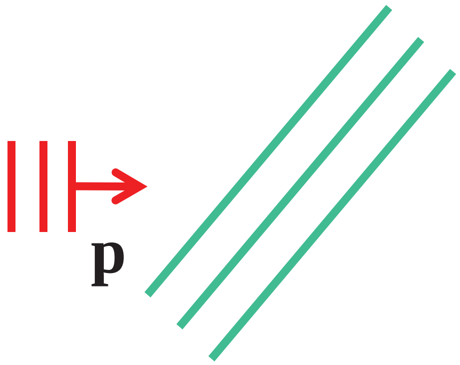
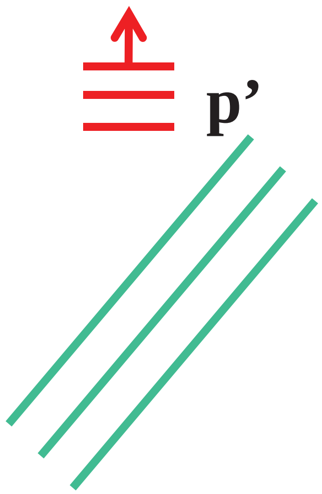
Superconductivity

Attraction mechanism in the metallic state



Attraction mechanism in the metallic state



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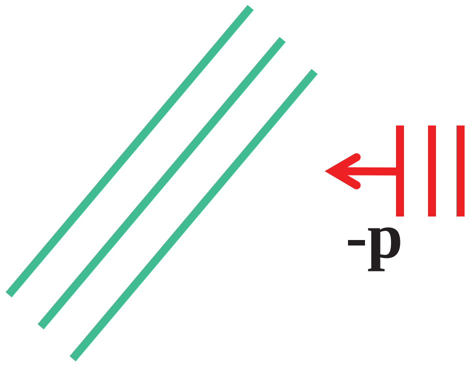
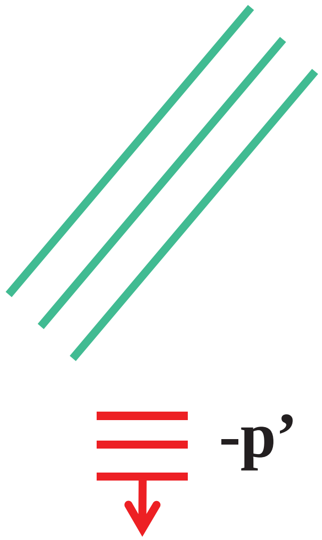
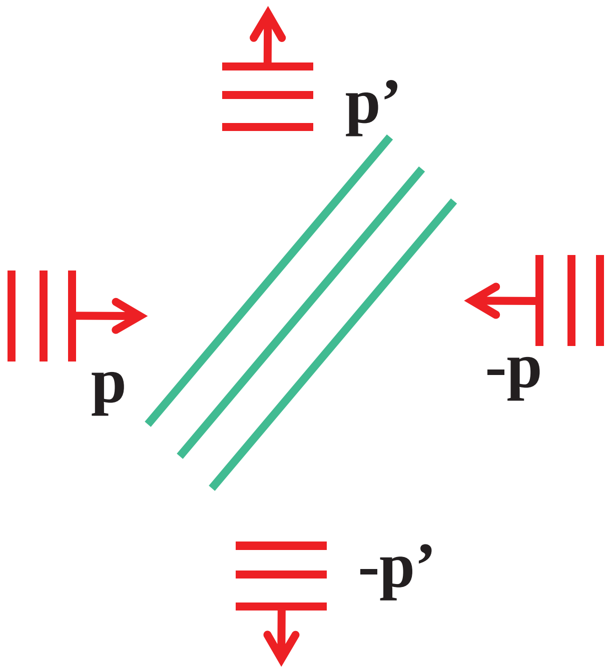
Attraction mechanism in the metallic state



Attraction mechanism in the metallic state



Attraction mechanism in the metallic state



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#1 Cooper pair, #2 Phase coherence

$$E_P = \sum_{\mathbf{p},\mathbf{p}'} U_{\mathbf{p}-\mathbf{p}'} \psi_{\mathbf{p}\uparrow,-\mathbf{p}\downarrow} \psi^*_{\mathbf{p}'\uparrow,-\mathbf{p}'\downarrow}$$

$$E_P = \sum_{\mathbf{p},\mathbf{p}'} U_{\mathbf{p}-\mathbf{p}'} \left(\langle \psi_{\mathbf{p}\uparrow,-\mathbf{p}\downarrow} \rangle \psi^*_{\mathbf{p}'\uparrow,-\mathbf{p}'\downarrow} + \psi_{\mathbf{p}\uparrow,-\mathbf{p}\downarrow} \left\langle \psi^*_{\mathbf{p}'\uparrow,-\mathbf{p}'\downarrow} \right\rangle \right)$$

$$|\mathrm{BCS}(\theta)\rangle = \ldots + e^{iN\theta}|N\rangle + e^{i(N+2)\theta}|N+2\rangle + \ldots .$$

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Giovanni Sordi

Patrick Sémon

Superfluid stiffness $T = 0$

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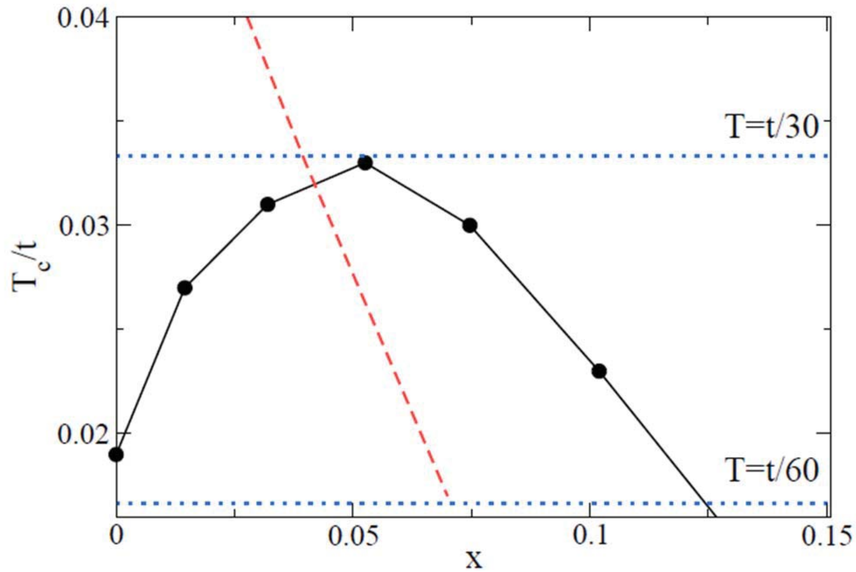
8 site cluster DCA $U = 6t$



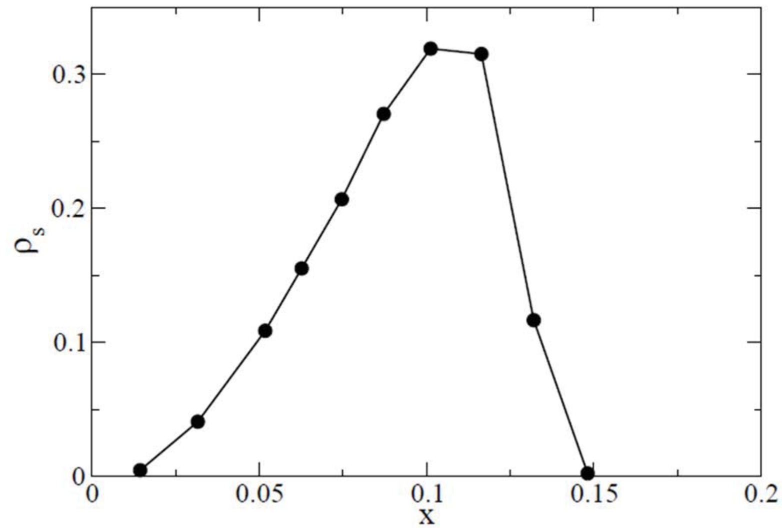


FIG. 8. Superfluid stiffness ρ_s determined in the superconducting state at $T = t/60$ from Eq. 15, as a function of doping.

E. Gull, A.J. Millis,
Phys. Rev. B **88**, 075127 (2013)

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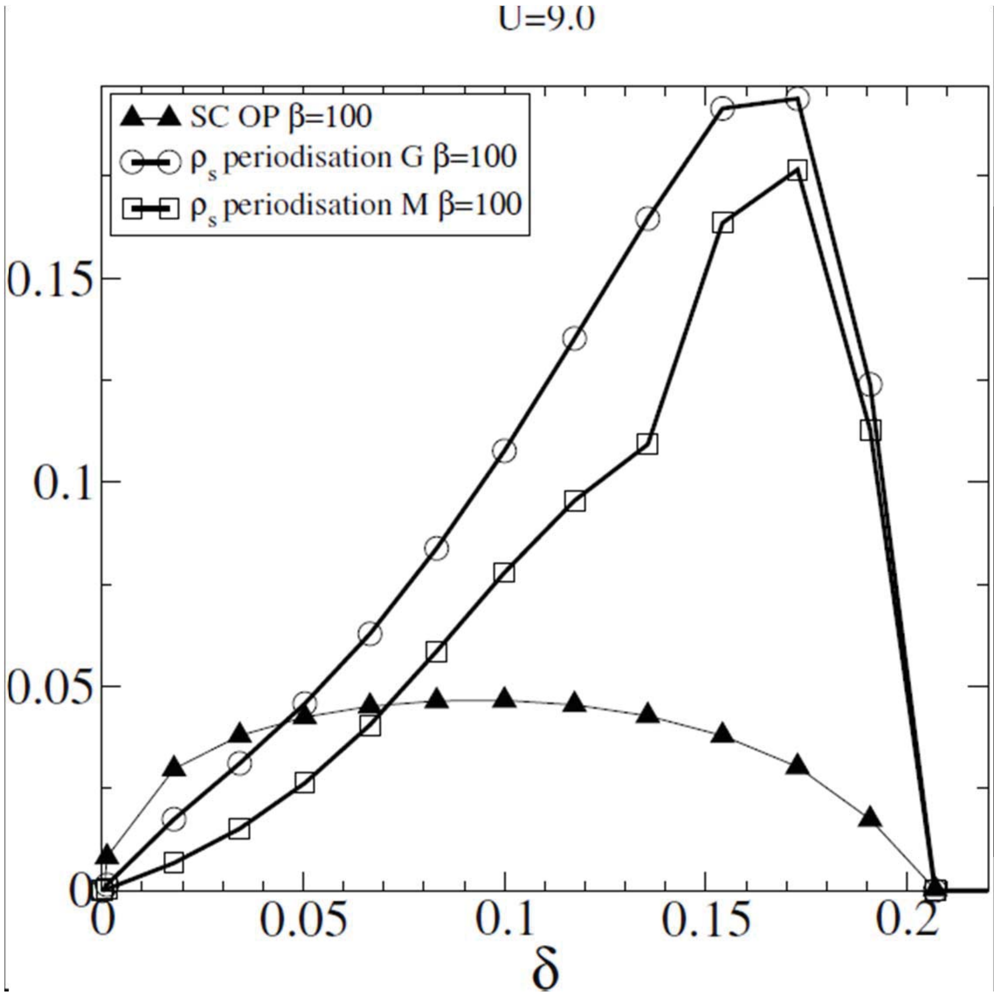
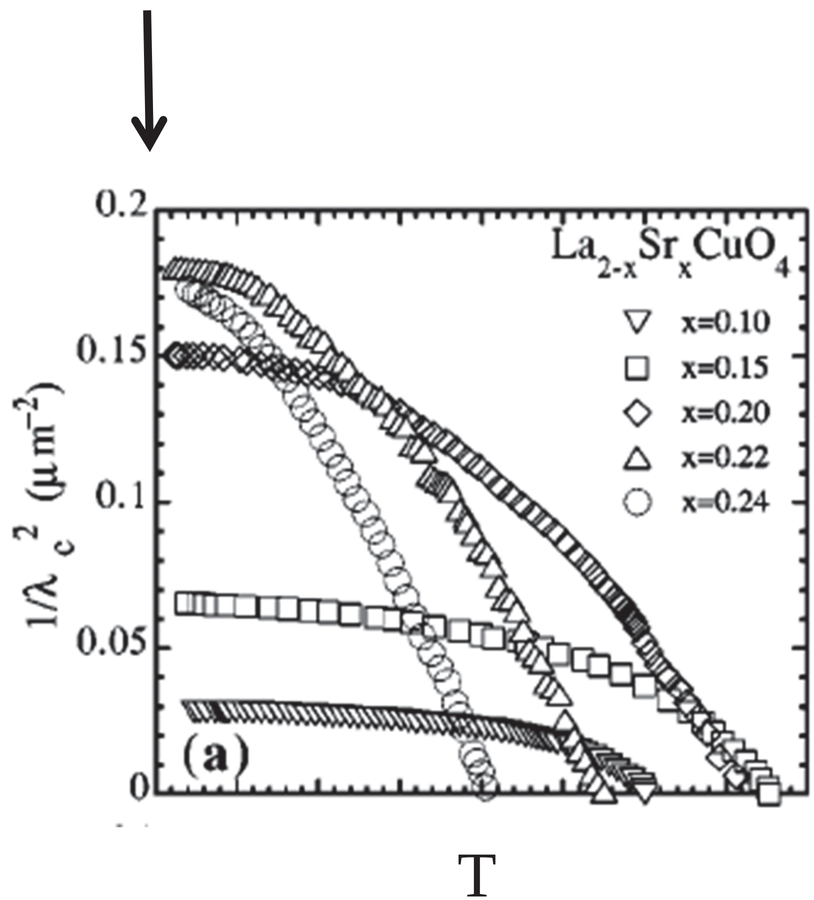
c-axis Superfluid stiffness *U = 9t, T=1/100*

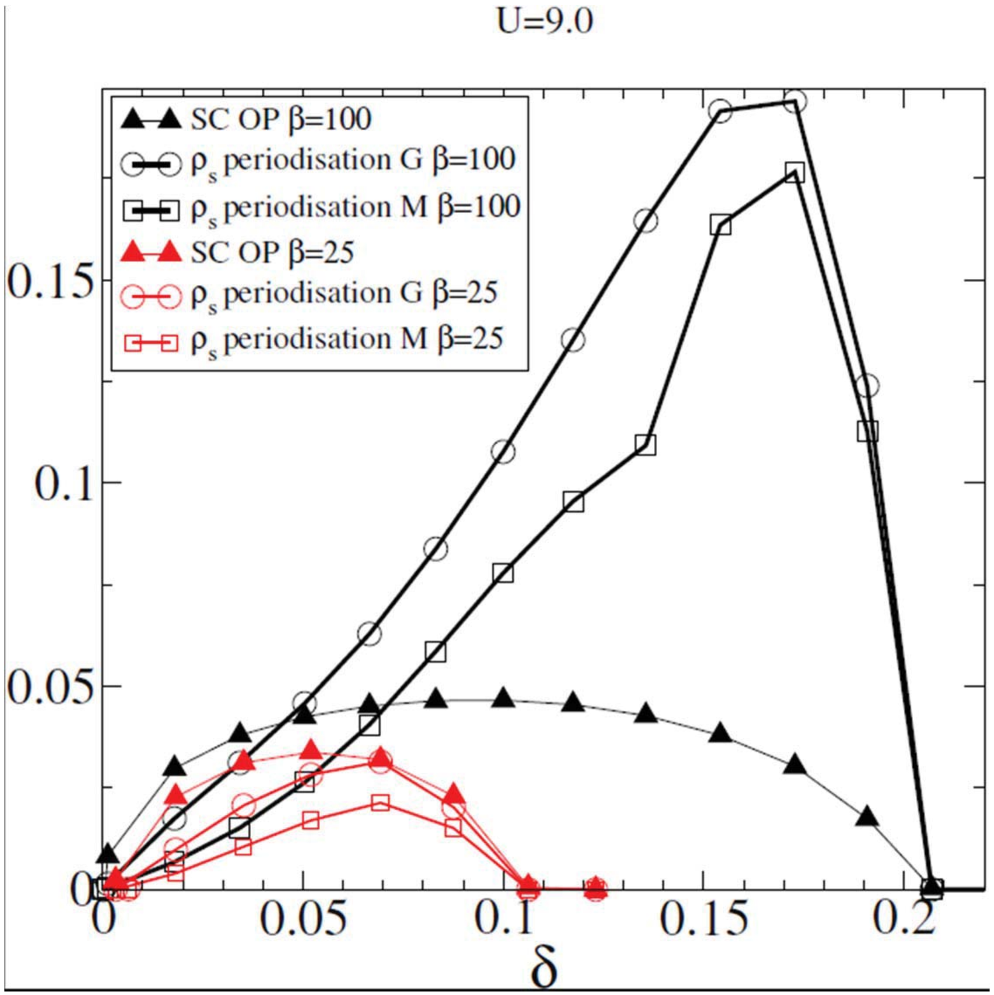


Sordi, Sémon unpublished

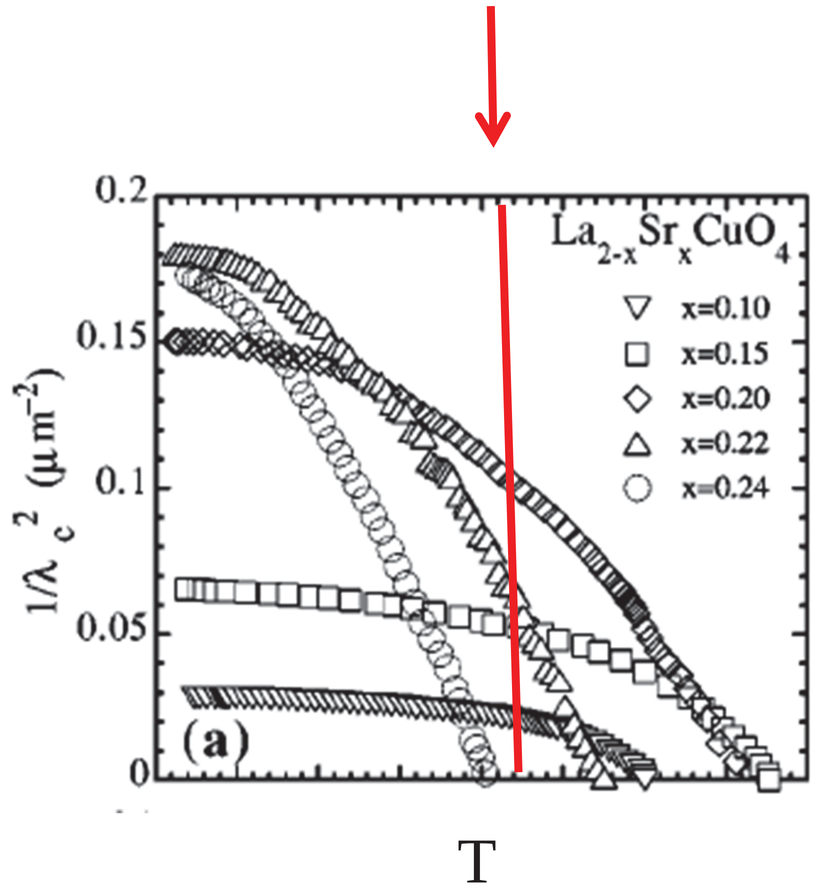


Panagopoulos et al. PRB 2000

c-axis Superfluid stiffness *U = 9t, T=1/100*



Sordi, Sémon, unpublished



Panagopoulos et al. PRB 2000

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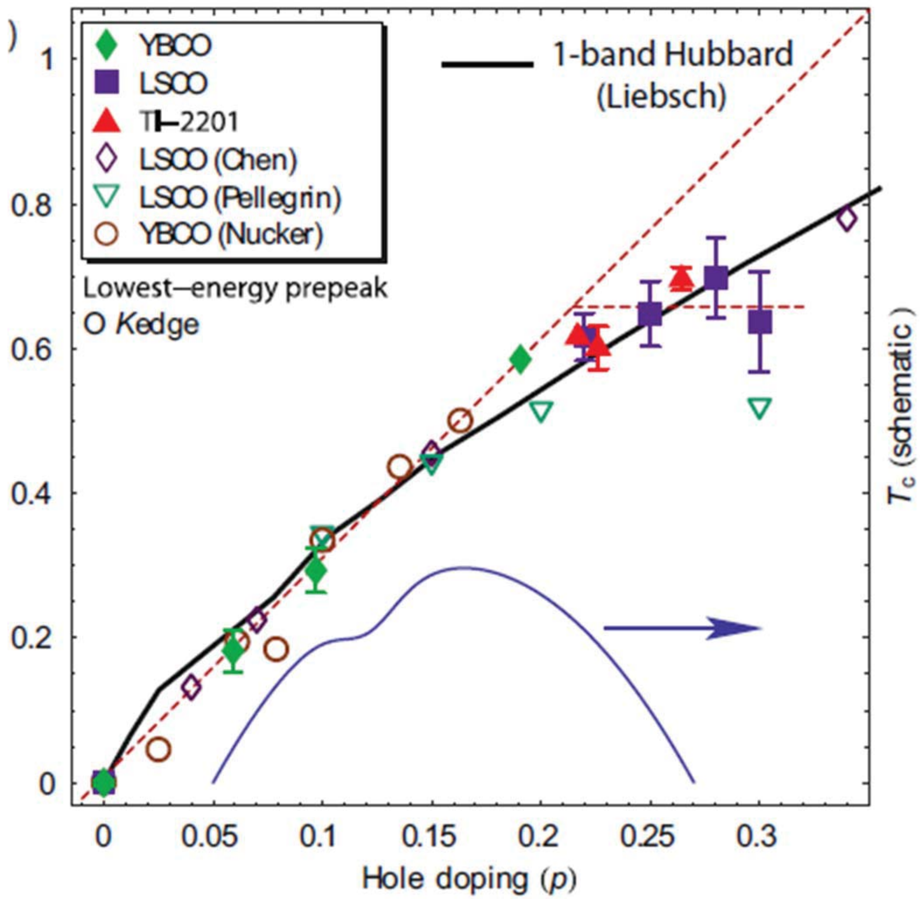
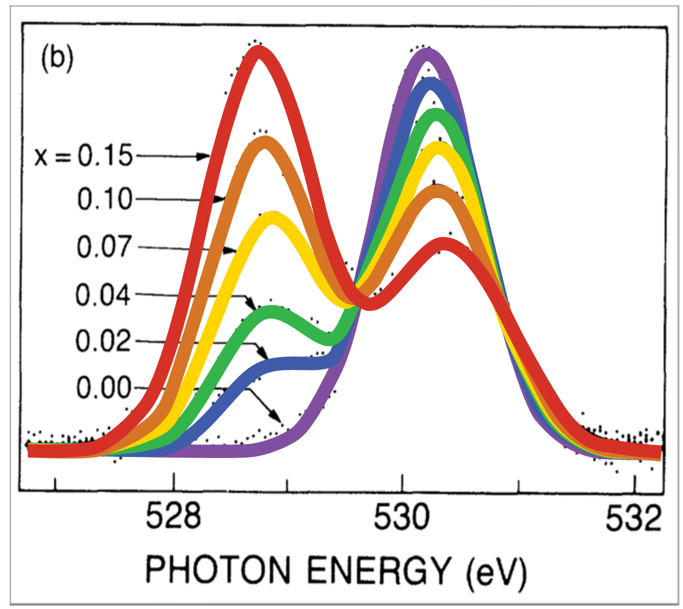
Compare with number of cariers





Peets et al. PRL 2009, Phillips and Jarrell, PRL 2010

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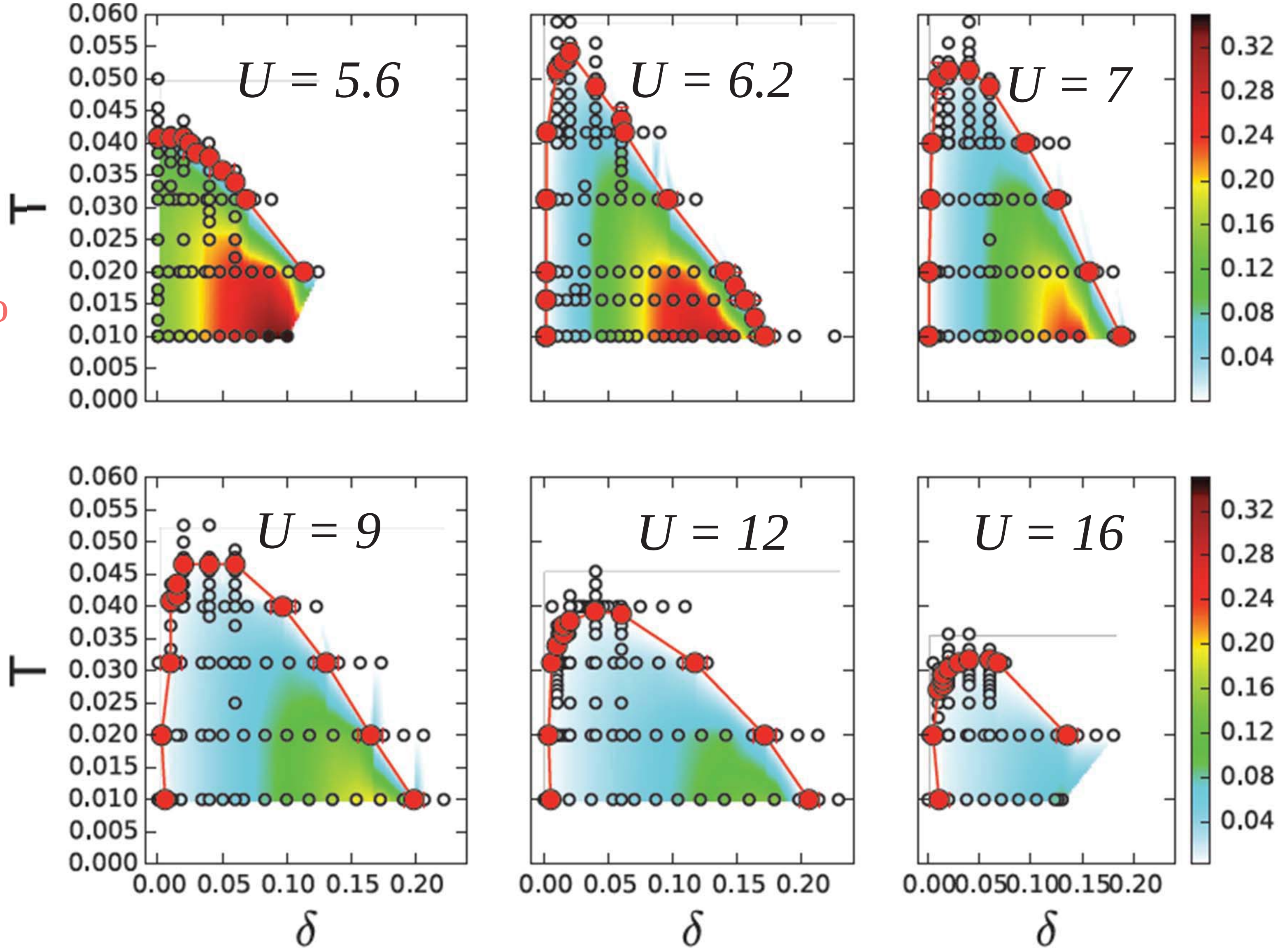
Superfluid stiffness

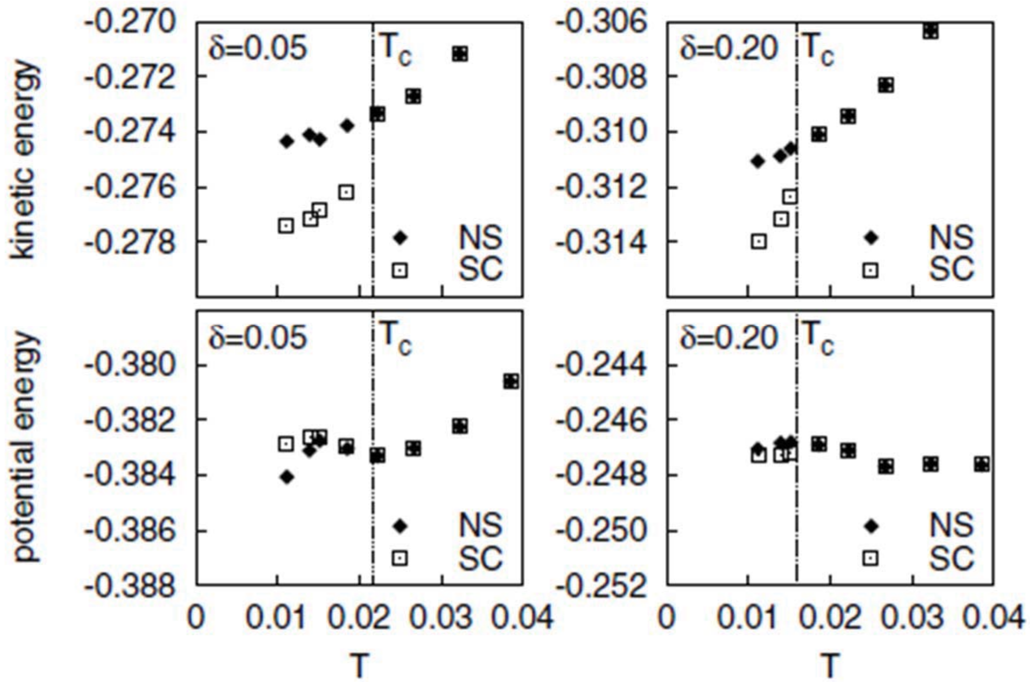
Lorenzo Fratino

Giovanni Sordi



Condensation energy

Condensation energy



U = 8t, 4 sites – DCA

Th. A. Maier, M. Jarrell, A. Macridin, and C. Slezak
PRL **92,** 027005 (2004)

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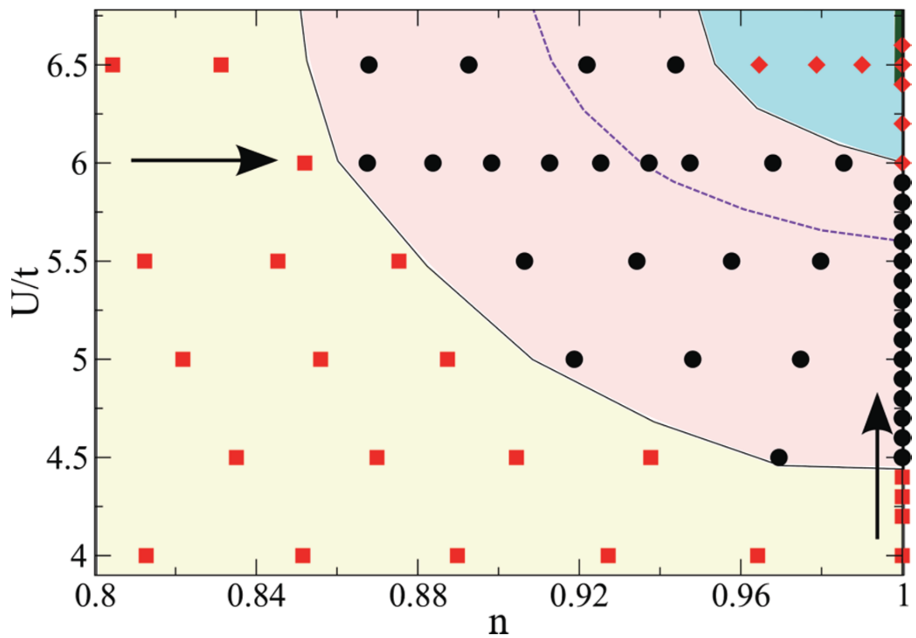
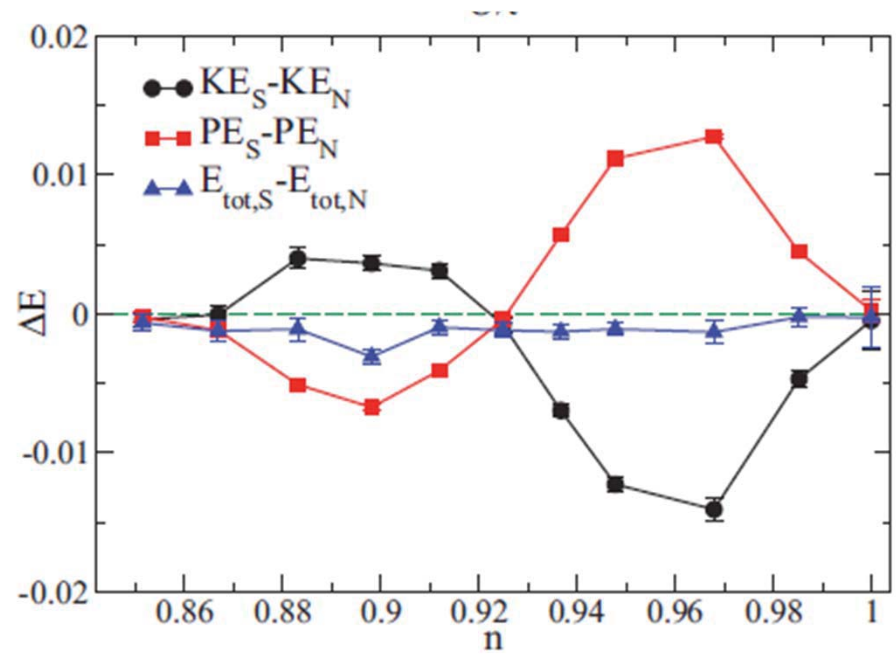
Condensation energy

Experiments: N. Bontemps et al. Annals of Physics 321 (2006) 1547–1558





U = 6t, T = 1/60, 8 sites – DCA

E. Gull, A. Millis, PRB **86**, 241106(R) (2012)
K. Haule, G. Kotliar EPL, 77 (2007) 27007

The glue

Superconductivity in general

Analog to weakly and strongly correlated antiferromagnets

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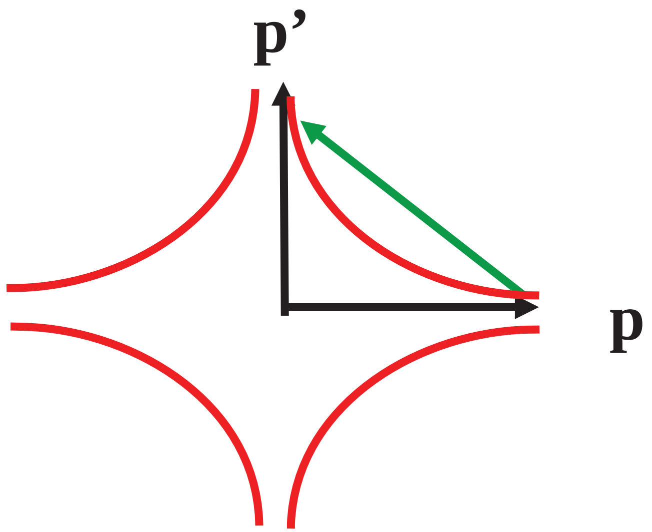
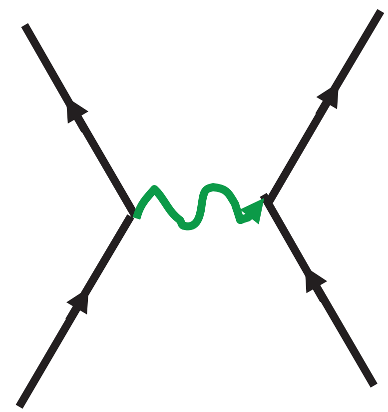
Cartoon « BCS » weak-correlation picture

$$\Delta_{\mathbf{p}} = -\frac{1}{2V}\sum_{\mathbf{p}'} U(\mathbf{p}-\mathbf{p}')\frac{\Delta_{\mathbf{p}'}}{E_{\mathbf{p}'}}\left(1-2n\left(E_{\mathbf{p}'}\right)\right)$$





Béal–Monod, Bourbonnais, Emery
P.R. B. **34**, 7716 (1986).

D. J. Scalapino, E. Loh, Jr., and J. E. Hirsch
P.R. B **34**, 8190-8192 (1986).

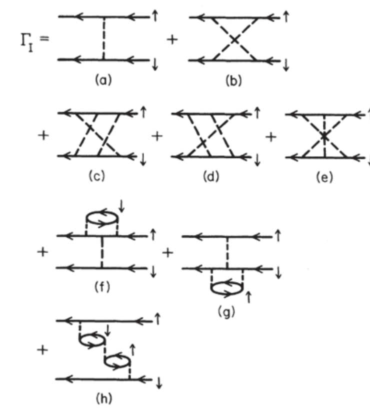
Kohn, Luttinger, P.R.L. **15**, 524 (1965).

Exchange of spin waves?
Kohn-Luttinger
T_c with pressure

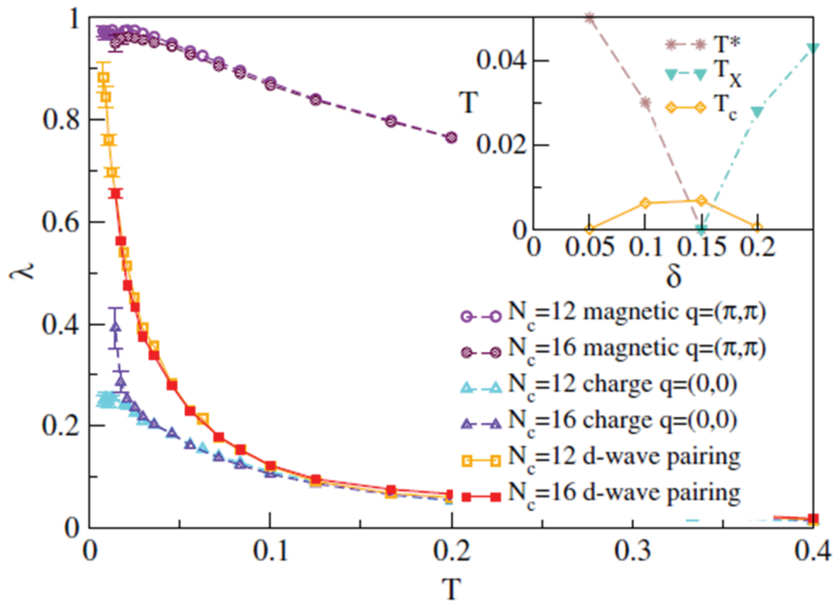
P.W. Anderson Science 317, 1705 (2007)

Detailed calculations

Bulut, Scalapino, White, PRB 47, 6157 (1993)
Maier, Jarrell, Scalapino PRL **96**, 047005 (2006)



$$\lambda_\alpha \phi_\alpha(p) = -\frac{T}{N} \sum \Gamma_I(p|p') G_\uparrow(p') G_\downarrow(-p') \phi_\alpha(p')$$



S.-X. Yang, H. Fotso, … J. Moreno,
J. Zaanen, and M. Jarrell PRL **106**, 047004 (2011)

DCA, *U=6t*, *N* = *12* and *16 sites*

U = *8t*, the « glue » approximation does not work so well

E. Khatami, A. Macridin, and M. Jarrell
Phys. Rev. B **80**, 172505 (2009)

A cartoon strong correlation picture

$$J \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j = J \sum_{\langle i,j \rangle} \left(\frac{1}{2} c_i^\dagger \vec{\sigma} c_i \right) \cdot \left(\frac{1}{2} c_j^\dagger \vec{\sigma} c_j \right)$$

$$d = \langle \hat{d} \rangle = 1/N \sum_{\vec{k}} (\cos k_x - \cos k_y) \langle c_{\vec{k},\uparrow} c_{-\vec{k},\downarrow} \rangle$$

$$H_{MF} = \sum_{\vec{k},\sigma} \varepsilon(\vec{k}) c_{\vec{k},\sigma}^\dagger c_{\vec{k},\sigma} - 4Jm\hat{m} - Jd(\hat{d} + \hat{d}^\dagger) + F_0$$

Pitaevskii Brückner:

Pair state orthogonal to repulsive core of Coulomb interaction

P.W. Anderson Science 317, 1705 (2007)

Miyake, Schmitt–Rink, and Varma P.R. B **34**, 6554-6556 (1986)

More sophisticated Slave Boson: Kotliar Liu PRB 1988

d-wave in mean-field

$$\hat{\mathcal{H}}_{mod\grave{e}le\ t-J} = -t \sum_{\langle i,j\rangle\ \sigma} \hat{P}\left(\hat{c}^\dagger_{i\sigma}\hat{c}_{j\sigma} + c.h\right)\hat{P} + J\sum_{\langle i,j\rangle}\left(\hat{\vec{S}}_i.\hat{\vec{S}}_j - \frac{1}{4}\hat{n}_i\hat{n}_j\right)$$

$$\begin{aligned}
J\hat{S}^z_i\hat{S}^z_j &= J(\hat{n}_{i\uparrow} - \hat{n}_{i\downarrow})(\hat{n}_{j\uparrow} - \hat{n}_{j\downarrow}) \\
&= J(\hat{c}^\dagger_{i\uparrow}\hat{c}_{i\uparrow} - \hat{c}^\dagger_{i\downarrow}\hat{c}_{i\downarrow})(\hat{c}^\dagger_{j\uparrow}\hat{c}_{j\uparrow} - \hat{c}^\dagger_{j\downarrow}\hat{c}_{j\downarrow}) \\
&= -J(\hat{c}^\dagger_{i\downarrow}\hat{c}_{i\downarrow}\hat{c}^\dagger_{j\uparrow}\hat{c}_{j\uparrow} + \hat{c}^\dagger_{i\uparrow}\hat{c}_{i\uparrow}\hat{c}^\dagger_{j\downarrow}\hat{c}_{j\downarrow}) + \ldots \\
&= -J(\hat{c}^\dagger_{j\uparrow}\hat{c}^\dagger_{i\downarrow}\hat{c}_{i\downarrow}\hat{c}_{j\uparrow} + \hat{c}^\dagger_{i\uparrow}\hat{c}^\dagger_{j\downarrow}\hat{c}_{j\downarrow}\hat{c}_{i\uparrow}) + \ldots
\end{aligned}$$

Hartree-Fock :

$$d^* = \langle \hat{c}^\dagger_{j\uparrow}\hat{c}^\dagger_{i\downarrow}\rangle_{\mathcal{H}_{mod\grave{e}le\ t-J}}$$

$$\langle J\hat{S}^z_i\hat{S}^z_j\rangle = -2J\,d^*d + \ldots$$

Miyake, Schmitt–Rink et Varma, PRB **34**, 6554-6556 (1986)

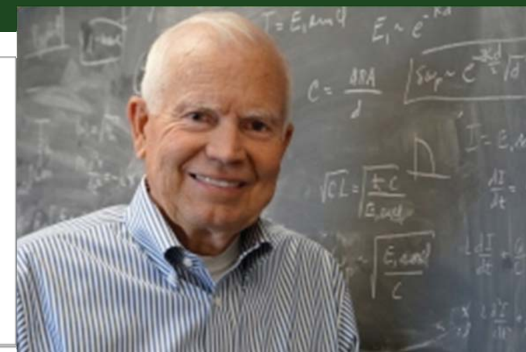
Anderson, Baskaran, Zou et Hsu, PRL **58**, 26 (1987)

P.W. Anderson

Raising the question

D.J. Scalapino

Is There Glue in Cuprate Superconductors?
Philip W. Anderson
Science **316**, 1705 (2007);
DOI: 10.1126/science.1140970



Is There Glue in Cuprate Superconductors?

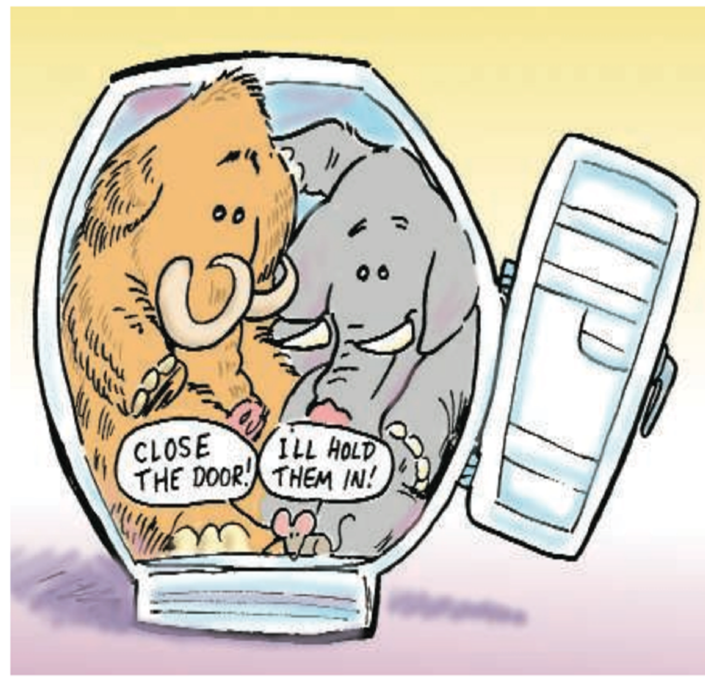
Philip W. Anderson

Many theories about electron pairing in cuprate superconductors may be on the wrong track.

Science e-letter, 5 and 10 Dec. 2007

Retardation

$$V_{él-ph}^{eff}(\vec{q},\omega) = \frac{e^2}{4\pi\varepsilon_0(q^2 + k_{TF}^2)}\left[1 + \frac{\omega_{ph}^2(\vec{q})}{\omega^2 - \omega_{ph}^2(\vec{q})}\right]$$



"We have a mammoth and an elephant in our refrigerator—do we care much if there is also a mouse?"

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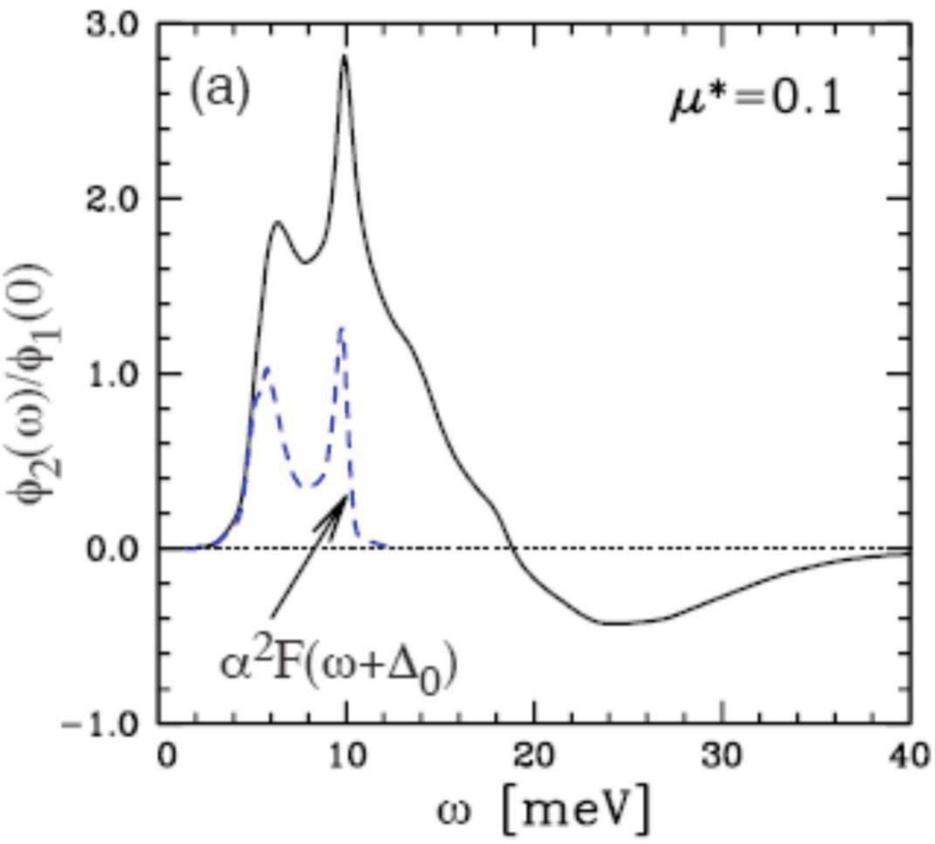
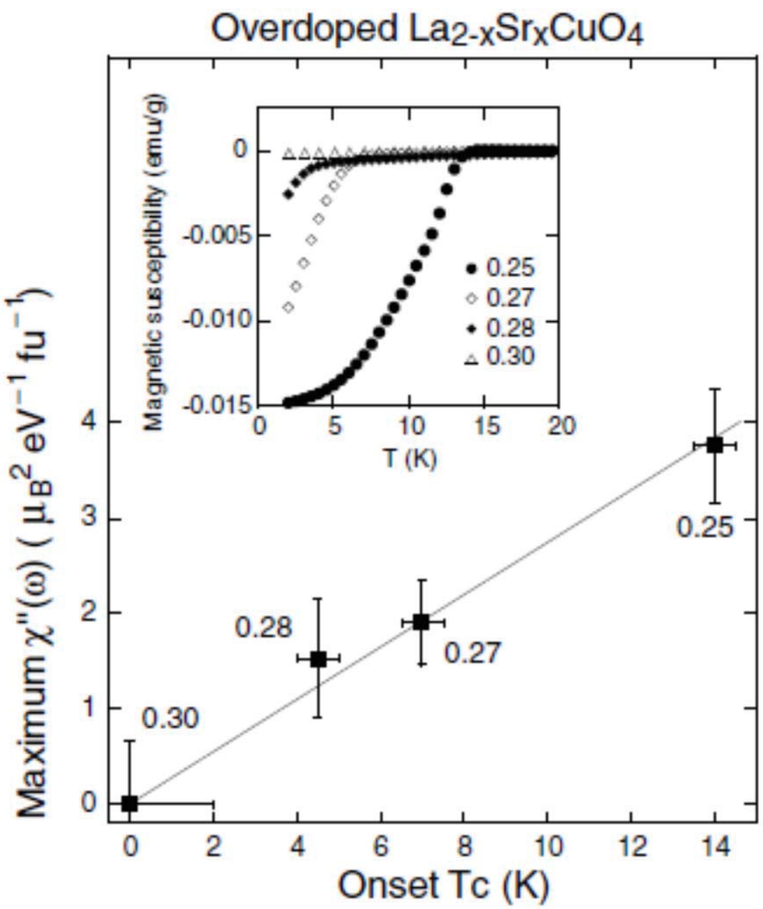
Im Σ_{an} and electron-phonon in Pb

Maier, Poilblanc, Scalapino, PRL (2008)

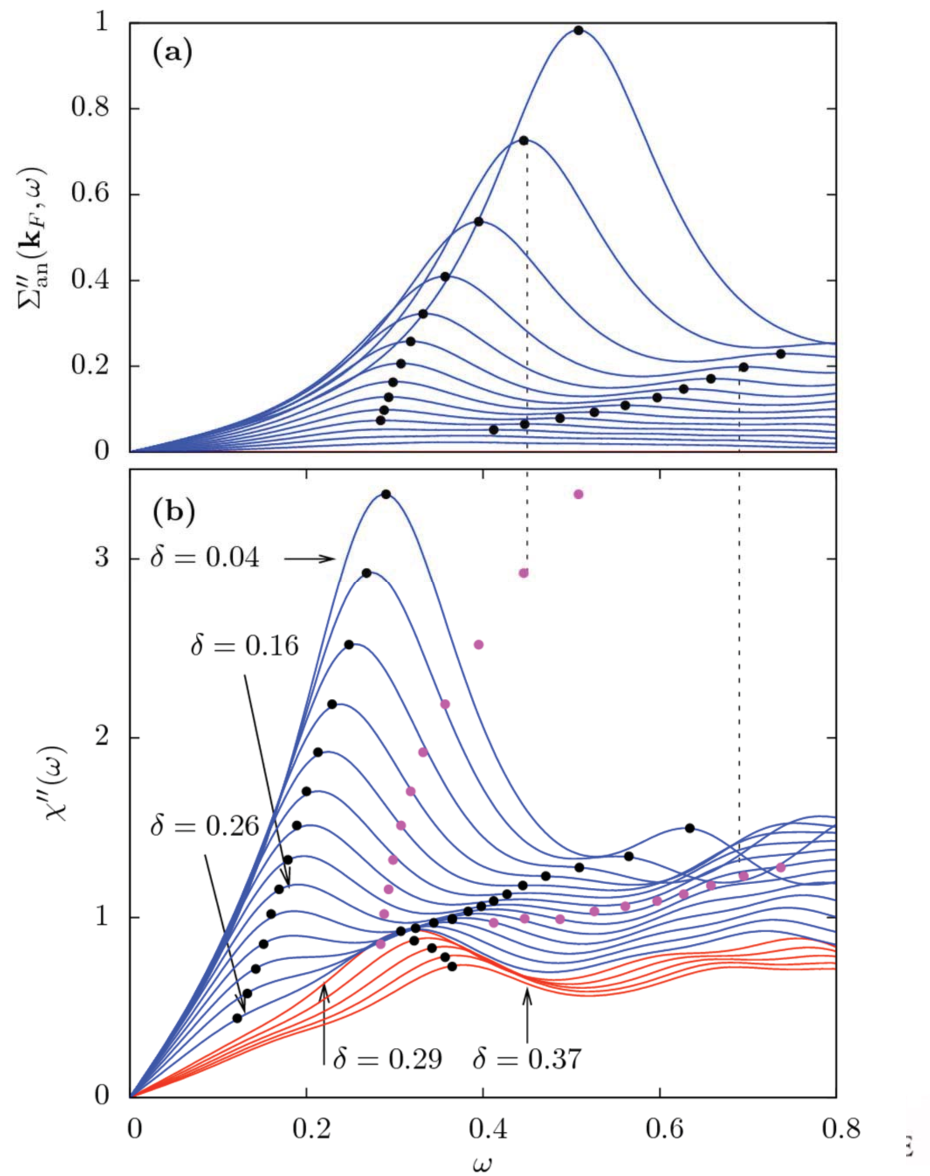


The glue

Kyung, Sénéchal, Tremblay, Phys. Rev. B **80,** 205109 (2009)



Wakimoto … Birgeneau PRL (2004)



The glue and neutrons

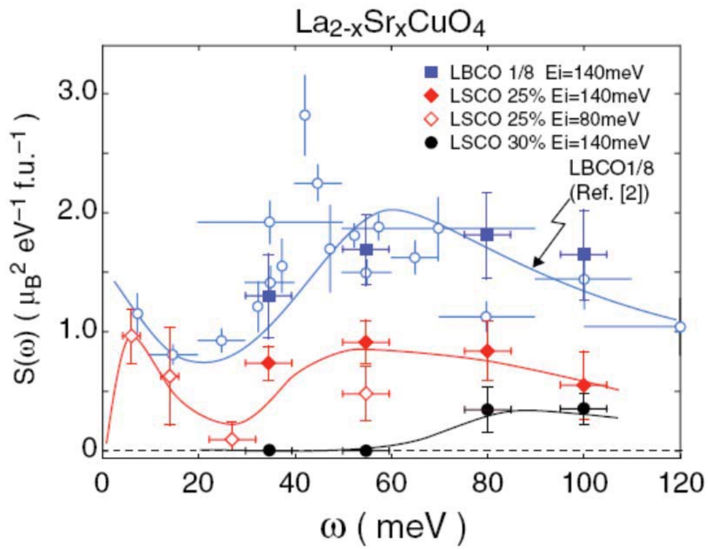
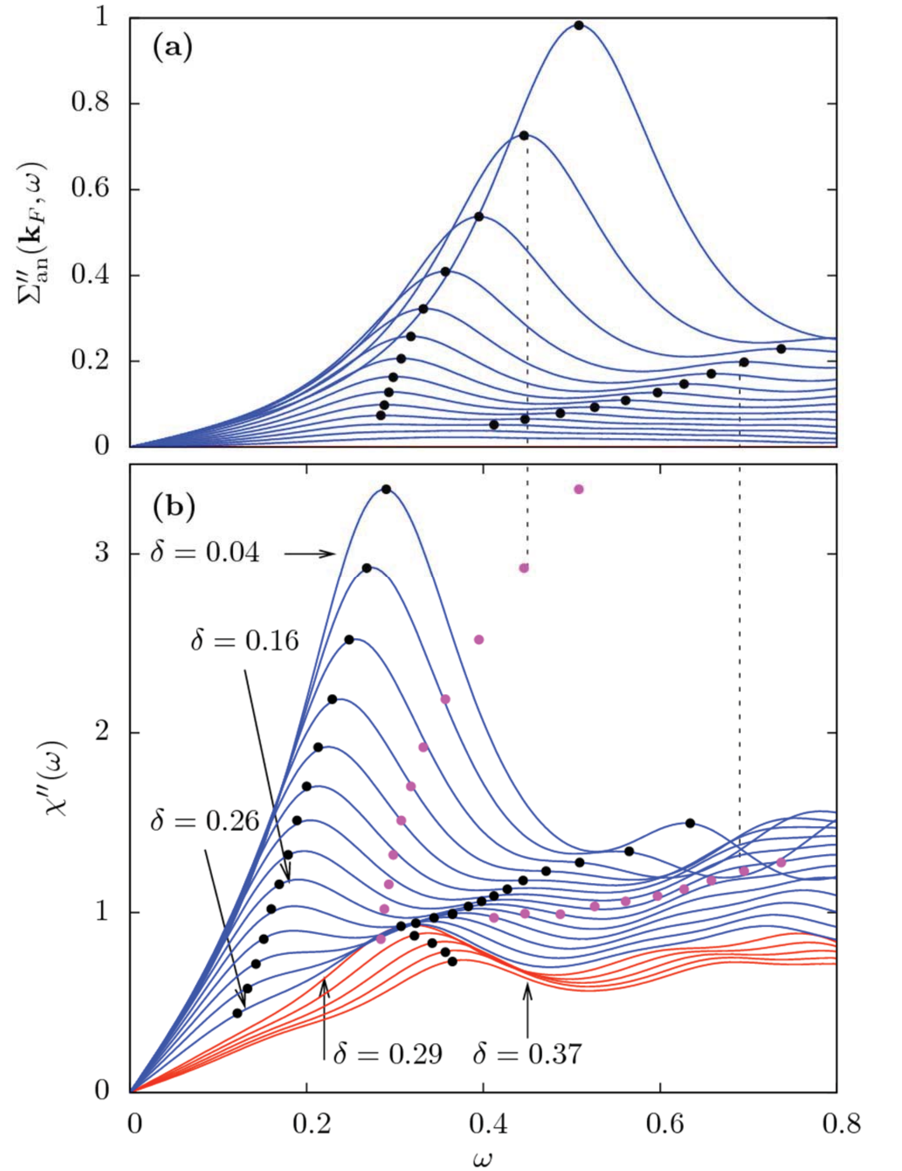


FIG. 3 (color online). **Q**-integrated dynamic structure factor $S(\omega)$ which is derived from the wide-H integrated profiles for LBCO 1/8 (squares), LSCO $x = 0.25$ (diamonds; filled for $E_i = 140$ meV, open for $E_i = 80$ meV), and $x = 0.30$ (filled circles) plotted over $S(\omega)$ for LBCO 1/8 (open circles) from [2]. The solid lines following data of LSCO $x = 0.25$ and 0.30 are guides to the eyes.

Wakimoto … Birgeneau PRL (2007); PRL (2004)



The glue in CDMFT and DCA

Th. Maier, D. Poilblanc, D.J. Scalapino, PRL (2008)

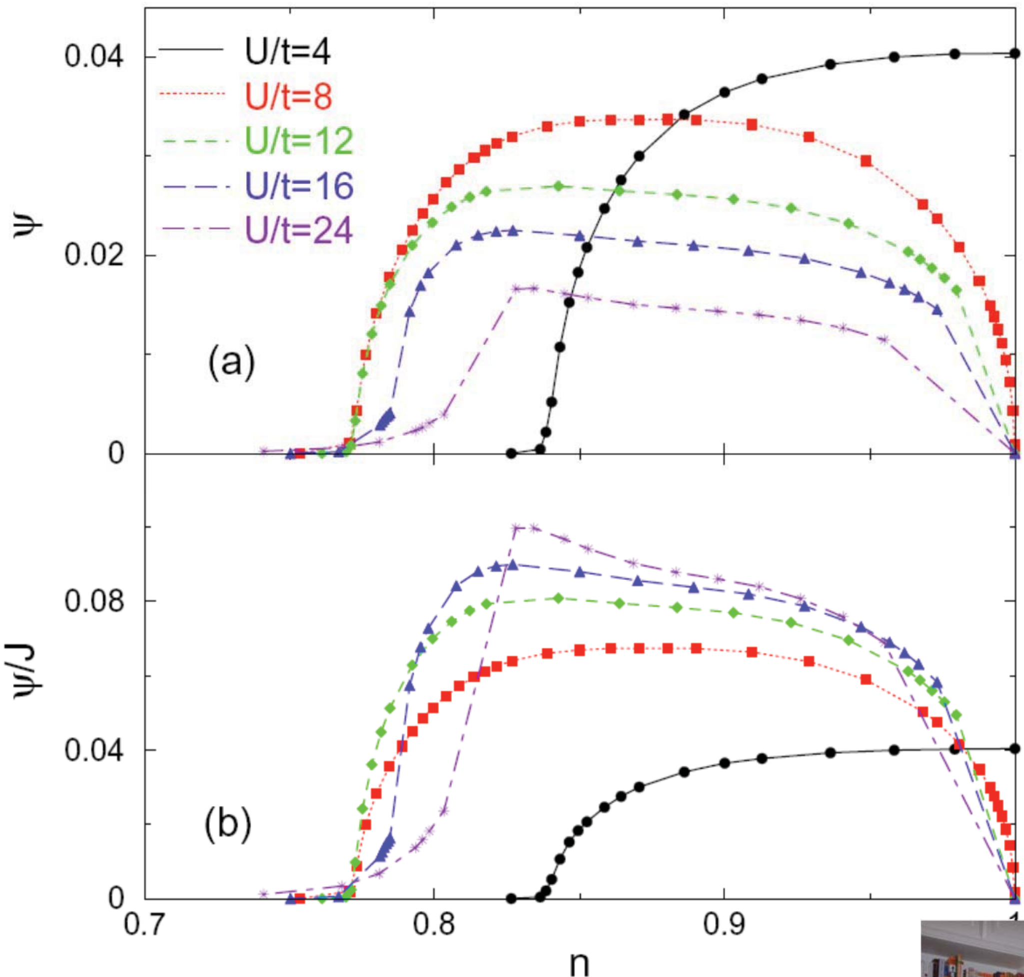
M. Civelli, PRL **103**, 136402 (2009)

M. Civelli PRB **79**, 195113 (2009)

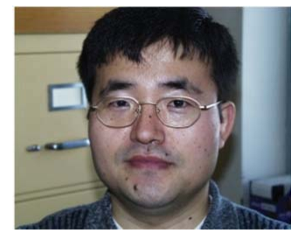
E. Gull, A. J. Millis PRB 90, 041110(R) (2014)

S. Sakai, M. Civelli, M. Imada arXiv:1411.4365

Dome vs Mott (CDMFT)



Kancharla, Kyung, Civelli, Sénéchal, Kotliar AMST Phys. Rev. B (2008)



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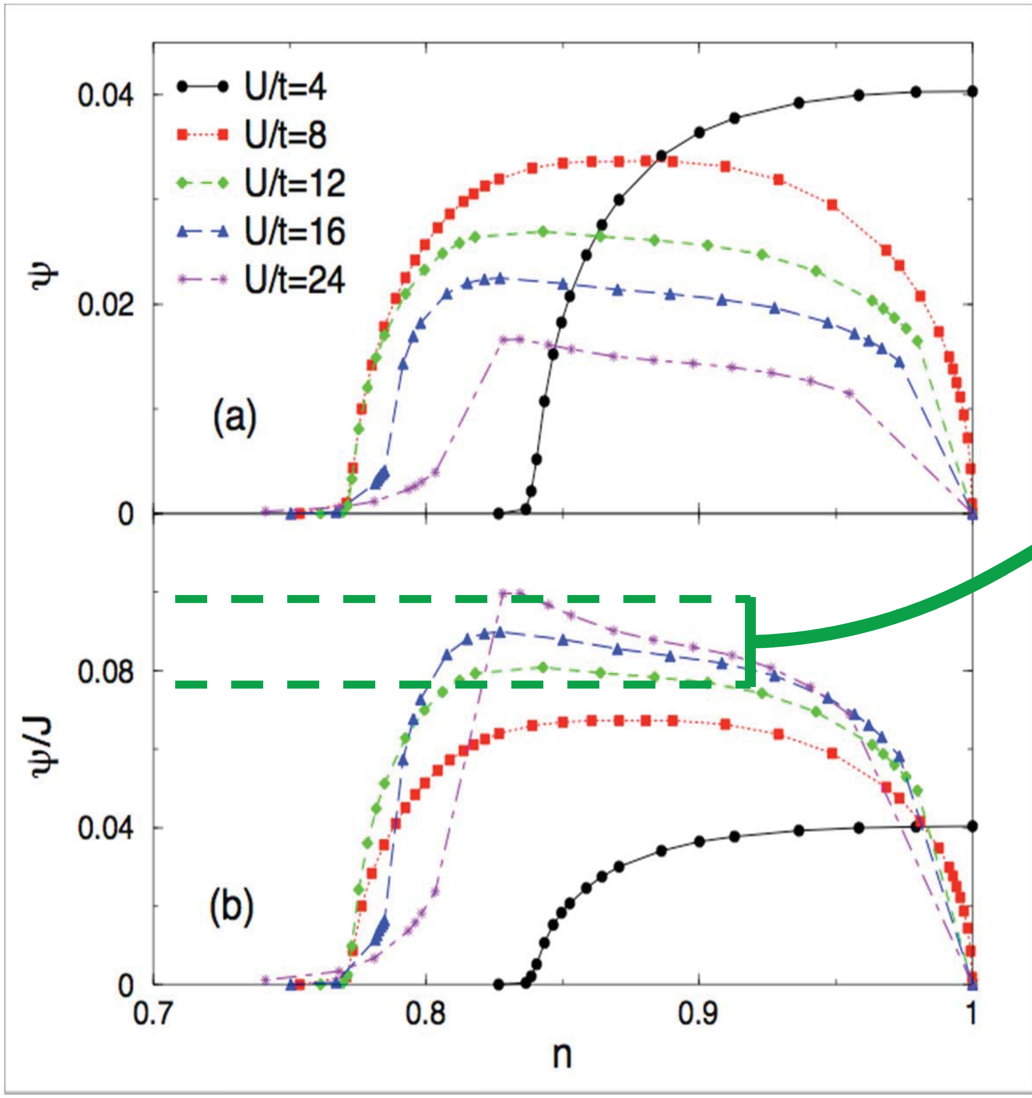
Strength of pairing: cuprates



$$J = \frac{4t^2}{U}$$

The superconducting order parameter scales like *J*

Frequencies important for pairing

Bumsoo Kyung

David Sénéchal

Anomalous Green function

$$[\mathcal{F}_{an}(t)]_{lm} = -i\theta(t)\,\langle\{\hat{c}_{l\uparrow}(t), \hat{c}_{m\downarrow}(0)\}\rangle_{\mathcal{H}_{AIM}}$$

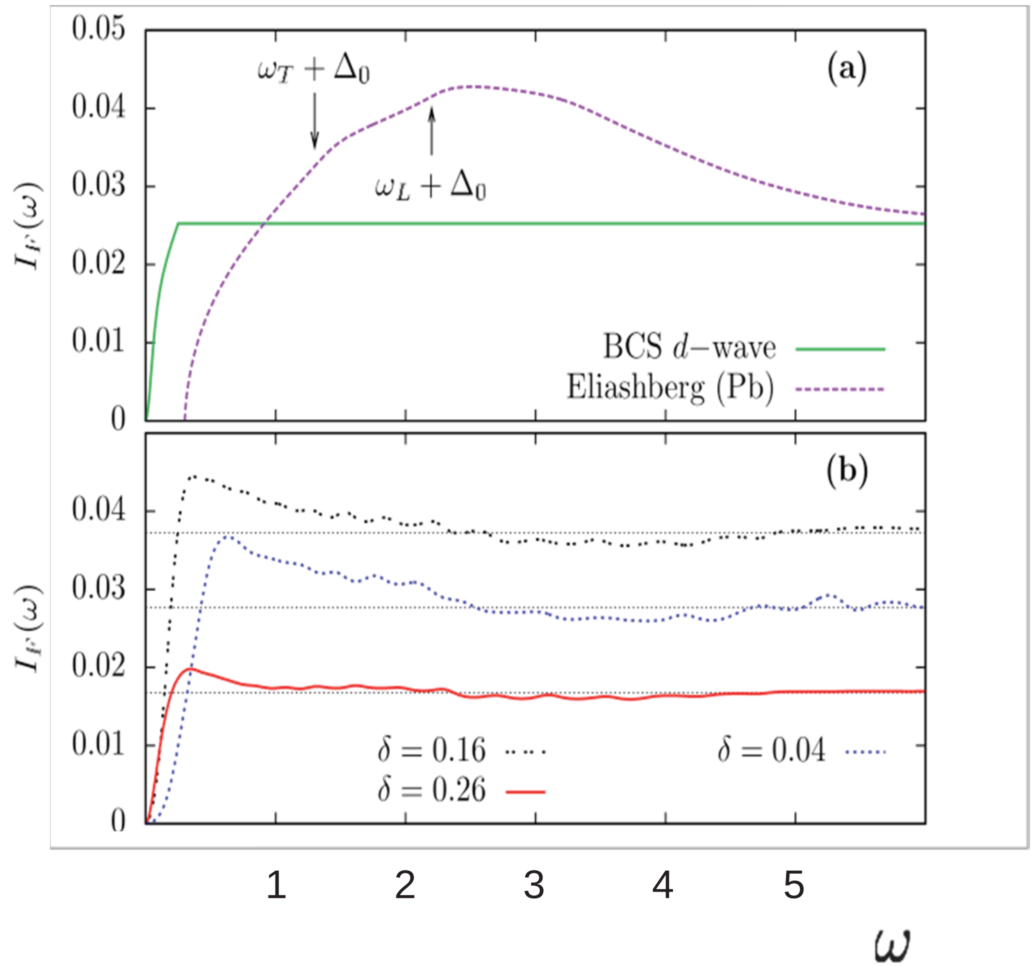
Anomalous spectral function

$$[\mathcal{A}_{an}(\omega)]_{lm} = -\frac{1}{\pi}\,\mathrm{Im}\,[\mathcal{F}_{an}(\omega)]_{lm}$$

Cumulative order parameter:

$$I_{\mathcal{F}}(\omega) = -\int_0^{\omega} \frac{d\omega'}{\pi}\,\mathrm{Im}\,[\mathcal{F}_{an}(\omega')]_{lm}$$

$$I_{\mathcal{F}}(\omega) \xrightarrow[\omega\to+\infty]{} \langle \hat{c}_{l\uparrow}\hat{c}_{m\downarrow}\rangle_{\mathcal{H}_{AIM}}$$



Scalapino, Schrieffer, Wilkins,
Phys. Rev. **148** (1966)

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Resilience to near-neighbor repulsion *V* (Scalapino)

$$\hat{\mathcal{H}}_{Hubbard} = -\sum_{\langle i,j\rangle_{1,2,3}\,\sigma}\left(t_{ij}\,\hat{c}^{\dagger}_{i\sigma}\hat{c}_{j\sigma} + c.h\right) + U\sum_{i}\hat{n}_{i\uparrow}\hat{n}_{i\downarrow} + V\sum_{\langle i,j\rangle}\hat{n}_i\hat{n}_j - \mu\sum_{i\,\sigma}\hat{n}_{i\sigma}$$

$YBa_2Cu_3O_7$: $t = 1$ $t' = -0.3$ $t'' = 0.2$

We expect superconductivity to disappear when:

$V > \frac{U^2}{W}$ **In weakly correlated case U/W < 1**

$V > J$ **In mean-field strongly correlated case**

In cuprates:

$V = 400\,\text{meV}$

$J = 130\,\text{meV}$

$U_c = V_c/[1 + N(0)V_c\ln(E_F/\omega_c)]$ Anderson-Morel

S. Onari, R. Arita, K. Kuroki et H. Aoki, PRB **70**, 094523 (2004)

S. Raghu, E. Berg, A. V. Chubukov et S. A. Kivelson, PRB **85**, 024516 (2012)

S. Sorella, et al. Phys. Rev. Lett. 88, 117002 (2002)

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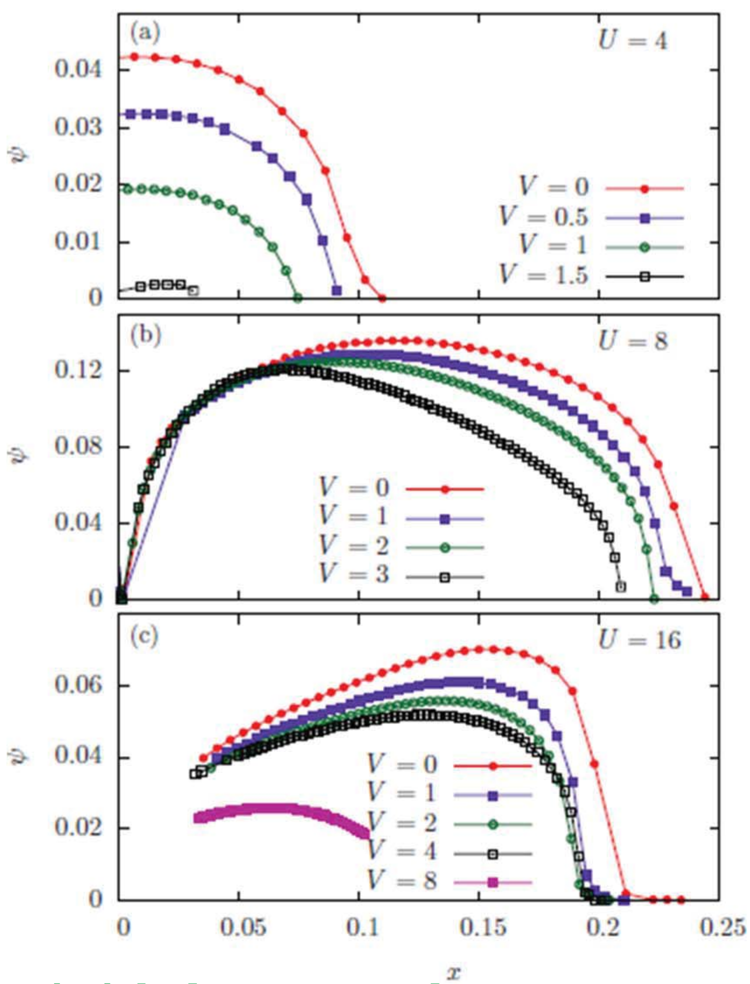
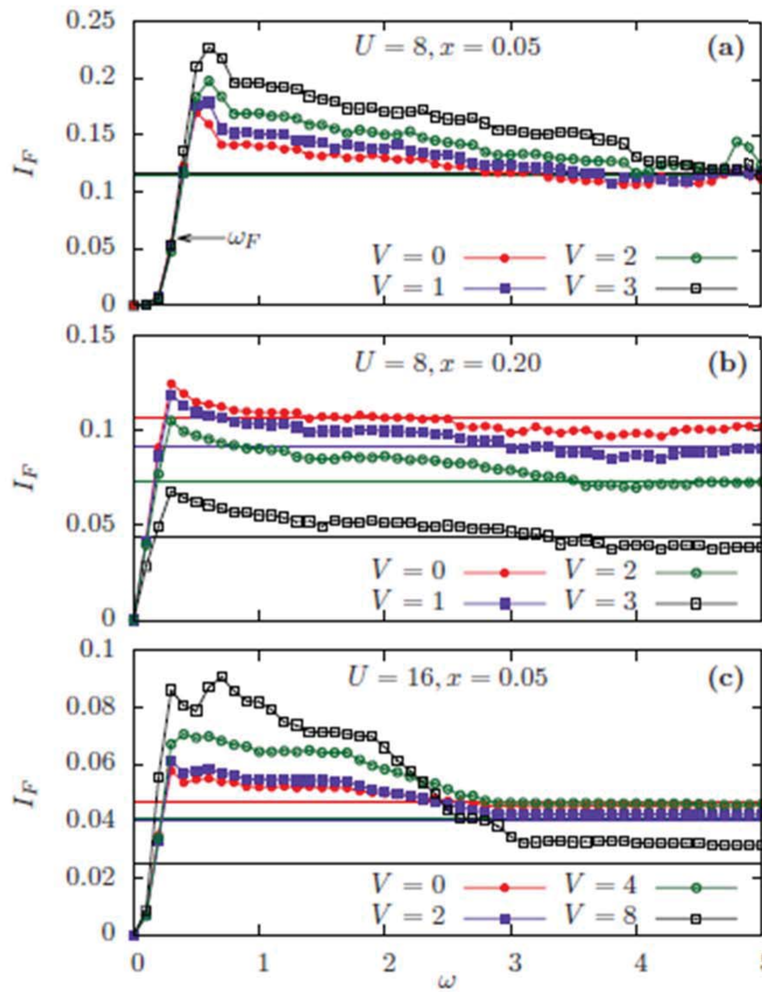
Resilience to near-neighbor repulsion

David Sénéchal

Alexandre Day

Vincent Bouliane





Sénéchal, Day, Bouliane, AMST PRB **87**, 075123 (2013)

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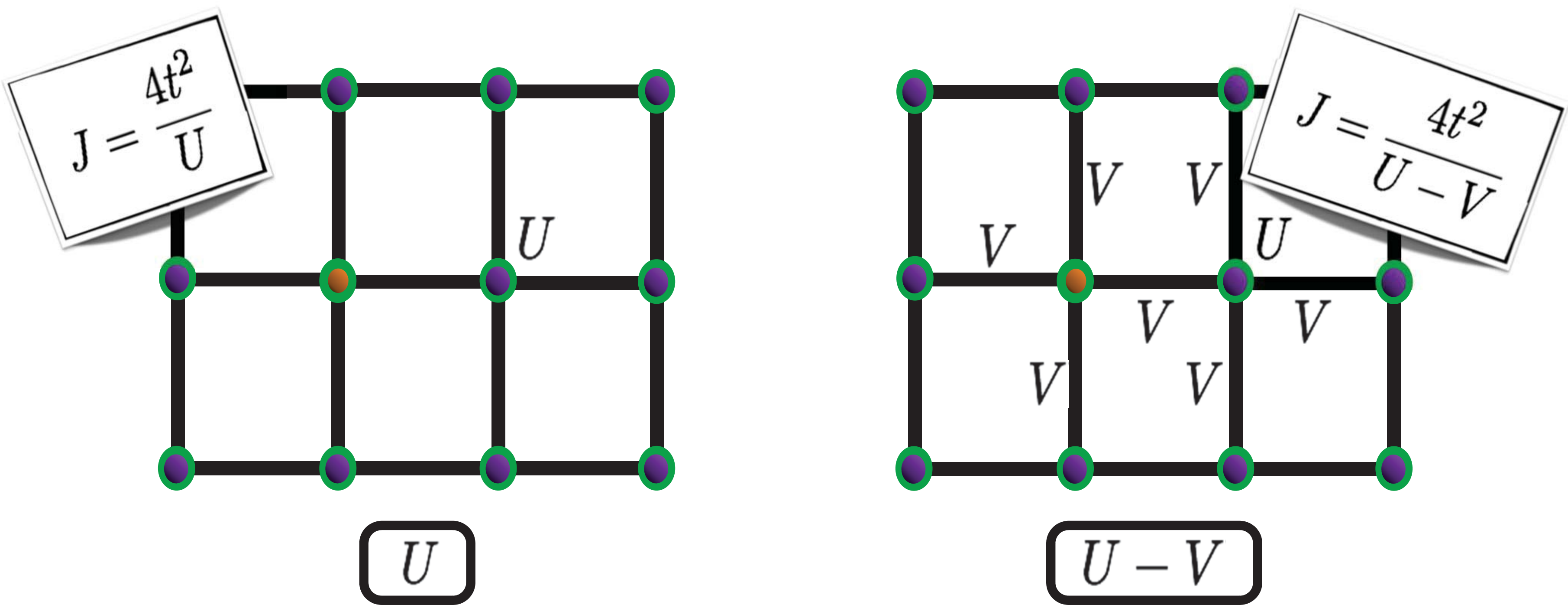
V also increases J

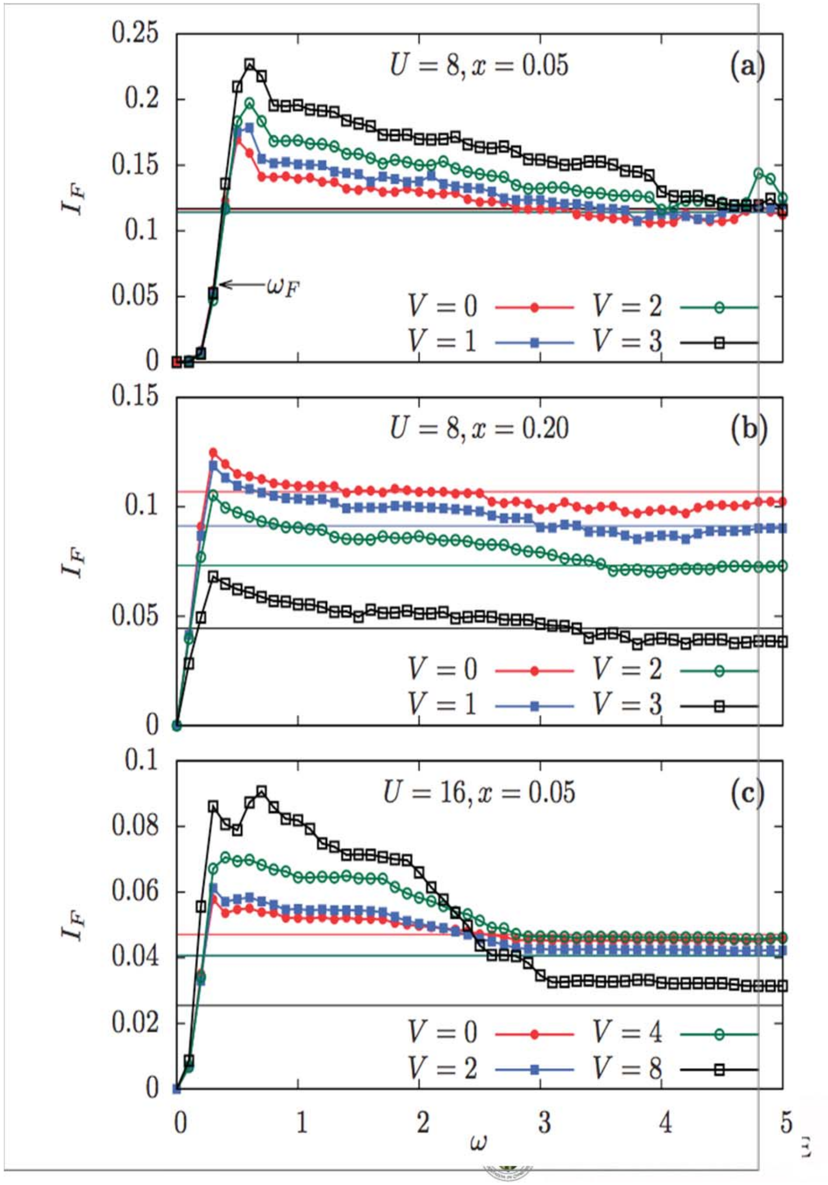


Binding aspects of V

$$J = \frac{4t^2}{U - V}$$

J **increases with** ***V*** **explaining better pairing at low frequency**

But V also induces more repulsion at high frequency, explaining the negative impact at high frequency on binding



Two gaps in underdoped regime of cuprates

Le Tacon *et al.* Nature Physics 2, 537 - 543 (2006)

….

Sakai et al. PRL 111, 107001 (2013)

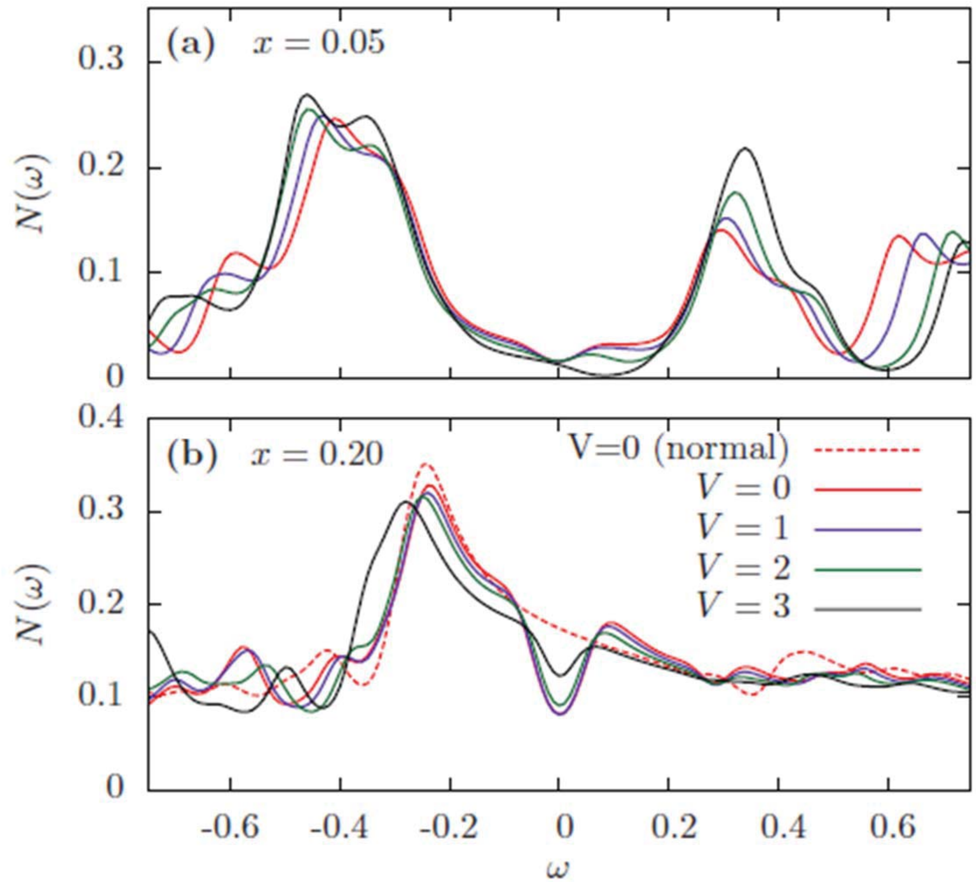
Resilience to near-neighbor repulsion

David Sénéchal

Alexandre Day

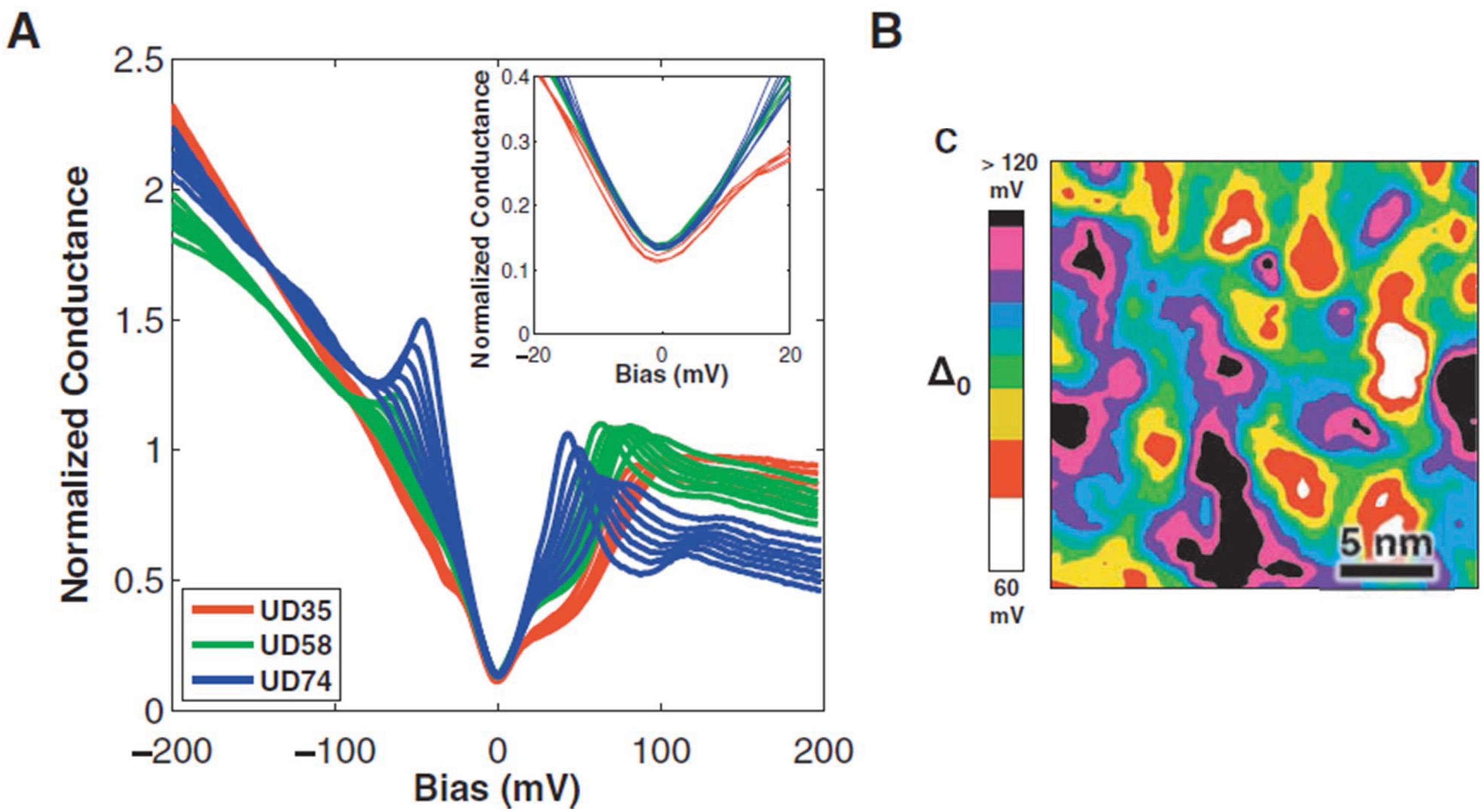
Vincent Bouliane

$U = 8t$



Sénéchal, Day, Bouliane, AMST PRB **87**, 075123 (2013)

Superconducting gap in STM



A. Pushp, Parker, … A. Yazdani,
Science **364**, 1689 (2009)

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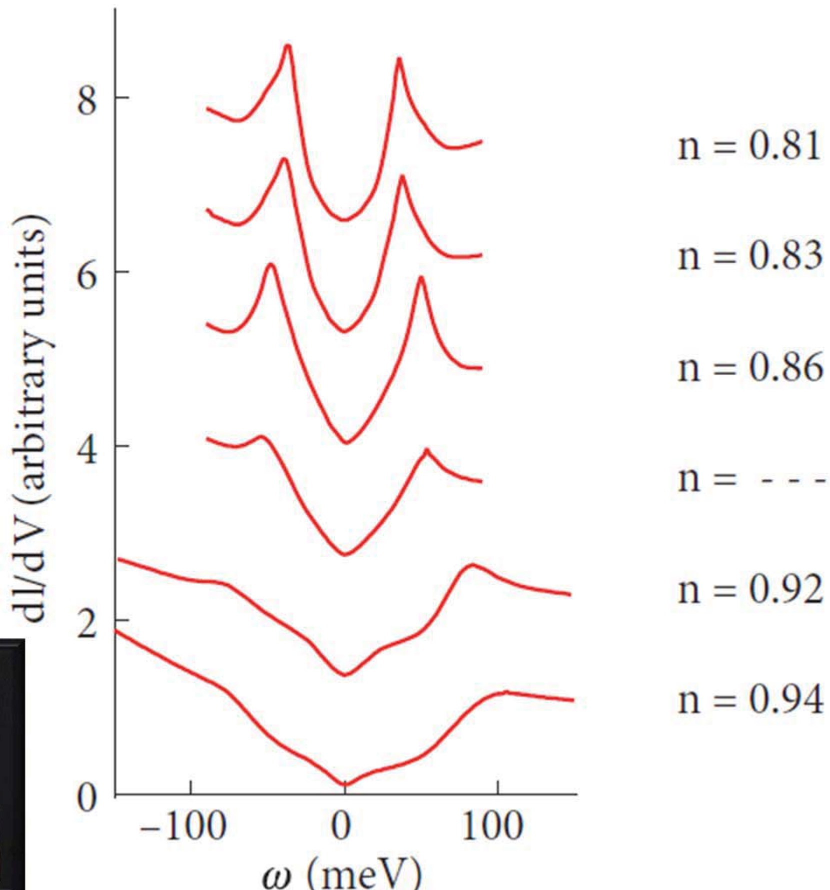
Experiment vs Theory, STM

Simon Verret

Jyotirmoy Roy

STM data

Kohsaka *et al.*, Nature **454** 1072 (2008)



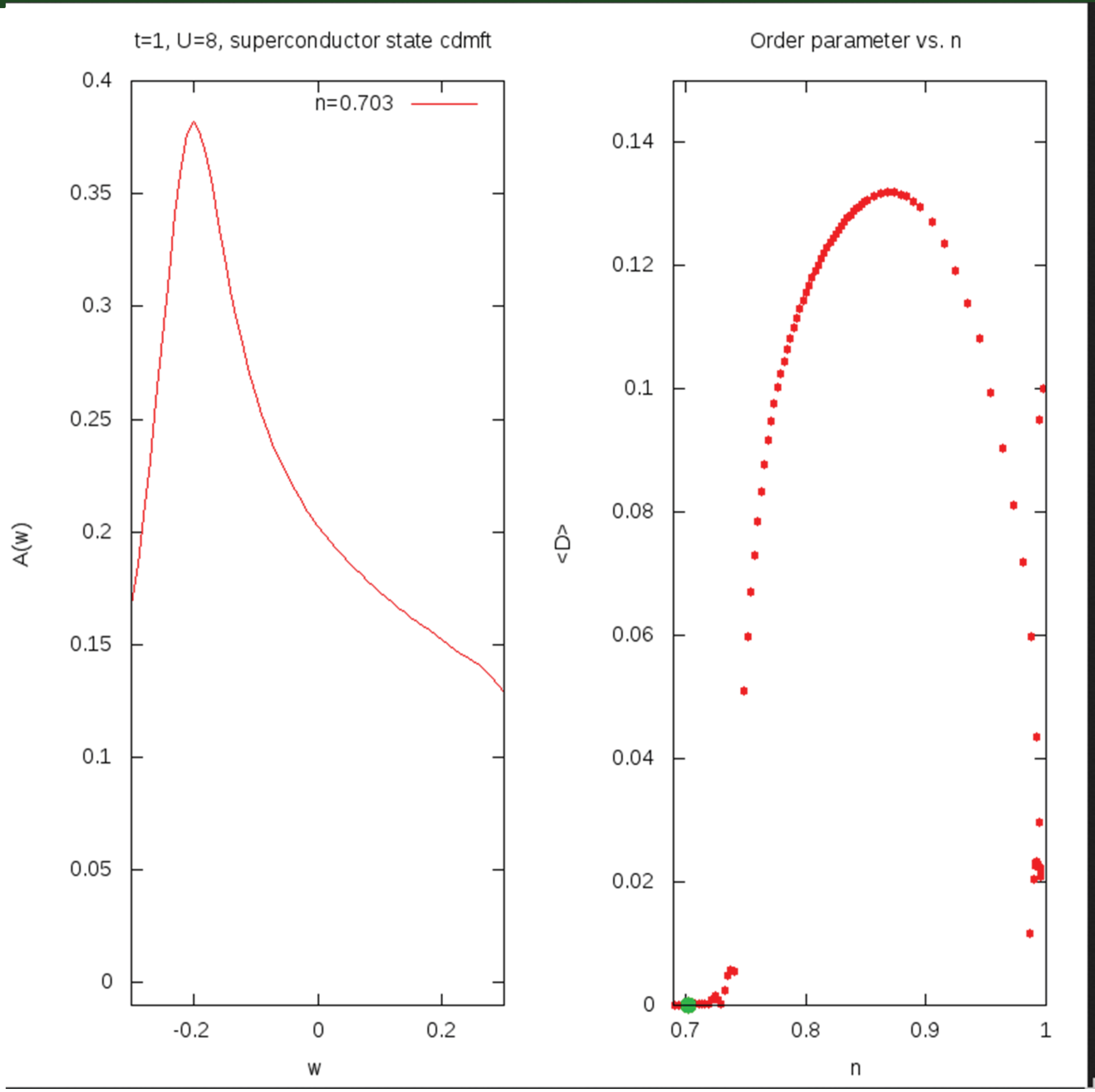
CDMFT

Unpublished

David Sénéchal

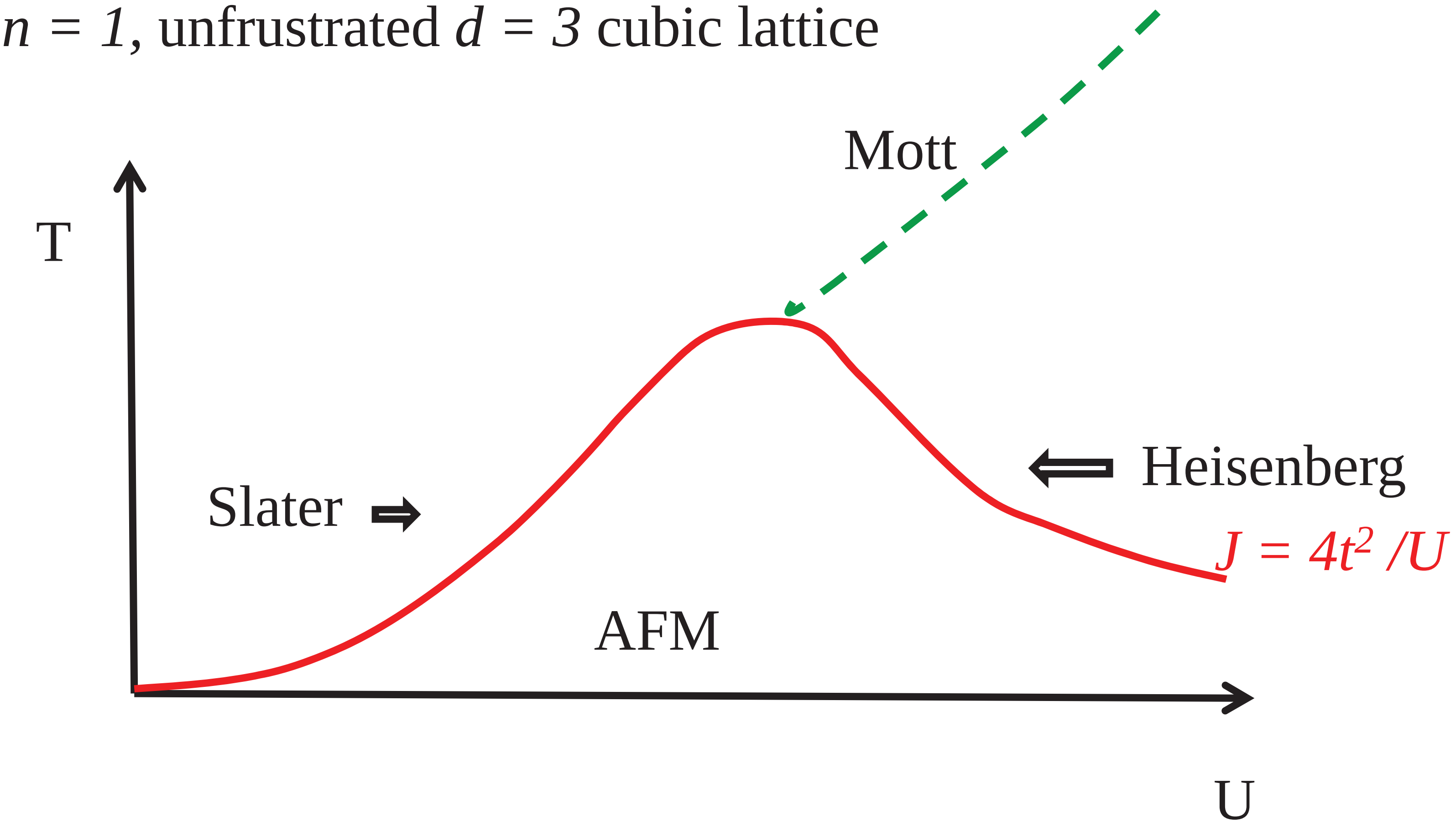
Evolution of SC gap and pseudogap with *n*

$t' = -0.3\ t$
$t'' = 0.2\ t$
$U=8t$



Local moment and Mott transition

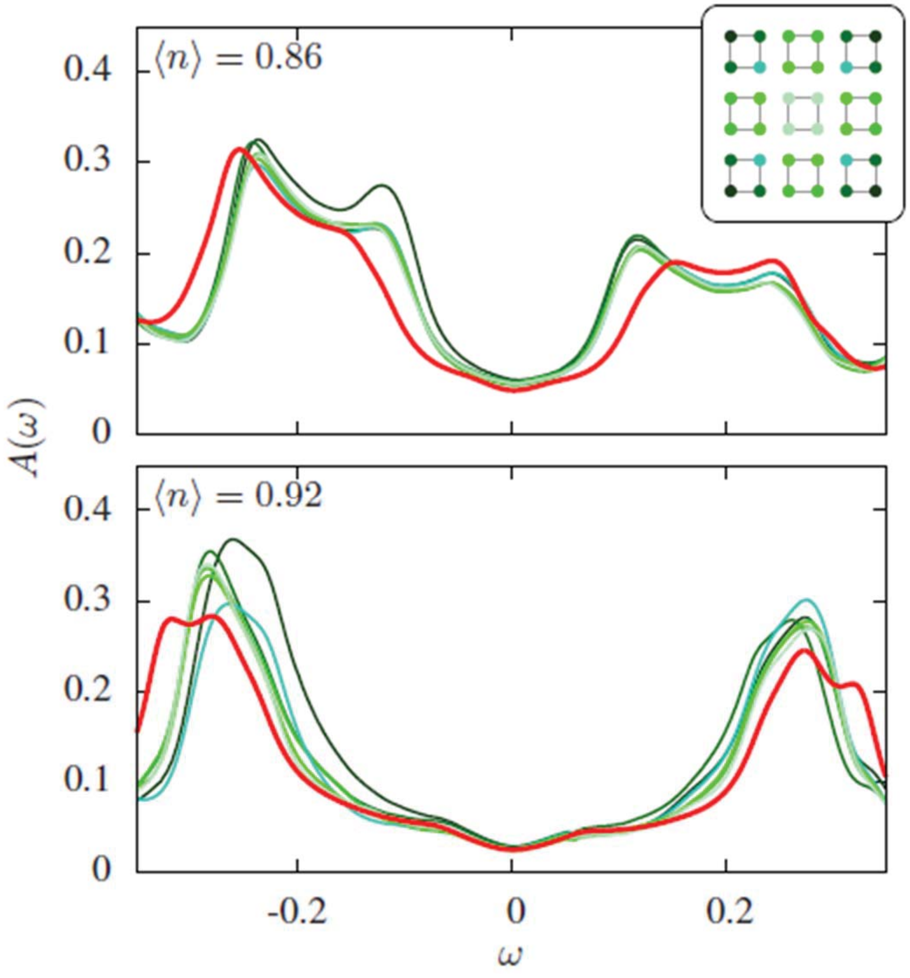
$n = 1$, unfrustrated $d = 3$ cubic lattice

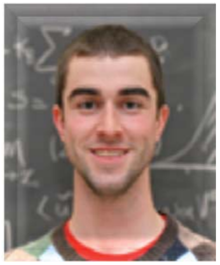


Effect of disorder

Alexandre Prémont Foley

David Sénéchal





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unpublished

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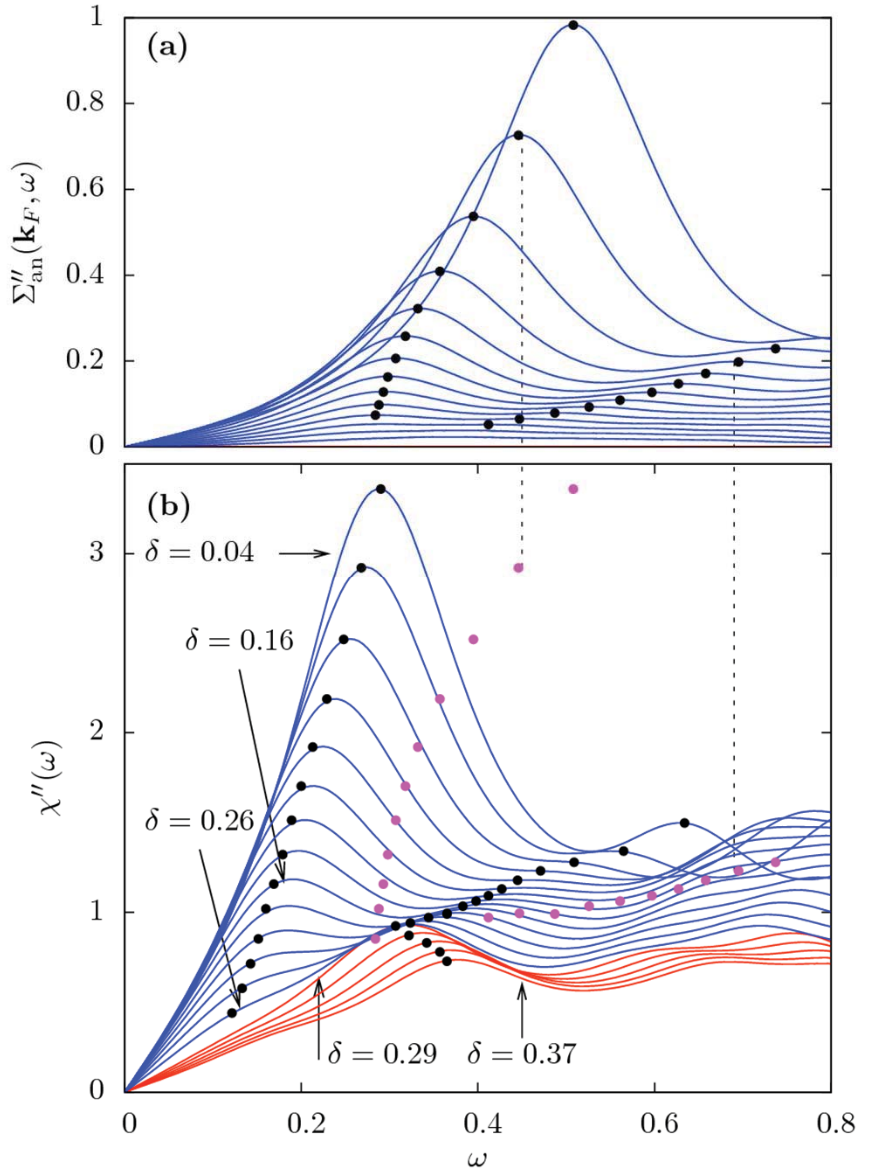
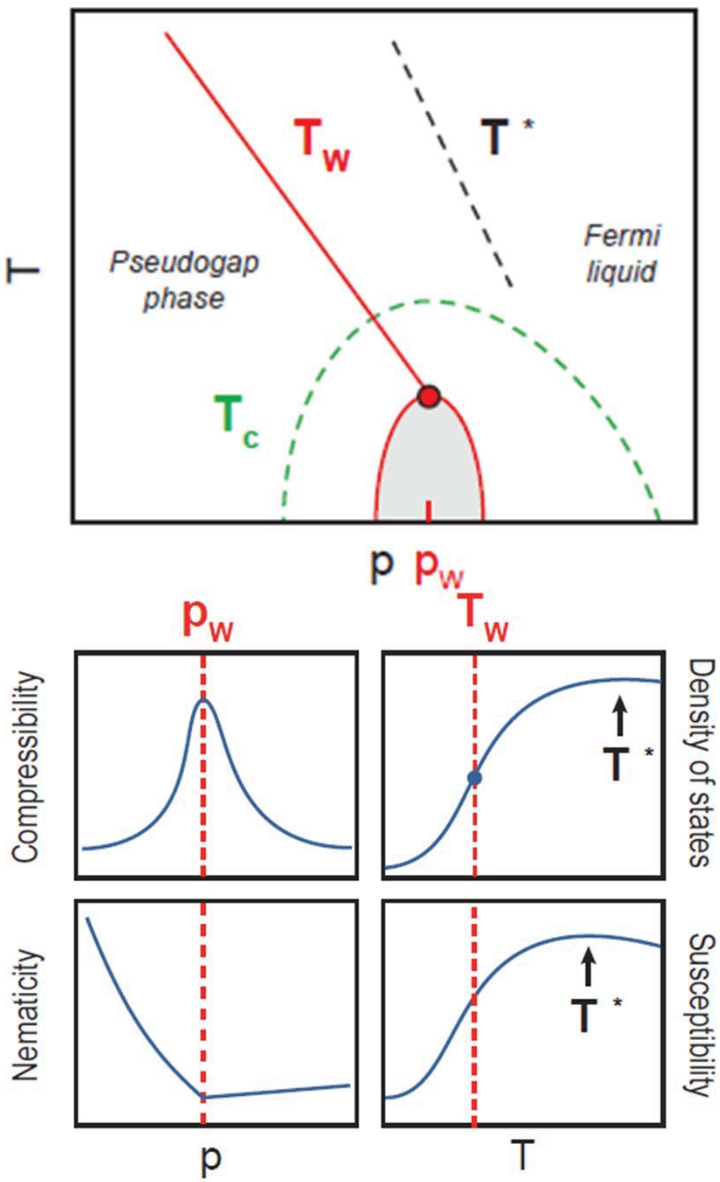
Summary

- There is retardation
- Strongly and weakly correlated SC differ
  - Penetration depth
  - Resilience to *V*



Organizing principle



- Is the pseudogap (PG) a crossover or a phase transition ?
- Relation between CDW and the PG ?
- Why CDW peaked at 12% doping ?
- Origin of nematicity ?
- Why a dome of SC ?
- Why superconducting ?
- Does a one-band model capture the key physics ?
- AFM QCP important?
- Lessons from other SC?

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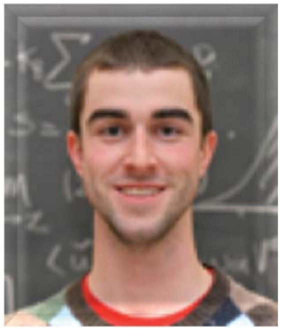
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Merci

Thank you

A.-M.S. Tremblay

"Strongly correlated superconductivity"

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