Imaging flow within the Earth’s core

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Motivations

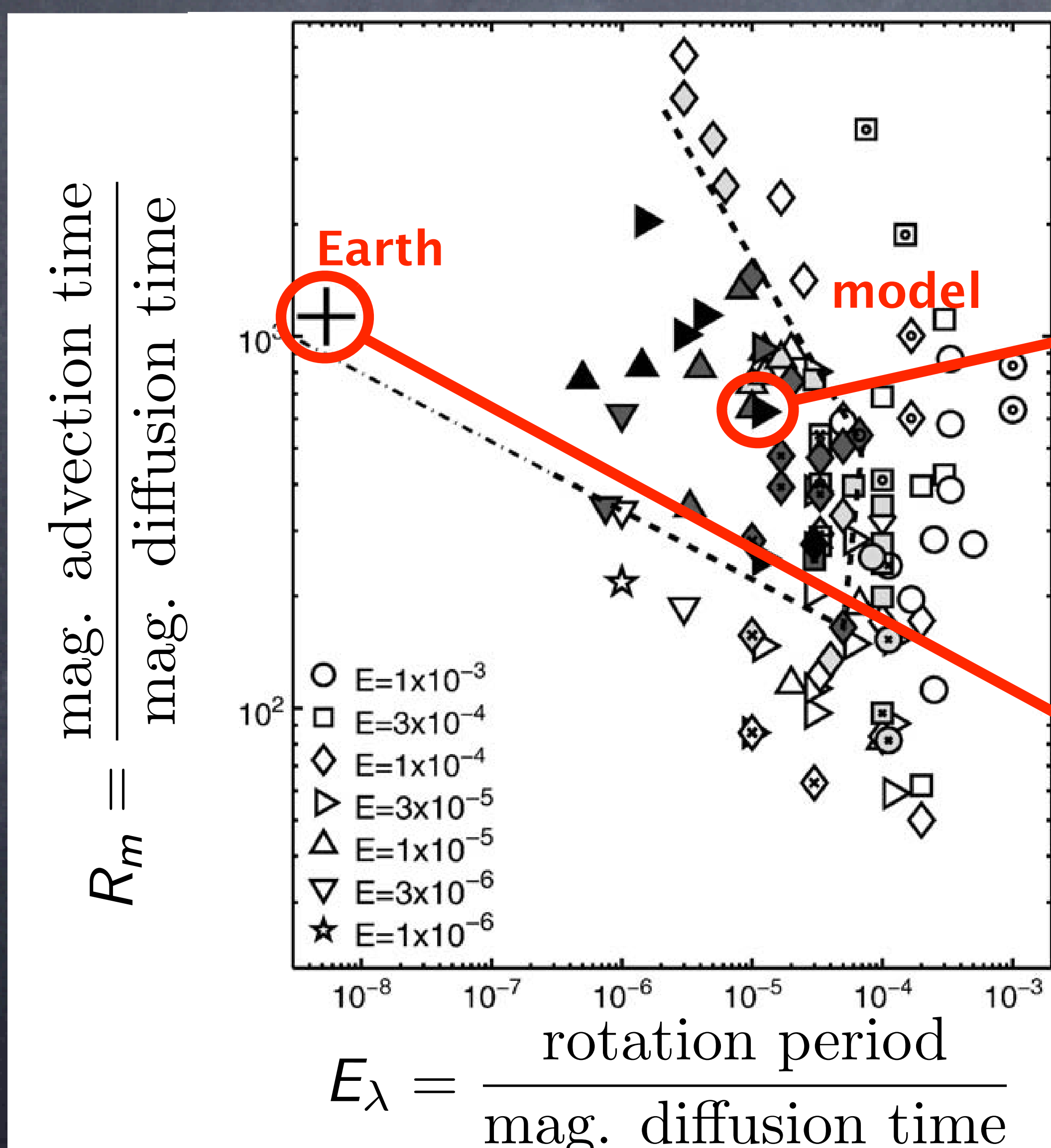
- We wish to estimate the deep flow structure of Earth’s core from the available surface magnetic observations.
- Our goals:
  - understand the dynamical regime of the core,
  - obtain a state vector for data assimilation practice.

Underdetermination

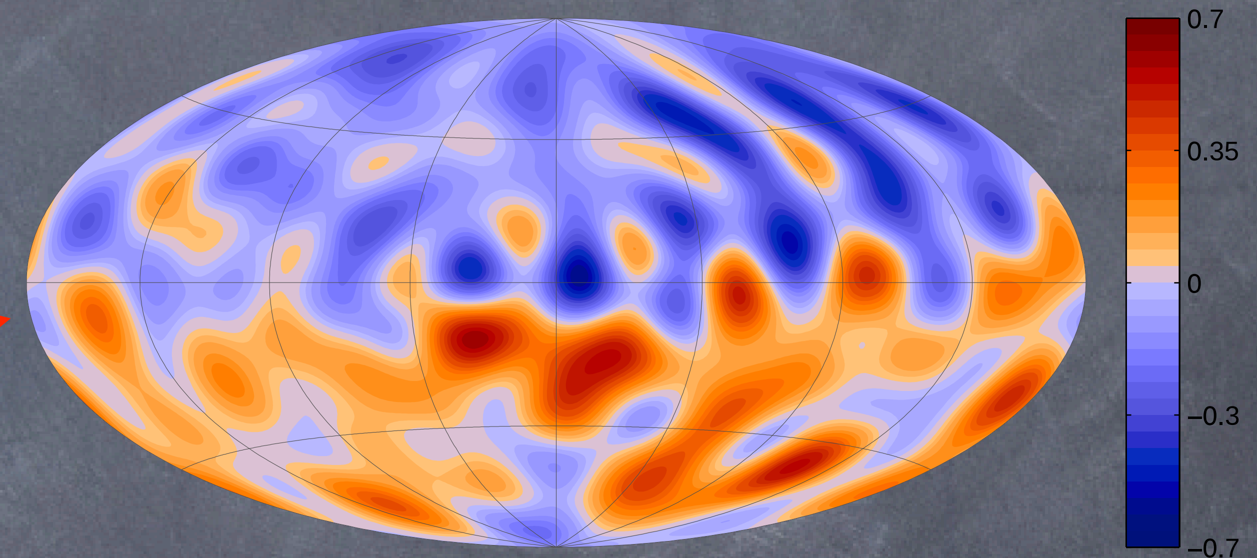
- State vector $\mathbf{x}(t_i) = [u^p_{lm}(r_j, t_i), u^t_{lm}(r_j, t_i), B^p_{lm}(r_j, t_i),$
$$B^t_{lm}(r_j, t_i), C_{lm}(r_j, t_i)]^T,$$
  on the nodes of a numerical discrete grid.
- Typical size is a million elements (in a 3D numerical model of the geodynamo)
- A set of magnetic observations is 200 elements (magnetic field and SV coefficients up to degree 8-13 of a typical geomagnetic field model).
- A prior is needed.

Using a numerical dynamo as a prior

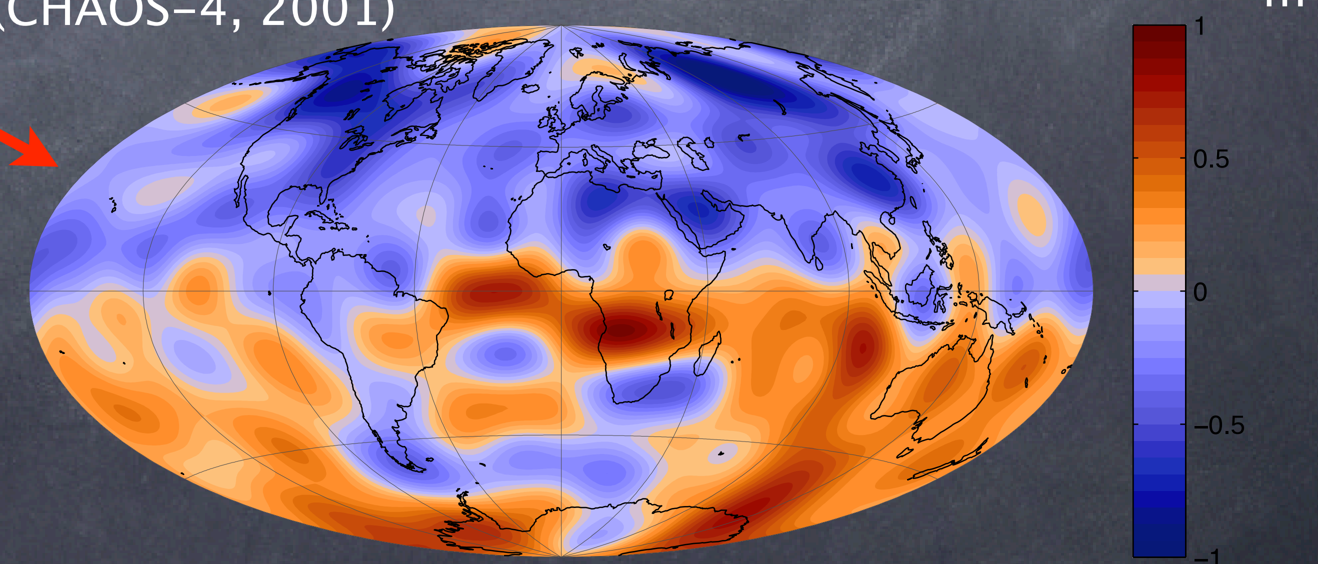
- Recent parameter space explorations and scaling relationships partially bridge the parameter gap between models and Earth.



Christensen, Aubert, Hulot 2010

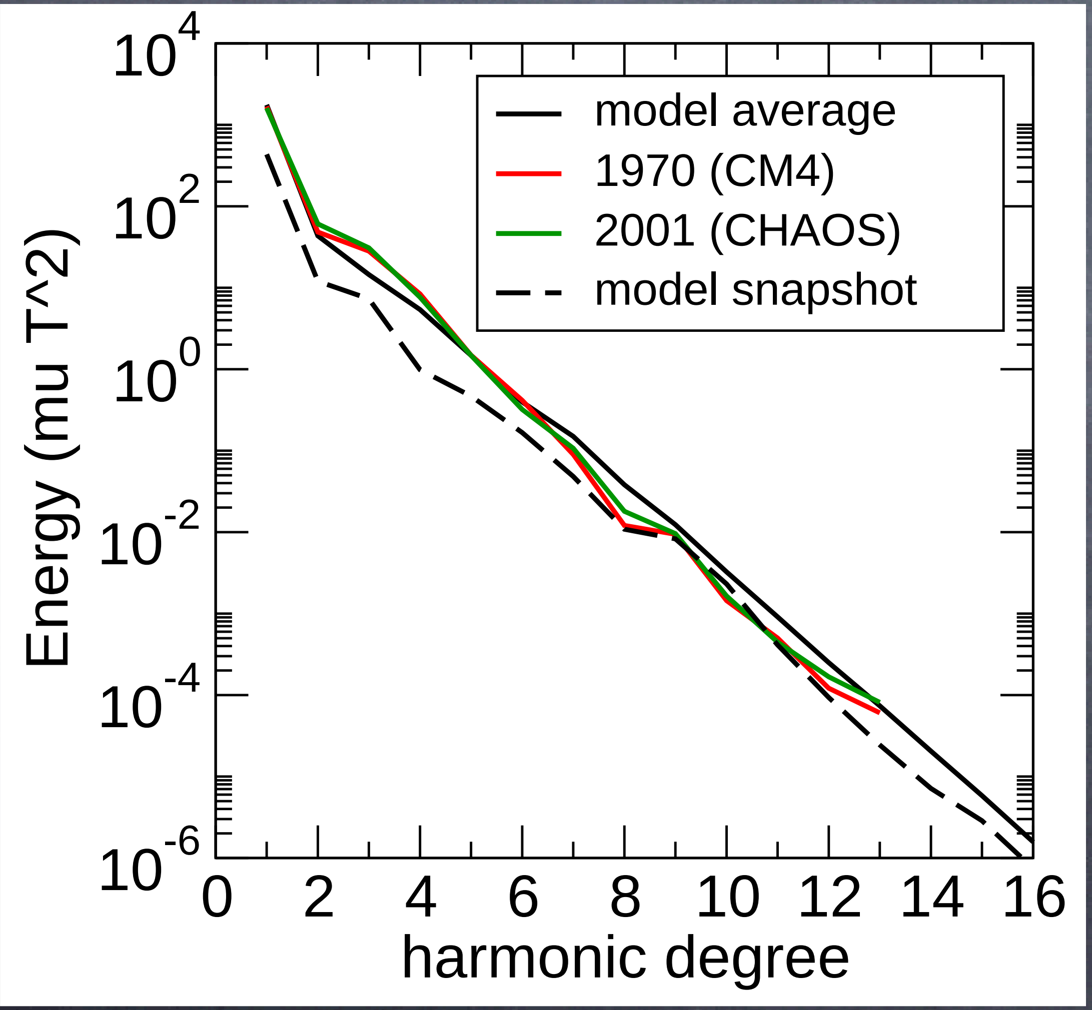


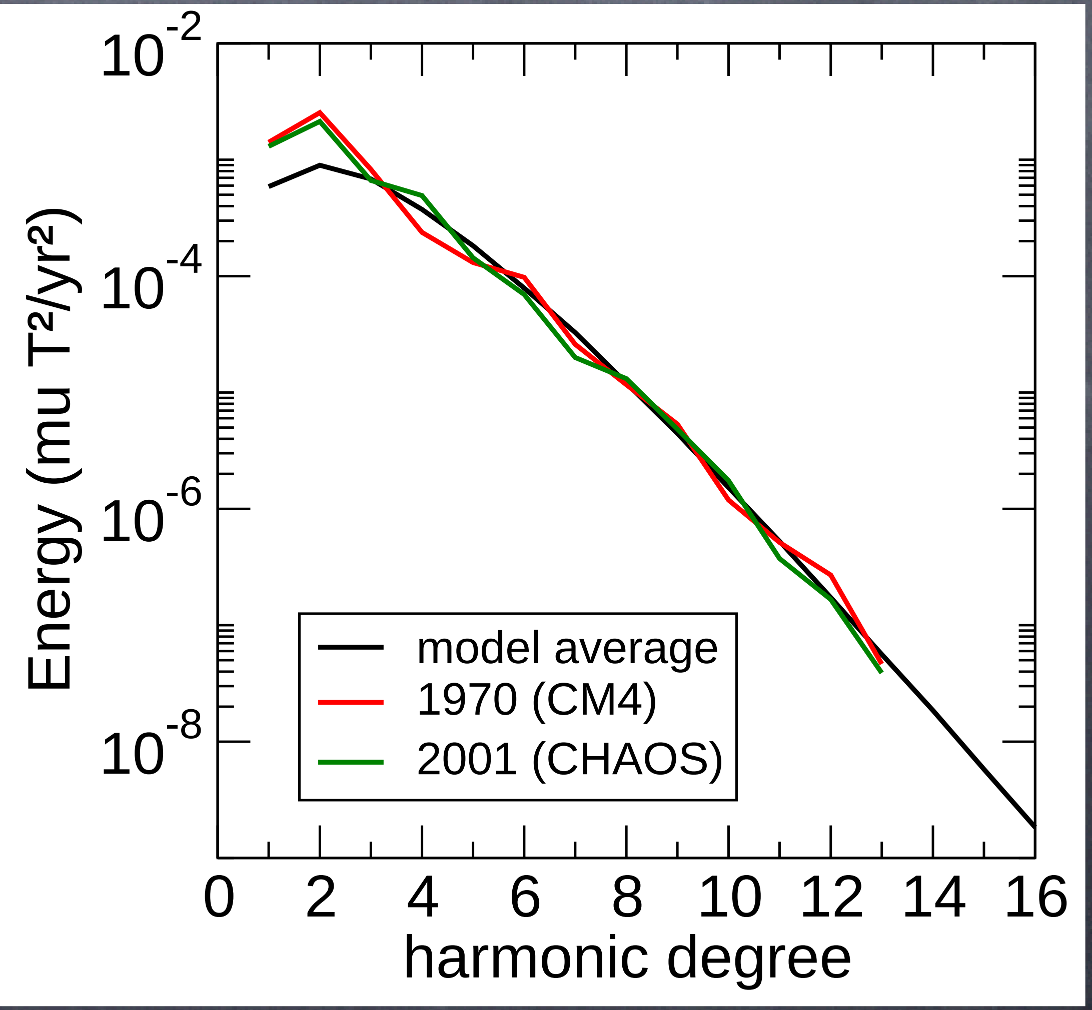
(CHAOS-4, 2001)



Scaling the model output

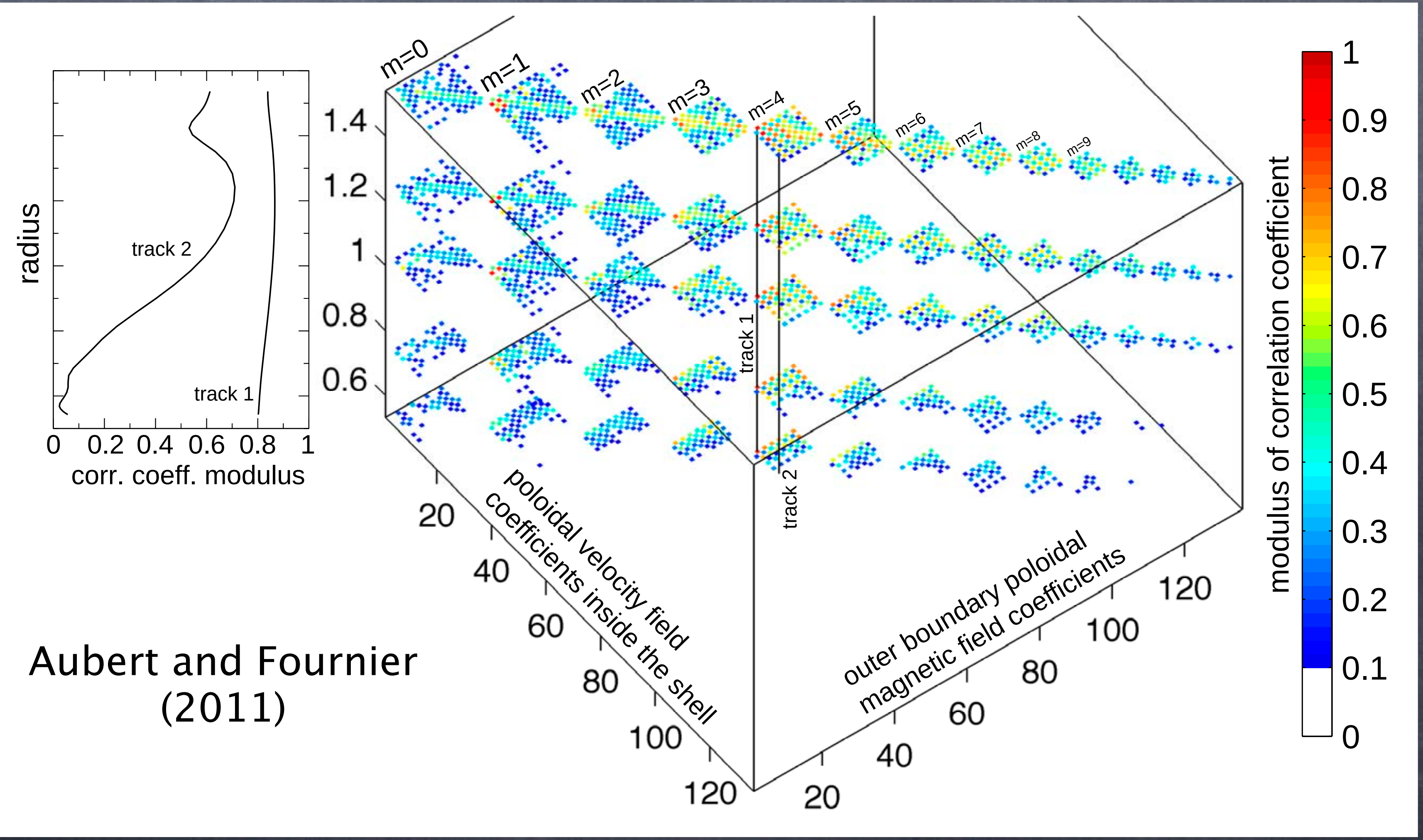
- Use scaling principles known (thought) to hold in the model (Earth)
    - magnetic field: power scaling (Christensen and Aubert 2006)
    - Time: advective time scaling (Lhuillier et al. 2011)





A numerical dynamo covariance matrix

- Prior information provided by the numerical dynamo is embedded in a statistical covariance matrix **P** which we directly compute using O(1000) decorrelated snapshots of the numerical dynamo.
- Probability to obtain **x** is $\mathcal{P}(\mathbf{x}) \propto \exp(-\mathbf{x}'\mathbf{P}^{-1}\mathbf{x}/2)$



Aubert and Fournier (2011)

Core surface flow inversions constrained by a numerical dynamo

- Below the CMB we invert the frozen-flux induction equation

$$\frac{\partial B_r}{\partial t} = -\nabla_H(\mathbf{u_{fs}} B_r)$$

- Can be formalised (in the spectral space) as

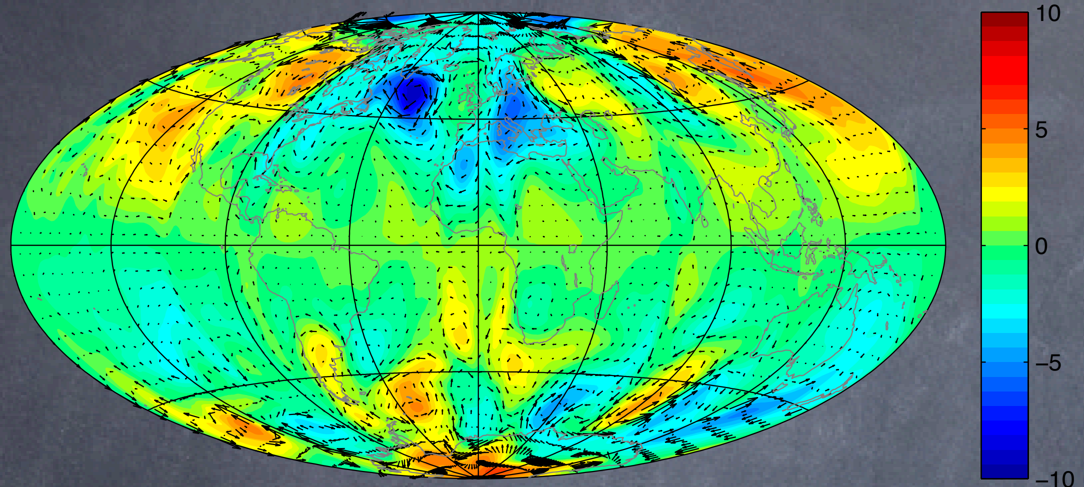
$$\mathbf{sv} = \mathbf{M} \cdot \mathbf{u_{fs}} + \epsilon^{\mathbf{o}}$$

- The inverse constrained by a numerical dynamo is

$$\mathbf{u_{fs}} = \mathbf{K} \cdot \mathbf{sv} \ , \ \mathbf{K} = \mathbf{P}\mathbf{M}' \left(\mathbf{M}\mathbf{P}\mathbf{M}' + \mathbf{R}\right)^{-1}$$

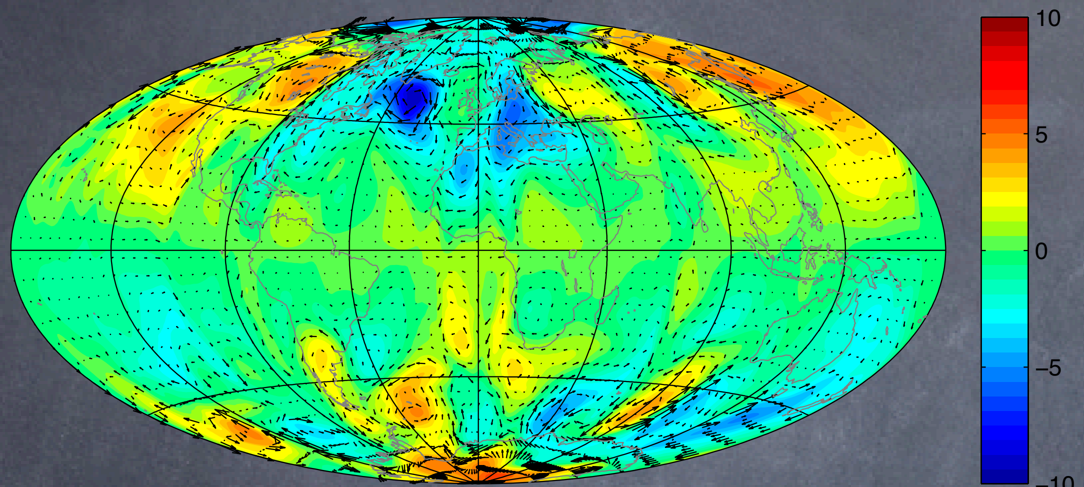
Validation on synthetics

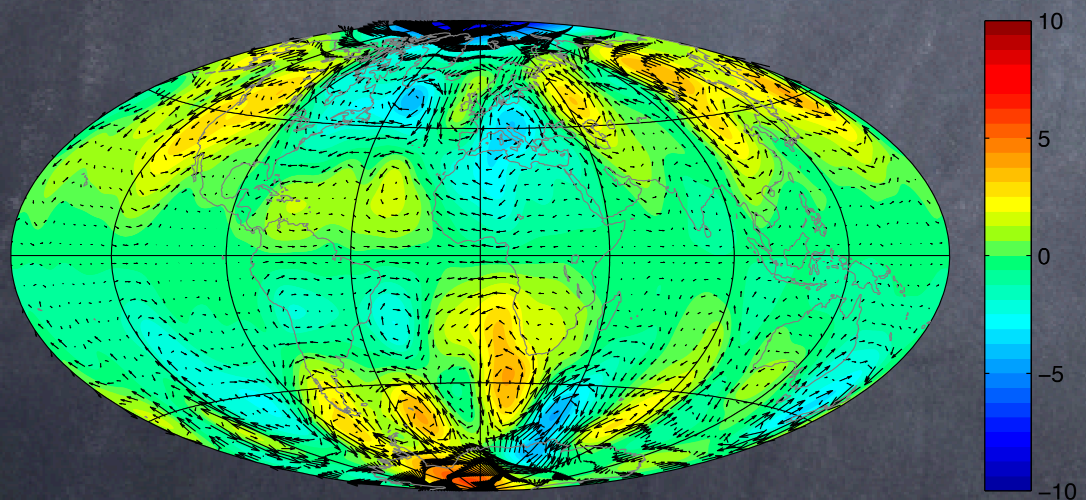
Synthetic reference velocity field

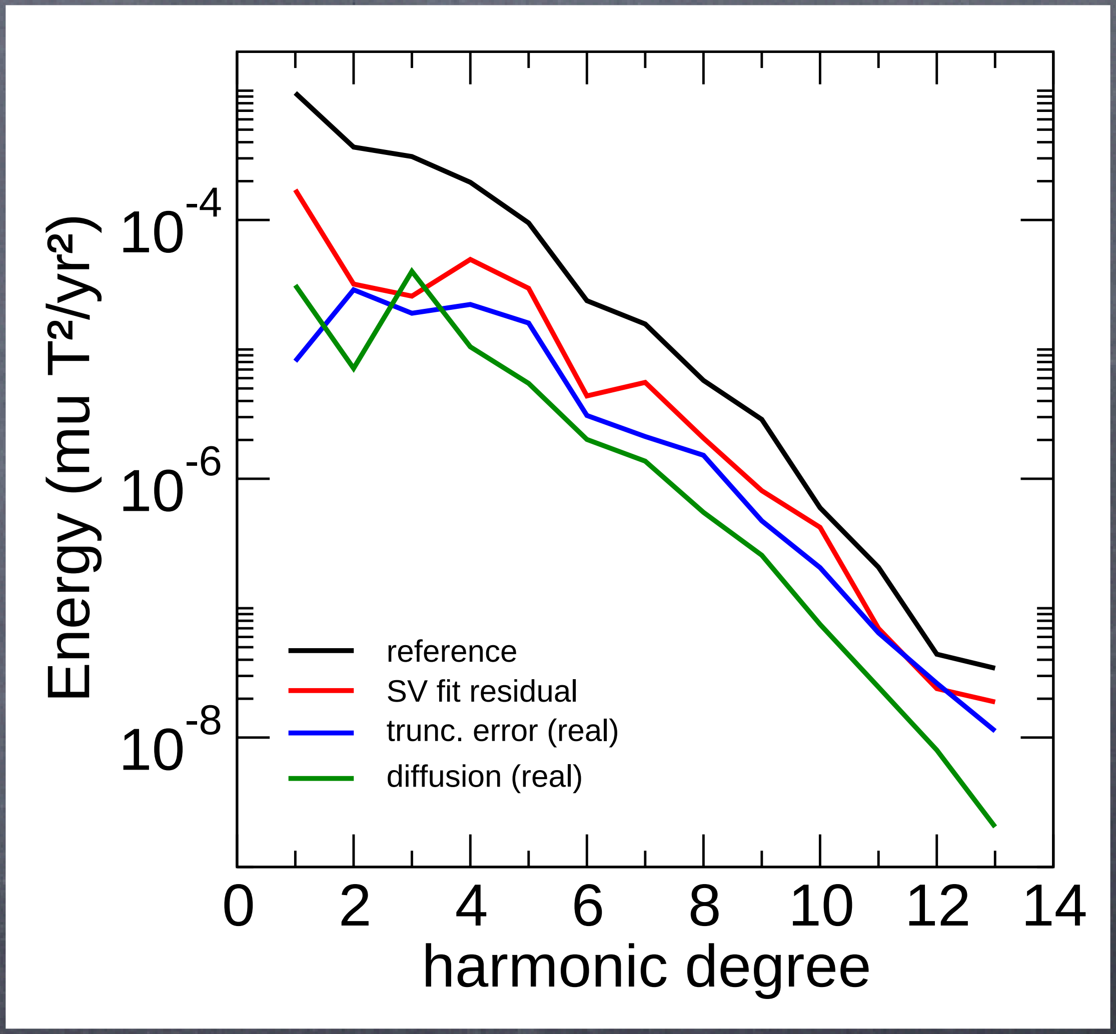


Validation on synthetics

Synthetic reference velocity field



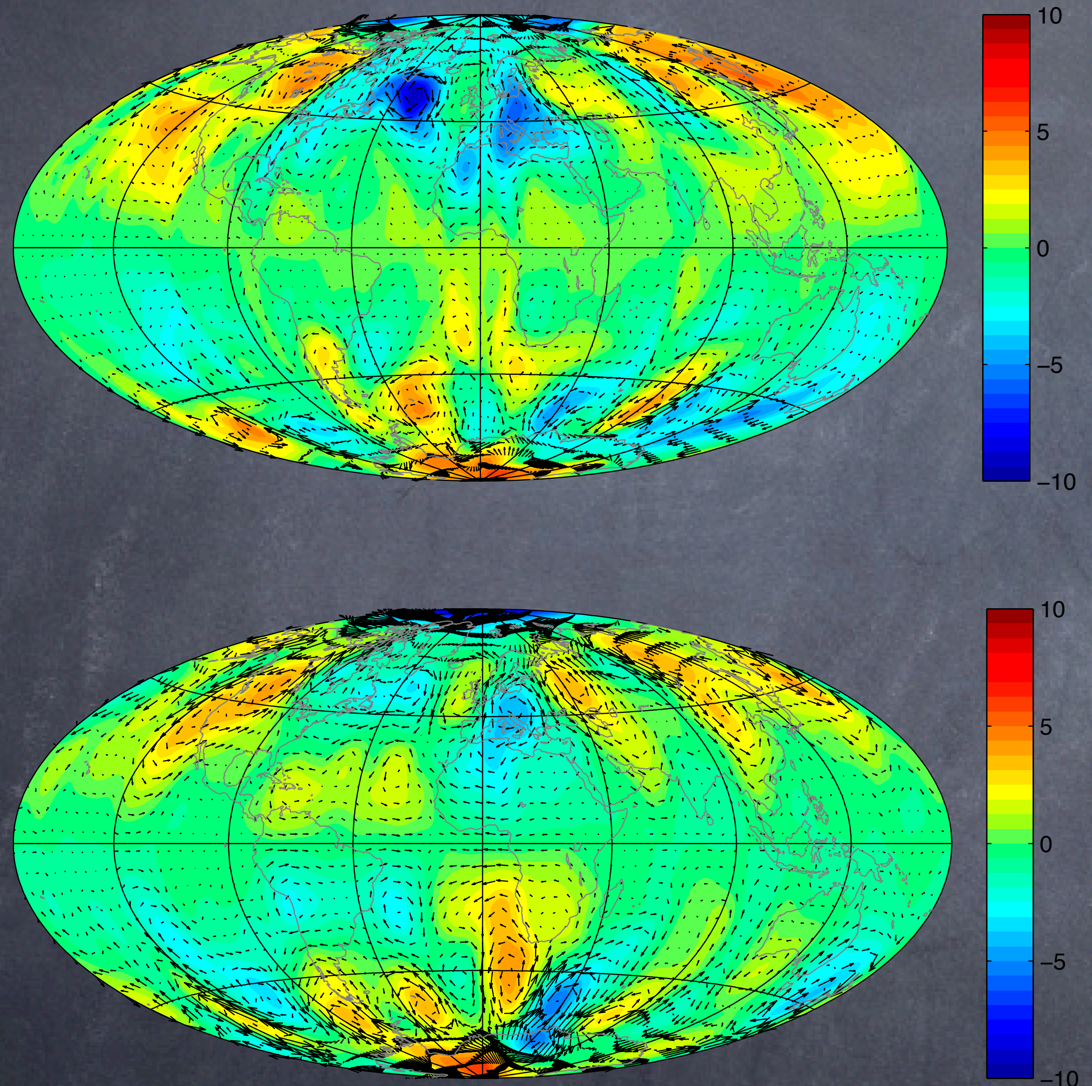


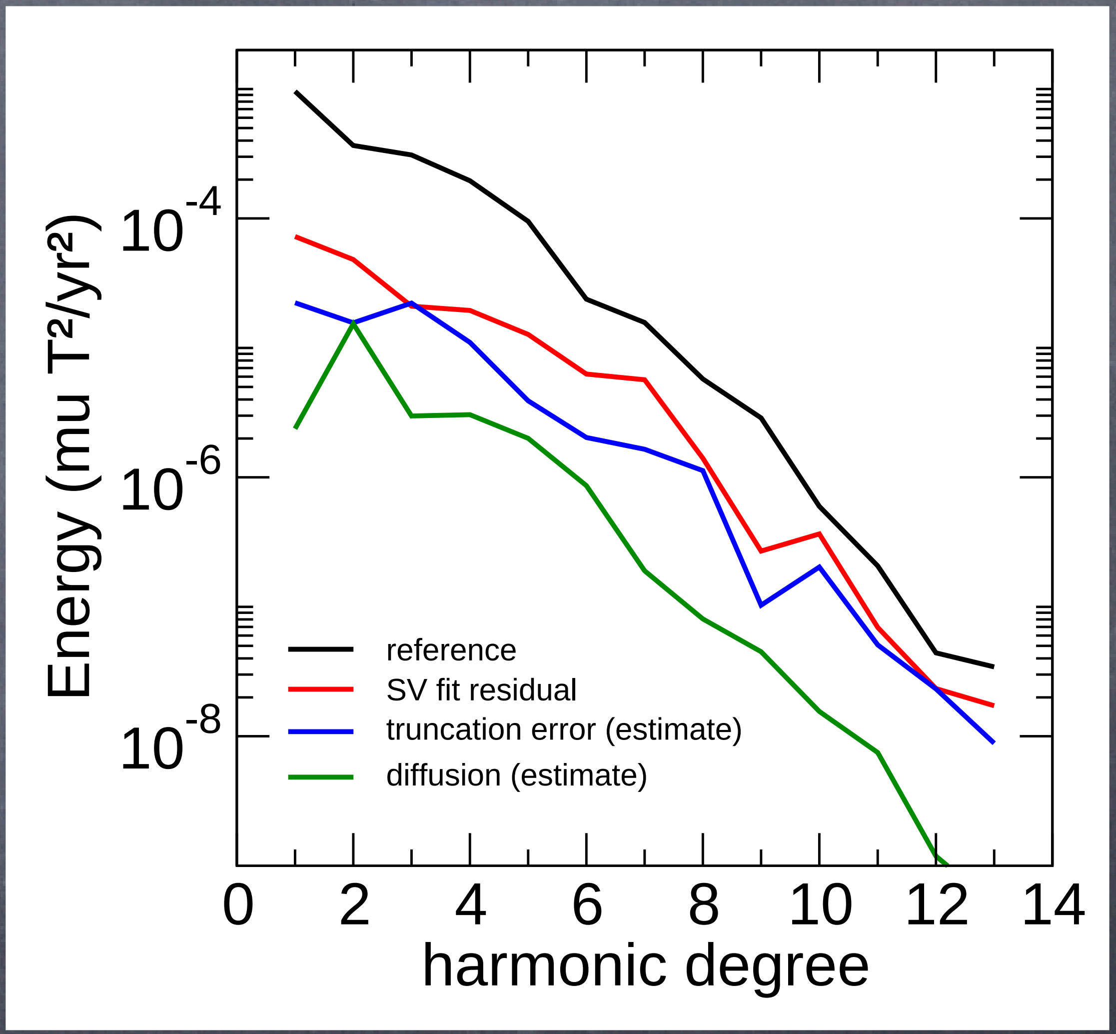


Inversion result if B and SV are known up to degree 13
true errors (= obtained knowing the reference)

Validation on synthetics

Synthetic reference velocity field





Inversion result if B and SV are known up to degree 13

estimated errors

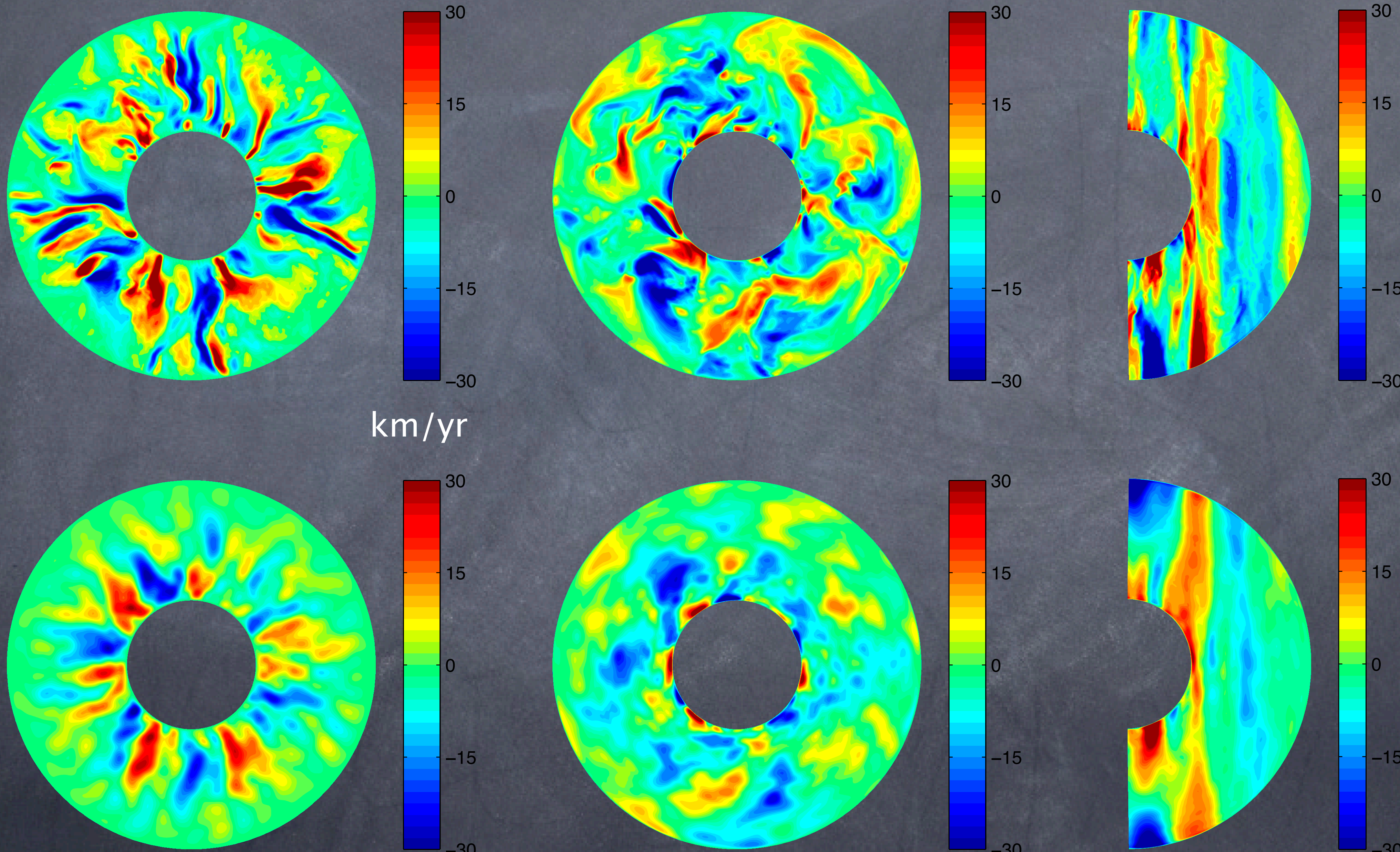
Inverting for the full core flow

- Observation operator

$$
\underbrace{\begin{pmatrix}
\ldots 0 & 1 & 0 & & & & & & & \\
 & \ddots & \ddots & \ddots & & & & & & \\
 & & 0 & 1 & 0 & \ldots & & & & \\
 & & & & \ldots & 0 & 1 & 0 & & \\
 & & & & & & \ddots & \ddots & \ddots & \\
 & & & & & & & 0 & 1 & 0\ldots
\end{pmatrix}}_{\mathbf{H}}
\underbrace{\begin{pmatrix}
u^p_{lm}(r_1) \\ \vdots \\ u^p_{10}(r_{fs}) \\ \vdots \\ u^p_{l_{max}l_{max}}(r_{fs}) \\ \vdots \\ u^p_{lm}(r_n) \\ u^t_{lm}(r_1) \\ \vdots \\ u^t_{10}(r_{fs}) \\ \vdots \\ u^t_{l_{max}l_{max}}(r_{fs}) \\ \vdots \\ u^t_{lm}(r_n)
\end{pmatrix}}_{\mathbf{u}}
=
\underbrace{\begin{pmatrix}
u^p_{10}(r_{fs}) \\ \vdots \\ u^p_{l_{max}l_{max}}(r_{fs}) \\ u^t_{10}(r_{fs}) \\ \vdots \\ u^t_{l_{max}l_{max}}(r_{fs})
\end{pmatrix}}_{\mathbf{u_{fs}}}
$$

- Direct problem $\mathbf{Hu} = \mathbf{u_{fs}}$
- Inverse solution $\mathbf{u} = \mathbf{Ku_{fs}}\ ,\ \mathbf{K} = \mathbf{PH}'\left(\mathbf{HPH}'\right)^{-1}$
- This solution is called the best linear unbiased estimate (BLUE), and can estimate other fields as well.

Validation on synthetics: flow inside the core

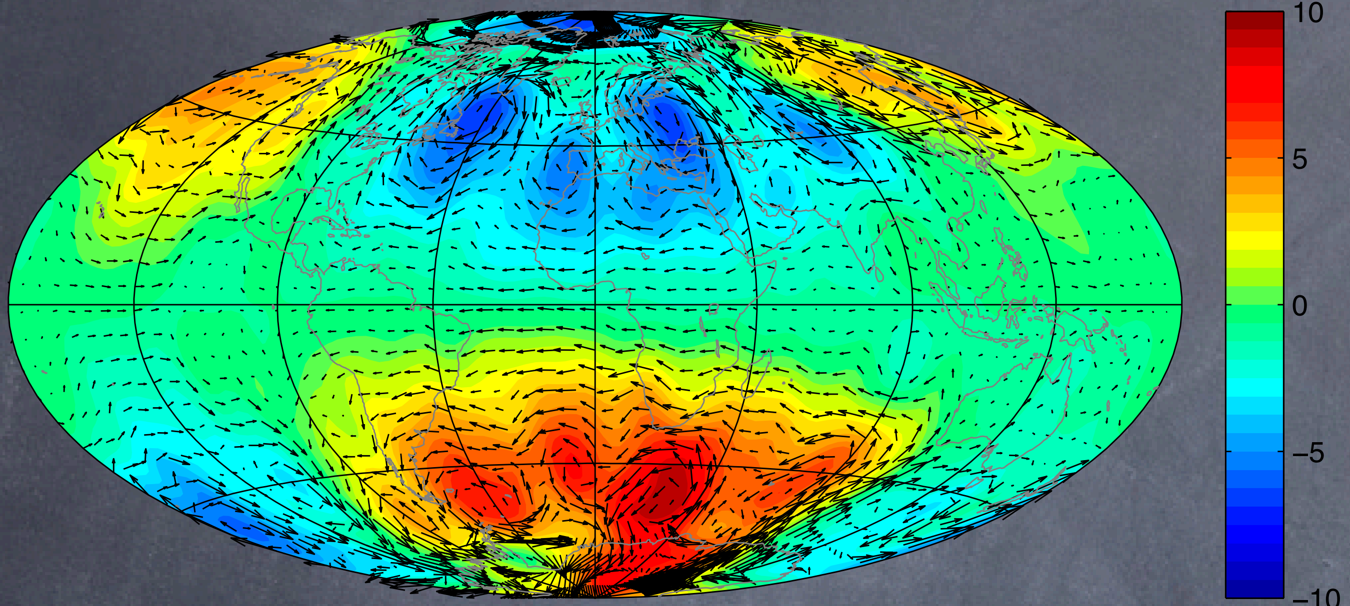


radial velocity
in equatorial plane

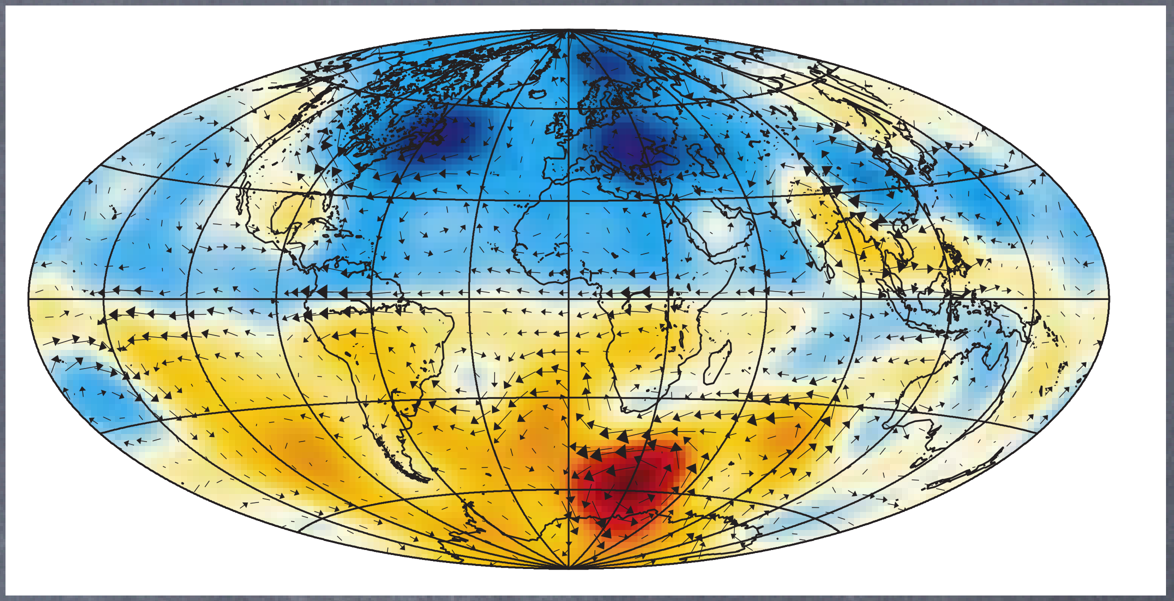
azimuthal velocity
in equatorial plane

azimuthal velocity
meridional cut

Inversions of geomagnetic field models



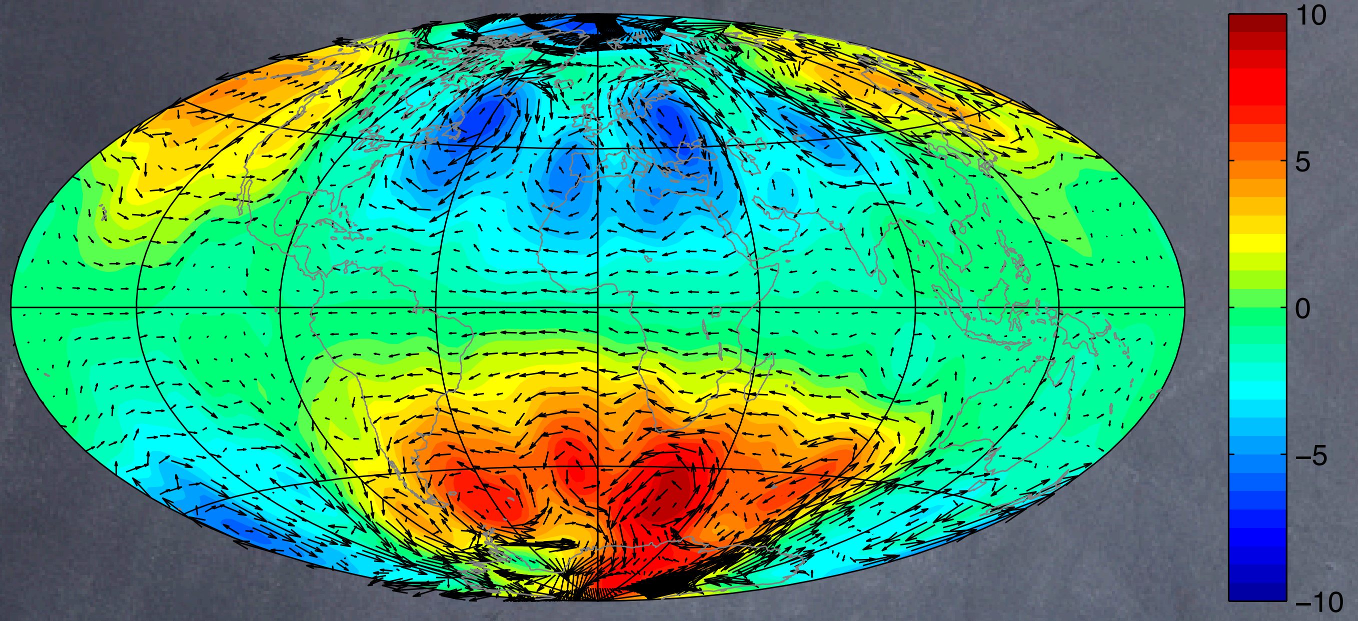
Flow in 1990 from CM4 (max 41 km/yr)



Hulot et al 2002 for 1990 (max 50km/yr)

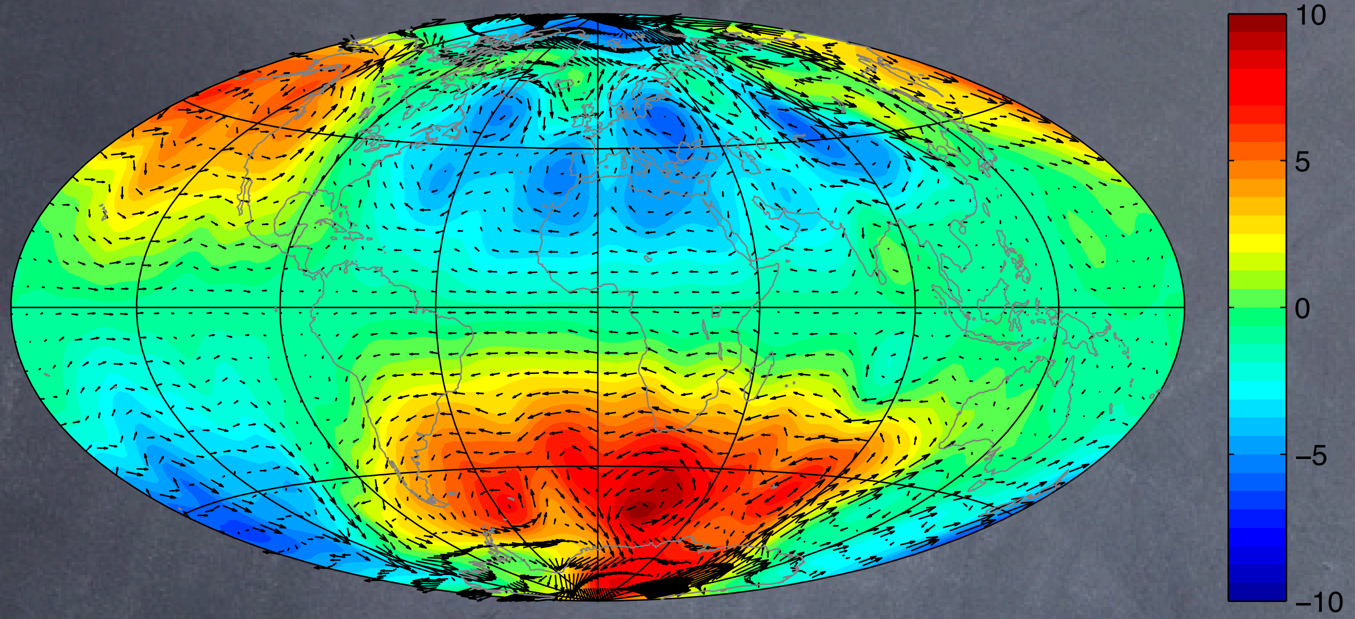
- large-scale vortices recovered
- added vortices as a result of a higher degree of equatorial symmetry
- westward drift in the Atlantic
- secondary structures in the Pacific

Inversions of geomagnetic field models

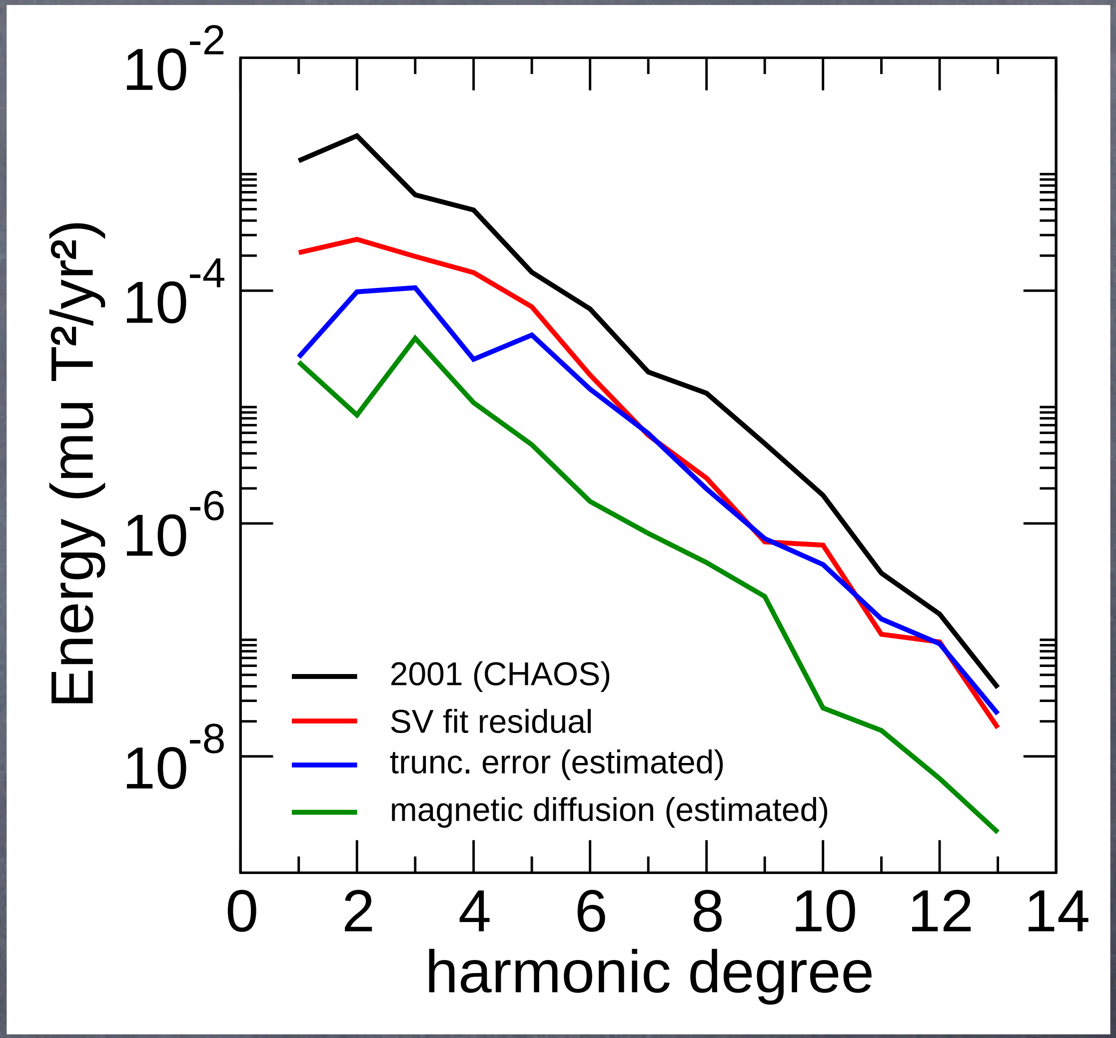


Flow in 1990 from CM4 (max 41 km/yr)

Inversions of geomagnetic field models



Flow in 2001 from CHAOS-4 (max 42 km/yr)



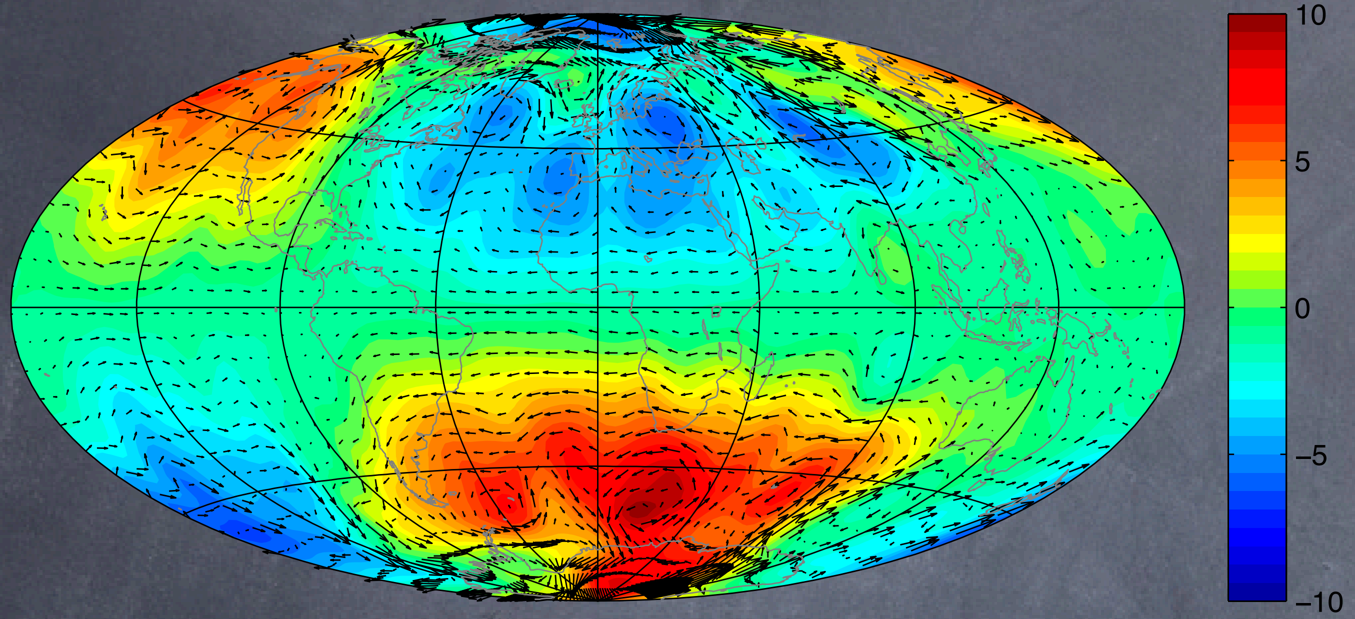
Normalised misfit to data

$$\sqrt{\frac{(\mathbf{sv}-\mathbf{M}\mathbf{u}_{\mathbf{fs}})'\mathbf{R}^{-1}(\mathbf{sv}-\mathbf{M}\mathbf{u}_{\mathbf{fs}})}{N_{sv}}}=1.1$$

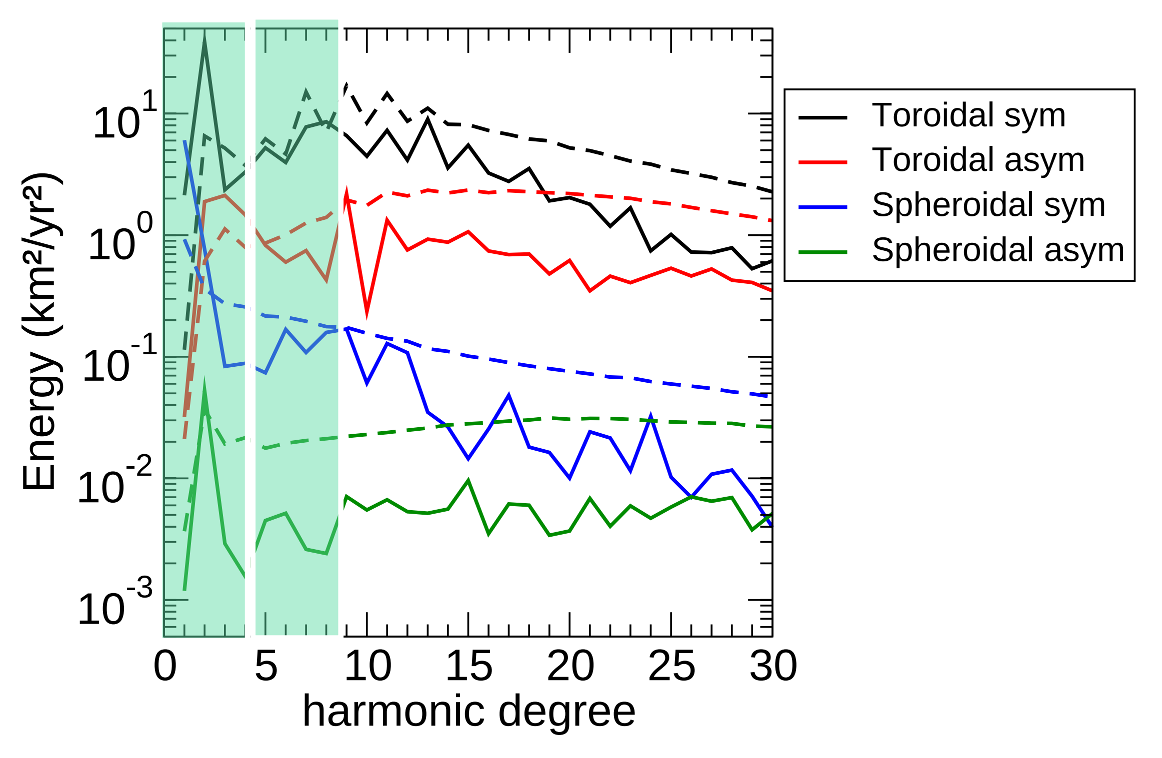
Normalised distance to numerical model time average

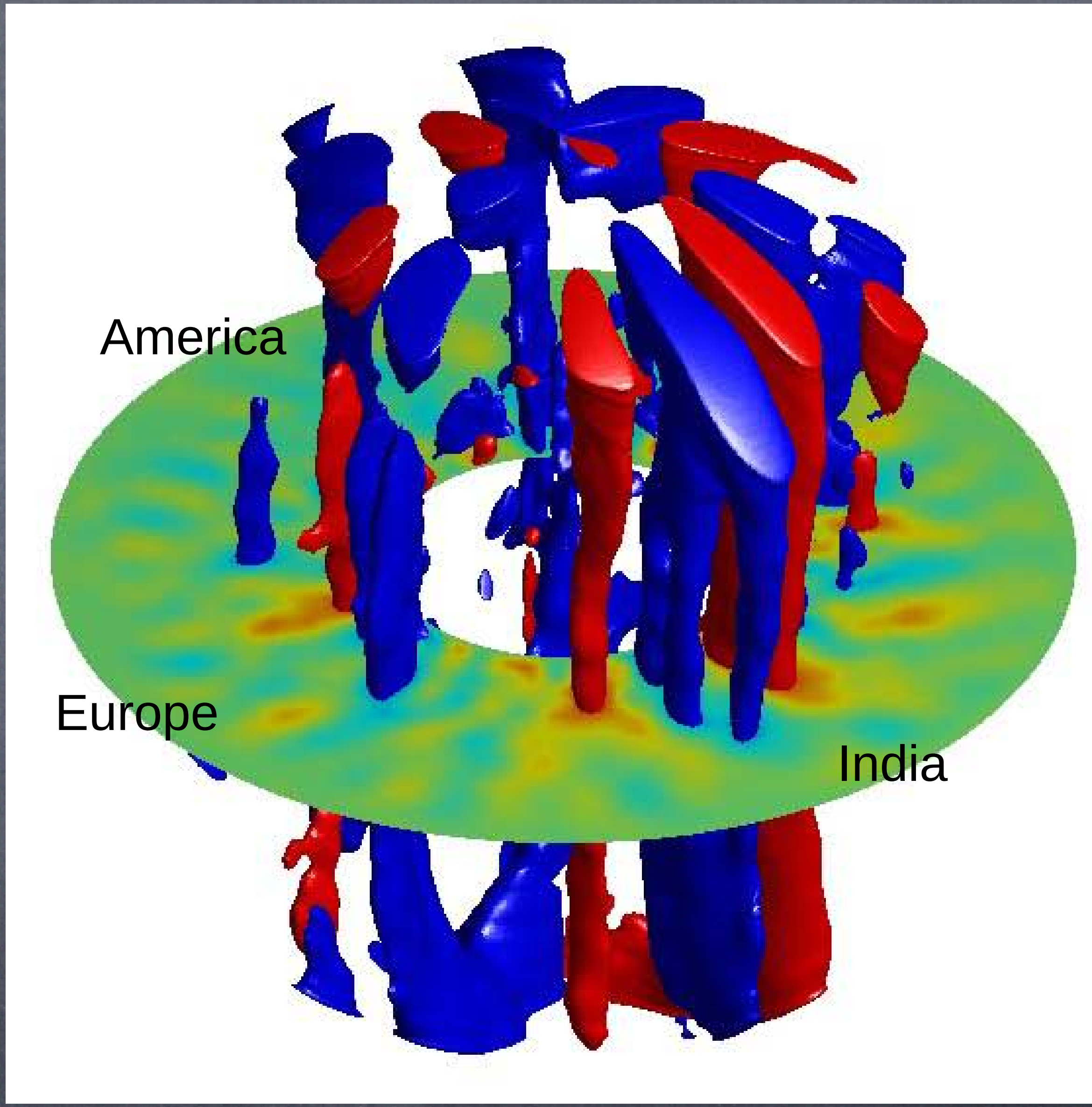
$$\sqrt{\frac{\mathbf{u}_{\mathbf{fs}}{}'\mathbf{P}^{-1}\mathbf{u}_{\mathbf{fs}}}{N_u}}=0.5$$

Inversions of geomagnetic field models



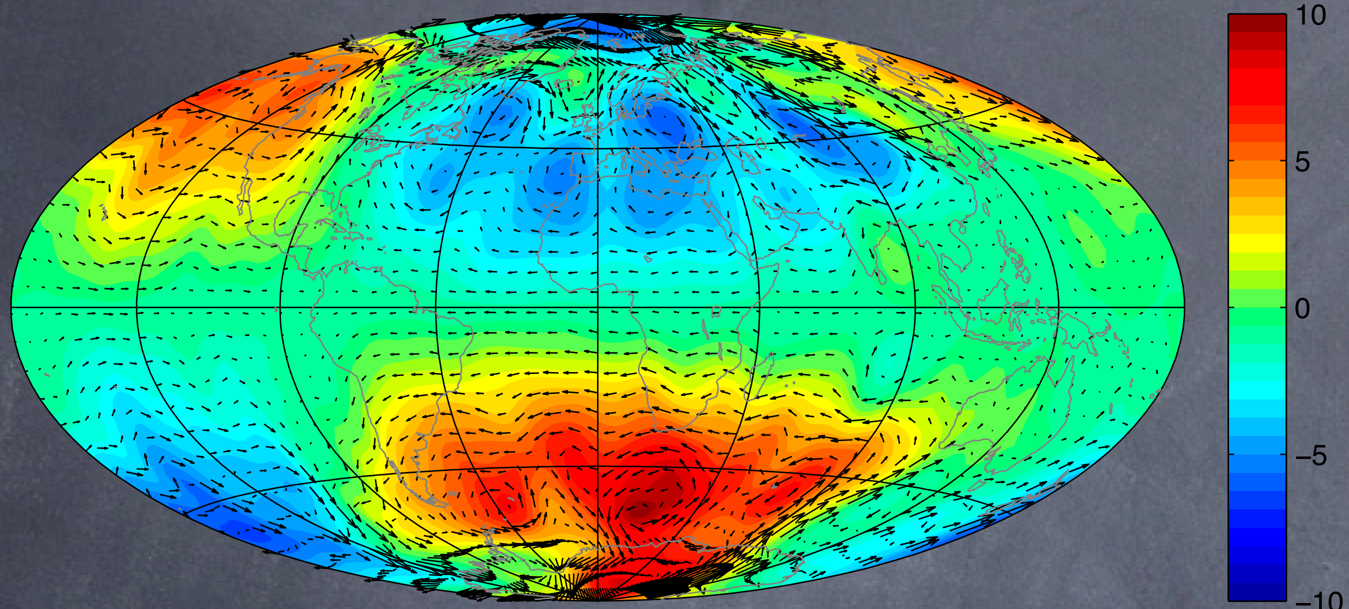
Flow in 2001 from CHAOS-4 (max 42 km/yr)





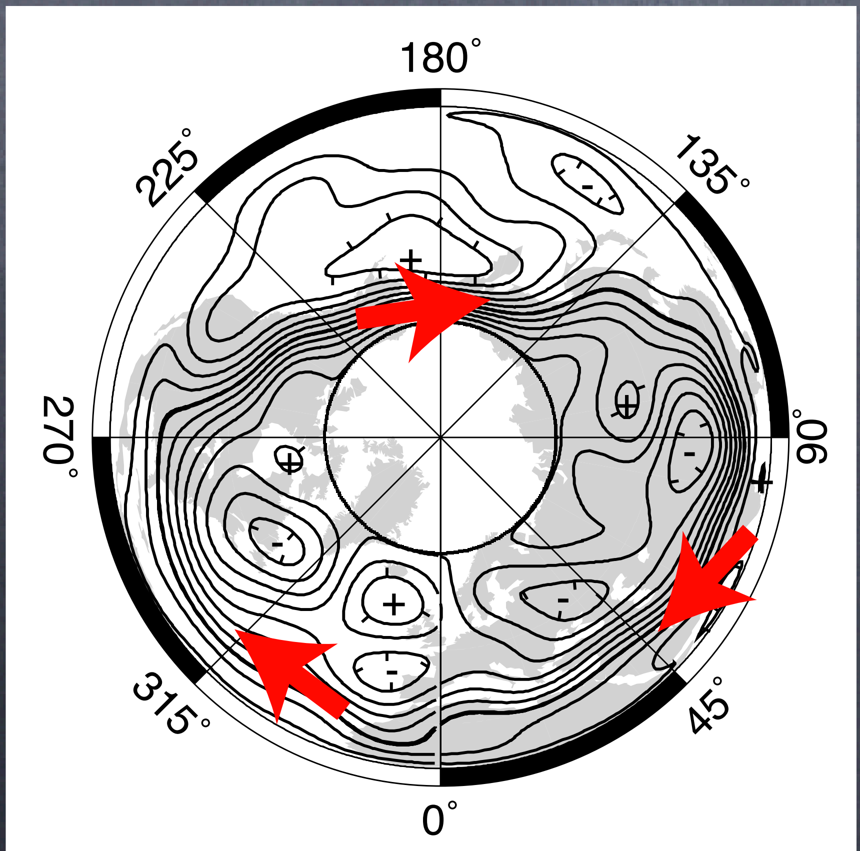
Cylindrical radial (axial) velocity

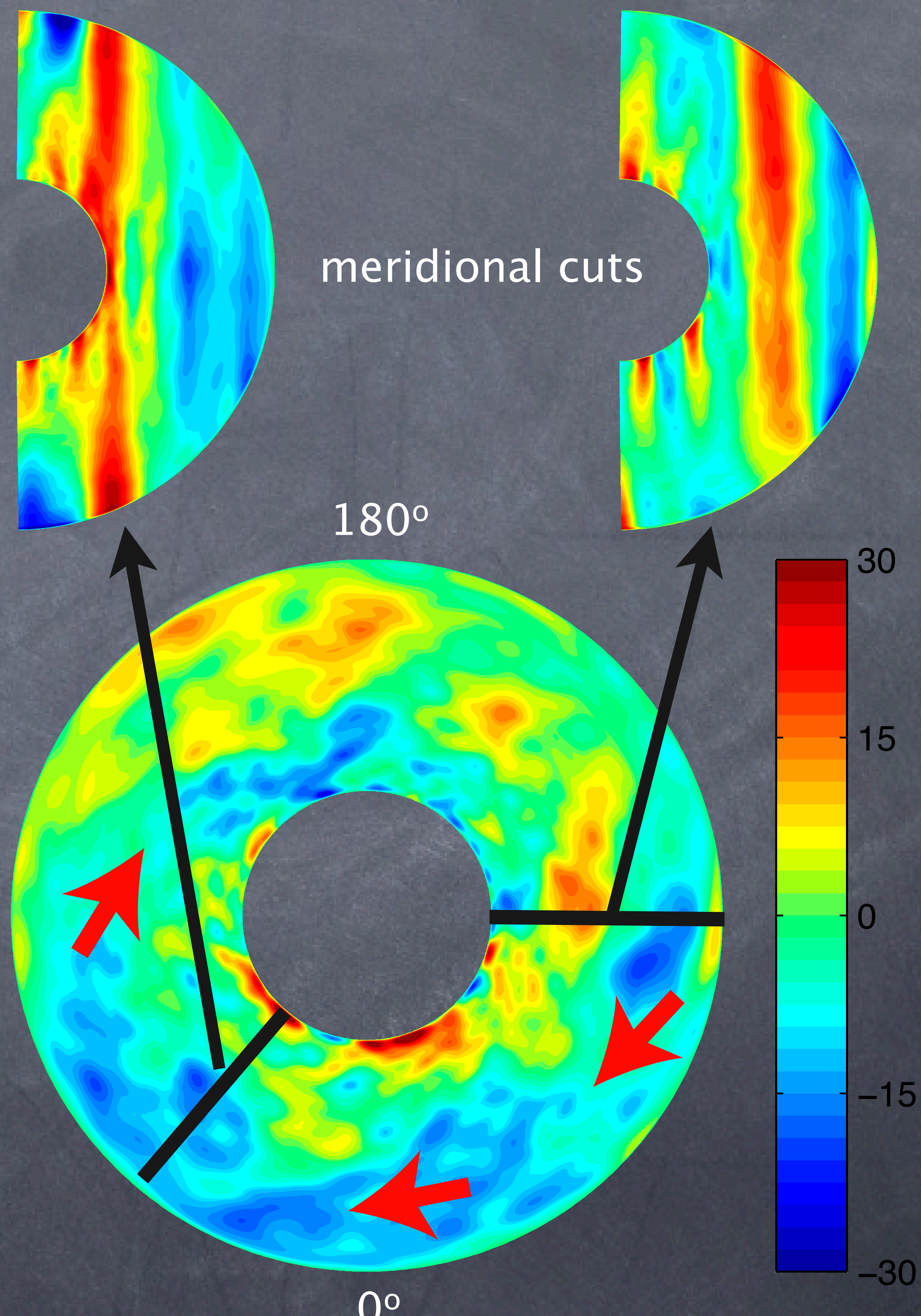
Inversions of geomagnetic field models



Flow in 2001 from CHAOS-4 (max 42 km/yr)

Pais and Jault 2008

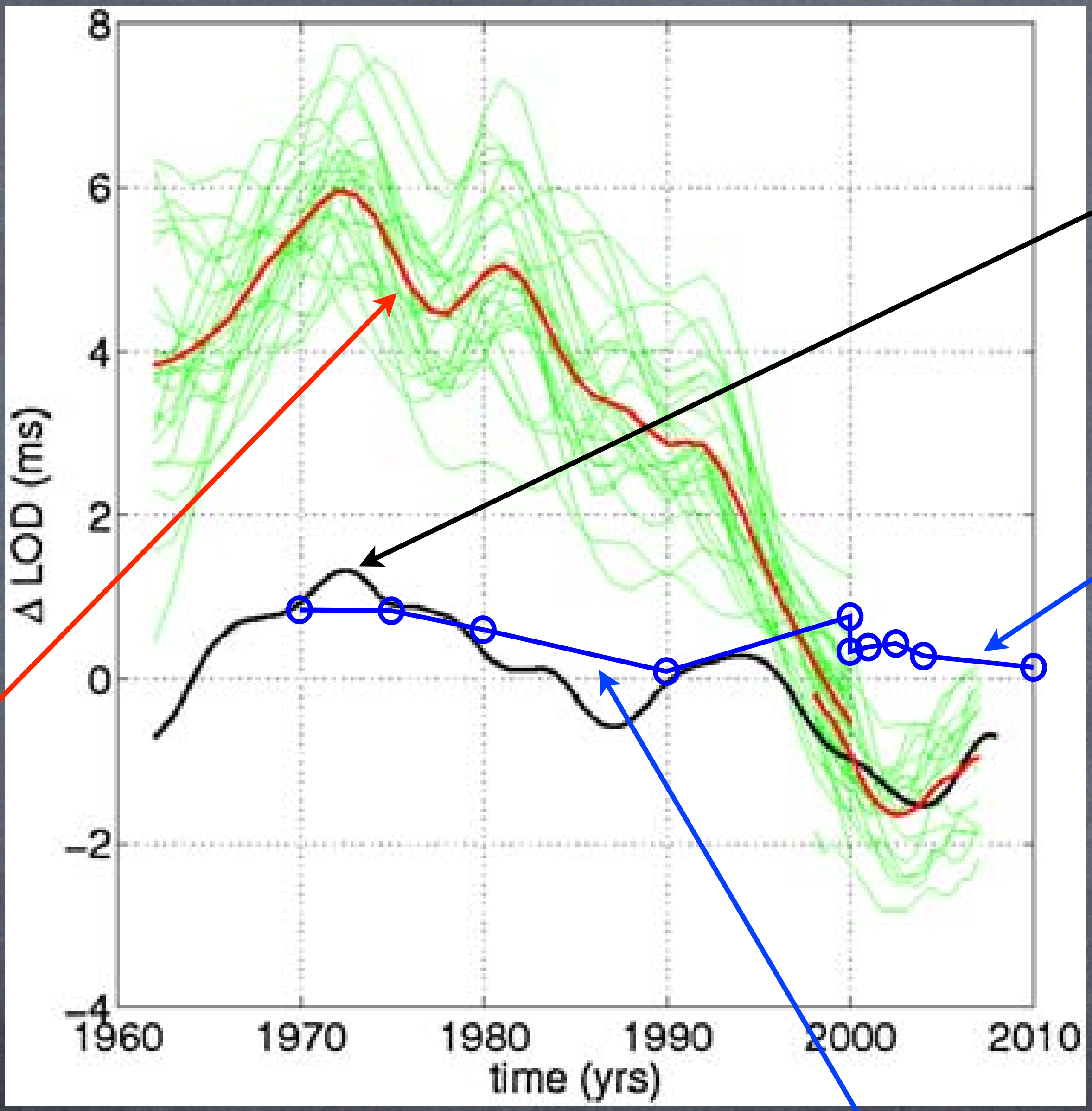




Azimuthal velocity in equatorial plane

Length of day variations

Background figure from Gillet et al. 2009

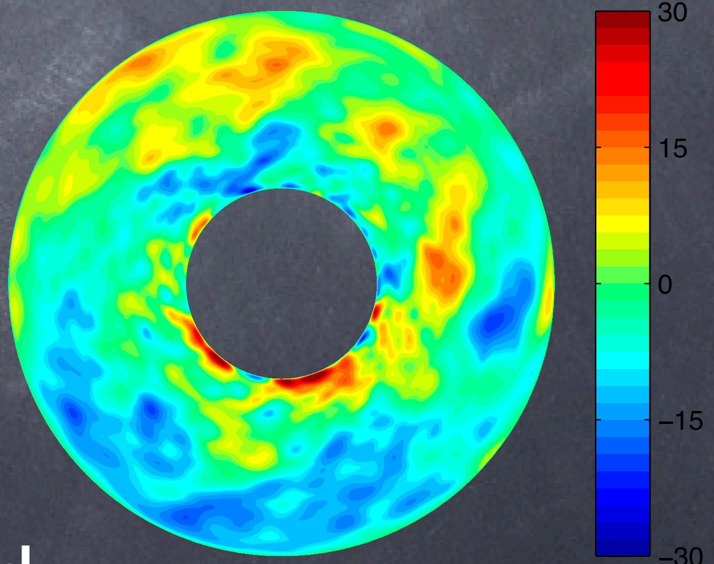


LOD series

This study (CHAOS-4)

This study (CM4)

Quasigeostrophic (ensemble) core flow inversions (Gillet et al. 2009)

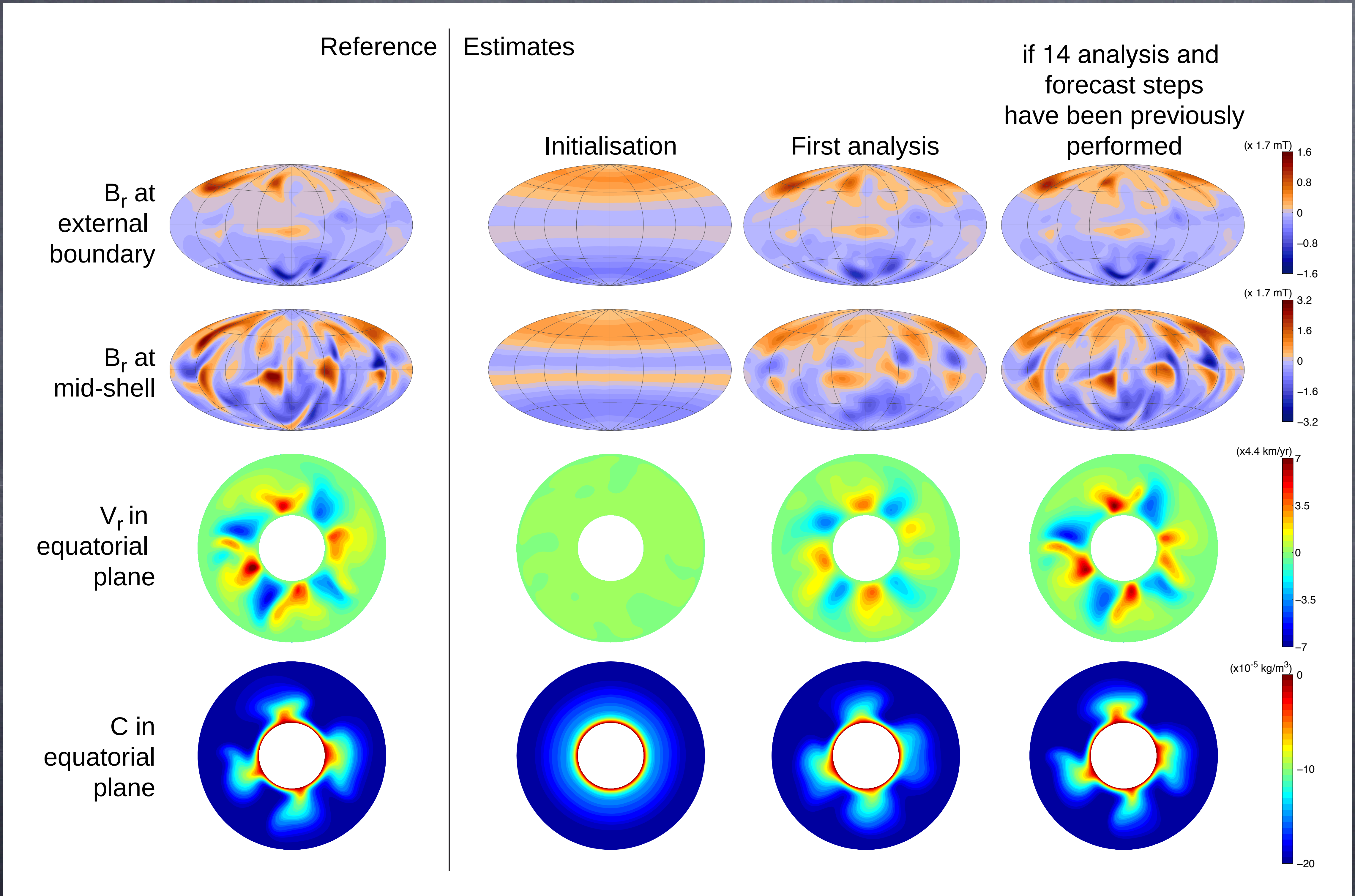


Azimuthal velocity in equatorial plane

Conclusions

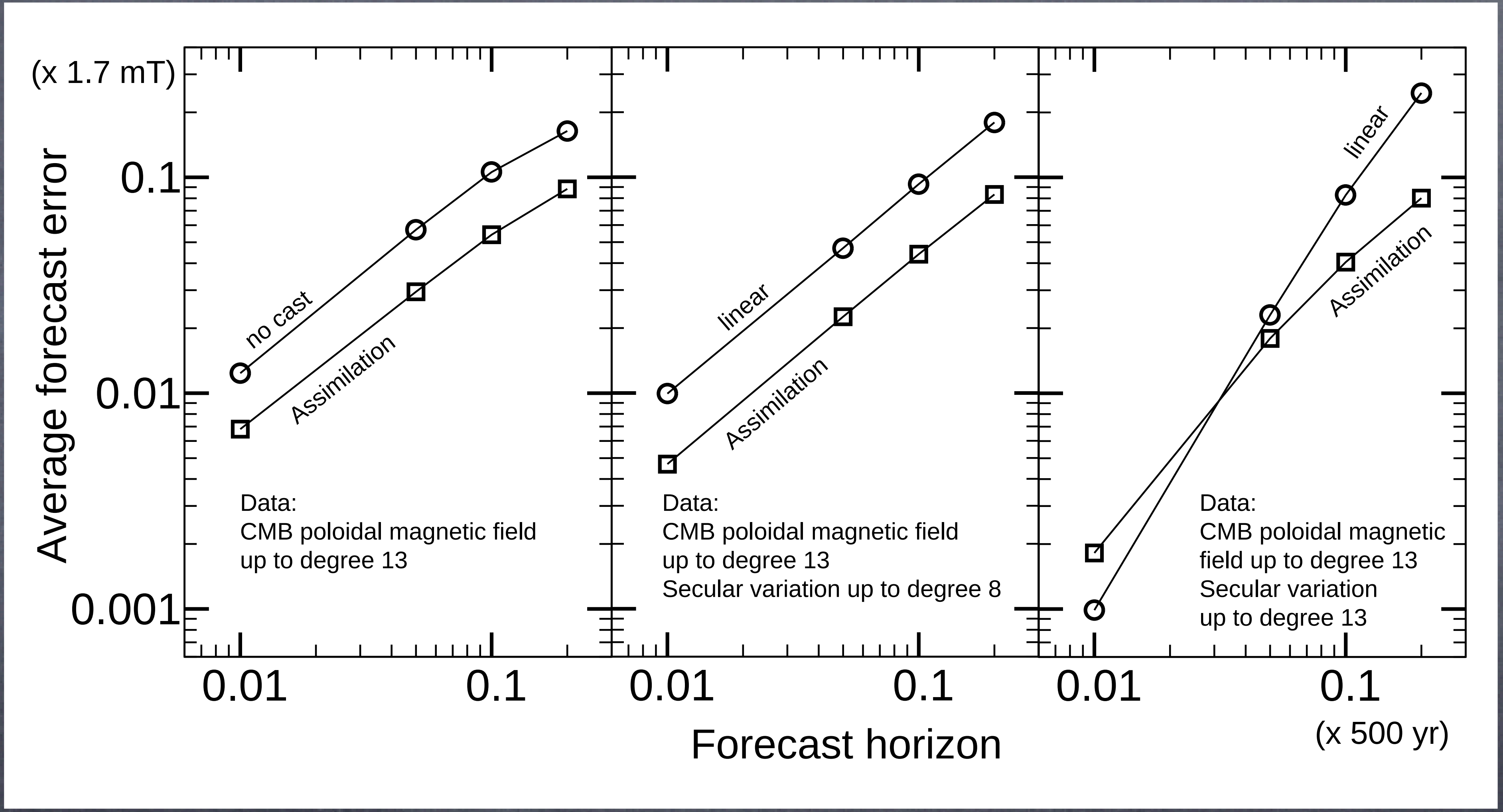
- geomagnetic data based flow inversions in the entire volume of Earth's core can be constrained with an Earth-like numerical dynamo model, with good model/data compatibility.
- The obtained flows have a columnar and large-scale character which exceeds that of the prior, thus strongly supporting columnar, large-scale convection in Earth's core.
- We thus start to see convergence between 3D and QG dynamical approaches.
- The presence of a giant eccentric retrograde equatorial gyre is confirmed.
- Predicted length-of-day variations are smaller than those induced by purely quasigeostrophic flows.

Outlook: geomagnetic data assimilation



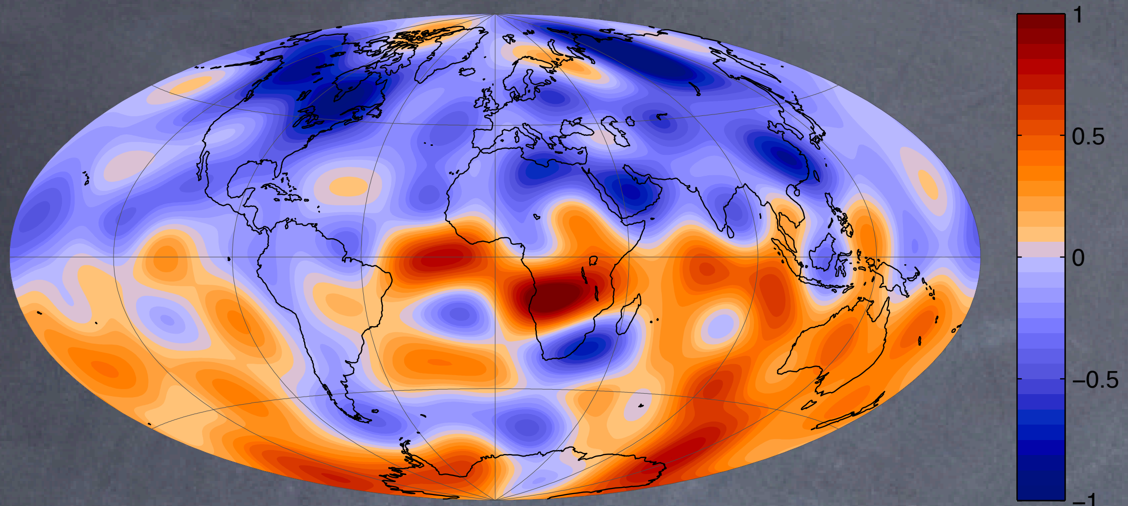
Aubert and Fournier (2011)

Outlook: geomagnetic data assimilation

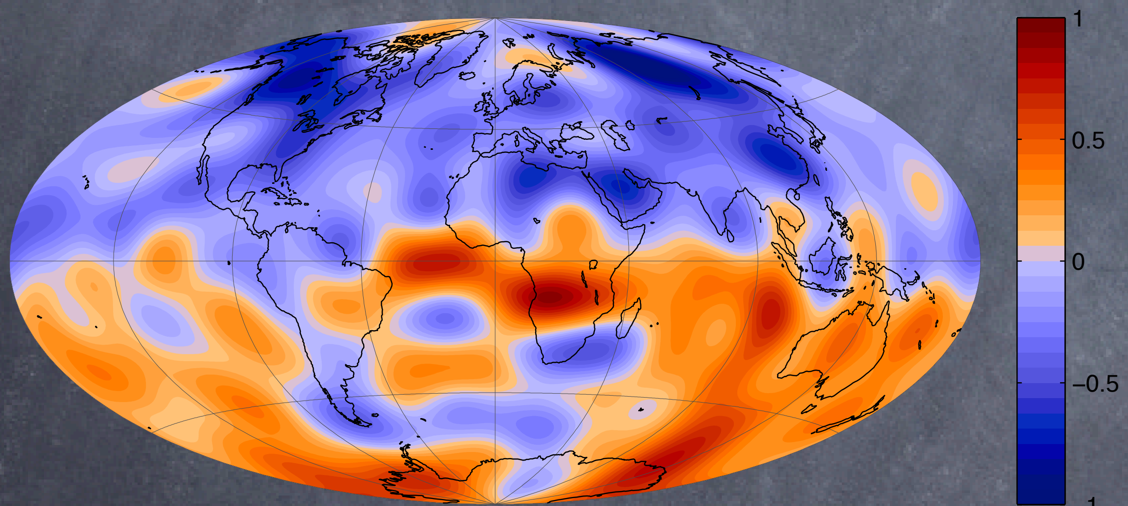


Aubert and Fournier (2011)

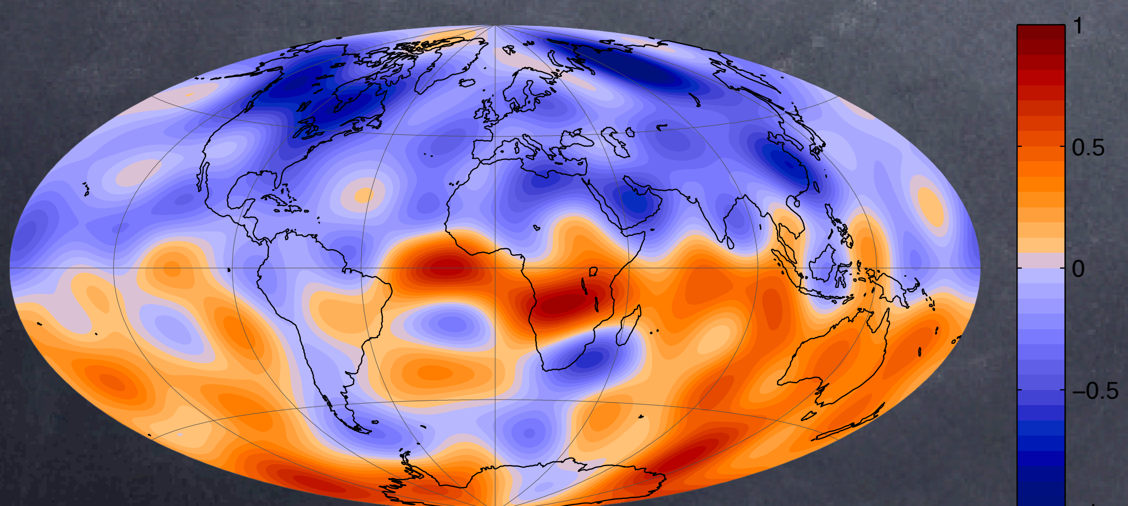
Long-term forecast of the field



Linear extrapolation in 2000 from 1970
rms error @CMB 0.11 mT

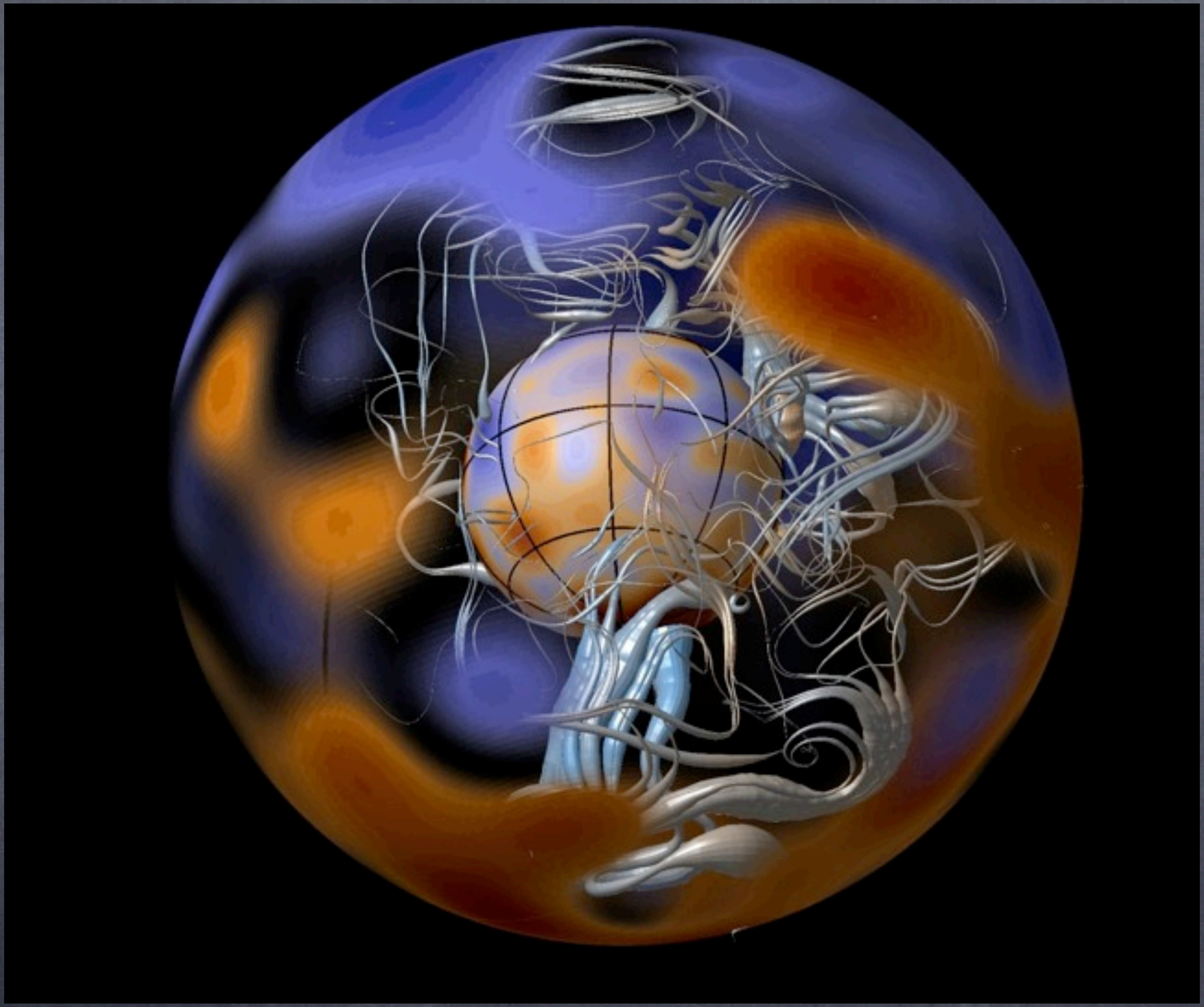


Reference: CHAOS-4 in 2000



Assimilation forecast in 2000 from 1970
rms error @CMB 0.07 mT

Imaging the interior magnetic structure



Magnetic state of the model in 2000
(viewed from south Atlantic)

ANR AVSGeomag

- 4-year ANR program 2011-2015
- IPGP/ISTerre
- PI Alex Fournier
- Progress in fundamental research on data assimilation in 3D and quasigeostrophic models of the geodynamo
- Acquire operational forecasting capacity by 2015, in time for IGRF 2015
- Acquire retrospective analysis capacity (improving knowledge of the past geomagnetic field)
- Stay tuned! `http://avsgeomag.ipgp.fr`