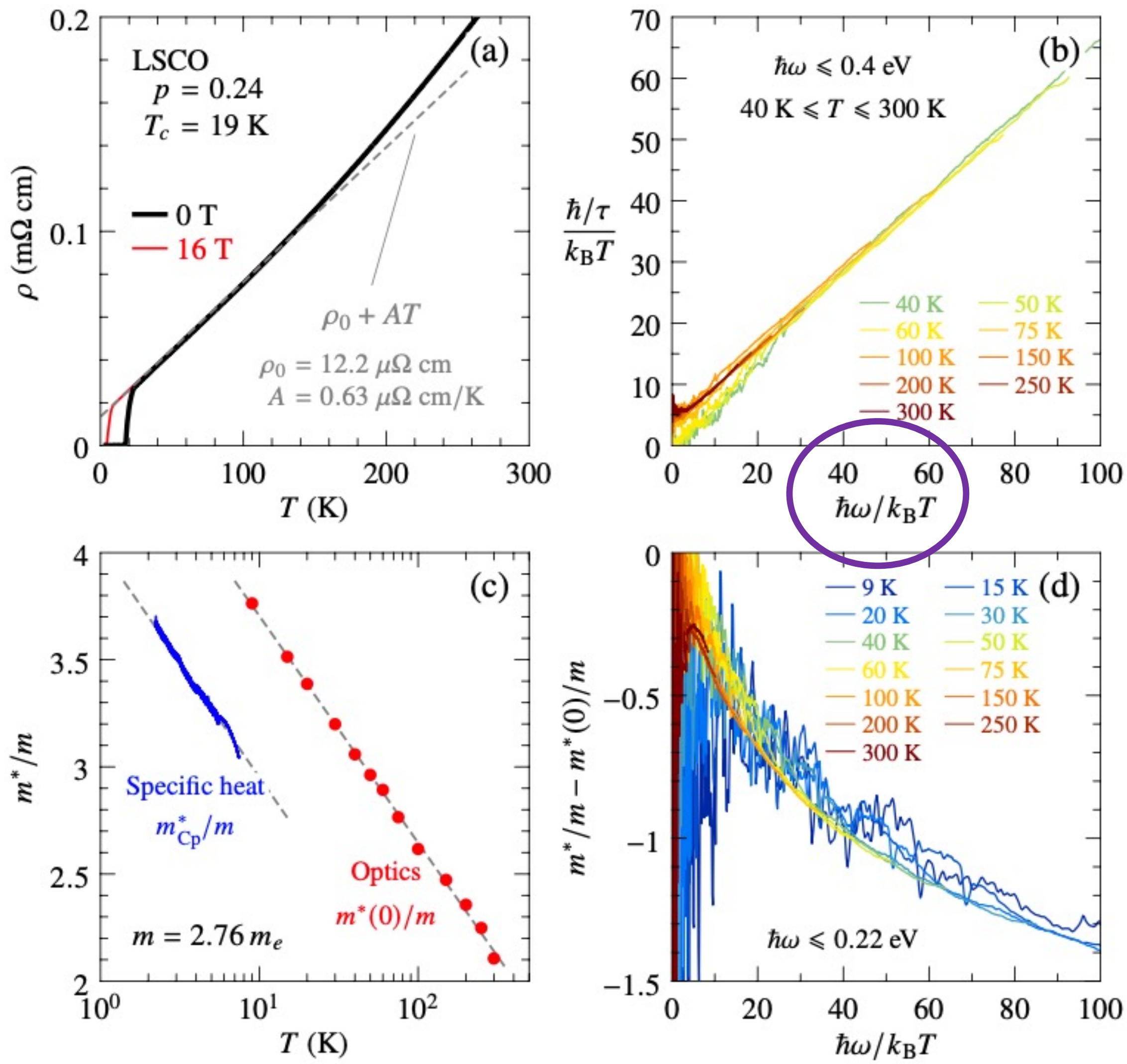






Alexandre Jaoui et al.
(D.Efetov's group, Barcelona)
arXiv:2108.07753 Nat Phys 2022

Quantum Critical Scaling Reveals a Single Energy Scale: k_BT



Recent optical conductivity data

Michon, van der Marel, Berthod, AG et al.
To Appear on arXiv.
See also future lectures and June 2-3 workshop

Early discoveries:

- Van der Marel et al. Nature 425 271 (2003)
- El Azrac et al. PRB 49, 9846 (1994); Baraduc et al. PRB 58, 11631 (1998)

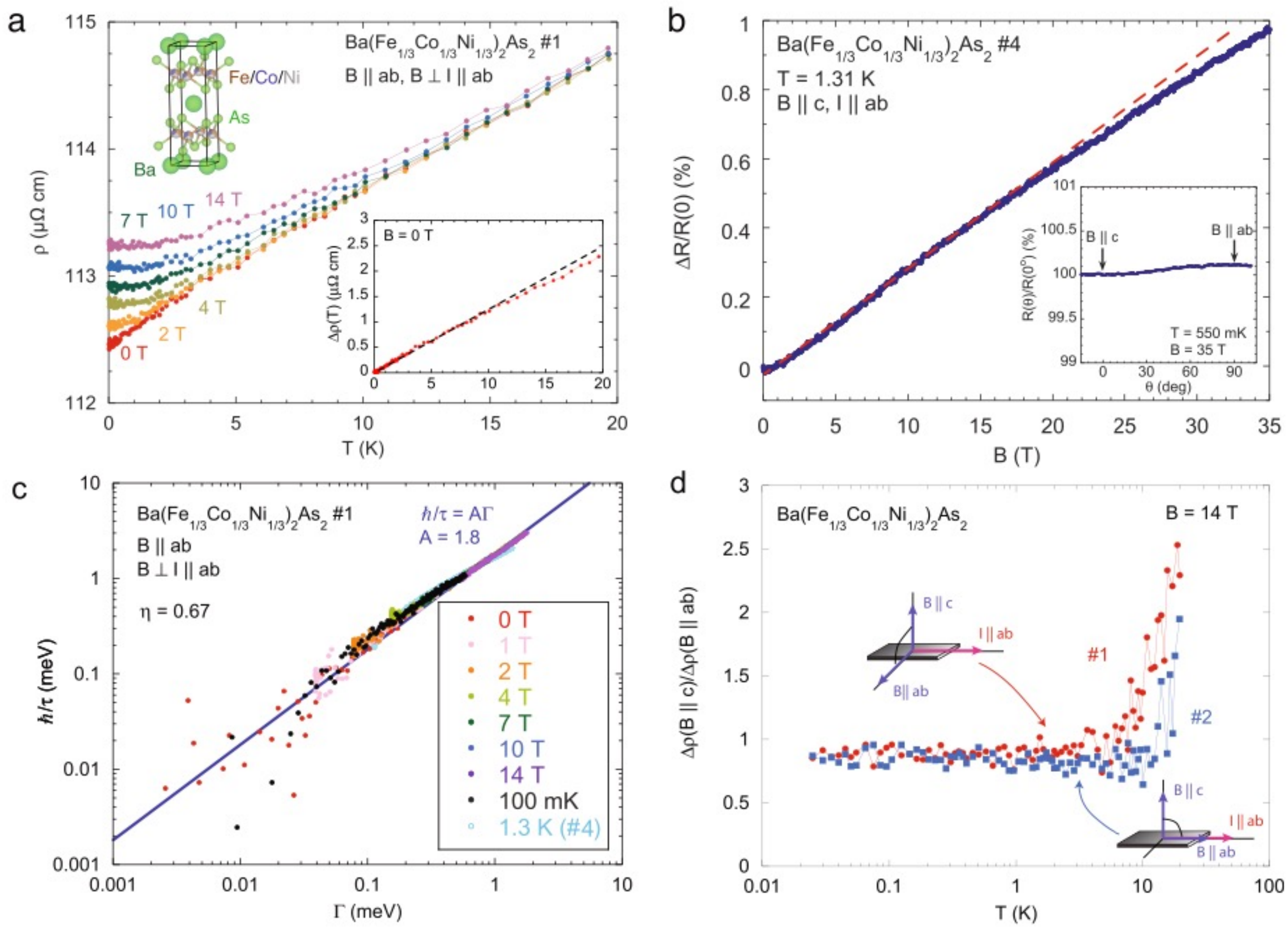
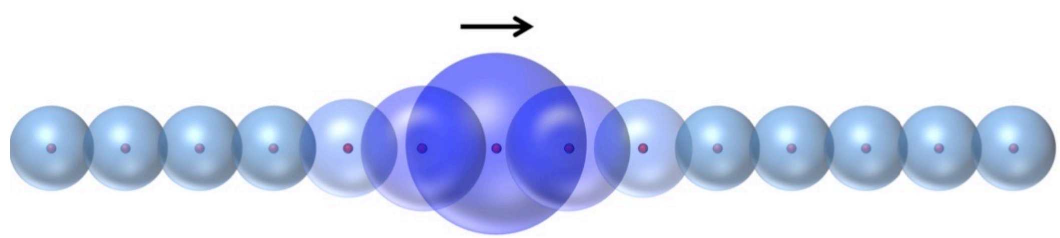


Fig. 1 Scale invariance in the resistivity of $Ba(Fe_{1/3}Co_{1/3}Ni_{1/3})_2As_2$. **a** Temperture dependence of resistivity for $Ba(Fe_{1/3}Co_{1/3}Ni_{1/3})_2As_2$ in the configuration of $B \parallel ab$, $B \perp I \parallel ab$. Upper inset: crystal structure for $Ba(Fe_{1/3}Co_{1/3}Ni_{1/3})_2As_2$ (ref. [46]). Lower inset: $\Delta\rho(T) = \rho(T) - \rho(0))$ as a function of T at $B = 0$ T for $Ba(Fe_{1/3}Co_{1/3}Ni_{1/3})_2As_2$. A dashed line is a guide to the eye to highlight quasi-linear-in-T dependence of resistivity. **b** Magnetic field dependence of $\Delta R(B)/R(0) \equiv (R(1.31\,\mathrm{K}, B) - R(1.31\,\mathrm{K}, 0))/R(1.31\,\mathrm{K}, 0)$ at $T = 1.31$ K. A red dashed line is a guide to the eye to highlight sublinear-in-B behavior of magnetoresistance. Inset: angular dependence of magnetoresistance at $T = 550$ mK and $B(\parallel c \perp I) = 35$ T. **c** Inelastic scattering rate $\hbar/\tau$ as a function of $\Gamma = \sqrt{(k_B T)^2 + (\eta \mu_B B)^2}$, with $\eta = 0.67$, suggestive of a universal scale invariance in the scattering mechanism in $Ba(Fe_{1/3}Co_{1/3}Ni_{1/3})_2As_2$. A blue sold line is a linear fit to data using $\hbar/\tau = A\Gamma$ with $A = 1.8$. **d** Temperature dependence of anisotropy of magnetoresistance between $\Delta\rho(B \parallel c)$ and $\Delta\rho(B \parallel ab \perp I$; sample #1) and between $\Delta\rho(B \parallel c)$ and $\Delta\rho(B \parallel ab \parallel I$; sample #2) at $B = 14$ T, showing lack of anisotropy in the scattering rate.

Why is T-linear Resistivity Puzzling?

Shaking up the established dogma of `Landau Fermi liquid theory'

In usual metals, current is transported by entities that `look like' free electrons (`quasi-particles')



Lev Landau, 1938

This constrains the available phase-space for excitations, implying that **resistivity depends on temperature quadratically (T^2), NOT linearly (T)**
→ T-linear resistivity may signal the collapse of Landau's paradigm!

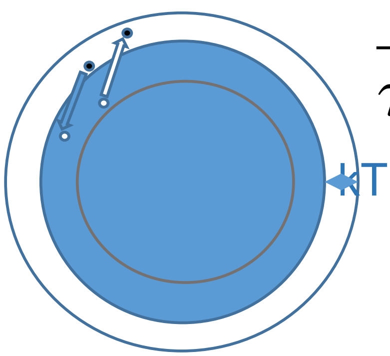
Low-Energy Many Body Spectrum

according to Landau Fermi Liquid Theory

$$\boxed{E\left[\{\delta n\}\right] - E_0 \;=\; \sum_{\alpha} \varepsilon_{\alpha}^{qp}\,\delta n_{\alpha} + \sum_{\alpha\beta} F_{\alpha\beta}\,\delta n_{\alpha}\delta n_{\beta}}$$

Low-energy excited states correspond to a small number of quasiparticle excitations (low-density gas of quasiparticles)

Fermi Golden Rule:



$$\frac{\hbar}{\tau_{qp}} \sim U^2 \;\times\; \rho(\varepsilon_F) \;\times\; \left(\frac{kT}{\varepsilon_F}\right)^2 \ll kT, \hbar\omega_{qp}$$

Square of Interaction strength

Density of states

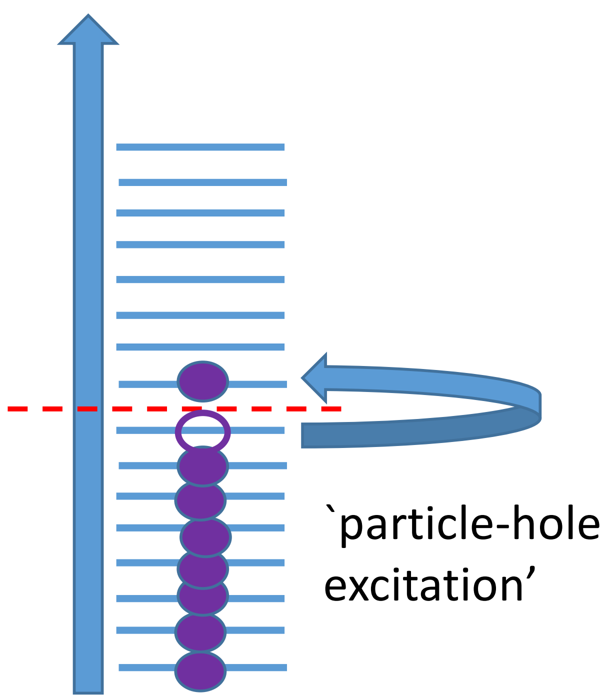
Fraction of phase space constrained by the Pauli principle

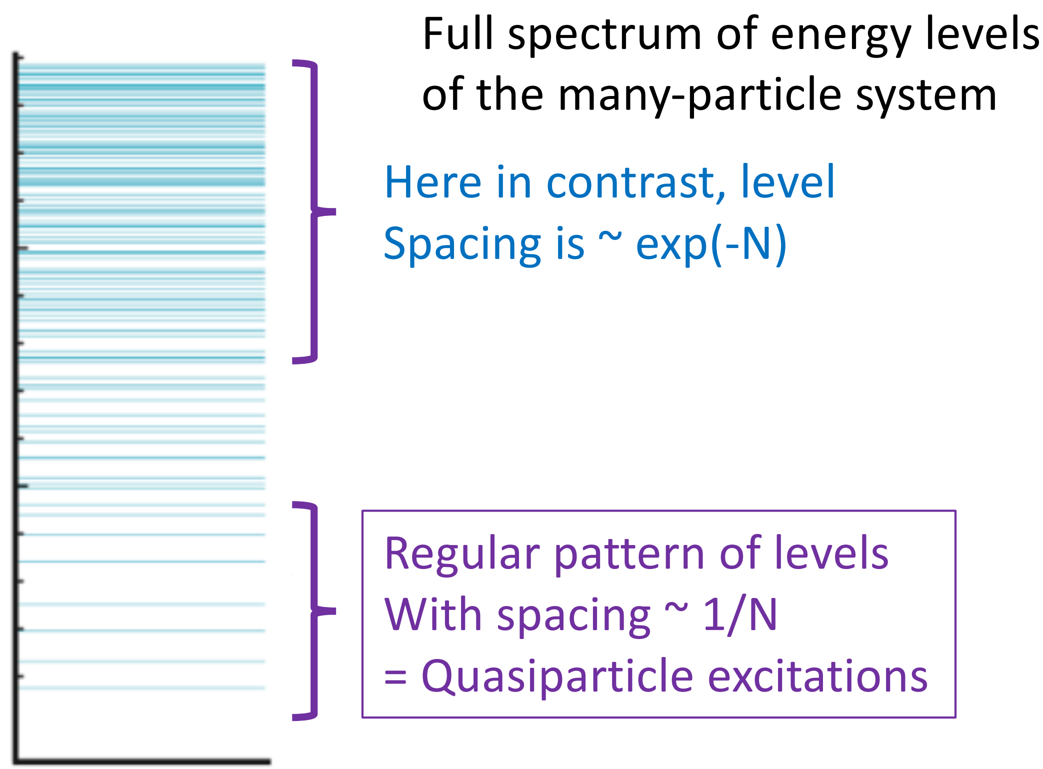
The Concept of Quasiparticles is a Very General One

- Not at all restricted to Landau Fermi Liquid Theory
- General idea: the low-lying spectrum of many-body states can be described by combining together elementary excitations
- These excitations can be very different from physical electrons, e.g. in Luttinger liquids: bosonic spin and charge density modes
- They can also obey unusual statistical/combinatorial rules
- FQHE
- Overscreened Kondo Models

The `Barcode' Signature of a Quantum System: Diagnosing Quasiparticles

According to Landau, the low-energy excitations of a collection of interacting electrons can be described as simple quantum transitions promoting an electron into an excited energy level





How Fast Can a Quantum System Equilibrate?
`Planckian Dissipation'

Consider a quantum system in equilibrium at a temperature T
and perturb/shake it
How fast can it come back to equilibrium?
Related questions:
How fast can chaos/scrambling develop (in some loose sense)?
At what maximum rate can a system send information?

Heisenberg uncertainty principle:

$$\Delta E \cdot \Delta t \gtrsim \hbar$$

$$\boxed{\Delta E \sim k_B T \Rightarrow \Delta t \gtrsim \frac{\hbar}{k_B T}}$$

Fastest `Planckian' time (because it contains Planck constant and follows from QM)

Quite fast timescale: ~ 7 10^{-12} seconds (7 picoseconds) at T= 1Kelvin

From equilibration/scrambling to transport/resistivity time-scale
(somewhat of a leap of faith – not obvious)

Resistivity according to Drude (~1900)

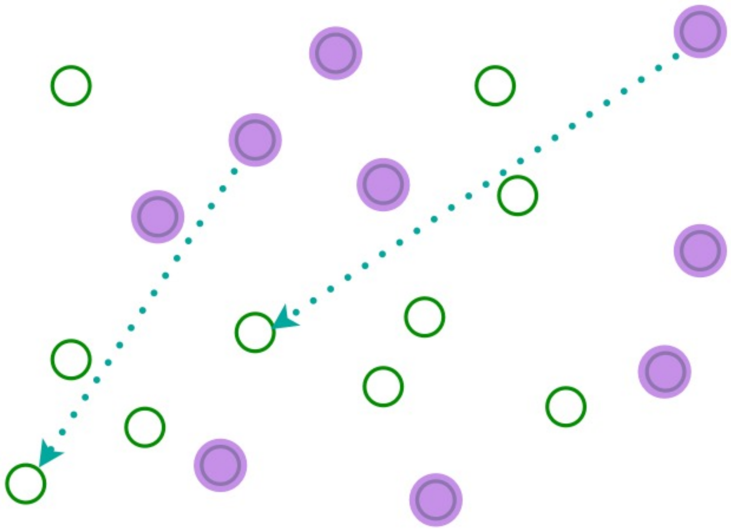
$$\boxed{\rho = \frac{1}{\omega_p^2\,\tau_{scatt}}}$$

IF we identify the scattering/collision time with an equilibration/scrambling time, we conclude that materials displaying T-linear resistivity **have the fastest possible equilibration time allowed by quantum mechanics: `Planckian Metals'**

$$\tau = \alpha\,\frac{\hbar}{k_B T} \;\; , \;\; \alpha = O(1)$$

Don’t understand something?
→ Make It Random!
A toy model: the SYK Model

S.Sachdev
J.Ye
A.Kitaev



Fermions (e.g. electrons) on N sites
Interacting pairwise with RANDOM Interactions
(Kitaev’s formulation – 2015 in a viral KITP video ☺)

$$H_4 = \frac{1}{(2N)^{3/2}} \sum_{ijk\ell=1}^{N} U_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell - \mu \sum_i c_i^\dagger c_i$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

$$\mathcal{Q} = \frac{1}{N} \sum_i \langle c_i^\dagger c_i \rangle.$$

Recent review article:
D.Chowdhury, AG, O.Parcollet, S.Sachdev
arXiv:2109.05037 – to appear in Rev Mod Phys

Volume 35B, number 1 PHYSICS LETTERS 26 April 1971

Precursors in Nuclear Physics / Random Matrix Theory

SOME RANDOM-MATRIX LEVEL AND SPACING DISTRIBUTIONS FOR FIXED-PARTICLE-RANK INTERACTIONS *

J. B. FRENCH
Department of Physics and Astronomy, University of Rochester, Rochester, New York, USA

and

S. S. M. WONG
Department of Physics, University of Toronto, Toronto, Canada
and Department of Physics and Astronomy, University of Rochester, Rochester, New York, USA

Received 16 March 1971

Ensemble spectra are given for random matrices constrained to describe k-body interactions in f^7. The transition from semicircular to Gaussian, as k decreases, is demonstrated. The nearest neighbor spacing distributions agree well in all cases with the Wigner spacing law.

Volume 34B. number 4 PHYSICS LETTERS 1 March 1971

TWO-BODY RANDOM HAMILTONIAN AND LEVEL DENSITY

O. BOHIGAS and J. FLORES ‡
Institut de Physique Nucléaire, Division de Physique Théorique ‡, 91 - *Orsay - France*

Received 22 December 1970

It is shown numerically that strong deviations from Wigner's semi-circle law for the level probability density of a random matrix are obtained if the two-body nature of the hamiltonian is taken into account. Instead of Wigner's law the probability density approaches closely a normal (gaussian) distribution.

The Sachdev-Ye Model (1992)

~ 1400 citations… (05/2022)

VOLUME 70, NUMBER 21 PHYSICAL REVIEW LETTERS 24 MAY 1993

Gapless Spin-Fluid Ground State in a Random Quantum Heisenberg Magnet

Subir Sachdev and Jinwu Ye
Departments of Physics and Applied Physics, P.O. Box 2157, Yale University, New Haven, Connecticut 06520
(Received 22 December 1992)

We examine the spin-S quantum Heisenberg magnet with Gaussian-random, infinite-range exchange interactions. The quantum-disordered phase is accessed by generalizing to SU(M) symmetry and studying the large M limit. For large S the ground state is a spin glass, while quantum fluctuations produce a spin-fluid state for small S. The spin-fluid phase is found to be generically gapless—the average, zero temperature, local dynamic spin susceptibility obeys $\bar{\chi}(\omega) \sim \ln(1/|\omega|) + i(\pi/2)\mathrm{sgn}(\omega)$ at low frequencies.

Heisenberg Model on a fully connected lattice with random all to all interactions:

$$H = \sum_{ij} J_{ij} \vec{S}_i \cdot \vec{S}_j \;\; , \;\; \overline{J_{ij}} = 0 \, , \; \overline{J_{ij}^2} = \frac{J^2}{N}$$

SU(M), large M, fermionic representations of spin:
NO spin-glass order – Gapless spin-liquid!

$$\chi(t) = \langle \vec{S}(0) \cdot \vec{S}(t) \rangle \sim \frac{1}{t}$$

Local spin dynamics Analogous to that of a \`Marginal Fermi Liquid'!

$$J\chi''(\omega, T=0) = c\, sign(\omega) + \cdots$$

$$\boxed{J\chi''(\omega, T) = \frac{\sqrt{\pi}}{2} \tanh \frac{\omega}{2T} + \cdots}$$

SYK: Exactly Solvable (for infinite size N)
SY: Exactly Solvable for large-M SU(M)

Counting of energy levels:

Non-zero Entropy at T=0 in the large-N limit!

Signals exponential accumulation of energy levels at low energy

`GPS' Entropy
2000
(O.Parcollet
PhD thesis)

$$\mathcal{S}(\Theta) = \int_{-\pi/4}^{\Theta} d\theta \, \ln \frac{\sin(\pi/4+\theta)}{\sin(\pi/4-\theta)} \frac{\partial \mathcal{Q}}{\partial \theta}$$

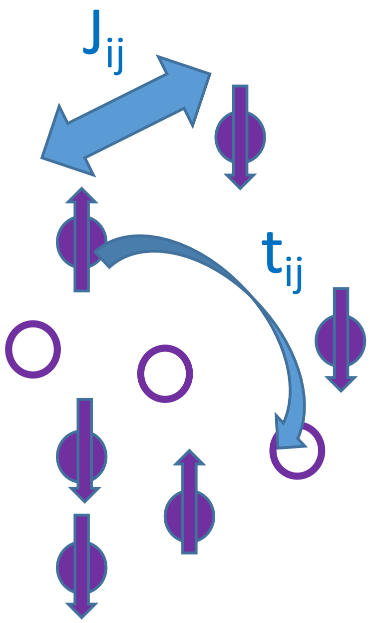
$$\mathcal{Q}(\Theta) = \frac{1}{2} - \frac{\Theta}{\pi} - \frac{\sin 2\Theta}{4}$$

Level spacing is e^{-N}, NOT 1/N
NO QUASIPARTICLE EXCITATIONS

AND: Fastest (Planckian) dissipation rate: $k_B T/\hbar$

Plus many other cool properties such as conformal invariance, omega/T scaling etc.

Introducing Mobile Charge Carriers in the SY Model



O.Parcollet and AG, Phys Rev B 59, 5341 (1999) Large-M
Part of O.Parcollet PhD thesis
Several recent works for SU(2)

$$H_{tJ} = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} \sum_{\alpha} t_{ij} \, \mathcal{P} c^{\dagger}_{i\alpha} c_{j\alpha} \mathcal{P} - \mu \sum_{i\alpha} c^{\dagger}_{i\alpha} c_{i\alpha} + \frac{1}{\sqrt{N}} \sum_{1 \le i < j \le N} J_{ij} \, \boldsymbol{S}_i \cdot \boldsymbol{S}_j \qquad (7$$

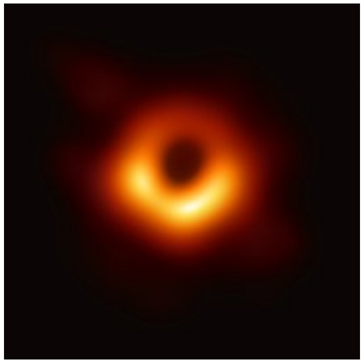
Lectures 4 and 5 will be devoted to the physics of this \`doped' SY model

Surprising Connections:
SYK Models
and The Quantum Physics of
Black Holes

→ See Lectures by Subir Sachdev

Black Holes: The Fastest Scramblers

One-slide introduction to black holes…



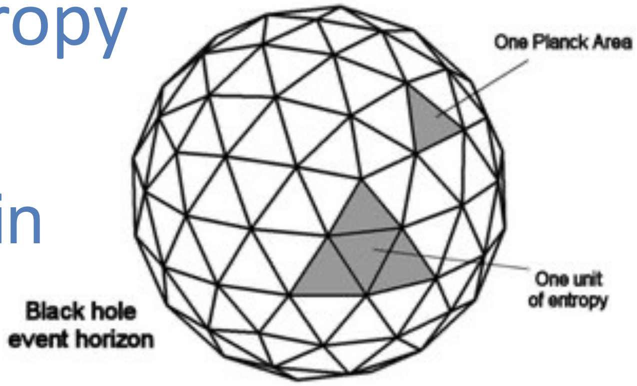
- Horizon Radius:

  $R = 2GM/c^2 \ (\simeq 9\text{mm for } M = M_{Earth}!)$
- Relaxation to Equilibrium (classical) : $\tau \sim 8\pi GM/c^3$
- Quantum Black Holes emit radiation and matter: Hawking temperature $T_H = \hbar c^3/(8\pi GMk_B)$
- Planckian Relaxation! $\tau \sim \hbar/k_B T_H$ (hbar cancels out)
- Quantum black holes have a finite entropy proportional to their surface area

$$S_{BH} \sim A/4\ell^2_{Planck}$$

$$\ell_{Planck} = \sqrt{\hbar G/c^3} \simeq 1.6\,10^{-35} m$$

Bekenstein
Hawking
1971-72



LIGO/VIRGO (2021)

Black holes are the fastest dissipators/ information transmitters

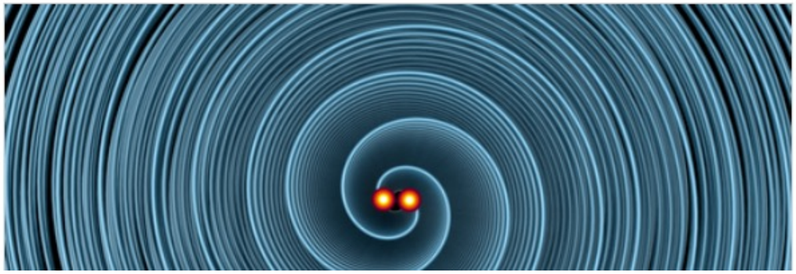
APS physics | Physics

SYNOPSIS

Black Holes Obey Information-Emission Limits

April 22, 2021 • *Physics* 14, s47

An analysis of the gravitational waves emitted from black hole mergers confirms that black holes are the fastest known information dissipaters.



PHYSICAL REVIEW LETTERS **126,** 161102 (2021)

Editors' Suggestion | Featured in Physics

Bekenstein-Hod Universal Bound on Information Emission Rate Is Obeyed by LIGO-Virgo Binary Black Hole Remnants

Gregorio Carullo,[1,2] Danny Laghi,[1,2] John Veitch,[3] and Walter Del Pozzo[1,2]

[1]*Dipartimento di Fisica "Enrico Fermi," Università di Pisa, Pisa I-56127, Italy*

[2]*INFN sezione di Pisa, Pisa I-56127, Italy*

[3]*Institute for Gravitational Research, University of Glasgow, Glasgow, G12 8QQ, United Kingdom*

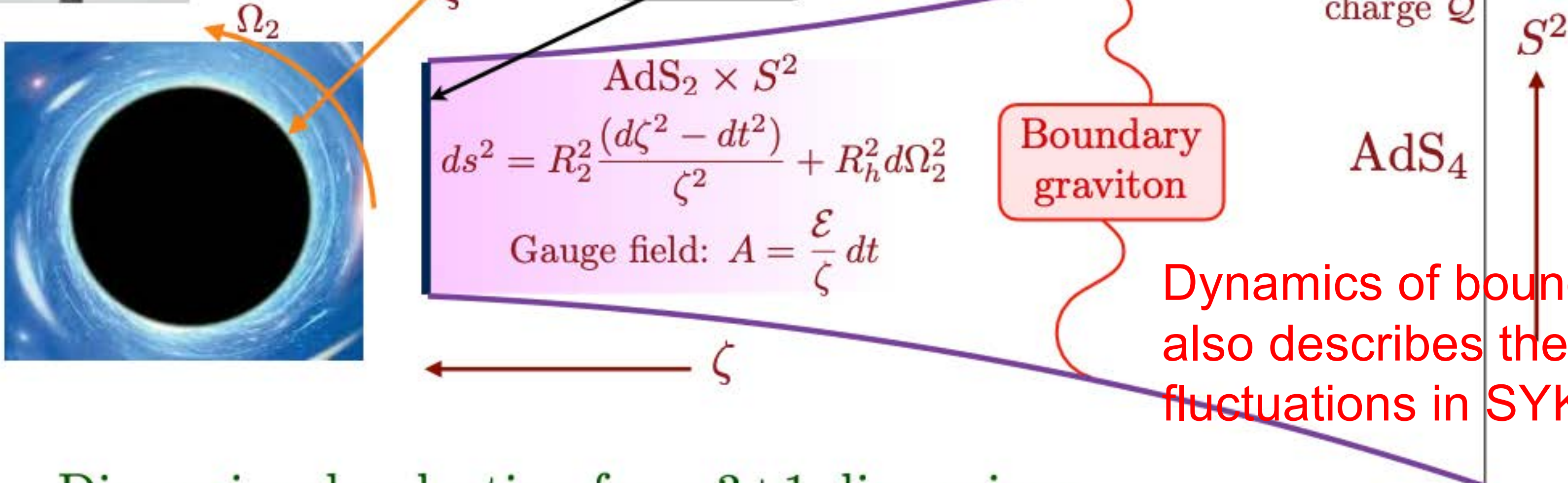
(Received 12 December 2020; revised 5 February 2021; accepted 9 March 2021; published 22 April 2021)

Causality and the generalized laws of black hole thermodynamics imply a bound, known as the Bekenstein-Hod universal bound, on the information emission rate of a perturbed system. Using a time-domain ringdown analysis, we investigate whether remnant black holes produced by the coalescences observed by Advanced LIGO and Advanced Virgo obey this bound. We find that the bound is verified by the astrophysical black hole population with 94% probability, providing a first confirmation of the Bekenstein-Hod bound from black hole systems.

Strange Metals and… Black Holes

The SYK model is connected to quantum black hole physics!

By Holography ~ Duality



Dynamics of boundary graviton also describes the fluctuations in SYK model

Dimensional reduction from 3+1 dimensions to 1+1 dimensions (AdS_2) at low energies!

\`GPS' Entropy of the SYK model = Bekenstein-Hawking entropy of AdS_2 charged black hole!

Slide: Courtesy S.Sachdev

COLLÈGE DE FRANCE 1530

Chaire de Physique
de la Matière Condensée
Antoine Georges

From Strange Metals to Black Holes: Sachdev-Ye-Kitaev (SYK) Models and related topics

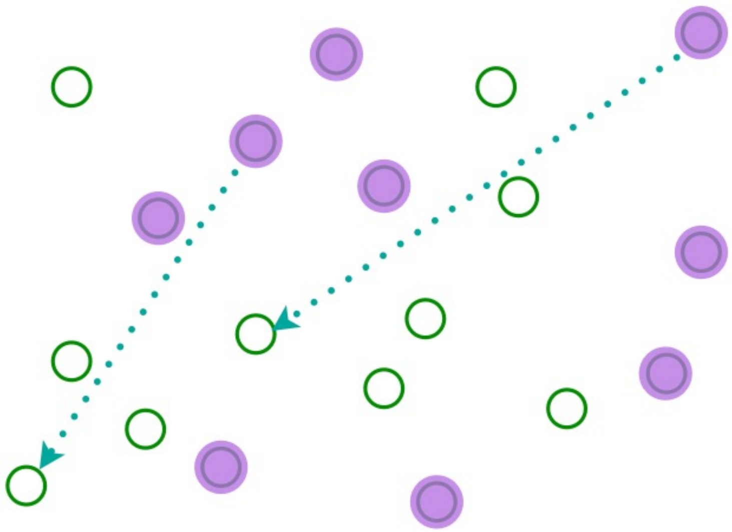
Lectures 2 and 3 – Introduction to the SYK Model

Mostly on blackboard – see video on website

A few supporting slides below.

2021-2022 Lectures
May 10 and 17, 2022

The SYK Model



S.Sachdev
J.Ye
A.Kitaev

Fermions (e.g. electrons) on N sites
Interacting pairwise with RANDOM Interactions
(Kitaev's formulation – 2015 in a viral KITP video ☺)

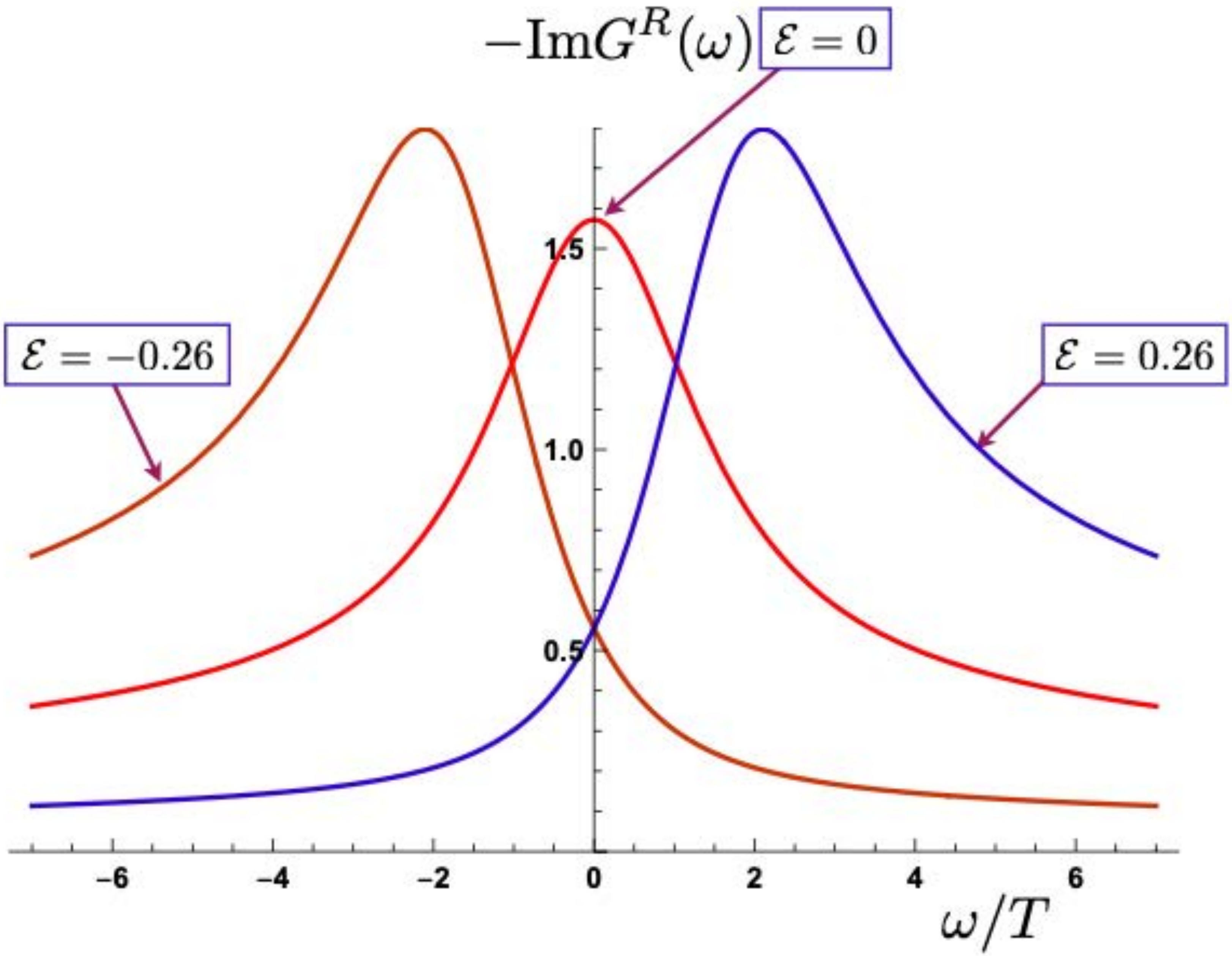
$$H_4 = \frac{1}{(2N)^{3/2}} \sum_{ijk\ell=1}^{N} U_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell - \mu \sum_i c_i^\dagger c_i$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

$$\mathcal{Q} = \frac{1}{N} \sum_i \langle c_i^\dagger c_i \rangle.$$

Can be viewed as
a big `quantum dot'
or multi-orbital atom
with random interactions

Recent review article:
D.Chowdhury, AG, O.Parcollet, S.Sachdev
arXiv:2109.05037 – to appear in Rev Mod Phys



$-\mathrm{Im}G^R(\omega)$
$\mathcal{E} = 0$
$\mathcal{E} = -0.26$
$\mathcal{E} = 0.26$
1.5
1.0
0.5
−6
−4
−2
0
2
4
6
ω/T

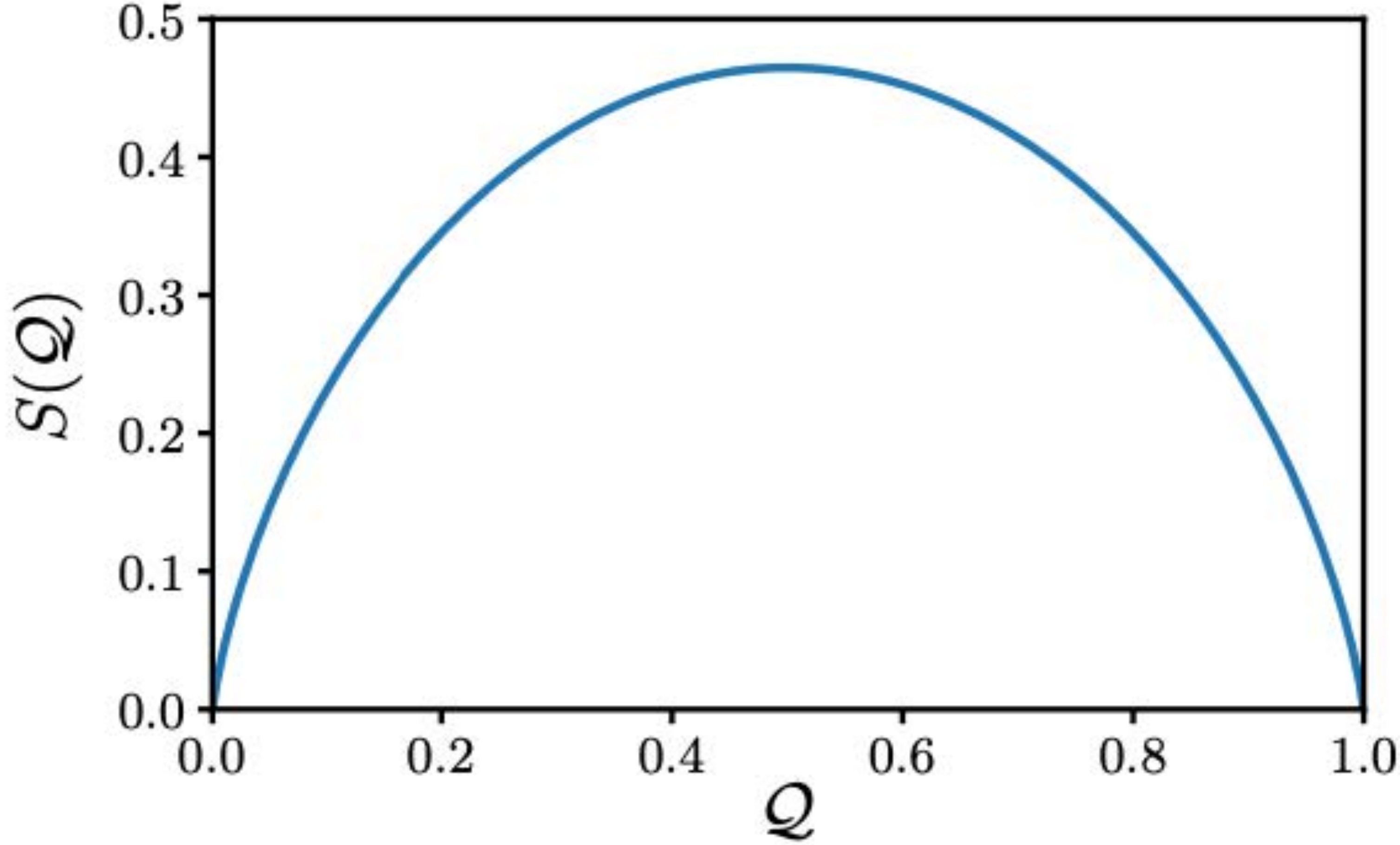


FIG. 9 $T = 0$ entropy density $\mathcal{S}$ vs. $\mathcal{Q}$ (Georges *et al.*, 2001).

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Non-Fermi-liquid regime of a doped Mott insulator

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Overscreened multichannel SU(N) Kondo model: Large-N solution and conformal field theory

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Mean Field Theory of a Quantum Heisenberg Spin Glass

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Quantum fluctuations of a nearly critical Heisenberg spin glass

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Finite Size Spectra

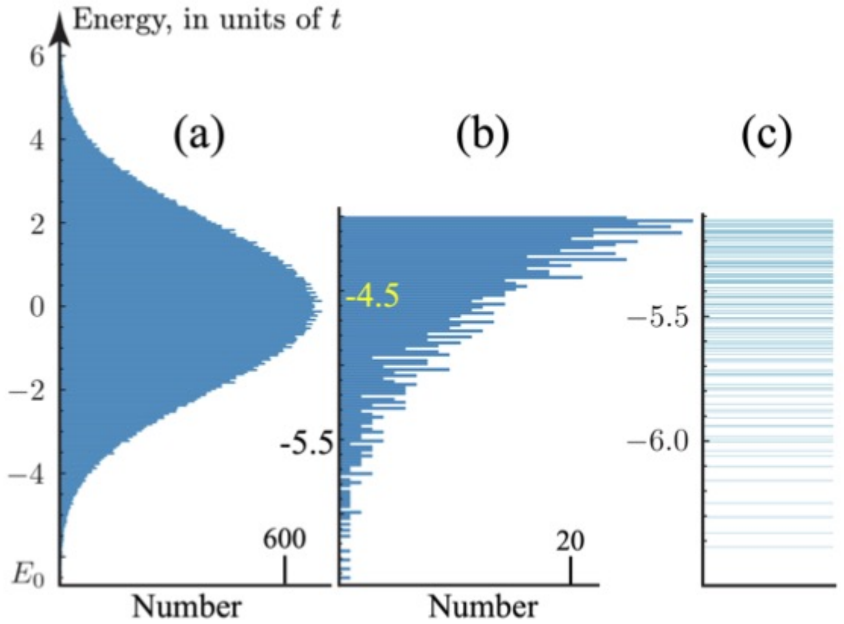

FIG. 5 65536 many-body eigenvalues of a $N = 32$ Majorana matrix model with random $q = 2$ fermion terms. $\mathcal{N}(E)$ is plotted in (a) and (b) in 200 and 100 bins, (b) and (c) zoom into the bottom of the band. Individual energy levels are shown in (c), and these are expected to have spacing $1/(N\rho(\mu))$ at the bottom of the band as $N \to \infty$.

Free Fermions
(Random hopping)

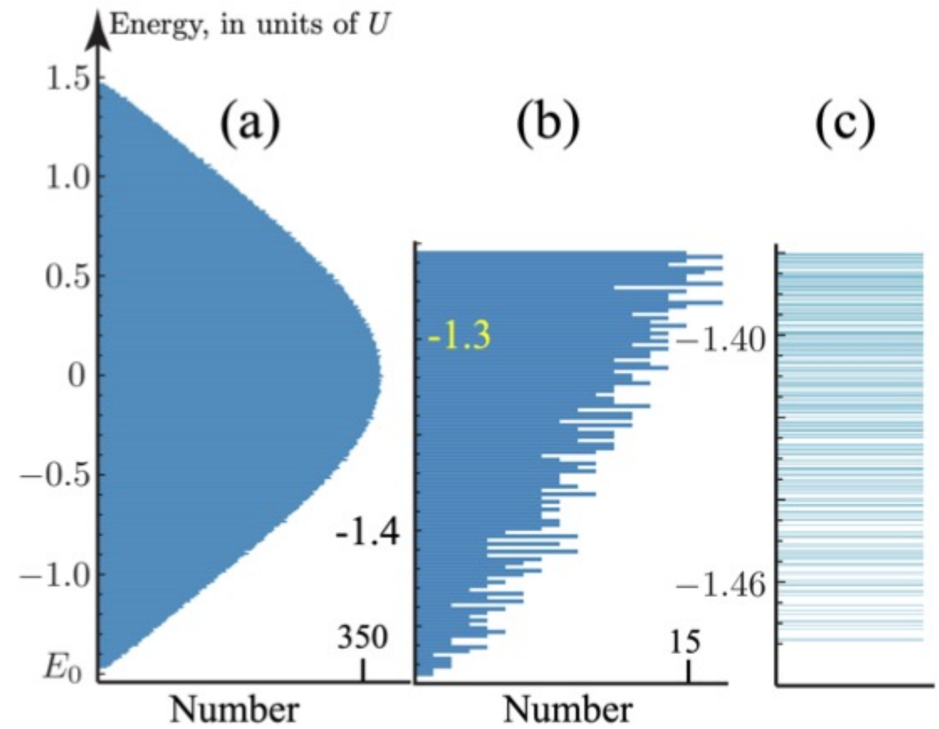

FIG. 6 65536 many-body eigenvalues of a $N = 32$ Majorana SYK Hamiltonian with random $q = 4$ fermion terms. $\mathcal{N}(E)$ is plotted in (a) and (b) in 200 and 100 bins, (b) and (c) zoom into the bottom of the band. Individual energy levels are shown in (c), and these are expected to have spacing e^{-NS} at the bottom of the band as $N \to \infty$. Compare to Fig. 5 for the random matrix model, which has a much sparser spacing $\sim 1/N$ at the bottom of the band.

SYK Model