



COLLÈGE  
DE FRANCE  
— 1530 —

*Chaire de Physique de la Matière Condensée*

***Des oxydes supraconducteurs  
aux atomes froids  
- la matière à fortes corrélations quantiques -***

Antoine Georges

**Cycle 2009-2010  
Cours 7 – 16 juin 2010**

# Cours 7: Transition métal-isolant de Mott (II): le point de vue de la théorie de champ moyen dynamique

## Séminaire:



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Andrew J. MILLIS  
Columbia University

*Illuminating the Strong Correlation Problem:  
Optical conductivity of manganites, nickelates and cuprates.*

# OUTLINE

- 1. Limitations of Brinkman-Rice
- 2. Introduction to Dynamical Mean-Field Theory
- 3. The effective hybridization function - a quantum generalization of the 'Weiss field' concept
- 4. Phase diagram: Metal-insulator transition, magnetic phases
- 5. Nature of the metallic phase
- 6. Fragility of the Fermi-liquid state: the quasiparticle coherence scale
- 7. Beyond Fermi liquid: transport, spectroscopies (transfers of spectral weight)
- 8. The Mott critical endpoint

# Qualitative features of Brinkman-Rice theory: (lecture 6)

- Quasiparticle weight vanishes as transition is reached:  $Z \sim U_c - U$  (BC) or  $Z \sim \delta$  (FC)
- Drude weight  $\sim Z$
- Effective mass  $m^*/m = 1/Z$  : *quasiparticles become heavy as insulator is reached*
- **Insulator is incompressible**: jump in chemical potential  $\Delta\mu \sim (U - U_c)^{1/2}$
- Local susceptibility diverges at the transition  $\chi_{loc} \sim 1/Z \rightarrow$  **insulator has local moments** (ln2 entropy)
- Optical gap of insulator and uniform susceptibility not so well-defined in this theory (sometimes identified to  $\Delta\mu$  and  $\chi_{loc}$ , respectively, but see below.

# The simplest self-energy which makes all this possible:

$$\Sigma(\omega) = \Sigma(0) + \omega \left( 1 - \frac{1}{Z} \right)$$

1)  $\Sigma(0)$  is in charge of making Luttinger happy by insuring a Large Fermi surface:

$$\mu[n] - \Sigma[\omega = 0; n] = \mu_{U=0}[n]$$

2) All the action is in  $Z$  !

no k-dependence  $\rightarrow \frac{m^*}{m} = 1 - \frac{\partial \Sigma}{\partial \omega} = \frac{1}{Z}$

3) This self-energy is both extremely simple and a bit crazy:

- Crazy high-frequency behavior
- Only quasiparticles are described and they have infinitely long lifetime ( $\Sigma'' = 0$ )
- Total spectral weight is  $Z \rightarrow$  incoherent part not included
- Don't even think of checking Kramers-Kronig...

# Limitations of Brinkman-Rice :

*cf. lecture 6 - quite clear from the experimental results on Titanates-*

- → Must describe lifetime of quasiparticles  
→ transport, optics
- Excited states: beyond quasiparticles (Hubbard satellites)
- Transfers of spectral weight
- Superexchange provides a cutoff to the divergence of effective mass (clear from entropic arguments)

## 2. Introduction to the Dynamical Mean-Field Theory framework

Correlated electrons in large dimensions:

W.Metzner & D.Vollhardt, 1989 (then@Aachen)

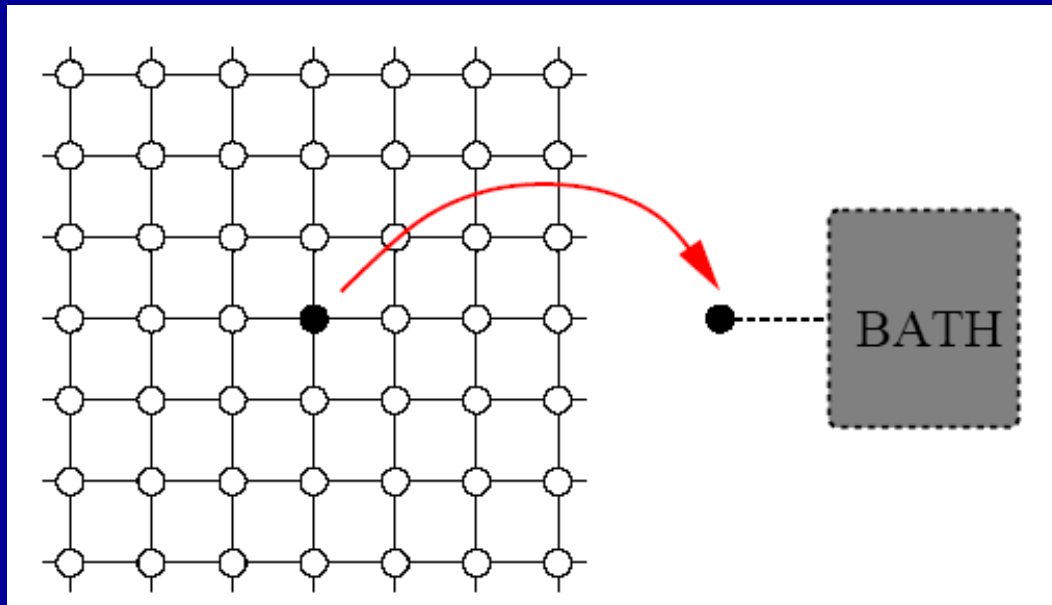
Dynamical Mean-Field Theory:

AG & G.Kotliar, 1991 (then@Princeton & Rutgers)

# DMFT:

An “effective atom” approach.

*Replace the full solid by an effective atom hybridized, in a **self-consistent manner**, to an **energy-dependent environment** (effective medium)*





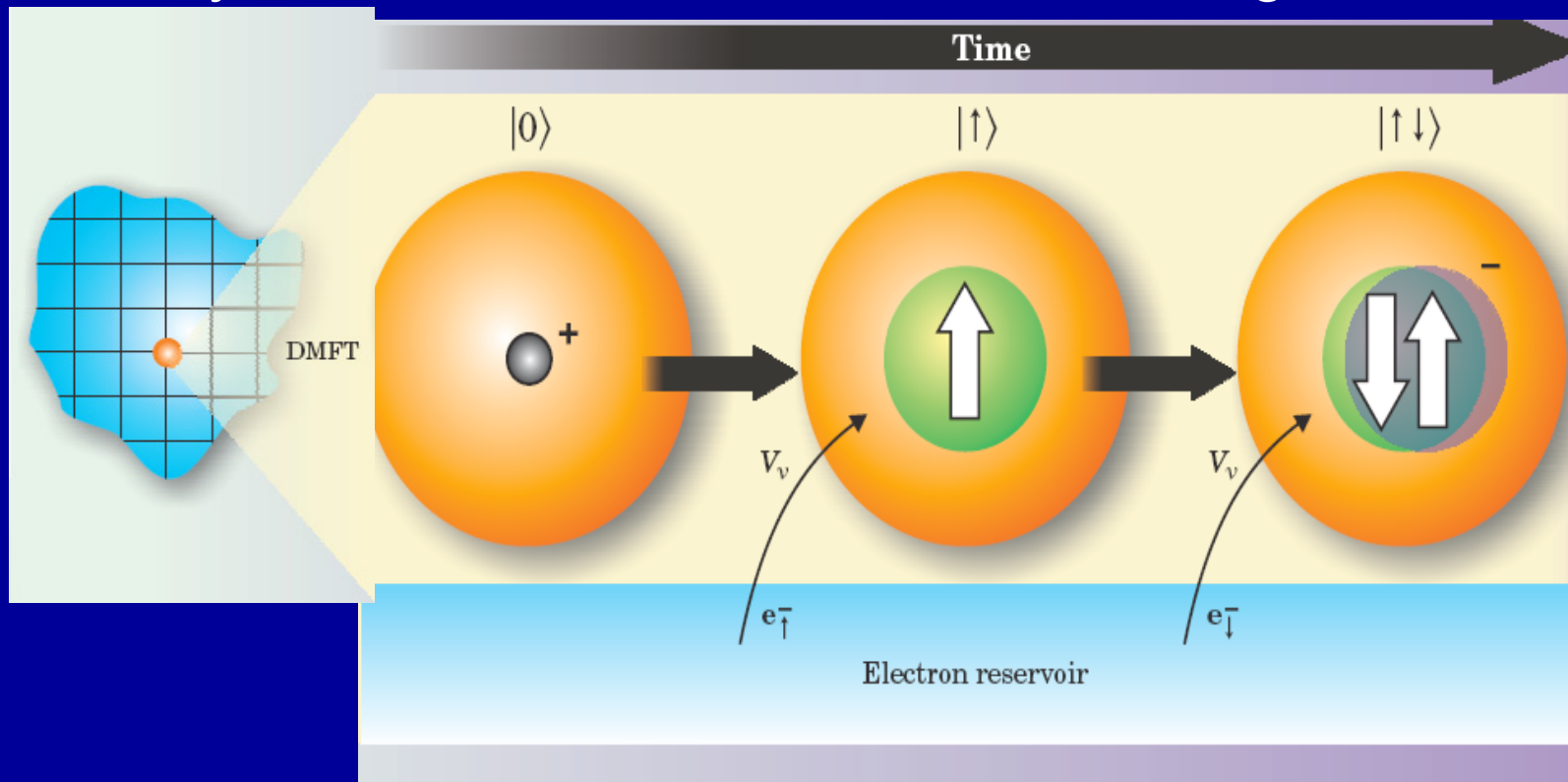
# A simple example: the Hubbard model

$$H = - \sum_{\mathbf{R}\mathbf{R}'\sigma} t_{\mathbf{R}\mathbf{R}'} f_{\mathbf{R}\sigma}^\dagger f_{\mathbf{R}'\sigma} + \sum_{\mathbf{R}} H_{\text{atom}}^{\mathbf{R}}$$

Focus on a given lattice site:

Atom can be in 4 possible configurations:  $|0\rangle, |\uparrow\rangle, |\downarrow\rangle, |\uparrow\downarrow\rangle$

Describe "history" of fluctuations between those configurations



Atom is coupled to the environment  
by exchange of electrons:  
the dynamics of these histories is that of an  
Anderson impurity problem (cf. lectures 1-2)

As we have seen in lectures 1-2, an AIM is entirely  
specified by:

- 1) The position of the atomic level  $\varepsilon_d$
- 2) The hybridization function  $\Delta(\omega)$

$$\begin{aligned} S_{\text{eff}} &= \int_0^\beta d\tau \int_0^\beta d\tau' d_\sigma^\dagger(\tau) [(-\partial_\tau - \varepsilon_d)\delta(\tau - \tau') - \Delta(\tau - \tau')] d_\sigma(\tau') \\ &+ U \int_0^\beta d\tau n_\uparrow n_\downarrow \end{aligned}$$

At this stage, we have not yet specified how to choose  $\varepsilon_d$  and  $\Delta$

Focus on energy-dependent local observable :

$$G_{RR}(\omega) \equiv G_{\text{loc}}$$

On-site Green's function (or spectral function) of the `solid`

Use atom-in-a-bath as a reference system to represent this observable:

→ IMPOSE that  $\varepsilon_d$  and  $\Delta$  should be chosen such that:

$$G_{\text{imp}}[\omega; \varepsilon_d, \Delta(\omega)] = G_{\text{loc}}(\omega)$$

At this point, given  $G_{\text{loc}}$  of the lattice Hubbard model, we have just introduced an exact local representation of it

$G_{RR}$  is related to the exact self-energy of the lattice (solid) by:

$$G_{RR}(\omega) = \sum_{\mathbf{k}} \frac{1}{\omega + \mu - \varepsilon_{\mathbf{k}} - \Sigma(\mathbf{k}, \omega)} = G_{loc}(\omega)$$

In which  $\varepsilon_{\mathbf{k}}$  is the tight-binding band (FT of the hopping  $t_{RR}$ ),

High-frequency  $\rightarrow$   $\varepsilon_d = -\mu + \sum_k \varepsilon_k (= -\mu)$

Let us now make the **APPROXIMATION** that the lattice self-energy is **k-independent** and coincides with that of the effective atom (impurity problem):

$$\Sigma(\mathbf{k}, \omega) \simeq \Sigma_{imp}(\omega)$$

This leads to the following self-consistency condition:

$$G_{imp}[i\omega; \Delta] = \sum_{\mathbf{k}} \frac{1}{G_{imp}[i\omega; \Delta]^{-1} + \Delta(i\omega) - \varepsilon_{\mathbf{k}}}$$

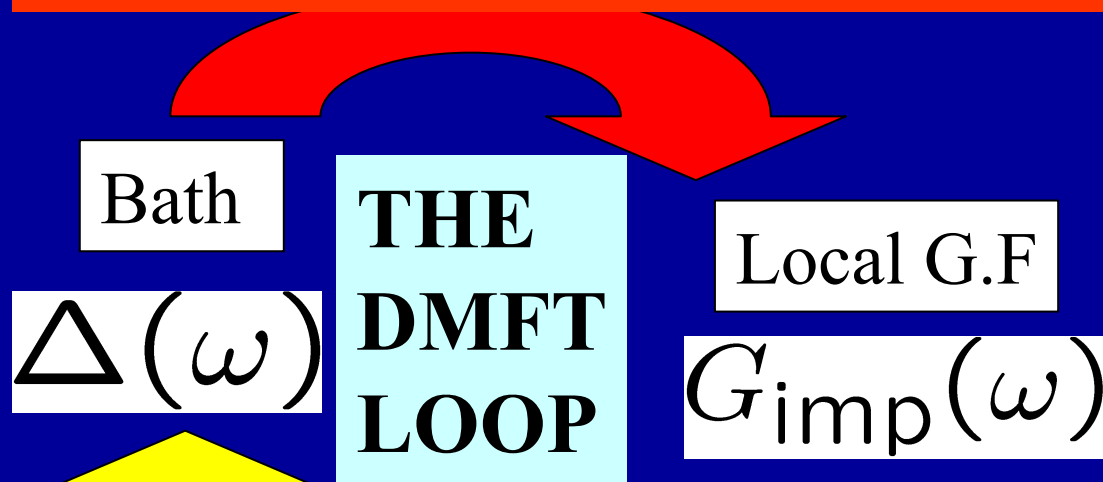
# DMFT equations:

**embedded atom + self-consistency**

→ fully determines both the local  $G$  and  $\Delta$ :

$$G_{\text{imp}}[i\omega; \Delta] = \sum_{\mathbf{k}} \frac{1}{G_{\text{imp}}[i\omega; \Delta]^{-1} + \Delta(i\omega) - \varepsilon_{\mathbf{k}}}$$

**EFFECTIVE QUANTUM IMPURITY PROBLEM**



**SELF-CONSISTENCY CONDITION**

Weiss mean-field theory  
 Density-functional theory  
 Dynamical mean-field theory

rely on similar  
 conceptual basis

**TABLE 2.** Comparison of theories based on functionals of a local observable

Theory	MFT	DFT	DMFT
Quantity	Local magnetization $m_i$	Local density $n(x)$	Local GF $G_{ii}(\omega)$
Equivalent system	Spin in effective field	Electrons in effective potential	Quantum impurity model
Generalised Weiss field	Effective local field	Kohn-Sham potential	Effective hybridisation

- Exact energy functional of local observable
- Exact representation of local observable:
- Generalized “Weiss field”
- Self-consistency condition, later approximated

see e.g:  
 A.G  
 arXiv cond-mat  
 0403123

# Check two simple limits ...

- Non-interacting case:  $\Sigma=0$

$$G_{loc}^{U=0} = \mathcal{G}_0 = \sum_{\mathbf{k}} \frac{1}{i\omega_n + \mu - \epsilon_{\mathbf{k}}}$$

- “Atomic” limit ( $t=0$ ):

4 states on each site:  $|0\rangle, |\uparrow\rangle, |\downarrow\rangle, |\uparrow\downarrow\rangle$

Obviously, in this limit:  $\Sigma_{ij}^{at} = \Sigma^{at}(i\omega_n) \delta_{ij}$

(Exact) atomic Green’s function:

$$G_{at} = \frac{1 - n/2}{i\omega_n + \mu} + \frac{n/2}{i\omega_n + \mu - U}$$

$$\Delta(i\omega_n) = 0 \quad \mathcal{G}_0^{-1} = \frac{1}{i\omega_n + \mu}$$

# The limit of large lattice connectivity

(Metzner&Vollhardt, 1989)

-The DMFT scheme becomes EXACT in the limit of large lattice connectivity (large number of spatial dimensions).

- Requires scaling of hopping amplitude

$$t_{ij} \propto 1/\sqrt{z}$$

- Proof: e.g. ``cavity construction'' or perturbative inspection of Baym-Kadanoff functional.



# Phases with long-range order

e.g self-consistency condition in commensurate  $Q=(\pi, \dots, \pi)$  antiferromagnetic phase

A- and B- sublattice, symmetry:  $G_{A\sigma}(i\omega_n) = G_{B,-\sigma}(i\omega_n)$

Green's function matrix:

$$\begin{pmatrix} \zeta_{A\sigma} & -\epsilon_{\mathbf{k}} \\ -\epsilon_{\mathbf{k}} & \zeta_{B\sigma} \end{pmatrix}$$

$$\zeta_{A\sigma} = i\omega_n + \mu - \sigma h_s - \Sigma_{A\sigma}$$

Self-consistency:

$$G_{\alpha\sigma} = \zeta_{\bar{\alpha}\sigma} \int_{-\infty}^{\infty} d\epsilon \frac{D(\epsilon)}{\zeta_{A\sigma} \zeta_{B\sigma} - \epsilon^2}$$

# Solving the effective quantum impurity problem: computational techniques

Several numerical algorithms, or semi-analytical approximation schemes have been developed over the years to this aim, starting in the early days of the Kondo effect (Anderson impurity model)

[e.g: Hirsch-Fye auxiliary field QMC,  
Resummed perturbation theories,  
Numerical Renormalisation Group]

***Recent key advance:***

***continuous time QMC → cf seminar 1 by O.Parcollet***

*(A.Rubtsov et al.: pert. series in  $U$ ,*

*P.Werner et al: pert. series around atomic limit)*

# $\Delta(\omega)$ : generalizing the Weiss field to the quantum world



Pierre Weiss  
1865-1940  
« *Théorie du  
Champ  
Moléculaire* »  
(1907)

*Einstein, Paul Ehrenfest, Paul Langevin, Heike Kammerlingh-Onnes, and Pierre Weiss at Ehrenfest's home, Leyden, the Netherlands. From Einstein, His Life and Times, by Philipp Frank (New York: A.A. Knopf, 1947). Photo courtesy AIP Emilio Segrè Visual Archives.*

# Low-frequency behavior of $\Delta(\omega)$ determines nature of the phase

- $\Delta(\omega \rightarrow 0)$  finite  $\rightarrow$  local moment is screened. `Self-consistent' Kondo effect. Gapless metallic state.
- $\Delta(\omega)$  gapped  $\rightarrow$  no Kondo effect, degenerate ground-state, insulator with local moments

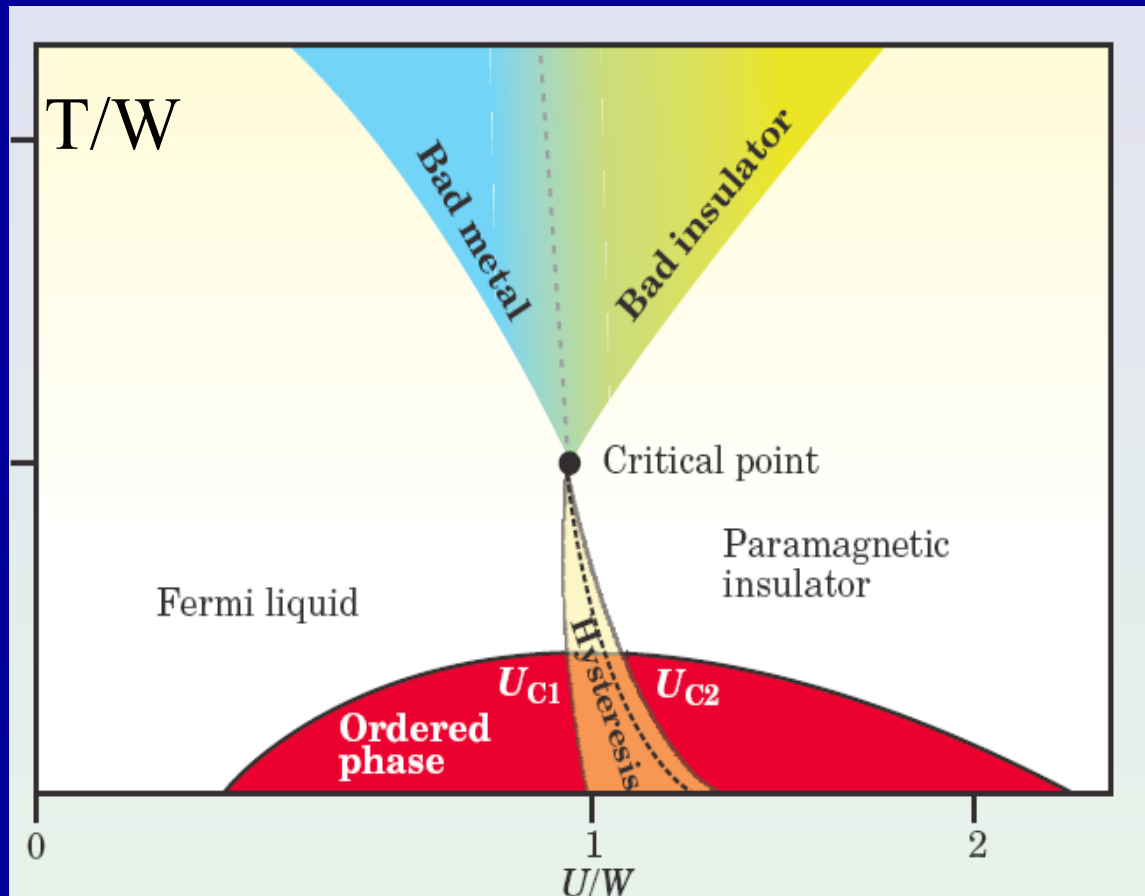
# A- Phase diagram

Will focus in the following (for lack of time) on:

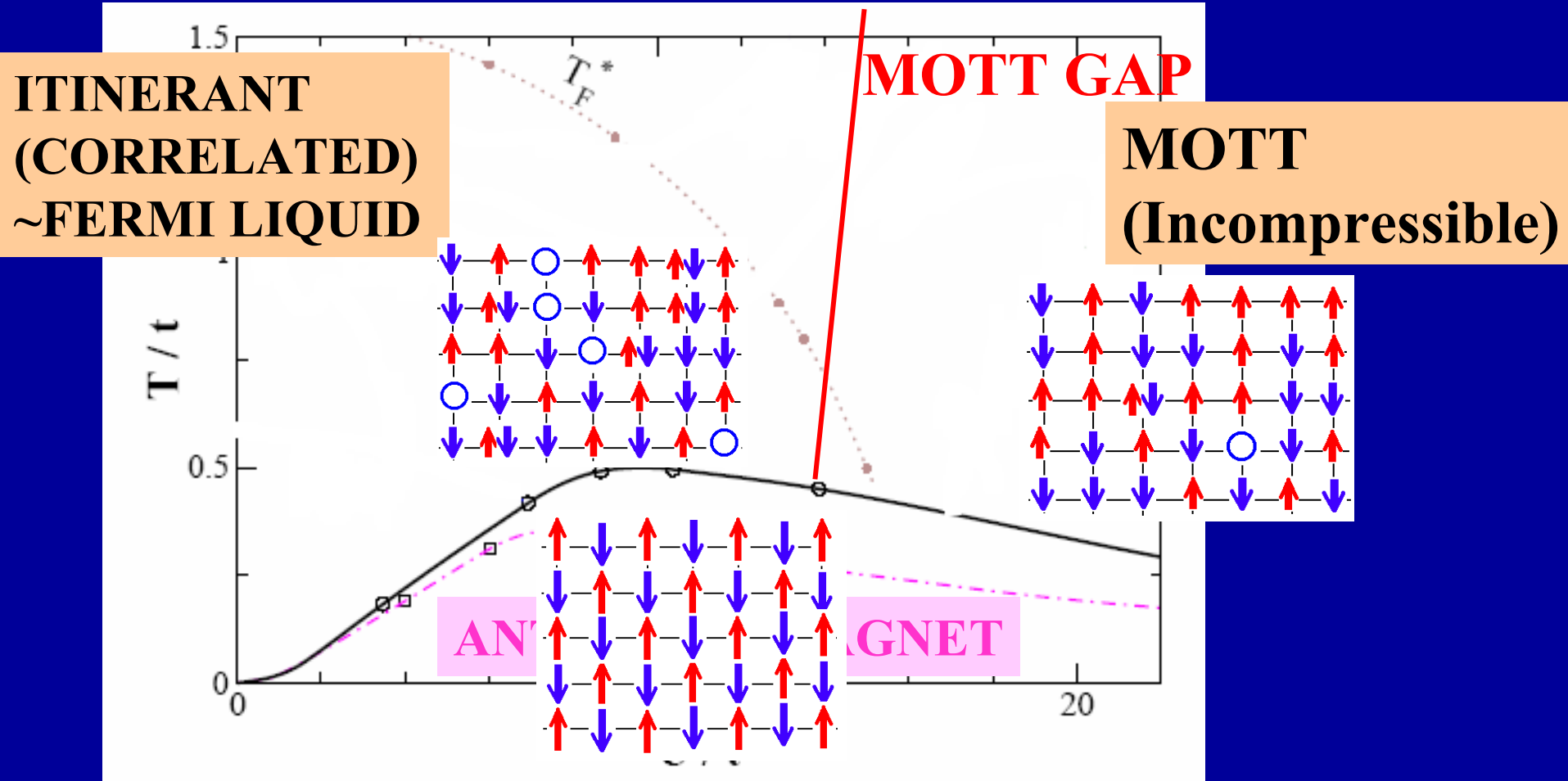
- $1/2$  -filled Hubbard model

- mostly paramagnetic solutions

# Generic phase diagram (Hubbard at $\frac{1}{2}$ -filling, schematic)



# Unfrustrated $\frac{1}{2}$ -filled model: phases with long-range order and crossovers, $d=3$ cubic lattice

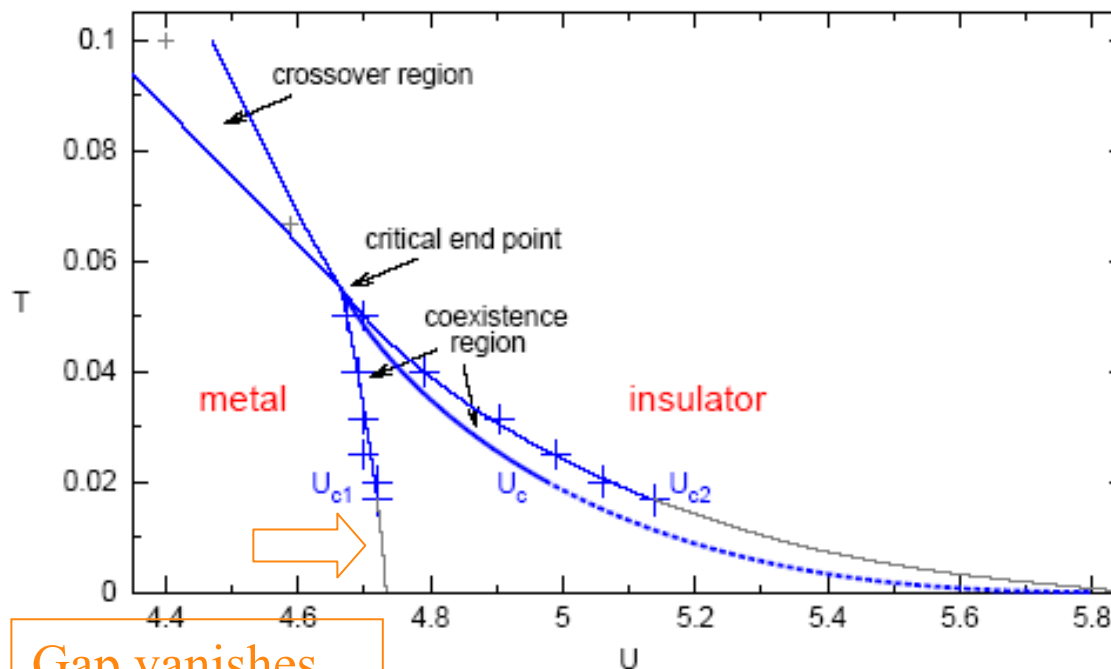


Critical boundary calculated for a 3D cubic lattice using:  
- Quantum Monte Carlo (Staudt et al. Eur. Phys. J. B17 (2000) 411)  
- Dynamical Mean-Field Theory approximation

# Phase diagram : zoom on paramagnetic solutions

Hubbard model, Bethe lattice, homog. phase,  $n = 1$ , e.g., DMFT(QMC)

[Blümer '02]



- coexistence region  $[U_{c1}; U_{c2}]$ , first-order transition
- crossover above critical region

Blümer et al. Units here are  $4D=2*\text{bandwidth}$



# B-Nature of the metallic phase

- At (possibly very) low  $T, \omega$ : a Fermi liquid

$$\text{Re}\Sigma(\omega + i0^+) = U/2 + (1 - 1/Z)\omega + O(\omega^3),$$

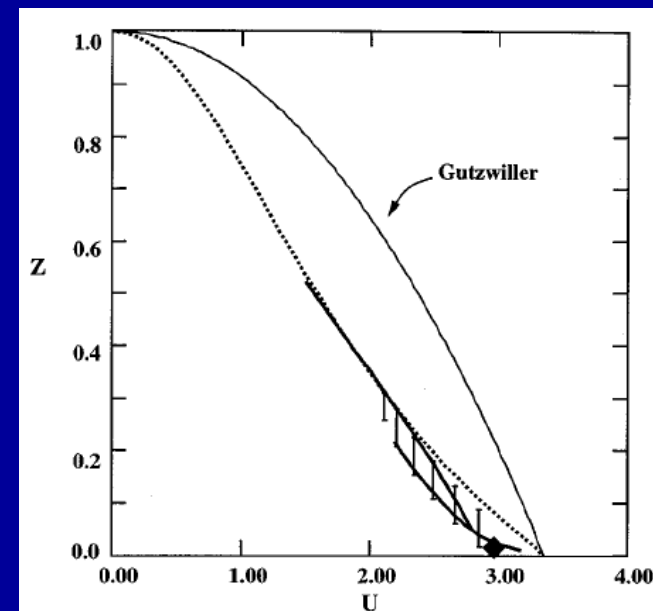
$$\text{Im}\Sigma(\omega + i0^+) = -B\omega^2 + O(\omega^4).$$

- At  $U_{c2}$  transition:  $Z \rightarrow 0$  ( $\sim$  Brinkman-Rice)

- Heavy quasiparticles:

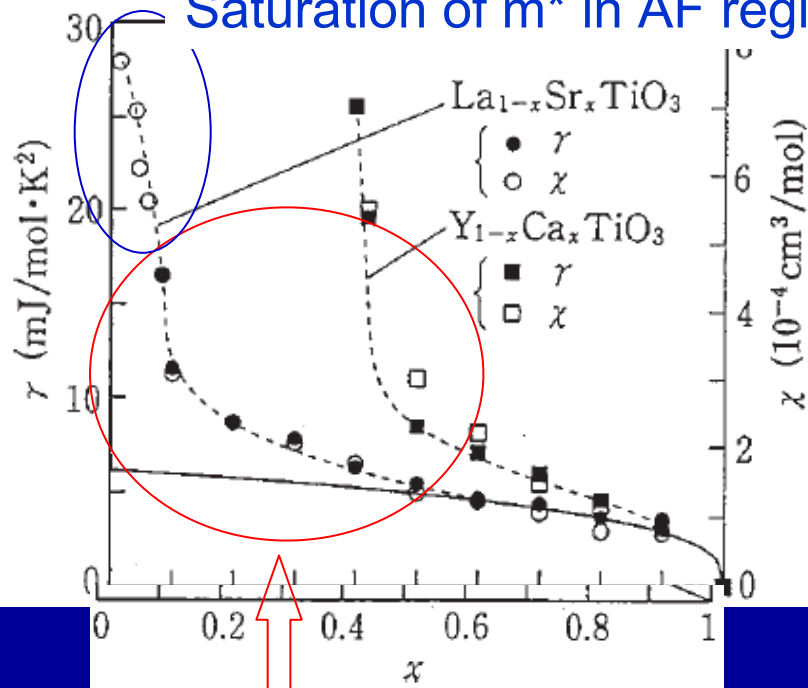
$$m^*/m = 1/Z$$

(divergence reflects  
large entropy of insulator,  
see below)



# Approach to the Mott state in titanates

Saturation of  $m^*$  in AF regime



Increase of effective mass

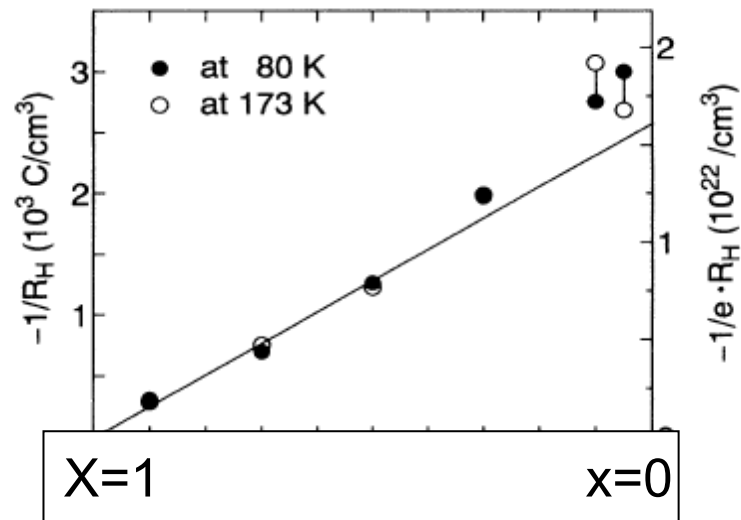
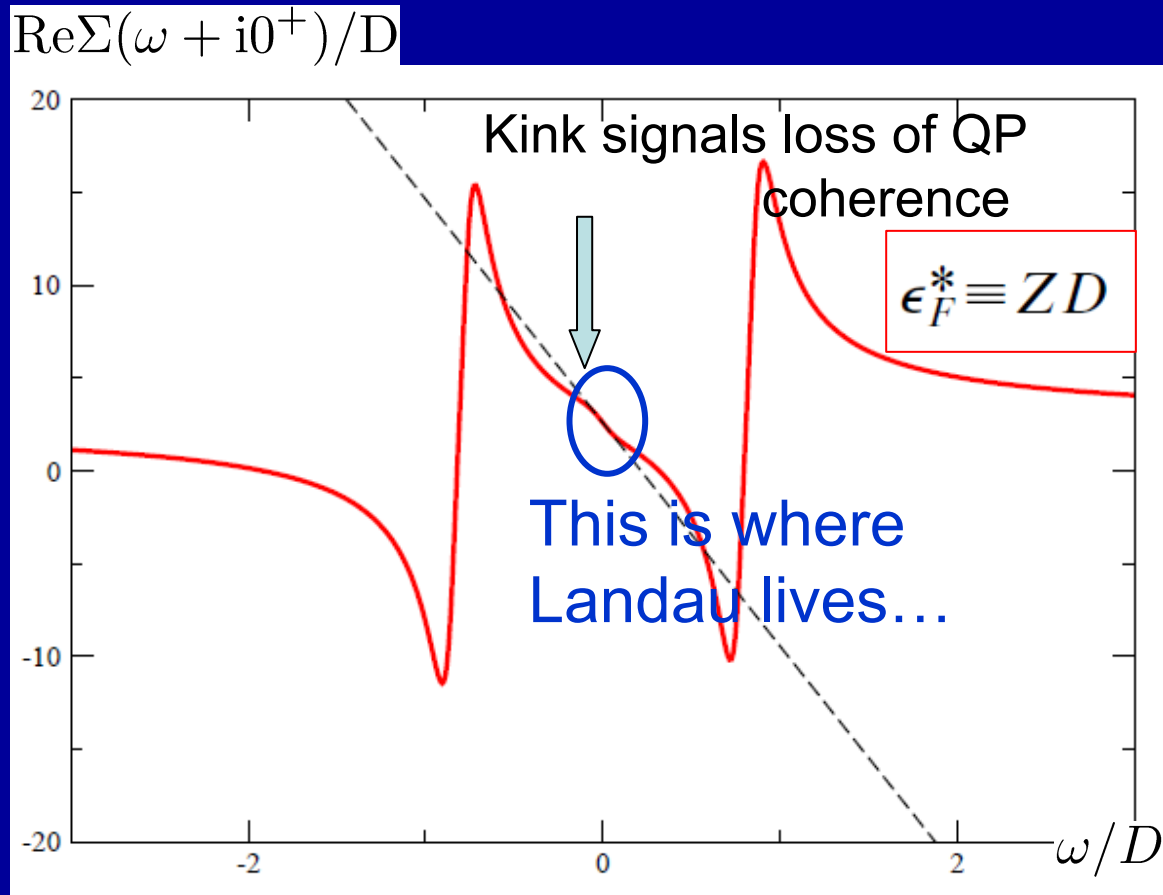


FIG. 2. The filling ( $x$ ) dependence of the inverse of Hall coefficient ( $R_H^{-1}$ ) in  $Sr_{1-x}La_xTiO_3$ . Open and closed circles represent the values measured at 80 K and 173 K, respectively. A solid line indicates the calculated one based on the assumption that each substitution of a  $Sr^{2+}$  site with  $La^{3+}$  supplies the compound with one electron-type carrier per Ti site.

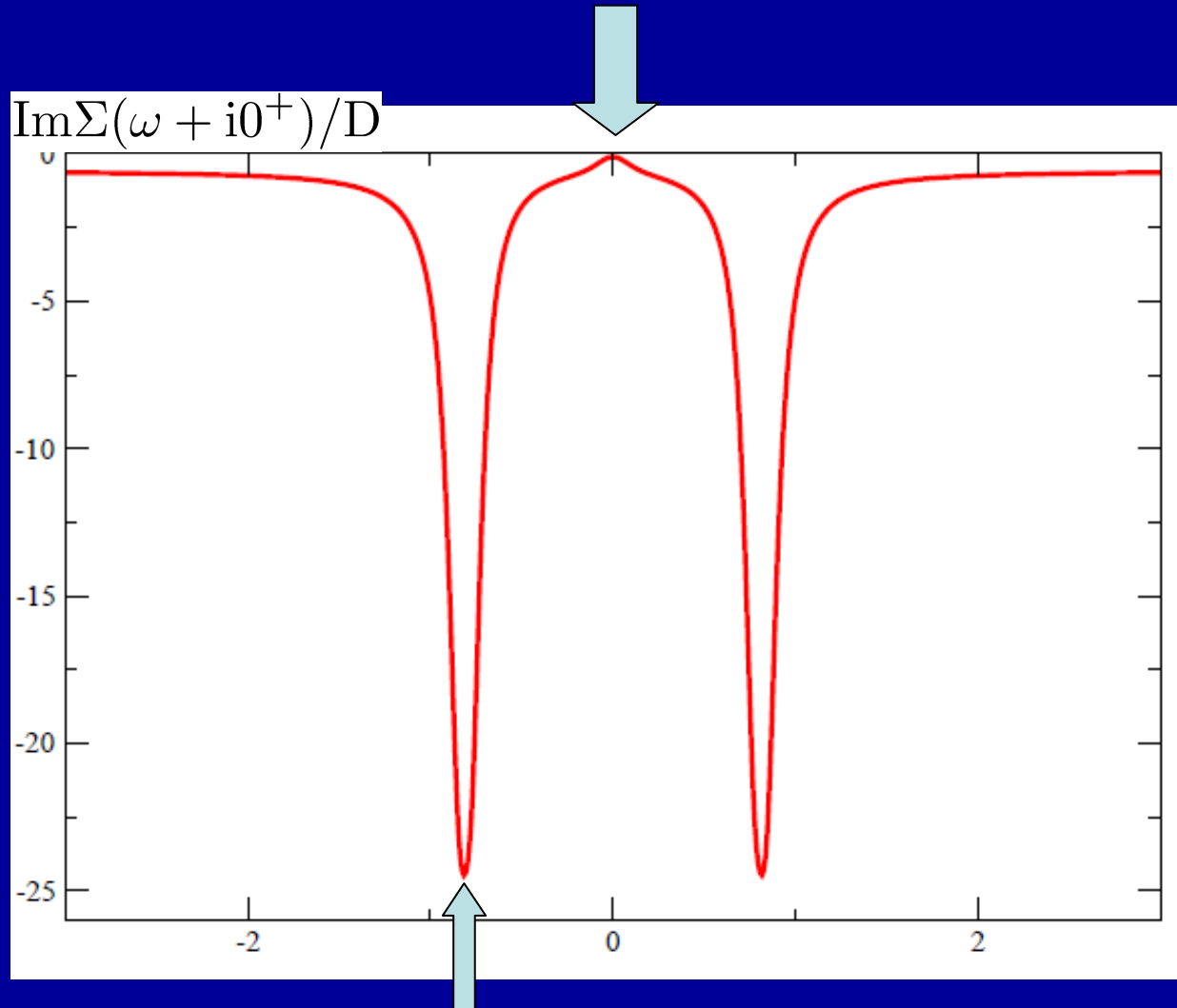
$R_H$  reported as  $\sim T$ -independent and consistent w/ large Fermi surface

# But... there is (plenty of) life beyond the Fermi-liquid regime



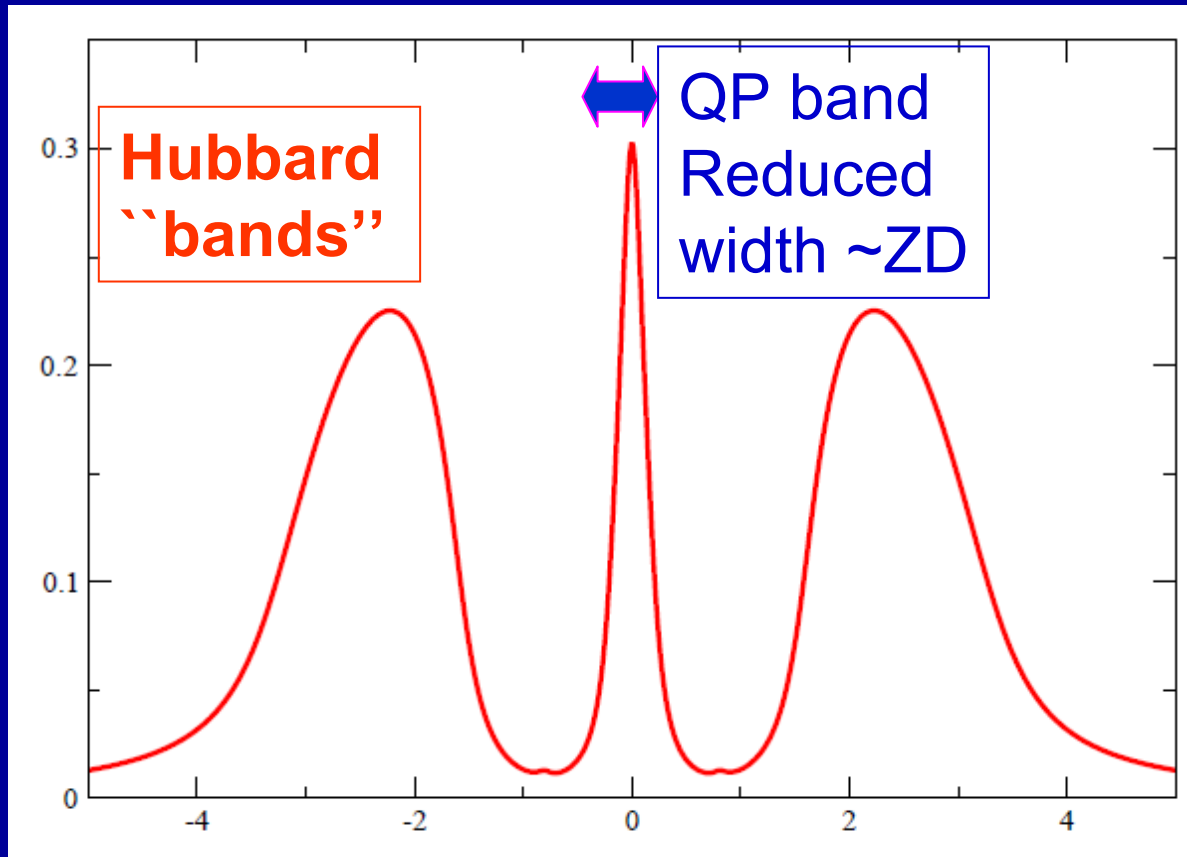
CTQMC+Analytical continuation (Pade),  
courtesy M.Ferrero, compares perfectly to NRG

$B\omega^2$  applies only below coherence scale  
B-coefficient is enhanced  $\sim 1/Z^2$



These 2 peaks will coalesce into a pole at  $\omega=0$   
as insulator is reached

# k-integrated spectral function (total d.o.s) :



Value of  $A(\omega=0)$  is pinned at  $U=0$  value due to Luttinger theorem

→ Low-energy quasiparticles and incoherent Hubbard bands Coexist in one-particle spectrum of correlated metal

# k-integrated spectral Function:

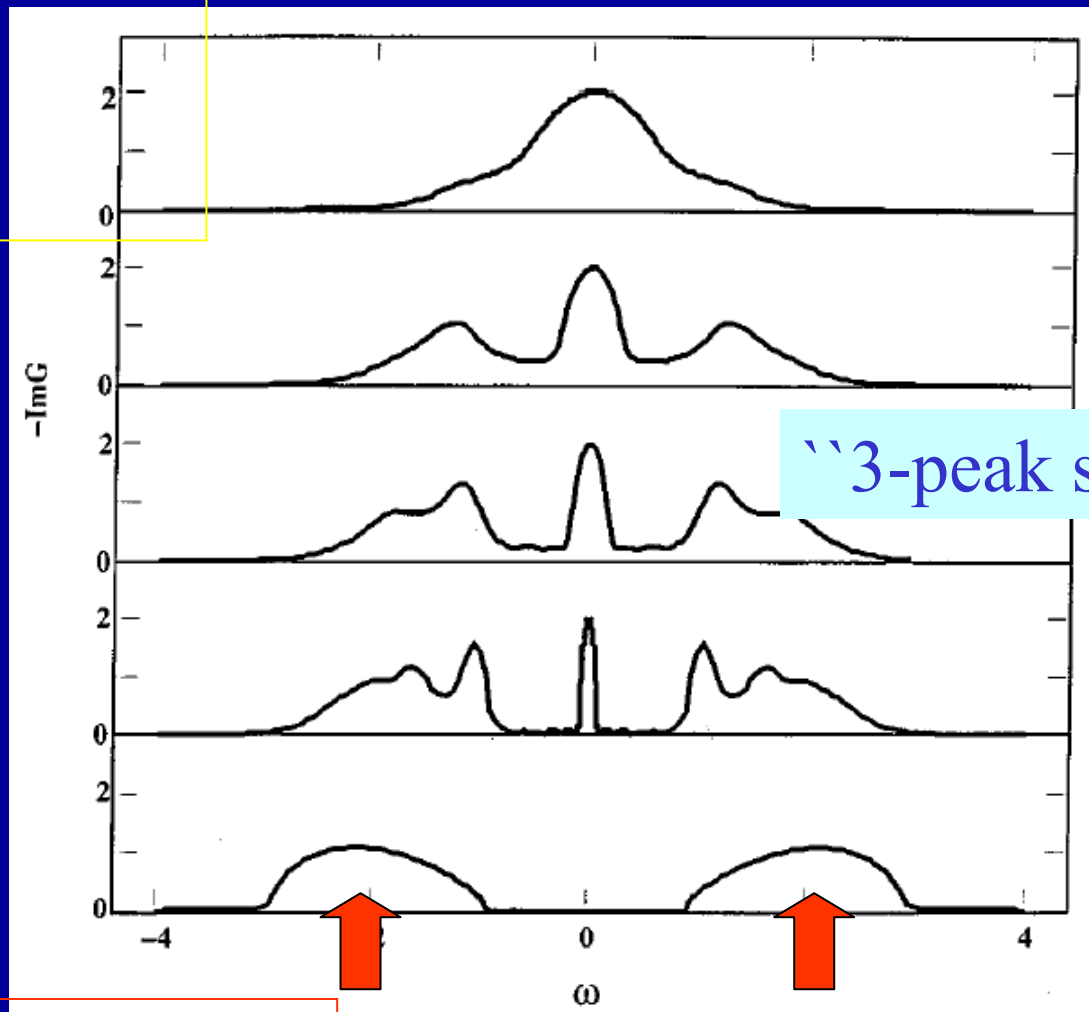
## early DMFT results

(AG&G.Kotliar 1992,  
AG&W.Krauth,  
M.Rozenberg et al.  
1992-94)

Increasing U

Quasiparticle peak

QP bandwidth  $\mathcal{E}_F^*$



“3-peak structure”

Hubbard bands

> GAP <

## Quasiparticle excitations

Wave-like

Momentum (k-) space

## Atomic-like excitations (Hubbard satellites)

Particle-like  
(adding/removing charges  
locally)

Real (R-) space

Spectral weight transfers

**Are treated on equal footing within DMFT**

A.Fujimori et al.

**Low-energy quasiparticles  
and  
high-energy Hubbard bands  
coexist**

**in a strongly-correlated  
metal:**

**early evidence from  
photoemission (1992).**

**Independent theoretical  
evidence from DMFT (1991)**

**Tremendous  
experimental progress  
over the last ~ 12 years !**

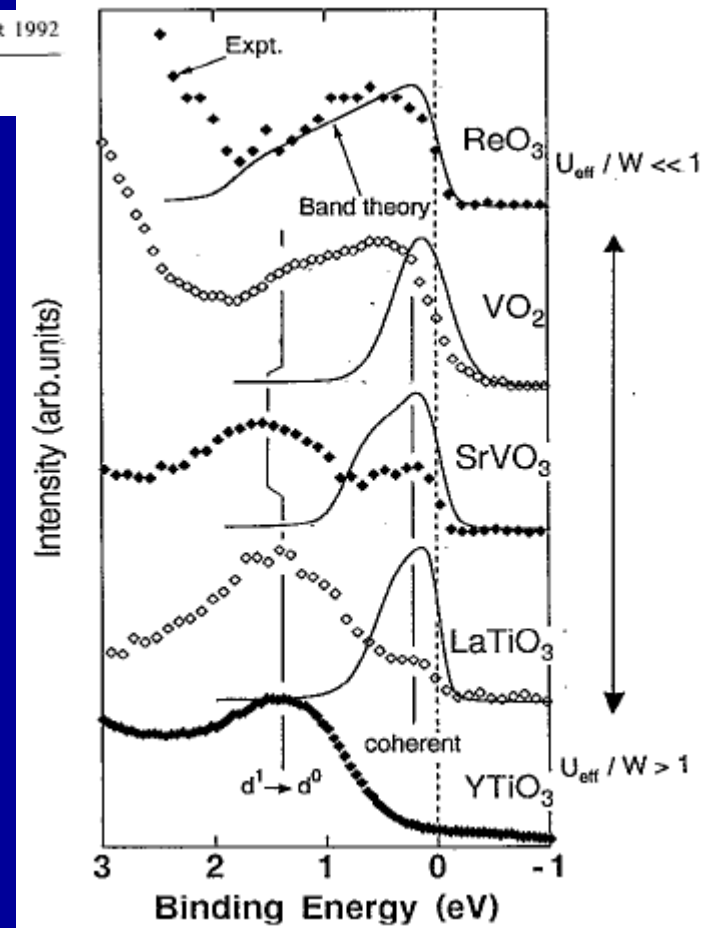
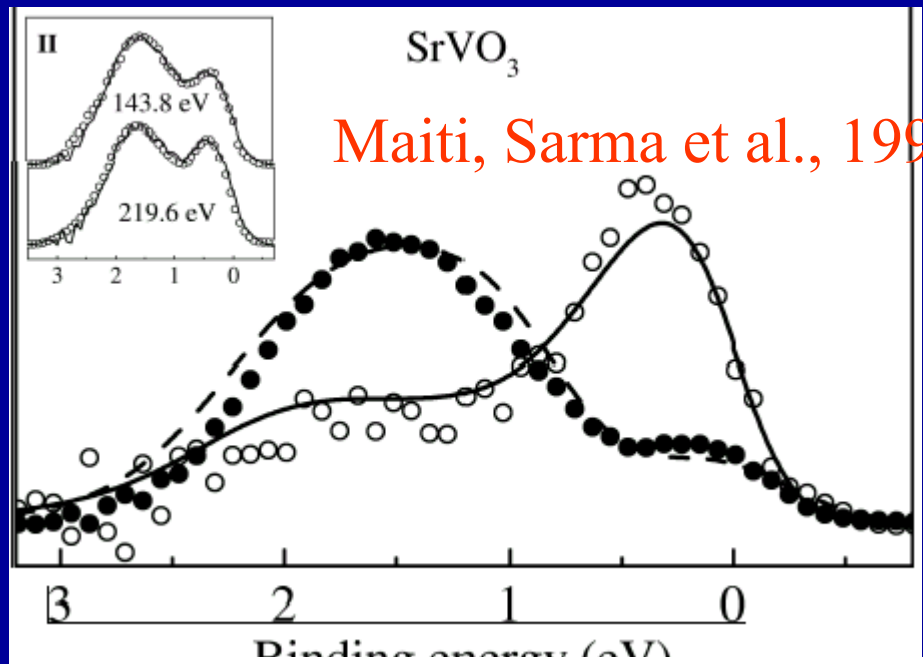
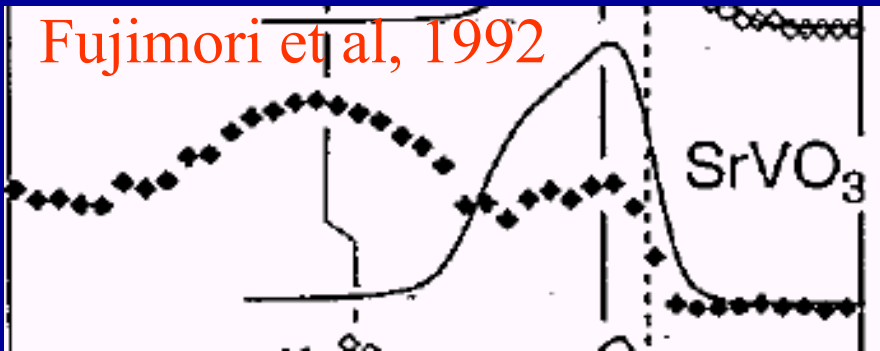


FIG. 2. Photoemission spectra (diamond symbols) of  $\text{YTiO}_3$  ( $h\nu=21.2$  eV),  $\text{LaTiO}_3$  ( $h\nu=48$  eV), and  $\text{SrVO}_3$  ( $h\nu=55$  eV) in the  $d$ -band region. The spectra of  $\text{VO}_2$  in the metallic phase ( $h\nu=60$  eV) and  $\text{ReO}_3$  ( $h\nu=40.8$  eV) are taken from Refs. [9] and [14], respectively. They are compared with the DOS given by band-structure calculations [13,15] (solid curves). The instrumental resolution is  $\sim 0.5$  eV for  $\text{VO}_2$  and  $\sim 0.2$ – $0.3$  eV otherwise.

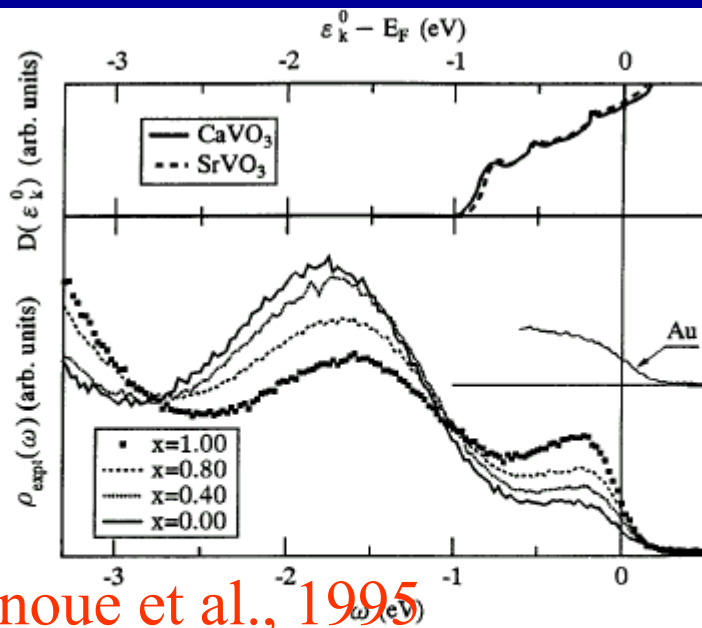


# PHOTOEMISSION... A 12 years (success) story

Fujimori et al., 1992

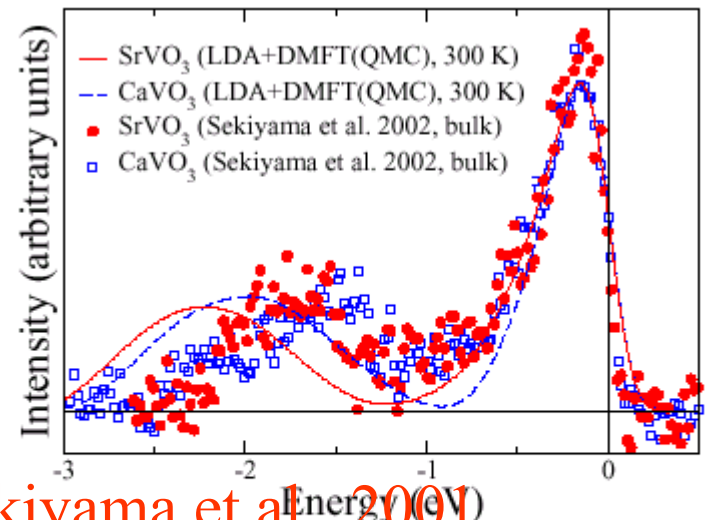


Maiti, Sarma et al., 1999



Inoue et al., 1995

FIG. 1. Top: DOS  $D(\epsilon_k^0)$  of  $\text{CaVO}_3$  and  $\text{SrVO}_3$  calculated by the APW method. Bottom: measured photoemission spectra  $\rho_{\text{expt}}(\omega)$  of  $\text{Ca}_{1-x}\text{Sr}_x\text{VO}_3$  taken with  $h\nu = 50$  eV. A spectrum of Au is also shown as a reference to  $E_F$  and the instrumental resolution.



Sekiyama et al., 2001

FIG. 4: Comparison of the calculated, parameter-free LDA+DMFT(QMC) spectra of  $\text{SrVO}_3$  (solid line) and  $\text{CaVO}_3$  (dashed line) with bulk-sensitive high-resolution PES ( $\text{SrVO}_3$ : circles;  $\text{CaVO}_3$ : rectangles) [4]. Horizontal line: experimental subtraction of the background intensity.

# Quasiparticle peak revealed in recent high-energy photoemission experiments !

VOLUME 90, NUMBER 18

PHYSICAL REVIEW LETTERS

week ending  
9 MAY 2003

## Prominent Quasiparticle Peak in the Photoemission Spectrum of the Metallic Phase of $V_2O_3$

S.-K. Mo,<sup>1</sup> J. D. Denlinger,<sup>2</sup> H.-D. Kim,<sup>3</sup> J.-H. Park,<sup>4</sup> J. W. Allen,<sup>1</sup> A. Sekiyama,<sup>5</sup> A. Yamasaki,<sup>5</sup> K. Kadono,<sup>5</sup> S. Suga,<sup>5</sup> Y. Saitoh,<sup>6</sup> T. Muro,<sup>7</sup> P. Metcalf,<sup>8</sup> G. Keller,<sup>9</sup> K. Held,<sup>10</sup> V. Eyert,<sup>11</sup> V. I. Anisimov,<sup>12</sup> and D. Vollhardt<sup>9</sup>

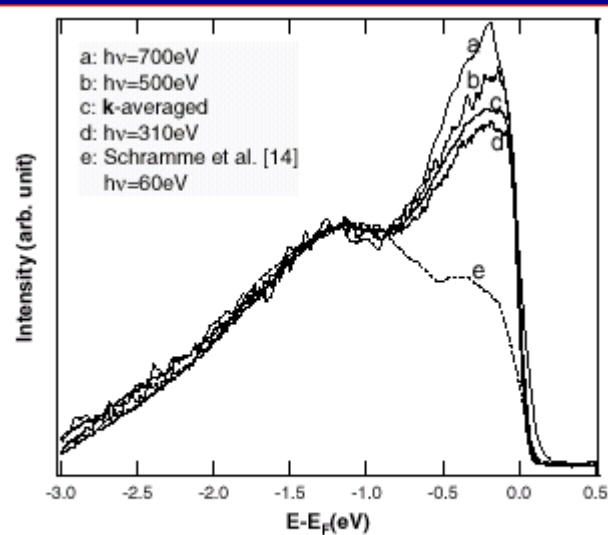
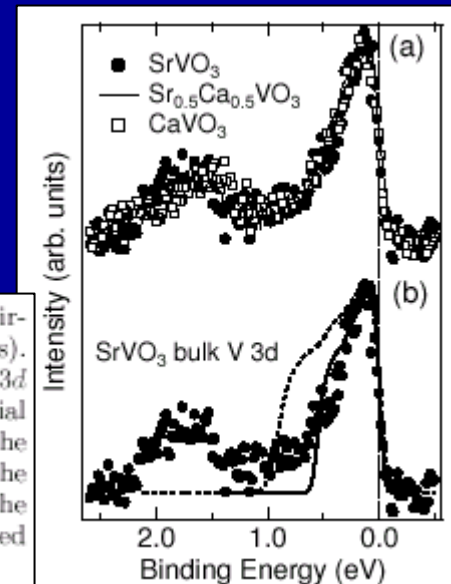


FIG. 2. PES spectra taken with various  $h\nu$ , the largest of which yields the greatest bulk sensitivity.

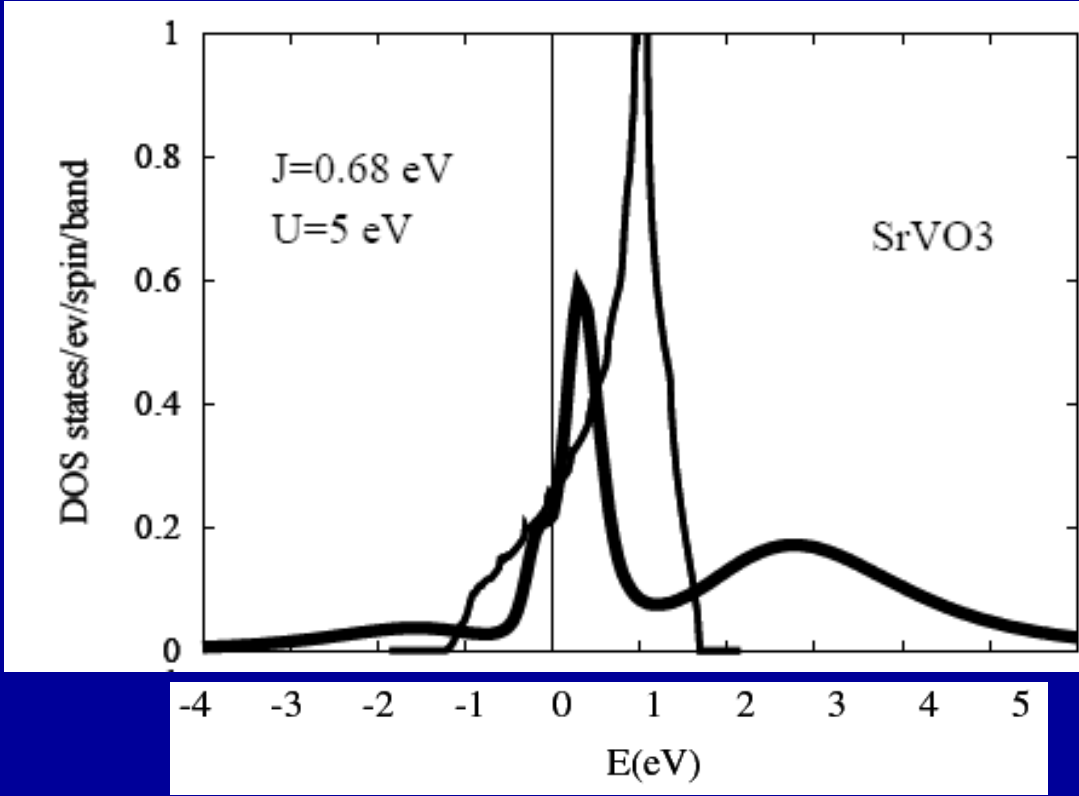
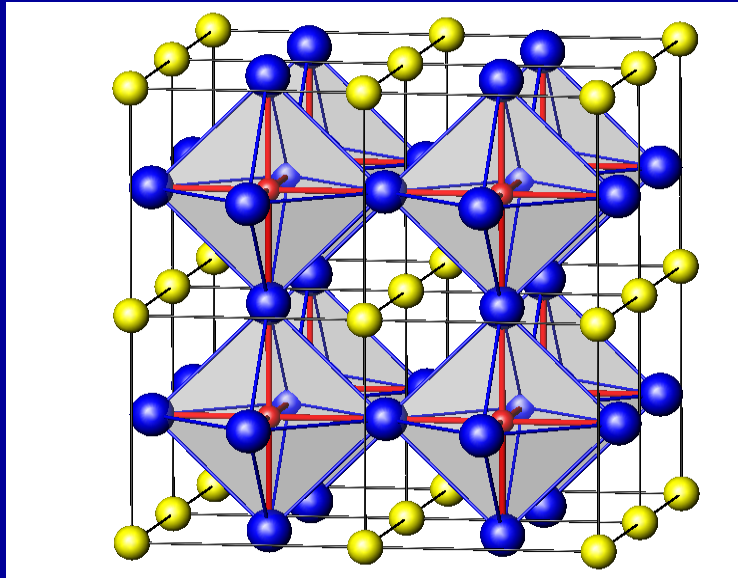
FIG. 3: (a) Bulk V 3d spectral functions of  $SrVO_3$  (closed circles),  $Sr_{0.5}Ca_{0.5}VO_3$  (solid line) and  $CaVO_3$  (open squares). (b) Comparison of the experimentally obtained bulk V 3d spectral function of  $SrVO_3$  (closed circles) to the V 3d partial density of states for  $SrVO_3$  (dashed curve) obtained from the band-structure calculation, which has been broadened by the experimental resolution of 140 meV. The solid curve shows the same V 3d partial density of states but the energy is scaled down by a factor of 0.6.



## Genuine Electronic States Insensitive to the Distortion in Perovskite Vanadium Oxides Revealed by High-Energy Photoemission

A. Sekiyama,<sup>1,\*</sup> H. Fujiwara,<sup>1</sup> S. Imada,<sup>1</sup> H. Eisaki,<sup>2,†</sup> S. I. Uchida,<sup>2</sup> K. Takegahara,<sup>3</sup> H. Harima,<sup>4</sup> Y. Saitoh,<sup>5</sup> and S. Suga<sup>1</sup>

# Realistic DMFT Calculation of spectrum for an oxide: SrVO<sub>3</sub>



## Mott Transition and Suppression of Orbital Fluctuations in Orthorhombic 3d<sup>1</sup> Perovskites

E. Pavarini,<sup>1</sup> S. Biermann,<sup>2</sup> A. Poteryaev,<sup>3</sup> A. I. Lichtenstein,<sup>3</sup> A. Georges,<sup>2</sup> and O. K. Andersen<sup>4</sup>

# The strongly correlated metal at low-energy: “single-parameter scaling”

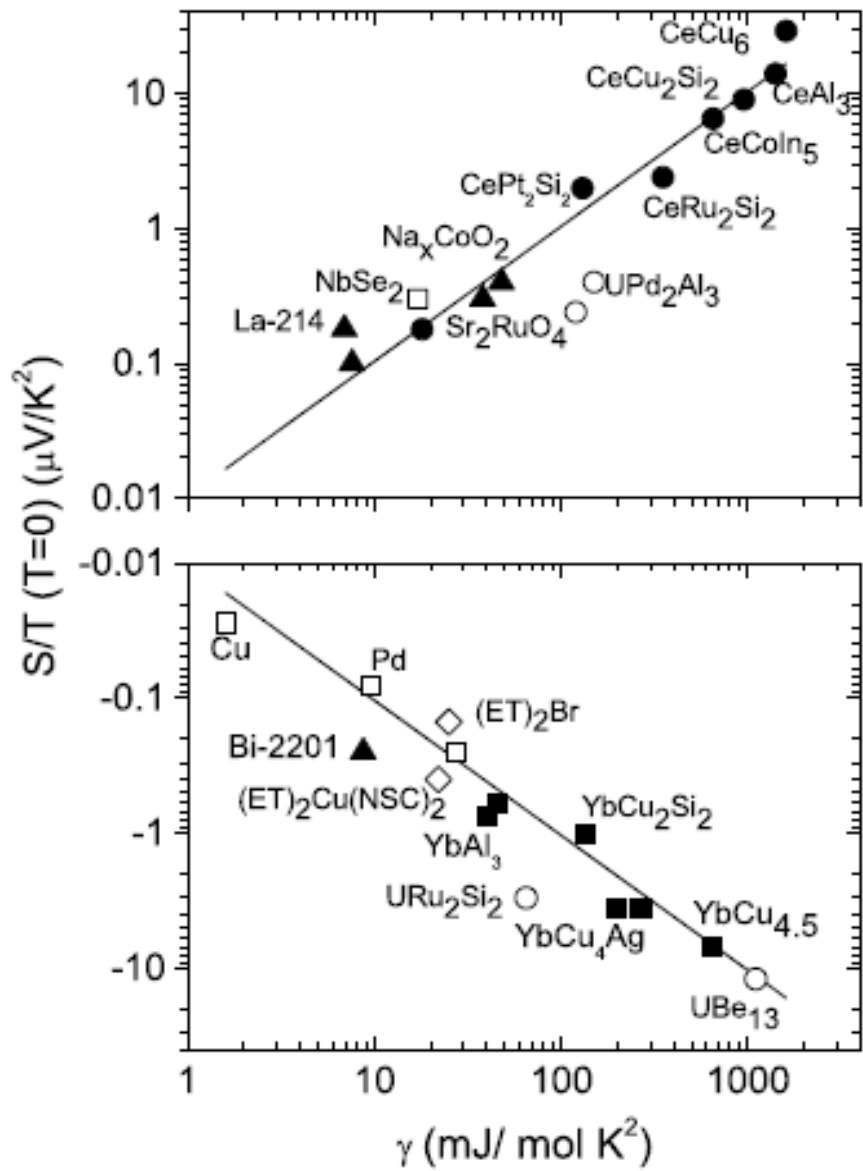
(if only  $\omega \sim ZD$  is considered  $\rightarrow$  can be extended to a 2-parameter scaling up to scale  $\sim Z^{1/2} D$ )

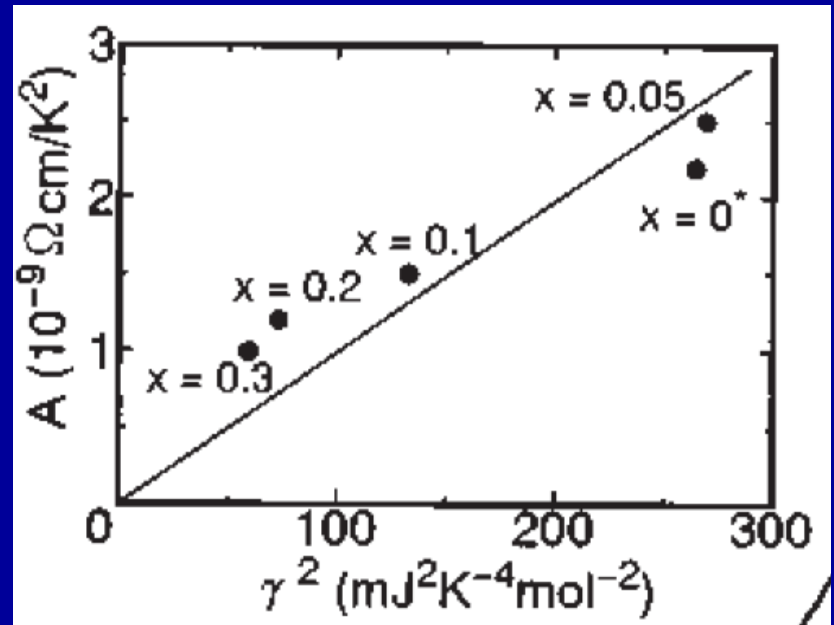
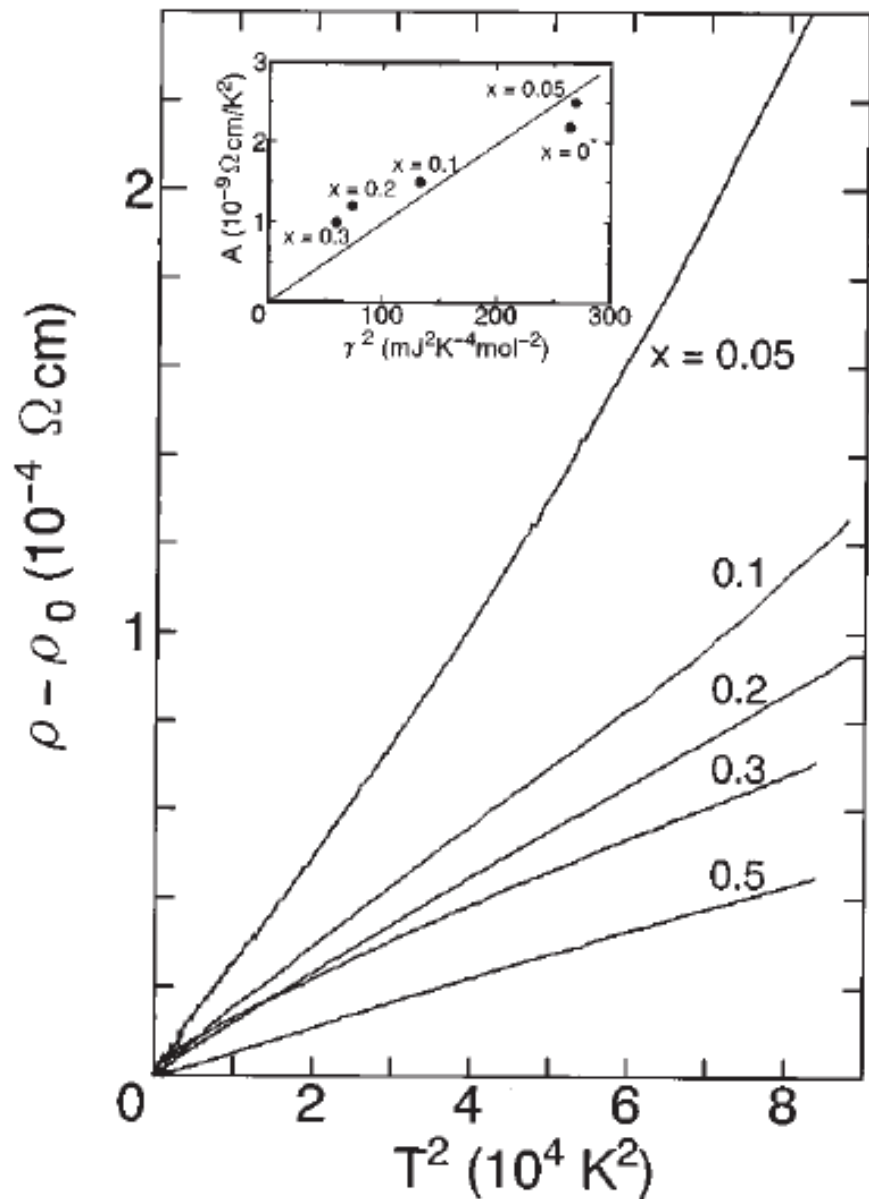
$$\text{Im}\Sigma(\omega) = c D \phi\left(\frac{\omega}{ZD}\right) \sim c \frac{\omega^2}{Z^2 D} + \dots$$

## Consequences:

- Kadowaki-Woods ratio for resistivity vs. specific heat
- Behnia-Jaccard-Flouquet ratio for Seebeck vs. sp. heat

# The Behnia-Jaccard-Flouquet ratio: $S/T\gamma$





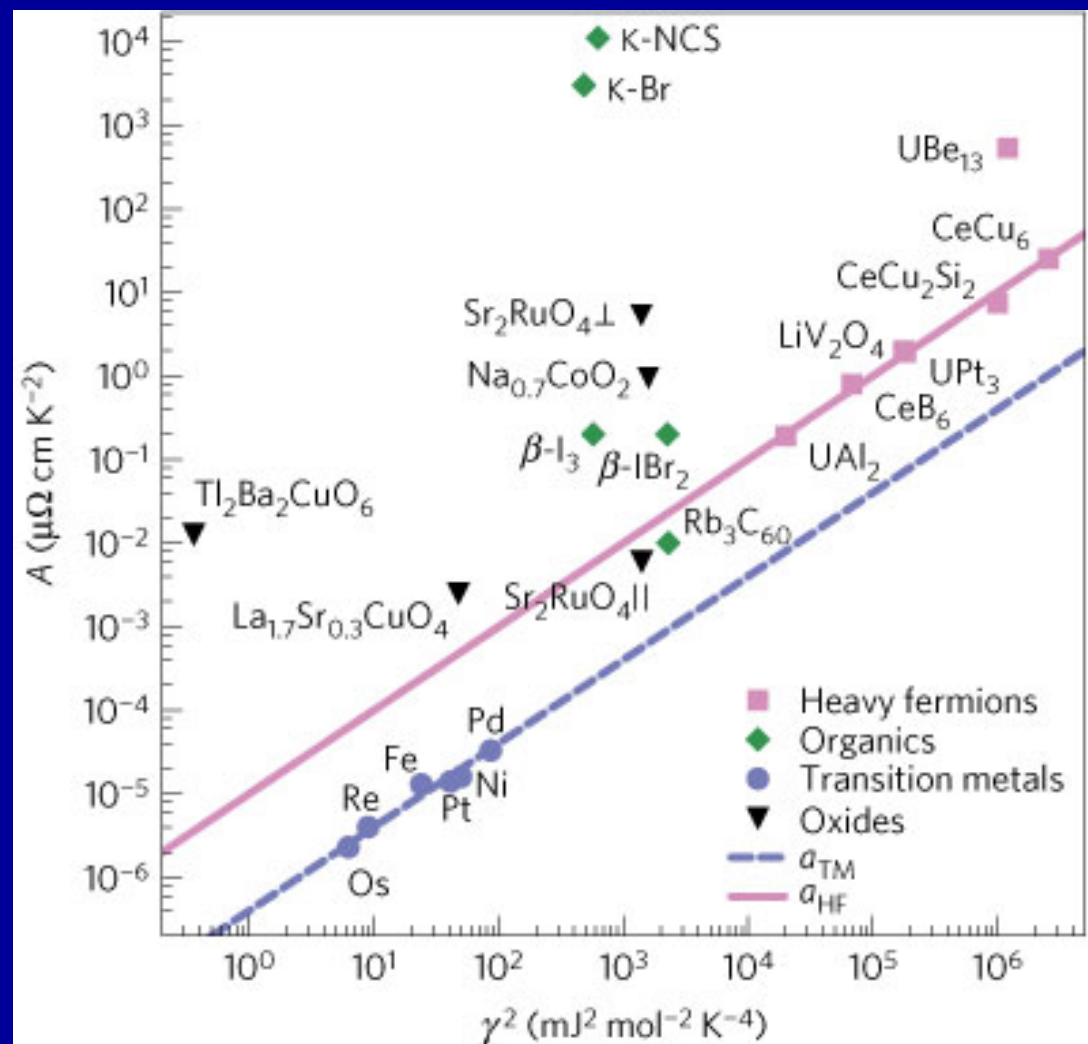
### Titanates/transport:

$$\rho_{dc} = AT^2 + \dots$$

$$A/\gamma^2 \sim \text{const.}$$

Fermi liquid behavior observed  
Below  $\sim 100\text{K}$  @ 5% doping

# Kadowaki-Woods ratio



## C- The paramagnetic Mott insulator DMFT viewpoint (i.e realistic for $T$ not too low, weak magnetic correlations)

- Local moments
- $\ln 2$  entropy per site
- Diverging local susceptibility
- Magnetic correlations show up in 2-particle quantities, not 1-particle (limitation of DMFT)
- Finite uniform  $q=0$  susceptibility  $\sim 1/J$
- Pole in  $\Sigma(\omega)$  related to gap



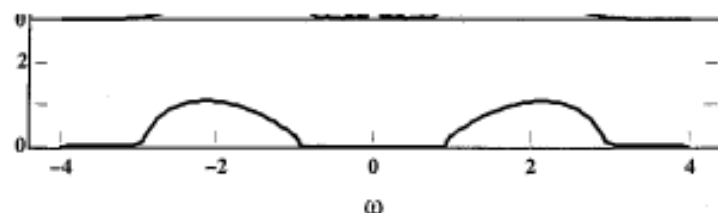
$$\text{Im}\Sigma(\omega + i0^+) = -\pi\rho_2\delta(\omega) \quad \text{for } \omega \in [-\Delta_g/2, \Delta_g/2]$$

$$\text{Re}\Sigma(\omega + i0^+) - U/2 = \frac{\rho_2}{\omega} + O(\omega).$$

$$\frac{1}{\rho_2} = \int_{-\infty}^{+\infty} d\epsilon \frac{\rho(\epsilon)}{\epsilon^2}.$$

- Insulating solution** :  $\Delta(0) = 0$  : gapped bath  $\Rightarrow$  no Kondo effect

Spectral function ( $U/D=4$ )

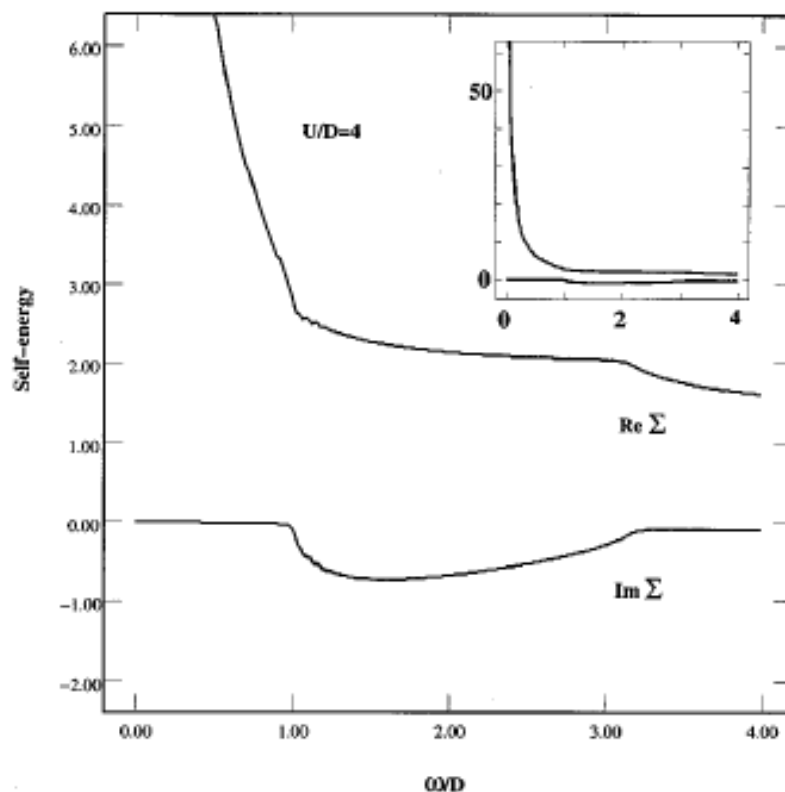


Atomic limit

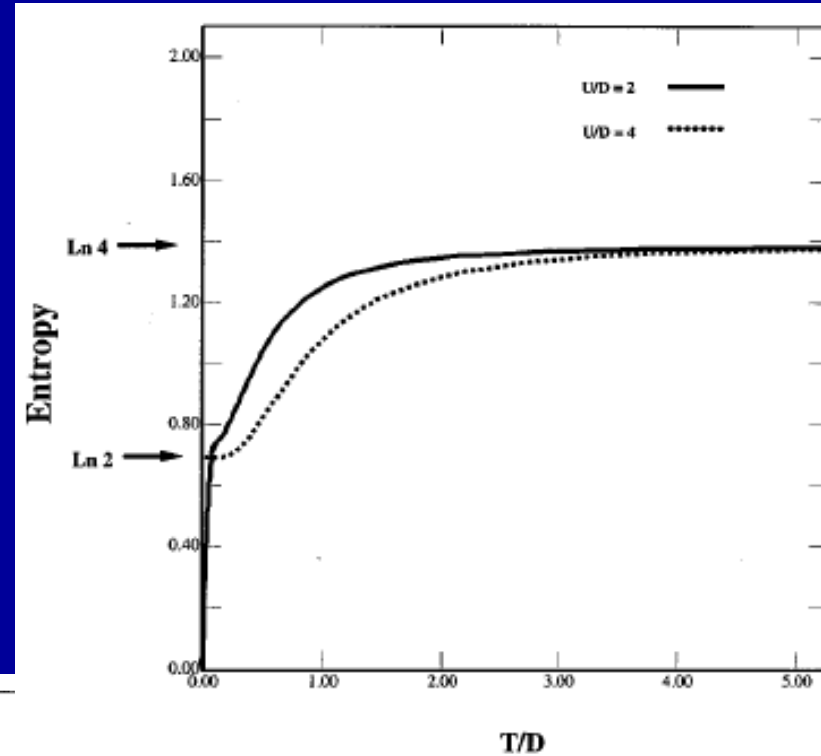
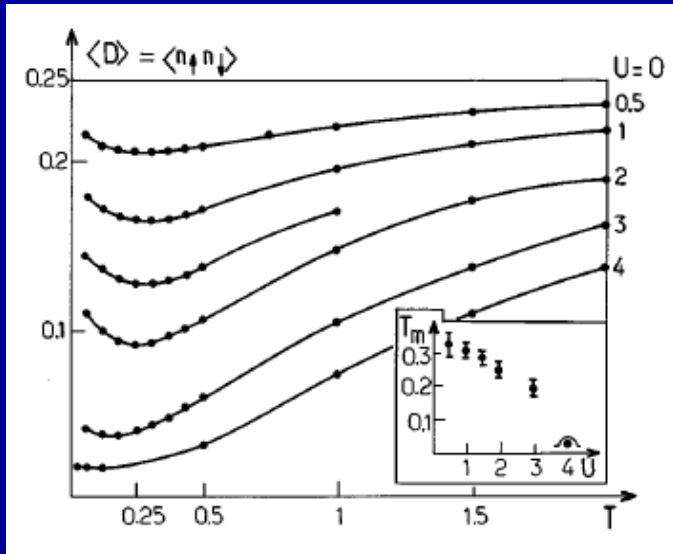
$$G(i\omega_n) = \frac{1}{2} \left( \frac{1}{i\omega_n + U/2} + \frac{1}{i\omega_n - U/2} \right)$$

$$\Sigma(i\omega_n) = \frac{U^2}{2i\omega_n}$$

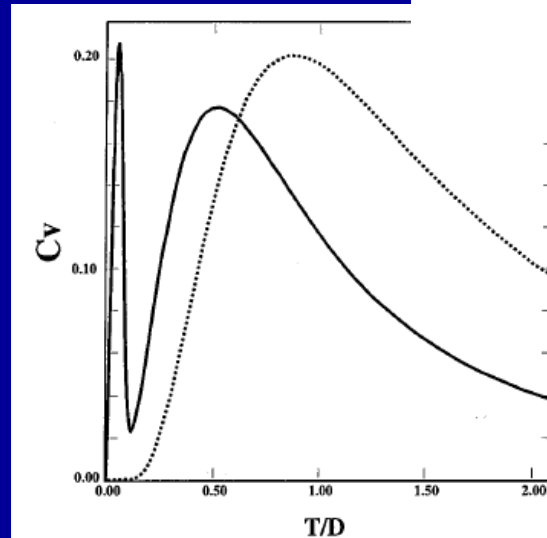
Self-energies in insulator



# D- Thermodynamics :



**Pomeranchuk effect  
 In metal !**



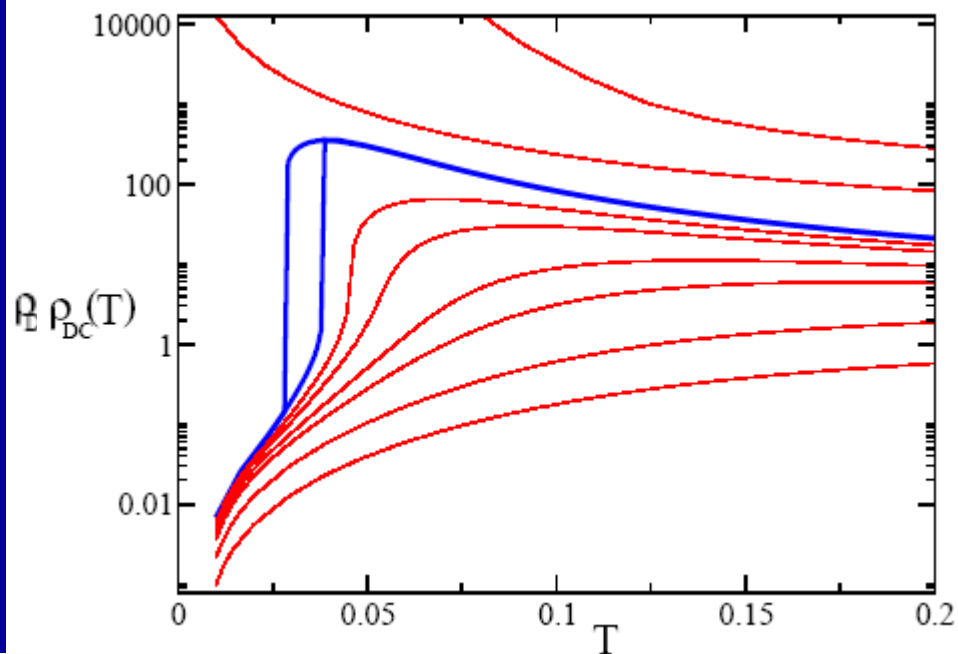
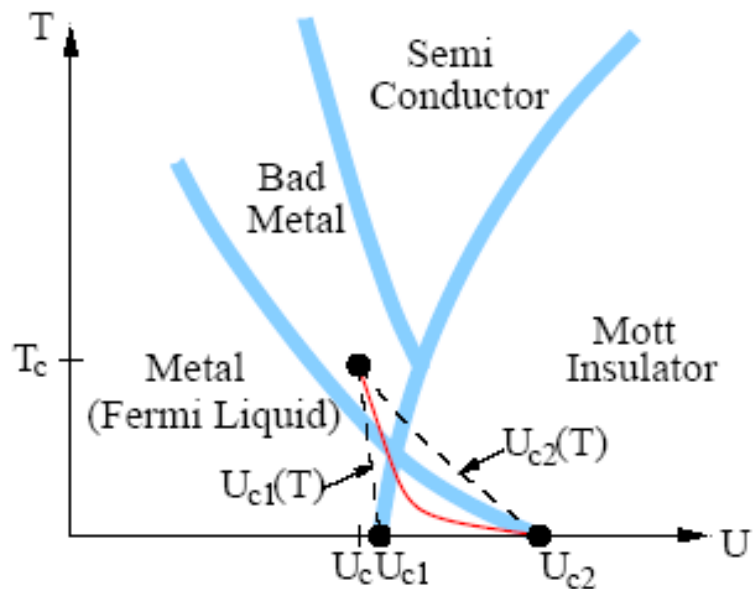
E- Fragile quasiparticles: materials with small quasiparticle coherence scale (e.g close to Mott transition):

- Large spectral weight transfers upon changing temperature
- Unconventional transport

QP coherence scale  $\sim$  width of quasiparticle band

$$\epsilon_F^* \sim ZD$$

→ Because DMFT describes BOTH low-energy quasiparticles and incoherent Hubbard bands, these issues can be addressed and computed



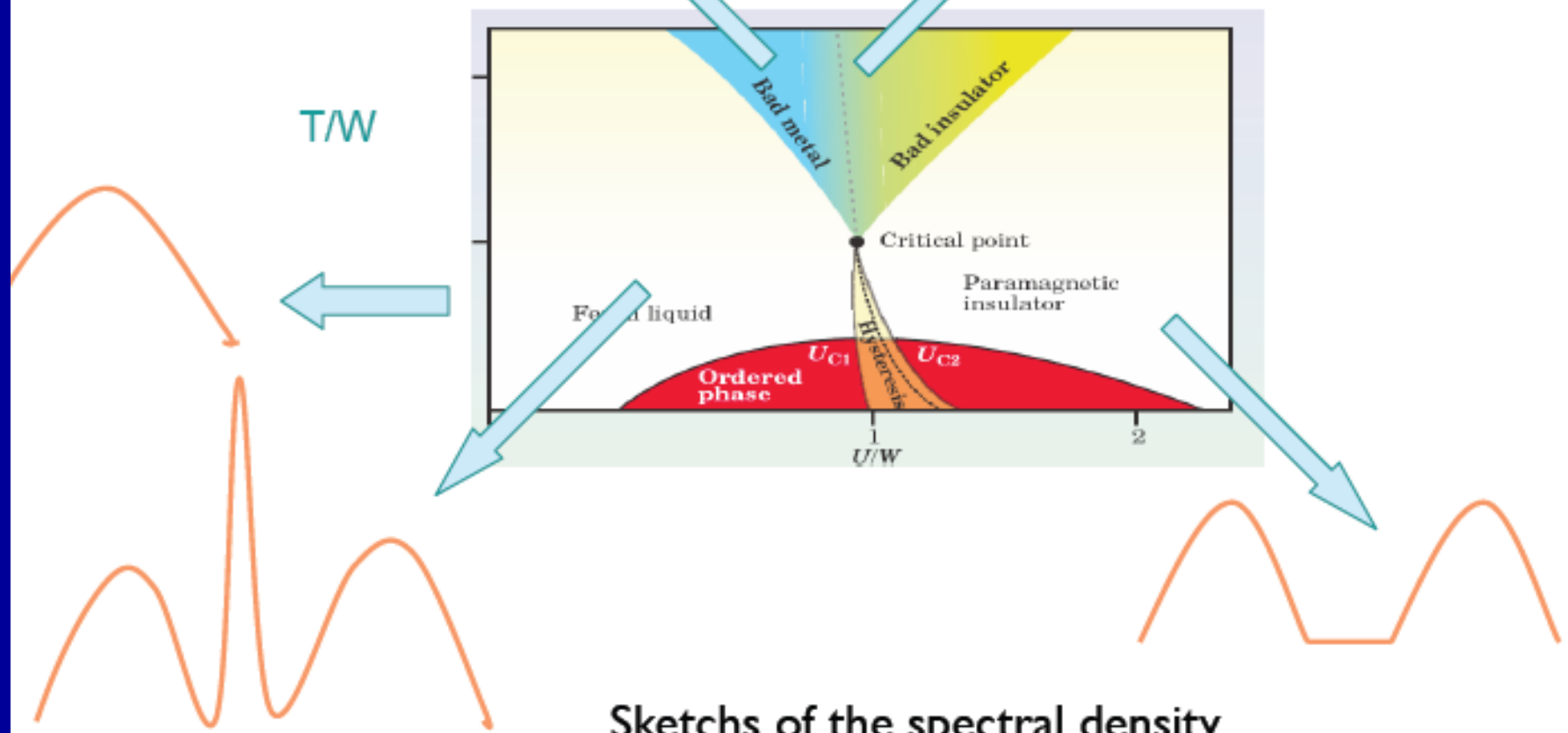
- *Fermi-liquid regime*  $T \ll \epsilon_F^*$ . Coherent quasiparticles. QP peak in the spectral function. Resistivity:

$$\rho = \rho_M \left( \frac{T}{\epsilon_F^*} \right)^2 \quad (1)$$

Note: prefactor enhanced by correlations.  $\rho_M \propto ha/e^2$  Mott value.

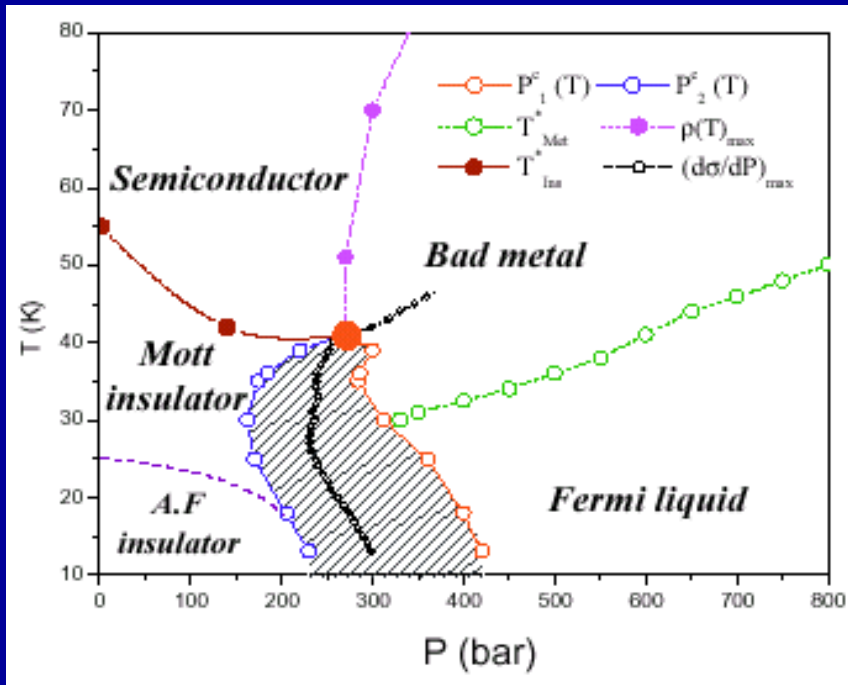
- *Incoherent ("bad") metal*  $T \sim \epsilon_F^*$ . QP peak quickly suppressed upon increasing  $T$  above  $\epsilon_F^*$ . Resistivity is quasi-linear, and *greater than Mott limit*.
- *Insulating-like*  $\epsilon_F^* \ll T \ll \Delta_g$ . QP peak is gone. (Pseudo-) gap in local d.o.s. Transport is insulating-like (Mott gap).

# Various regimes around the critical point



Sketchs of the spectral density

# Mott transition and transport crossovers in the organic compound $\kappa\text{-(BEDT-TTF)}_2\text{Cu}[\text{N}(\text{CN})_2]\text{Cl}$



P. Limelette et al., PRL 91 (2003) 016401

DMFT/NRG calculations  
of resistivity (S.Florens.T.Costi.A.G)

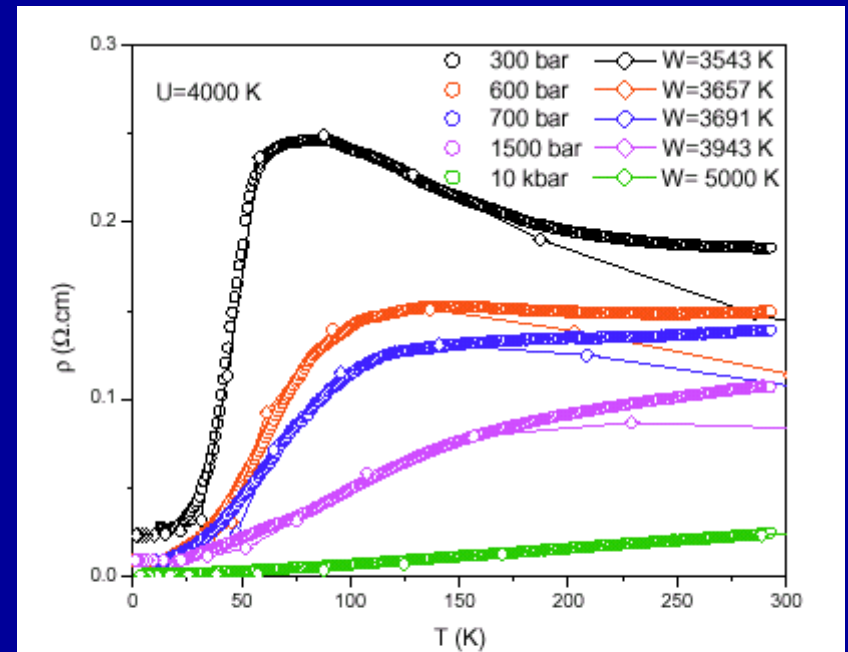
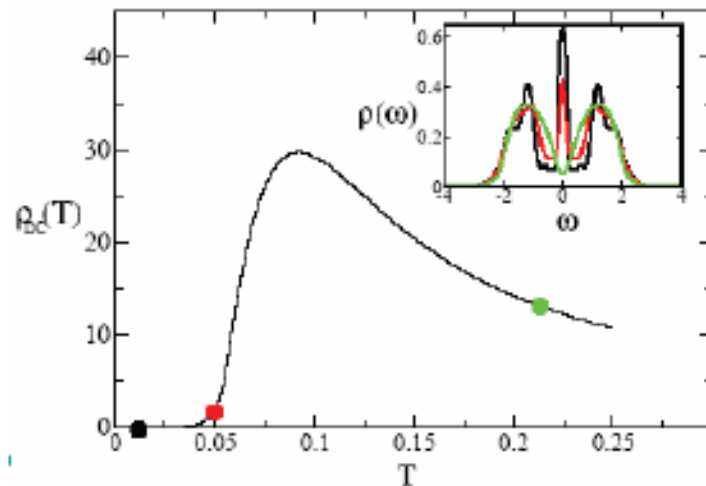
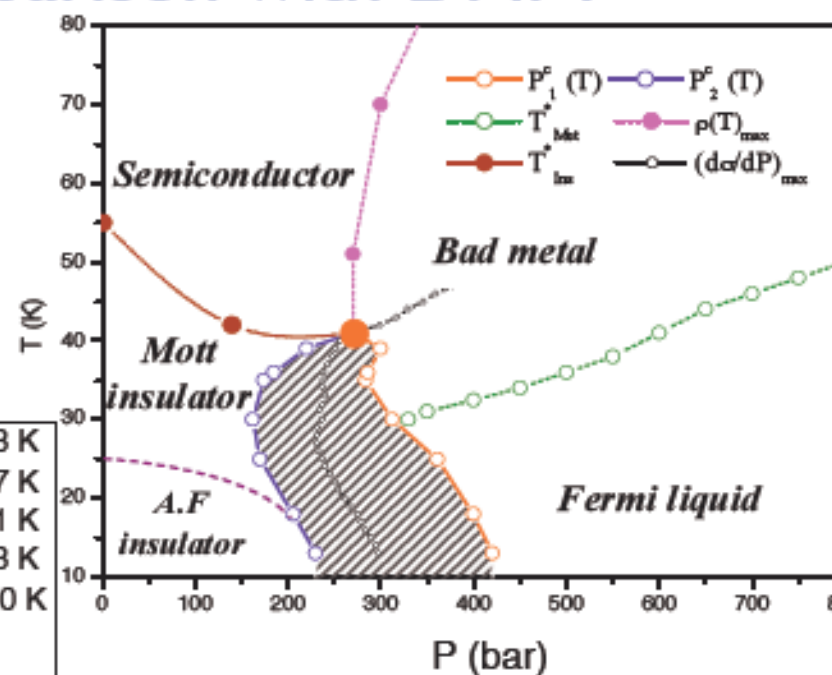
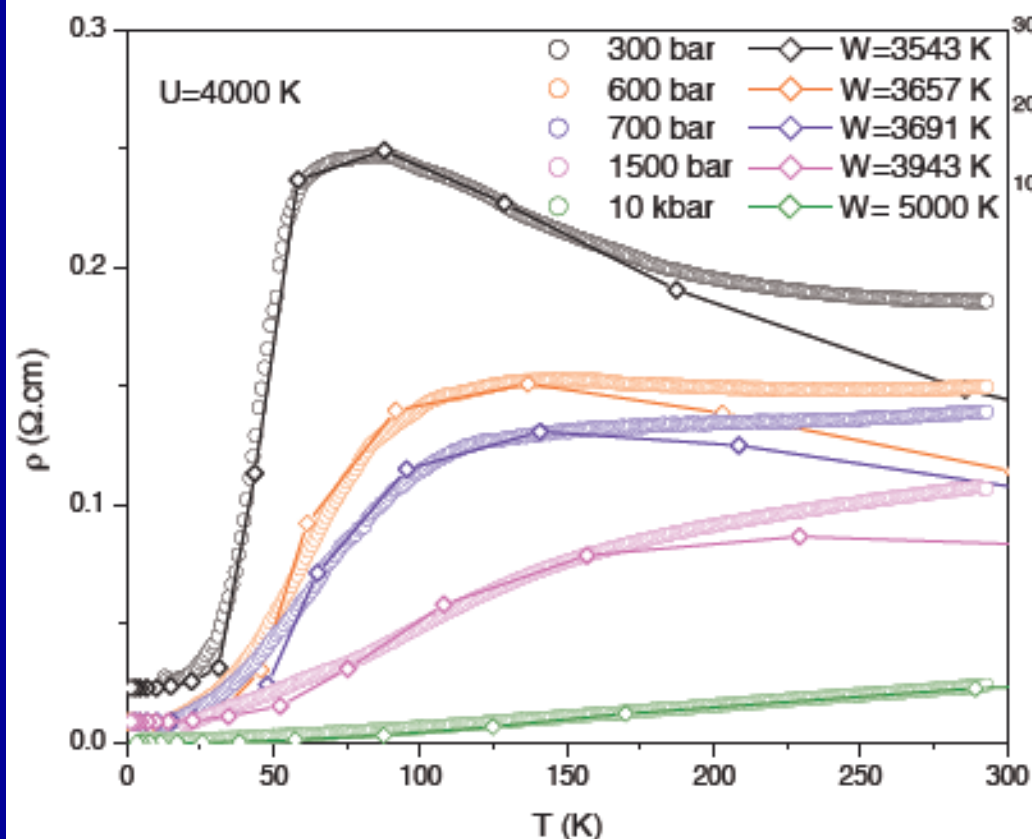


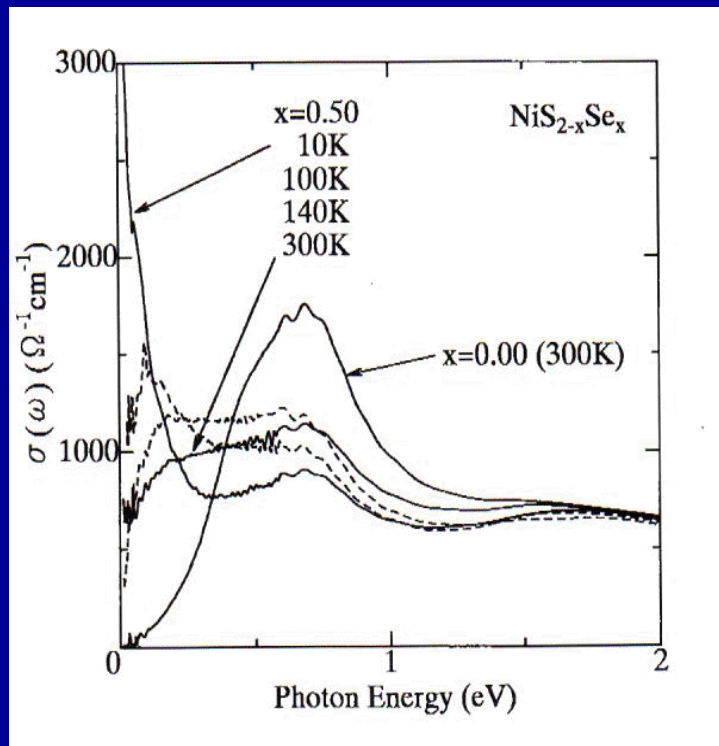
FIG. 6: Temperature-dependence of the resistivity at different pressures. The data (circles) are compared to a DMFT-NRG calculation (diamonds), with a pressure dependence of the bandwidth as indicated. The measured residual resistivity  $\rho_0$  has been added to the theoretical curves.

# Bad metal regime. Comparison with DMFT

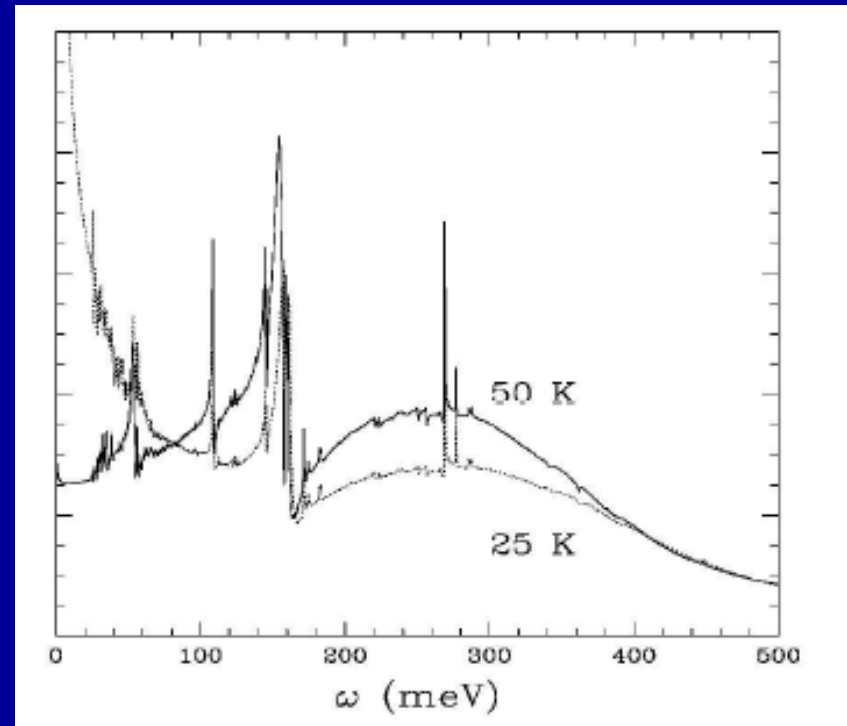
- Bethe lattice, NRG solver
- Adjusted parameters :  
D,  $\rho(T=0)$ , global scale and U.



# Large transfers of spectral weight seen in all spectroscopies, e.g optics:



Miyasaka&Takagi, 2000



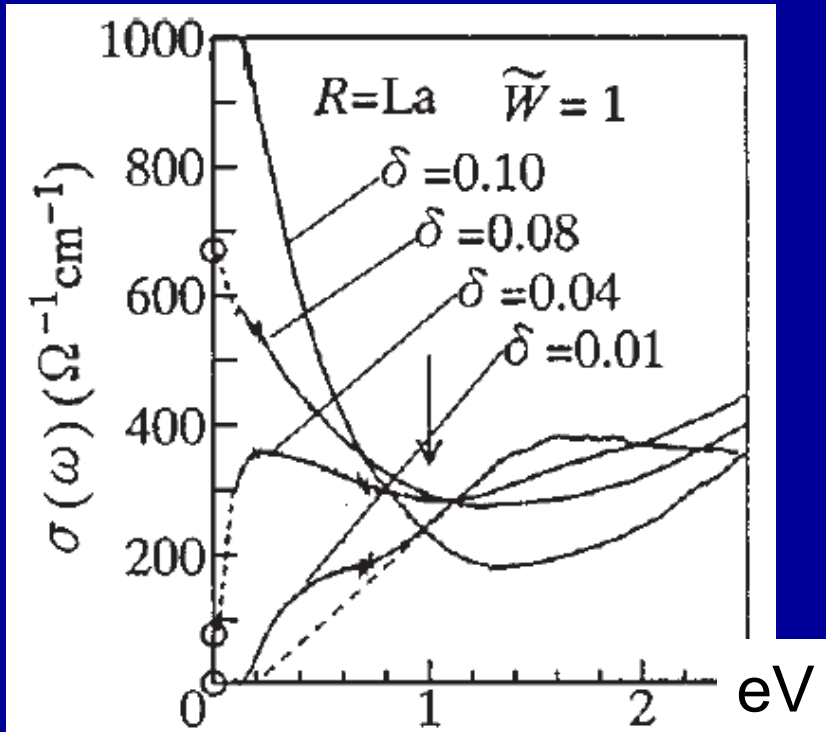
Optical conductivity of  $\kappa\text{-(BEDT-TTF)}_2\text{Cu[N(CN)}_2\text{]Br}$  at ambient pressure [100], for  $T = 25\text{ K}$  and  $T = 50\text{ K}$ .



# Optical conductivity

Drude weight  $\sim$  doping

FIG. 108.  $N_D$  to  $N_{D0}$  as a function of  $\delta$  (Katsufuji, Okimoto, and Tokura, 1995) for  $\text{La}_{1-x}\text{Sr}_x\text{TiO}_3$ .



Large transfers of spectral weight

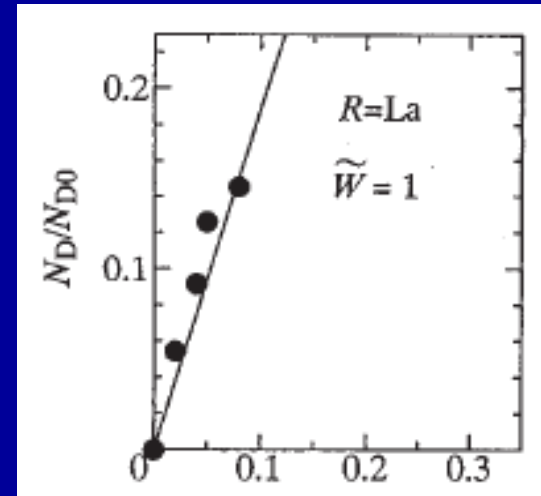
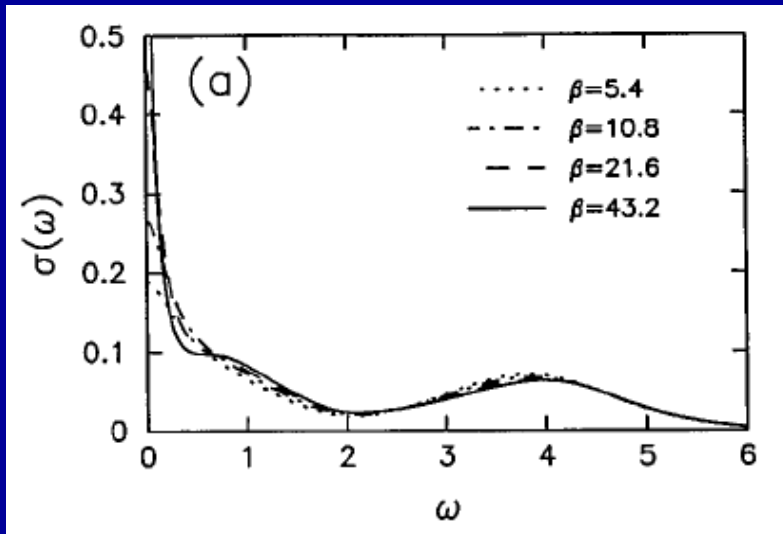


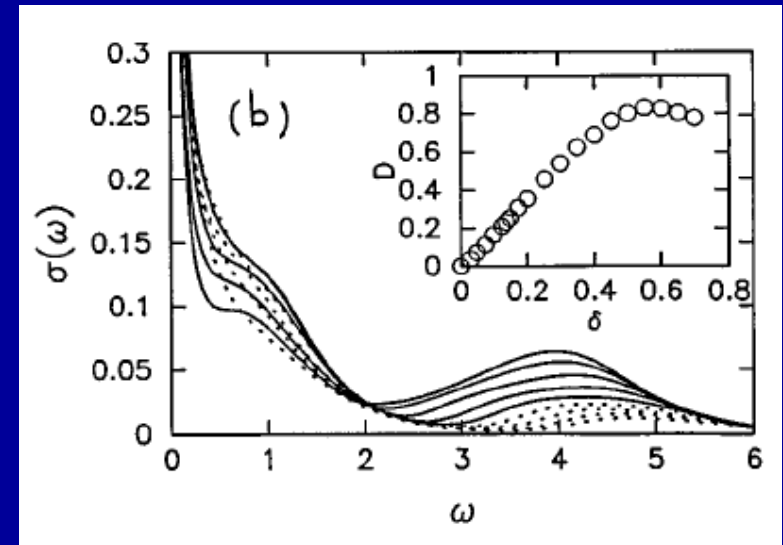
FIG. 107. Optical conductivity spectra in  $R_{1-x}\text{Sr}_x\text{TiO}_{3+y}$  or  $R_{1-x}\text{Ca}_x\text{TiO}_{3+y}$  ( $R=\text{La}, \text{Nd}, \text{Sm},$  and  $\text{Y}$ ). From Katsufuji, Okimoto, and Tokura, 1995.

# Optics and transfers of spectral weight from DMFT calculations...



T-dependence

e.g. Rozenberg et al., 1995  
Jarrell et al., 1995 (curves above)



Doping-dependence

Old results  
→ For much more recent work,  
see seminar by AJ Millis

# Critical behaviour at the Mott critical endpoint

A liquid-gas transition ?

**Insulator:**

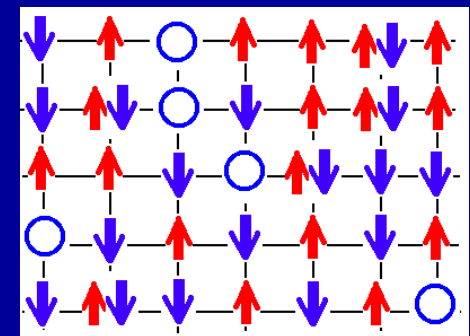
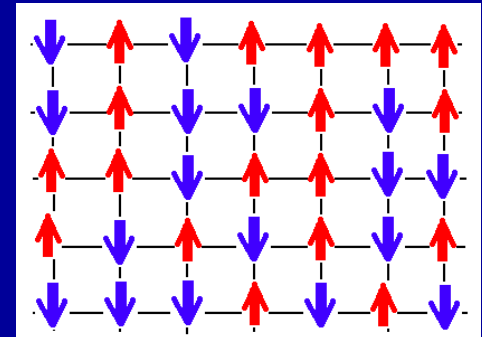
low-density of doubly occupied sites  $\longrightarrow$  GAS

**Metal:**

High-density  $\longrightarrow$  LIQUID

+ cf. early ideas of Castellani et al.

+ Recent DMFT/Landau theory approach: **scalar** order parameter



# Critical behaviour near $T_c, U_c$

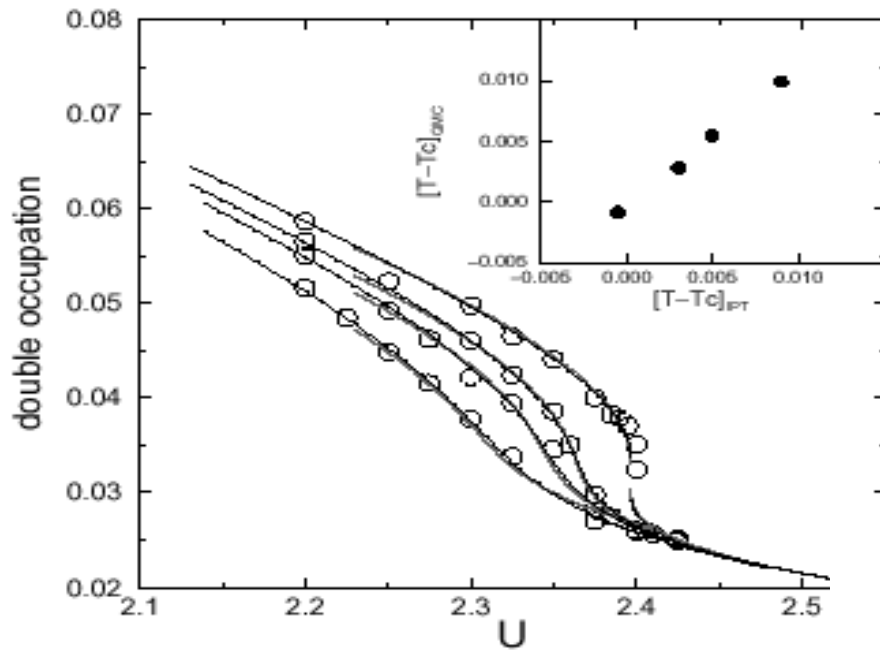


FIG. 1. Double occupation  $\langle d \rangle$  as a function of  $U$  for different temperatures. The thin lines denote IPT results for  $T_1$  0.0469, 0.05, 0.052, 0.056 (top to bottom). The thick line is a fit to the IPT data using the LG theory. The circles are data obtained at  $T_{QMC} = 1/40, 1/35, 1/32, 1/25$  [8]. The results were shifted by a constant  $-0.07$  along the  $U$  axis by  $-0.003$  along the  $\langle d \rangle$  axis. The curves for the three temperatures are above  $T_c$  and the lowest temperature ones (branches) are just below. The inset shows the scaling of reduced temperatures  $[T - T_c]_{QMC}$  versus  $[T - T_c]_{IPT}$ .

Kotliar et al. PRL 84 5180 (2000)

Cf. LIQUID-GAS transition

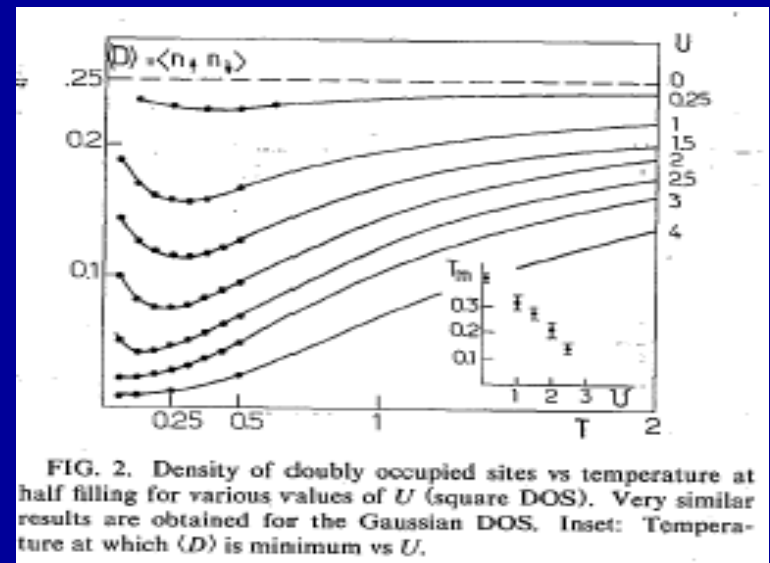


FIG. 2. Density of doubly occupied sites vs temperature at half filling for various values of  $U$  (square DOS). Very similar results are obtained for the Gaussian DOS. Inset: Temperature at which  $\langle D \rangle$  is minimum vs  $U$ .

Georges and Krauth PRL 69 1240 (1992)

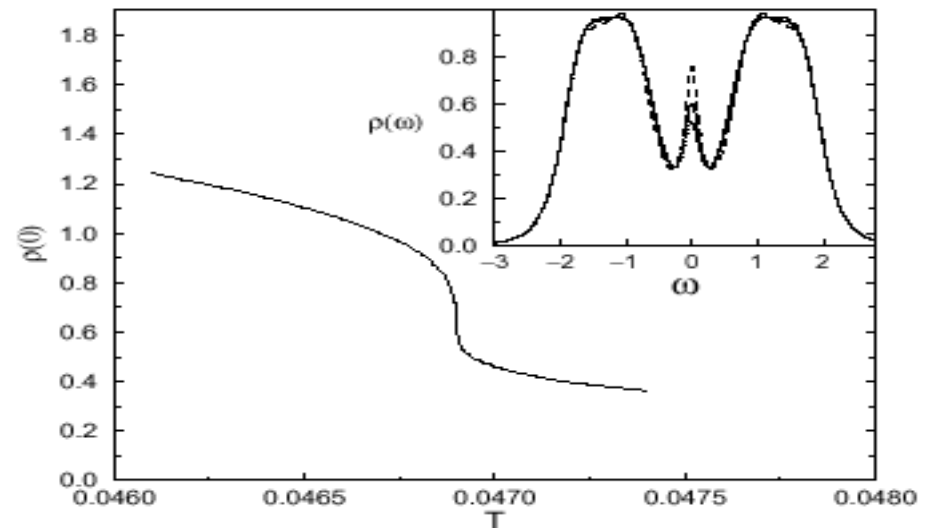
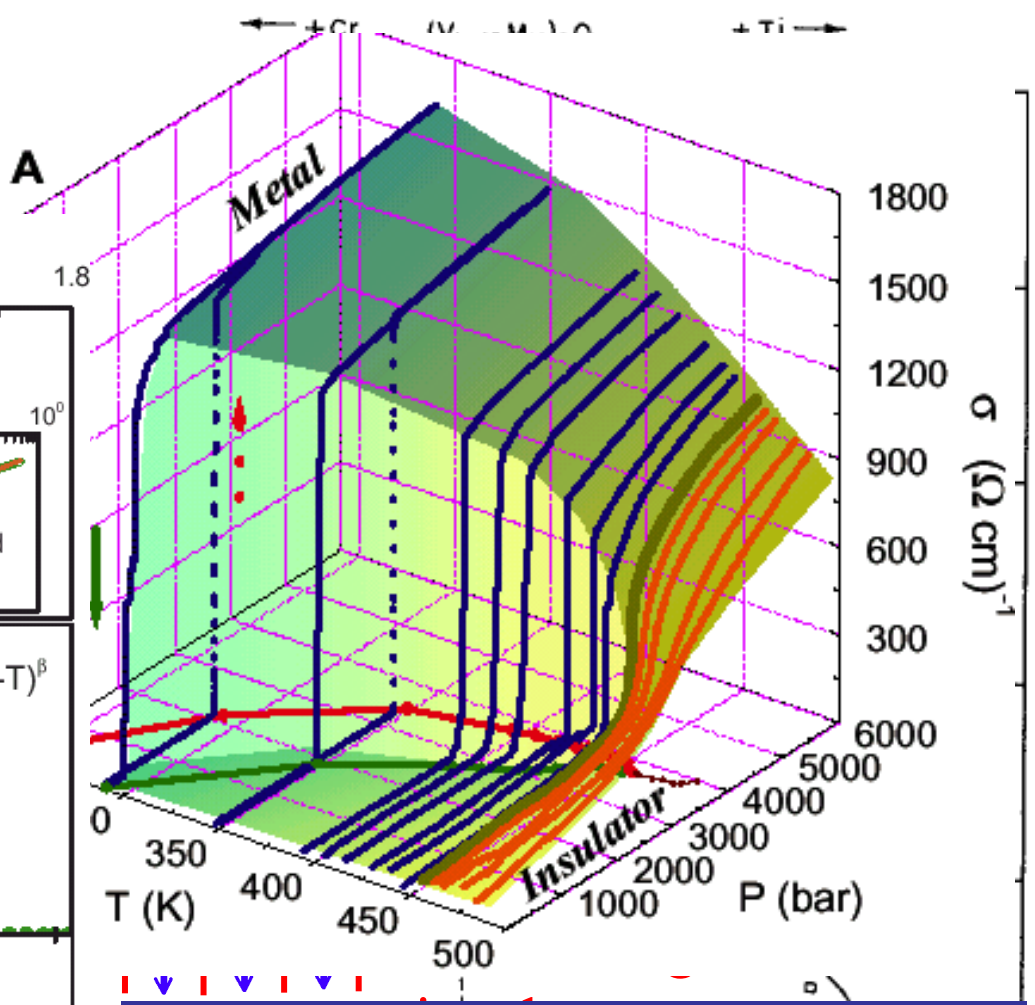
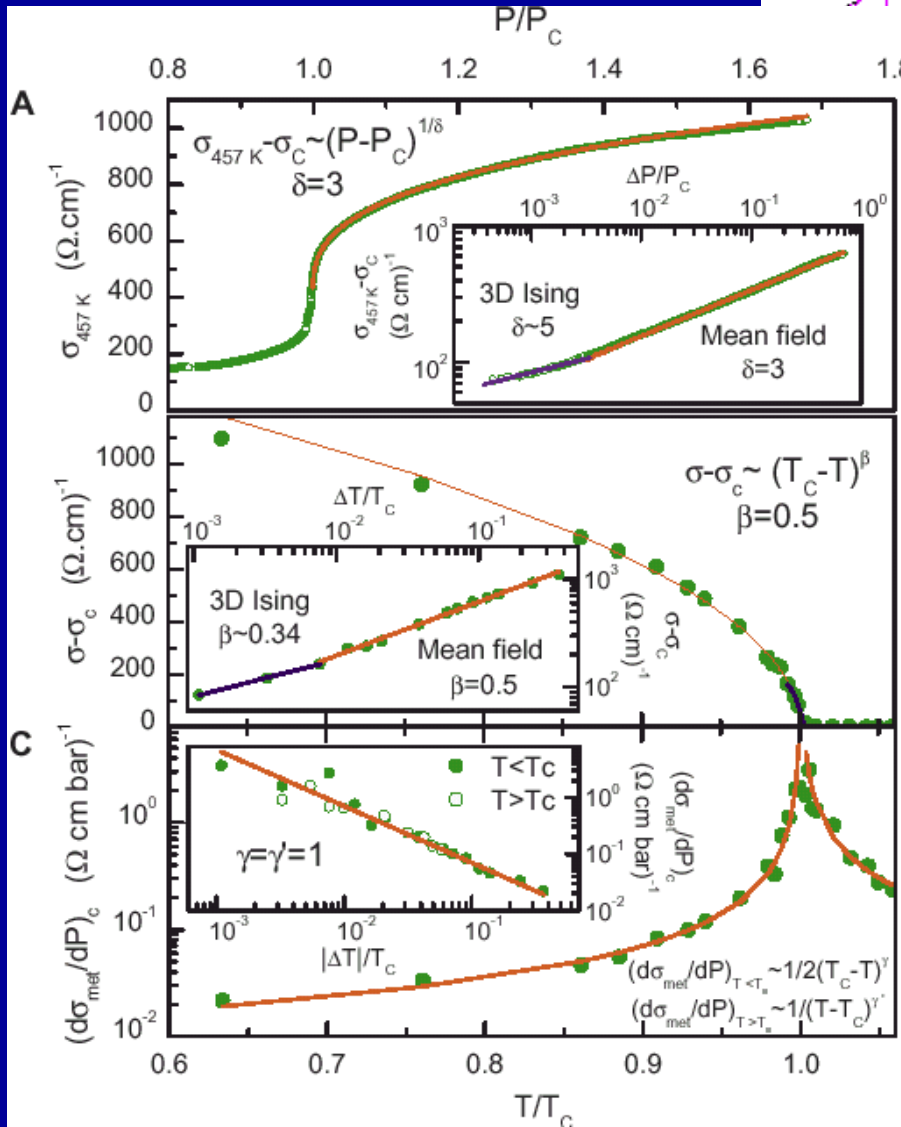


FIG. 2. The density of states at the Fermi energy  $\rho(0) \equiv A_0$  as a function of temperature in the critical region ( $U = 2.46316 \approx U_c$ ). The singular behavior of the slope at  $T_c \approx 0.046897$  can be clearly appreciated. The inset shows the variation of the spectral function for  $U \approx U_c$  in the vicinity of  $T_c$ : dashed line for  $T - T_c/T_c = -0.00025$ , solid line for  $T - T_c/T_c = 0.00006$ , and dotted line for  $T - T_c/T_c = 0.00049$  [11].

# C. Liquid-gas critical endpoint

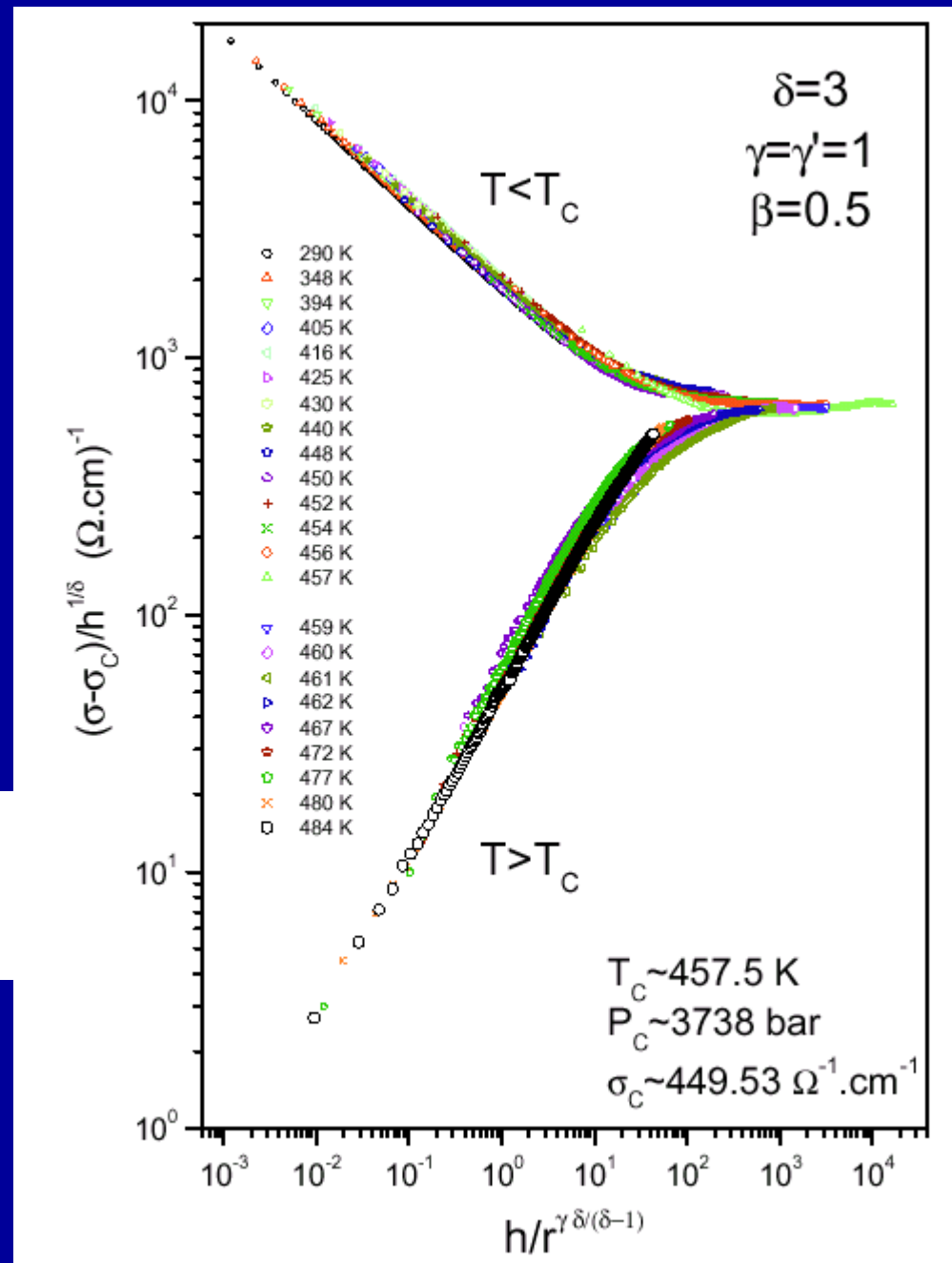


Limelette et al, Science  
 2003: conductivity under  
 variable pressure

# Scaling: Universal form of the "equation of state"

$$\sigma_{met}(P, T) - \sigma_c = (\delta h)^{1/\delta} f_{\pm} \left( \frac{\delta h}{|r|^{\gamma\delta/(\delta-1)}} \right)$$

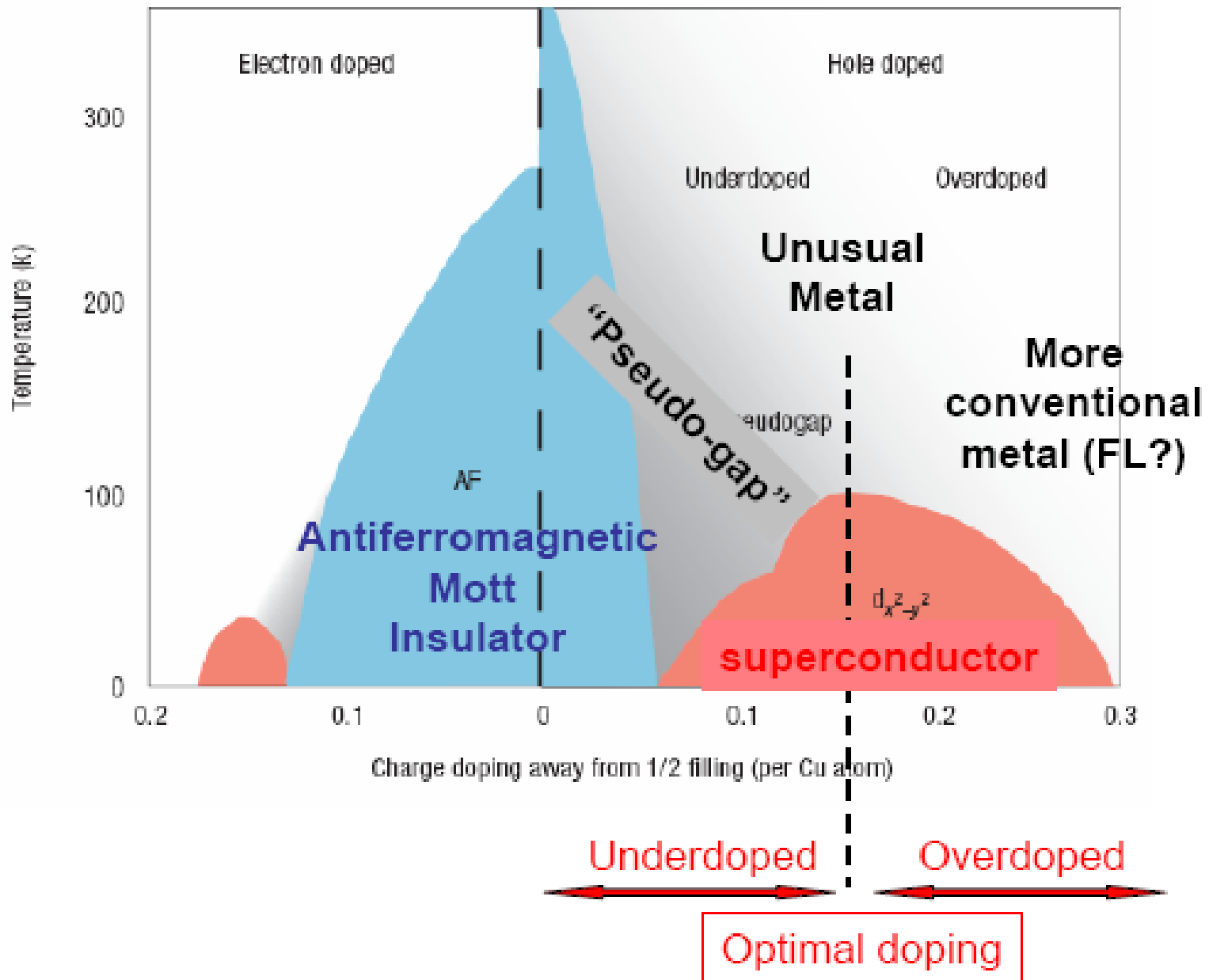
Cf also: Kagawa et al.  
(Kanoda's group)  
on the BEDT organics



# Beyond DMFT...

- Spatial correlations, when sizeable, influence quasiparticle properties  
(e.g. cuts-off divergence of effective mass)

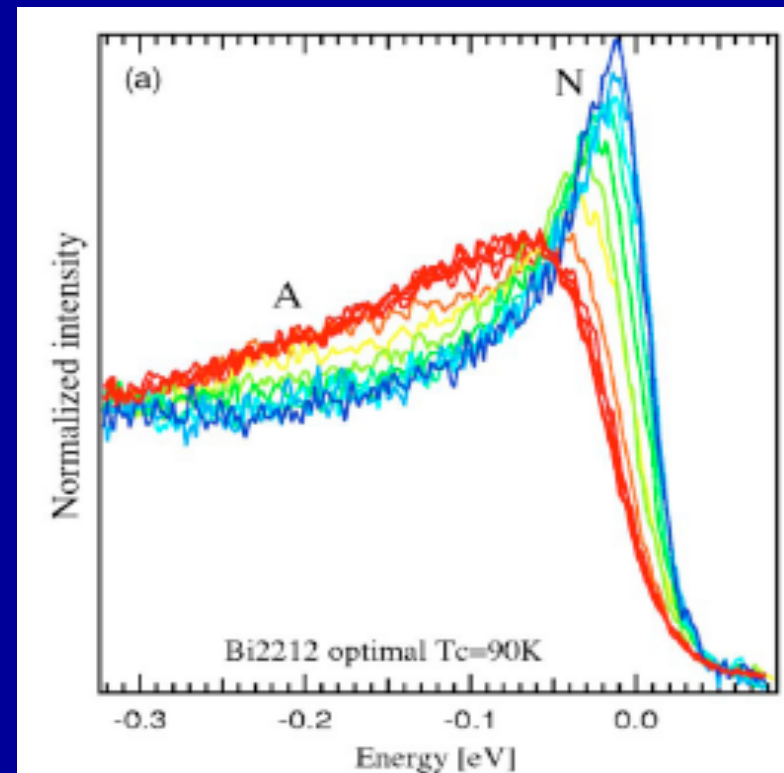
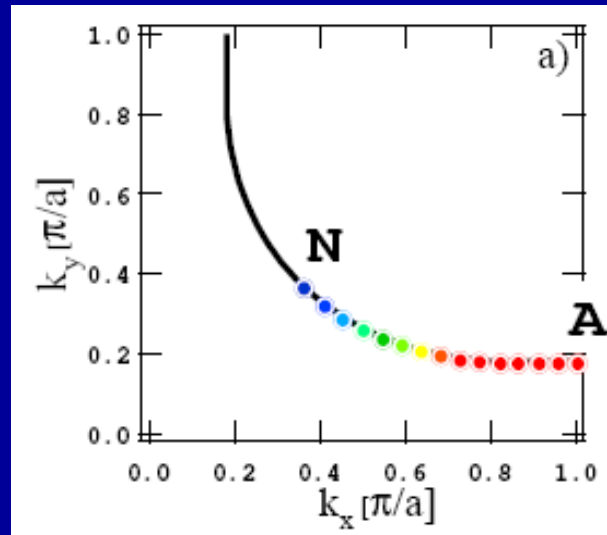
Some materials, especially hi-Tc cuprates at low doping levels, are strongly non-Brinkman Rice and follow a quite peculiar route to the Mott transition with strong momentum-space differentiation





# NORMAL state:

- “Nodal” regions display reasonably coherent quasiparticles
- In contrast, excitations in the “antinodal” regions e.g.  $(0,\pi)$  are much more incoherent  
AND they are (pseudo-) gapped below  $T^*$

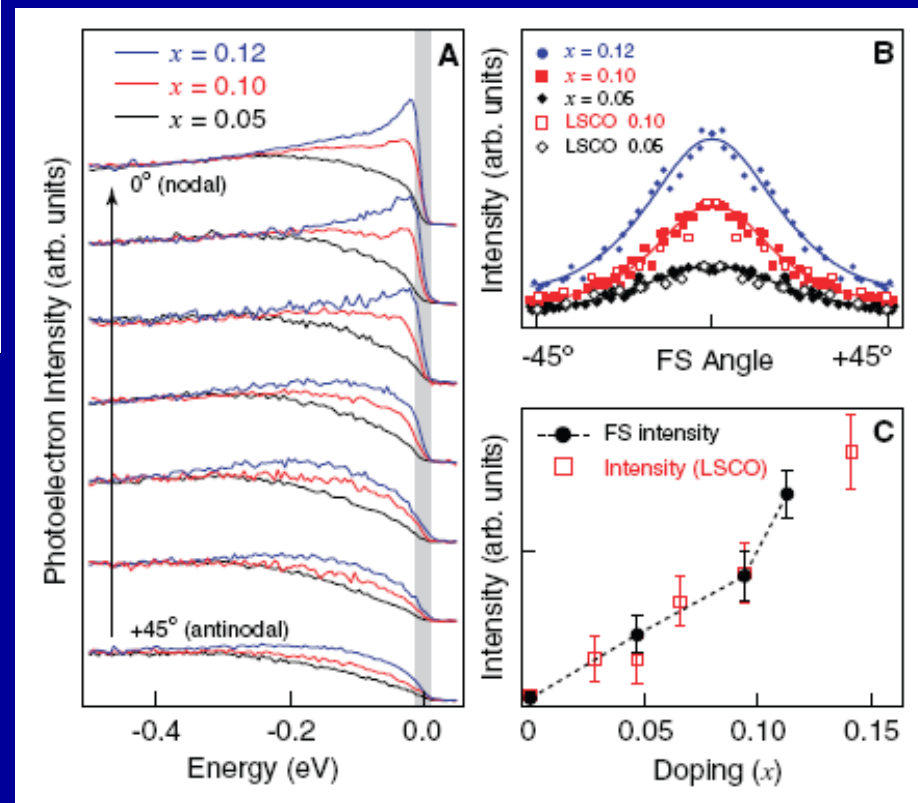
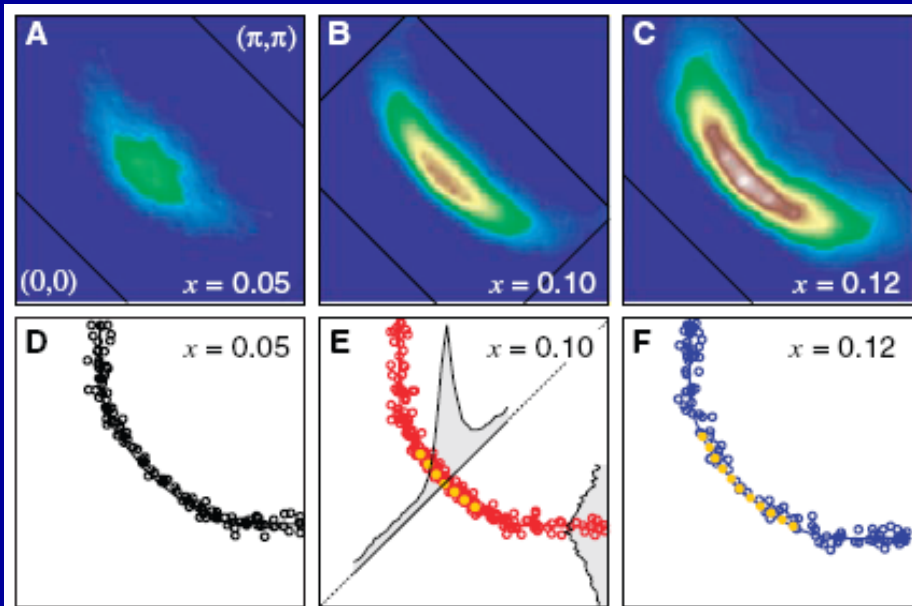


Kaminski et al., 2004 Bi2212  
 $T_c=90K @ T=140K$

# ARPES sees « Fermi arcs »



K. Shen et al. Science 2007



→ Next year's lectures !  
(Fall 2010)