

# La dynamique atmosphérique des Céphéides, la rotation de la Voie Lactée et l'étalonnage des échelles de distance dans l'Univers



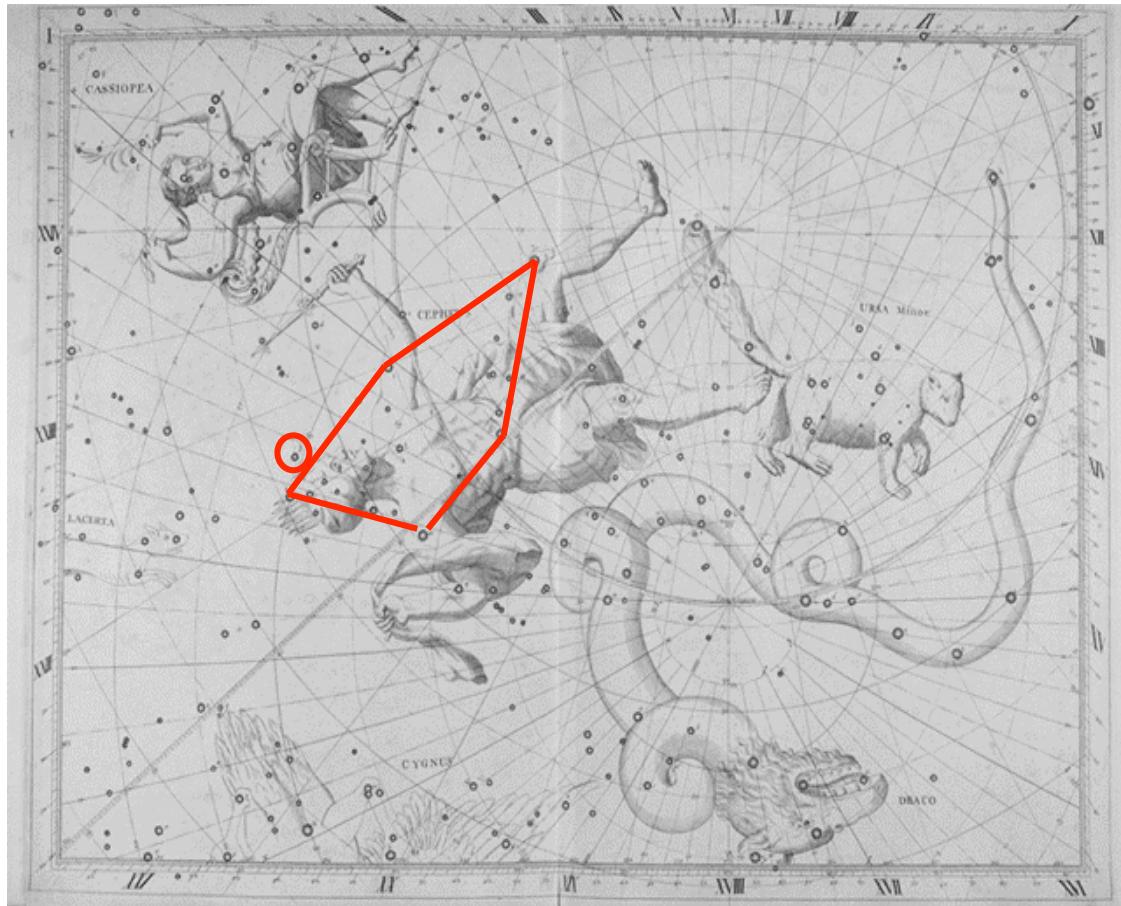
## Sommaire:

- 1/ Les Céphéides : petit historique
- 2/ Etalonnage des échelles de distance dans l'Univers
- 3/ Structure Vélocimétrique de la Voie Lactée
- 4/ Enveloppe circumstellaire
- 5/ Perspectives : la spectro-interférométrie

Nicolas Nardetto

Post-Doc à l'Université de Concepcion, Chili

# Découverte de la première Céphéide : $\delta$ Cephei



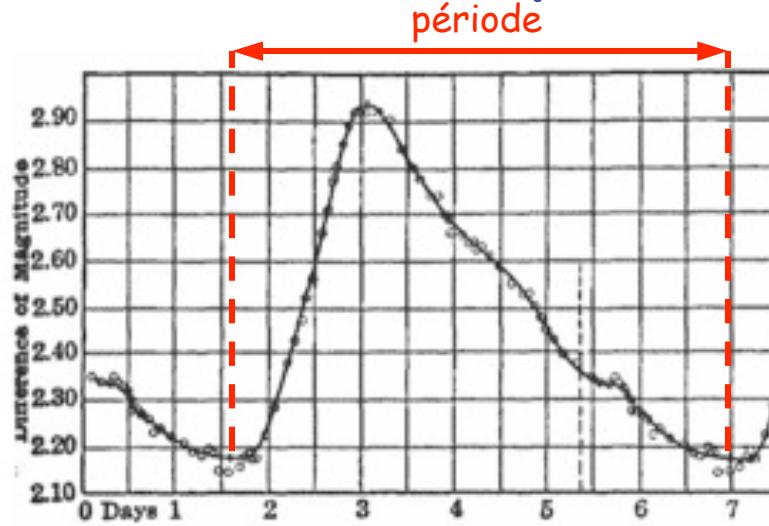
John Goodrick (1764-1786)

$M=5$  Ms  
 $R=41.6$  Rs  
 $L=2000$  Ls  
 $Teff=5500-6800$  K

$P=5.36$  jours  
 $\Delta R/R=10\%$

# Découverte de la relation période-luminosité (PL)

16 Céphéides dans  
les Nuages de  
Magellan



H. Leavitt (1908)

magnitude

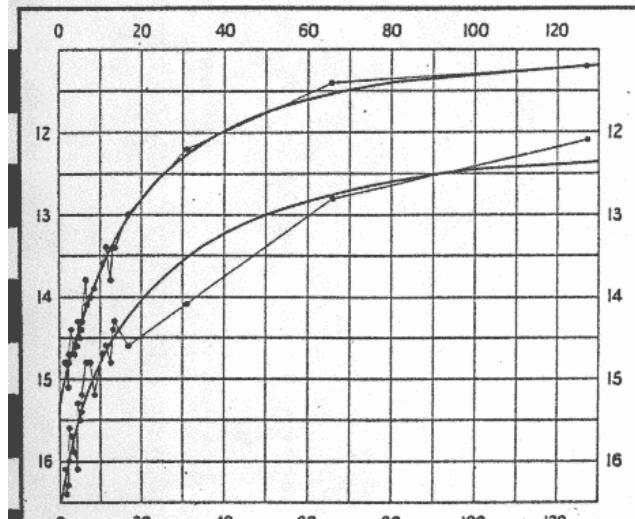


FIG. 1.

Période (jours)

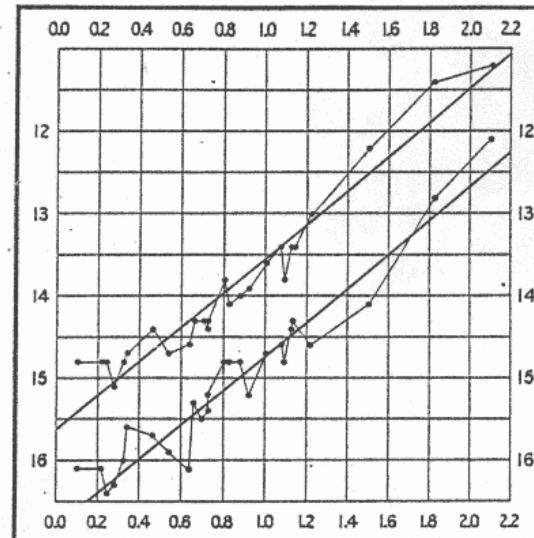


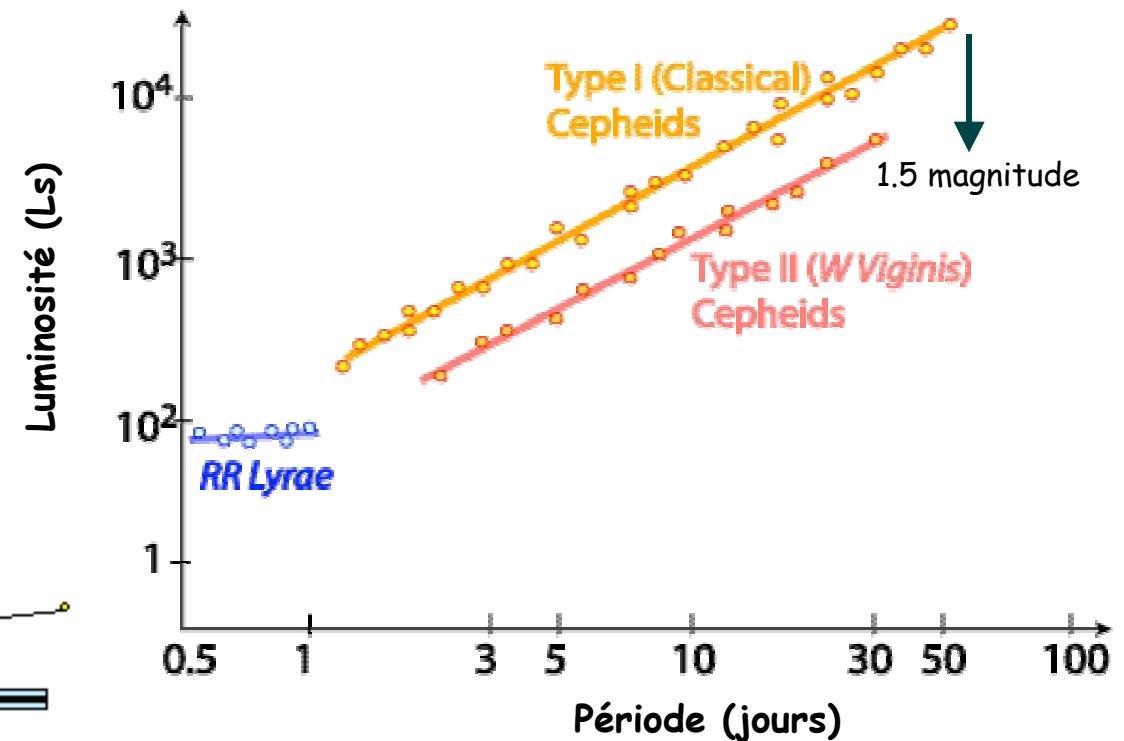
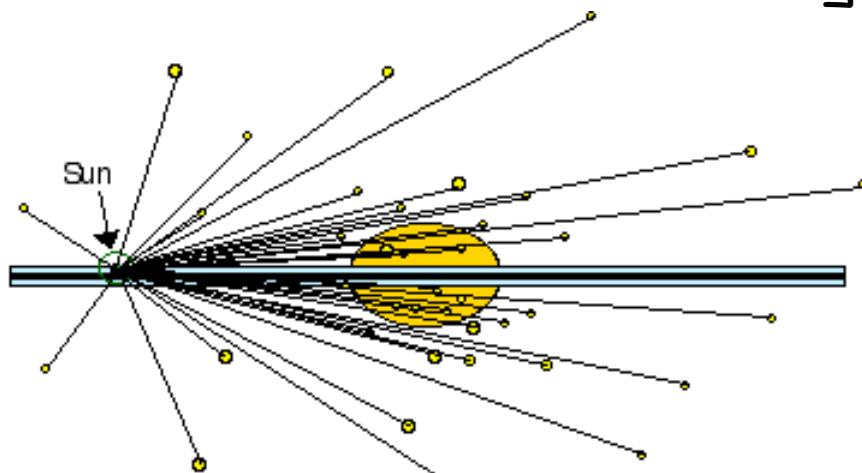
FIG. 2.

log P (jours)

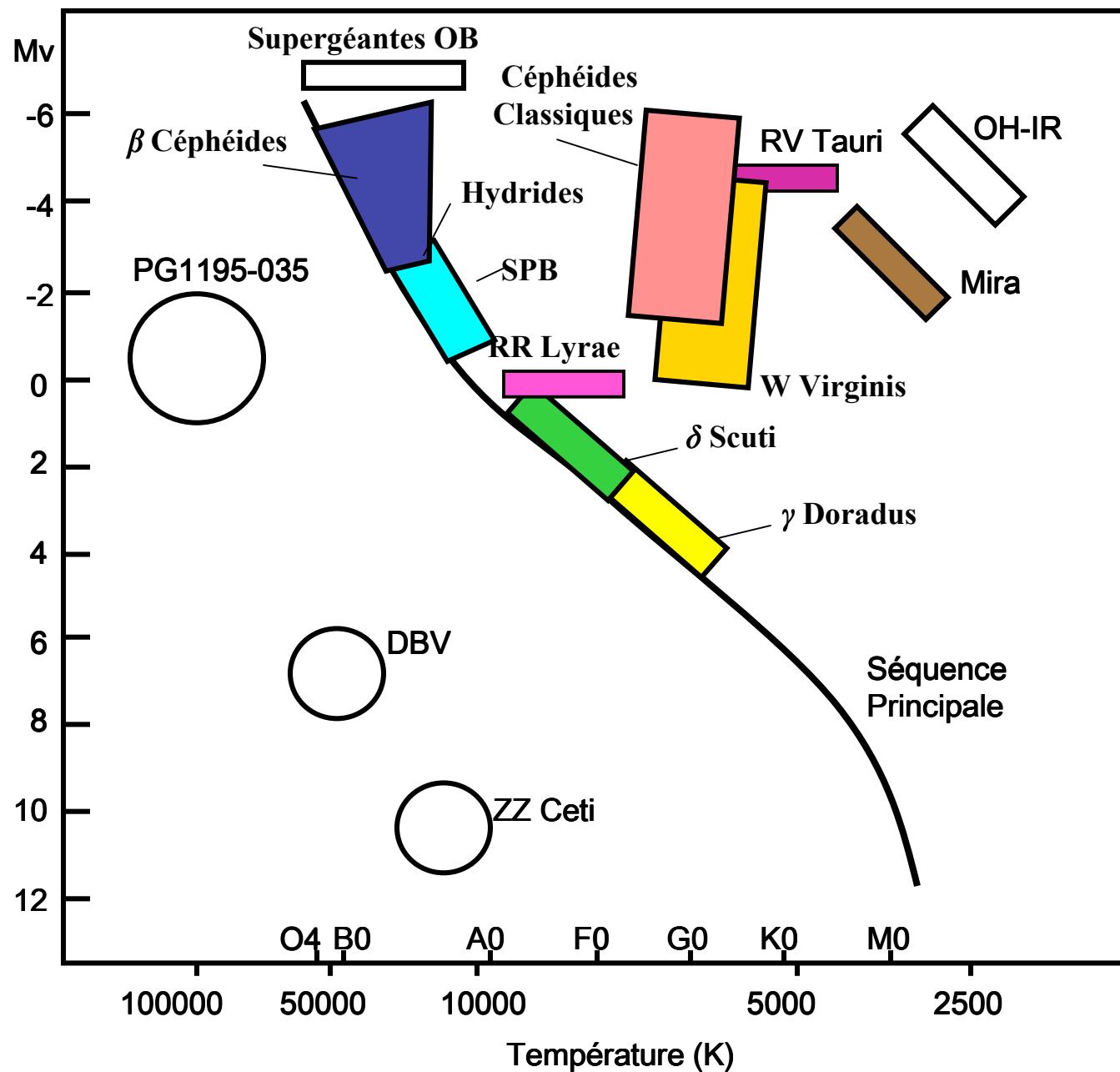
# Premiers étalonnages de la relation PL

Hertzsprung (1913) :  $\langle M \rangle = a + b \log_{10} P$  (3000 a.l. pour le SMC au lieu de 210000!)

Shapley (1920) : il confond les Céphéides de type I et de type II (ou W Virginis)



# Les différents types d'étoiles pulsantes

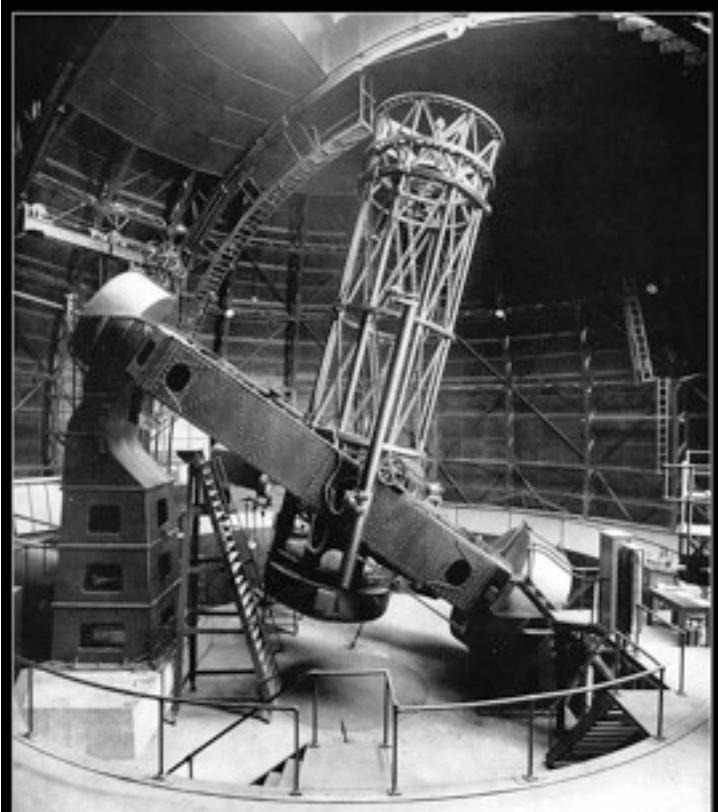


# La distance à la Galaxie d'Andromède (M31)

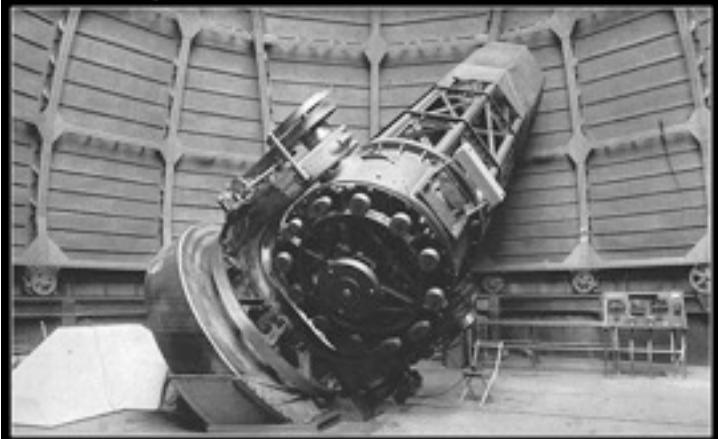
Hubble (1925) : distance de M31, 2 fois trop près! Il observe les Céphéides de type I et utilise la relation de Shapley (valable pour les W Virginis).



Baade (1944) : distinction entre les Céphéides et les W Virginis

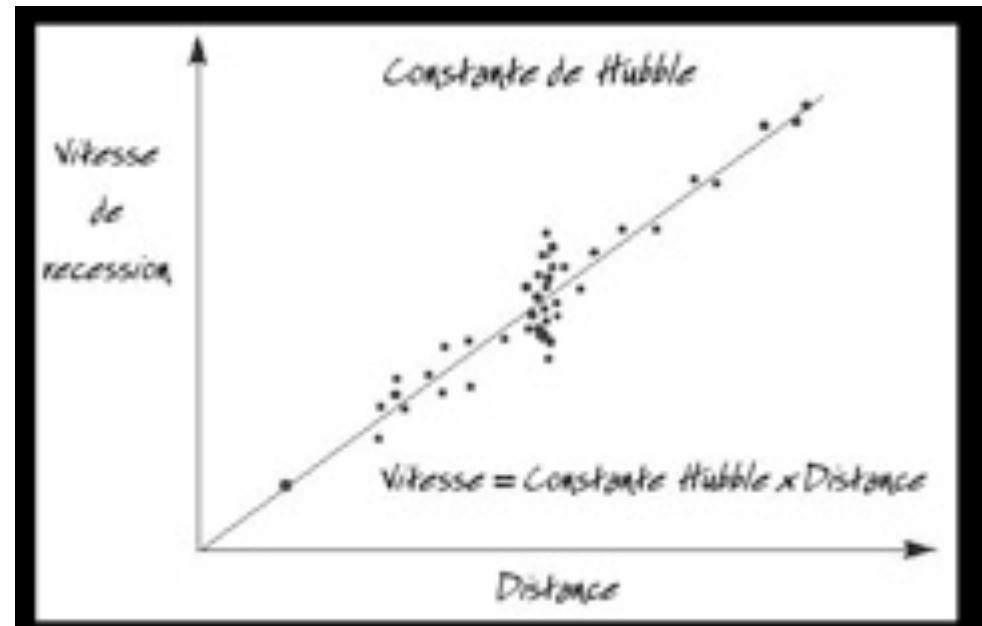


Le télescope Hooker de 2,5m de l'Observatoire du Mont Wilson.



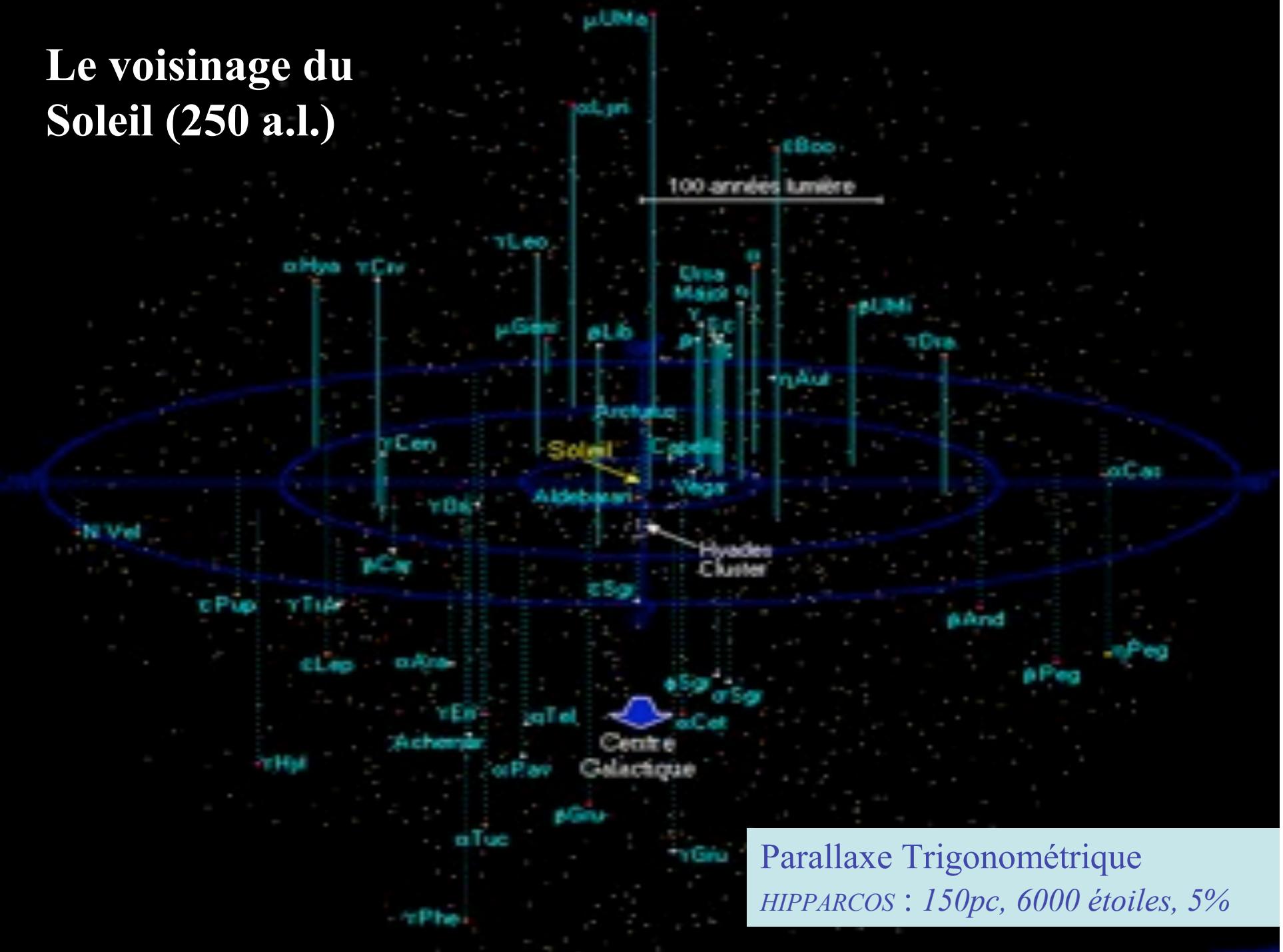
Le télescope équatorial de 1m52 de l'Observatoire du Mont Wilson.

## La constante de Hubble

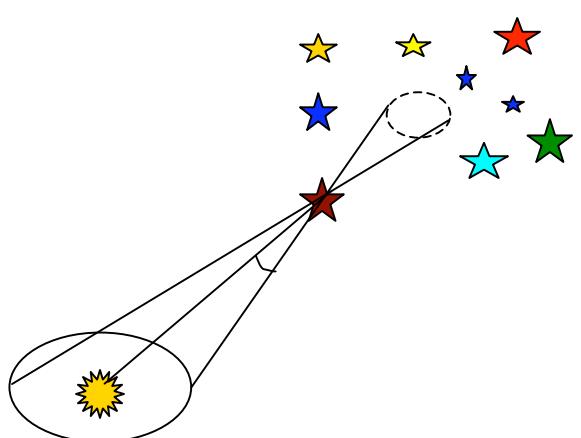
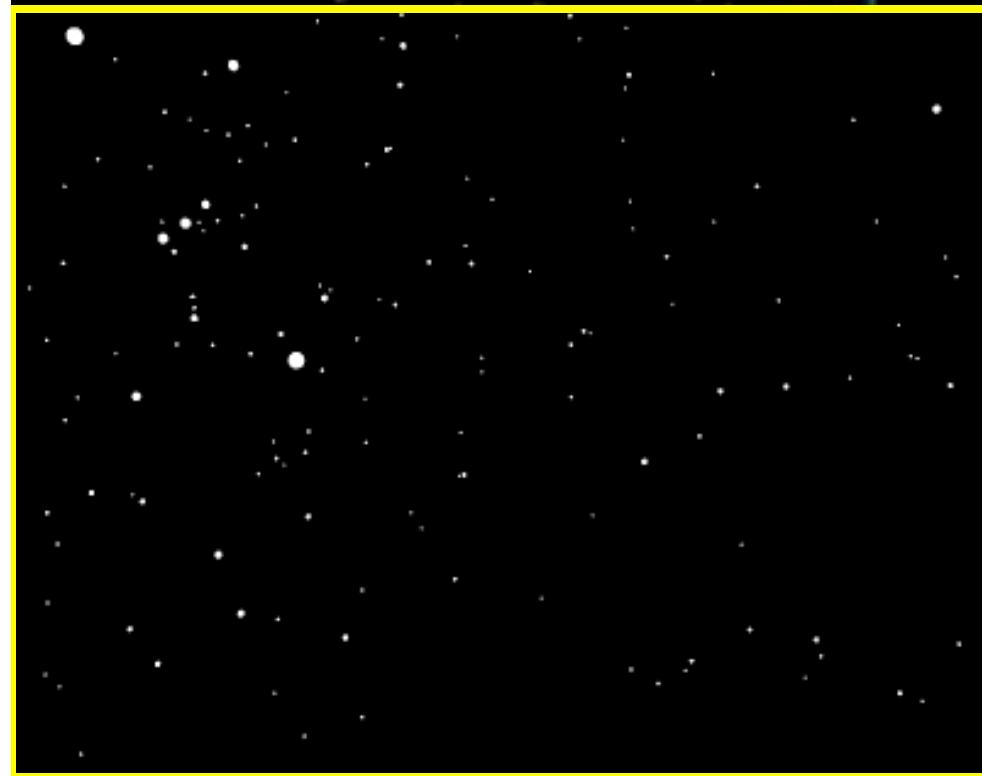


Hubble, 1929

# Le voisinage du Soleil (250 a.l.)

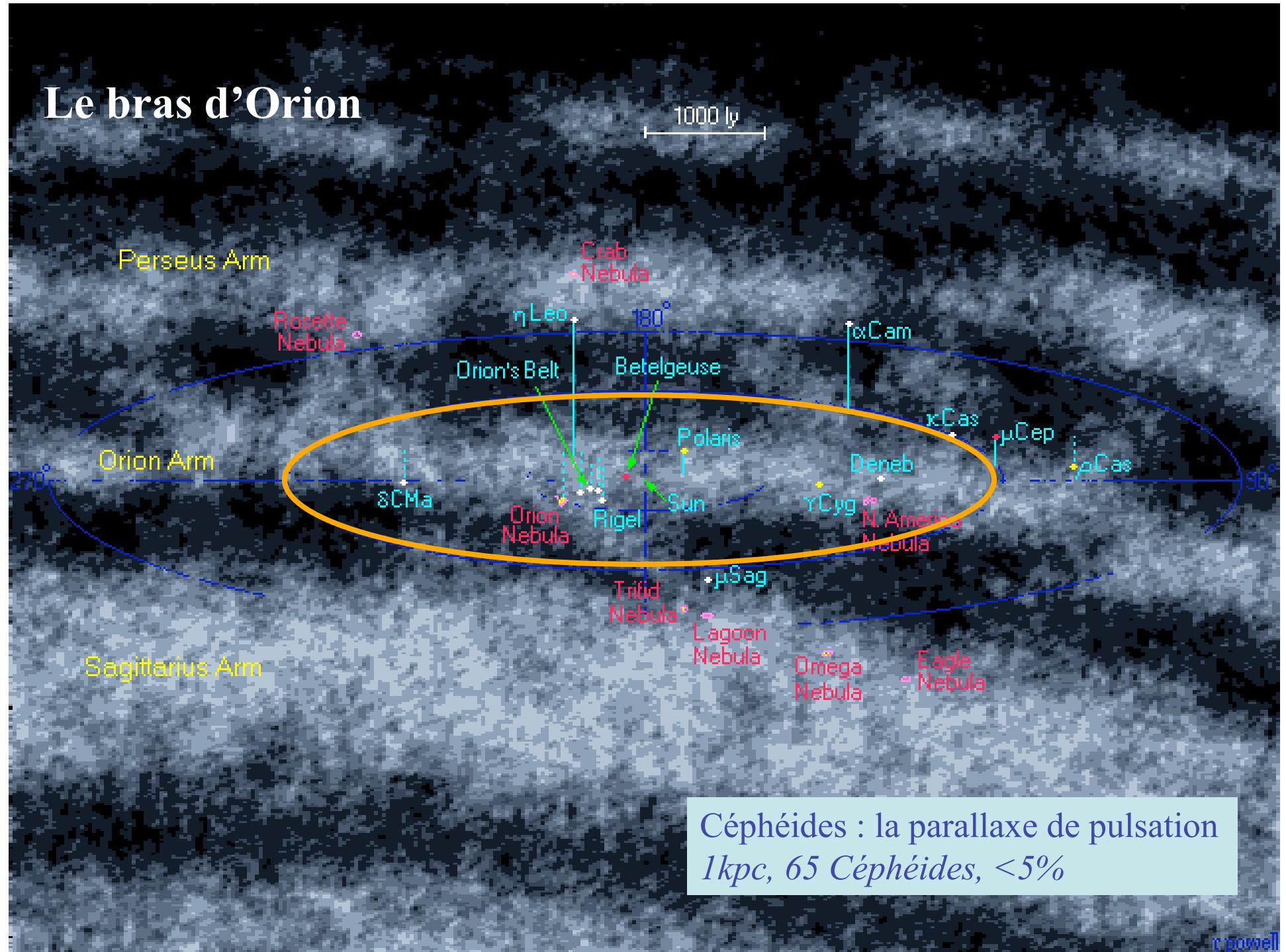


# Le voisinage du Soleil (250 a.l.)

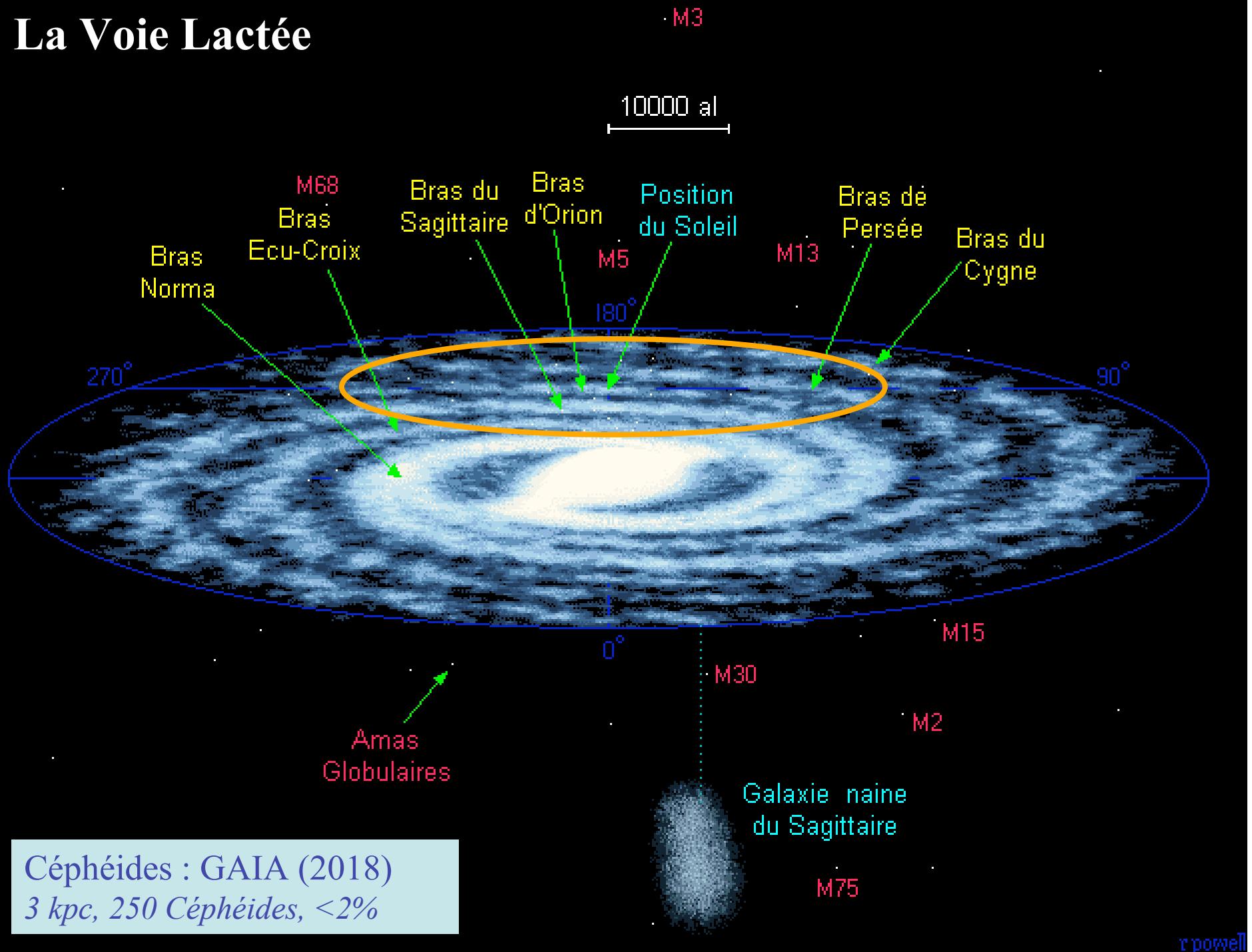


Parallaxe Trigonométrique  
HIPPARCOS : 150pc, 6000 étoiles, 5%

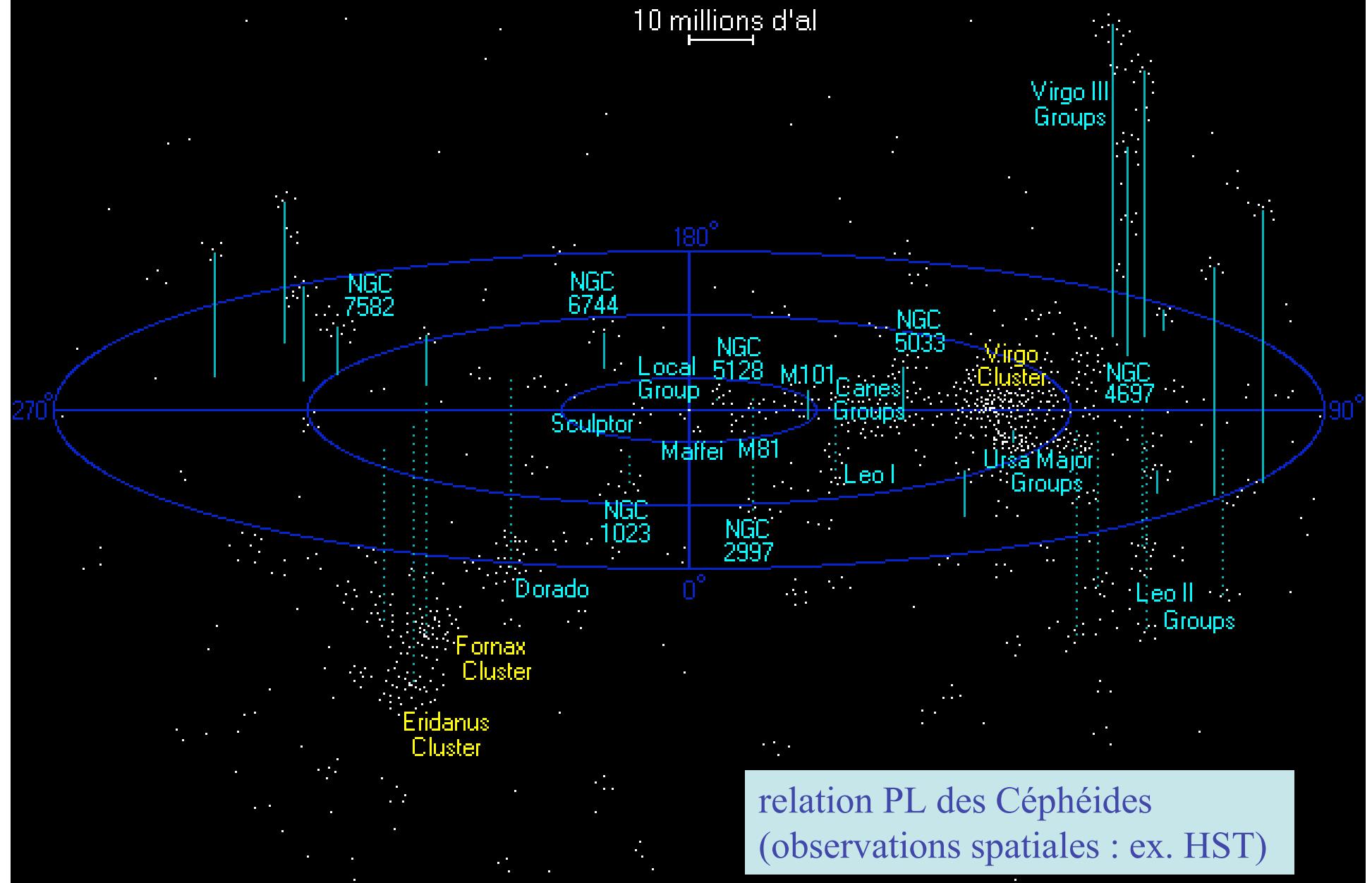
# Le bras d'Orion



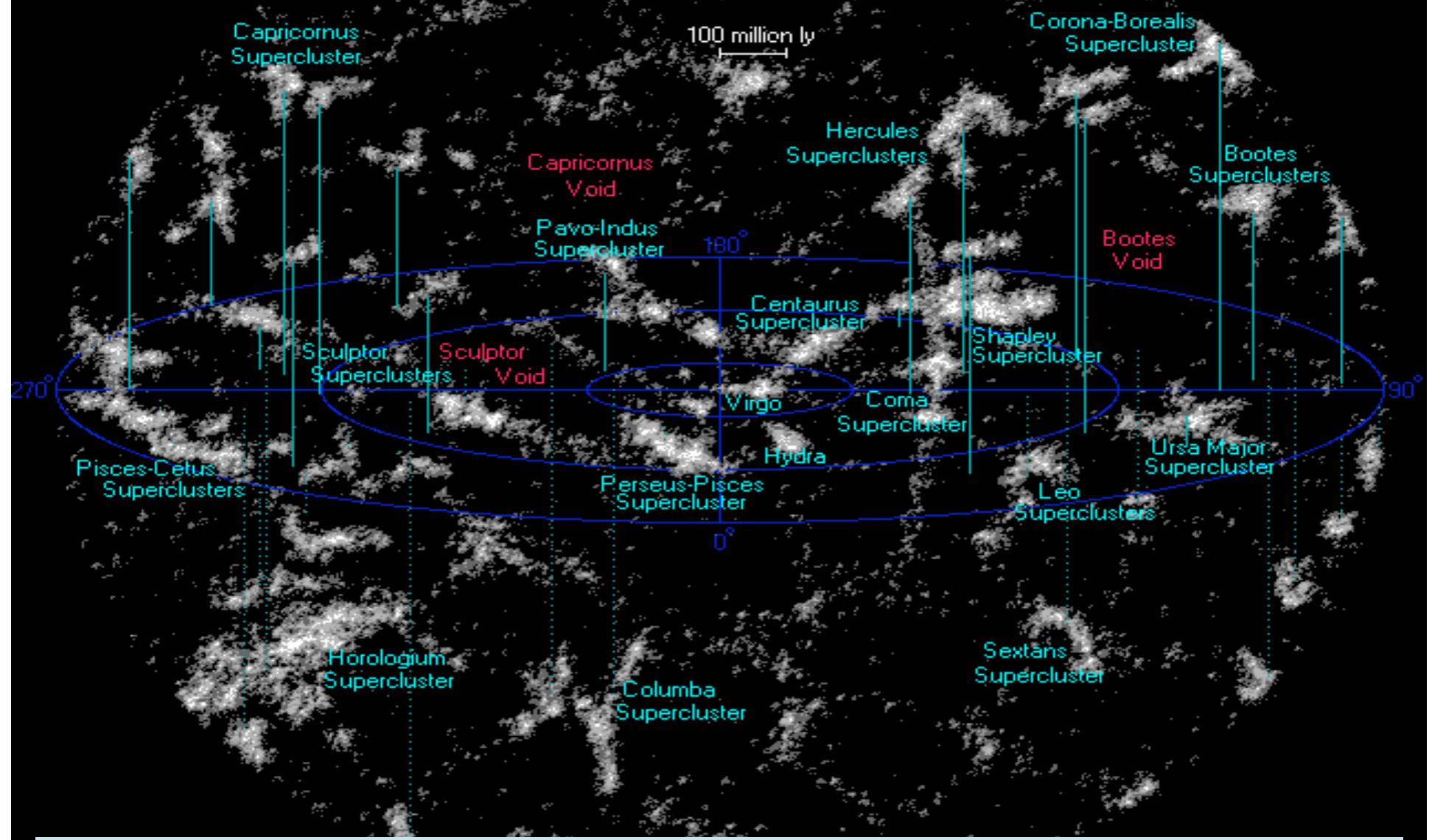
# La Voie Lactée



# Le Superamas de la Vierge

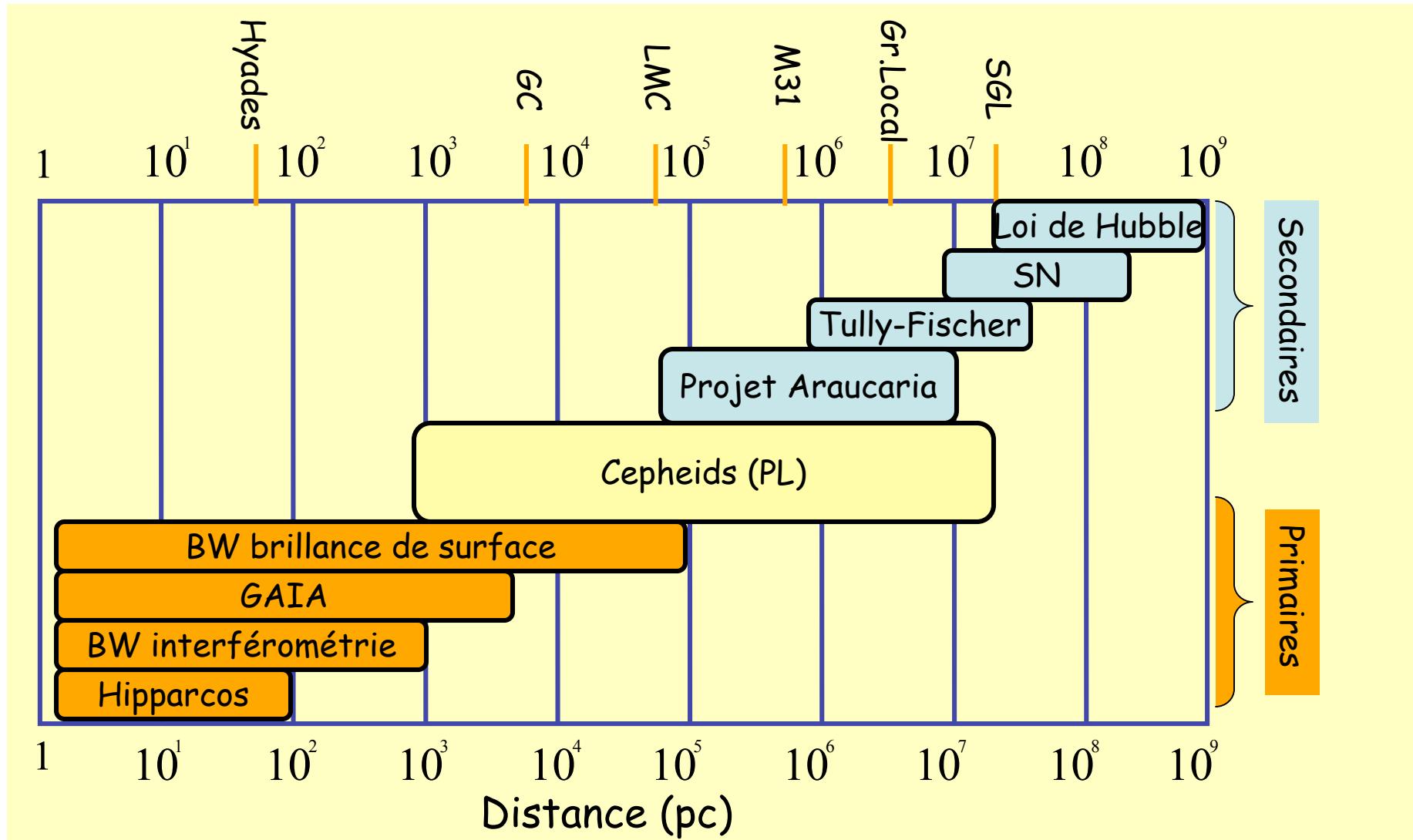


# Les Superamas voisins



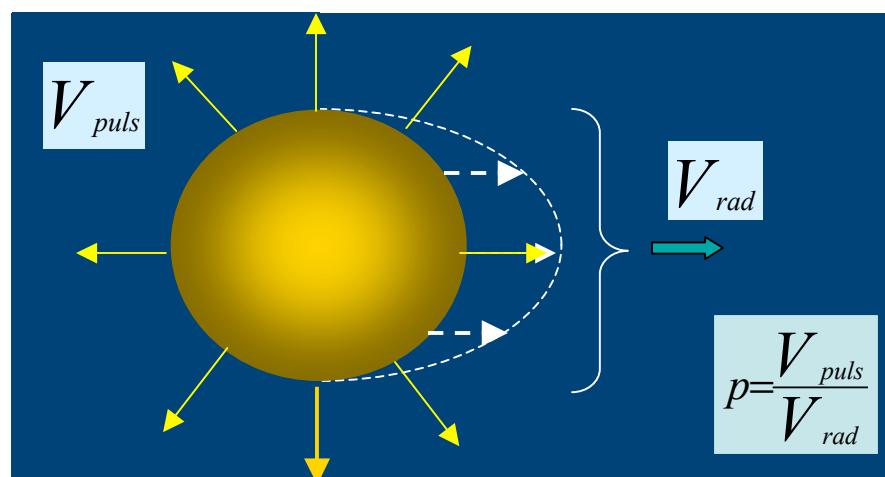
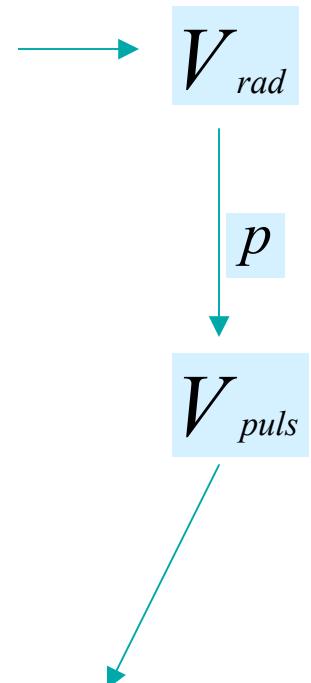
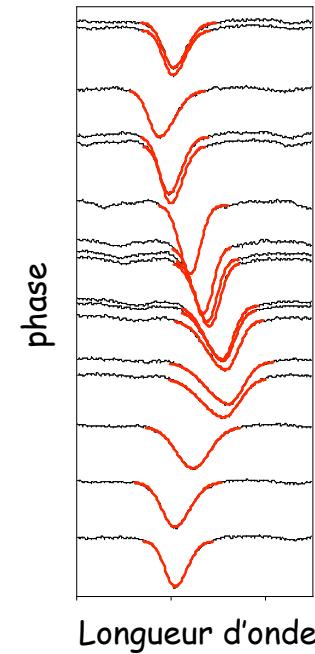
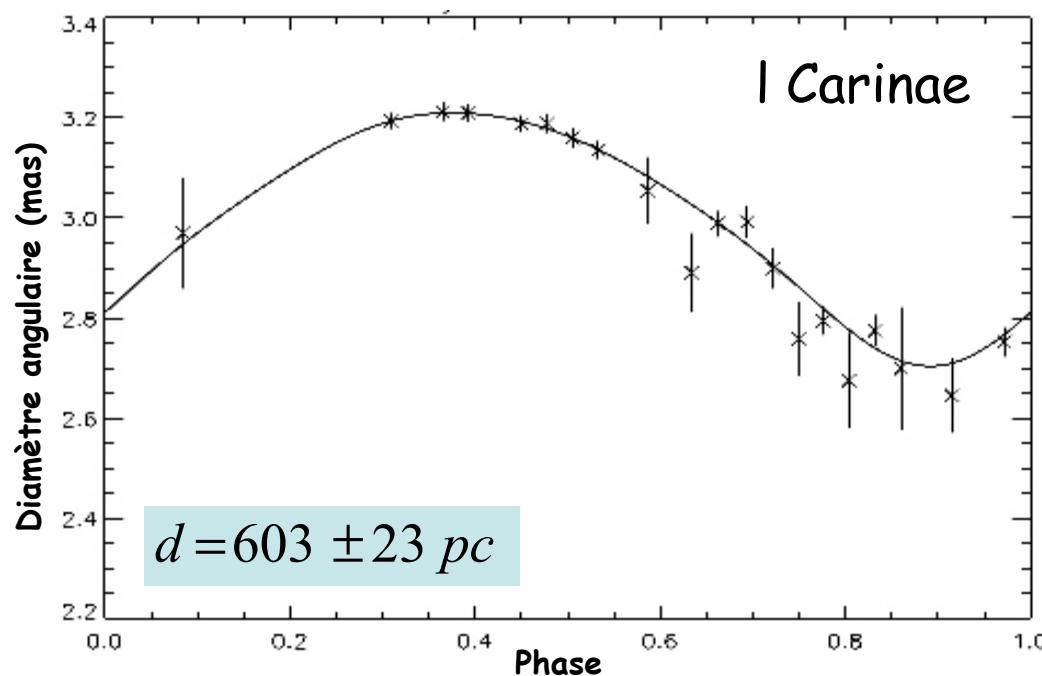
Tully-Fischer, Faber-Jackson, Brilliance de surface des Galaxies, Supernovae, Loi de Hubble

# L'étalonnage des échelles de distance



# Les méthodes de type Baade-Wesselink

Cepheid distances from infrared long-baseline interferometry I. VINCI/VLTI observations of seven Galactic Cepheids  
P. Kervella, N. Nardetto, D. Bersier, D. Mourard and V. Coudé du Foresto, 2004, A&A, 416, 941K

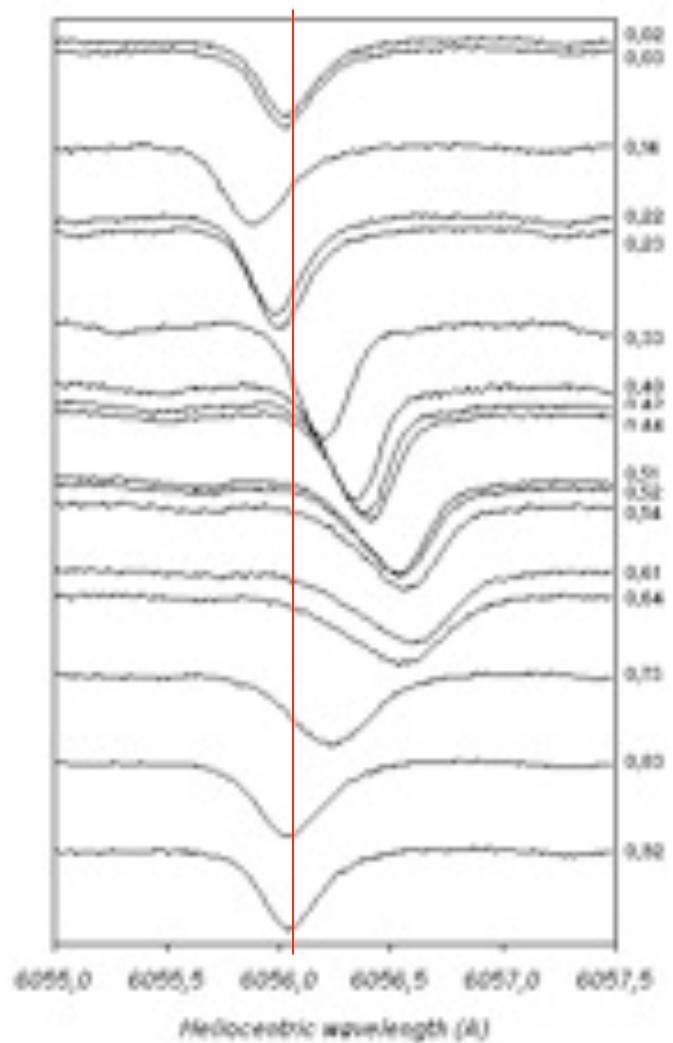


$$\theta(\phi) = \overline{\theta}_{LD} + 9.305 \left( \int_0^P V_{puls} dt \right) \frac{1}{d}$$



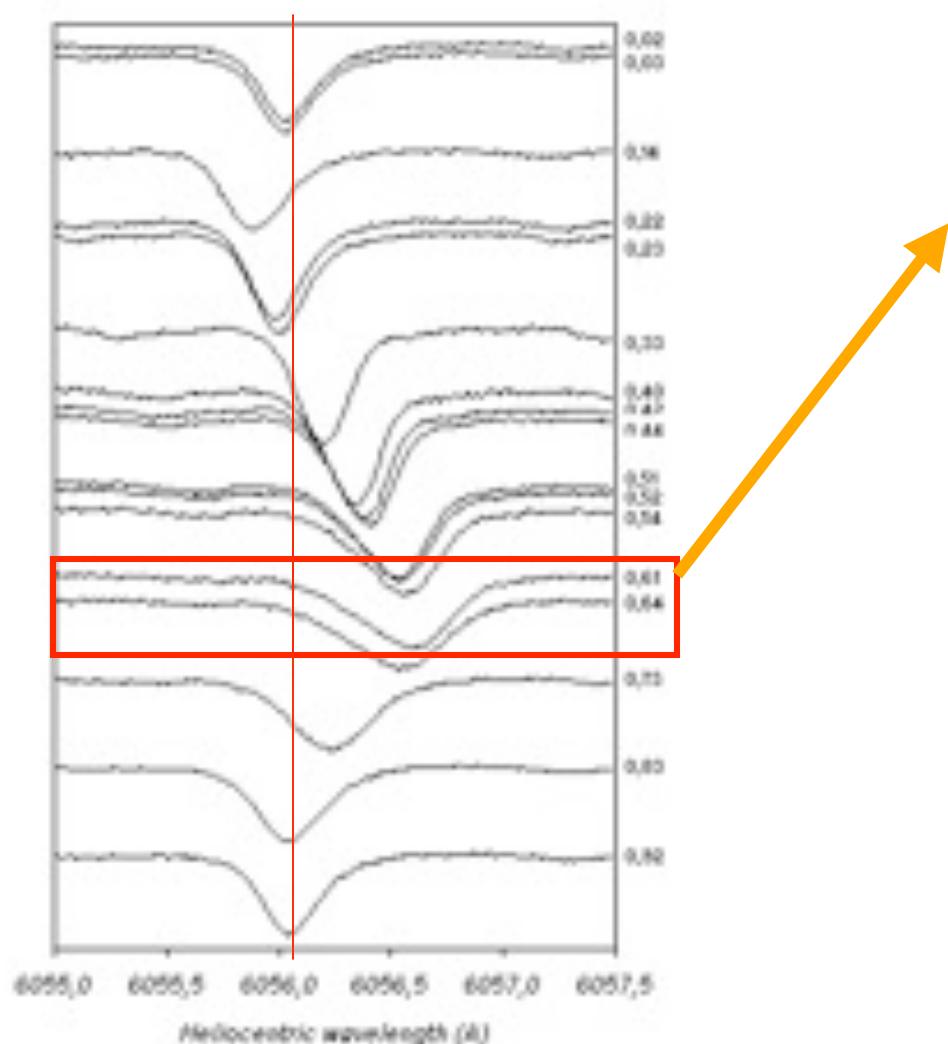
$$\Delta R \Leftrightarrow \Delta \theta$$

# La définition de la vitesse radiale $V_{\text{rad}}$ et de $p_o$ (1/3)



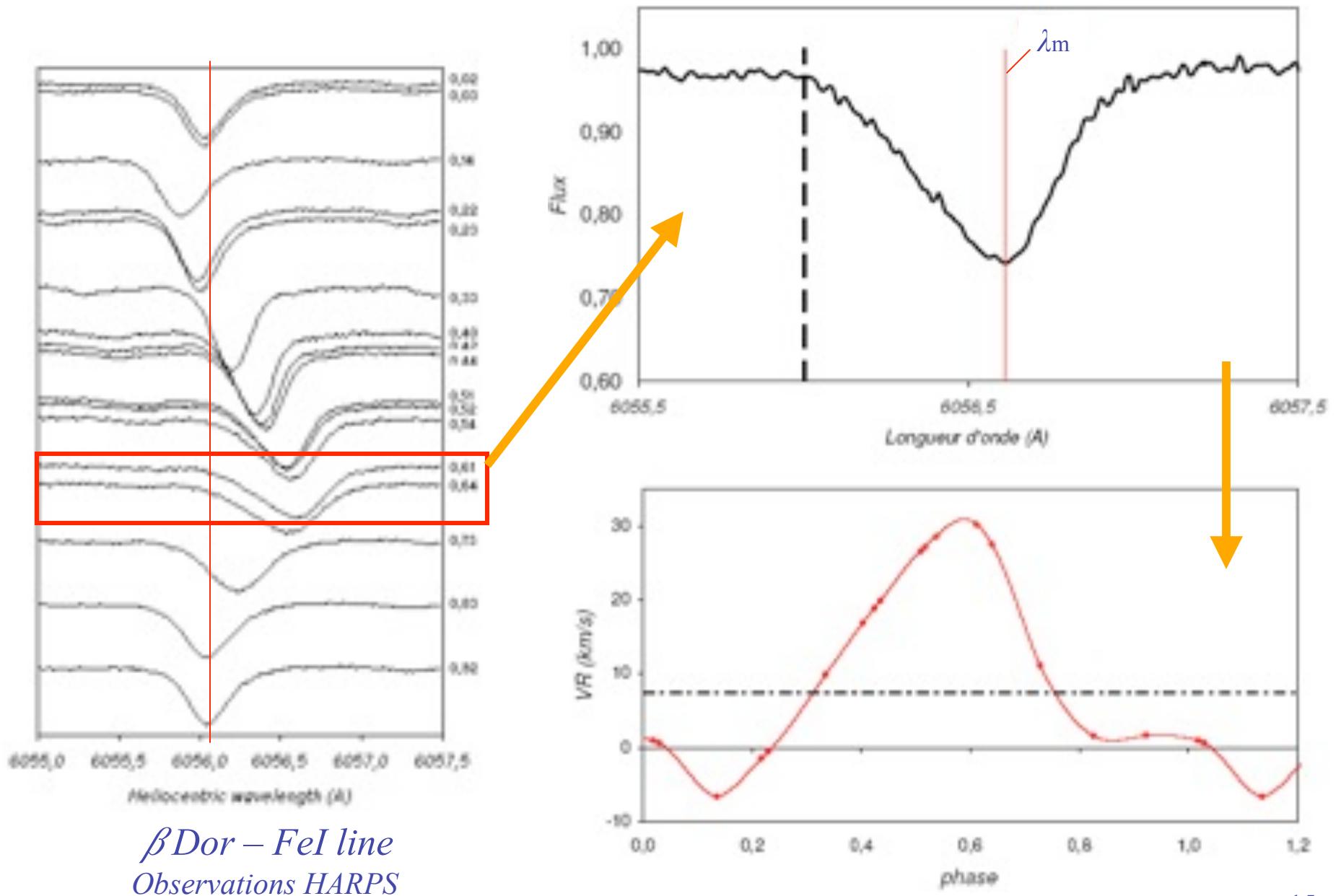
*$\beta$ Dor – FeI line  
Observations HARPS*

# La définition de la vitesse radiale $V_{\text{rad}}$ et de $p_o$ (1/3)

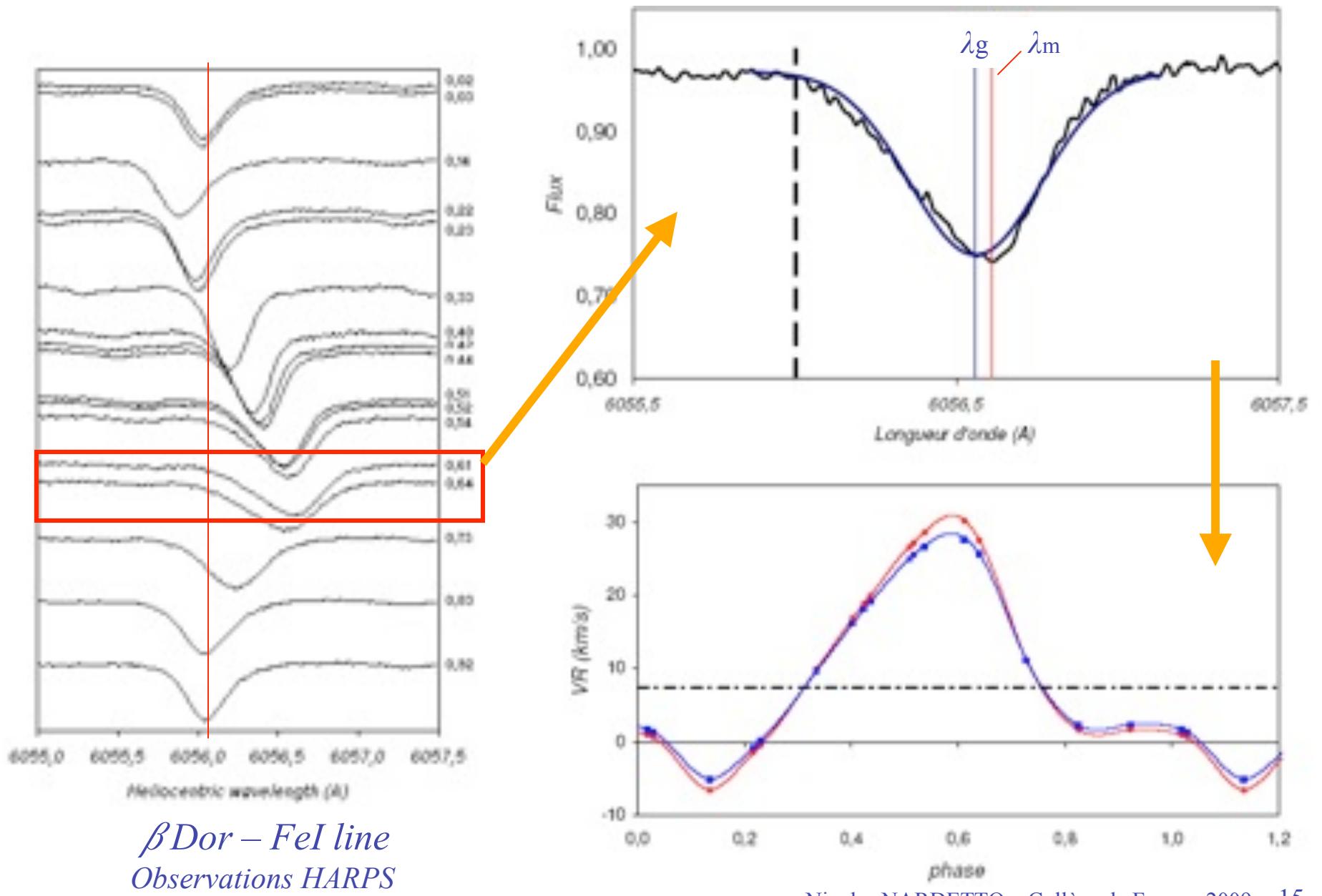


$\beta$ Dor – FeI line  
Observations HARPS

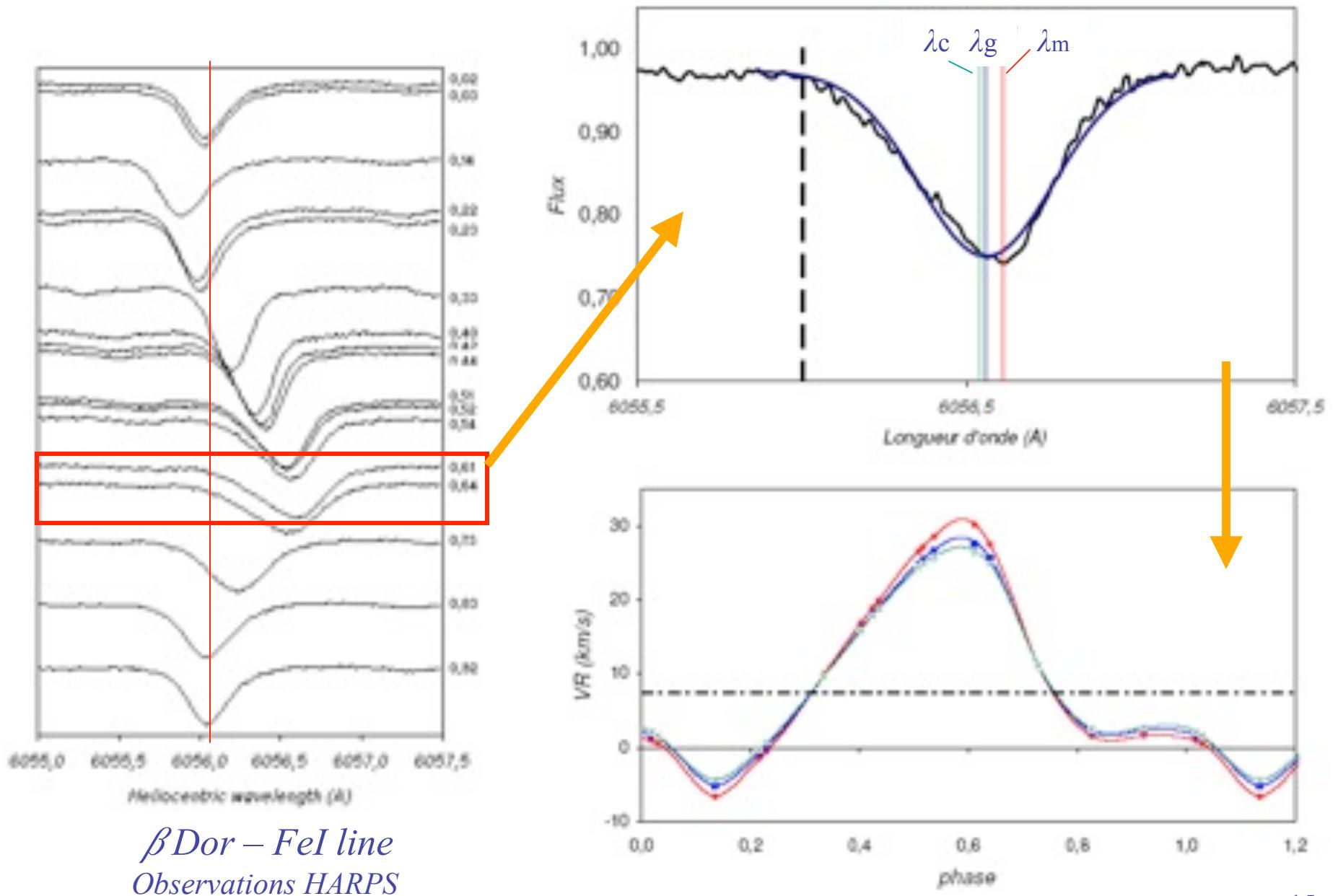
# La définition de la vitesse radiale $V_{\text{rad}}$ et de $p_o$ (1/3)



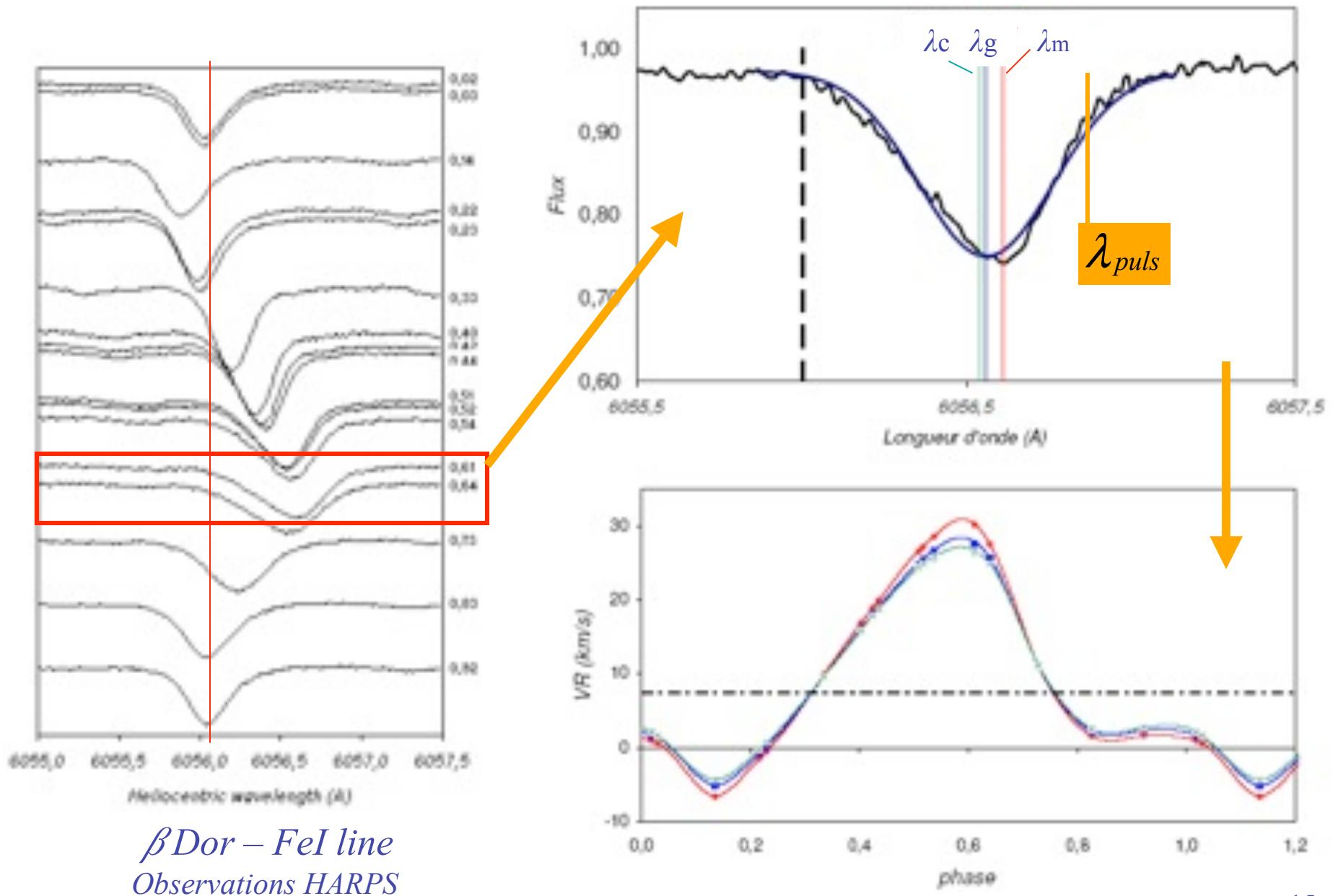
# La définition de la vitesse radiale $V_{\text{rad}}$ et de $p_o$ (1/3)



# La définition de la vitesse radiale $V_{\text{rad}}$ et de $p_o$ (1/3)

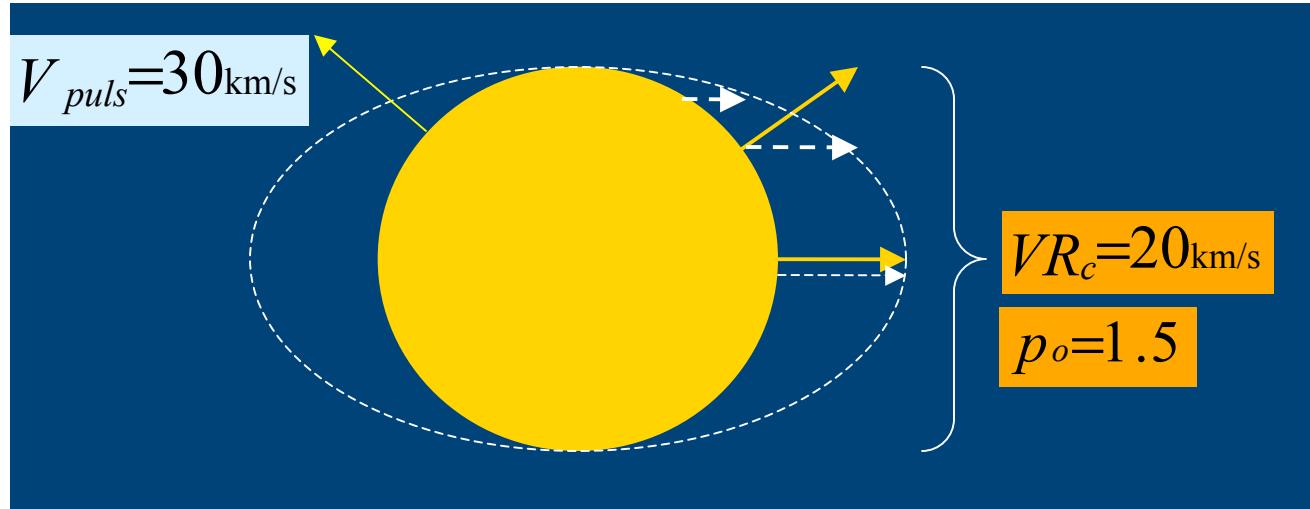


# La définition de la vitesse radiale $V_{\text{rad}}$ et de $p_o$ (1/3)



# La définition de la vitesse radiale $V_{\text{rad}}$ et de $p_o$ (2/3)

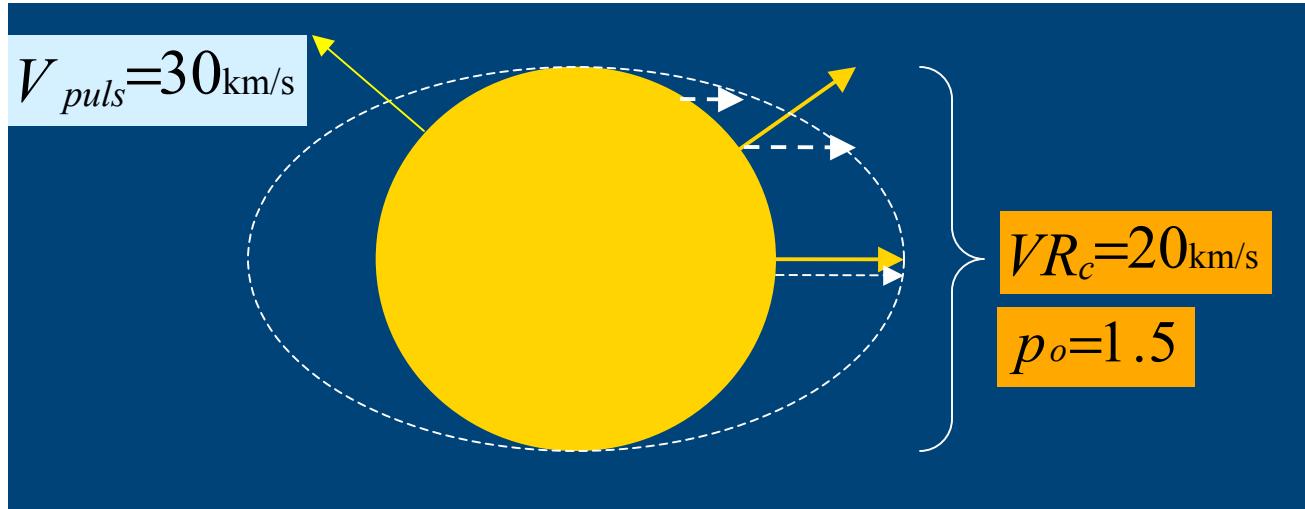
## • Effet de projection géométrique



ACB  
 $uV=0$   
Dispersion Doppler  
 $\sigma_c=0 \text{ A}$   
Rotation  
 $V_{rot} \sin(i)=0 \text{ km/s}$

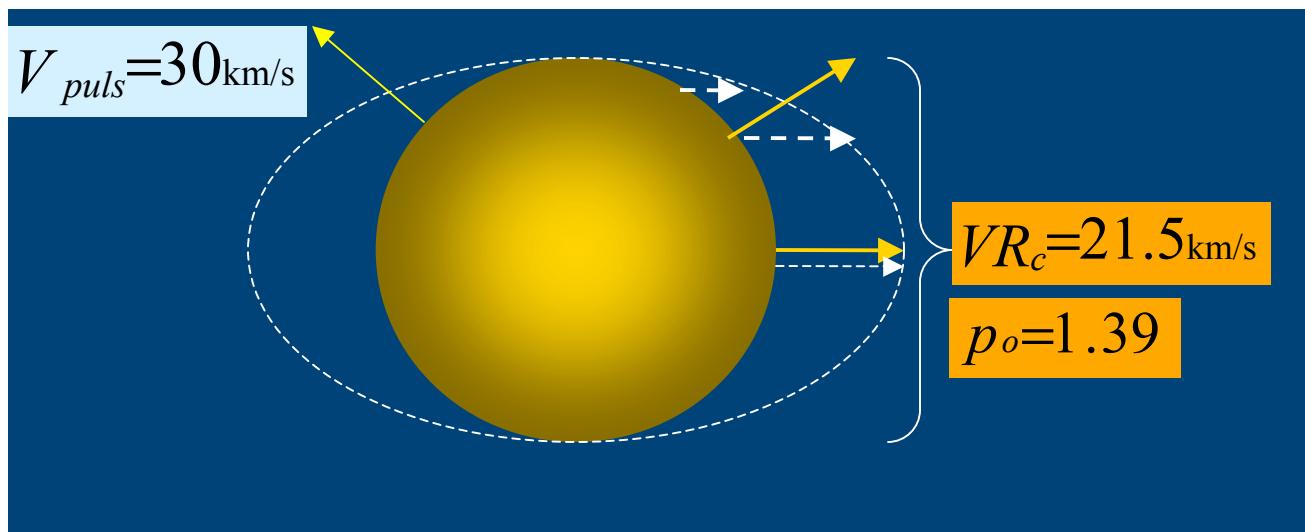
# La définition de la vitesse radiale $V_{\text{rad}}$ et de $p_o$ (2/3)

## • Effet de projection géométrique



ACB  
 $uV=0$   
Dispersion Doppler  
 $\sigma_c=0 \text{ A}$   
Rotation  
 $V_{\text{rot}} \sin(i)=0 \text{ km/s}$

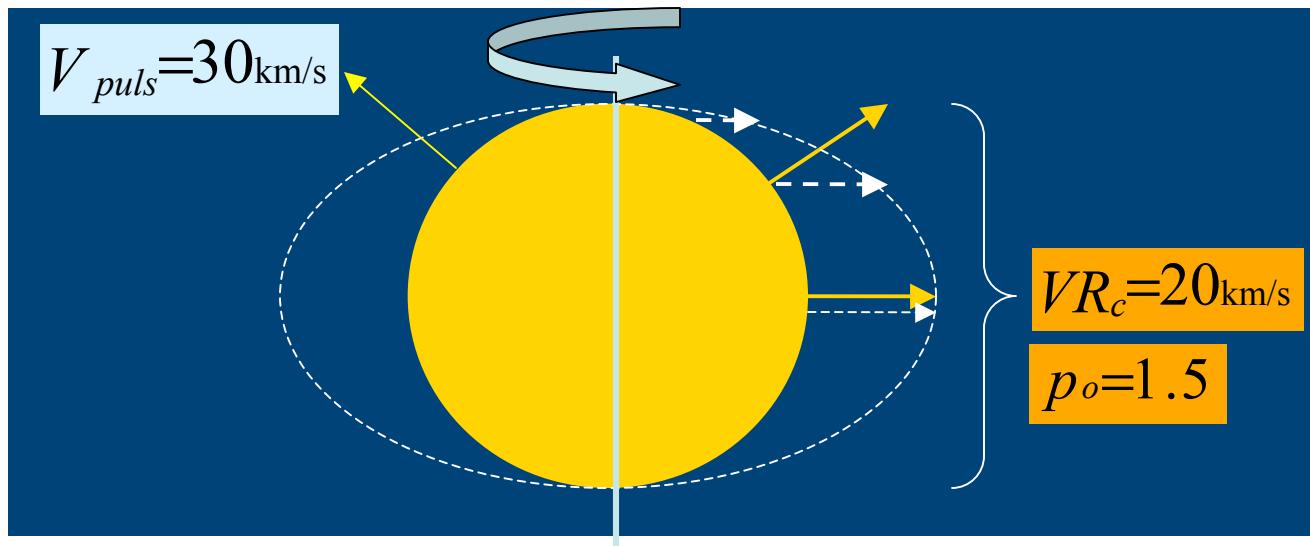
## • Effet de l'assombrissement centre-bord



ACB  
 $uV=0.7$   
Dispersion Doppler  
 $\sigma_c=0 \text{ A}$   
Rotation  
 $V_{\text{rot}} \sin(i)=0 \text{ km/s}$

# La définition de la vitesse radiale $V_{rad}$ et de $p_o$ (3/3)

## • Effet de la rotation



ACB

$u=0$

Dispersion Doppler

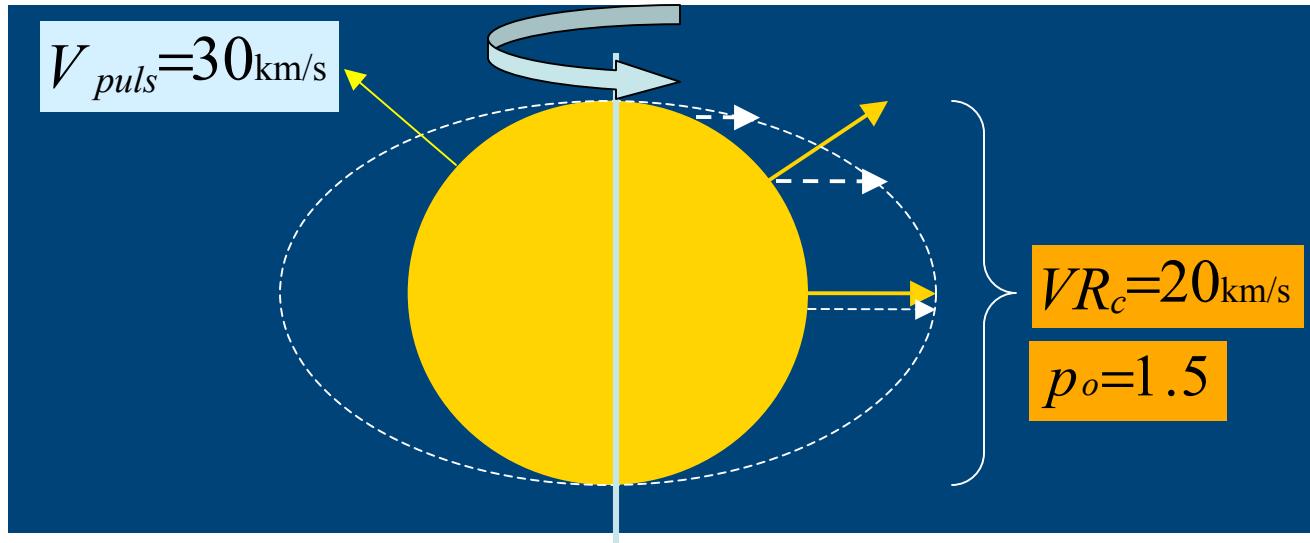
$\sigma_c=0 \text{ A}$

Rotation

$V_{rot} \sin(i)=10 \text{ km/s}$

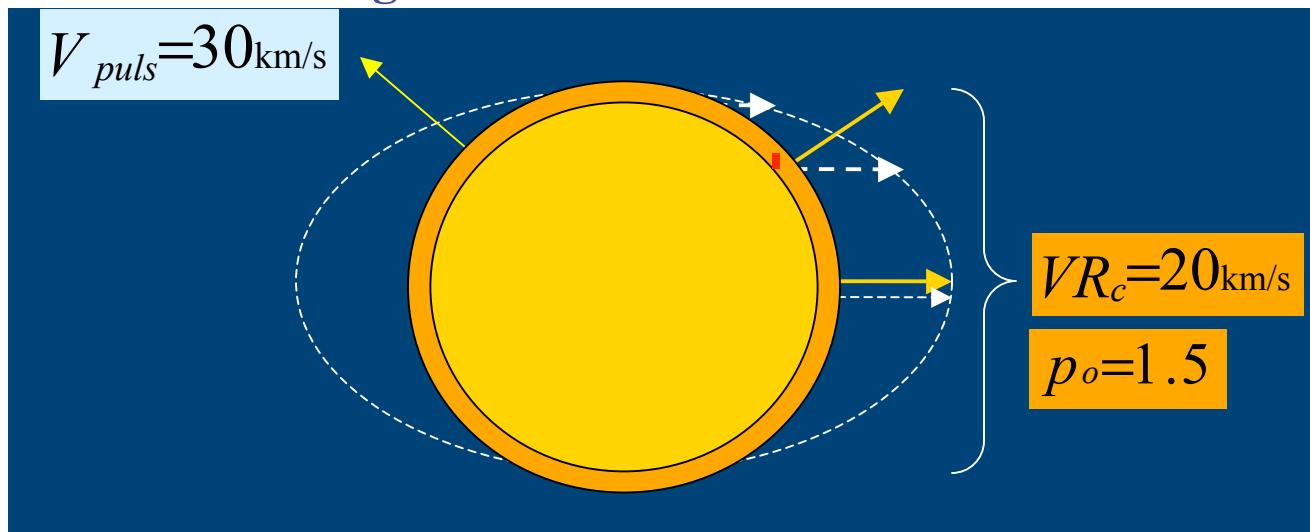
# La définition de la vitesse radiale $V_{rad}$ et de $p_o$ (3/3)

- Effet de la rotation



ACB  
 $u=0$   
 Dispersion Doppler  
 $\sigma_c = 0 \text{ A}$   
 Rotation  
 $V_{rot} \sin(i) = 10 \text{ km/s}$

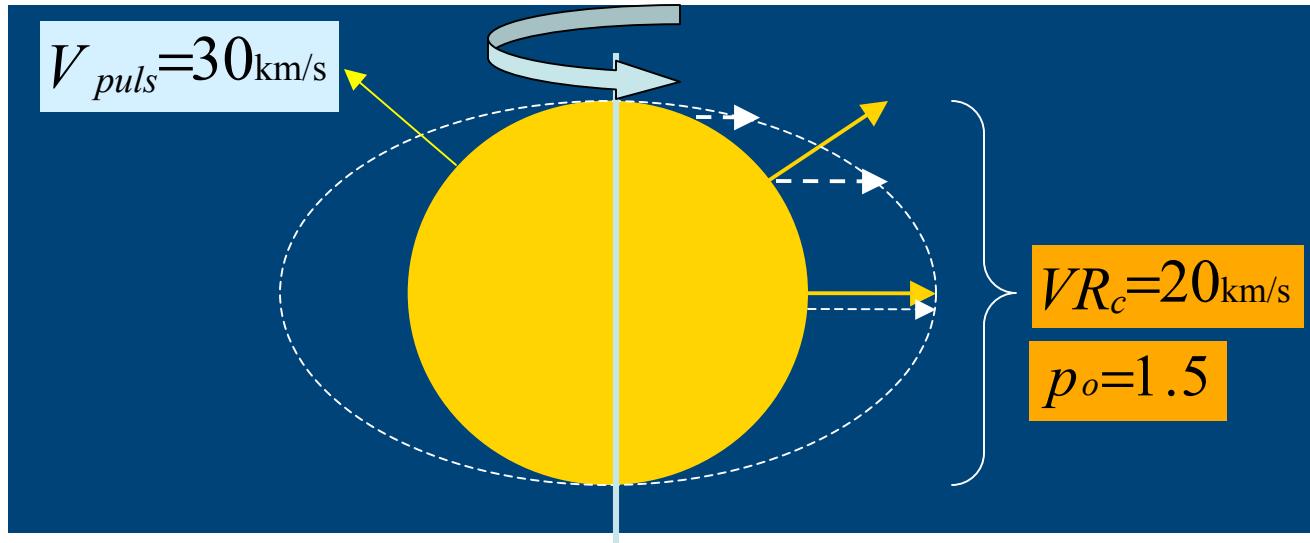
- Effet de la largeur de la raie  $\sigma_c$



ACB  
 $u=0$   
 Dispersion Doppler  
 $\sigma_c = 0.30 \text{ A}$   
 Rotation  
 $V_{rot} \sin(i) = 0 \text{ km/s}$

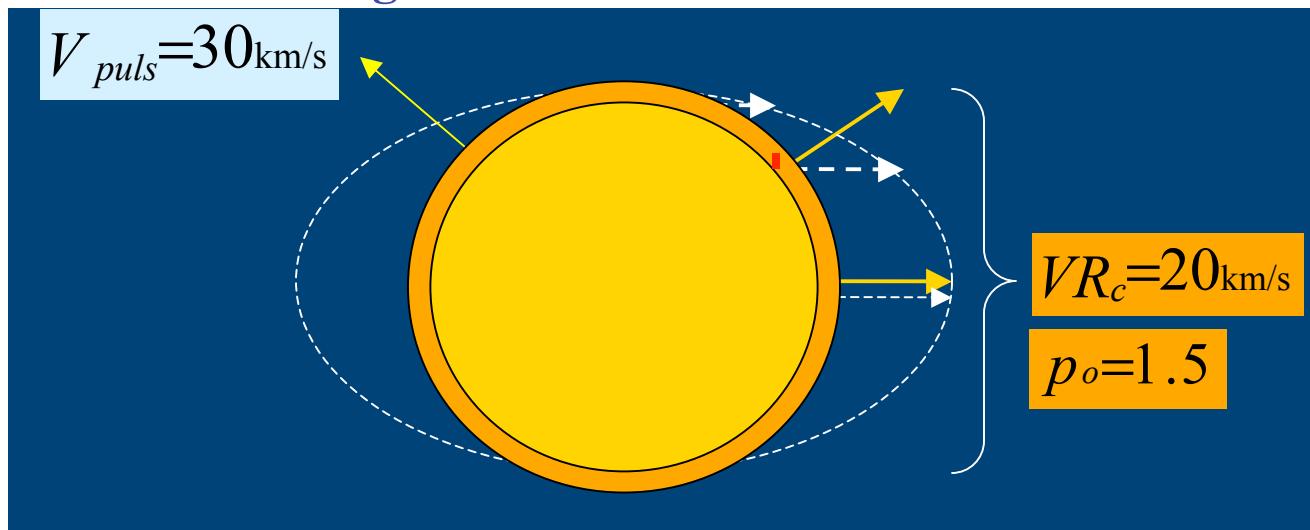
# La définition de la vitesse radiale $V_{rad}$ et de $p_o$ (3/3)

- Effet de la rotation



ACB  
 $u=0$   
 Dispersion Doppler  
 $\sigma_c = 0 \text{ A}$   
 Rotation  
 $V_{rot} \sin(i) = 10 \text{ km/s}$

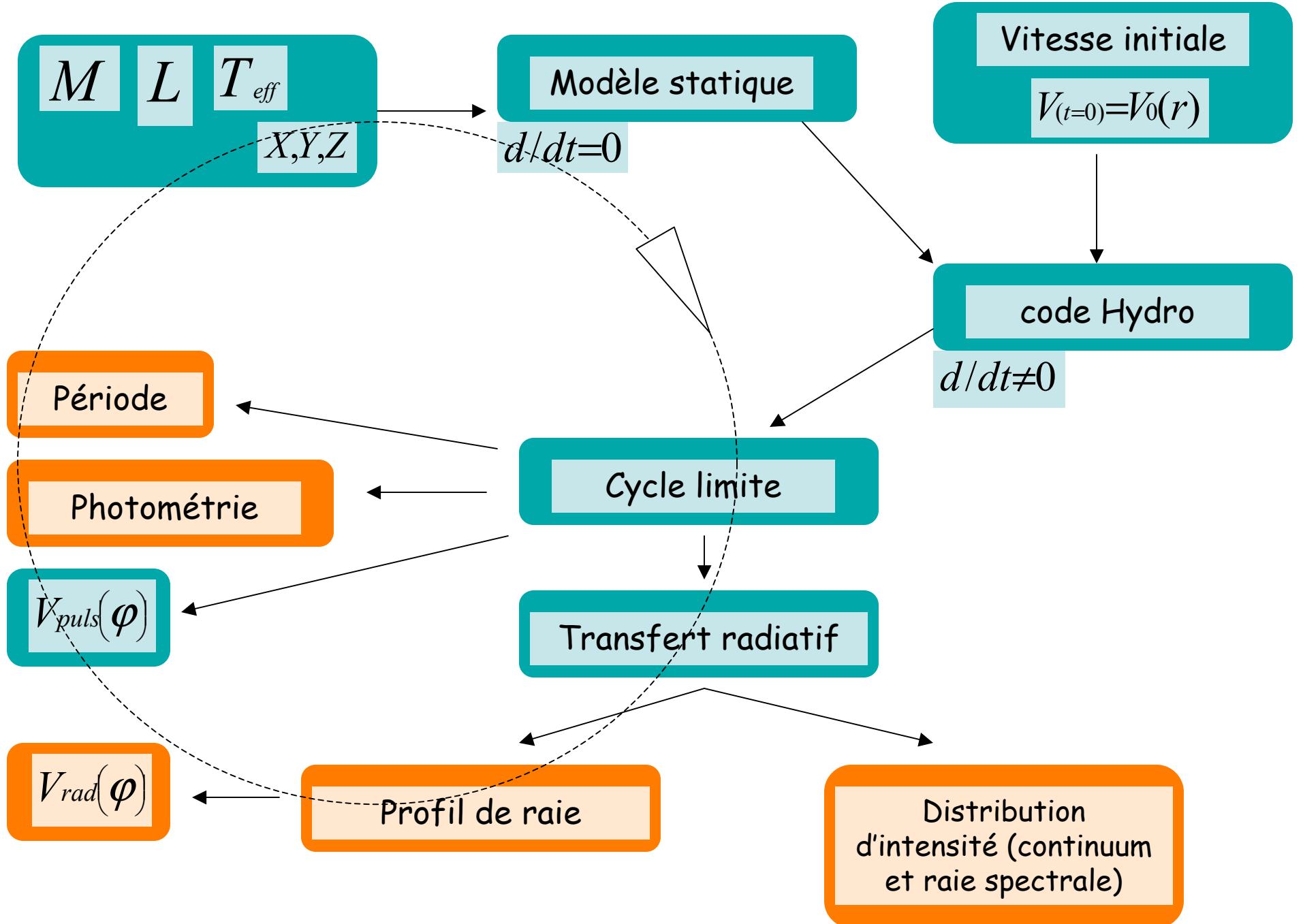
- Effet de la largeur de la raie  $\sigma_c$



ACB  
 $u=0$   
 Dispersion Doppler  
 $\sigma_c = 0.30 \text{ A}$   
 Rotation  
 $V_{rot} \sin(i) = 0 \text{ km/s}$

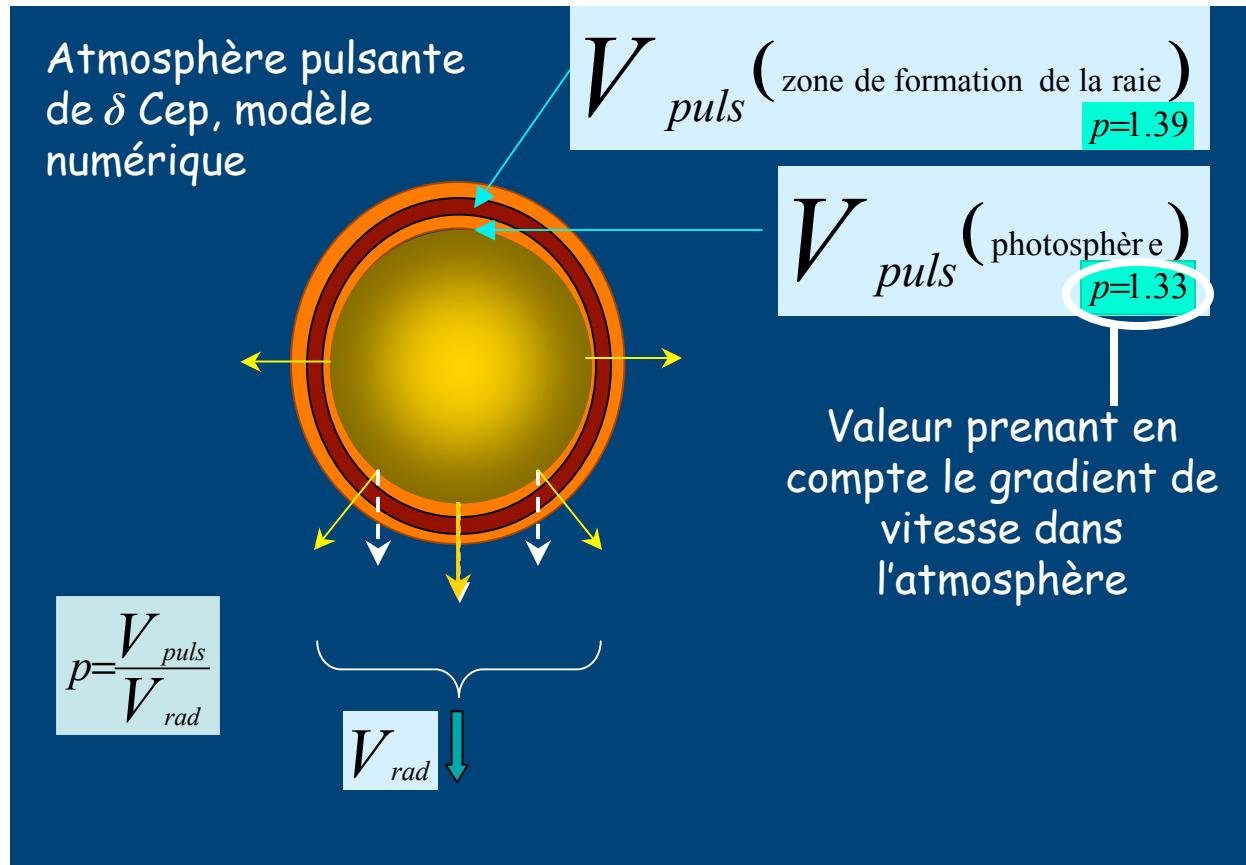
La Vitesse Radiale déduite de la méthode du premier moment est indépendante de la largeur de la raie et de la rotation

Nicolas NARDETTO – Collège de France, 2009 17



# Effet de la dynamique atmosphérique

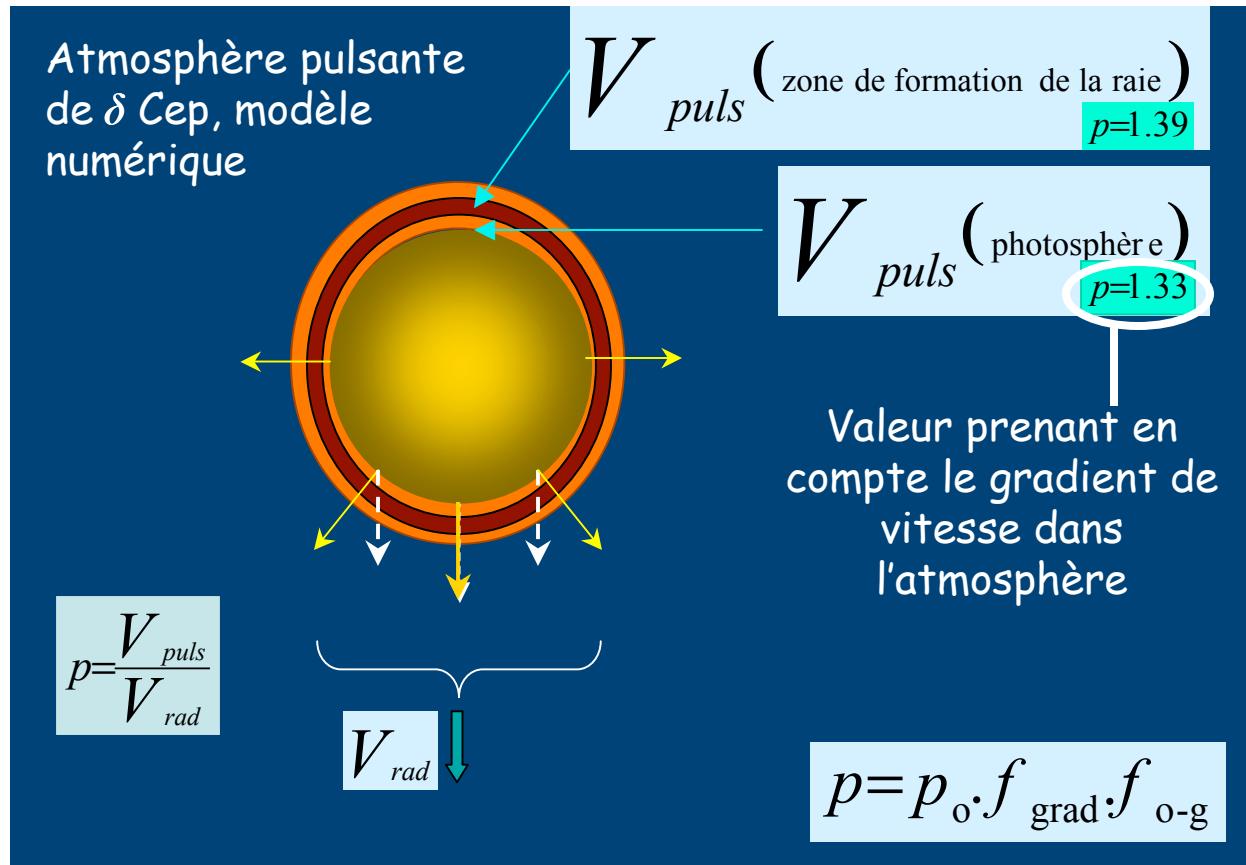
Self consistent modelling of the projection factor  
for interferometric distance determination  
N. Nardetto, A. Fokin, D. Mourard, Ph. Mathias,  
P. Kervella, D. Bersier, 2004, A&A, 428, 131



Surestimation des distances de 6% avec VINCI :  
résultat confirmé par le HST + CHARA (Mérand et al. 2005)

# Effet de la dynamique atmosphérique

Self consistent modelling of the projection factor  
for interferometric distance determination  
N. Nardetto, A. Fokin, D. Mourard, Ph. Mathias,  
P. Kervella, D. Bersier, 2004, A&A, 428, 131



Surestimation des distances de 6% avec VINCI :  
résultat confirmé par le HST + CHARA (Mérand et al. 2005)

# Observations HARPS de 10 Céphéides (P=3j à P=42j)

300 spectres

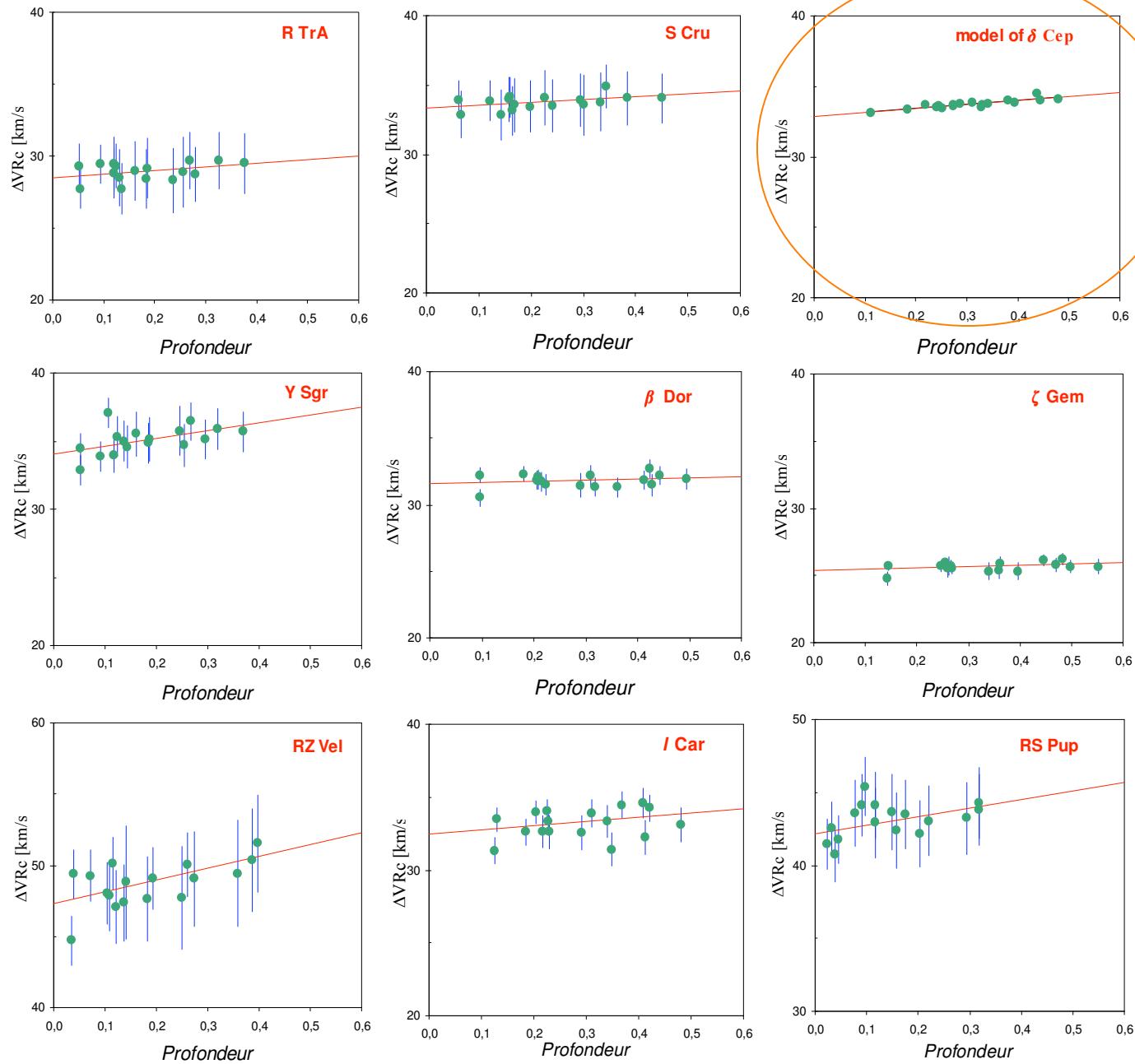
Des milliers  
de raies

17 retenues

Name	Wavelength (Å)
Fe I	4683.560
Fe I	4896.439
Fe I	5054.643
Ni I	5082.339
Fe I	5367.467
Fe I	5373.709
Fe I	5383.369
Ti II	5418.751
Fe I	5576.089
Fe I	5862.353
Fe I	6024.058
Fe I	6027.051
Fe I	6056.005
Si I	6155.134
Fe I	6252.555
Fe I	6265.134
Fe I	6336.824

$$\Delta VR_c = a_0 D + b_0$$

# Observations HARPS de 10 Céphéides (P=3j à P=42j)



300 spectres

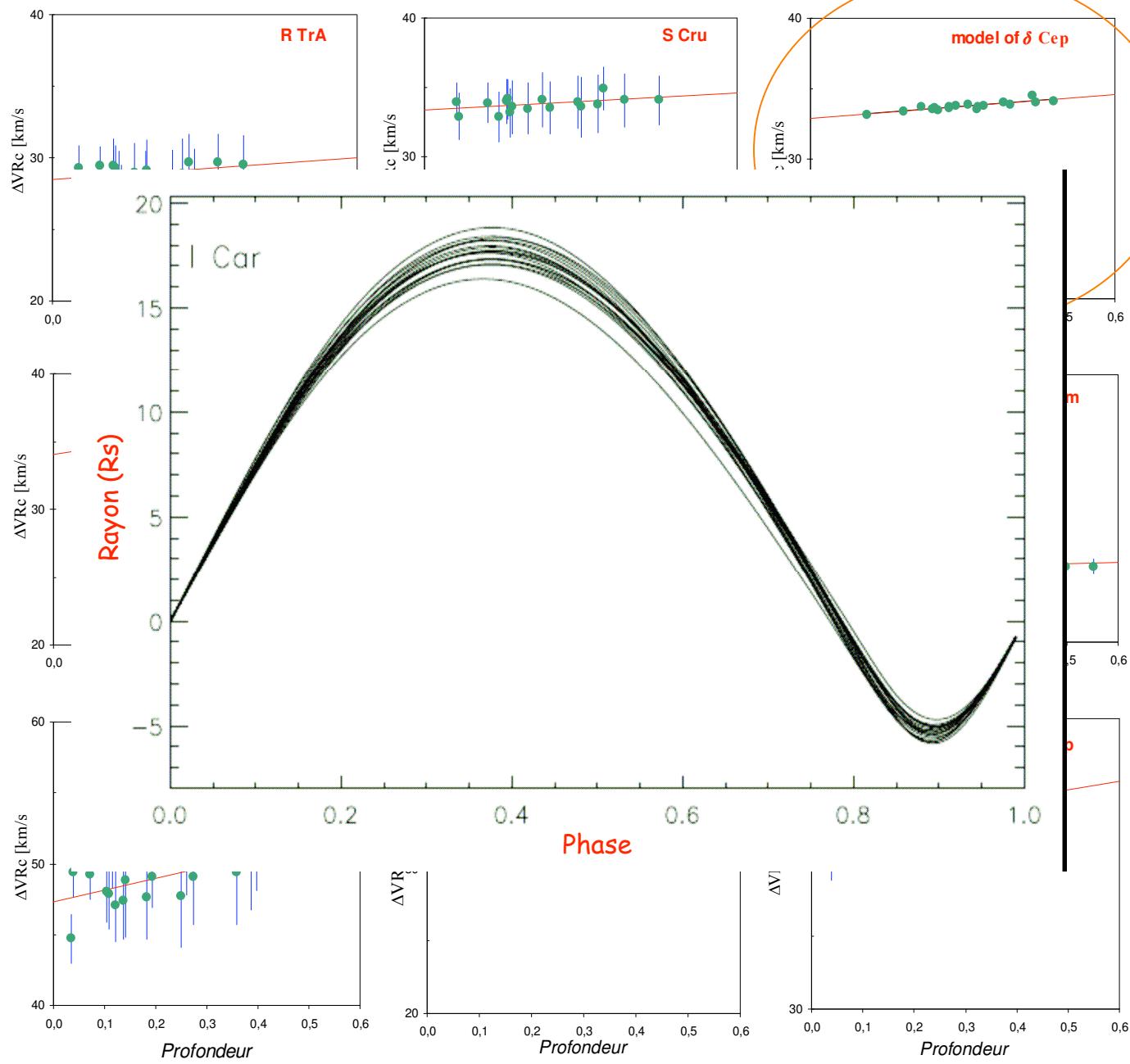
Des milliers  
de raies

17 retenues

Name	Wavelength (Å)
Fe I	4683.560
Fe I	4896.439
Fe I	5054.643
Ni I	5082.339
Fe I	5367.467
Fe I	5373.709
Fe I	5383.369
Ti II	5418.751
Fe I	5576.089
Fe I	5862.353
Fe I	6024.058
Fe I	6027.051
Fe I	6056.005
Si I	6155.134
Fe I	6252.555
Fe I	6265.134
Fe I	6336.824

$$\Delta VR_c = a_0 D + b_0$$

# Observations HARPS de 10 Céphéides (P=3j à P=42j)



300 spectres

Des milliers  
de raies

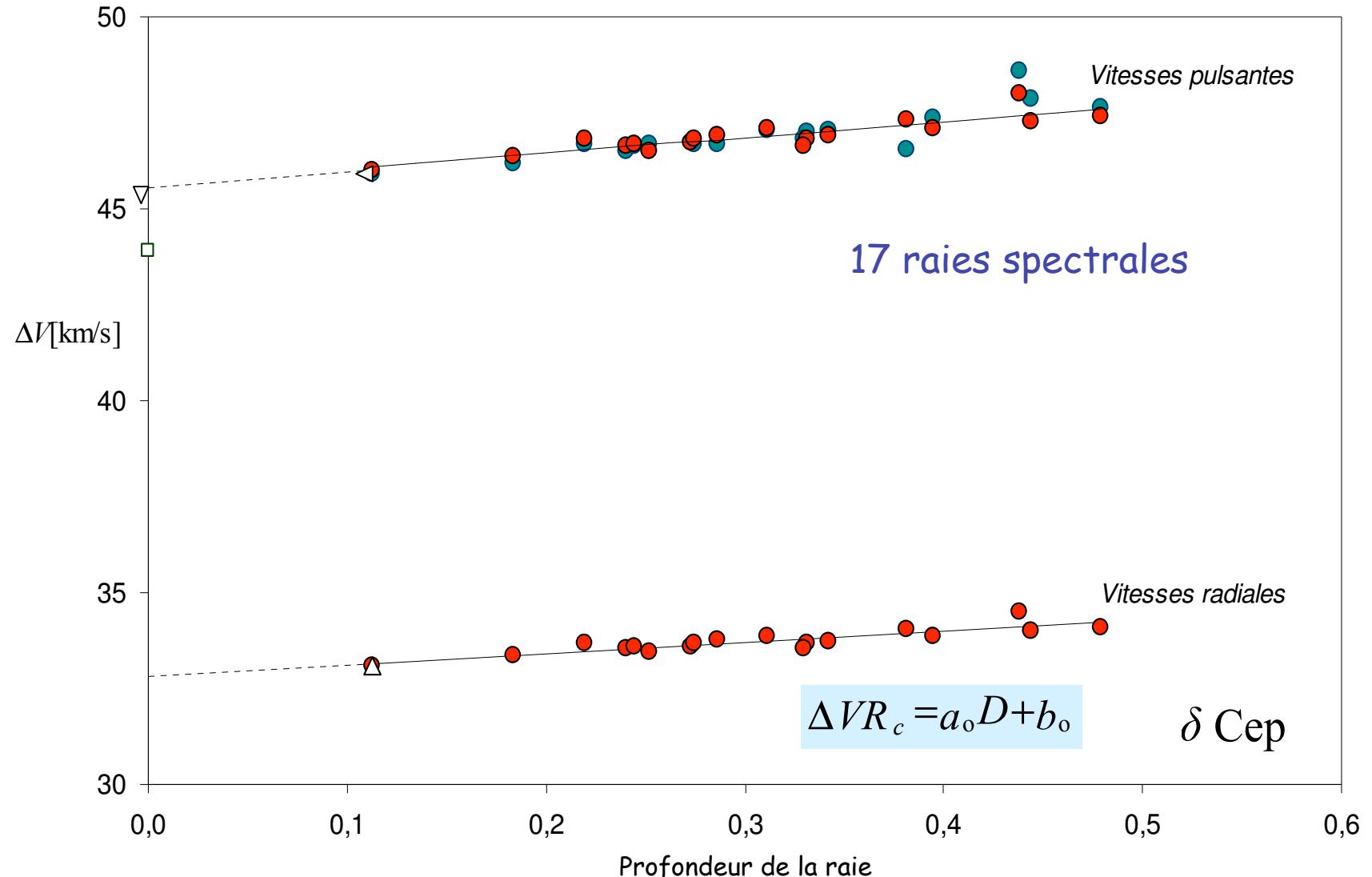
17 retenues

Name	Wavelength (Å)
Fe I	4683.560
Fe I	4896.439
Fe I	5054.643
Ni I	5082.339
Fe I	5367.467
Fe I	5373.709
Fe I	5383.369
Ti II	5418.751
Fe I	5576.089
Fe I	5862.353
Fe I	6024.058
Fe I	6027.051
Fe I	6056.005
Si I	6155.134
Fe I	6252.555
Fe I	6265.134
Fe I	6336.824

$$\Delta VR_c = a_0 D + b_0$$

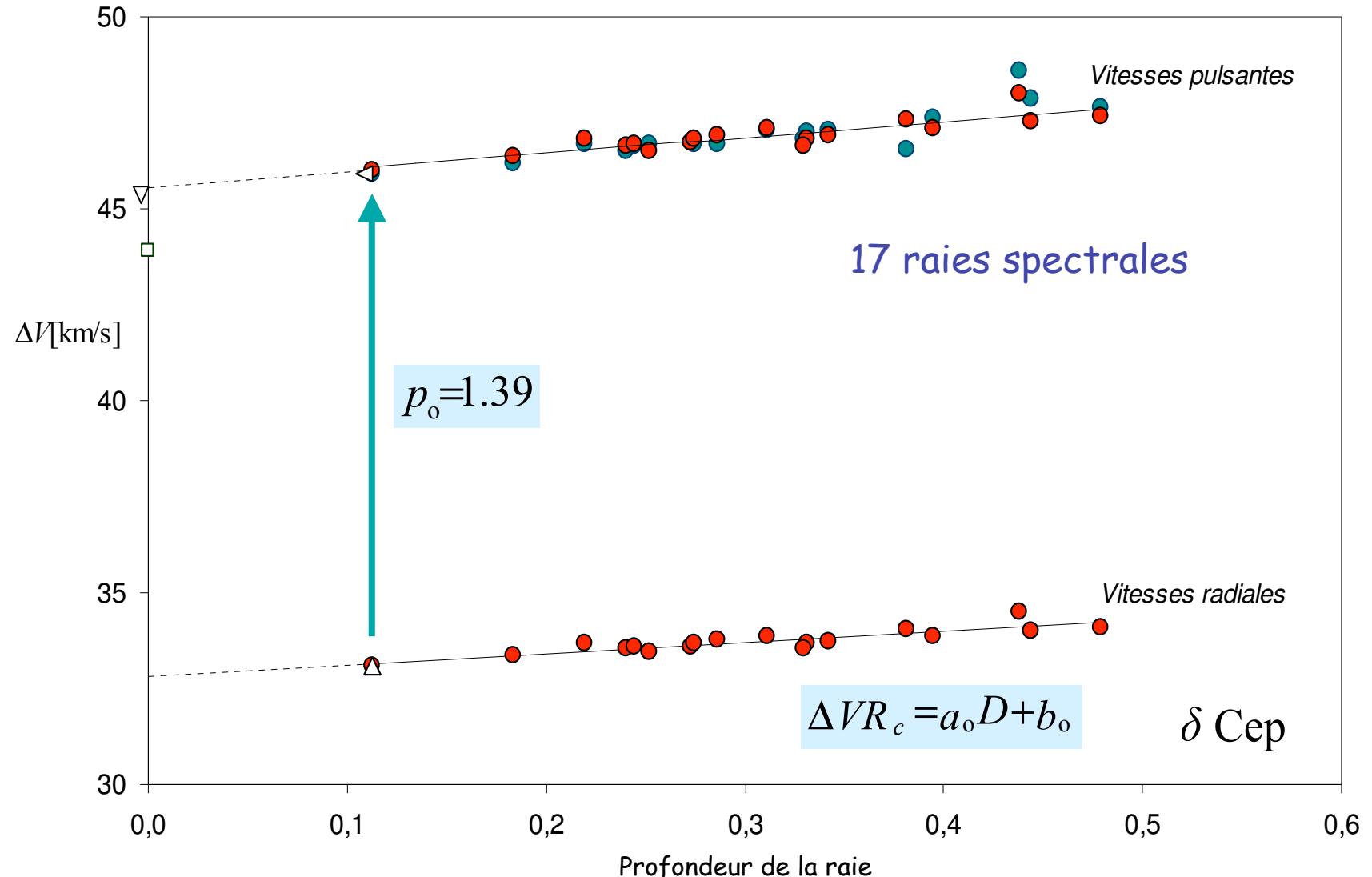
# Décomposition du facteur de projection

$$p = p_o \cdot f_{\text{grad}} \cdot f_{\text{o-g}}$$



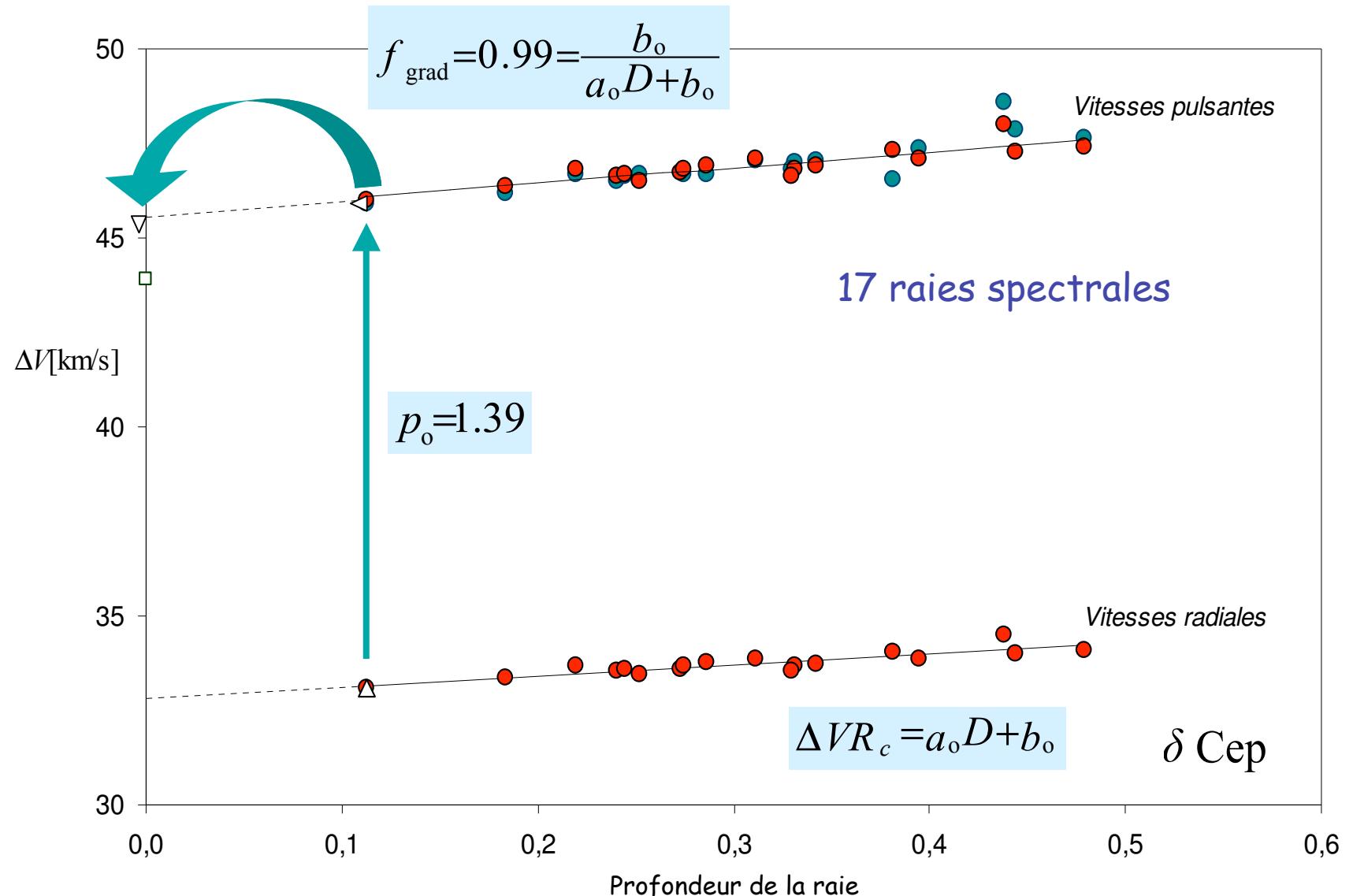
# Décomposition du facteur de projection

$$p = p_o \cdot f_{\text{grad}} \cdot f_{\text{o-g}}$$



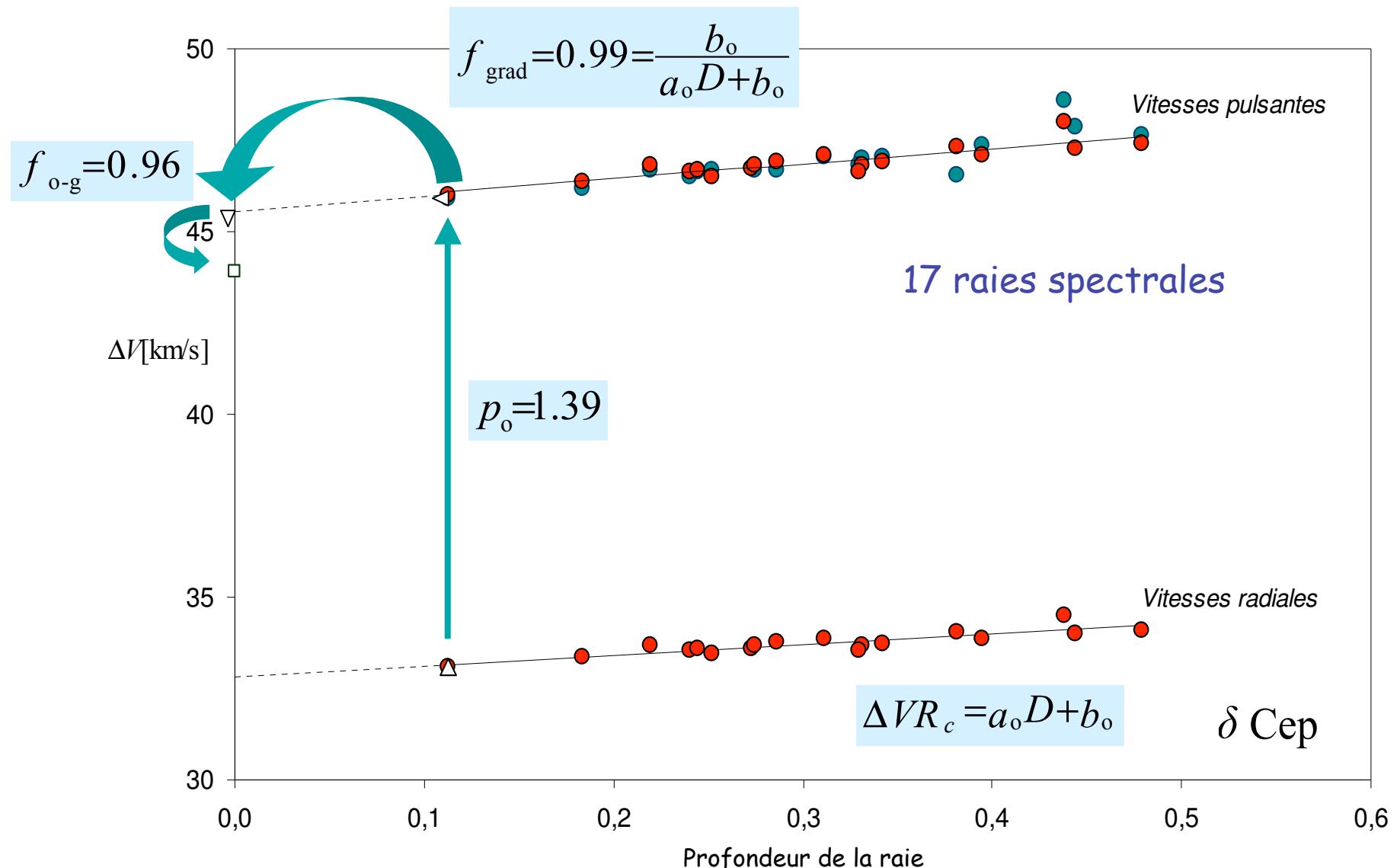
# Décomposition du facteur de projection

$$p = p_o \cdot f_{\text{grad}} \cdot f_{\text{o-g}}$$



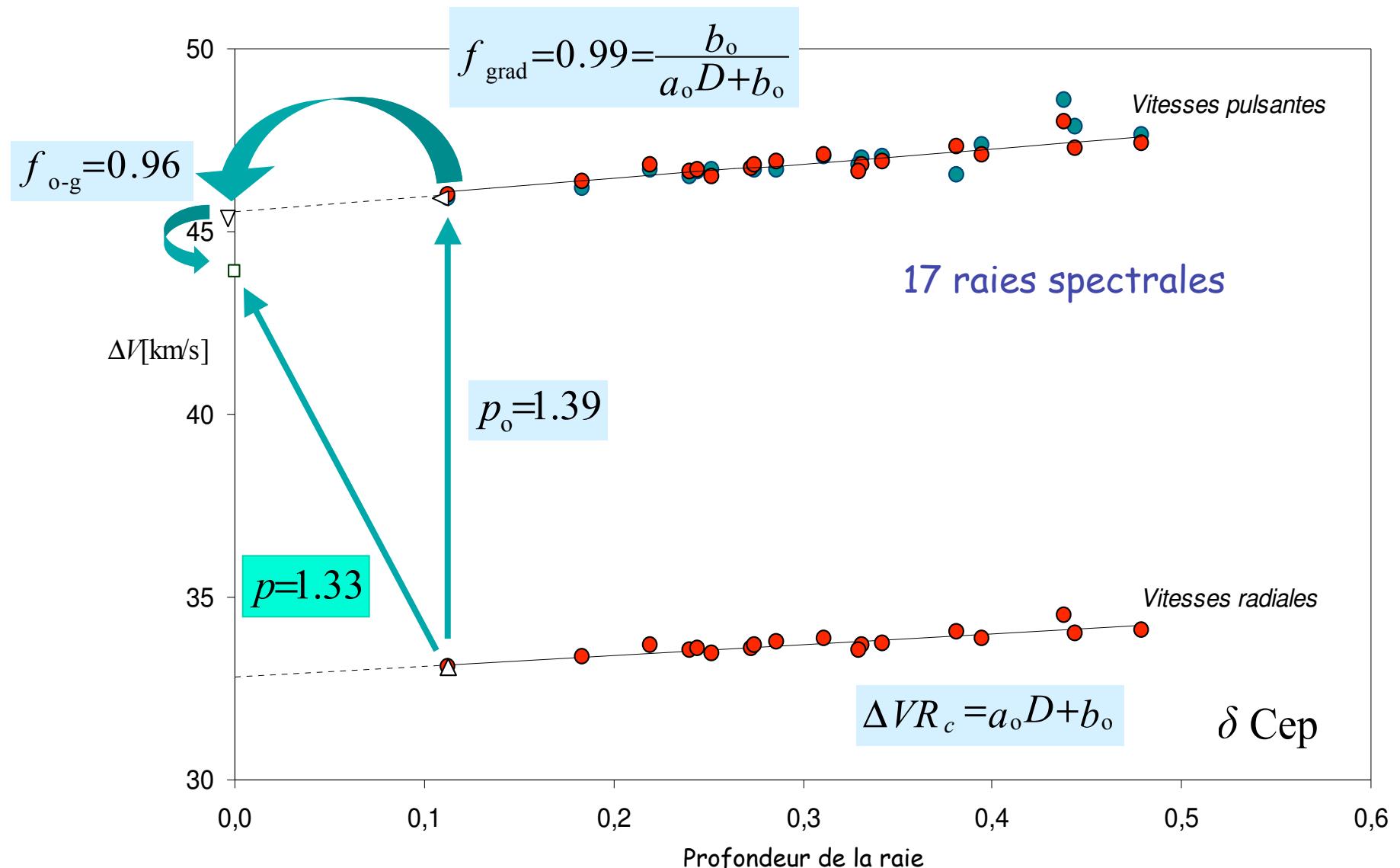
# Décomposition du facteur de projection

$$p = p_o \cdot f_{\text{grad}} \cdot f_{\text{o-g}}$$



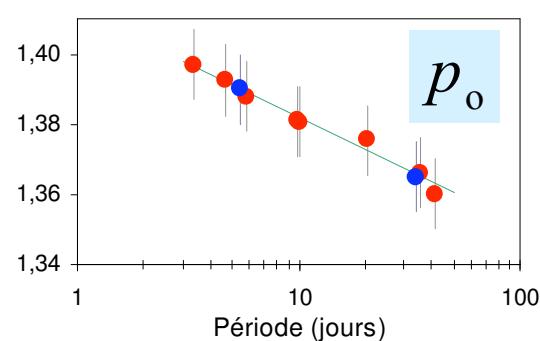
# Décomposition du facteur de projection

$$p = p_o \cdot f_{\text{grad}} \cdot f_{\text{o-g}}$$

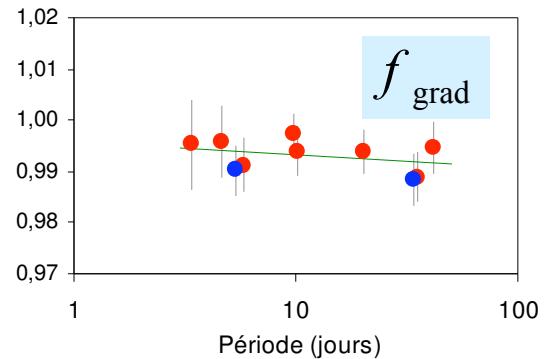


# La relation Pp

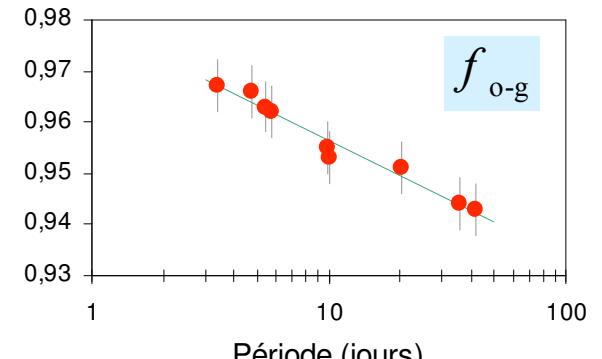
High resolution spectroscopy for Cepheids distance determination II. A period-projection factor relation  
N. Nardetto, D. Mourard, Ph. Mathias, A. Fokin, D. Gillet, 2007, A&A, 471, 661



Modélisation  
Géométrique



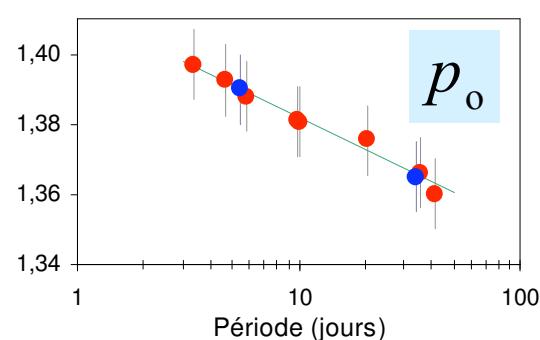
Observations HARPS



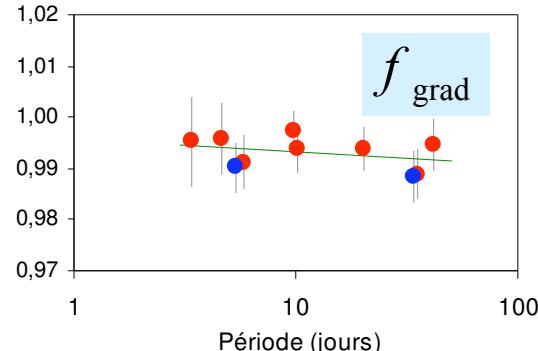
Modélisation  
Hydrodynamique

# La relation Pp

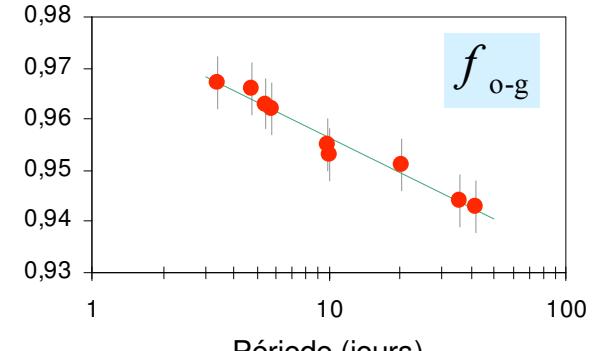
High resolution spectroscopy for Cepheids distance determination II. A period-projection factor relation  
 N. Nardetto, D. Mourard, Ph. Mathias, A. Fokin, D. Gillet, 2007, A&A, 471, 661



Modélisation  
Géométrique

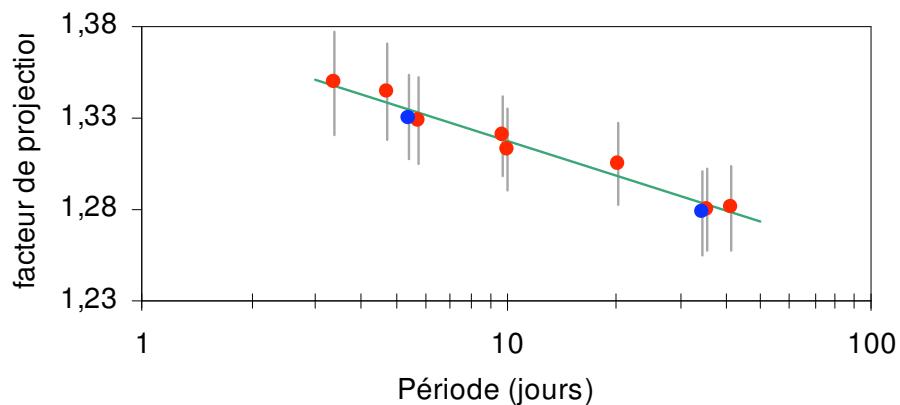


Observations HARPS



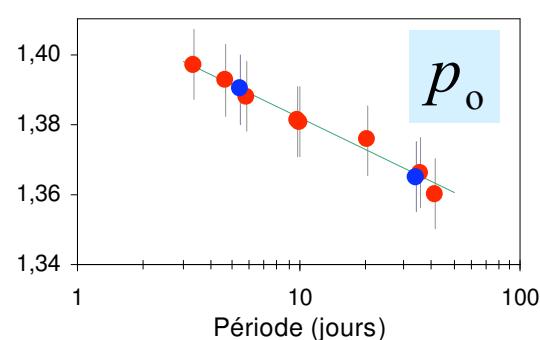
Modélisation  
Hydrodynamique

## FeI 4896A

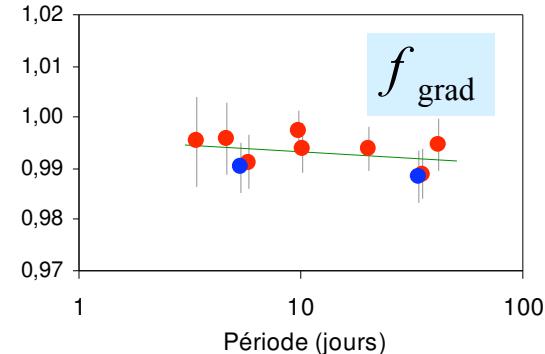


# La relation Pp

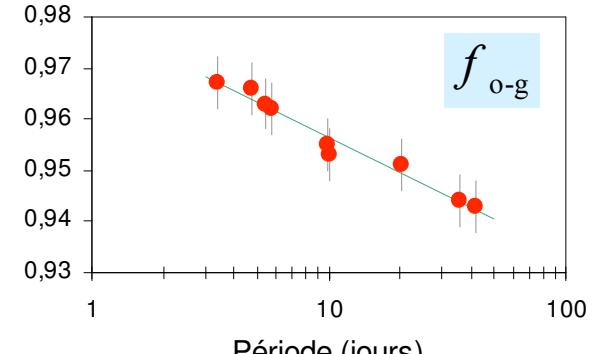
High resolution spectroscopy for Cepheids distance determination II. A period-projection factor relation  
 N. Nardetto, D. Mourard, Ph. Mathias, A. Fokin, D. Gillet, 2007, A&A, 471, 661



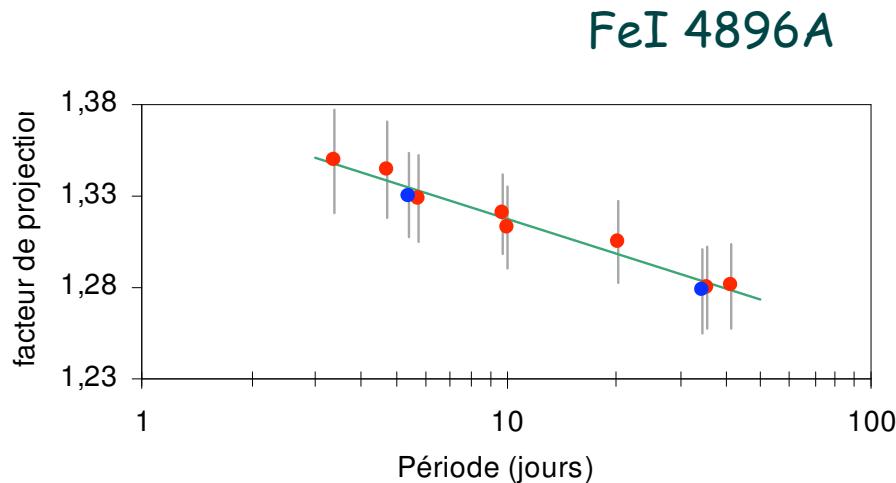
Modélisation  
Géométrique



Observations HARPS



Modélisation  
Hydrodynamique



Découverte d'une relation Pp.  
 Résultats confirmés par le HST



A new calibration of Galactic Cepheid  
 Period-Luminosity relations based on four  
 different methods

P. Fouqué, P. Arriagada, J. Storm , T.G.  
 Barnes, N. Nardetto, A. Mérand, P. Kervella,  
 W. Gieren, D. Bersier & G. F. Benedict & B.  
 E. Mc Arthur 2007, A&A, 476, 73

## Implications sur la CONSTANTE DE HUBBLE

Freedman (2001)  
Programme clef du HST

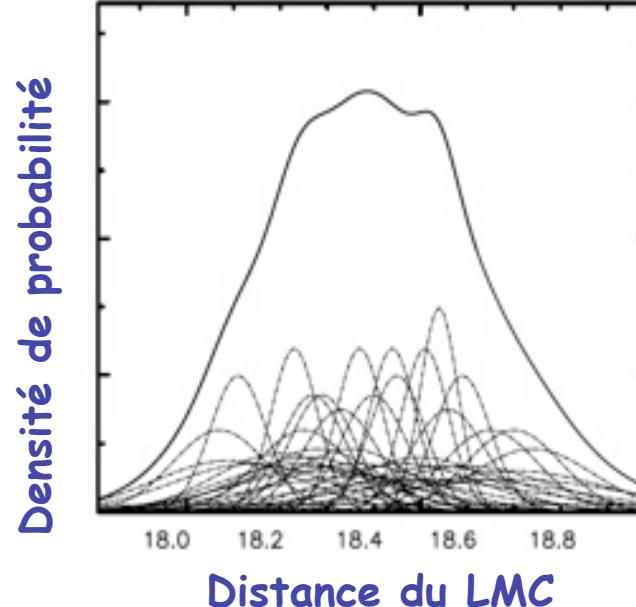


Méetallicité (pente):  
4% syst. sur  $H_0$



pente (Voie Lactée) = pente (LMC)

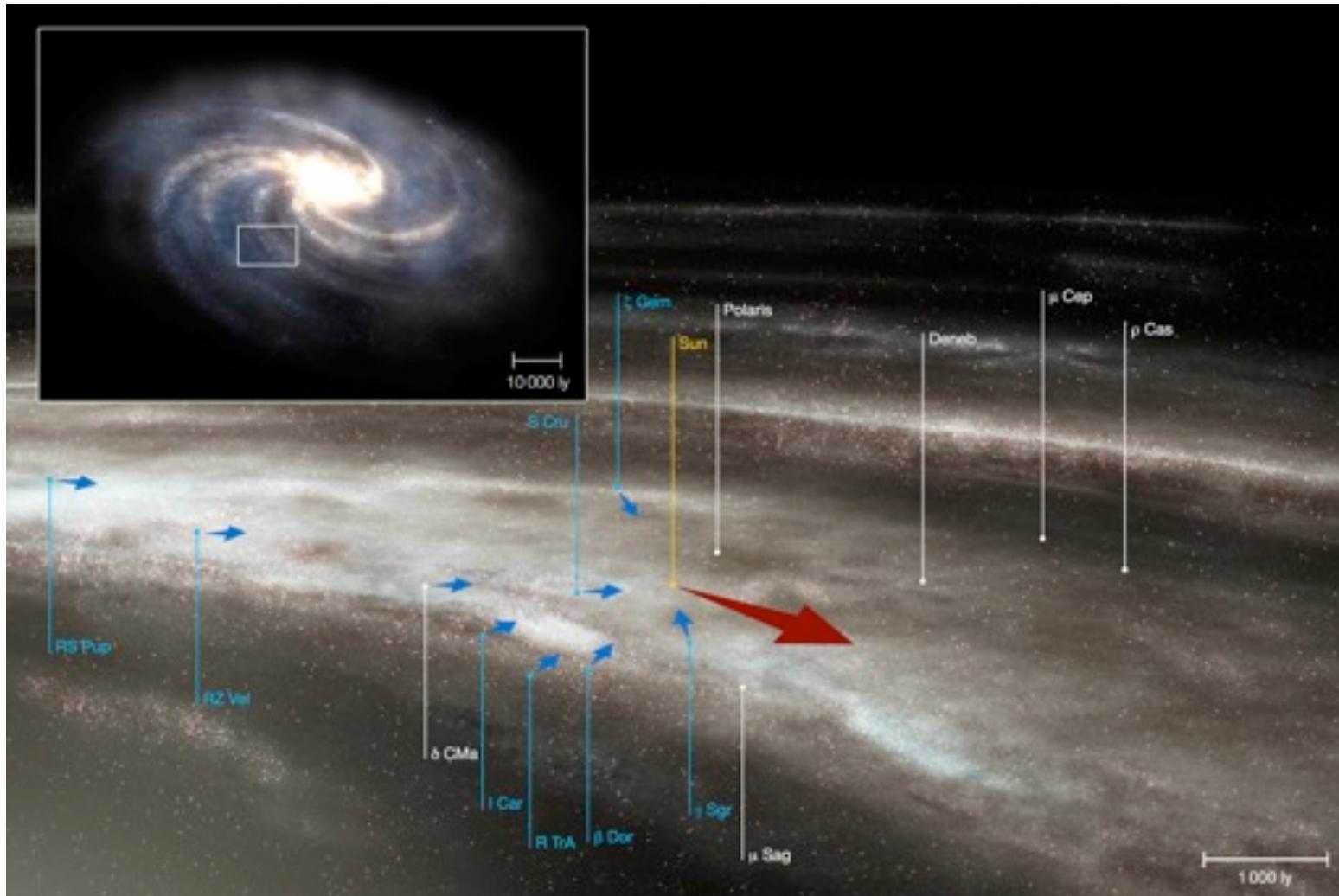
Distance du LMC  
(Point-zéro):  
5% syst. sur  $H_0$   
 $_0^{(LMC)} = 18.50 \pm 0.10 \text{ mag}$



Objectif : précision et  
exactitude de 0.01 mag

# La rotation de la Voie Lactée

Le problème du « K-term » des Céphéides : Joy (1939)



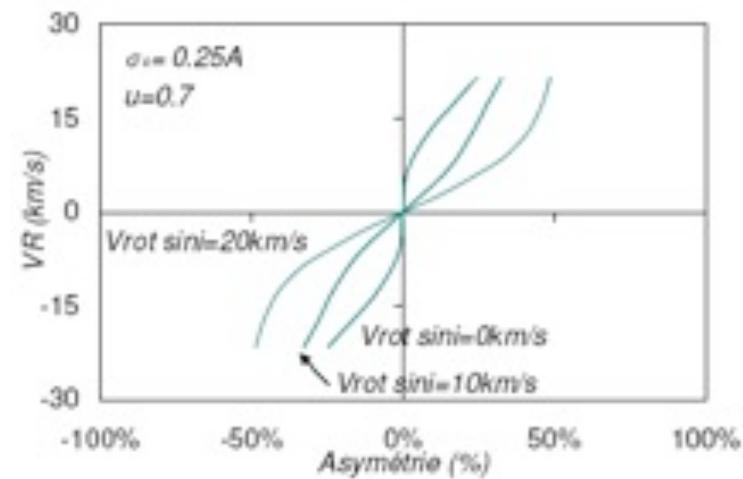
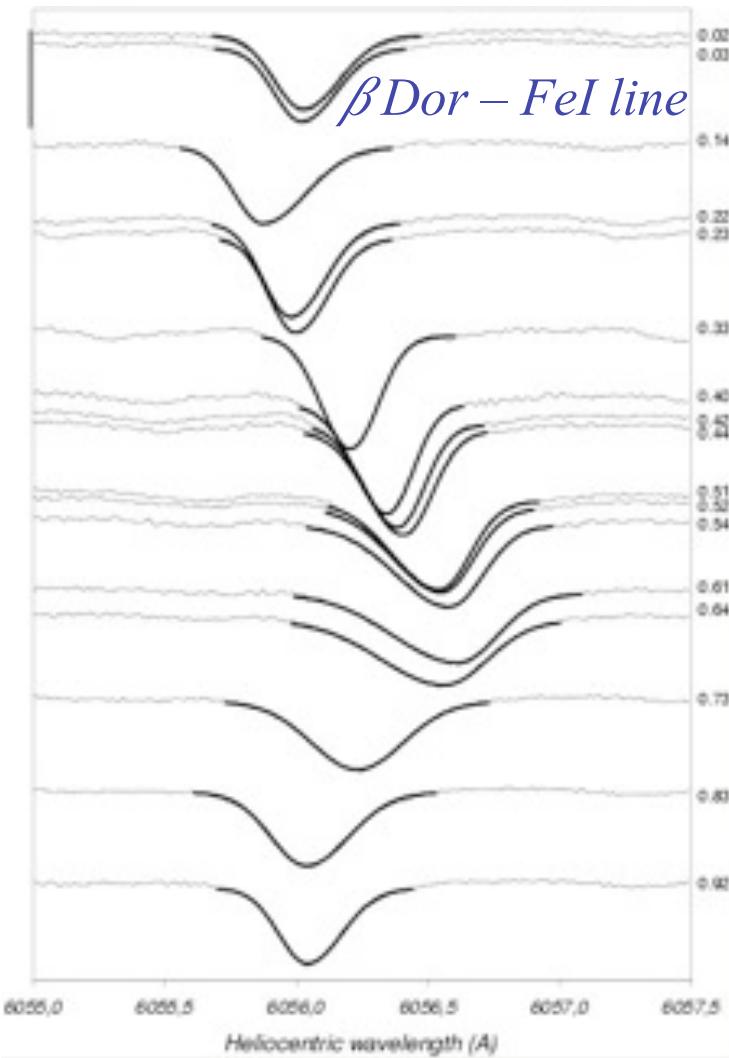
Un débat de 70 ans: Parenego (1947), Stibbs (1956), Wielen (1974), Caldwell & Coulson (1987), Moffett & Barnes (1987), Wilson et al. (1991), Pont, Mayor & Burki (1994)

Dynamique atmosphérique ou structure cinématique de la Voie Lactée

# Les " $\gamma$ - asymétries" (1/2)

High resolution spectroscopy for Cepheids distance determination I. Line asymmetry

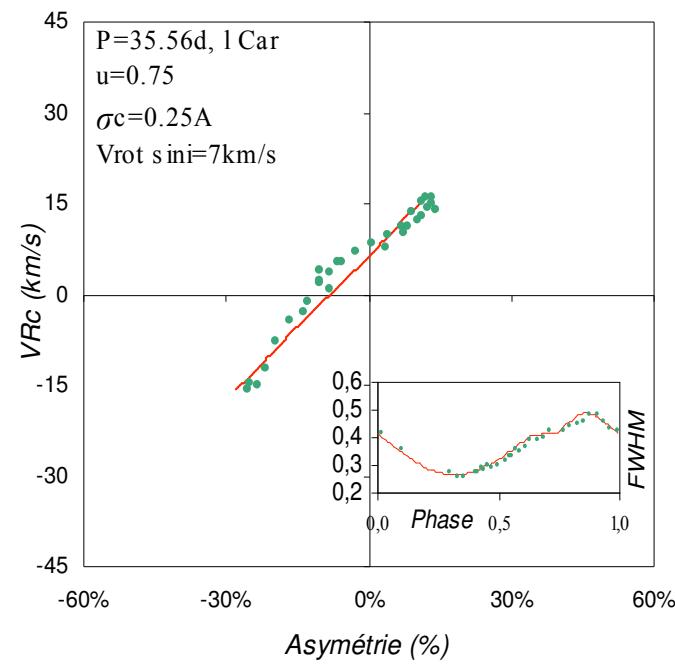
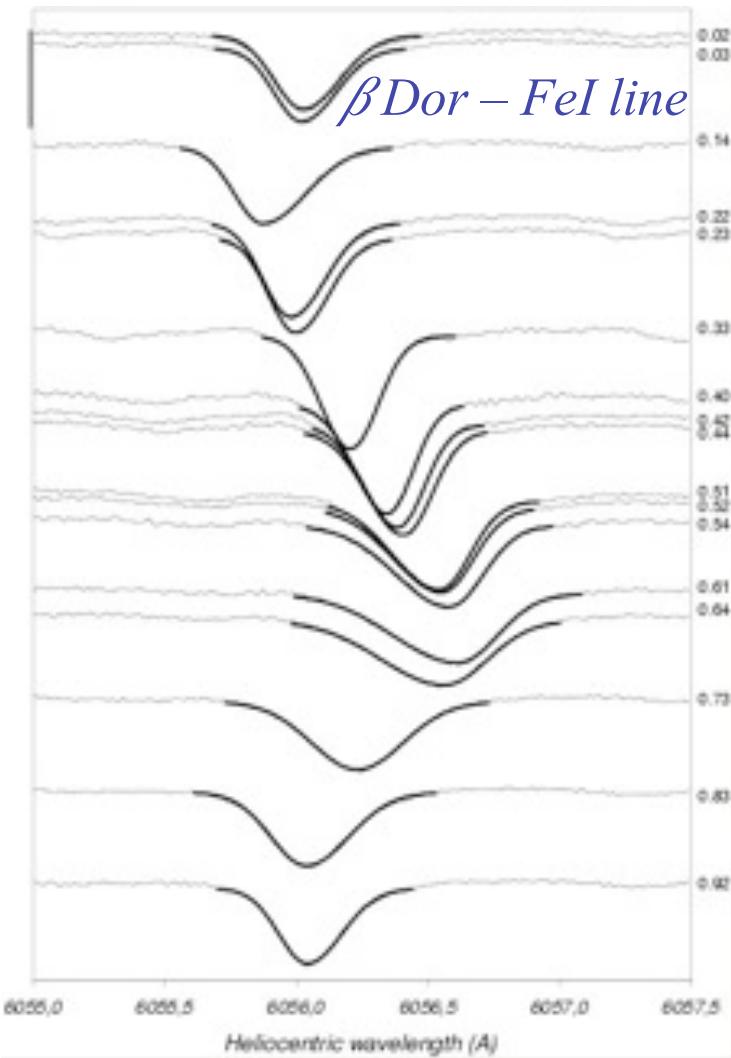
N. Nardetto, D. Mourard, P. Kervella, Ph. Mathias, A. Mérand, D. Bersier., 2005, A&A, 453, 409



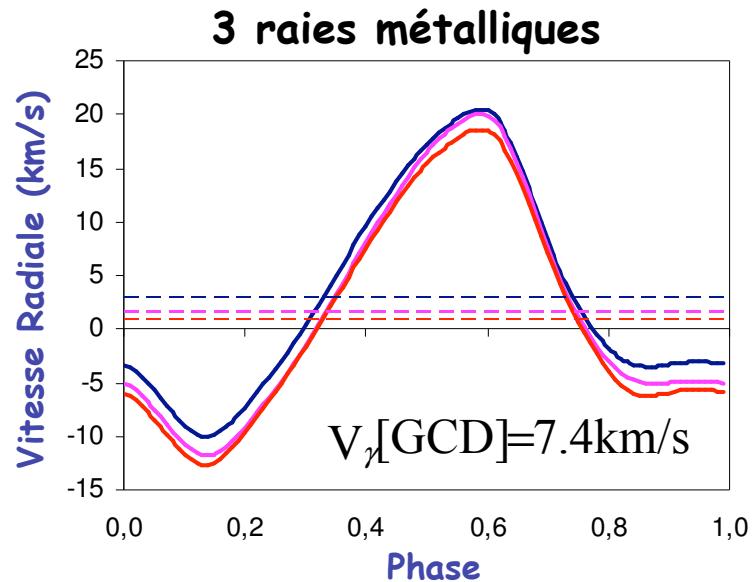
# Les " $\gamma$ - asymétries" (1/2)

High resolution spectroscopy for Cepheids distance determination I. Line asymmetry

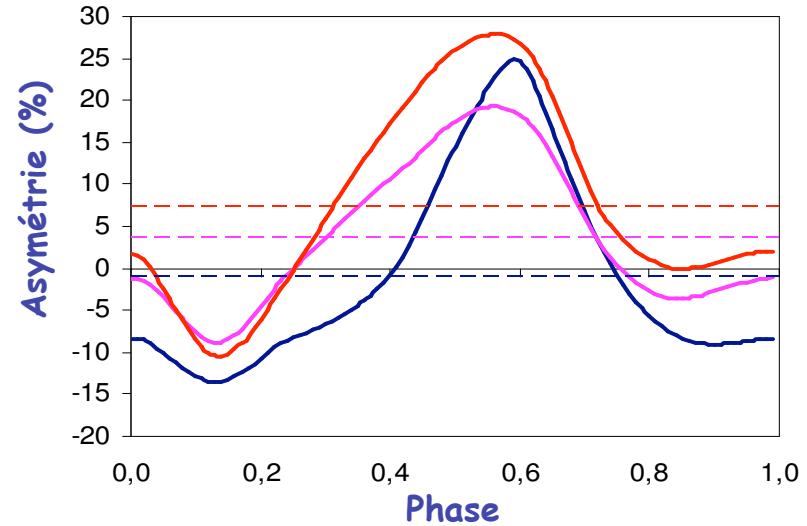
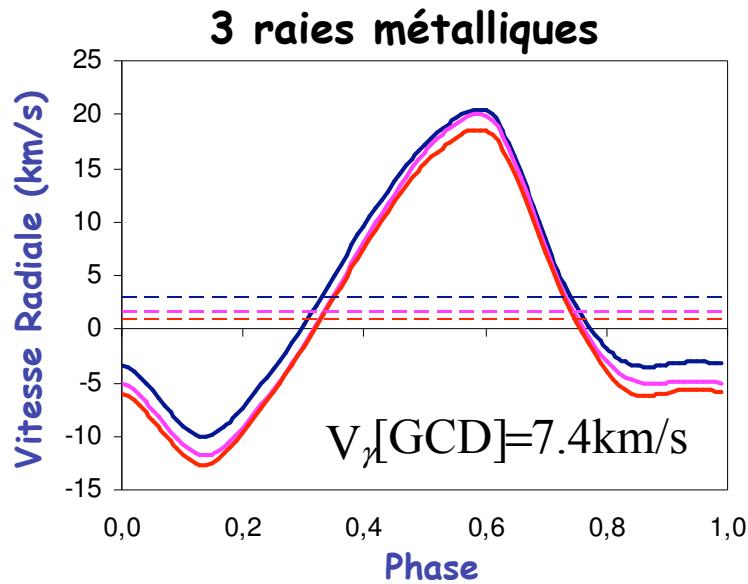
N. Nardetto, D. Mourard, P. Kervella, Ph. Mathias, A. Mérand, D. Bersier., 2005, A&A, 453, 409



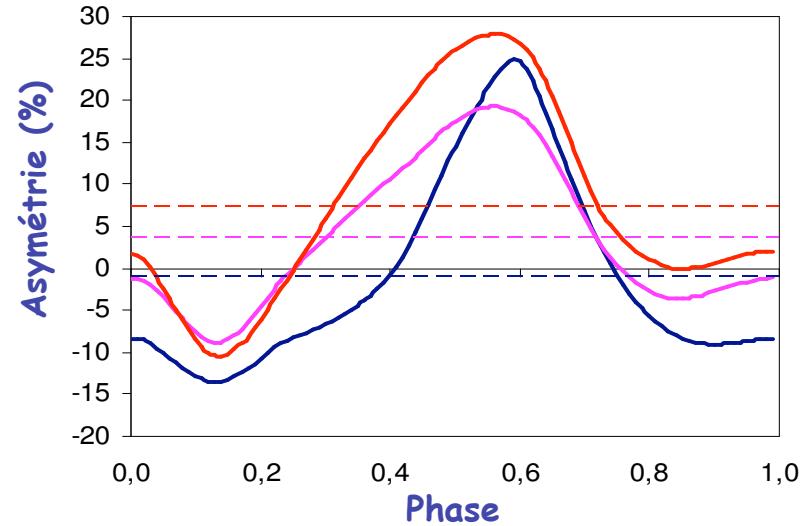
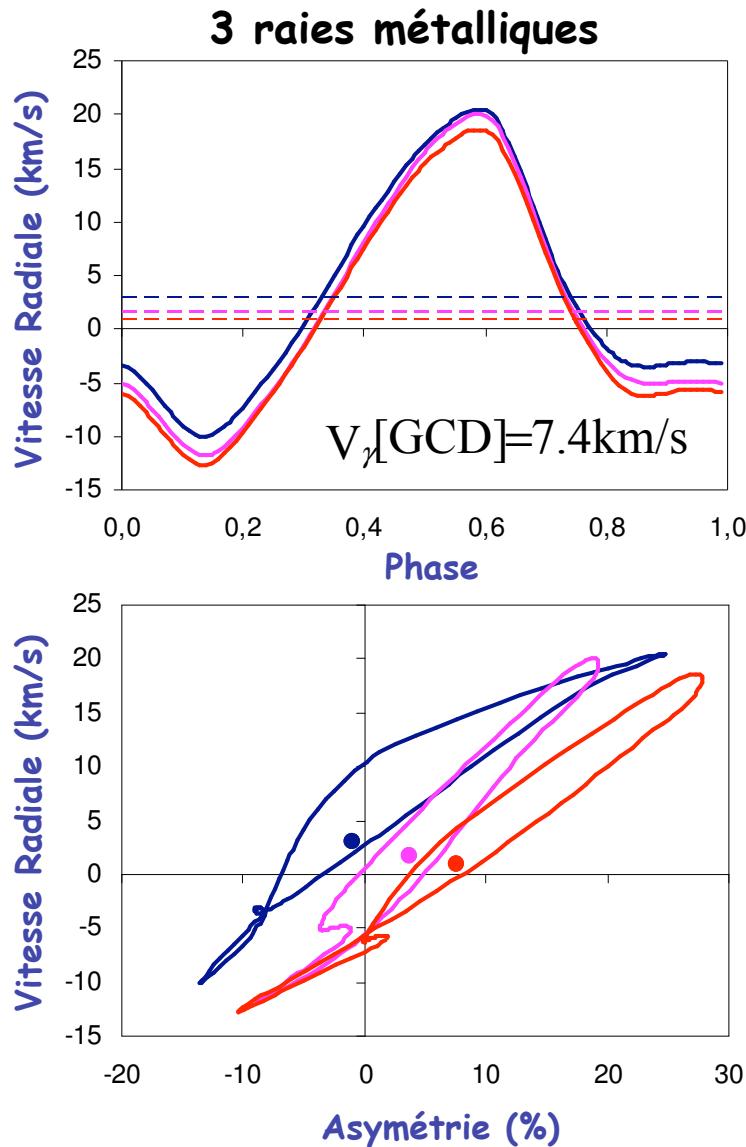
## Les " $\gamma$ - asymétries" (2/2)



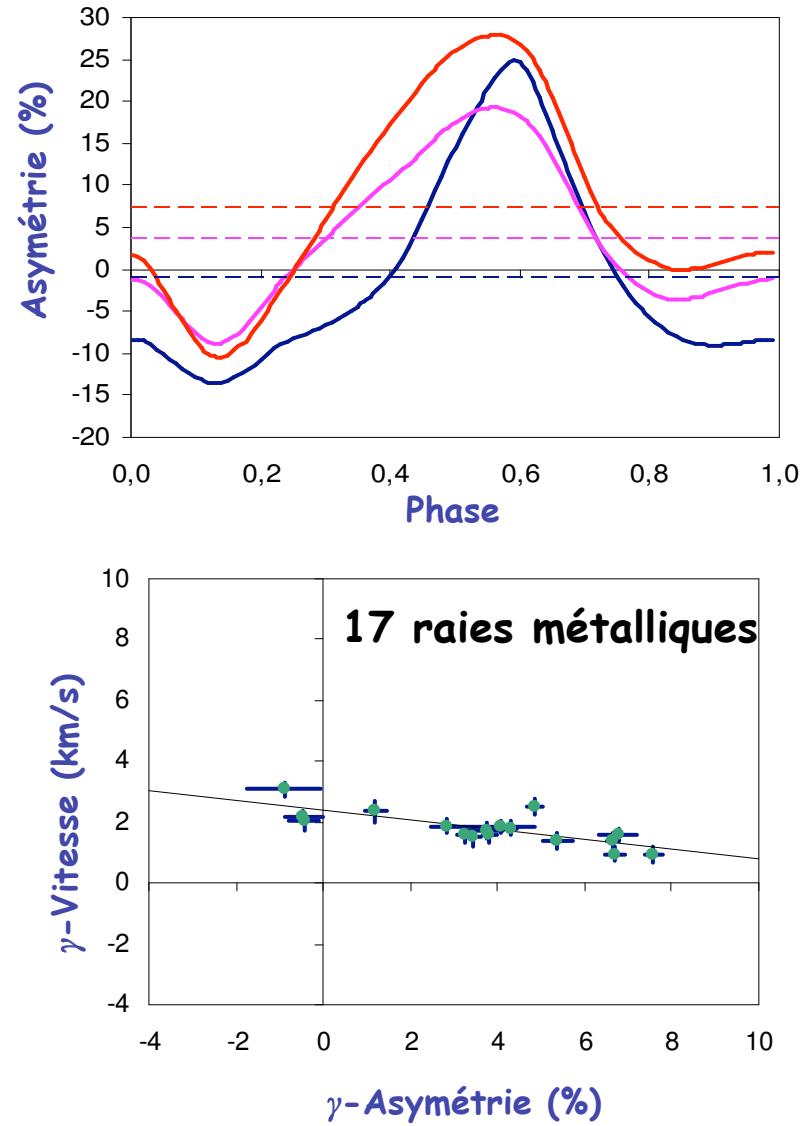
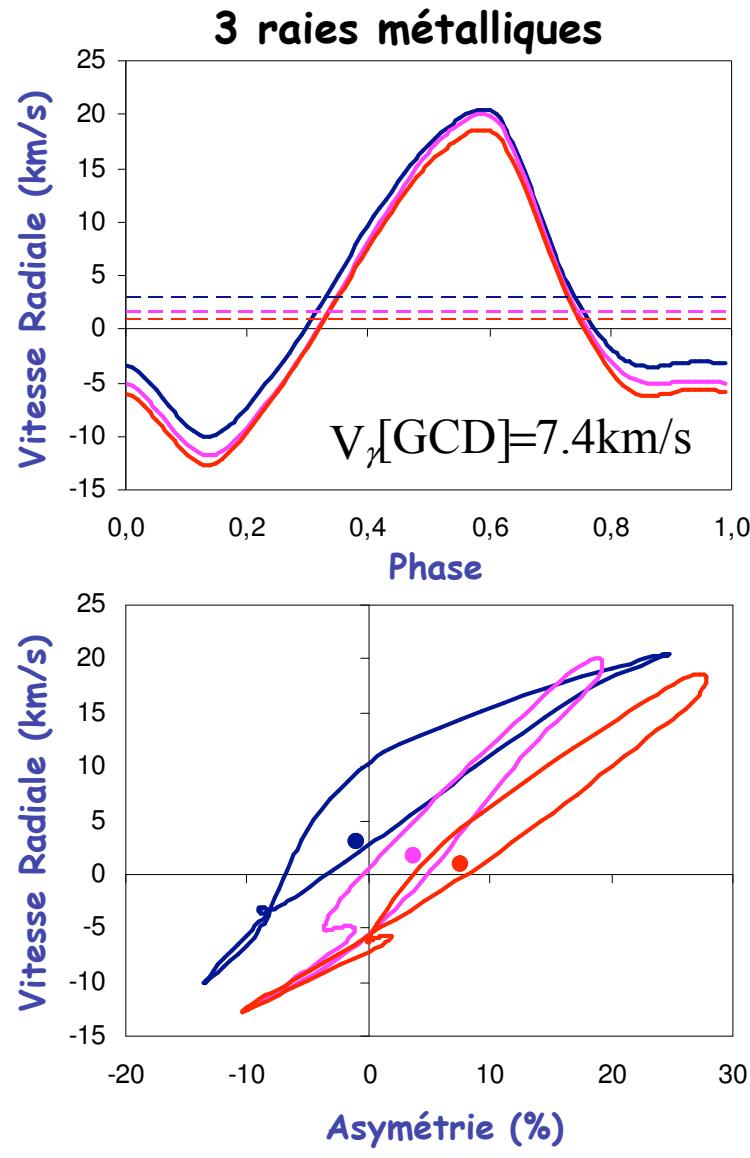
## Les " $\gamma$ - asymétries" (2/2)



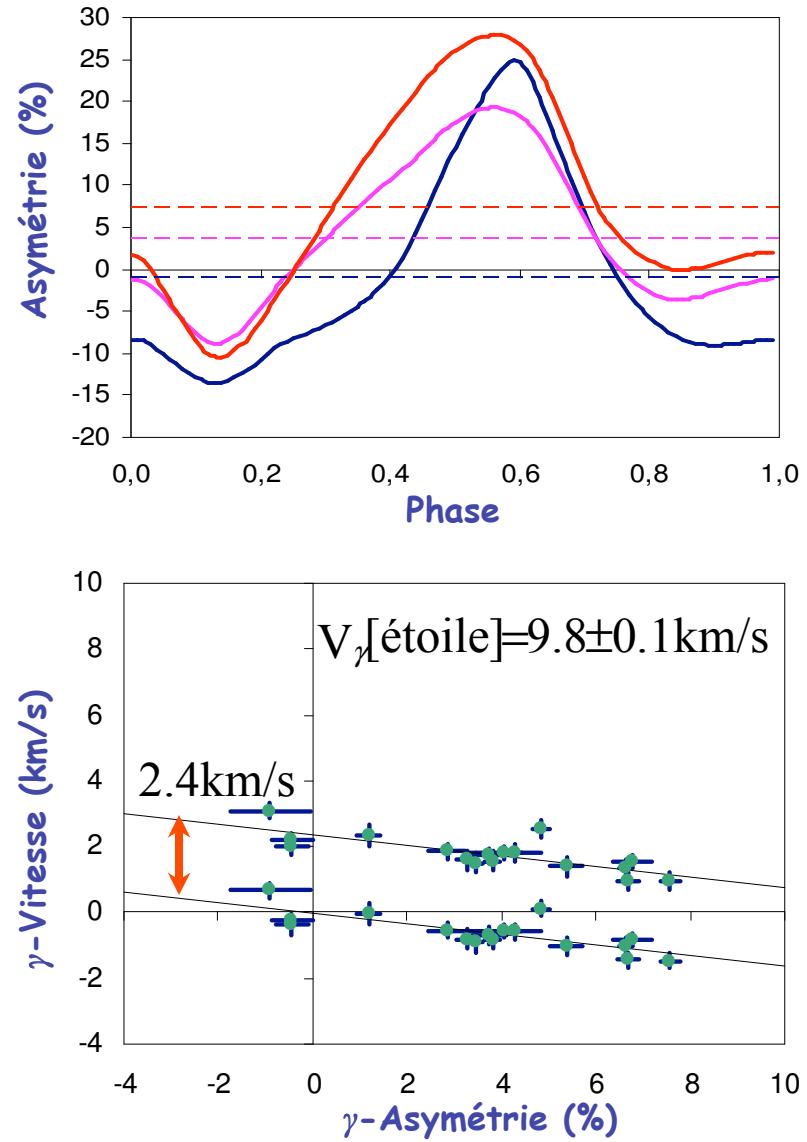
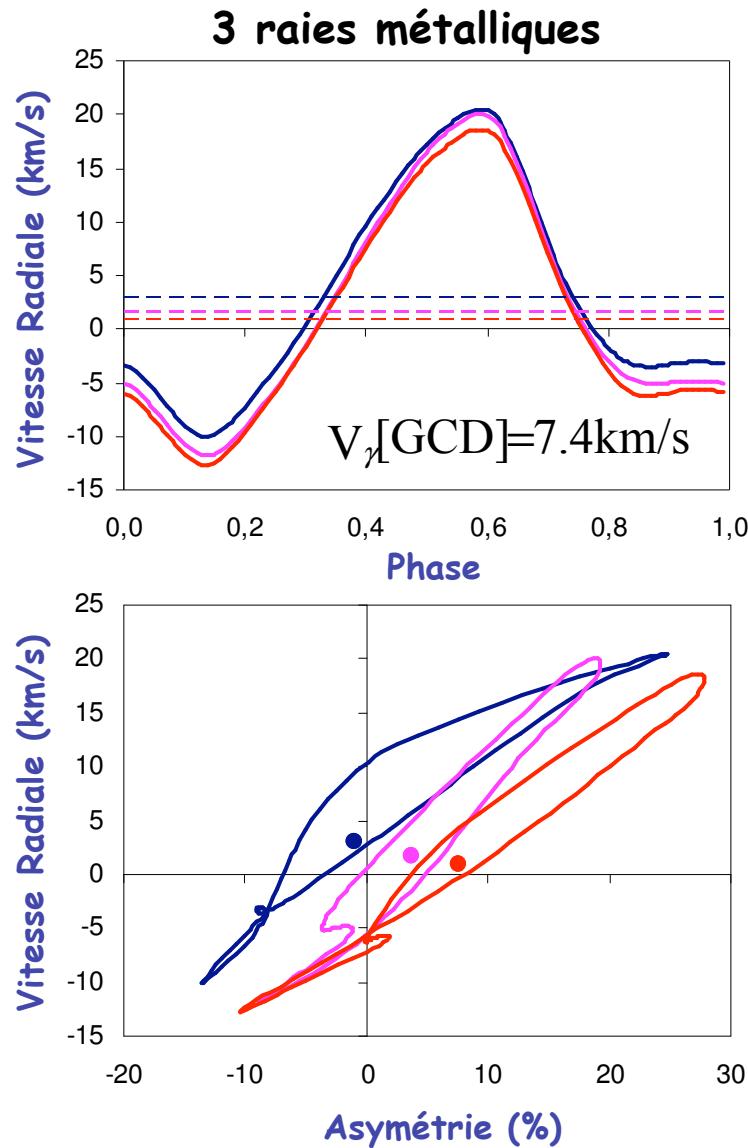
## Les " $\gamma$ - asymétries" (2/2)



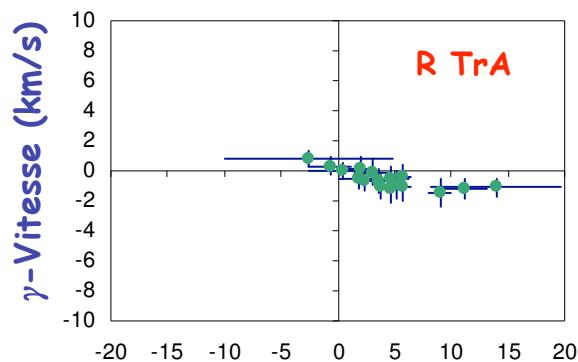
## Les " $\gamma$ - asymétries" (2/2)



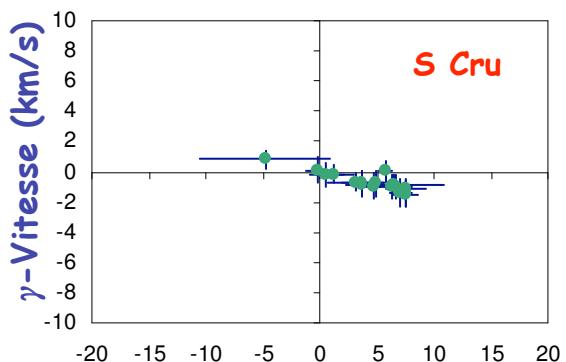
## Les " $\gamma$ - asymétries" (2/2)



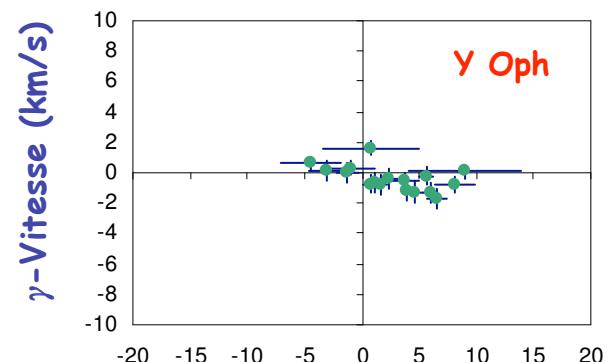
**High resolution spectroscopy for Cepheids distance determination**  
**III. A relation between  $\gamma$ -velocities and  $\gamma$ -asymmetries**  
 N. Nardetto, A. Stoekl, D. Bersier, T. Barnes, 2008, A&A, 489, 1255



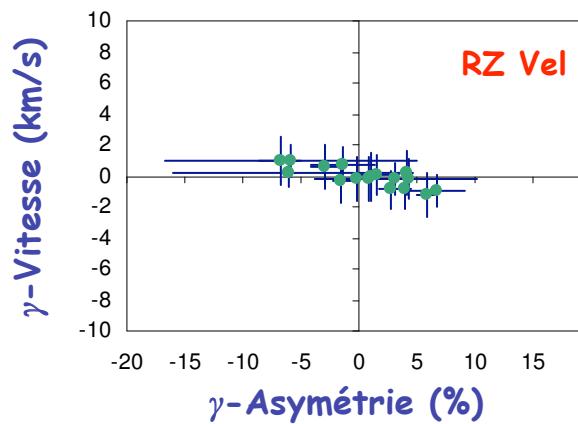
$\gamma$ -Asymétrie (%)



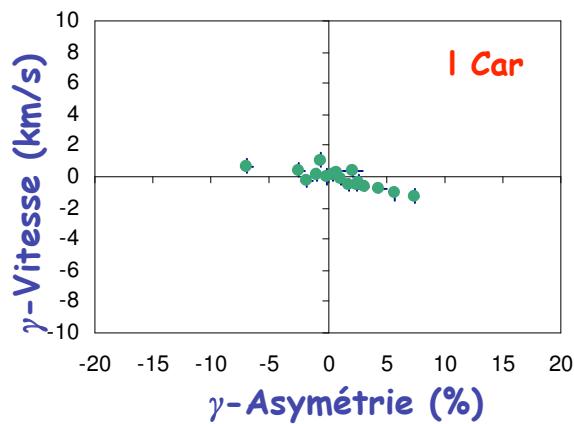
$\gamma$ -Asymétrie (%)



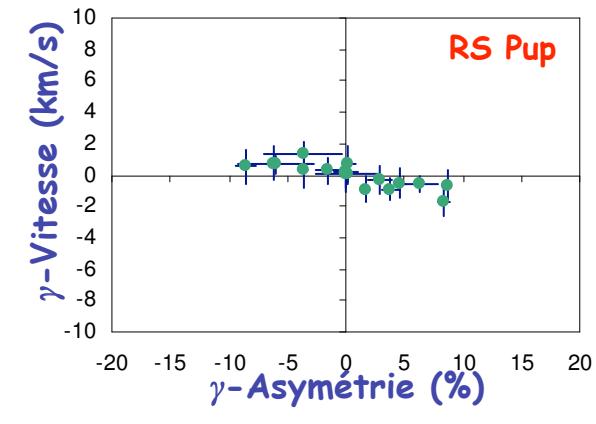
$\gamma$ -Asymétrie (%)



$\gamma$ -Asymétrie (%)

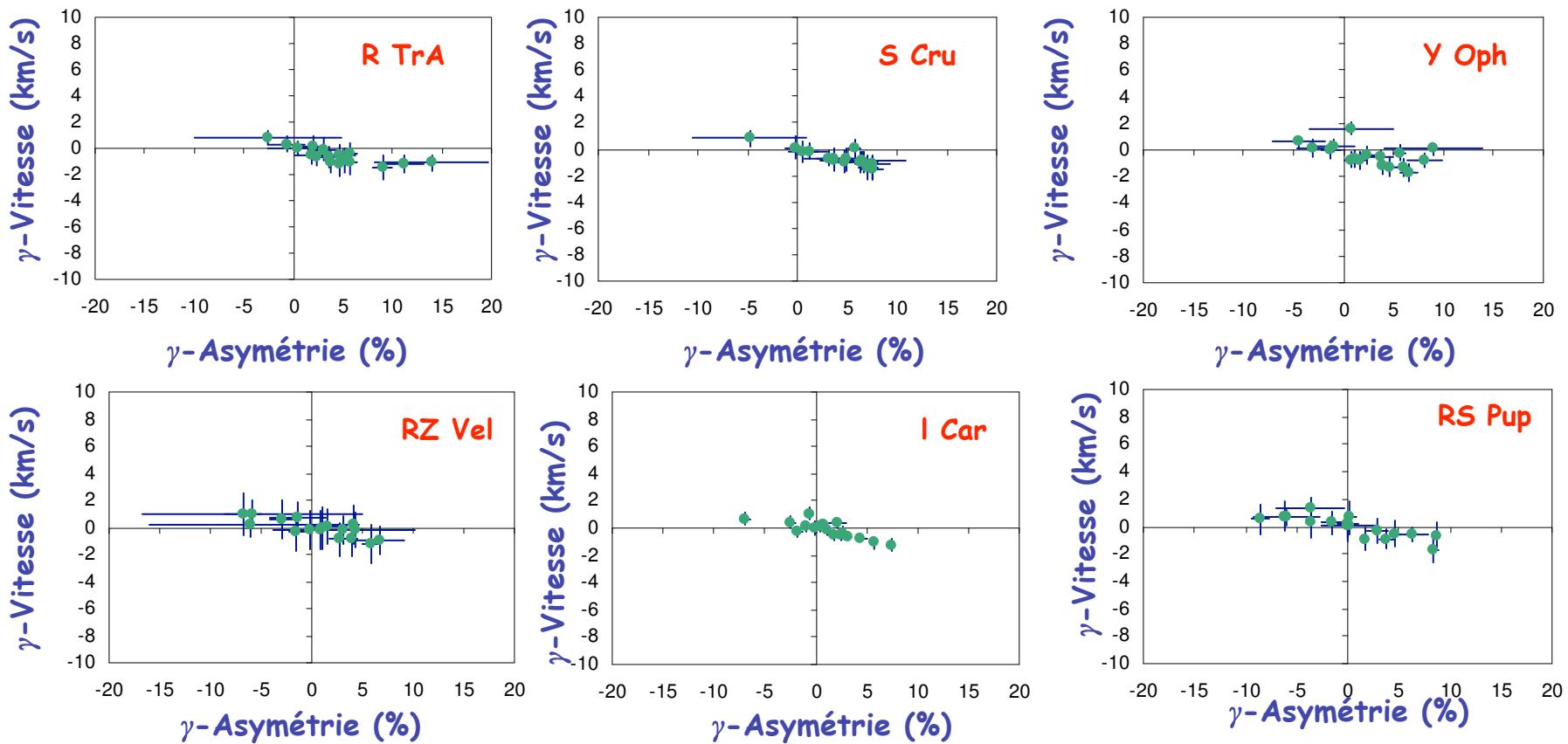


$\gamma$ -Asymétrie (%)



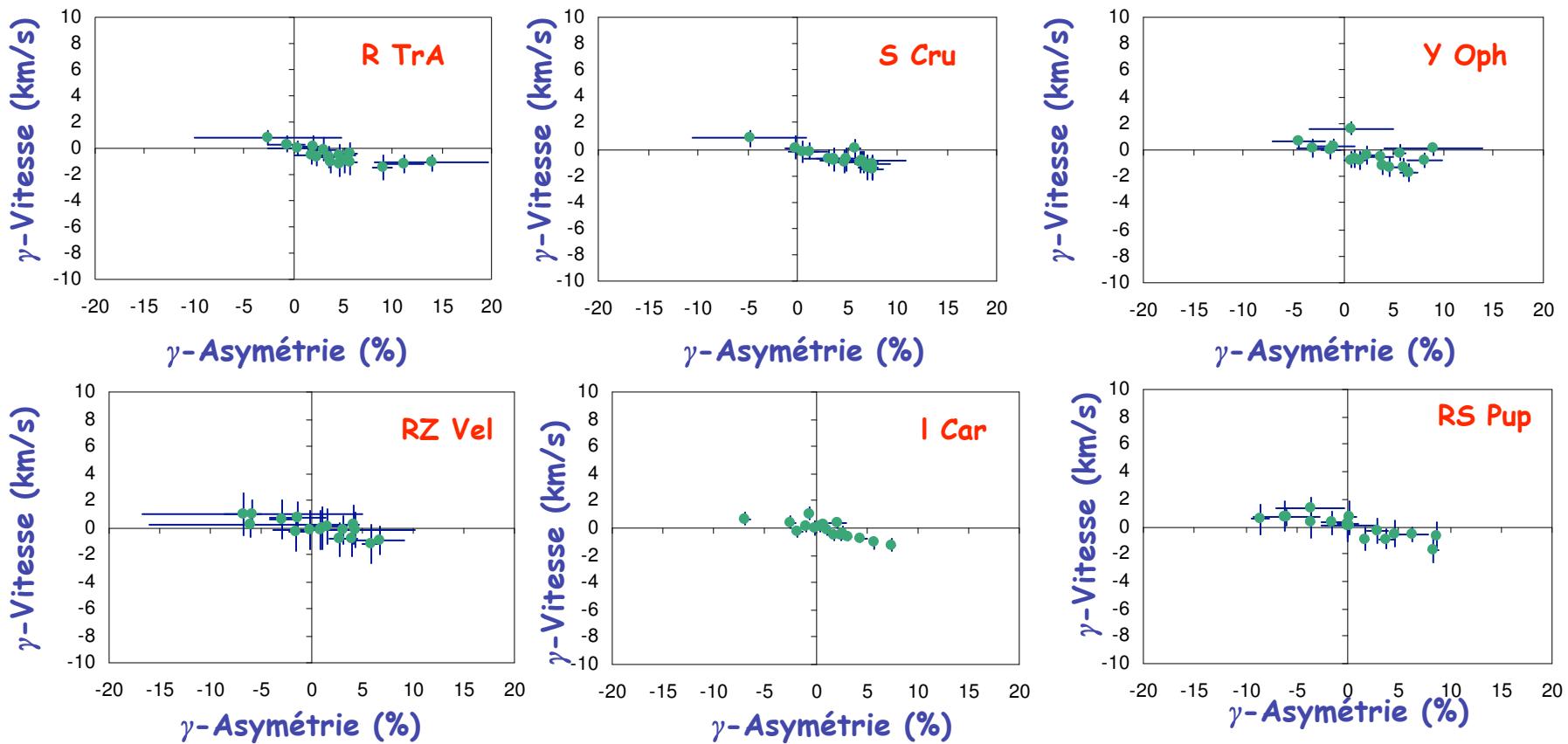
$\gamma$ -Asymétrie (%)

**High resolution spectroscopy for Cepheids distance determination**  
**III. A relation between  $\gamma$ -velocities and  $\gamma$ -asymmetries**  
 N. Nardetto, A. Stoekl, D. Bersier, T. Barnes, 2008, A&A, 489, 1255

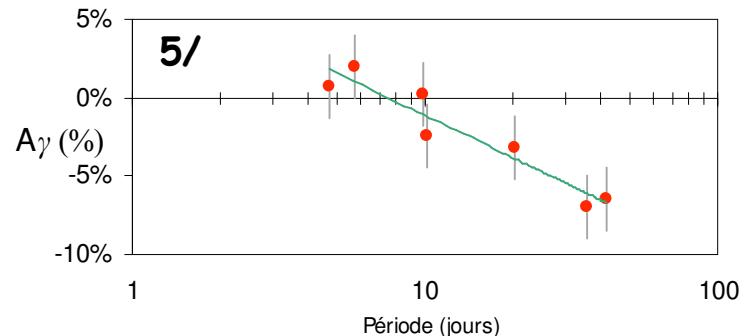


On trouve pour les 8 étoiles une correction moyenne (redshift) de **1.8+-0.2km/s!**

**High resolution spectroscopy for Cepheids distance determination**  
**III. A relation between  $\gamma$ -velocities and  $\gamma$ -asymmetries**  
 N. Nardetto, A. Stoekl, D. Bersier, T. Barnes, 2008, A&A, 489, 1255

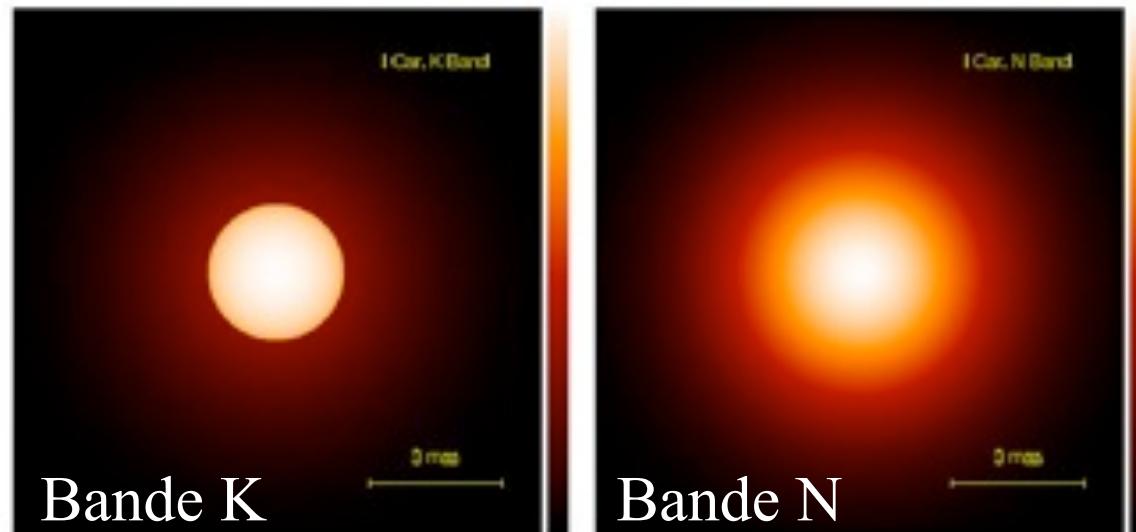


On trouve pour les 8 étoiles une correction moyenne (redshift) de  $1.8 \pm 0.2 \text{ km/s}!$



## L'enveloppe circumstellaire des Céphéides (CHARA)

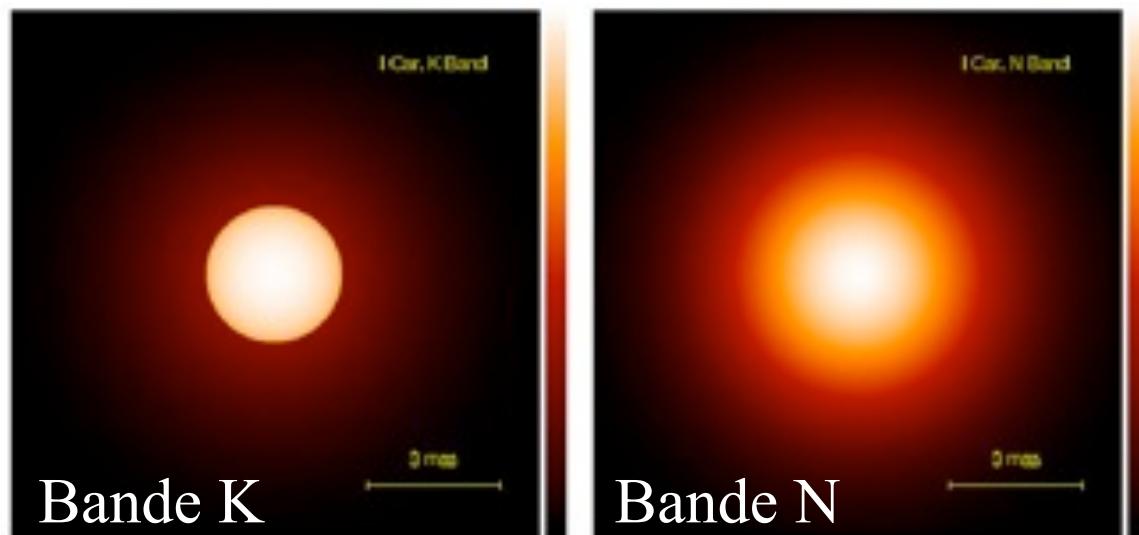
1 Car



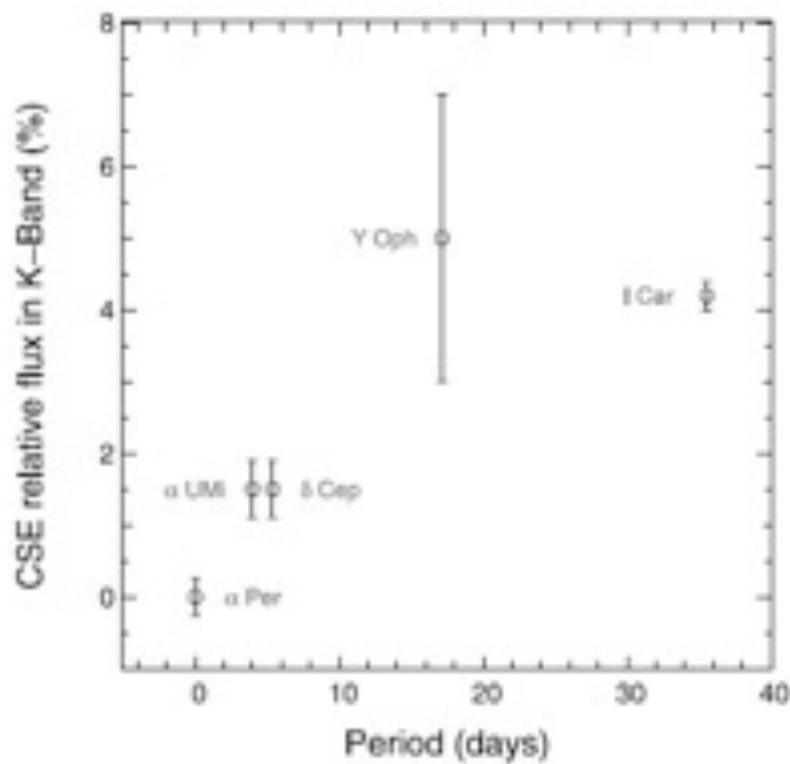
Kervella et al. 2006, *A&A*, 448, 623

# L'enveloppe circumstellaire des Céphéides (CHARA)

1 Car

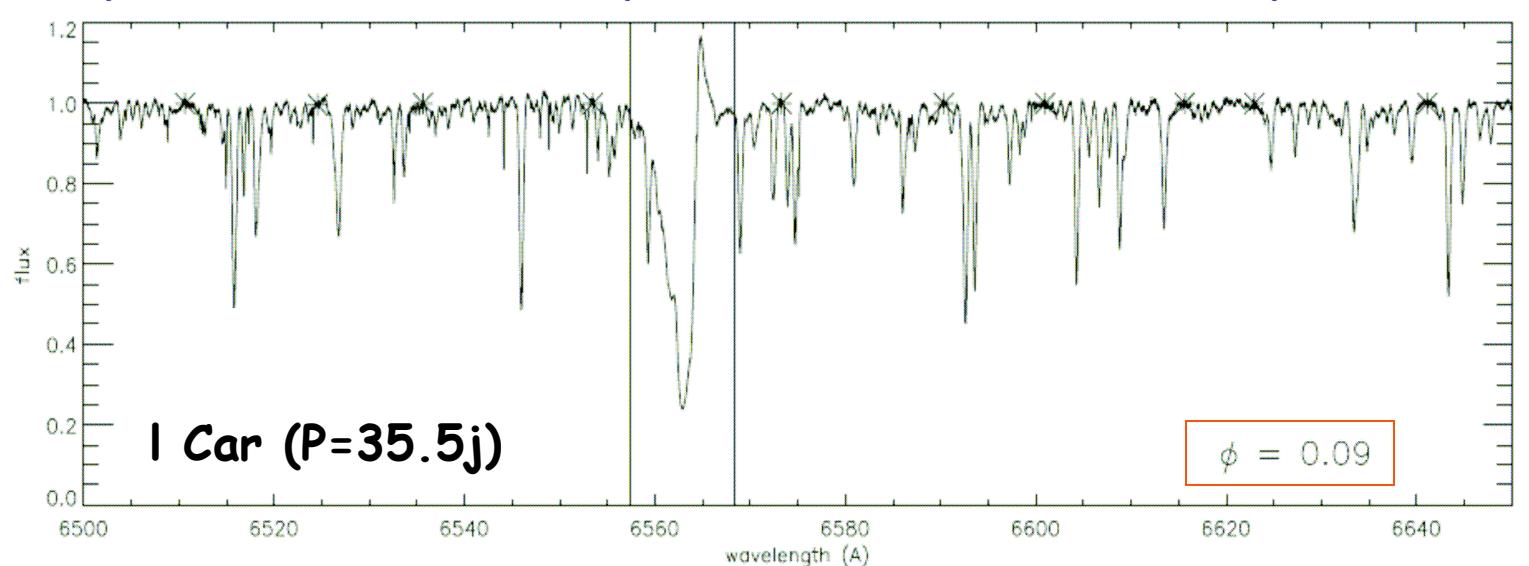


Kervella et al. 2006, A&A, 448, 623

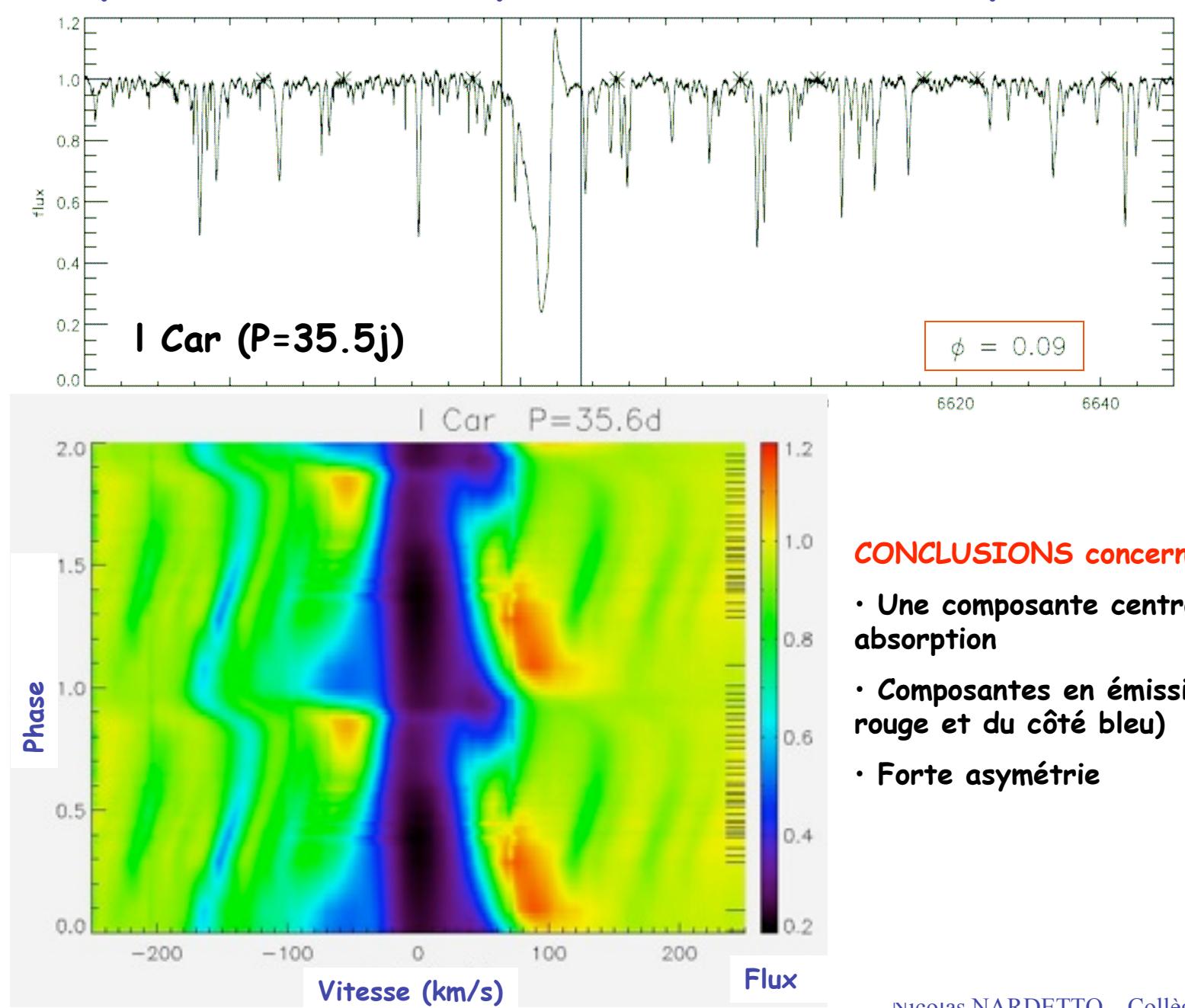


Mérand et al. 2007, A&A, 453, 155

## Les profiles H $\alpha$ des Céphéides (1/3) : le cas particulier de / Car



## Les profiles H $\alpha$ des Céphéides (1/3) : le cas particulier de I Car

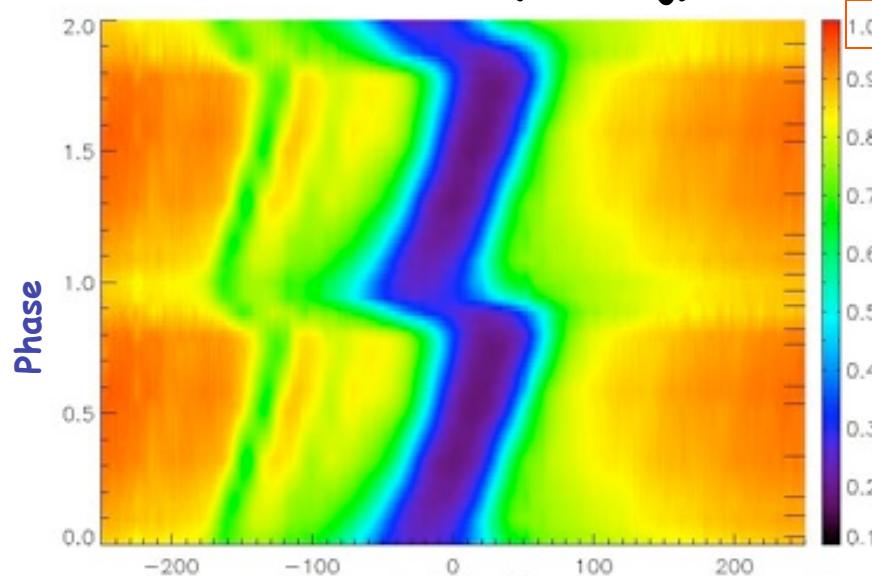


### CONCLUSIONS concernant I Car:

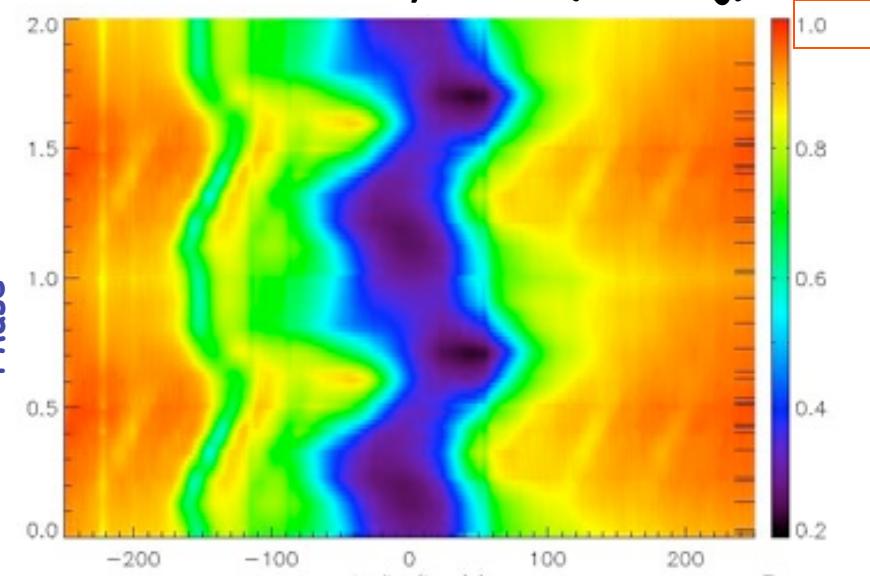
- Une composante centrale en absorption
- Composantes en émission (du côté rouge et du côté bleu)
- Forte asymétrie

## Les profiles Ha (2/3)

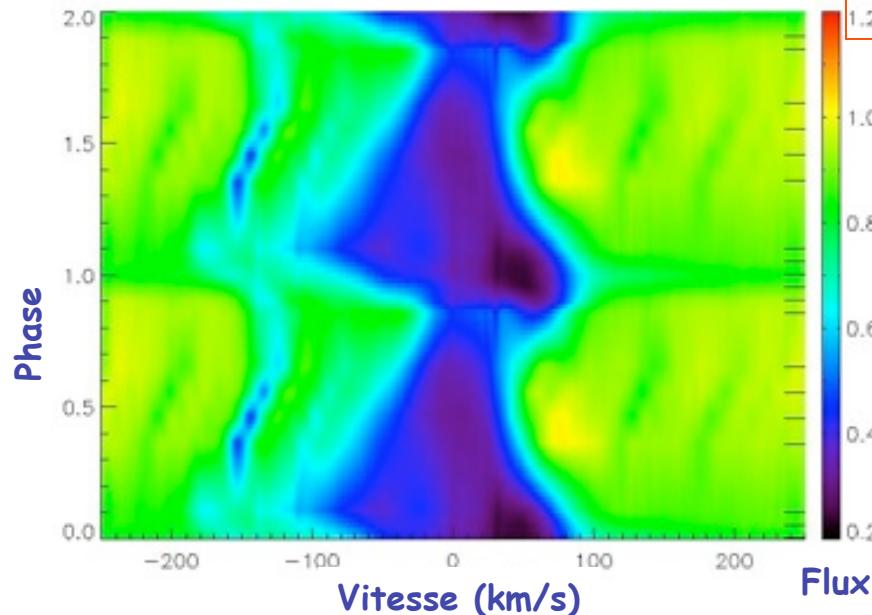
## S Cru (P=4.7j)



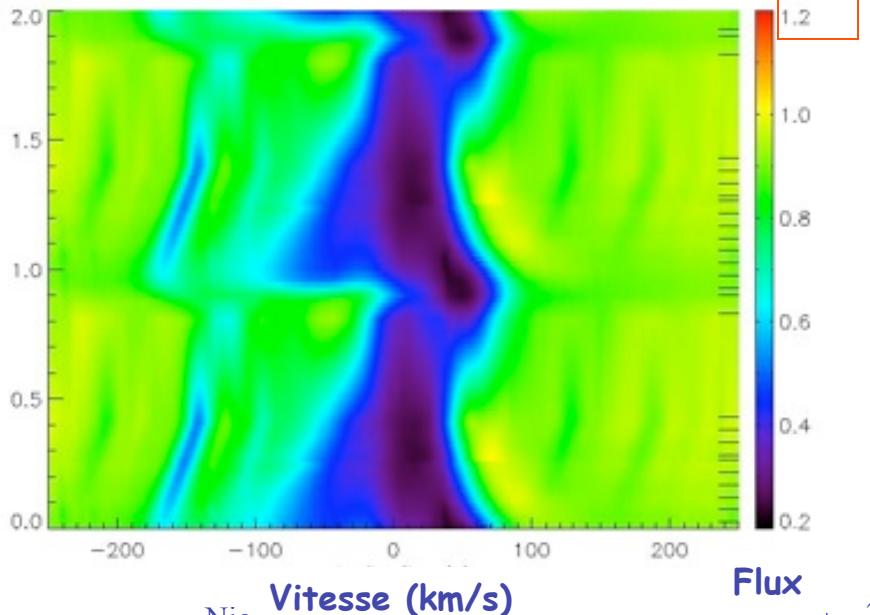
## $\beta$ Dor (P=9.8j)



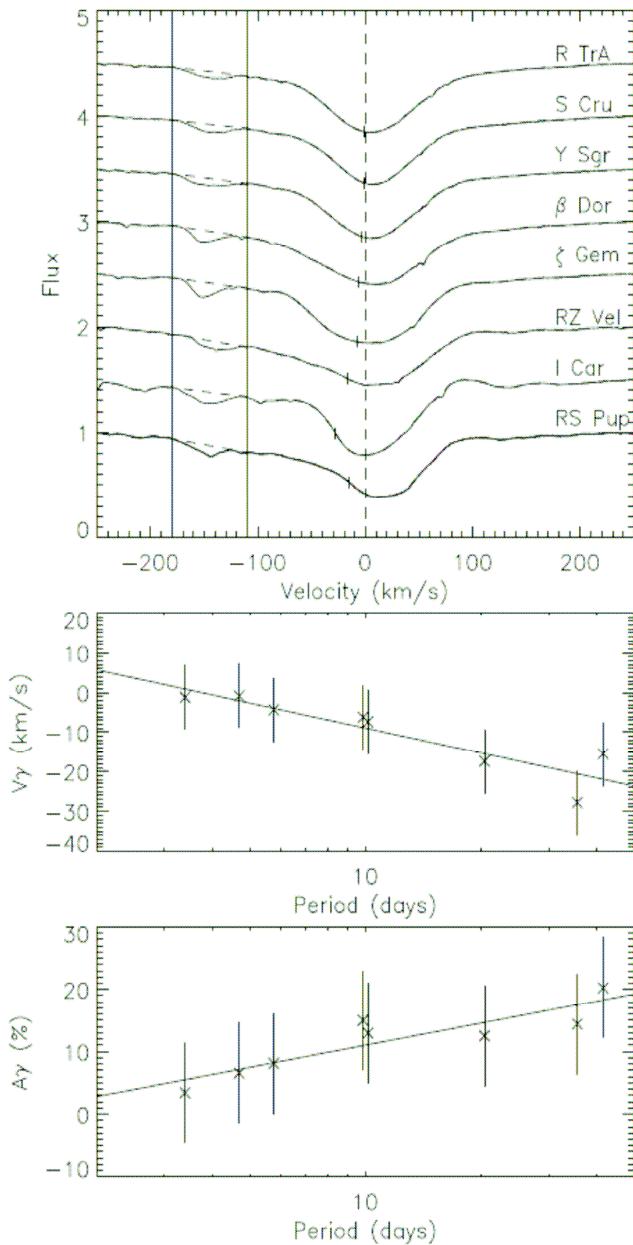
## Vitesse (km/s) RZ Vel (P=20.4j)



## Vitesse (km/s) RS Pup (P=41.5j)



# Les profiles H $\alpha$ des Céphéides (3/3): une relation "Période-A $\gamma$ "



High resolution spectroscopy for Cepheids distance determination

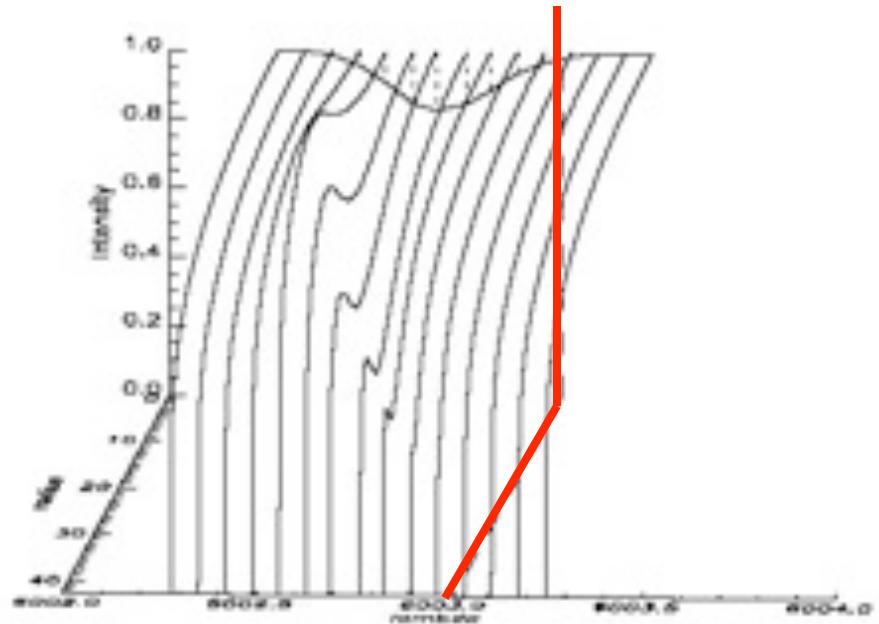
## IV. Times series of H $\alpha$ line profiles

N. Nardetto, J. Groh, S. Kraus, F. Millour, D. Gillet,  
2008, A&A, 489, 1263

## CONCLUSIONS générales:

- 1/ Une relation entre la période et la perte de masse?
- 2/ Modélisation de l'enveloppe circumstellaire
- 3/ Observations spectro-interférométriques (AMBER & VEGA)

## Perspectives : spectro-interférométrie

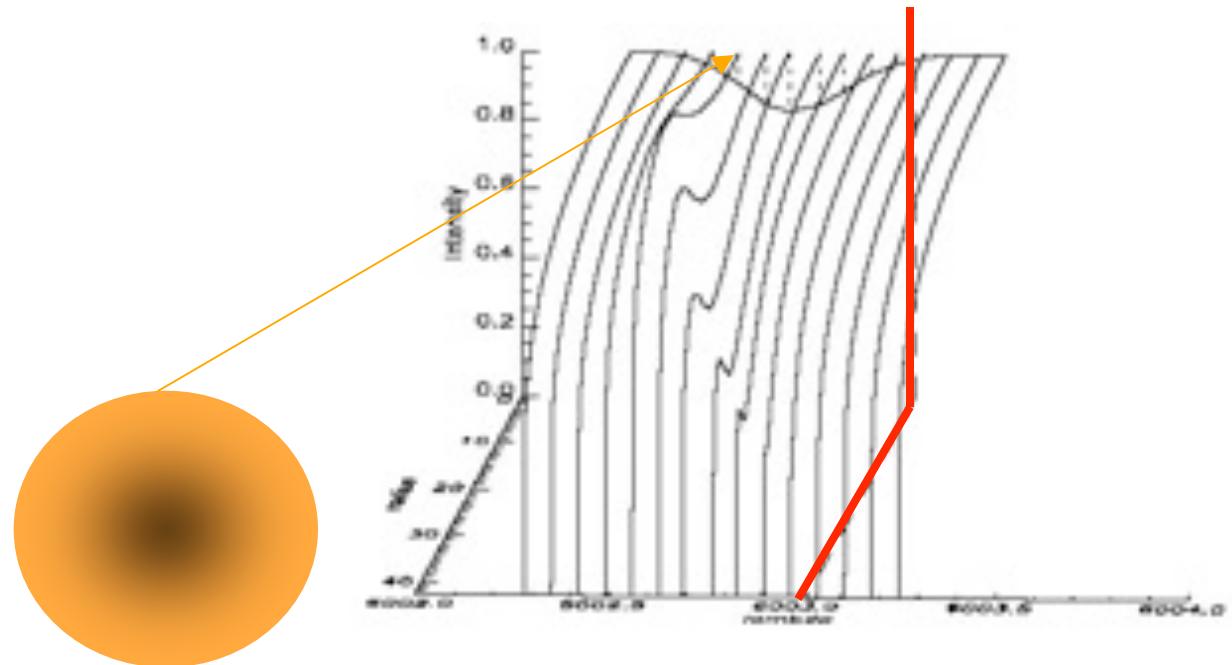


$$\varphi=0$$

$$V_{puls}=20 \text{ km/s}$$

$$V_{rot} * \sin i = 0 \text{ km/s}$$

## Perspectives : spectro-interférométrie

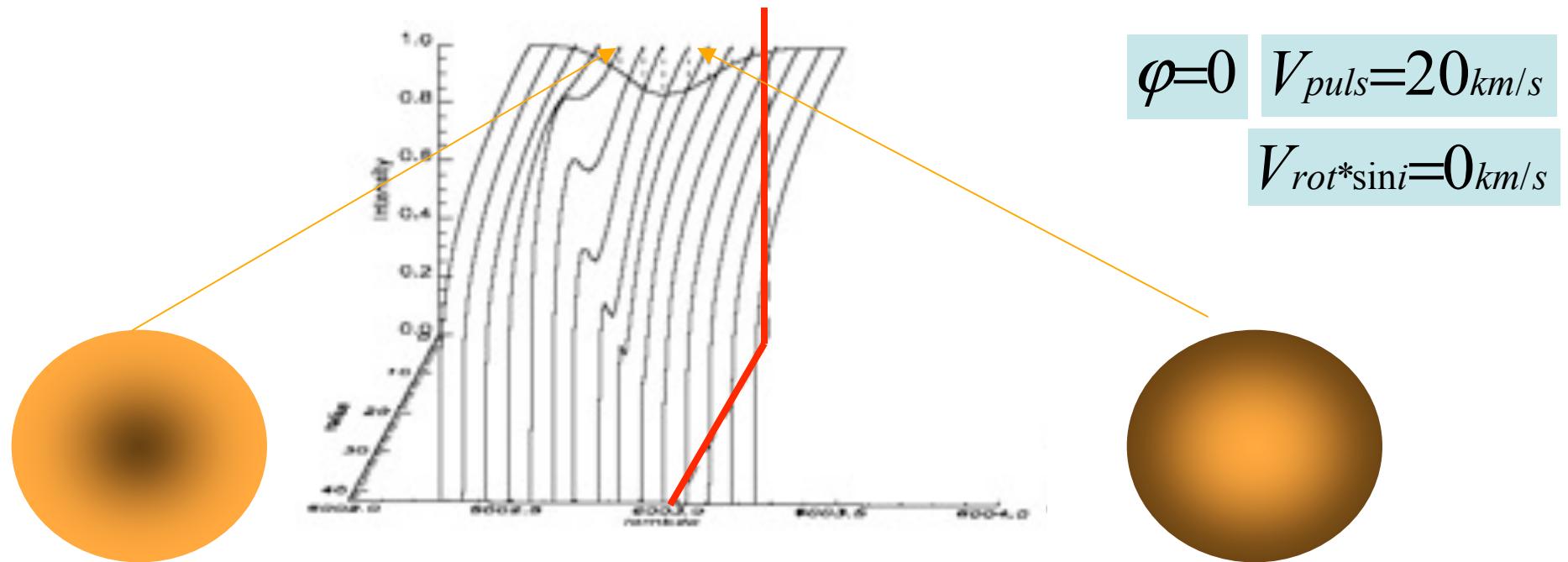


$$\varphi=0$$

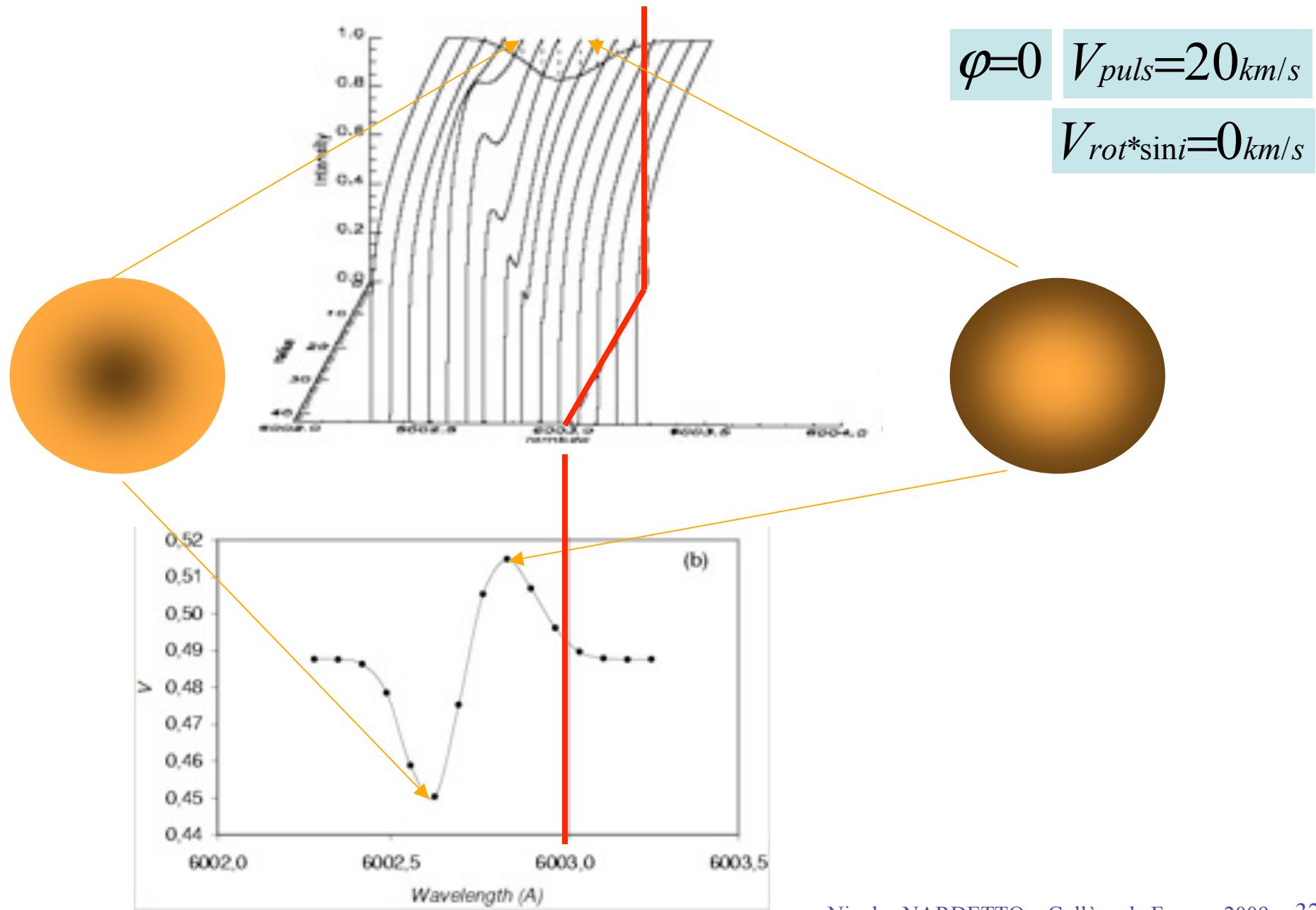
$$V_{puls}=20 \text{ km/s}$$

$$V_{rot} * \sin i = 0 \text{ km/s}$$

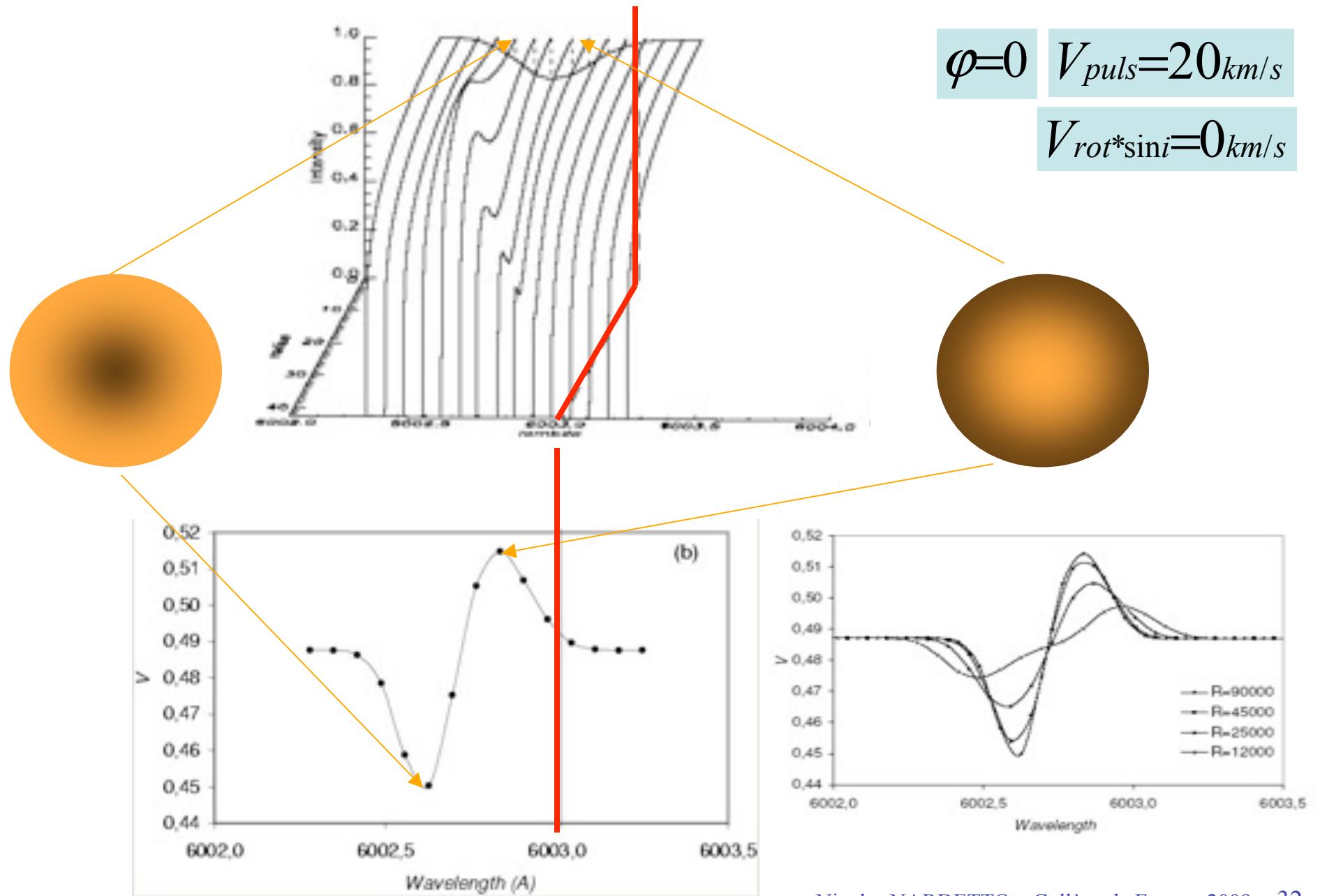
# Perspectives : spectro-interférométrie



# Perspectives : spectro-interférométrie

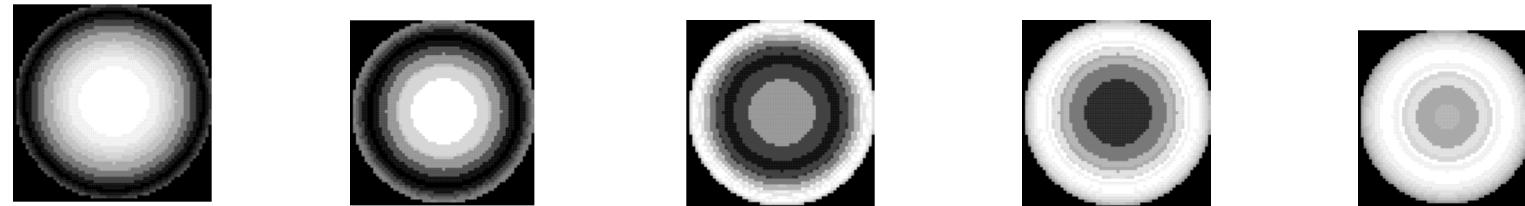


# Perspectives : spectro-interférométrie



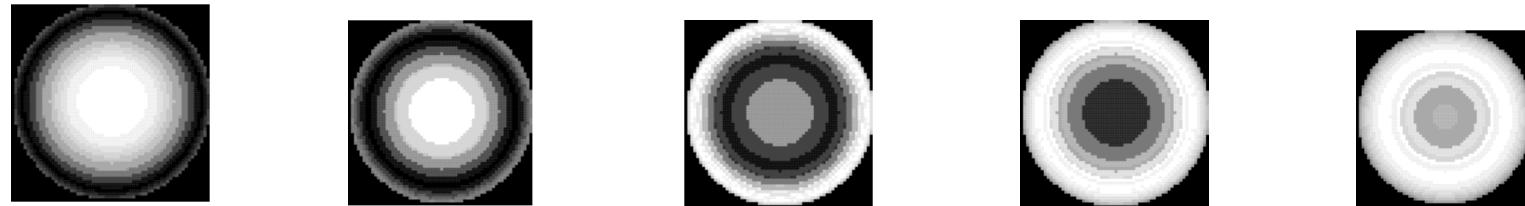
## Perspectives : spectro-interférométrie

$$ACB \quad \sigma_c \quad V_{puls} \quad \cancel{V_{rot}\sin i} \quad + \quad D \quad \theta$$

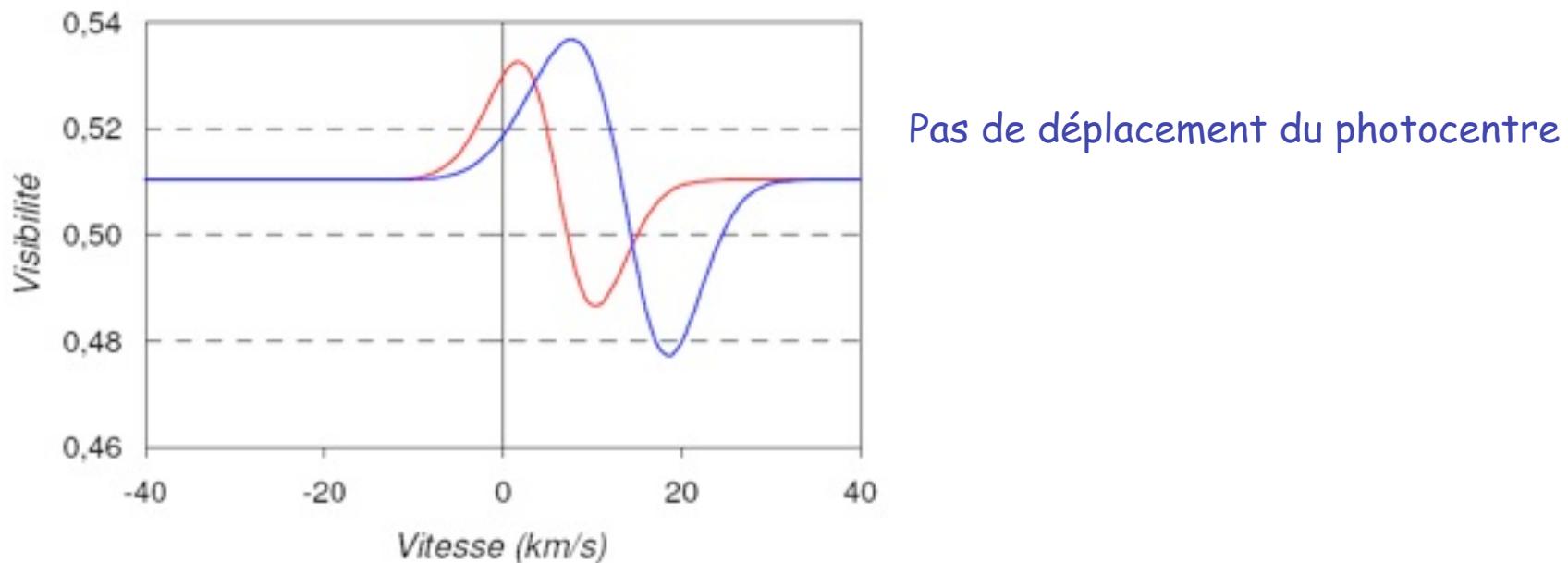


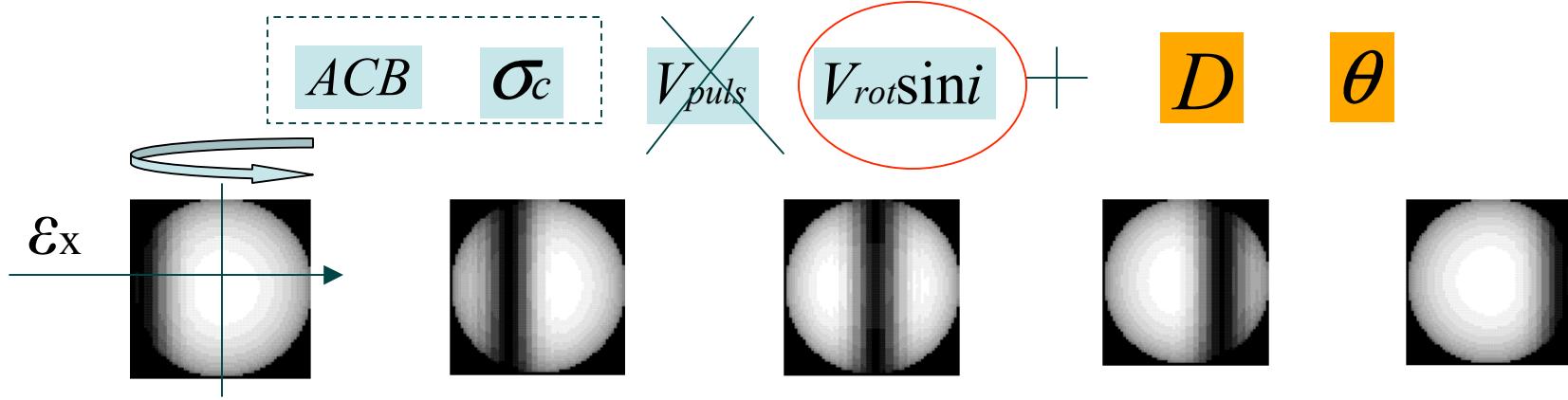
## Perspectives : spectro-interférométrie

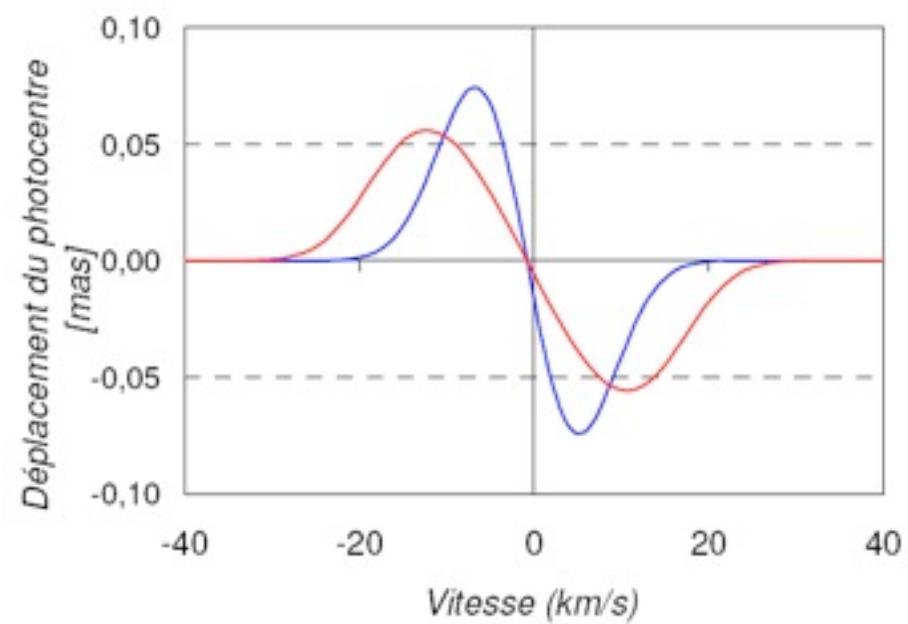
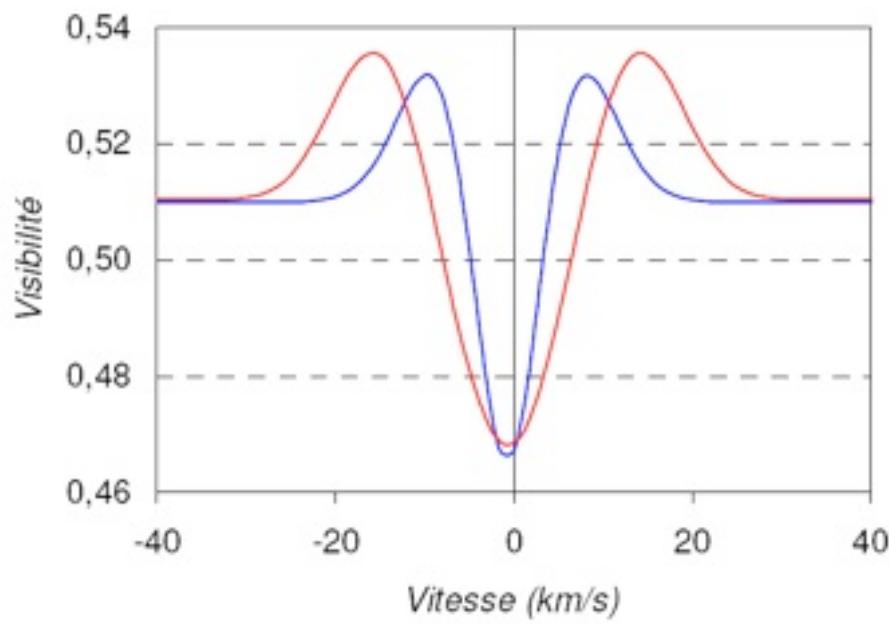
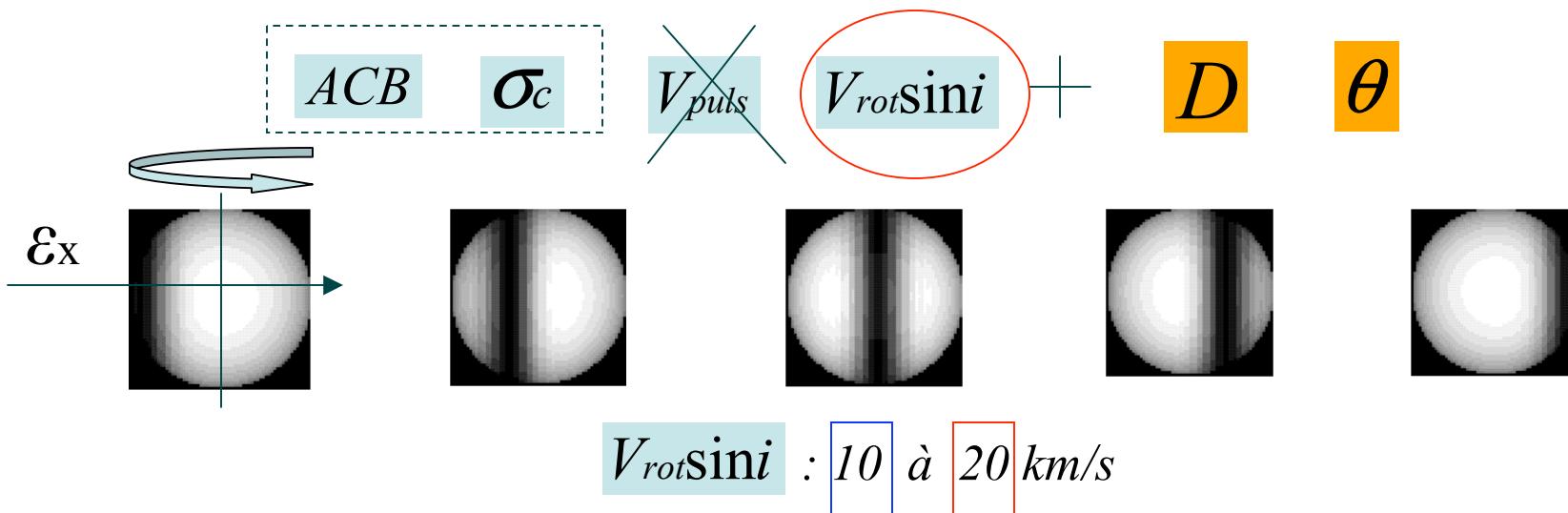
$$ACB \quad \sigma_c \quad V_{puls} \quad \cancel{V_{rot}\sin i} \quad + \quad D \quad \theta$$

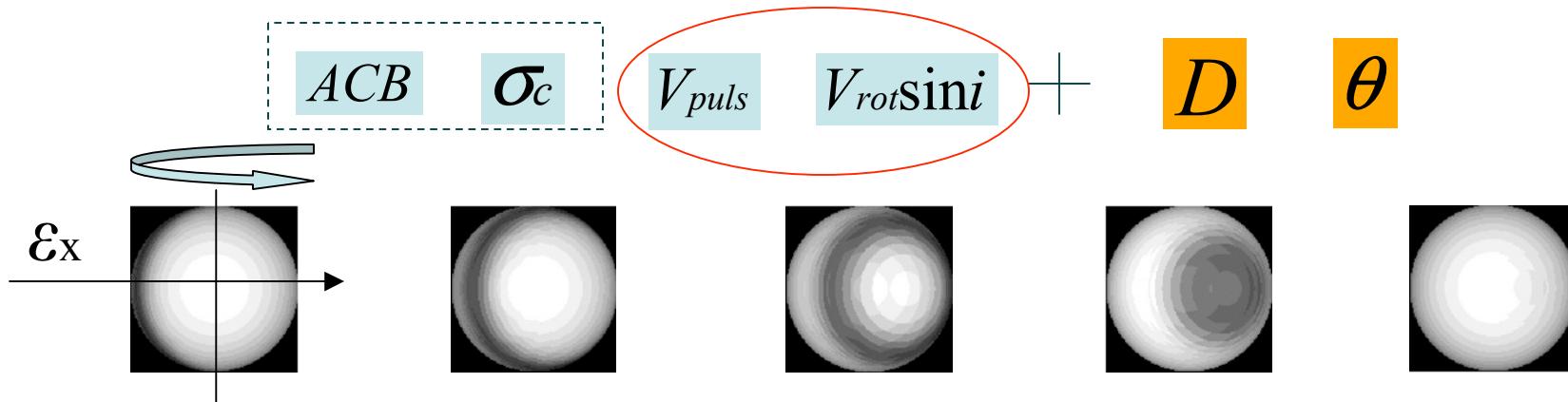


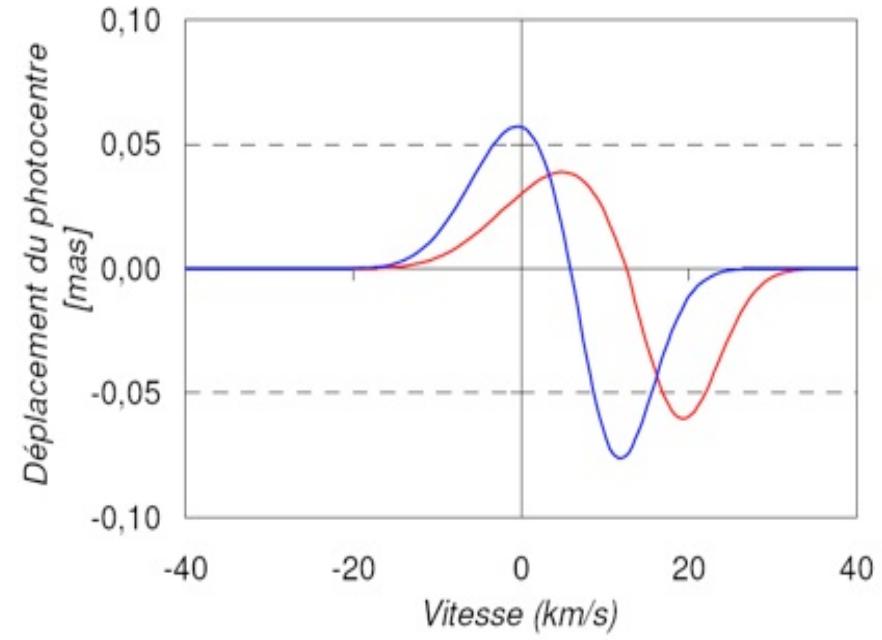
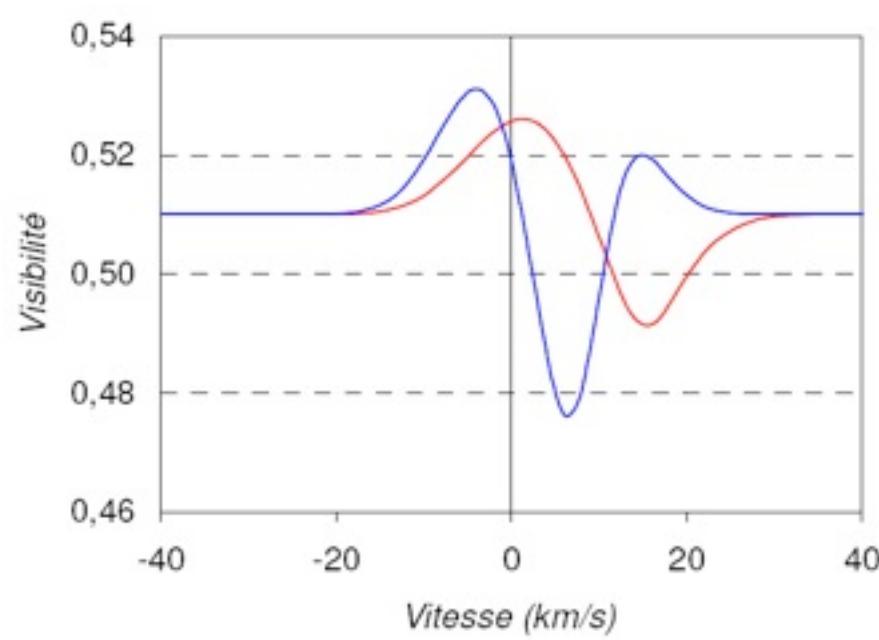
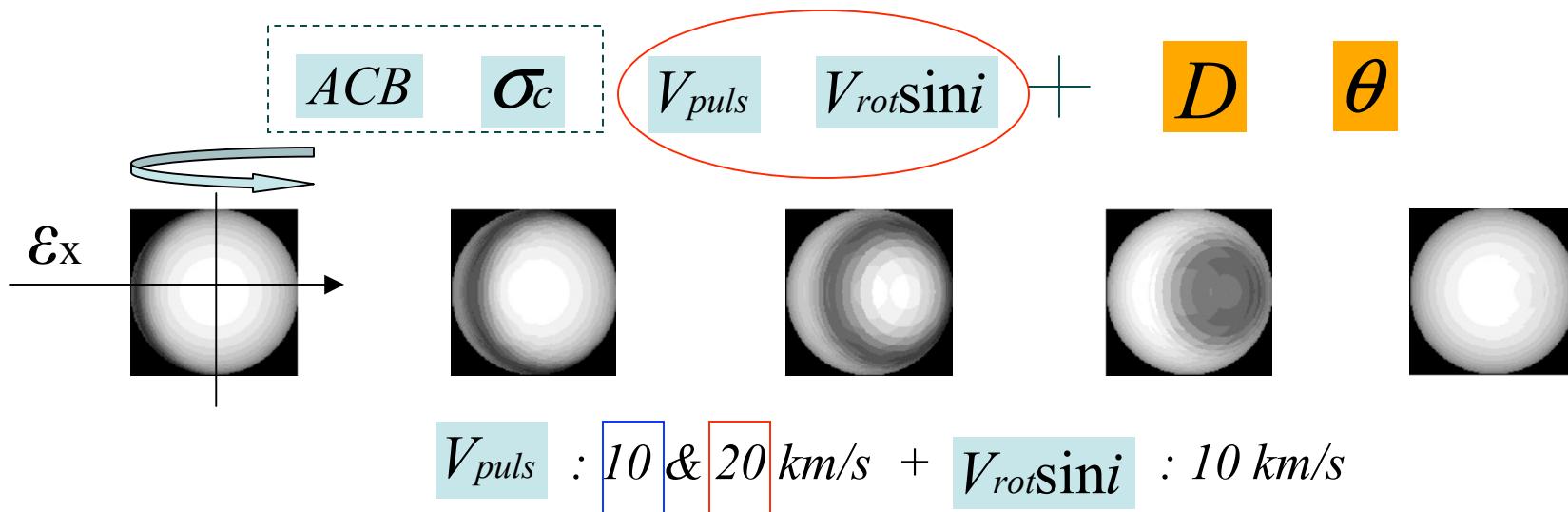
$V_{puls}$  : 10 & 20 km/s

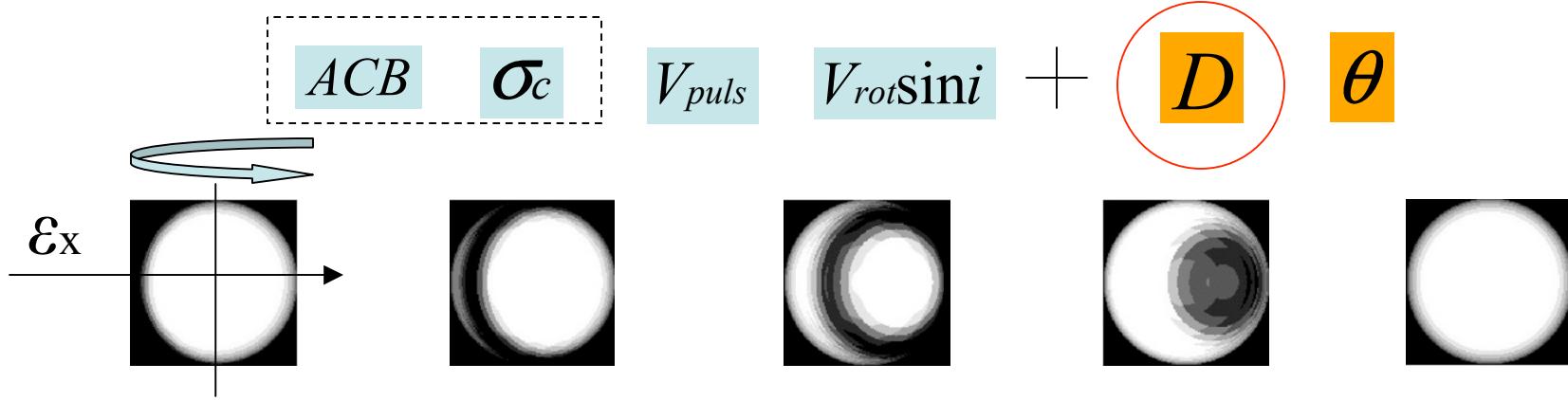


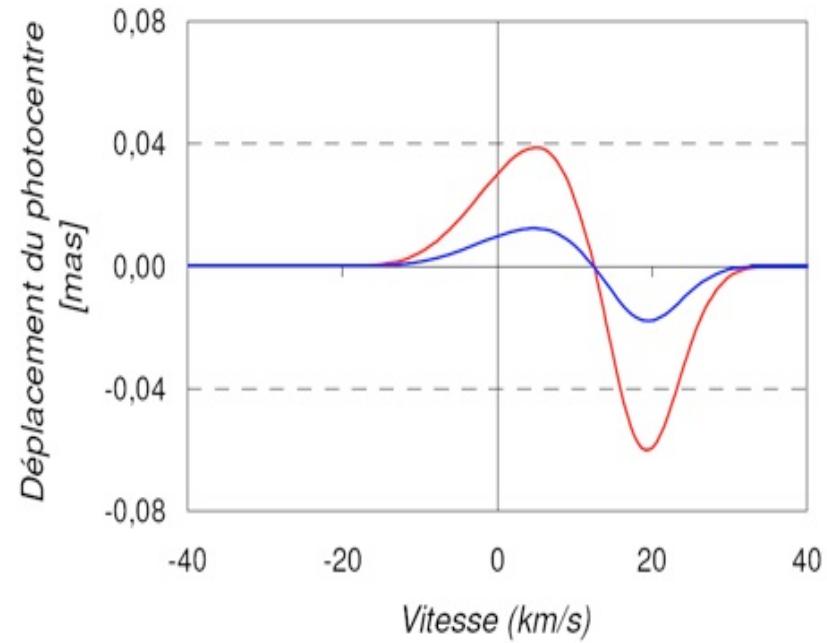
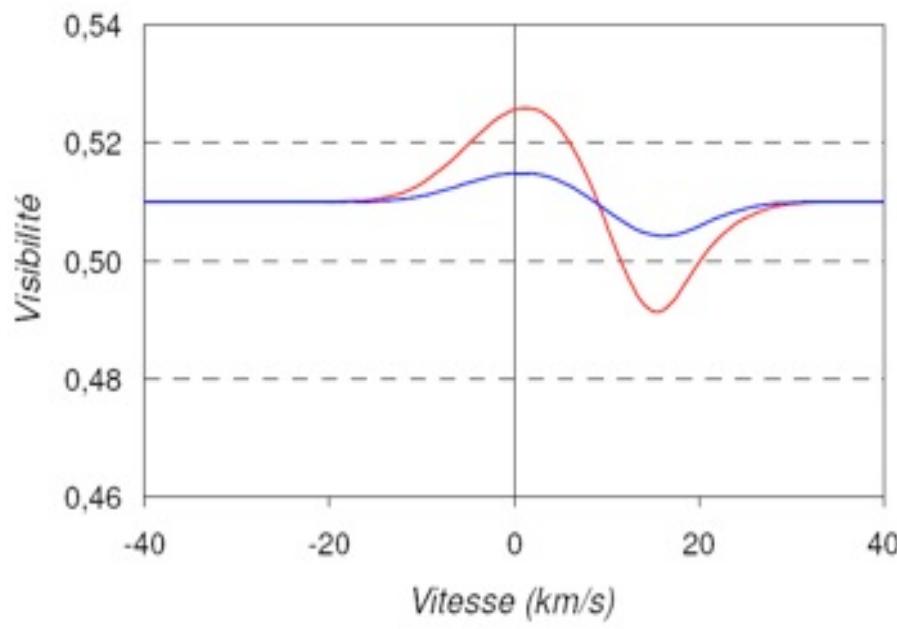
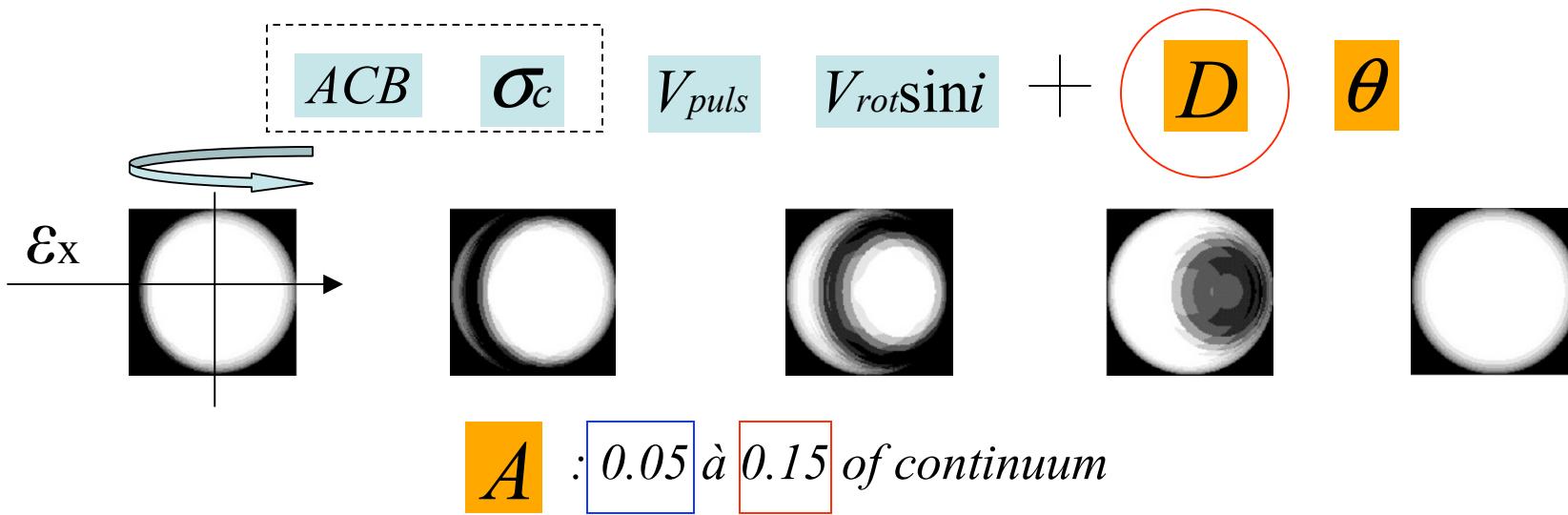


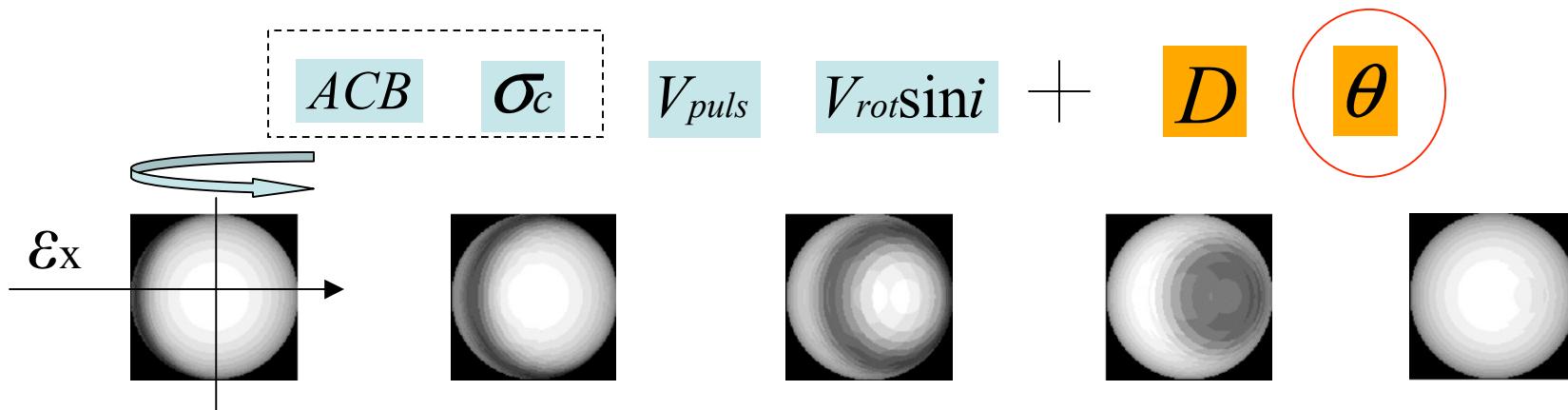


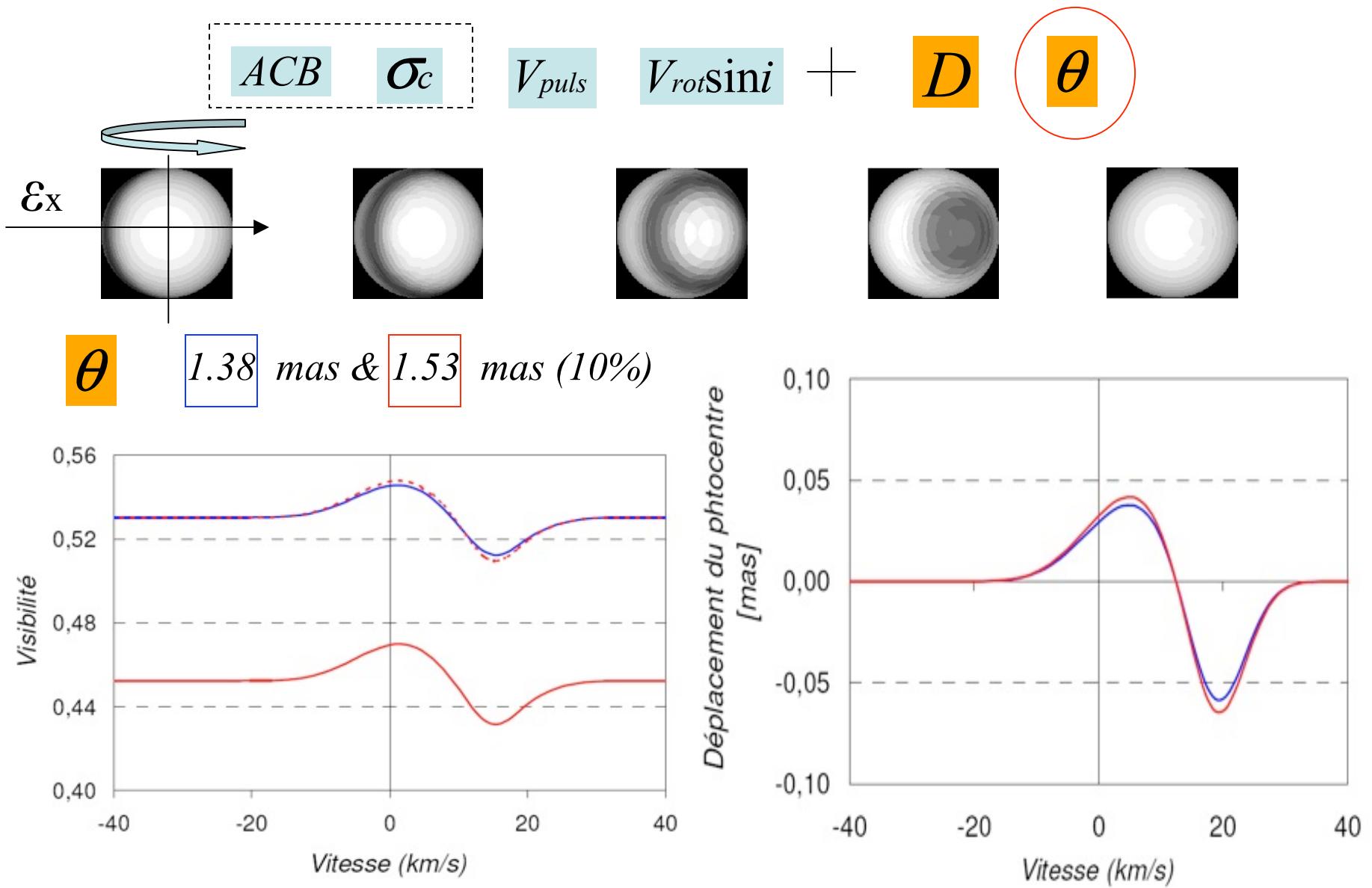










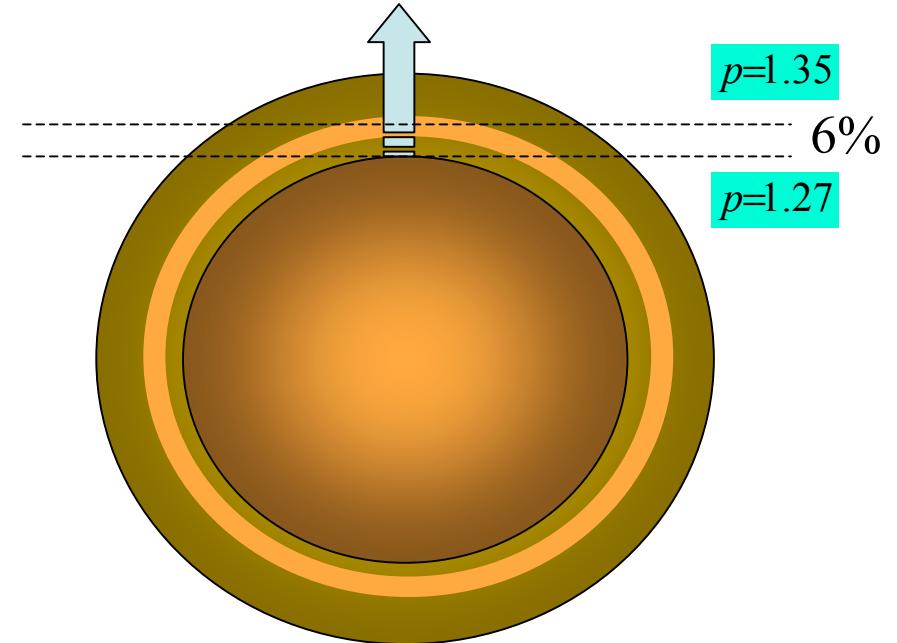
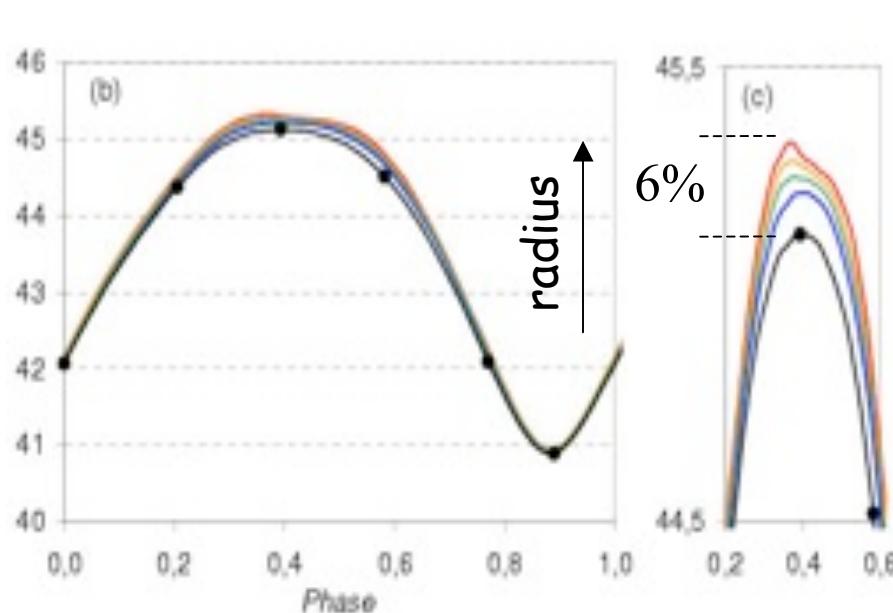


Precision at  $1 \mu m$  &,  $B=200m$  :  $1 \mu as$  (AMBER/VLTI)  
 Precision at  $0.6 \mu m$  &,  $B=300m$  :  $0.5 \mu as$  (VEGA/CHARA)

# Perspectives : spectro-interférométrie

« Probing the dynamical structure of Cepheid's atmosphere »

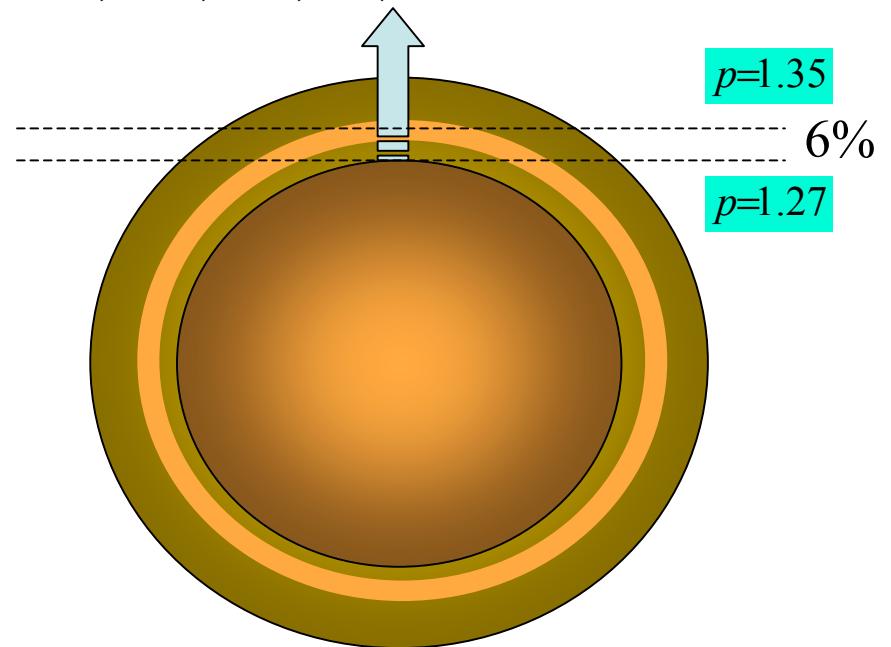
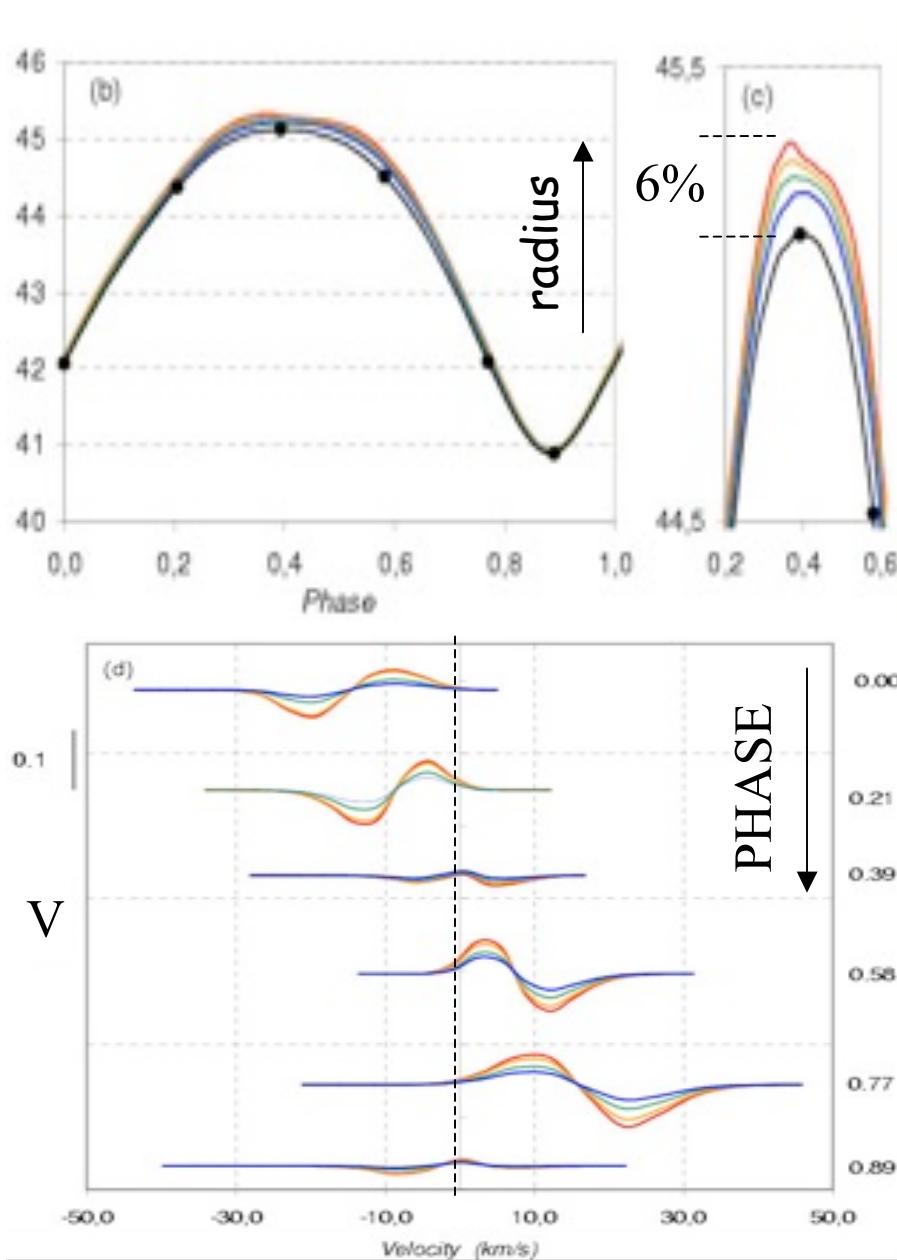
N. Nardetto, F. Fokin, D. Mourard, Ph. Mathias, 2005, A&A, 454, 327



# Perspectives : spectro-interférométrie

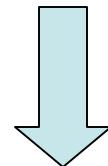
« Probing the dynamical structure of Cepheid's atmosphere »

N. Nardetto, F. Fokin, D. Mourard, Ph. Mathias, 2005, A&A, 454, 327



# Conclusion:

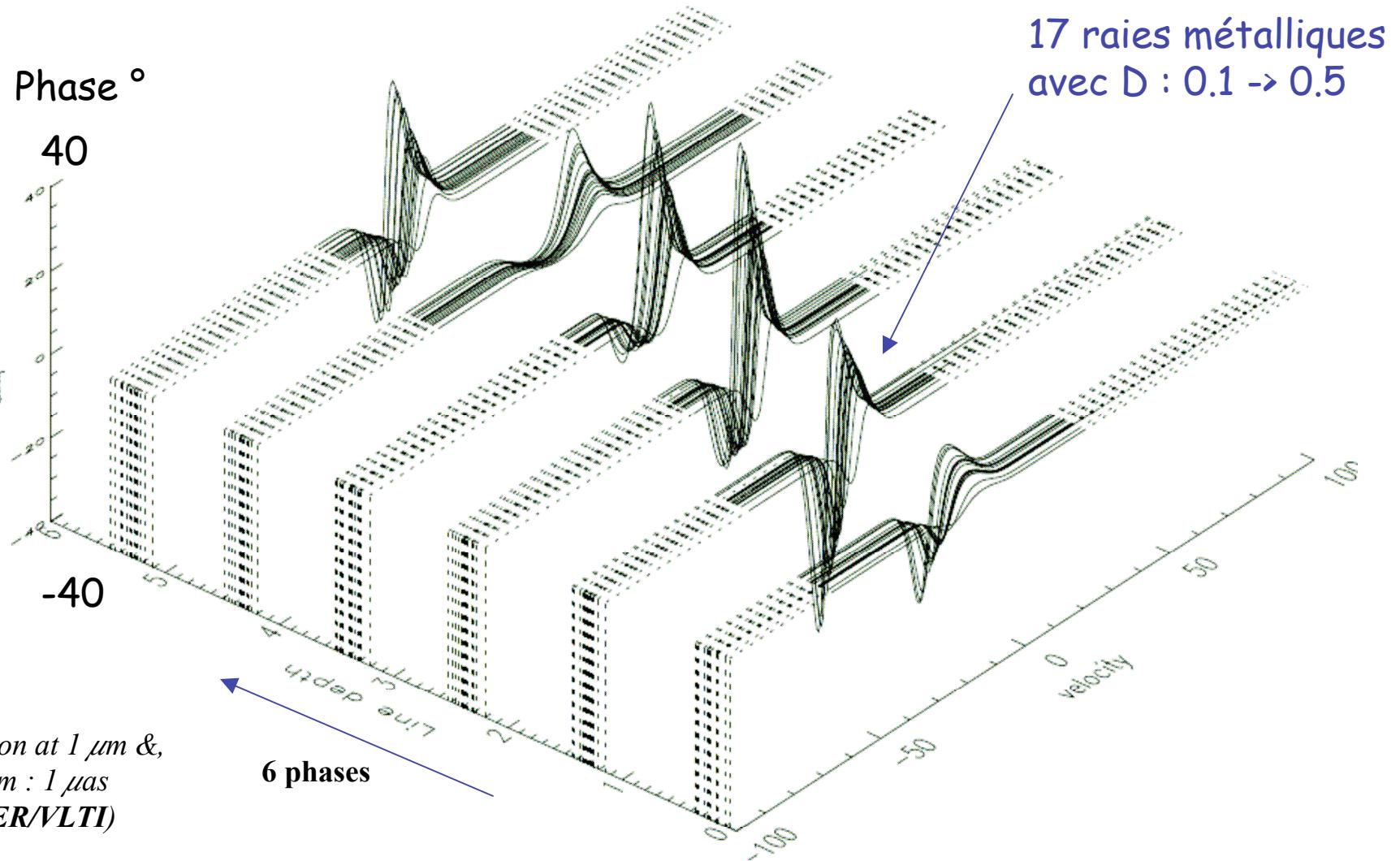
PHYSIQUE DES CEPHEIDES



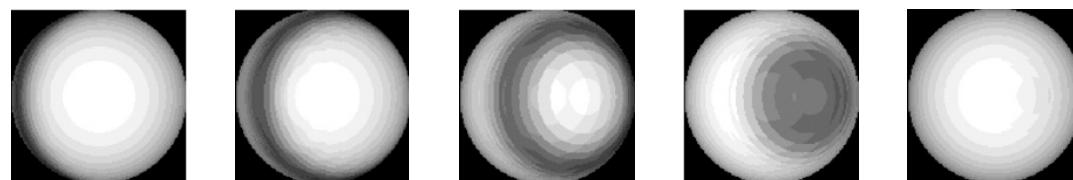
DISTANCES GALACTIQUES & EXTRAGALACTIQUES  
CONSTANTE DE HUBBLE



## p-facteur & spectro-interférométrie(3/3)



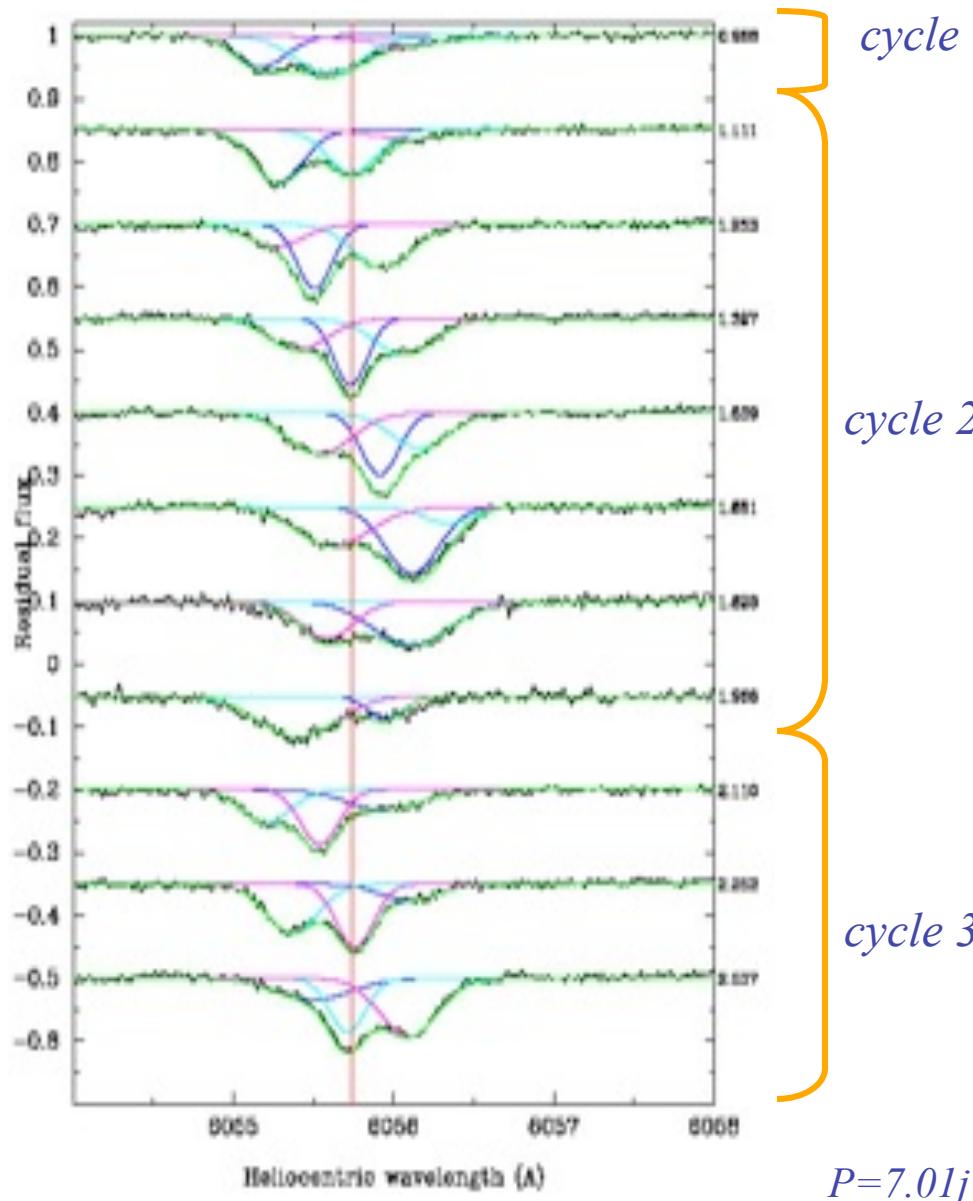
Precision at  $0.6 \mu\text{m}$  &,  $B=300\text{m}$  :  
 $0.5 \mu\text{as}$   
(VEGA/CHARA)



# Le cas atypique de X Sgr

Multiple shock waves in the atmosphere of the Cepheid X Sagittarii?

Mathias, P.; Gillet, D.; Fokin, A. B.; Nardetto, N.; Kervella, P.; Mourard, D., A&A, 2006, 457, 575

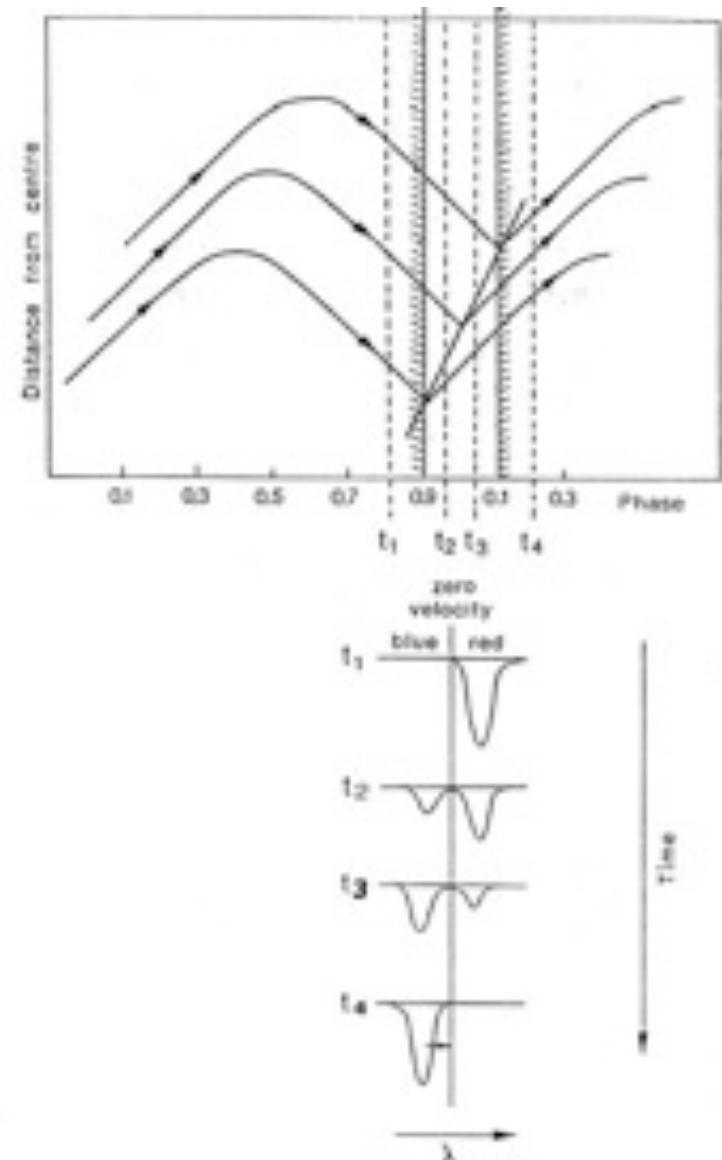
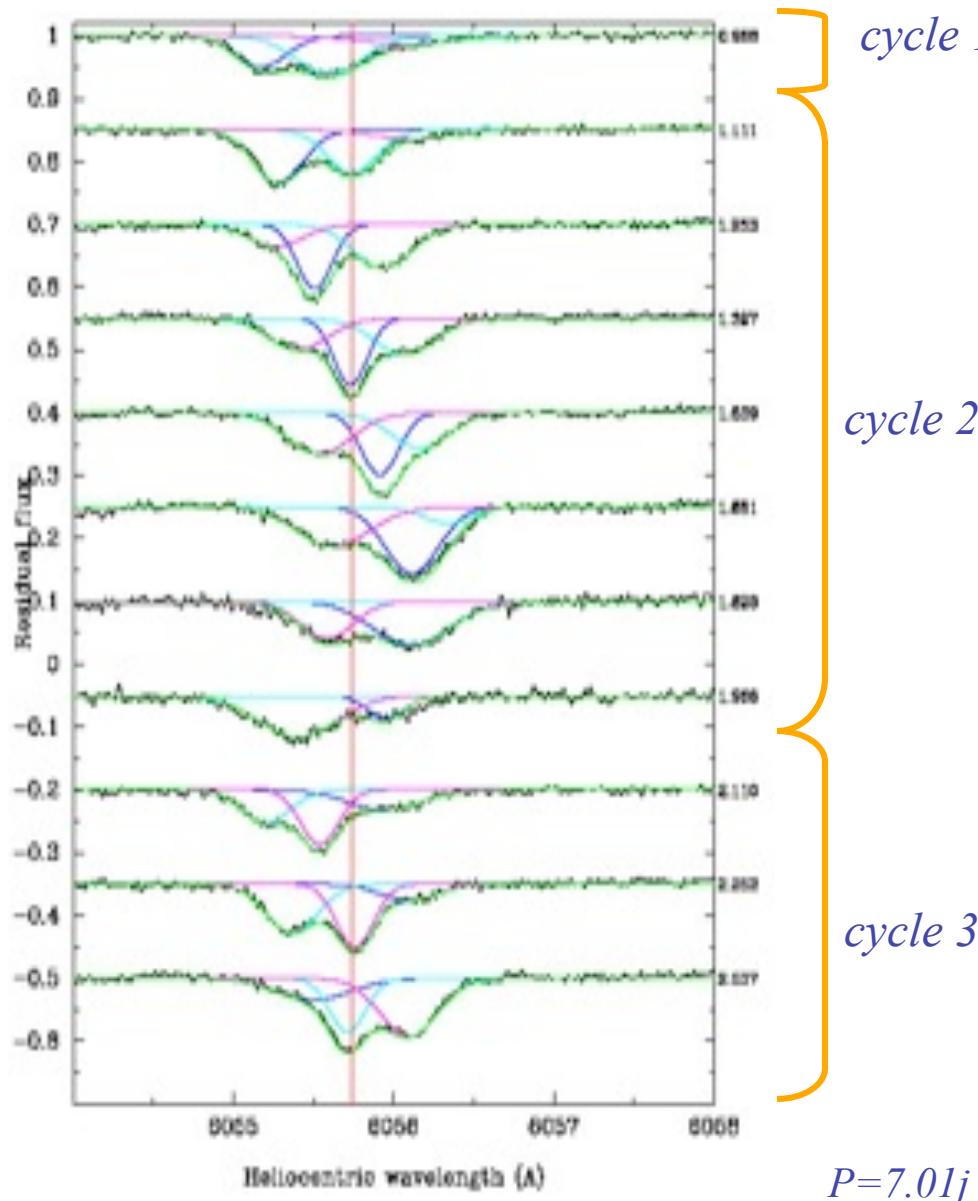


$$P=7.01j$$

# Le cas atypique de X Sgr

Multiple shock waves in the atmosphere of the Cepheid X Sagittarii?

Mathias, P.; Gillet, D.; Fokin, A. B.; Nardetto, N.; Kervella, P.; Mourard, D., A&A, 2006, 457, 575



## OBERVATIONS versus MODELISATION : perspectives

### 2. Asymétrie des raies métalliques non expliquée

P-«dynamique atmosphérique»

- **Grille adaptative:** Vienne
- **Convection:** Lyon, Vienne

### 1. Profils d'hydrogène non expliqués

P- «perte de masse»  
P-«env. circumstellaire»

- **Modélisation de l'enveloppe circumstellaire**

Extension aux étoiles pulsantes du diag. HR

## Physique des Céphéides

- **Transfert radiatif hors-ETL...**  
Boston, Moscou, ...

P-«assombrissement centre-bord»

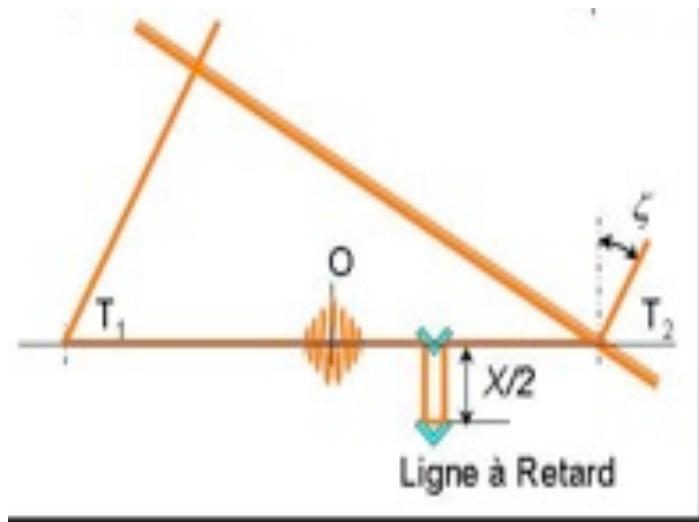
### 3. Lois d'ACB incompatibles (moyenne et variation)

- **Evolution/pulsation:**  
Lyon, Rome, Budapest, Nice (CESAM)

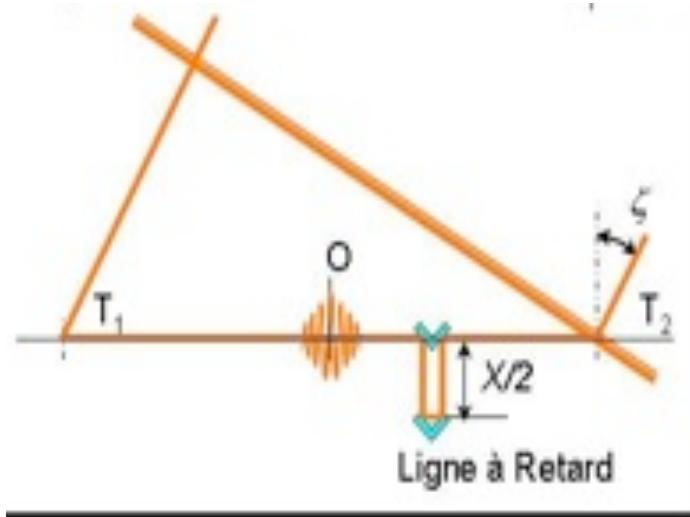
P-«masse»  
paramètres fondamentaux

### 4. Masses incompatibles (10%)

# Principe de l'interférométrie



# Principe de l'interférométrie

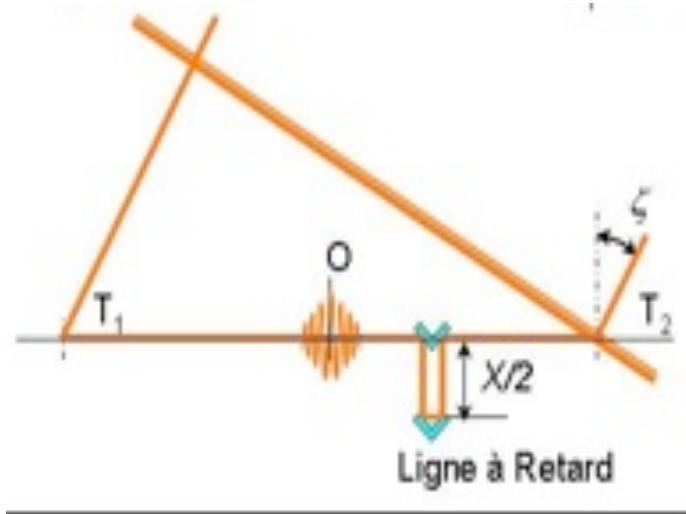


$$I = |\Psi_1 + \Psi_2 e^{i\theta}|^2$$

$$I = |\Psi_1|^2 + |\Psi_2|^2 + 2\Psi_1 \Psi_2^* \cos(\theta)$$

$$I = (I_1 + I_2) * \left( 1 + \frac{2\sqrt{I_1 I_2}}{I_1 + I_2} * \frac{\Psi_1 \Psi_2^*}{\sqrt{|\Psi_1|^2 |\Psi_2|^2}} * \cos(\theta) \right)$$

# Principe de l'interférométrie



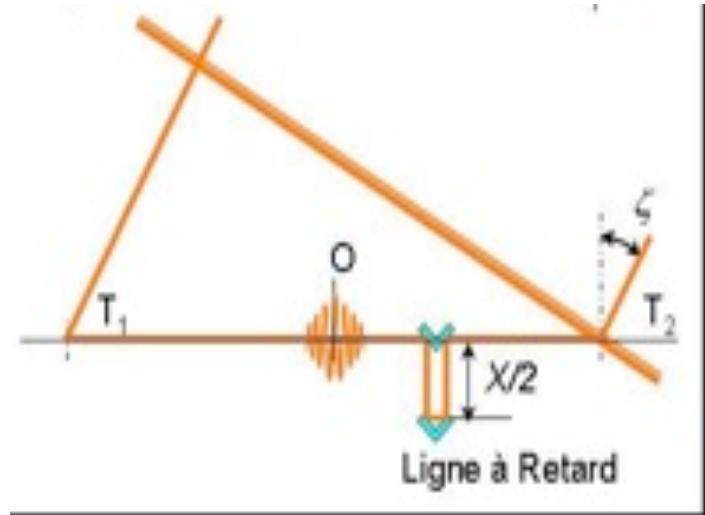
$$I = |\Psi_1 + \Psi_2 e^{i\theta}|^2$$

$$I = |\Psi_1|^2 + |\Psi_2|^2 + 2\Psi_1 \Psi_2^* \cos(\theta)$$

$$I = (I_1 + I_2) * \left( 1 + \frac{2\sqrt{I_1 I_2}}{I_1 + I_2} * \frac{\Psi_1 \Psi_2^*}{\sqrt{|\Psi_1|^2 |\Psi_2|^2}} * \cos(\theta) \right)$$

$$\gamma_{12} = \frac{\Psi_1 \Psi_2^*}{\sqrt{|\Psi_1|^2 |\Psi_2|^2}} = \frac{\left| \tilde{O}\left(\frac{B}{\lambda}\right) \right|}{\left| \tilde{O}(0) \right|}$$

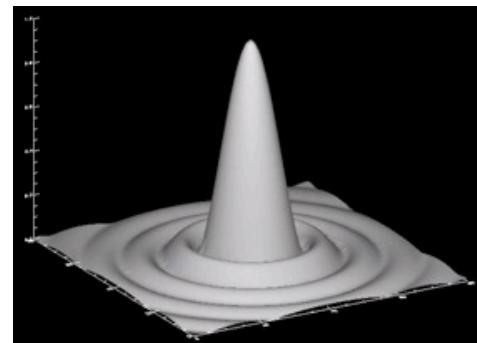
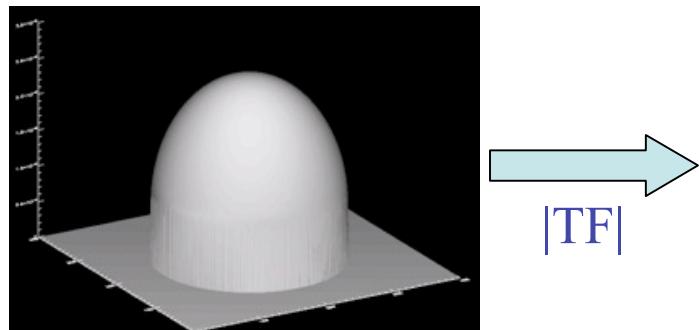
# Principe de l'interférométrie



$$I = |\Psi_1 + \Psi_2 e^{i\theta}|^2$$

$$I = |\Psi_1|^2 + |\Psi_2|^2 + 2\Psi_1 \Psi_2^* \cos(\theta)$$

$$I = (I_1 + I_2) * \left( 1 + \frac{2\sqrt{I_1 I_2}}{I_1 + I_2} * \frac{\Psi_1 \Psi_2^*}{\sqrt{|\Psi_1|^2 |\Psi_2|^2}} * \cos(\theta) \right)$$



$\theta$  PHOTOSPHERE  
ACB

$$\gamma_{12} = \frac{\Psi_1 \Psi_2^*}{\sqrt{|\Psi_1|^2 |\Psi_2|^2}} = \frac{\left| \tilde{O}\left(\frac{B}{\lambda}\right) \right|}{\left| \tilde{O}(0) \right|}$$