Climate Change, Directed Innovation, and Energy Transition: The Long-run Consequences of the Shale Gas Revolution

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Abstract

We look at the short and long term effects of a shale gas boom in an economy where energy can be produced with coal, or shale gas, or a clean energy source. In the short run, a shale gas revolution has counteracting effects on CO2 emissions: on the one hand it allows countries to substitute away from coal which in turn reduces CO2 emissions everything else equal; on the other hand the shale gas boom may increase pollution as it increases the scale of aggregate production. In the long run a shale gas boom tends to increase CO2 emissions as it induces firms to direct innovation away from clean innovation towards shale gas innovation, and we show the possibility of an infinitely delayed switch from shale gas to clean energy. We then derive conditions on the parameters under which, as a result of the above trade-off, the shale gas revolution reduces emissions in the short-run but increases emissions in the long-run. We then use data on electricity production to calibrate the model.

1 Introduction

Technological progress in shale gas extraction (specifically the combination of hydraulic fracturing and horizontal drilling) has led to a boom in the natural gas industry in the United States. As shown in Figure 1, shale gas production in the United States increased more than threefold between January 2005 and January 2010, and it has increased close to 5 times more from January 2010 to December 2018. This shale gas boom has revolutionized energy production in the United States. Figure 2.A shows that natural gas started displacing coal at a much faster rate from 2009 so that today natural gas is more important than coal in electricity production. Panel.B shows the effect of the shale gas boom on CO_2 emissions in the electricity sector. Natural gas causes much less emissions than coal, as a result the CO_2 emission intensity of the electricity sector has declined by around a quarter in a few years. In fact, CO_2 emissions from the electricity sector peaked in 2007 and have kept declining since.[1]

Interestingly, at the time of the shale gas boom, innovation in clean technologies in electricity has collapsed. Figure 3 shows that patenting in renewables or more generally in green energy (which includes renewables, biofuels and nuclear) has collapsed with the shale gas boom, both as a share of total patents and as a ratio relative to patents in fossil fuel electricity generation.[2] For instance, patenting in renewables in the US has gone from representing 0.4% of total patents in 2009 to close to 0.1% in 2015. If the shale gas boom reduced emissions in the short-run at the cost of displacing innovation toward truly green technologies, then its overall effect on emissions and climate change is much less clear.

This paper investigates the short and long term effects of a shale gas boom in an economy where energy can be produced with coal, natural gas, or a clean energy source. In the first part of the paper we develop a simple framework to highlight the key trade-offs involved in allowing for improvements in the intermediate source of energy (specifically, in the extraction technology of natural gas). The final good is produced with an intermediate input and with energy. Energy is itself produced using coal, and/or natural gas, and/or a green source of energy (think of eolian). Fossil fuel energy - coal or natural gas - are produced using a combination of resource use and an energy input (think of a power plant). The green energy is produced using only energy input. Resource use in energy production in turn generates pollution, with higher pollution propensity for coal than for shale gas.

The model delivers the following insights. In the short run, there are two effects of a shale gas boom: a substitution effect and a scale effect. First, the substitution effect: a shale gas boom helps substitute natural gas energy for both, coal energy (this reduces emission) and green energy (this increases emission). The overall substitution effect leads to a reduction in aggregate pollution when coal use is sufficiently more polluting than natural gas use. Second, the scale effect: a shale gas boom makes overall energy production cheaper which leads to an increase in overall energy consumption and therefore to an increase in aggregate pollution. Overall, in the short-run a shale gas boom will reduce pollution when the substitution effect is sufficiently negative and large compared to the scale effect. This in turn occurs when coal is sufficiently more polluting than natural gas at the margin.

In the long run, a shale gas boom tends to postpone the switch toward green innovation, i.e. towards innovating in the energy input production technology for clean energy. In fact,

[1] All data here are taken from the EIA. The pattern of Figure 2.B also applies for the entire economy. See Appendix 7.

[2] Details on data construction are given in section 2

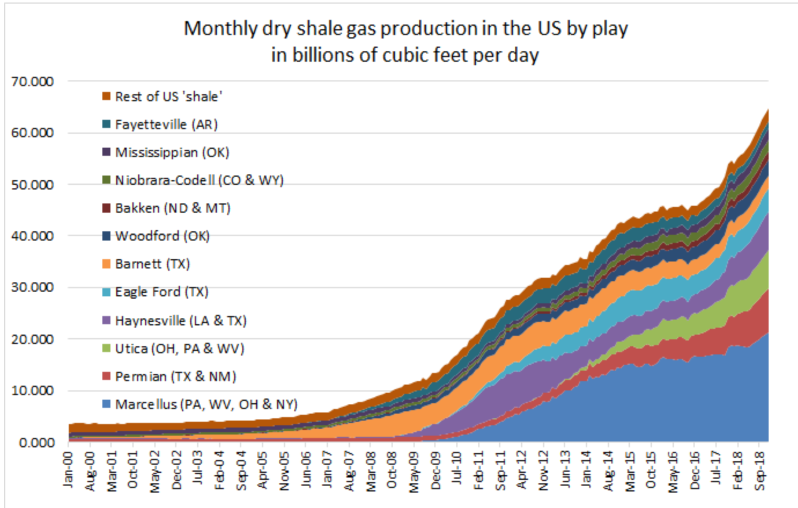


Figure 1: The shale gas boom

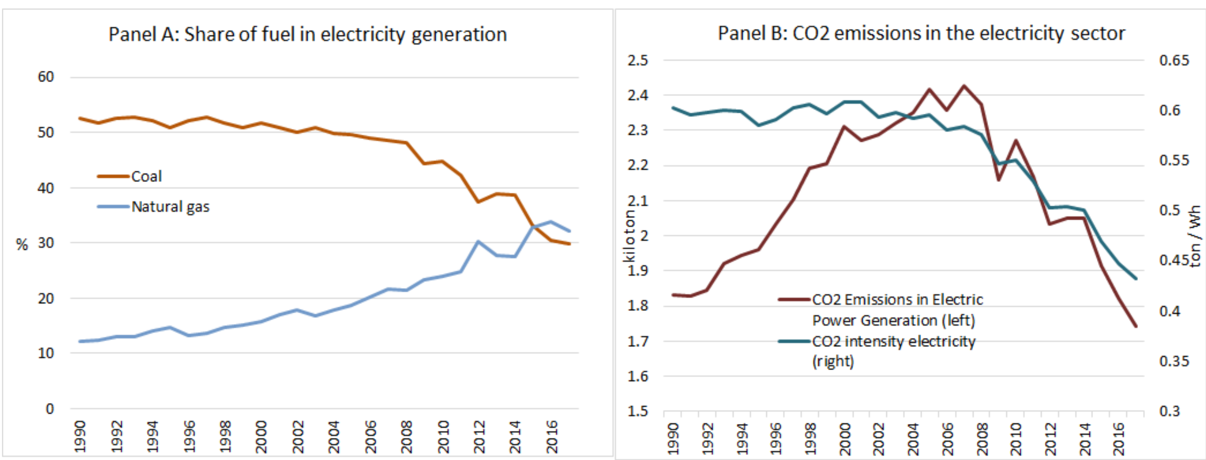


Figure 2: The shale gas boom in electricity

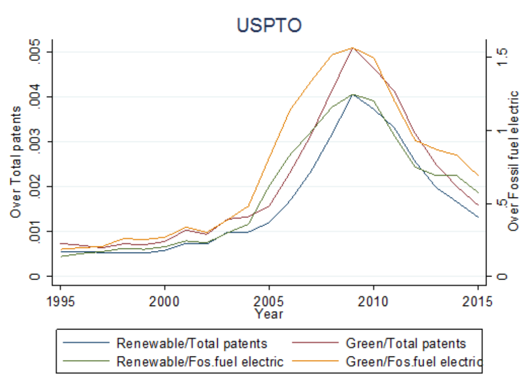


Figure 3: The collapse of clean innovation in electricity

we provide sufficient conditions under which a shale gas boom results in the economy getting trapped in fossil fuels, which in turn results in a permanent increase in aggregate emissions whereas in the absence of the shale gas boom emissions would have converged to zero in the long run.

To assess the short-run and long-run impacts of improving the shale extraction technology, we move to a quantitative analysis. We first calibrate the static version of our model to the electricity sector in the United States. We use data on electricity production and costs according to the energy source (coal, gas and the different renewable energies), and estimates from the literature on the elasticity of substitution across fuels in order to estimate the initial technology levels. Our preliminary results indicate that, for the United States, a reduction in the price of natural gas (akin to the "shale gas revolution") may lead to a decrease in CO_2 emissions (i.e. the intermediate technology has a positive environmental effect in the short-run). We then simulate a dynamic economy and show that for reasonable parameter values, the shale gas revolution decreases innovations in green technologies but increases emissions in the medium- and long-run.

This project belongs to the developing literature on macroeconomics and climate change. The first strand of that literature focuses on "integrated assessment models" (IAMs), which consist of dynamic models of the economy and the climate to evaluate the impact of climate change policies on welfare. This literature has been pioneered by Nordhaus (1991; 1994), whose seminal global DICE model is a benchmark in the literature and one of three models used by the U.S. government to value the social cost of carbon emissions (Interagency Working Group, 2011). The literature building on DICE has analyzed issues ranging from climate tipping points (Lemoine and Traeger, 2014) and uncertaintly (Cai, Judd, and Lontzek, 2018) to general equilibrium dynamics (Golosov et al., 2014,), fiscal policy interactions (Barrage, 2018), and the role of intertemporal preferences (Stern, 2007, and Gerlagh and Liski, 2018), among others. The broader IAM literature now also features numerous modeling groups and frameworks (e.g., RICE, Nordhaus and Boyer, 2000; MERGE, Manne and Richels, 2005; PAGE, Hope, 2011; FUND, Anthoff and Tol, 2013) that consider details such as multi-regional and sectorally differentiated climate impacts. However, with few exceptions (Popp, 2004; WITCH Model, e.g., Bosetti et al., 2007), this literature has largely taken as given the evolution of technology.

A second strand focuses on endogenous technological change. In particular, Acemoglu et al. (2012, henceforth AABH), showed that, in a 2-sector model, market forces would naturally favor the sector which is already the more advanced. As a result, the social planner needs to redirect innovation from the dirty to the clean sector in order to reduce emissions in the long-run. Hémous (2016) pursued this type of analysis in a multi-country model. Both papers conduct numerical simulations but are essentially theoretical projects and do not carry out a comprehensive calibration exercise. Similarly to this paper, Lemoine (2018) extends AABH by modelling separately the resource used in energy production and the complementary inputs necessary to produce energy. Acemoglu et al. (2016) expands on these ideas and calibrates a quantitative model of transition from dirty to clean technologies using firm-level data and Fried (2018) calibrates a model featuring directed technical change using a oil price shocks. None of these papers feature "bridge" technologies. Aghion et al. (2016) provide empirical evidence for directed technical change between clean and dirty technologies and path dependence in the car industry (see also Newell, Jaffe and Stavins, 1999, Popp, 2002, Calel and Dechezleprêtre, 2012 or Meng, 2019).

A third strand of literature builds computational energy-economic or detailed electricity sector models which can be used to simulate the implications of changes in resource prices and policies. Leading examples include the U.S. Energy Information Administration's NEMS model, the MIT EPPA Model (Paltsev et al., 2005; McFarland et al., 2004), and the RFF HAIKU Model, inter alia.[3] Applications of these frameworks to study the impacts of the shale gas boom have found mixed results. Several studies project significant declines in short- and medium-run greenhouse gas emissions from the electricity sector due to fuel substitution away from coal (e.g., Burtraw et al., 2012; Venkatesh et al., 2012). Brown and Krupnick (2010) project higher overall CO_2 emissions in 2030 due to the shale gas revolution (in the absence of climate policy), and that 2030 electricity generation will include higher natural gas consumption along with lower use of coal, nuclear, and renewables. A recent multi-model comparison study finds estimates of the CO_2 emissions impacts of the shale gas revolution ranging from -2% to +11% (McJeon et al., 2014). Our analysis seeks to add to this literature in two main dimensions. On the one hand, while these models are often extraordinarily detailed in their representations of the electricity sector, their complexity make them black-box and prevent from deriving general lessons. Our paper makes a step in that direction while retaining the ability to derive analytical results. On the other hand, though several models account for learning-by-doing effects (i.e., a reduction in capital costs of power plants of a new technology with increased past construction), they typically take progress in the technological frontier as exogenously given. Our analysis focuses on this channel and its implications for the greenhouse gas emissions impacts of shale gas in addition to the fuel switching and scale effects at play.

Finally, a recent, mostly empirical, literature has aimed at establishing the short-term effect of the shale gas revolution on emissions: Linn and Muehlenbachs (2018) and Cullen and Mansur (2017) model the US electricity sector and deliver estimates which can be compared to our short-run estimate (see also Knittel, Metaxoglou, and Trinade, 2015, or Holladay and Jacob LaRiviere, 2017).

2 Motivational evidence: the decline in green innovations

Our first contribution is to document the decline in green innovation in electricity generation since the shale gas boom. We use the World Patent Statistical Database (PATSTAT) from 2018 which contains the bibliographical information of patents from most patent offices in the world. A patent gives the right to use a technology exclusively in a given market and filing a patent in each country involves costs. Therefore, the location of the patent office at which a specific innovation is protected indicates how profitable a market is for the innovator. Patents are classified using different technological codes. We use the IPC classification (and the CPC which is a simple extension of the IPC). Each patent may contain several codes. To identify patents relevant to the generation of electricity using fossil fuels, we use Lanzi, Verdolini and Hascic (2011), who identified IPC codes corresponding to fossil fuel technologies for electricity generation.[4] To identify green innovations, we directly rely on the CPC classification, which contains a technological subclass Y02 for the reduction of greenhouse gases. Innovation in renewable electricity (geothermal, hydro, tidal, solar thermal, photovoltaic and wind) is contained in the main group Y02E10. To define green innovation, we add the main groups Y02E30 (which corresponds to nuclear energy) and Y02E50 (which corresponds to biofuels and

[3] Add E2 Model; GCAM Model; ADAGE Mode;; Boehringer and Rutherford, 2009.

[4] We use the full list of codes given in their Appendix A.1.

fuel from waste).[5] We use patent applications from 1995 to 2015 to give a long time period before the shale gas boom and because the data for the most recent years are incomplete.

Figure 3 in the introduction plots patent applications at the USPTO (where the date of application is the date of first filing).[6] To give an idea of the order of magnitude, there are on average 3264 renewable electricity patents at the USPTO per year between 1995 and 2015. Figure 4 then plots the ratio of renewable to fossil fuel electric patent applications at USPTO, the Canadian, the French and the German patent offices both for all patents (left figure) and for patents where the inventor is local (right figure)—knowing that when inventors from multiple locations are listed, we count patents fractionally. The figures reveal that the pattern observed at the USPTO generalizes to other countries: while patents in renewables were quickly catching up and even overtook patents in fossil fuel electricity until 2009, the pattern has since sharply reversed. Moreover, the reversal occurred sooner for the United States and Canada, the two countries which have exploited shale gas. The pattern is particularly strong for US patents which were following quite closely German patents until 2009. The pattern is similar if one looks at the share of renewable out of total patents.

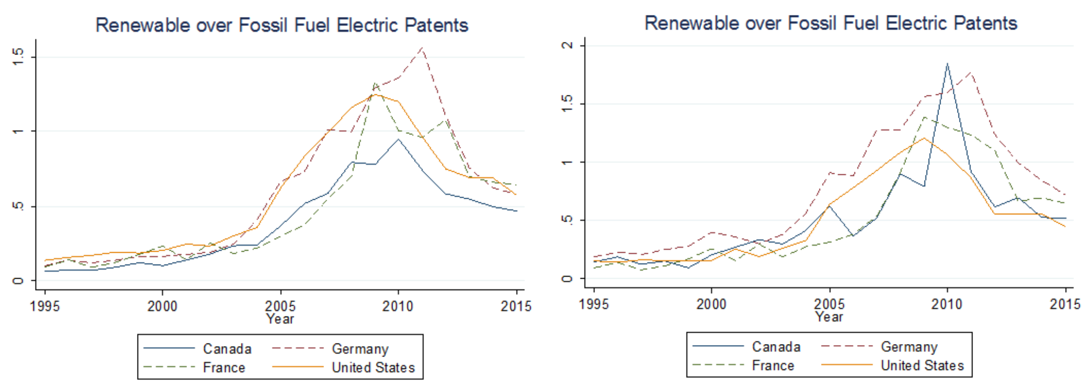


Figure 4: Ratio of renewables to fossil fuel electric patents at different patent offices. For all patents (left) and domestic patents (right).

One may wonder how the shale gas boom could have affected green patenting in Germany and France by local innovator though. There are at least three reasons: First, even domestic innovators may have their incentives shaped by foreign markets, so that German innovators may be less likely to undertake research in renewable energy because the US market is less profitable, leading to a decline in German patents. Second, if innovation in the US is redirected away from renewable energy, the relative amount of spillovers in renewables should decline. Third, innovation is forward looking and as the shale gas revolution unfolded in the US, there was an active political debate in Europe about the exploitation of shale gas.

Nevertheless, to further assess whether the United States and Canada have experienced a

[5]Nuclear energy poses environmental and safety concern but is considered as "green" when it comes to greenhouse gases. Biofuels are used for transportation but also for electricity generations. We do not include innovation aiming at making fossil fuel electricity less polutting (Y02E20), at improving the efficiency of the grid (Y02E40), or at improving electricity storage (Y02E60), since those are not technologies which compete with fossil fuel technologies directly.

[6]We use patent applications instead of granted patents because we want to use recent years for which only few patents are already granted.

decline in green patenting relative to fossil fuel electric patents, we conduct a simple difference-in-difference exercise, where we compute the ratio of renewable or green patents to fossil fuel electric patents for the most important patent offices. We date the shale gas boom from 2009, following Holladay and LaRiviere (2017) who estimate a structural break in natural gas prices in the US on December 5^{th} 2008—in addition natural gas prices in Canada follow closely US ones and drop in 2009 as well. For a subset of countries, we are also able to assess whether and when the exploitation of shale gas is banned (see Appendix 7 for a full list and data source). Finally, we control for GDP per capita in some of our regressions (using OECD data).

Table 1 reports the results both for all patents and when we restrict to patents by domestic inventors. The baseline regression in column (1) suggests that after the shale gas boom the ratio of renewable to fossil fuel electric patents declined by 0.56. The coefficient on the shale gas boom is always negative and significant when we use the larger set of countries that is when we do not control for a shale gas ban. When, we control for a shale gas ban, the coefficient stays negative but is not always significant. The coefficient on the shale gas ban is not significant but has the right sign.

Table 1: Effect of shale gas boom on electricity innovation across patent offices

	Patent Office: all				Patent Office: domestic			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Panel A: Renewable / Fossil fuel electric							
Shale Gas Boom	-0.560**	-0.674**	-0.920**	-0.456*	-0.861**	-1.113	-1.116**	-0.778
	(0.23)	(0.29)	(0.37)	(0.23)	(0.35)	(0.71)	(0.44)	(0.70)
Ban		0.396		0.527		0.452		0.659
		(0.52)		(0.43)		(0.85)		(0.72)
	Panel B: Green / Fossil fuel electric							
Shale Gas Boom	-0.540**	-0.699*	-0.998**	-0.423	-0.814**	-1.106	-1.084**	-0.752
	(0.26)	(0.34)	(0.46)	(0.27)	(0.37)	(0.76)	(0.47)	(0.76)
Ban		-0.699*		-0.423		0.261		0.481
		(0.52)		(0.41)		(0.81)		(0.66)
FEs (C, T)	Y	Y	Y	Y	Y	Y	Y	Y
Control ln(GDPCap)			Y	Y			Y	Y

Note: Difference-in-difference regressions. The shale gas boom is dated from 2009. Standard errors are clustered at the country-level. Column (2), (4), (6), (8) include AU, CA, CH, CL, CZ, DE, DK, ES, FR, GB, HU, IE, JP, NL, US, the other columns also include CN, TW, AT, BE, IS, IL, EE, FI, GR, IT, KR, LV, LT, LU, MX, NO, PL, PT, NZ, SK, SI, SE, TR.

We conduct several robustness exercises in Appendix 7: we show that the results of the difference-in-difference exercise are similar when we consider only granted patents, a shorter sample period or add a year lag between the shale gas boom or bans and their effect on patenting. We also reproduce a similar analysis, where we look at EPO or USPTO patents and allocate patents to countries depending on the nationality of their inventors. We find similar results for EPO patents but the coefficients are not significant at the USPTO suggesting that foreign inventors also patented less in the US and Canada after the shale gas boom.

Finally, we use data on natural gas prices indexed from the International Energy Agency (IEA) from a group of 12 countries to conduct a panel analysis. We regress the log ratio of renewable or green patents over fossil fuel electric patents at the patent offices of the different countries on the log price index, country and year fixed effects and GDP per capita with a 2 year lag. Table 2 shows a positive correlation between these ratio and natural gas prices, with a significant coefficient when considering all patents.

Table 2: Natural gas price and innovation in electricity

	Patent Office: all		Patent Office: domestic	
	(1)	(2)	(3)	(4)
	Panel A: Renewable / Fossil fuel electric			
ln(Price Index)	0.443***	0.432***	0.295	0.301
	(0.14)	(0.13)	(0.22)	(0.23)
	Panel B: Green / Fossil fuel electric			
ln(Price Index)	0.460**	0.454**	0.216	0.225
	(0.16)	(0.15)	0.19)	(0.20)
FEs (C, T)	Y	Y	Y	Y
Control ln(GDPCap)		Y		Y

Note: Independent variable lagged 2 periods. Standard errors are clustered at the country-level. Includes AU, BE, CA, FR, GR, JP, KR, MX, NZ, CH, GB, US.

Overall, this section shows that innovation in the electricity sector has been sharply redirected away from renewable and green electricity at the time of the shale gas revolution in the US. We provide suggestive evidence that the shale gas revolution may have been a factor behind this trend. A more thorough empirical exercise is beyond the scope of this paper.

3 Short-run and long-run effects of the shale gas boom

We now develop a simple tractable model which we will use to develop our main theoretical intuitions. We first describe the model, then solves for the static equilibrium and look at the short-term effects of the shale gas revolution, before analyzing the dynamic equilibrium and the long-run effects.

3.1 Model description

Production technology. Time is discrete and the economy comprises a continuum of researchers and a continuum of identical individuals whose utility depends positively on consumption and negatively on aggregate pollution. The final (consumption) good is produced according to:

$$Y_t = \left((1-\nu)\, Y_{Pt}^{\frac{\lambda-1}{\lambda}} + \nu \left(\widetilde{A}_{Et} E_t \right)^{\frac{\lambda-1}{\lambda}} \right)^{\frac{\lambda}{\lambda-1}},$$

where E_t is an energy composite, Y_{Pt} is a production input produced according to $Y_{Pt} = A_{Pt} L_{Pt}$ and A_{Pt} and $\widetilde{A}_{Et}$ represent respectively productivity in goods production and energy efficiency.

The energy composite is produced according to

$$E_t = \left(\kappa_c E_{c,t}^{\frac{\varepsilon-1}{\varepsilon}} + \kappa_s E_{s,t}^{\frac{\varepsilon-1}{\varepsilon}} + \kappa_g E_{g,t}^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}}.$$

where each $E_{i,t}$ denotes a specific electricity type: $E_{c,t}, E_{s,t}$, and $E_{g,t}$ denote coal, natural gas and green (wind for example) energy respectively. The $\kappa' s$ are share parameters and ε is the elasticity of substitution between electricity types.

The production of energy $i \in \{c, s, g\}$ is given by

$$E_{i,t} = \min\left(Q_{it}, R_{it}\right), \tag{1}$$

where Q_{it} represents an energy input and R_{it} is a resource use corresponding to that particular source of energy (coal, natural gas, and "wind"). We then immediately get: $E_{it} = Q_{it} = R_{it}$. Wind is free but the extraction of natural gas and coal is costly.

Each resource i at date t involves a pollution intensity ξ_{it} so that: $P_{i,t} = \xi_i R_{i,t}$ with $\xi_c > \xi_s > 0 = \xi_g$. In other words, using the green resource does not pollute the atmosphere, and the use of natural gas pollutes the atmosphere but less than that of the coal resource. Aggregate pollution is then given by

$$P_t = \xi_g R_{i,t} + \xi_s R_{s,t} + \xi_c R_{c,t} = \xi_s R_{s,t} + \xi_c R_{c,t}. \tag{2}$$

We take the ξ's to remain fixed over time.

There are two dimensions of technical change: the first one is in the energy input production (which represents technological progress in power plants) and the other one is in the extraction technology.

Let us first formalize technological progress in the energy input production. We assume that the energy input i is produced at time t according to

$$Q_{it} = \exp\left(\int_0^1 \ln q_{ijt} dj \right) \tag{3}$$

where q_{ijt} is the intermediate input produced by local monopolist j in energy sector i. The production of this intermediate input occurs according to the linear technology:

$$q_{ijt} = A_{ijt} l_{ijt}^q, \tag{4}$$

where l^q_{ijt} denotes labor hired for the production of the intermediate and A_{ijt} denotes productivity in the production of intermediate j for energy sector i. Since coal and natural gas power plants share certain technologies and inputs (for instance steam turbines), we will assume that a share of the intermediates are common to both sectors.

Next we model technological progress in the extraction technology as follows. To produce one unit of resource, one needs to spend one unit of extraction input. Without loss of generality we denote the extraction input by R_{it} as well. We model the extraction technology exactly as the power plant technology. That is we write the production function as

$$R_{it} = \exp\left(\int_0^1 \ln r_{ijt} dj\right).$$

Each extraction input is produced according to

$$r_{ijt} = B_{ijt} l^r_{ijt}.$$

where l^r_{ijt} denotes labor hired for extraction input j and B_{ijt} denotes productivity in the production of intermediate j for extraction sector i. Coal and natural gas are in infinite supply so that the cost of the resource is simply equal to the cost of extraction.

Innovation. There is vertical innovation in A_{ijt} and B_{ijt} over time. The current monopolist has access to the latest vintage of the technology while its competitors have access to the previous vintage, which is γ times less productive.

$$A_{ijt} = \gamma A_{ijt-1}$$

if innovation occurs at date t in energy intermediate input ij and similarly

$$B_{ijt} = \gamma B_{ijt=1}$$

if innovation occurs at date t in energy extraction input ij.

We define the average productivities in energy production and resource extraction in sector i as:

$$\ln A_{it} = \int_0^1 \ln A_{ijt} dj \text{ and } \ln B_{it} = \int_0^1 \ln B_{ijt} dj. \tag{5}$$

We assume that there is a mass 1 of scientists who can decide to allocate their research efforts between the three energy input sectors (improving A_{ct}, A_{st} or A_{gt}) and the two resource extraction sectors (B_{ct} and B_{st}). In this theory section and for simplicity we consider that innovation in the extraction sector is exogenous. Each scientist has a probability of success given by $\eta_i s_{it}^{-\psi} A_{it}^{-\zeta_i}$, where η_i represents research productivity in sector i, ψ denotes a stepping-on-the toe externality and ζ_i represents decreasing returns to innovation. Finally, to reflect the fact that several inputs in coal and natural gas power plants are similar, we will assume that a share of innovations in fossil fuel technologies apply to both A_{ct} and A_{st}. Energy efficiency $\widetilde{A}_{Et}$ and productivity in the rest of the economy A_{Pt} evolve exogenously.

3.2 The short-run effects of the shale gas revolution

Static equilibrium. We now solve for the static equilibrium given productivity vectors A_{ijt}. For simplicity, we drop the subscript t in this subsection. The Leontief technology imposes that the price of electricity of type i is given by

$$p_i = p^q_i + p^r_i, \tag{6}$$

where p_i^q is the price of the energy input and p_i^r is the price of the resource extraction input (with $p_g^r = 0$ since extraction is free in green technologies). Maximization by the producer of the energy input i implies that

$$p_{ij}^q y_{ij} = p_i^q Q_i,$$

where p_{ij}^q is the price of the energy intermediate input ij. Following Bertrand competition, we immediately obtain:

$$p_{ij}^q = \frac{\gamma w}{A_{ij}} \text{ so that } l_{ij}^q = \frac{p_i^q Q_i}{\gamma w},$$

where w is the wage. This leads to equilibrium profits:

$$\pi_{ij}^q = \left(1 - \frac{1}{\gamma}\right) p_i^q Q_i.$$

Aggregating across intermediates, the price of energy input i obeys:

$$p_i^q = \frac{\gamma w}{A_i}. \tag{7}$$

Following the same logic in the resource extraction sector, we obtain that

$$p_{it}^r = \frac{\gamma w}{B_{ij}},\ l_{it}^r = \frac{p_i^r R_i}{\gamma w} \text{ and } \pi_{ij}^r = \left(1 - \frac{1}{\gamma}\right) p_i^r R_i$$

and the resource price is

$$p_i^r = \frac{\gamma w}{B_i}. \tag{8}$$

We denote by C_i the harmonic mean of A_i and B_i, which is the overall productivity in the production of electricity of type i, so that the price of electricity of type i is simply given by

$$p_i = \frac{\gamma w}{C_i} \text{ where } \frac{1}{C_i} \equiv \frac{1}{A_i} + \frac{1}{B_i}. \tag{9}$$

Then, profits maximization for the energy composite producer implies that the quantity of energy i is given by:

$$E_i = \kappa_i^\varepsilon \left(\frac{C_{it}}{C_{Et}}\right)^\varepsilon E_t, \tag{10}$$

where C_{Et} is the overall productivity of the energy sector:

$$C_{Et} \equiv \left(\kappa_c^\varepsilon C_{ct}^{\varepsilon-1} + \kappa_s^\varepsilon C_{st}^{\varepsilon-1} + \kappa_g^\varepsilon C_{gt}^{\varepsilon-1}\right)^{\frac{1}{\varepsilon-1}}. \tag{11}$$

The price of the energy composite is given by

$$p_E = \frac{\gamma w}{C_E}, \tag{12}$$

and we have that total energy production is given by

$$E = C_E L_E, \tag{13}$$

where L_E is total labor hired by the energy sector.

The relative sizes of the energy sectors (in revenues) are given by

$$\Theta_i = \frac{p_i E_i}{p_E E} = \kappa_i^{\varepsilon} \left(\frac{C_i}{C_E} \right)^{\varepsilon - 1}. \tag{14}$$

To solve for labor allocation, we look at the maximization problem of the final good producer. We assume that the intermediate input Y_P is also sold at a mark-up γ.[7] Then, taking the ratio of the two first order conditions with respect to E and L_P we get

$$L_E = \frac{\nu^{\lambda} \widetilde{A}_{Et}^{\lambda-1} C_E^{\lambda-1}}{\nu^{\lambda} \widetilde{A}_{Et}^{\lambda-1} C_E^{\lambda-1} + (1-\nu)^{\lambda-1} A_P^{\lambda-1}} L. \tag{15}$$

Define

$$\xi_E \equiv \xi_c \kappa_c^{\varepsilon} \left(\frac{C_c}{C_E} \right)^{\varepsilon} + \xi_s \kappa_s^{\varepsilon} \left(\frac{C_s}{C_E} \right)^{\varepsilon} \tag{16}$$

as the average emission intensity of energy production. Then the equilibrium level of pollution is given by:

$$P = \xi_E E \tag{17}$$

The shale gas revolution. We can now derive conditions under which an increase in natural gas extraction productivity B_s increases or decreases contemporaneous aggregate pollution P. The increase in natural gas extraction productivity is akin to the shale gas revolution. In the subsequent sections on dynamics we look at the long-run consequences of the shale gas revolution. The model allows us to decompose the overall effect of an improvement in shale gas technology on pollution into a substitution effect and a scale effect.

We can write the effect of an increase in natural gas extraction technology as

$$\frac{\partial \ln P}{\partial \ln B_s} = \underbrace{\frac{\partial \ln \xi_E}{\partial \ln B_s}}_{\text{substitution effects}} + \underbrace{\frac{\partial \ln E}{\partial \ln B_s}}_{\text{scale effect}}, \tag{18}$$

the first term corresponds to substitution effects in energy production (a change in extraction technology will affect the average pollution intensity), and the second effect is the Jevons scale effect (a change in extraction technology will increase the scale of the energy sector).

$$\begin{aligned} \frac{\partial \ln \xi_E}{\partial \ln B_s} &= \varepsilon \frac{C_s}{B_s} \frac{\kappa_s^{\varepsilon} \left(-\xi_c \kappa_c^{\varepsilon} C_c^{\varepsilon} C_s^{\varepsilon-1} + \xi_s C_s^{\varepsilon} \left(\kappa_c^{\varepsilon} C_c^{\varepsilon-1} + \kappa_g^{\varepsilon} C_g^{\varepsilon-1} \right) \right)}{C_E^{\varepsilon-1} \left(\xi_c \kappa_c^{\varepsilon} C_c^{\varepsilon} + \xi_s \kappa_s^{\varepsilon} C_s^{\varepsilon} \right)} \\ &= \varepsilon \frac{C_s}{B_s} \left(\frac{P_s}{P} - \Theta_s \right), \end{aligned}$$

where P_s represents pollution generated by natural gas. Therefore the substitution effect is negative when the revenue share of natural gas Θ_s in the energy sector is larger than its emission share P_s/P. This holds whenever:

$$\frac{\xi_c C_c}{\xi_s C_s} > 1 + \left(\frac{\kappa_g}{\kappa_c} \right)^{\varepsilon} \left(\frac{C_g}{C_c} \right)^{\varepsilon-1}. \tag{19}$$

[7] Implicitly, the intermediate input Y_P is also an aggregate of intermediates which are sold by monopolists engaged in Bertrand competition.

The terms $\xi_i C_i$ correspond to the pollution intensity per unit of input. If $\xi_c C_c > \xi_s C_s$, then natural gas is effectively cleaner than coal, so that the substitution effect away from coal reduces average emissions. This is not enough to ensure that the overall substitution effect is negative because the substitution effect away from green is positive. To ensure that average emissions decrease following the shale gas boom, it must be that the coal technologies are sufficiently dirtier than natural gas compared to the backwardness of green technologies relative to coal (the term $\left(\frac{\kappa_g}{\kappa_c}\right)^{\varepsilon}\left(\frac{C_g}{C_c}\right)^{\varepsilon-1}$).

The scale effect is given by

$$\begin{aligned}\frac{\partial \ln E}{\partial \ln B_s} &= \frac{C_s}{B_s}\left(\lambda+(1-\lambda)\frac{\nu^{\lambda}\widetilde{A}_{Et}^{\lambda-1}C_E^{\lambda-1}}{\nu^{\lambda}\widetilde{A}_{Et}^{\lambda-1}C_E^{\lambda-1}+(1-\nu)^{\lambda-1}A_P^{\lambda-1}}\right)\frac{\kappa_s^{\varepsilon}C_s^{\varepsilon-1}}{C_E^{\varepsilon-1}}\\ &= \frac{C_s}{B_s}\Theta_s\left(\lambda+(1-\lambda)\frac{L_E}{L}\right),\end{aligned}$$

so that, given Θ_s and the labor share L_E/L, the scale effect is smaller when the energy input is more complement to production input (that is for λ low). The lower is λ, the more labor gets reallocated to the production input following an increase in extraction technology B_s.

Thus the overall effect of the shale gas boom on pollution is given by:

$$\frac{\partial \ln P}{\partial \ln B_s} = \frac{C_s}{B_s}\left(\underbrace{\varepsilon\left(\frac{P_s}{P}-\Theta_s\right)}_{\text{substitution effect}}+\underbrace{\Theta_s\left(\lambda+(1-\lambda)\frac{L_E}{L}\right)}_{\text{scale effect}}\right).$$

Since $\varepsilon > 1$ and $\lambda < 1$, the substitution effect may dominate the scale effect. In fact, we obtain that $\partial \ln P/\partial \ln B_s < 0$ if and only if

$$\frac{\xi_s}{\xi_c} < \frac{\kappa_c^{\varepsilon}C_c^{\varepsilon}\left[\varepsilon-\left(\lambda+(1-\lambda)\frac{\nu^{\lambda}\widetilde{A}_{Et}^{\lambda-1}C_E^{\lambda-1}}{\nu^{\lambda}\widetilde{A}_{Et}^{\lambda-1}C_E^{\lambda-1}+(1-\nu)^{\lambda-1}A_P^{\lambda-1}}\right)\right]}{\left[\kappa_s^{\varepsilon}C_s^{\varepsilon}\left(\lambda+(1-\lambda)\frac{\nu^{\lambda}\widetilde{A}_{Et}^{\lambda-1}C_E^{\lambda-1}}{\nu^{\lambda}\widetilde{A}_{Et}^{\lambda-1}C_E^{\lambda-1}+(1-\nu)^{\lambda-1}A_P^{\lambda-1}}\right)+\varepsilon C_s\left(\kappa_c^{\varepsilon}C_c^{\varepsilon-1}+\kappa_g^{\varepsilon}C_g^{\varepsilon-1}\right)\right]}. \tag{20}$$

We summarize our discussion in the following Proposition:

Proposition 1 *A shale gas boom (that is a one time increase in B_s) leads to a decrease in emissions in the short-run provided that the natural gas is sufficiently clean compared to coal (for ξ_s/ξ_c small enough that (20) is satisfied).*

3.3 The innovation effect of a shale gas boom

We now solve for the allocation of innovation in laissez-faire, and look at how this allocation is affected by a shale gas boom. A first finding is that under suitable assumptions a shale gas boom induces firms to direct innovation away from clean innovation towards shale gas innovation. A second finding is that there exists a non empty set of parameter values such that a shale gas boom delays the switch towards clean innovation, with the possibility of an infinitely delayed switch.

For simplicity, we assume here that all energy inputs are common to the natural gas and the coal power plants (but the productivities of the intermediates may differ by a constant). Moreover, we assume that there are no decreasing returns to scale: $\zeta_i = 0$. Therefore innovators must decide whether they want to innovate in the green energy input or in the fossil fuel energy input. If an innovator innovates in the green energy input, she obtains expected profits

$$\Pi_{gt} = \eta_g s_g^{-\psi} \left(1 - \frac{1}{\gamma}\right) p_g E_g \tag{21}$$

If she innovates in fossil fuel energy inputs, she obtains expected profits

$$\begin{aligned} \Pi_{ft} &= \eta_f s_f^{-\psi} \left(1 - \frac{1}{\gamma}\right) \left(p_c^y Y_c + p_s^y Y_s\right) \\ &= \eta_f s_f^{-\psi} \left(1 - \frac{1}{\gamma}\right) \left(\frac{C_c}{A_c} p_c E_c + \frac{C_s}{A_s} p_s E_s\right). \end{aligned} \tag{22}$$

In equilibrium, expected profits in green and fossil fuel innovations must be the same. Therefore, using (14), we get:

$$\frac{\Pi_{gt}}{\Pi_{ft}} = \frac{\eta_g s_{gt}^{-\psi} \kappa_g^{\varepsilon} C_{gt}^{\varepsilon-1}}{\eta_f s_{ft}^{-\psi} \left(\kappa_c^{\varepsilon} \frac{C_{ct}^{\varepsilon}}{A_{ct}} + \kappa_s^{\varepsilon} \frac{C_{st}^{\varepsilon}}{A_{st}}\right)} = 1. \tag{23}$$

As shown in Appendix 8.1, the allocation of innovation is uniquely determined by this equation provided that the following Assumption, which we maintain for the rest of the section, holds:

Assumption 1 $(\ln \gamma) \max\left(\eta_g, \eta_f\right) < \psi / \left((\varepsilon - 1)(1 - \psi)\right)$.

Proposition 2 *Under Assumption 1 the equilibrium allocation of innovation is unique.*

For γ or η_g and η_f small enough, we get:

$$\left(\frac{s_{gt}}{s_{ft}}\right)^{\psi} \approx \frac{\eta_g \kappa_g^{\varepsilon} C_{gt-1}^{\varepsilon-1}}{\eta_f \left(\frac{1}{A_{ct-1}} \kappa_c^{\varepsilon} \left(\frac{1}{A_{ct-1}} + \frac{1}{B_{ct}}\right)^{-\varepsilon} + \frac{1}{A_{st-1}} \kappa_d^{\varepsilon} \left(\frac{1}{A_{st-1}} + \frac{1}{B_{st}}\right)^{-\varepsilon}\right)}. \tag{24}$$

This expression highlights that, as in AABH, the innovation allocation features some form of path dependence. A higher green productivity at time $t-1$ $A_{g(t-1)} = C_{g(t-1)}$ increases the relative size of the green energy sector and favors innovation in that sector at time t. Similarly higher productivity levels in the fossil fuel technologies A_{ct-1} and A_{st-1} tend to favor innovation in fossil fuel technologies. Yet, this is only the case as long as $B_{ct}/A_{c(t-1)}$ and $B_{st}/A_{s(t-1)}$ are not too low: otherwise the return to innovation in fossil fuel technologies A_{ct} or A_{st} decreases as such innovation would have little effect on the overall productivities of coal and natural gas technologies C_{ct} or C_{st}.

In addition, the right-hand of (24) is decreasing in B_{st}, so that an increase in B_{st} leads to a reallocation of scientists away from the green technology. Intuitively, this is due to two effects: first, progress in extraction technology is complementary with progress in the associated energy input because the two are linked in a Leontief way; second, progress in extraction technology makes fossil fuel overall more advanced than green technologies, which induces further innovation in fossil fuels (since the two are substitute).

Therefore, a shale gas boom at time $t = 1$ (an increase in B_{s1}) reduces innovation in green technologies contemporaneously (s_{g1} decreases). This leads to higher levels of A_{c1} and A_{s1} and a lower level for the green technology C_{g1}, which, under certain assumptions,[8] will then further reduce innovation in clean technologies at $t = 2$. More precisely, in Appendix 8.1, we prove the following Proposition:

Proposition 3 *Assume that Assumption 1 holds. Then, a shale gas boom (an increase in B_{s1}) leads to reduced innovation in green technologies at $t = 1$ (i.e., to a decrease in s_{g1}). Moreover, if $\min\left(B_{ct}/A_{c(t-1)},\, B_{st}/A_{s(t-1)}\right) > \gamma^{\eta_f}/\left(\varepsilon - 1\right)$ for all $t > 1$, then green innovation declines for all $t \geq 1$.*

The Proposition states sufficient conditions under which a shale gas boom at time 1 (which increases the extraction technology B_{s1}) shifts innovation toward fossil fuel technologies for all $t \geq 1$. If the shale gas boom shifts extraction technology B_{st} up for all $t \geq 1$, then its negative effects on green innovation cumulate over time. That is, green innovation at time t, s_{gt}, will decrease not only because B_{st} moves up, but also because there is path dependence in the direction of innovation and the shale gas boom will have reduced green innovation in preceding periods.

To describe the overall dynamics of pollution following a shale gas boom, we need to make assumptions on the dynamic path followed by extraction technologies and by the production technology A_{Pt}. We proceed to do so in the next sections under two polar cases.

3.4 Long-run equilibrium with fast progress in extraction technologies

We first consider the case where the extraction technologies grow exogenously at a fast rate. Specifically, we assume that $\eta_c = \eta_f = \eta$ and that B_{ct} and B_{st} grow both grow at factor rate γ^{η}. We also assume that A_{Pt} grow exogenously at the same factor. These assumptions ensure that in the long-run, the economy will grow at rate γ^{η} in all possible scenarii. We define a shale gas boom as a one time increase in B_{s1}, such that the entire path B_{st} is moved up by a constant factor.

In this case, productivity in extraction technologies must grow weakly faster than in power plant technologies, so that if $\min\left(B_{ct}/A_{c(t-1)},\, B_{st}/A_{s(t-1)}\right) > \gamma^{\eta}/\left(\varepsilon - 1\right)$ for $t = 1$, then this is also true for $t > 1$. Using Proposition 3, we get that a shale gas boom will lead to a reallocation of innovation away from green technologies for all $t \geq 1$.

Since the extraction technologies in fossil fuel must grow at least as fast as the power plant technologies, then the innovation allocation problem looks asymptotically similar to that in AABH and features path dependence. That is, the innovation allocation is asymptotically "bang-bang" with either all researchers working on green innovation or all researchers working on fossil fuel innovation (except for a knife-edge case). More specifically, there exists a threshold value $\overline{A}_{g0}\left(A_{s0}, A_{c0}, B_{s1}, B_{c1}\right)$, which depends on the initial productivities in fossil fuel technologies, such that if the initial green productivity is below that threshold, i.e. if

[8] As noted below equation (24), an increase in $A_{c(t-1)}$ or $A_{s(t-1)}$ may have a negative effect on fossil fuel innovation if the extraction technologies are too much behind the power plant technologies. The assumption that $\min\left(B_{ct}/A_{c(t-1)},\, B_{st}/A_{s(t-1)}\right) > \gamma^{\eta_f}/\left(\varepsilon - 1\right)$ ensures that this is not the case.

$A_{g0} < \overline{A}_{g0}$, then the economy is on a “fossil-fuel” path where eventually all innovation occurs in fossil fuel technologies. The opposite occurs if the initial green technology is above the threshold, i.e. if $A_{g0} > \overline{A}_{g0}$.[9]

By favoring innovation in fossil fuel technologies, a shale gas boom moves the threshold value $\overline{A}_{g0}$ upwards. For intermediate values of the initial green productivity A_{g0}, the economy will move from a "green" path to a "fossil fuel" path. On a fossil fuel path emissions grow asymptotically at factor γ^{η} while on a green path emissions decrease toward 0.[10] Hence, switching from green innovation to fossil fuel innovation has dramatic consequences on the emission path. In the Appendix we prove:

Proposition 4 *Assume that Assumption 1 holds, that B_{ct} and B_{st} grow exogenously at factor γ^{η} and that $\min(B_{c1}/A_{c0},\ B_{s1}/A_{s0}) > \gamma^{\eta}/(\varepsilon - 1)$. Then a shale gas boom at $t = 1$ leads to a decrease in green innovations for all $t \geq 1$. For small enough initial green productivity A_{g0}, emissions will grow forever regardless of a shale gas boom and for large enough initial A_{g0} emissions will converge to zero in either case, but for an intermediate range of A_{g0}, emissions will grow forever following a shale gas boom while they converge to zero over time absent a shale gas boom.*

This proposition deals with the extreme case in which the shale gas boom may lead to (much) higher emissions in the long-run. It is interesting to note that this may occur even for parameters such that the initial effect of the shale gas boom is to reduce emissions. Indeed, the latter occurs whenever coal is sufficiently polluting compared to natural gas, but how polluting the two technologies are, has no bearing on the allocation of innovation, which is driving the result here.

3.5 Long-run equilibrium with no progress in extraction technologies

We now consider the polar case where B_{st} and B_{ct} remain constant over time (except for a possible shift of the B_{st} schedule following a shale gas boom). We maintain the assumptions that $\eta_c = \eta_f$ and that A_{Pt} grows at by factor γ^{η}.

When B_{st} and B_{ct} remain constant, it eventually becomes unprofitable for firms to innovate in energy input production technologies for coal or natural gas. In other words, in this case innovation will always end up occurring on green energy production. Intuitively, since extraction technologies do not improve and since extraction and power plant inputs are complements, the share of income within the fossil fuel sector going to power plant inputs goes to 0, which discourages innovation in fossil fuel power plant technologies. Emissions will then always asymptote 0.

[9] It is not possible to derive analytical expressions for the threshold $\overline{A}_{g0}$. Yet, a sufficient condition to be on the fossil fuel path is $\kappa_g^{\varepsilon} A_{g0}^{\varepsilon-1} \leq \frac{\kappa_c^{\varepsilon}}{A_{c0}} \left(\frac{1}{A_{c0}} + \frac{\gamma^{\eta}/2^{1-\psi}}{B_{c1}} \right)^{-\varepsilon} + \frac{\kappa_s^{\varepsilon}}{A_{s0}} \left(\frac{1}{A_{s0}} + \frac{\gamma^{\eta}/2^{1-\psi}}{B_{s1}} \right)^{-\varepsilon}$, which ensures that $s_{g1} \leq 1/2$. Similarly $\kappa_g^{\varepsilon} A_{g0}^{\varepsilon-1} > \kappa_c^{\varepsilon} A_{c0}^{\varepsilon-1} + \kappa_s^{\varepsilon} A_{s0}^{\varepsilon-1}$ is a sufficient condition to ensure that the economy is on a green path (regardless of the value of the extraction technology).

[10] On a fossil fuel path, C_{st} and C_{ct} grow asymptotically at factor γ^{η}, which ensures that C_{Et} grow at the same rate and that ξ_E tends toward a constant (16). Since A_{Pt} also grow at the same rate, L_E approaches a positive constant (see 15) and pollution grows asymptotically at factor γ^{η}. On a green path, $C_{Et} \to \kappa_g^{\varepsilon/(\varepsilon-1)} C_{gt}$ and both asymptotically grow at factor γ^{η}, L_E still approaches a positive constant but the emission rate ξ_E now tends toward 0. Using (13), (16) and (17), we then get $P_t \to (\xi_c \kappa_c^{\varepsilon} C_{ct}^{\varepsilon} + \xi_s \kappa_s^{\varepsilon} C_{st}^{\varepsilon})\, \kappa_g^{-\varepsilon} C_{gt}^{1-\varepsilon} L_E \to 0$,since C_{ct} and C_{st} do not grow exponentially.

Yet, by making extraction technologies more productive, a shale gas boom still favors innovation in fossil fuel technologies, which will have the effect of delaying the switch toward green innovations. Formally we establish (proof in Appendix 8.3):[11]

Proposition 5 *Assume that Assumption 1 holds and that B_{ct} and B_{st} are constant over time. i) Then there exists a time t_{switch} such that for all $t > t_{switch}$, $s_{gt} > 1/2$ and eventually all innovations occurs in green technologies. If $\varepsilon \geq 2$, a shale gas boom at $t = 1$ delays the time t_{switch} and reduces green innovation until then. ii) In addition for $\varepsilon \geq 2$ and for $\ln \gamma$ small, emissions are reduced in the long-run.*

Overall, for suitable parameter values, a shale gas boom reduces emissions in the short-run, but it delays or (in the case of the previous subsection) prevents the switch towards clean innovation. As a result, the shale gas boom permanently lowers the clean technology and increases the fossil fuel technologies. In the long-run, clean technologies are still the most developed, so that coal's main competitor is clean energy and the negative effect on emissions coming from the substitution of coal with natural gas is dominated. As a result, emissions increase in the long-run following the shale gas boom.[12]

4 Calibration

We now calibrate our model to the US electricity sector. Subsection 4.1 extends the basic model studied so far. Subsection 4.2 explains the calibration and section 4.3 contains results on the static effect of the shale gas boom. Finally, subsection 4.4 presents a dynamic calibration of our model. The extended model is calibrated so as to match the evolution of the US electricity sector. Looking first at the static effects of a shale gas boom, we find a small reduction in emissions following a shale gas boom (equal to -0.168%) which is in line with the magnitudes generated by exisiting empirical exercises. Then, looking at the dynamic effects of a shale gas boom, we find that innovation gets redirected away from green technologies as a result of the boom, and consequently emissions end up increasing in the long-run. In the baseline calibration, emissions increase within 15 years folowing the shale gas boom. Finally we find that the effects are quantitatively sensitive to changing parameter values.

4.1 From the basic to the calibrated model

To bring the model to the data, we relax two assumptions we have made so far. First, we allow for a different elasticity of substitution between green electricity and fossil fuel electricity on one hand and within the fossil fuel electricity nest on the other. That is, we assume that the energy composite is produced according to:

$$E_t = \left(\left(\kappa_c E_{c,t}^{\frac{\sigma-1}{\sigma}} + \kappa_s E_{s,t}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}\frac{\varepsilon-1}{\varepsilon}} + \kappa_g E_{g,t}^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}},$$

[11] The assumption $\varepsilon \geq 2$ is a sufficient condition and plays a role similar to the assumption $\min\left(B_{ct}/A_{c(t-1)},\, B_{st}/A_{s(t-1)}\right) > \gamma^{\eta_f}/\left(\varepsilon-1\right)$ in Proposition 3.

[12] To establish the result formally, we require that the innovation step, $\ln\gamma$, is small. This assumption is made for analytical tractability, numerical simulations suggest that the result is robust to removing it.

with $1 < \varepsilon \leq \sigma$, so that natural gas and coal electricity may be more substitutable with each other than with green electricity—reflecting for instance the fact that some green resources are intermittent. Second, we relax the assumption that labor is the only factor of production and introduce capital. We assume that the production input Y_P is produced according to

$$Y_{Pt} = A_{Pt} L_{Pt}^{\varphi} K_{Pt}^{1-\varphi},$$

where K_P is the capital used and φ is the labor share in the production sector. The intermediate energy and extraction inputs are produced according to

$$q_{ijt} = A_{ijt} \left(l_{ijt}^{q}\right)^{\phi} \left(k_{ijt}^{q}\right)^{1-\phi} \text{ and } r_{ijt} = B_{ijt} \left(l_{ijt}^{r}\right)^{\phi} \left(k_{ijt}^{r}\right)^{1-\phi},$$

where k_{ijt}^{q} and k_{ijt}^{r} denote the capital used in the production of intermediate energy and extraction inputs and ϕ is the labor share in the energy sector.

Solving for the equilibrium follows the same steps as in section 3. In the price of the energy inputs or the resource, the wage is replaced by the input bundle price,

$$c_{Et} = \left(\frac{w_t}{\phi}\right)^{\phi} \left(\frac{\rho_t}{1-\phi}\right)^{1-\phi},$$

where ρ_t is the interest rate. Therefore (7), (8) and (9) are replaced with

$$p_{it}^{y} = \frac{\gamma c_{Et}}{A_{it}},\ p_{it}^{r} = \frac{\gamma c_{Et}}{B_{it}} \text{ and } p_{it} = \frac{\gamma c_{Et}}{C_{it}}. \tag{25}$$

Similarly the price of the production input Y_{Pt} is now given by

$$p_{Pt} = \frac{\gamma c_{Pt}}{A_{Pt}} \text{ where } c_{Pt} = \left(\frac{w_t}{\varphi}\right)^{\varphi} \left(\frac{\rho_t}{1-\varphi}\right)^{1-\varphi}.$$

Profits are still a share $1 - 1/\gamma$ of the revenues generated by a sector.

The effective productivity of energy C_{Et} is now given by

$$C_{Et} \equiv \left(C_{ft}^{\varepsilon-1} + \kappa_g^{\varepsilon} A_{gt}^{\varepsilon-1}\right)^{\frac{1}{\varepsilon-1}} \text{ with } C_{ft} \equiv \left(\kappa_c^{\sigma} C_{ct}^{\sigma-1} + \kappa_s^{\sigma} C_{st}^{\sigma-1}\right)^{\frac{1}{\sigma-1}}. \tag{26}$$

C_{ft} is the effective productivity of the fossil fuel bundle: $E_f \equiv \left(\kappa_c E_{c,t}^{\frac{\sigma-1}{\sigma}} + \kappa_s E_{s,t}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$. We then obtain that the price of electricity in laissez-faire obeys

$$p_{Et} = \frac{\gamma c_{Et}}{C_{Et}}, \tag{27}$$

and that the quantity of energy composite produced is given by

$$E_t = C_{Et} L_{Et}^{\phi} K_{Et}^{1-\phi}, \tag{28}$$

where K_{Et} and L_{Et} are aggregate quantity of capital and labor involved in energy production.

Propositions 1 and Proposition 3 can be extended to this set-up under certain assumptions (see Appendix 8.4 for formal statements and proofs). In particular, we can still decompose the effect of an increase in extraction technology between a substitution effect and a scale effect as in (18), and we can decompose the substitution effect between substitution away from green technologies and within fossil fuels:

$$\frac{\partial \ln \xi_{Et}}{\partial \ln (B_{st})} = \frac{\theta_{sft} C_{st}}{B_{st}} \left[\underbrace{\varepsilon \Theta_{gt}}_{\text{substitution away from green}} - \underbrace{\sigma \frac{P_{ct}}{P_t} \left(1 - \frac{\xi_s C_{st}}{\xi_c C_{ct}} \right)}_{\text{substitution within fossil fuels}} \right] \tag{29}$$
$$= \frac{C_{st}}{B_{st}} \left[\sigma \frac{P_{st}}{P_t} - (\sigma - \varepsilon)\, \theta_{sft} - \varepsilon \Theta_{st} \right]$$

where Θ_{gt} is the revenue share of the green industry in the energy sector, θ_{sft} is the revenue share of the gas industry within the fossil fuel energy subsector, and $P_{c,t}$ denotes emissions from coal energy. The substitution effect away from green is always positive (substituting away from green always increases pollution). As before, the substitution effect within fossil fuels is negative as long as the pollution intensity in terms of input units is larger for coal electricity than for natural gas ($\xi_{s,t} C_{st} < \xi_{c,t} C_{ct}$). In this case, a shale gas boom is more likely to lead to a reduction in pollution emissions when: the share of emissions caused by coal is large, the elasticity of substitution between fossil fuels is large relative to that with green electricity ($\sigma > \varepsilon$), green technologies are relatively less advanced (Θ_{gt} is low) and the scale effect is small.

4.2 Setting up the calibration

The calibration utilizes both the literature and matches selected moments in the data, as summarized in Table 3.

Parameter	Value(s)	Sources and Notes
ε	1.8561	Papageorgiou et al. (2013) avg. estimate of elasticity of subs. btw. clean, dirty inputs in electricity production
σ	2	Bosetti et al. (2007) calibration of fossil fuel electricity sub-nest elasticity based on empirical Ko and Dahl (2001), Sonderholm (1991)
κ_c, κ_s κ_g	0.3785, 0.3382 0.2833	Rationalize electricity demand equations (30),(31) at base year (2011) generation data (EIA) and prices (levelized costs, NREL, 2010, 2012)
ϕ	0.403	Barrage (2018)
φ	0.67	Standard
λ	0.5	Literature (Chen et al., 2017; Hassler, Krusell, Olosvsso, 2012; Van der Werf, 2008; Böringer and Rutherford, 2008; Bosetti et al., 2007); See also discussion in Appendix.
υ	0.5	Normalized (without loss of generality)
γ	1.07	Match 2004-2014 profits for Petroleum and Coal, Durable Manuf., Wholesale (U.S. Census)
$\widetilde{A}_{E,0}$	3.4486e+05	Rationalize final goods producer's electricity demand (35) in base year (2008) at observed GDP F_0 (BEA)
ξ_c, ξ_s	1.001, 0.429	Billion metric tons of CO2 / trillion kWh (EIA, 2016)
$A_{g,0}, A_{c,0}, A_{s,0}, B_{c,0}$, $B_{s,0}, C_{f,0}, C_{E,0}, A_{P,0}$		Match 13 equilibrium conditions at observed employment $L_{E,0}$ and $L_{P,0}$ (BEA), capital K_0 (BEA), and levelized cost elements ($p^y_{i,t}$, $p^r_{i,t}$) (NREL) (See Appendix B)

Table 3: Summary of Calibration Method

First, the benchmark substitution elasticities (ε, σ) are calibrated externally based on empirical estimates and other studies in the literature (Papageorgiou et al., 2013; Bosetti et al., 2007). Next, the $\kappa' s$ are chosen to rationalize base year (2008) electricity generation data (EIA) at baseline prices, for which we use levelized cost ("LCOE") estimates from NREL (2010, 2012):

Electricity	$E_{i,0}$ (tril. kWh)	$p_{i,0}$ (\$/MWh)
Coal	1.986	57.38
Gas	0.883	64.40
Green (w/ hydro)	1.187	80.86
Green (w/o hydro)	0.932	73.74
Note: $p_{g,0}$ includes nuclear power and represents quantity-weighted avg. LCOE across green types. (Sources: NREL (2010, 2012) and EIA (2017))		

Table 4: Base Year Data

Given these moments in the data, we solve for the $\kappa' s$ jointly with the price of the fossil composite's initial price $p_{f,0}$ and the initial fossil composite $E_{f,0}$ through five equations in five unknowns, namely:

- (i) profit-maximizing fossil electricity input demands,

$$\frac{E_{c,t}}{E_{s.t}} = \left(\frac{\kappa_c}{\kappa_s}\frac{p_{st}}{p_{ct}}\right)^{\sigma} \Rightarrow \kappa_c = \kappa_s \left(\frac{E_{c,t}}{E_{s.t}}\right)^{\frac{1}{\sigma}} \frac{p_{ct}}{p_{st}}; \tag{30}$$

- (ii) profit-maximizing green versus fossil electricity input demands,

$$\frac{E_{g,t}}{E_{f.t}} = \left(\kappa_g \frac{p_{ft}}{p_{gt}}\right)^{\varepsilon} \Rightarrow \kappa_g = \left(\frac{E_{gt}}{E_{ft}}\right)^{\frac{1}{\varepsilon}} \frac{p_{gt}}{p_{ft}}; \tag{31}$$

- (iii) the fossil composite's price index,

$$p_{ft} = \left(\kappa_c^{\sigma} p_{ct}^{1-\sigma} + \kappa_s^{\sigma} p_{st}^{1-\sigma}\right)^{\frac{1}{1-\sigma}}; \tag{32}$$

- (iv) the fossil composite's production definition,

$$E_f \equiv \left(\kappa_c E_{c,t}^{\frac{\sigma-1}{\sigma}} + \kappa_s E_{s,t}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}; \tag{33}$$

and

- (v)

$$1 = \kappa_c + \kappa_g + \kappa_s \tag{34}$$

We can then back out the initial electricity composite quantity and price:

$$\begin{aligned} p_{E,0}(\$2009\ bil./tril.kWh - eq) &= \left(\kappa_g^{\varepsilon} p_{g,0}^{1-\varepsilon} + p_{f,0}^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}} = 179.974 \\ E_0(tril.kWh - eq) &= \left(E_{f,0}^{\frac{\varepsilon-1}{\varepsilon}} + \kappa_g E_{g,0}^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}} = 1.331 \end{aligned}$$

Next, we solve for $\widetilde{A_{E,0}}$ based on the final goods producer's electricity first order condition:

$$p_{E,0} = \frac{\partial Y_0}{\partial E_0} = [Y_0]^{\frac{1}{\lambda}} v \widetilde{A_{E0}}^{\frac{\lambda-1}{\lambda}} E_0^{\frac{-1}{\lambda}} \tag{35}$$

where we bring in base year GDP Y_0 from the BEA. In order to calibrate λ, we again refer to the literature with appropriate adjustments as our model focuses on electricity, whereas empirical estimates commonly measure elasticities of substitution between overall energy and a capital-labor composite. On the one hand, commonly utilized values for general energy-capital labor elasticities range around 0.4 to 0.5 (MIT EPPA Model, e.g., Chen et al., 2017; Böringer and Rutherford, 2008; Bosetti et al., 2007; Van der Werf, 2008), and electricity-other energy elasticities of 0.5 (e.g., Chen et al., 2017; Bosetti et al., 2007). We thus use $\lambda = 0.5$ as a benchmark. On the other hand, we also consider lower values as new empirical evidence from Hassler, Krusell, and Olovsson (2012) finds near-zero substitution elasticities. We set $\nu = 0.5$ without loss of generality since different values of ν can be accommodated by adjusting the level of $\widetilde{A_{E0}}$.

In order to calibrate the remaining parameters, we obtain the following additional data. First, we collect employment shares for the extraction and electricity sectors from NIPA tables for the calibration base year 2008 (see Appendix B). Normalizing the total labor force size to $L_0 = 1$ then yields values $L_{E,0} = 0.011044$ and $L_{P,0} = 0.98896$. Next, we obtain the aggregate initial capital stock $K_0 = \$50,584.6$ billion (BEA, 'Fixed Assets and Consumer Durables,' \$2008). Finally, we take advantage of the fact that our levelized cost estimates provide a break-down into fuel and non-fuel (O&M, capital) components to distinguish input prices $p^q_{i,t}$ and resource costs $p^r_{i,t}$ faced by electricity producers. That is, we match the model (6) to the data via:

$$\underbrace{p_{coal-elec,t}}_{LCOE_t} = \underbrace{p^q_{c,t}}_{LCOE_{O\&M,K,coal,t}} + \underbrace{p^r_{c,t}}_{LCOE_{Fuel,coal,t}}$$

Next, we set the labor shares in the non-electricity sector to the standard value $\phi = 0.67$, and choose the labor share in electricity and resource production to $\varphi = 0.403$ based on estimates from Barrage (2018). We calibrate $\gamma = 1.07$ based on profit data from the U.S. Census Bureau (Quarterly Financial Reports) specifically to match that profits are a share $1 - 1/\gamma$ of sectoral income in laissez faire. Given that profits in the model base year of 2008 and subsequent recession were abnormally low, we target average profit margins 2004-2014. Details are provided in Appendix B.

Next, given these values, we then solve for the remaining 13 parameters and unknown variables in initial equilibrium $(A_{g,0}, A_{c,0}, A_{s,0}, B_{c,0}, B_{s,0}, C_{f,0}, C_{E,0}, A_{P,0}, K_{E,0}, K_{P,0}, c_{E,0}, w_0, \rho_0)$ through a system of equilibrium conditions (given in Appendix 9.4). Lastly, pollution intensities ξ_c and ξ_s could be calibrated based either upon the inherent carbon content of coal and gas resource inputs R_{it}, or based on the benchmark pollution intensity of each type of electricity generation E_{it}. We take the latter approach at this stage (since resource input usage is proportional to electricity in the model but not in reality) and calibrate $\xi's$ based on average emissions rates of U.S. electricity generation from coal vs. natural gas generators (EIA, 2016).

4.3 Static results

This subsection presents quantitative estimates for the static effects of improvements in shale gas extraction technology. We specifically consider increases in $B_{s,0}$ of 10%, and 50%. The latter approximately corresponds to the observed natural gas price decline of around 67% from 2008 to 2012 via (25) (ignoring general equilibrium effects). The impact of changing B_s on the average effective emissions rate per unit of electricity $\xi_{E,t}$ can be directly computed from (46) using also (26). In order to compute the change in overall energy demand, we then solve for the new macroeconomic equilibrium (see Appendix 8.4 for details). Table 5 presents the results. As expected, the net effect of an improvement in shale extraction technology on contemporaneous carbon emissions is generally negative, but could be positive depending on its magnitude and the parameters. In particular, a larger elasticity of substitution within fossil fuels is associated with larger declines in CO_2 emissions, as in that case natural gas is a better competitor to coal. In contrast, a higher elasticity of substitution ε between fossil fuels and green technologies is associated with lower declines in CO_2 emissions, as then the substitution effect of natural gas away from clean technologies is stronger. Finally, a lower value for the elasticity of substitution between the production input and energy is associated with a larger decline in CO_2 emissions since it limits the scale effect (as C_{Et} increases, more workers get reallocated toward the production input).

Total Effects of Improved Shale Extraction Technology			
	$\%\Delta\xi_E$	$\%\Delta E$	$\%\Delta CO_2$
Baseline Parameters			
+10% Increase in $B_{s,0}$	-1.03%	+0.79%	-0.244%
+50% Increase in $B_{s,0}$	-3.51%	+3.46%	-0.168%
Higher $\varepsilon = 2$			
+10% Increase in $B_{s,0}$	-0.940%	+0.79%	-0.157%
+50% Increase in $B_{s,0}$	-3.17%	+3.47%	+0.185%
Lower $\varepsilon = 1.5$			
+10% Increase in $B_{s,0}$	-1.24%	+0.79%	-0.461%
+50% Increase in $B_{s,0}$	-4.35%	+3.44%	-1.052%
Higher $\sigma = 2.2$			
+10% Increase in $B_{s,0}$	-1.24%	+0.79%	-0.460%
+50% Increase in $B_{s,0}$	-4.33%	+3.52%	-0.961%
Lower $\sigma = 1.8$			
+10% Increase in $B_{s,0}$	-0.81%	+0.79%	-0.029%
+50% Increase in $B_{s,0}$	-2.69%	+3.40%	+0.616%
Lower $\lambda = 0.3$			
+10% Increase in $B_{s,0}$	-1.03%	+0.48%	-0.546%
+50% Increase in $B_{s,0}$	-3.51%	+2.11%	-1.469%

Table 5: Static Effects of Shale Technology Improvements

Ideally, we would like to compare these simulation results to real data in order to validate the model. A simple comparison to emissions data would not be informative as the shale gas revolution coincided with the Great Recession, among other confounders. Instead, we thus turn to the empirical literature wherein a number of studies have produced micro-econometric estimates of the short-run effects of natural gas price changes on electricity producers, typically using power plant-level generation and emissions data and spatial variation in natural gas prices over the mid-2000's through 2012 period. Of course these studies' findings are not strictly comparable to our model's predictions as they represent short-run partial equilibrium estimates that hold various aggregate factors constant. They nonetheless represent the best available empirical evidence on the impacts of the shale gas revolution on electricity generation, and thus provide valuable benchmarks. Reassuringly, our model's results lie within the range of the most relevant empirical estimates. Linn and Muehlenbachs (2018) results imply that a 10% decrease in shale prices in 2008 would decrease the emissions intensity of electricity generation by -0.59%. Our corresponding benchmark estimate of -1.03% is larger, but on the same order of magnitude. On the other hand, Cullen and Mansur (2017) estimate that a 67% natural gas price decline from \$6/mmBTU to \$2/mmBTU (as observed from 2008 to 2012) would lead to a 10% decline in CO2 emissions levels (from electricity generation). Our corresponding estimates lie below this value both in the corresponding partial equilibrium (-3.51%) and general equilibrium (-0.168%) calculations. Overall, however, our estimates thus lie in between those of Linn and Muehlenbachs (2018) and Cullen and Mansur (2017).[13]

4.4 Dynamic simulation

We now look at the dynamic effects of the shale gas boom. We choose one period as corresponding to 5 years. For simplicity, we assume that the capital stock grows at 2% a year, $\widetilde{A}_{Et}$ is constant and A_{Pt} grows with a factor 1.02^{φ} per year. We assume that $\eta_f = \eta_g$ and choose η such that should innovation in energy occurs in green technology only, A_{gt} would grow with a factor 1.02^{ϕ} per year. These assumptions guarantee that the long-run growth rate of the economy is 2% a year (we have $\eta = 5\phi \ln 1.02/\ln\gamma = 0.5898$). We assume that $\psi = 0.5$. We start with the economy in 2008 calibrated as above and look at the effect of a shale gas revolution during the next period (2013). As in section 3.3, we assume that up to a constant productivity term the energy intermediates in fossil fuel power plant technologies q_{cj} and q_{sj} are identical.

We first look at the effect of the shale gas boom when there is no further innovation in extraction technologies. Figure 4.4 looks at the case of a 50% increase in B_s in 2013 in the baseline case. In line with Proposition 5, Panel A shows that the shale gas boom increases the share of scientists in fossil fuel innovations. Since B_{st} and B_{ct} are constant (after the boom), this share eventually goes toward 0. Panel B plots the resulting change in output and in emissions. The initial effect on emissions is small and negative but it turns positive by 2028 and it increases over time. The effect on output is positive but small.

[13] Several other important empirical studies on this topic exist but we do not compare their estimates to our model as they are not sufficiently comparable. Knittel, Metaxoglou, and Trinade (2015) who compare shale gas share and CO2 emissions responses to natural gas price variation between investor-owned utilities and independent power producers in vertically integrated and restructured electricity markets, but focus only on entities with both coal- and gas-fired capacity, rather than the overall generating system as represented in our model. Holladay and Jacob LaRiviere (2017) study the effects of natural gas price declines on electricity generators but focus on changes in *marginal* emissions rates in the very short run due to changes in the dispatch of existing generation capacity.

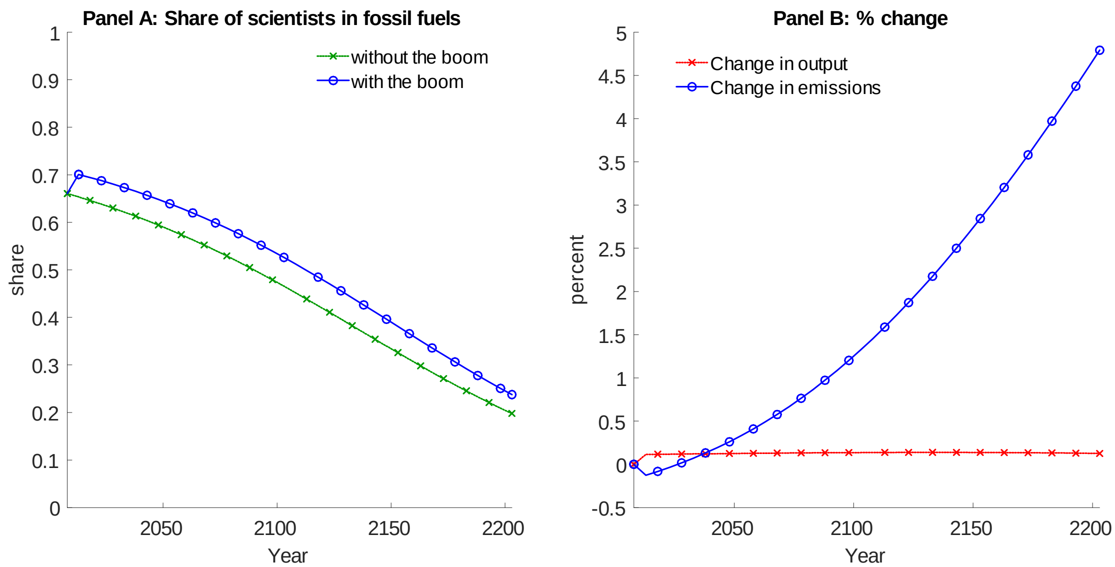


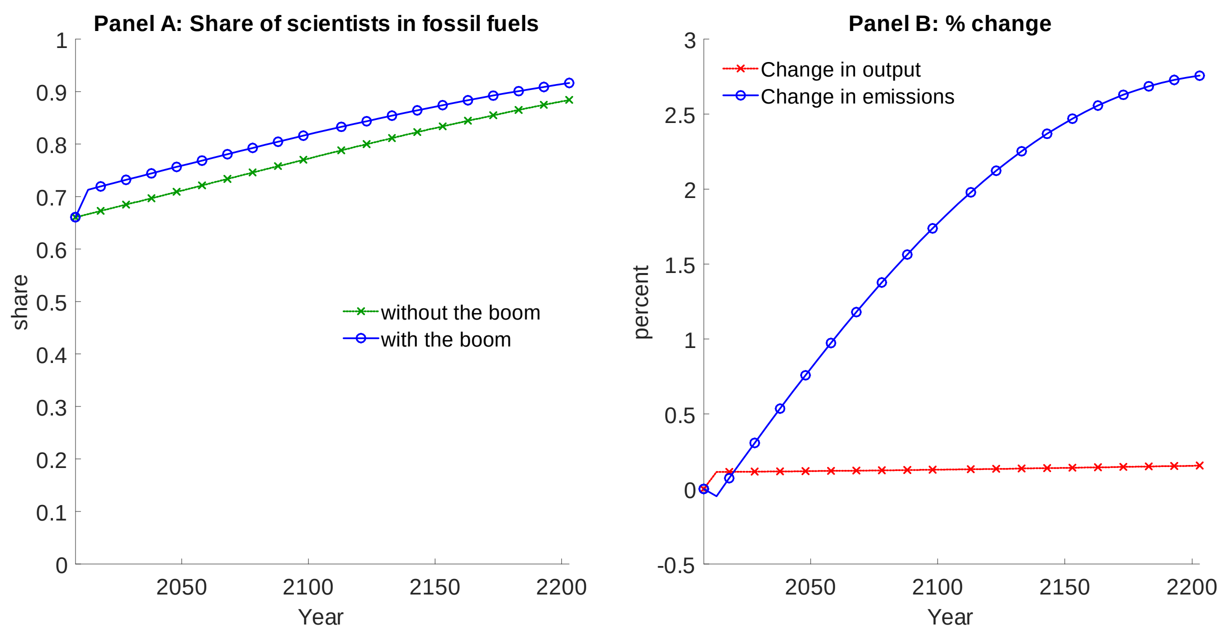
Table 4.4 shows the effect on the share of innovation in green technologies ($Innov_g = s_{gt}^{1-\psi} / \left(s_{ft}^{1-\psi} + s_{gt}^{1-\psi}\right)$) and on emissions in 2013 (short-term) and in 2063 (long-term), for the baseline parameters, varying the same elasticities as before and for a higher value of ψ (which corresponds to a less elastic innovation response). In the short-run, the comparative statics results are naturally in line with those of Table 5. The medium-run effects on emissions vary. The shale gas boom still reduces emissions 50 years ahead when the two fossil fuels are more substitute (σ is larger), or green technologies are a poorer substitute (ε is lower, in which case the innovation elasticity is lower too), or the scale effect is less strong (λ is low), or innovation is less elastic (ψ is higher). In all cases though, the shale gas boom leads to higher emissions in the long-run (beyond 2063).

Total Effects of Improved Shale Extraction Technology

	2013		2063	
	$\%\Delta Innov_g$	$\%\Delta CO_2$	$\%\Delta Innov_g$	ΔCO_2
Baseline Parameters	-6.2	-0.13	-6.18	0.49
Higher $\varepsilon = 2$	-6.99	0.26	-7.22	1.28
Lower $\varepsilon = 1.5$	-4.26	-1.07	-3.79	-1.25
Higher $\sigma = 2.2$	-5.94	-0.94	-5.75	-0.55
Lower $\sigma = 1.8$	-6.46	0.68	-6.6	1.54
Lower $\lambda = 0.3$	-6.2	-1.43	-6.17	-0.84
Higher $\psi = 2/3$	-2.89	-0.17	-2.77	-0.11

Dynamic effect of Shale boom for a 50% increase in extraction technology with constant extraction technologies thereafter

Figure 4.4 and Table 4.4 reproduce the same analysis when extraction technologies grow over time at the same rate as the maximal rate achievable for the power plant technologies (i.e. with a factor γ^{η}). Figure 4.4 is similar to Figure 4.4 except that since A_{g2008} is not large enough, innovation occurs increasingly more in the fossil fuel sector, whether the shale boom occurs or not. Table 4.4 shows that the comparative statics does not hinge upon whether the extraction technologies improve over time or remain constant. Yet, the medium run effect on emissions is systematically larger than when extraction technologies remain constant.



Total Effects of Improved Shale Extraction Technology				
	2013		2063	
	$\%\Delta Innov_g$	$\%\Delta CO_2$	$\%\Delta Innov_g$	ΔCO_2
Baseline Parameters	-6.31	-0.05	-7.73	1.08
Higher $\varepsilon = 2$	-7.11	0.32	-9	1.57
Lower $\varepsilon = 1.5$	-4.35	-0.97	-4.91	-0.15
Higher $\sigma = 2.2$	-6.06	-0.84	-7.47	0.27
Lower $\sigma = 1.8$	-6.55	0.73	-7.98	1.87
Lower $\lambda = 0.3$	-6.31	-1.35	-7.73	-0.28
Higher $\psi = 2/3$	-2.92	-0.09	-3.28	0.64

Dynamic effect of Shale boom for a 50% increase in extraction technology with growing extraction technologies thereafter

5 Conclusion

In this paper, we looked at the short and long term effects of a shale gas boom in an economy where energy can be produced with coal, natural gas, or a clean energy source. In the short run, a shale gas revolution has counteracting effects on CO2 emissions: on the one hand it allows countries to substitute away from coal which in turn reduces CO2 emissions everything else equal; on the other hand the shale gas boom may increase pollution as it increases the scale of aggregate production. In the long run a shale gas boom tends to increase CO2 emissions as it induces firms to direct innovation away from clean innovation towards shale gas innovation. A shale gas boom may even infinitely delay a switch from fossil fuel to clean energy.

To assess the short-run and long-run impacts of improving the shale extraction technology, we moved to a quantitative analysis. We first calibrated the static version of our model using US data on electricity production and the costs of producing electricity using coal, gas and the different types of renewable energies. Our preliminary results indicate that, for the United States, a reduction in the price of natural gas (akin to the "shale gas revolution") leads to a decrease in CO_2 emissions in the short-run. We then simulated the dynamic model with directed innovation, and showed that for reasonable parameter values, the shale gas revolution decreases innovations in green technologies and consequenly increases emissions in the medium- and long-run.

At the same time, the quantitative predictions of our models depend on parameters and initial conditions, which suggests that a shale gas boom could have quite different effects across countries. A next step for this research agenda would be to calibrate the model to different countries.

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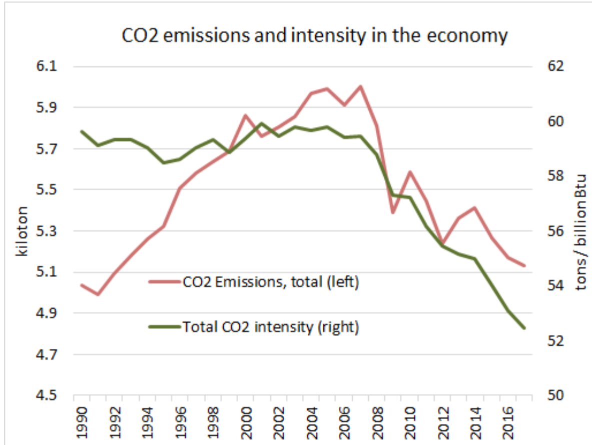

Figure 5: CO2 emissions and intensity for the whole economy

7 Appendix A: Details on the empirical analysis

Figure 5 reproduces Figure 2.B but for total emissions in the United States and for the CO2 intensity of primary energy consumption. The trends are similar.

Table 6 carries several robustness checks to the regressions of Table 1: in turn, we restrict attention to granted patents, we start the analysis in 2000 and we include a 1 year lag between the dependent and independent variables.

Table 7 moves the analysis at the level of the country of invention. We look in turn at EPO and USPTO patents and attribute patent applications to a country according to the nationality of its inventor.

Table 6: Robustness checks

$\frac{Renewable}{FossilFuelElectric}$	Patent Office: all				Patent Office: domestic			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
				Panel A: Granted patents				
Shale Gas Boom	-0.485***	-0.341*	-0.698***	-0.453**	-0.470	-0.421	-0.771*	-0.500
	(0.17)	(0.18)	(0.21)	(0.19)	(0.32)	(0.54)	(0.41)	(0.68)
Ban		0.700		0.633		0.176		0.127
		(0.43)		(0.45)		(0.31)		(0.39)
				Panel B: Starting year 2000				
Shale Gas Boom	-0.373*	-0.650**	-0.649**	-0.461	-0.753**	-1.020	-0.975**	-0.529
	(0.20)	(0.29)	(0.29)	(0.28)	(0.32)	(0.67)	(0.43)	(0.61)
Ban		0.275		0.385		-0.085		0.211
		(0.43)		(0.34)		(0.58)		(0.40)
				Panel C: Lag of 1 year				
Shale Gas Boom	-0.519**	-0.739**	-0.863**	-0.532*	-0.846**	-1.197	-1.112**	-0.814
	(0.21)	(0.34)	(0.32)	(0.26)	(0.37)	(0.81)	(0.46)	(0.78)
Ban		0.360		0.486		0.358		0.598
		(0.51)		(0.42)		(0.77)		(0.63)
FEs (C, T)	Y	Y	Y	Y	Y	Y	Y	Y
Control ln(GDPCap)			Y	Y			Y	Y

Note: Difference-in-difference regressions. The shale gas boom is dated from 2009. Standard errors are clustered at the country-level. Column (2), (4), (6), (8) include AU, CA, CH, CL, CZ, DE, DK, ES, FR, GB, HU, IE, JP, NL, US, the other columns also include CN, TW, AT, BE, IS, IL, EE, FI, GR, IT, KR, LV, LT, LU, MX, NO, PL, PT, NZ, SK, SI, SE, TR.

8 Appendix B: Theoretical results

8.1 Uniqueness of the equilibrium and proof of Proposition 3

We can rewrite (23) as:

$$f\left(s_{gt}, A_{ct-1}, B_{ct}, A_{st-1}, B_{st}, C_{gt-1}\right) = 1 \tag{36}$$

where the function f is defined as

$$f \equiv \frac{\eta_f \left(\frac{\gamma^{-\eta_f s_{ft}^{1-\psi}}}{A_{ct-1}} \kappa_c^{\varepsilon} \left(\frac{\gamma^{-\eta_f s_{ft}^{1-\psi}}}{A_{ct-1}} + \frac{1}{B_{ct}} \right)^{-\varepsilon} + \frac{\gamma^{-\eta_f s_{ft}^{1-\psi}}}{A_{st-1}} \kappa_s^{\varepsilon} \left(\frac{\gamma^{-\eta_f s_{ft}^{1-\psi}}}{A_{st-1}} + \frac{1}{B_{st}} \right)^{-\varepsilon} \right) s_{gt}^{\psi}}{\eta_g \kappa_g^{\varepsilon} C_{gt-1}^{\varepsilon-1} s_{ft}^{\psi} \gamma^{\eta_g s_{gt}^{1-\psi}(\varepsilon-1)}}.$$

Table 7: Regressions at the level of the inventor country

	EPO				USPTO			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Panel A: Renewable / Fossil fuel electric								
Shale Gas Boom	-0.889**	-0.867*	-0.979**	-0.833	-0.527	-0.725	-0.628	-0.387
	(0.39)	(0.45)	(0.44)	(0.50)	(0.33)	(0.52)	(0.41)	(0.60)
Ban		2.420		2.440		2.239		2.441
		(2.31)		(2.29)		(2.08)		(2.00)
Panel B: Green / Fossil fuel electric								
Shale Gas Boom	-1.135**	-1.114*	-1.264**	-1.001*	-0.641*	-0.765	-0.774	-0.244
	(0.43)	(0.52)	(0.50)	(0.56)	(0.37)	(0.60)	(0.48)	(0.73)
Ban		2.572		2.639		2.379		2.691
		(2.61)		(2.57)		(2.26)		(2.15)
FEs (C, T)	Y	Y	Y	Y	Y	Y	Y	Y
Control ln(GDPCap)			Y	Y			Y	Y

Note: Difference-in-difference regressions. The shale gas boom is dated from 2009. Standard errors are clustered at the country-level. Column (2), (4), (6), (8) include AU, CA, CH, CL, CZ, DE, DK, ES, FR, GB, HU, IE, JP, NL, US, the other columns also include CN, TW, AT, BE, IS, IL, EE, FI, GR, IT, KR, LV, LT, LU, MX, NO, PL, PT, NZ, SK, SI, SE, TR.

We then get that

$$\begin{aligned}
&\frac{\partial \ln f}{\partial \ln s_{gt}} \\
= \quad & \psi - \eta_g (\varepsilon - 1)(1-\psi)(\ln \gamma)\, s_{gt}^{1-\psi} + \psi \frac{s_{gt}}{s_{ft}} \\
& + \frac{\eta_f (1-\psi) \ln(\gamma)\, s_{ft}^{1-\psi} \frac{s_{gt}}{s_{ft}} \left(\kappa_c^{\varepsilon} \frac{C_{ct}^{\varepsilon}}{A_{ct}} \left(1 - \varepsilon \frac{B_{ct}}{B_{ct}+A_{ct}}\right) + \kappa_s^{\varepsilon} \frac{C_{st}^{\varepsilon}}{A_{st}} \left(1 - \varepsilon \frac{B_{st}}{B_{st}+A_{st}}\right)\right)}{\kappa_c^{\varepsilon} \frac{C_t^{\varepsilon}}{A_{ct}} + \kappa_s^{\varepsilon} \frac{C_{st}^{\varepsilon}}{A_{st}}} \\
\geq \quad & \psi - \eta_g (\varepsilon - 1)(1-\psi)(\ln \gamma)\, s_{gt}^{1-\psi} + \left(\psi - \eta_f (\varepsilon - 1)(1-\psi)(\ln \gamma)\, s_{ft}^{1-\psi}\right) \frac{s_{gt}}{s_{ft}}.
\end{aligned}$$

Therefore we get that $\frac{\partial \ln f}{\partial \ln s_{gt}} > 0$ if Assumption 1 holds. In that case since $f(0,.) = 0$ and $\lim_{s_g \to 1} f(s_g,.) = \infty$, we obtain that (23) defines a unique equilibrium innovation allocation.

We directly get that $\frac{\partial f}{\partial B_{st}} > 0$ which establishes that an increase in B_{s1} leads to a lower value for s_{g1}.

Further, we obtain that $\frac{\partial f}{\partial C_{gt-1}} < 0$, so that a higher value for C_{gt-1} leads to more clean innovation. Further, we get

$$\frac{\partial \ln f}{\partial \ln A_{ct-1}} = \frac{\frac{1}{A_{ct}}\kappa_c^{\varepsilon}C_{ct}^{\varepsilon}}{\frac{1}{A_{ct}}\kappa_c^{\varepsilon}C_{ct}^{\varepsilon} + \frac{1}{A_{st}}\kappa_s^{\varepsilon}C_{st}^{\varepsilon}}\left(\varepsilon\frac{B_{ct}}{B_{ct} + \gamma^{\eta_f s_{ft}^{1-\psi}}A_{ct-1}} - 1\right).$$

Therefore $\frac{\partial \ln f}{\partial \ln A_{ct-1}} \geq 0$ for all values of s_{ft} provided that $\frac{B_{ct}}{A_{ct-1}} > \frac{\gamma^{\eta_f}}{\varepsilon-1}$. Similarly, $\frac{\partial \ln f}{\partial \ln A_{st-1}} \geq 0$ for all values of s_{ft} provided that $\frac{B_{st}}{A_{st-1}} > \frac{\gamma^{\eta_f}}{\varepsilon-1}$. If these conditions are satisfied, then an increase in B_{s1} leads to higher values of A_{s1}, A_{c1} and a lower value of C_{g1}, which imply a lower value of s_{g2}. This in turns leads to even higher values of A_{s2}, A_{c2} and a lower value for C_{g2}. By iteration, we then get that all s_{gt} decrease for $t \geq 1$.

8.2 Proof of Proposition 4

We first note that as argued in the text, if $\min\left(B_{ct}/A_{c(t-1)},\, B_{st}/A_{s(t-1)}\right) > \gamma^{\eta}/\left(\varepsilon-1\right)$ at $t=1$, then this holds for all $t>1$, so that Lemma 3 applies.

We then prove the following lemma:

Lemma 1 *Assume that Assumption 1 holds, that B_{ct} and B_{st} grow exogenously at factor γ^{η} and that $\min\left(B_{c1}/A_{c0},\, B_{s1}/A_{s0}\right) > \gamma^{\eta}/\left(\varepsilon-1\right)$. Then the economy features $s_{gt}\to 1$ or $s_{gt}\to 0$ (except for a knife-edge case where $s_{gt}\to 1/2$).*

Proof. Assume that for some time period τ, $s_{g\tau}\leq 1/2$, we first establish that $s_{gt}<1/2$ for all $t>\tau$. For ease of notations, define $f_{\tau}\left(s_{g\tau}\right)\equiv f\left(s_{g\tau}, A_{c(\tau-1)}, B_{c\tau}, A_{s(\tau-1)}, B_{s\tau}, C_{g(\tau-1)}\right)$. We then get that:

$$\begin{aligned}
& f_{\tau+1}\left(s_{g\tau}\right) \\
= \quad & \frac{\frac{\gamma^{-\eta s_{f\tau}^{1-\psi}}}{A_{c\tau}}\kappa_c^{\varepsilon}\left(\frac{\gamma^{-\eta s_{f\tau}^{1-\psi}}}{A_{c\tau}}+\frac{1}{B_{c(\tau+1)}}\right)^{-\varepsilon}+\frac{\gamma^{-\eta s_{f\tau}^{1-\psi}}}{A_{s\tau}}\kappa_s^{\varepsilon}\left(\frac{\gamma^{-\eta s_{f\tau}^{1-\psi}}}{A_{s\tau}}+\frac{1}{B_{s(\tau+1)}}\right)^{-\varepsilon}}{\kappa_g^{\varepsilon}C_{g\tau}^{\varepsilon-1}\gamma^{\eta s_{g\tau}^{1-\psi}(\varepsilon-1)}}\left(\frac{s_{g\tau}}{s_{f\tau}}\right)^{\psi} \\
= \quad & \gamma^{(\varepsilon-1)\eta\left(s_{f\tau}^{1-\psi}-s_{g\tau}^{1-\psi}\right)}\frac{\frac{\kappa_c^{\varepsilon}\gamma^{-\eta s_{f\tau}^{1-\psi}}}{A_{c(\tau-1)}}\left(\frac{\gamma^{-\eta s_{f\tau}^{1-\psi}}}{A_{c(\tau-1)}}+\frac{\gamma^{-\eta\left(1-s_{f\tau}^{1-\psi}\right)}}{B_{c\tau}}\right)^{-\varepsilon}+\frac{\kappa_s^{\varepsilon}\gamma^{-\eta s_{f\tau}^{1-\psi}}}{A_{s(\tau-1)}}\left(\frac{\gamma^{-\eta s_{f\tau}^{1-\psi}}}{A_{s(\tau-1)}}+\frac{\gamma^{-\eta\left(1-s_{f\tau}^{1-\psi}\right)}}{B_{s\tau}}\right)^{-\varepsilon}}{\kappa_g^{\varepsilon}C_{g\tau-1}^{\varepsilon-1}\gamma^{\eta s_{g\tau}^{1-\psi}(\varepsilon-1)}}\left(\frac{s_{g\tau}}{s_{f\tau}}\right)^{\psi} \\
> \quad & f_{\tau}\left(s_{g\tau}\right)=1,
\end{aligned}$$

where we use that $s_{f\tau}<1$ so that $\gamma^{-\eta\left(1-s_{f\tau}^{1-\psi}\right)}<1$ and that $s_{f\tau}\geq s_{g\tau}$. Since $f_{\tau+1}$ is increasing then it must be that $s_{g(\tau+1)}<s_{g\tau}$, which implies that $s_{gt}>1/2$ for all $t>\tau$. Since s_{gt} is increasing (from τ), it must tend toward a constant s_g^* smaller than $1/2$. As a result, $\frac{\gamma^{-\eta_f s_{ft}^{1-\psi}}}{A_{ct-1}}\kappa_c^{\varepsilon}\left(\frac{\gamma^{-\eta_f s_{ft}^{1-\psi}}}{A_{ct-1}}+\frac{1}{B_{ct}}\right)^{-\varepsilon}+\frac{\gamma^{-\eta_f s_{ft}^{1-\psi}}}{A_{st-1}}\kappa_s^{\varepsilon}\left(\frac{\gamma^{-\eta_f s_{ft}^{1-\psi}}}{A_{st-1}}+\frac{1}{B_{st}}\right)^{-\varepsilon}$ will grow at factor $\gamma^{\eta(\varepsilon-1)\left(1-s_g^*\right)^{1-\psi}}$ while $C_{g(t-1)}^{\varepsilon-1}$ will grow less fast with a factor $\gamma^{\eta(\varepsilon-1)s_g^{*1-\psi}}$ if $s_g^*>0$ (or will not grow exponentially if $s_g^*=0$). As a result $f_t\left(s_t\right)\to\infty$ for s_t bounded above 0, therefore it must be that $s_g^*=0$. In other words, all innovation tend toward the fossil fuel sector.

Assume instead that for all $t's$, $s_{gt} > 1/2$. We want to establish that $\lim_{\infty} s_{gt} = 1/2$ is only possible for a knife-edge case. To do that consider A_{c0}, A_{s0} and C_{g0} such that $\lim_{\infty} s_{gt} = 1/2$. Now consider an alternative set-up where the initial green productivity $\widetilde{C}_{g0}$ is higher. Since $\min\left(B_{ct}/A_{c(t-1)}, B_{st}/A_{s(t-1)}\right) > \gamma^{\eta}/(\varepsilon-1)$ for all t, the reasoning of Appendix 8.1 applies and we get that under the alternative path (denoted with $\widetilde{\ }$), $\widetilde{s}_{gt} > s_{gt}$ so that $\widetilde{C}_{gt} > C_{gt}$, $\widetilde{A}_{ct} < A_{ct}$ and $\widetilde{A}_{st} < A_{st}$. In fact one gets:

$$\widetilde{A}_{st} < \gamma^{\eta\left(\widetilde{s}_{f1}^{1-\psi} - s_{f1}^{1-\psi}\right)} A_{st},\ \widetilde{A}_{ct} < \gamma^{\eta\left(\widetilde{s}_{f1}^{1-\psi} - s_{f1}^{1-\psi}\right)} A_{ct} \text{ and } \widetilde{C}_{gt} > \gamma^{\eta\left(\widetilde{s}_{g1}^{1-\psi} - s_{g1}^{1-\psi}\right)} C_{gt}.$$

Since B_{ct} and B_{st} grow faster than A_{ct} and A_{st}, we have that

$$f_t\left(s_{gt}\right) \sim \gamma^{\eta(\varepsilon-1)\left(s_{ft}^{1-\psi} - s_{gt}^{1-\psi}\right)} \frac{\left(\kappa_c^{\varepsilon} A_{c(t-1)}^{\varepsilon-1} + \kappa_s^{\varepsilon} A_{s(t-1)}^{\varepsilon-1}\right) s_{gt}^{\psi}}{\kappa_g^{\varepsilon} C_{g(t-1)}^{\varepsilon-1} s_{ft}^{\psi}},$$

since by assumption $\lim s_{gt} = 1/2$, then $\lim \frac{\kappa_c^{\varepsilon} A_{c(t-1)}^{\varepsilon-1} + \kappa_s^{\varepsilon} A_{s(t-1)}^{\varepsilon-1}}{\kappa_g^{\varepsilon} C_{g(t-1)}^{\varepsilon-1}} = 1$. Therefore (as B_{ct} and B_{st} still grow faster than $A_{c(t-1)}^{\varepsilon-1}$ and $A_{s(t-1)}^{\varepsilon-1}$).

$$\lim \widetilde{f}_t\left(\frac{1}{2}\right) = \lim \frac{\kappa_c^{\varepsilon} \widetilde{A}_{c(t-1)}^{\varepsilon-1} + \kappa_s^{\varepsilon} \widetilde{A}_{s(t-1)}^{\varepsilon-1}}{\kappa_g^{\varepsilon} \widetilde{C}_{g(t-1)}^{\varepsilon-1}} \leq \gamma^{\eta(\varepsilon-1)\left(\left(\widetilde{s}_{f1}^{1-\psi} - s_{f1}^{1-\psi}\right) - \left(\widetilde{s}_{g1}^{1-\psi} - s_{g1}^{1-\psi}\right)\right)} < 1.$$

Therefore $\lim \widetilde{s}_{gt} \neq 1/2$ as this would impose that $\lim \widetilde{f}_t\left(\frac{1}{2}\right) = 1$ which is impossible. A similar reasoning can be applied for an alternative path with a lower C_{g0}, which then results in $\widetilde{s}_{gt} \to 0$. In other words $s_{gt} \to 1/2$ corresponds to a knife-edge case.

Then, consider a path such that $s_{gt} > 1/2$ and $s_{gt} \not\to 1/2$. For t large enough, we get that:

$$f_t\left(s_{gt}\right) \sim \gamma^{\eta(\varepsilon-1)\left(s_{ft}^{1-\psi} - s_{gt}^{1-\psi}\right)} \frac{\left(\kappa_c^{\varepsilon} A_{c(t-1)}^{\varepsilon-1} + \kappa_s^{\varepsilon} A_{s(t-1)}^{\varepsilon-1}\right) s_{gt}^{\psi}}{\kappa_g^{\varepsilon} C_{g(t-1)}^{\varepsilon-1} s_{ft}^{\psi}}.$$

As C_{gt-1} grows faster than $A_{c(t-1)}^{\varepsilon-1}$ and $A_{s(t-1)}^{\varepsilon-1}$, (36) can then only be satisfied if $s_{gt} \to 1$. This achieves the proof of the lemma. ■

A sufficient condition to get that $s_{gt} \to 0$ is obtained for $s_{g1} \leq 1/2$ which corresponds to $f_1(1/2) \geq 1$, which is equivalent to

$$\kappa_g^{\varepsilon} A_{g0}^{\varepsilon-1} \leq \frac{\kappa_c^{\varepsilon}}{A_{c0}} \left(\frac{1}{A_{c0}} + \frac{\gamma^{\eta/2^{1-\psi}}}{B_{c1}}\right)^{-\varepsilon} + \frac{\kappa_s^{\varepsilon}}{A_{s0}} \left(\frac{1}{A_{s0}} + \frac{\gamma^{\eta/2^{1-\psi}}}{B_{s1}}\right)^{-\varepsilon}.$$

In contrast assume now that $\kappa_g^{\varepsilon} A_{g0}^{\varepsilon-1} > \kappa_c^{\varepsilon} A_{c0}^{\varepsilon-1} + \kappa_s^{\varepsilon} A_{s0}^{\varepsilon-1}$, then since

$$f_1\left(s_{g1}\right) < \gamma^{\eta(\varepsilon-1)\left(s_{f1}^{1-\psi} - s_{g1}^{1-\psi}\right)} \frac{\left(A_{c0}^{\varepsilon-1} + A_{s0}^{\varepsilon-1}\right) s_{g1}^{\psi}}{C_{g0}^{\varepsilon-1} s_{f1}^{\psi}},$$

we must have $s_{g1} > 1/2$. This ensures that $s_{gt} > 1/2$ for all $t's$. For t large enough, we then have

$$f_t\left(s_{gt}\right) \sim \gamma^{\eta(\varepsilon-1)\left(s_{ft}^{1-\psi}-s_{gt}^{1-\psi}\right)} \frac{\left(A_{c(t-1)}^{\varepsilon-1}+A_{s(t-1)}^{\varepsilon-1}\right) s_{gt}^{\psi}}{C_{g(t-1)}^{\varepsilon-1} s_{ft}^{\psi}} < \gamma^{\eta(\varepsilon-1)\left(s_{ft}^{1-\psi}-s_{gt}^{1-\psi}\right)} \frac{\left(A_{c0}^{\varepsilon-1}+A_{s0}^{\varepsilon-1}\right) s_{gt}^{\psi}}{C_{g0}^{\varepsilon-1} s_{ft}^{\psi}},$$

so that $s_{gt} \not\to 1/2$. Therefore for sufficiently low C_{g0} the economy will be on a path toward with $s_{gt} \to 0$ and for C_{g0} sufficiently high toward a path with $s_{gt} \to 1$. Since the only other possibility is that $s_{gt} \to 1/2$ and is obtained for a knife-edge case (where a higher C_{g0} leads to $s_{gt} \to 1$ and a lower one leads to $s_{gt} \to 0$), we get that there exists a C^* (which depends on the other parameters) so that for $C_{g0} > C^*$, $s_{gt} \to 1$, for $C_{g0} = C^*$, $s_{gt} \to 1/2$ and for $C_{g0} < C^*$, $s_{gt} \to 0$.

As already established, an increase in B_{s0} implies that s_{gt} decreases at all $t's$. Therefore, following the same reasoning that established that $s_{gt} \to 1/2$ is a knife-edge case for a given value of C_{g0}, if $s_{gt} \to 0$ prior to the increase it will still do so after the shale gas boom; if $s_{gt} \to 1/2$, it will tend toward 0; and if $s_{gt} \to 1$ it will either still do so, or tend toward 1/2 for a knife-edge case or tend toward 0 for a larger increase in B_{s0} (these latter two cases being only possible for intermediate values of C_{g0}).

8.3 Proof of Proposition 5

8.3.1 Proof of part i)

First, suppose that $s_{ft} \not\to 0$, so that A_{ct} and A_{st} are unbounded. With B_{ct} and B_{st} constant, we get that

$$f_t\left(s_{gt}\right) \sim \left(\kappa_c^{\varepsilon} \frac{B_{c0}^{\varepsilon}}{A_{ct-1}} + \kappa_s^{\varepsilon} \frac{B_{s0}^{\varepsilon}}{A_{st-1}}\right) \frac{\gamma^{-\eta s_{ft}^{1-\psi}} s_{gt}^{\psi}}{\kappa_g^{\varepsilon} C_{gt-1}^{\varepsilon-1} \gamma^{\eta_g s_{gt}^{1-\psi}(\varepsilon-1)} s_{ft}^{\psi}},$$

which tends toward 0 unless s_{ft} is arbitrarily small. Therefore, it must be that $s_{ft} \to 0$.

We then establish the existence of a time t_{switch} by showing that if $s_{gt} \geq 1/2$ then $s_{g(t+1)} > 1/2$. Assume that $s_{gt} \geq 1/2$, then one gets

$$\begin{aligned}
f_{t+1}\left(\frac{1}{2}\right) &= \frac{\frac{\gamma^{-\eta s_{ft}^{1-\psi}}}{A_{c(t-1)}} \kappa_c^{\varepsilon} \left(\frac{\gamma^{\eta 2^{\psi-1}} \gamma^{-\eta s_{ft}^{1-\psi}}}{A_{c(t-1)}} + \frac{1}{B_{c0}}\right)^{-\varepsilon} + \frac{\gamma^{-\eta s_{ft}^{1-\psi}}}{A_{s(t-1)}} \kappa_s^{\varepsilon} \left(\frac{\gamma^{\eta 2^{\psi-1}} \gamma^{-\eta s_{ft}^{1-\psi}}}{A_{s(t-1)}} + \frac{1}{B_{s0}}\right)^{-\varepsilon}}{\kappa_g^{\varepsilon} \gamma^{\eta s_{gt}^{1-\psi}(\varepsilon-1)} C_{g(t-1)}^{\varepsilon-1} \gamma^{\eta(\varepsilon-1) 2^{\psi-1}}} \\
&= \gamma^{\eta(\varepsilon-1)\left(s_{ft}^{1-\psi}-s_{gt}^{1-\psi}\right)} \frac{\frac{\kappa_c^{\varepsilon}}{A_{c(t-1)}} \left(\frac{\gamma^{\eta 2^{\psi-1}}}{A_{c(t-1)}} + \frac{\gamma^{\eta s_{ft}^{1-\psi}}}{B_{c0}}\right)^{-\varepsilon} + \frac{\kappa_s^{\varepsilon}}{A_{s(t-1)}} \left(\frac{\gamma^{\eta 2^{\psi-1}}}{A_{s(t-1)}} + \frac{\gamma^{\eta s_{ft}^{1-\psi}}}{B_{s0}}\right)^{-\varepsilon}}{\kappa_g^{\varepsilon} C_{g(t-1)}^{\varepsilon-1} \gamma^{\eta(\varepsilon-1) 2^{\psi-1}}} \\
&< \frac{\frac{\kappa_c^{\varepsilon}}{A_{c(t-1)}} \left(\frac{\gamma^{\eta 2^{\psi-1}}}{A_{c(t-1)}} + \frac{1}{B_{c0}}\right)^{-\varepsilon} + \frac{\kappa_s^{\varepsilon}}{A_{s(t-1)}} \left(\frac{\gamma^{\eta 2^{\psi-1}}}{A_{s(t-1)}} + \frac{1}{B_{s0}}\right)^{-\varepsilon}}{\kappa_g^{\varepsilon} C_{g(t-1)}^{\varepsilon-1} \gamma^{\eta(\varepsilon-1) 2^{\psi-1}}} = f_t\left(\frac{1}{2}\right) \leq 1.
\end{aligned}$$

Therefore $s_{g(t+1)} > 1/2$, which establishes the existence of a time t_{switch} ($t_{switch} = 1$ if $s_{g1} \geq 1/2$).

We now show that an increase in B_{s0} increases t_{switch}, to do that we establish that an increase in B_{s0} leads to an increase in s_{gt} as long as $s_{gt} \leq 1/2$. We define

$$\widehat{f}_t\left(s_{gt}, s_{g(t-1)}, ..., s_{g1}, B_{s0}\right) = \frac{s_{gt}^{\psi}}{\kappa_g^{\varepsilon} C_{g0}^{\varepsilon-1} s_{ft}^{\psi} \gamma^{\eta(\varepsilon-1)\sum_{\tau=1}^{t} s_{g\tau}^{1-\psi}}} \left(\begin{array}{c} \frac{\kappa_c^{\varepsilon}\gamma^{-\eta\sum_{\tau=1}^{t} s_{f\tau}^{1-\psi}}}{A_{c0}} \left(\frac{\gamma^{-\eta\sum_{\tau=1}^{t} s_{f\tau}^{1-\psi}}}{A_{c0}} + \frac{1}{B_{c0}} \right)^{-\varepsilon} \\ + \frac{\kappa_s^{\varepsilon}\gamma^{-\eta\sum_{\tau=1}^{t} s_{f\tau}^{1-\psi}}}{A_{s0}} \left(\frac{\gamma^{-\eta\sum_{\tau=1}^{t} s_{f\tau}^{1-\psi}}}{A_{s0}} + \frac{1}{B_{s0}} \right)^{-\varepsilon} \end{array} \right),$$

so that the equilibrium innovation allocation is still defined through $\widehat{f}_t\left(s_{gt}, s_{g(t-1)}, ..., s_{g1}, B_{s0}\right) = 1$ with $\widehat{f}_t$ increasing in s_{gt} and in B_{s0}. We obtain for $\widetilde{\tau} \in [1, t-1)$

$$\frac{\partial \ln \widehat{f}_t}{\partial \ln s_{g\widetilde{\tau}}} = \left[\frac{\begin{array}{c} \frac{\kappa_c^{\varepsilon}}{A_{ct}} \left(\frac{1}{A_{ct}} + \frac{1}{B_{c0}} \right)^{-\varepsilon} \left(1 - \varepsilon \frac{\frac{1}{A_{ct}}}{\frac{1}{A_{ct}} + \frac{1}{B_{c0}}} \right) \\ + \frac{\kappa_s^{\varepsilon}}{A_{st}} \left(\frac{1}{A_{st}} + \frac{1}{B_{s0}} \right)^{-\varepsilon} \left(1 - \varepsilon \frac{\frac{1}{A_{st}}}{\frac{1}{A_{st}} + \frac{1}{B_{s0}}} \right) \end{array}}{\frac{\kappa_c^{\varepsilon}}{A_{ct}} \left(\frac{1}{A_{ct}} + \frac{1}{B_{c0}} \right)^{-\varepsilon} + \frac{\kappa_s^{\varepsilon}}{A_{st}} \left(\frac{1}{A_{st}} + \frac{1}{B_{s0}} \right)^{-\varepsilon}} s_{f\widetilde{\tau}}^{-\psi} - (\varepsilon - 1) s_{g\widetilde{\tau}}^{-\psi} \right] s_{g\widetilde{\tau}} \eta (1-\psi) \ln \gamma.$$

Yet if $t < t_{switch}$, then $s_{f\widetilde{\tau}} > s_{g\widetilde{\tau}}$, so that

$$\frac{\partial \ln \widehat{f}_t}{\partial \ln s_{g\widetilde{\tau}}} < -\left[\varepsilon - 2 + \varepsilon \frac{\frac{\kappa_c^{\varepsilon}}{A_{ct}^2} \left(\frac{1}{A_{ct}} + \frac{1}{B_{c0}} \right)^{-\varepsilon-1} + \frac{\kappa_s^{\varepsilon}}{A_{st}^2} \left(\frac{1}{A_{st}} + \frac{1}{B_{s0}} \right)^{-\varepsilon-1}}{\frac{\kappa_c^{\varepsilon}}{A_{ct}} \left(\frac{1}{A_{ct}} + \frac{1}{B_{c0}} \right)^{-\varepsilon} + \frac{\kappa_s^{\varepsilon}}{A_{st}} \left(\frac{1}{A_{st}} + \frac{1}{B_{s0}} \right)^{-\varepsilon}} \right] s_{f\widetilde{\tau}}^{-\psi} s_{g\widetilde{\tau}} \eta (1-\psi) \ln \gamma.$$

Therefore if $\varepsilon \geq 2$, we have that $\frac{\partial \ln \widehat{f}_t}{\partial \ln s_{g\widetilde{\tau}}} < 0$.

Therefore, the shale gas boom reduces $\widehat{f}_1$ leading to a lower value for s_{g1}. It then reduces $\widehat{f}_2$ both directly and because of its negative effect on s_{g1}, leading to a lower value for s_{g2}. By iteration, the shale gas boom will reduce all s_{gt} at least until the switch toward green innovation occurs.

8.3.2 Proof of Part ii)

We prove that emissions in the long-run must be decreasing following a shale gas boom for $\ln \gamma$ sufficiently small. To establish this result, we first show the following Lemma:

Lemma 2 *For $t > t_{switch}$, $s_{gt} > s_{g(t-1)}$.*

Proof. To establish the result, define:

$$f_{\gamma,t}\left(s_{g(t-1)}, \gamma\right) = \frac{\left(\frac{\gamma^{-\eta s_{f(t-1)}^{1-\psi}}}{A_{ct-1}} \kappa_c^{\varepsilon} \left(\frac{\gamma^{-\eta s_{f(t-1)}^{1-\psi}}}{A_{ct-1}} + \frac{1}{B_c} \right)^{-\varepsilon} + \frac{\gamma^{-\eta s_{f(t-1)}^{1-\psi}}}{A_{st-1}} \kappa_s^{\varepsilon} \left(\frac{\gamma^{-\eta s_{f(t-1)}^{1-\psi}}}{A_{st-1}} + \frac{1}{B_s} \right)^{-\varepsilon} \right) s_{g(t-1)}^{\psi}}{\kappa_g^{\varepsilon} C_{g(t-1)}^{\varepsilon-1} \gamma^{\eta(\varepsilon-1) s_{g(t-1)}^{1-\psi}} s_{f(t-1)}^{\psi}}.$$

We then obtain:

$$\frac{\partial \ln f_{\gamma,t}}{\partial \ln \gamma}$$

$$= \frac{\left(\begin{array}{c} \frac{\gamma^{-\eta s_{f(t-1)}^{1-\psi}}}{A_{ct-1}}\kappa_c^{\varepsilon}\left(\frac{\gamma^{-\eta s_{f(t-1)}^{1-\psi}}}{A_{ct-1}}+\frac{1}{B_c}\right)^{-\varepsilon}\left(\varepsilon\frac{\frac{\gamma^{-\eta s_{f(t-1)}^{1-\psi}}}{A_{ct-1}}}{\frac{\gamma^{-\eta s_{f(t-1)}^{1-\psi}}}{A_{ct-1}}+\frac{1}{B_c}}-1\right) \\ +\frac{\gamma^{-\eta s_{f(t-1)}^{1-\psi}}}{A_{st-2}}\kappa_s^{\varepsilon}\left(\frac{\gamma^{-\eta s_{f(t-1)}^{1-\psi}}}{A_{st-2}}+\frac{1}{B_s}\right)^{-\varepsilon}\left(\varepsilon\frac{\frac{\gamma^{-\eta s_{f(t-1)}^{1-\psi}}}{A_{st-1}}}{\frac{\gamma^{-\eta s_{f(t-1)}^{1-\psi}}}{A_{st-1}}+\frac{1}{B_c}}-1\right) \end{array}\right)}{\frac{\gamma^{-\eta s_{f(t-1)}^{1-\psi}}}{A_{ct-1}}\kappa_c^{\varepsilon}\left(\frac{\gamma^{-\eta s_{f(t-1)}^{1-\psi}}}{A_{ct-1}}+\frac{1}{B_c}\right)^{-\varepsilon}+\frac{\gamma^{-\eta s_{f(t-1)}^{1-\psi}}}{A_{st-2}}\kappa_s^{\varepsilon}\left(\frac{\gamma^{-\eta s_{f(t-1)}^{1-\psi}}}{A_{st-2}}+\frac{1}{B_s}\right)^{-\varepsilon}}\eta s_{f(t-1)}^{1-\psi}-(\varepsilon-1)\,\eta s_{g(t-1)}^{1-\psi}$$

$$< (\varepsilon-1)\,\eta\left(s_{f(t-1)}^{1-\psi}-s_{g(t-1)}^{1-\psi}\right)\leq 0,$$

since $t-1 \geq t_{swicth}$ so that $s_{f(t-1)} \geq s_{g(t-1)}$. Note that $f_{\gamma,t}\left(s_{g(t-1)},1\right) = f_{t-1}\left(s_{g(t-1)}\right) = 1$, therefore $f_t\left(s_{g(t-1)}\right) < f_{\gamma,t}\left(s_{g(t-1)},\gamma\right) < 1$, so that $s_{g(t-1)} < s_{gt}$. ■

We then establish the following Lemma:

Lemma 3 *Consider a small increase in B_s. Denote by t_A the smallest t such that $d\ln A_{st_A} < 0$ and assume that $t_A < \infty$. Then $d\ln C_{gt_A} > d\ln A_{st_A}$.*

Proof. Since

$$\ln A_{ct} = \ln A_{c0} + \eta\left(\ln\gamma\right)\sum_{\tau=1}^{t} s_{f\tau}^{1-\psi} \text{ and } \ln A_{st} = \ln A_{s0} + \eta\left(\ln\gamma\right)\sum_{\tau=1}^{t} s_{f\tau}^{1-\psi}$$

we have that

$$d\ln A_{ct} = d\ln A_{st} = \eta\left(1-\psi\right)\left(\ln\gamma\right)\sum_{\tau=1}^{t} s_{f\tau}^{-\psi} ds_{f\tau}.$$

By definition of t_A, $d\ln A_{c(t_A-1)} > 0$ and $d\ln A_{ct_A} < 0$, therefore we must have $ds_{ft_A} < 0$. Since $ds_{ft} > 0$ for $t \leq t_{switch}$, we must have $t_A > t_{switch}$.
In addition, we have:

$$d\ln C_{gt} = -\eta\left(1-\psi\right)\left(\ln\gamma\right)\sum_{\tau=1}^{t} s_{g\tau}^{-\psi} ds_{f\tau}.$$

Therefore, we can write

$$d\ln A_{st_A} - d\ln C_{gt_A} = \eta\left(1-\psi\right)\left(\ln\gamma\right)\left(\sum_{\tau=1}^{t_A}\left(s_{f\tau}^{-\psi} - s_{g\tau}^{-\psi}\right)ds_{f\tau}\right).$$

We know that $ds_{ft} > 0$ for $t \leq t_{switch}$ and that $ds_{ft_A} < 0$, therefore ds_{ft} must change sign as t increases at least once. We indexes the times where ds_{ft} switches signs by t_{2p} and t_{2p+1}, such that ds_{ft} becomes negative at t_{2p+1} and positive at t_{2p} and p is a weakly positive integer in the integer set $[0, P-1]$ with $P \geq 1$. We denote by $t_0 = t_{switch} + 1$ and $t_{2P} = t_A + 1$. We can then write

$$\begin{aligned} & d \ln A_{st_A} - d \ln C_{gt_A} \qquad (37) \\ = \ & \eta (1-\psi) (\ln \gamma) \left(\begin{array}{c} \sum_{\tau=1}^{t_{switch}} \left(s_{f\tau}^{-\psi} - s_{g\tau}^{-\psi} \right) ds_{f\tau} \\ + \sum_{p=0}^{P-1} \left(\sum_{\tau=t_{2p}}^{t_{2p+1}-1} \left(s_{f\tau}^{-\psi} - s_{g\tau}^{-\psi} \right) ds_{f\tau} + \sum_{\tau=t_{2p+1}}^{t_{2p+2}-1} \left(s_{f\tau}^{-\psi} - s_{g\tau}^{-\psi} \right) ds_{f\tau} \right) \end{array} \right) \\ = \ & \eta (1-\psi) (\ln \gamma) \left(\begin{array}{c} \sum_{\tau=1}^{t_{switch}} \left(s_{f\tau}^{-\psi} - s_{g\tau}^{-\psi} \right) ds_{f\tau} \\ + \sum_{p=0}^{P-1} \left(\sum_{\tau=t_{2p}}^{t_{2p+1}-1} \left(1 - \frac{s_{f\tau}^{\psi}}{s_{g\tau}^{\psi}} \right) s_{f\tau}^{-\psi} ds_{f\tau} + \sum_{\tau=t_{2p+1}}^{t_{2p+2}-1} \left(1 - \frac{s_{f\tau}^{\psi}}{s_{g\tau}^{\psi}} \right) s_{f\tau}^{-\psi} ds_{f\tau} \right) \end{array} \right) \end{aligned}$$

Using that $s_{f\tau}^{-\psi} - s_{g\tau}^{-\psi} < 0$ for $\tau \leq t_{switch}$, that $\frac{s_{f\tau}^{\psi}}{s_{g\tau}^{\psi}}$ is decreasing for $\tau > t_{switch}$ (following lemma 2), that $ds_{f\tau} > 0$ on intervals $[t_{2p}, t_{2p+1} - 1]$ and negative otherwise, we get

$$d \ln A_{st_A} - d \ln C_{gt_A} < \eta (1-\psi) (\ln \gamma) \sum_{p=0}^{P-1} \left(1 - \frac{s_{ft_{2p+1}}^{\psi}}{s_{gt_{2p+1}}^{\psi}} \right) \sum_{\tau=t_{2p}}^{t_{2p+2}-1} s_{f\tau}^{-\psi} ds_{f\tau}$$

By definition t_A is the smallest t such that $\sum_{\tau=1}^{t_A} s_{f\tau}^{-\psi} ds_{f\tau} < 0$, therefore for any $t_X < t_A$, we have $\sum_{\tau=1}^{t_X} s_{f\tau}^{-\psi} ds_{f\tau} > 0$ and $\sum_{\tau=t_X+1}^{t_A} s_{f\tau}^{-\psi} ds_{f\tau} < 0$. Therefore, we get that

$$\begin{aligned} & \sum_{p=P-2}^{P-1} \left(1 - \frac{s_{ft_{2p+1}}^{\psi}}{s_{gt_{2p+1}}^{\psi}} \right) \sum_{\tau=t_{2p}}^{t_{2p+2}-1} s_{f\tau}^{-\psi} ds_{f\tau} \\ = \ & \left(1 - \frac{s_{ft_{2P-3}}^{\psi}}{s_{gt_{2P-3}}^{\psi}} \right) \sum_{\tau=t_{2P-4}}^{t_{2P-2}-1} s_{f\tau}^{-\psi} ds_{f\tau} + \left(1 - \frac{s_{ft_{2P-1}}^{\psi}}{s_{gt_{2P-1}}^{\psi}} \right) \sum_{\tau=t_{2P-2}}^{t_A} s_{f\tau}^{-\psi} ds_{f\tau} \\ < \ & \left(1 - \frac{s_{ft_{2P-3}}^{\psi}}{s_{gt_{2P-3}}^{\psi}} \right) \sum_{\tau=t_{2P-4}}^{t_A} s_{f\tau}^{-\psi} ds_{f\tau}. \end{aligned}$$

Iterating, we get

$$d \ln A_{st_A} - d \ln C_{gt_A} < \eta (1-\psi) (\ln \gamma) \left(1 - \frac{s_{ft_1}^{\psi}}{s_{gt_1}^{\psi}} \right) \sum_{\tau=t_{switch}+1}^{t_A} s_{f\tau}^{-\psi} ds_{f\tau} \leq 0.$$

Therefore $d \ln C_{gt_A} > d \ln A_{st_A}$, q.e.d. ■

We establish a symmetric lemma:

Lemma 4 *Consider a small increase in B_s. Denote by t_A the smallest t such that $d \ln C_{gt_A} > 0$ and assume that $t_A < \infty$. Then $d \ln C_{gt_A} > d \ln A_{st_A}$.*

Proof. The beginning of the proof is the same as in the previous lemma: $d\ln C_{gt_A} > 0$ requires that $ds_{ft_A} < 0$, which implies $t_A > t_{switch}$ and that ds_{ft} switches sign an odd number of times. We use (37) to write:

$$
\begin{aligned}
& d\ln A_{st_A} - d\ln C_{gt_A} \\
= \quad & \eta(1-\psi)(\ln\gamma)\left(\begin{array}{c} \sum_{\tau=1}^{t_{switch}}\left(s_{f\tau}^{-\psi}-s_{g\tau}^{-\psi}\right)ds_{f\tau} \\ +\sum_{p=0}^{P-1}\left(\sum_{\tau=t_{2p}}^{t_{2p+1}-1}\left(\frac{s_{g\tau}^{\psi}}{s_{f\tau}^{\psi}}-1\right)s_{g\tau}^{-\psi}ds_{f\tau}+\sum_{\tau=t_{2p+1}}^{t_{2p+2}-1}\left(\frac{s_{g\tau}^{\psi}}{s_{f\tau}^{\psi}}-1\right)s_{g\tau}^{-\psi}ds_{f\tau}\right)\end{array}\right) \\
< \quad & \eta(1-\psi)(\ln\gamma)\sum_{p=0}^{P-1}\left(\frac{s_{gt_{2p+1}}^{\psi}}{s_{ft_{2p+1}}^{\psi}}-1\right)\sum_{\tau=t_{2p}}^{t_{2p+2}-1}s_{g\tau}^{-\psi}ds_{f\tau},
\end{aligned}
$$

following the same logic as before. By definition t_A is the smallest t such that $\sum_{\tau=1}^{t_A} s_{g\tau}^{-\psi}ds_{g\tau} > 0$, therefore for any $t_X < t_A$, we have $\sum_{\tau=1}^{t_X} s_{g\tau}^{-\psi}ds_{g\tau} < 0$ and $\sum_{\tau=t_X+1}^{t_A} s_{g\tau}^{-\psi}ds_{g\tau} > 0$. Given that $ds_{g\tau} = -ds_{f\tau}$, then $\sum_{\tau=t_X+1}^{t_A} s_{g\tau}^{-\psi}ds_{g\tau} < 0$. Using exactly the same reasoning as before, we obtain:

$$d\ln A_{st_A} - d\ln C_{gt_A} < 0.$$

■

We can now establish the result. Using (15), (16) and (17), we get.

$$P = \left(\xi_c\kappa_c^{\varepsilon}\left(\frac{C_c}{C_E}\right)^{\varepsilon} + \xi_s\kappa_s^{\varepsilon}\left(\frac{C_s}{C_E}\right)^{\varepsilon}\right)C_E\frac{\nu^{\lambda}\widetilde{A}_E^{\lambda-1}C_E^{\lambda-1}}{\nu^{\lambda}\widetilde{A}_E^{\lambda-1}C_E^{\lambda-1}+(1-\nu)^{\lambda}A_P^{\lambda-1}}L.$$

Therefore, for a large t, as C_{gt} grows faster than C_{ct} or C_{st}, we get that:

$$P_t \to \frac{\xi_c\kappa_c^{\varepsilon}C_{ct}^{\varepsilon}+\xi_s\kappa_s^{\varepsilon}C_s^{\varepsilon}}{\kappa_g^{\varepsilon}}\frac{\nu^{\lambda}\widetilde{A}_E^{\lambda-1}\kappa_g^{\frac{\varepsilon}{\varepsilon-1}(\lambda-1)}C_{gt}^{\lambda-\varepsilon}}{\nu^{\lambda}\widetilde{A}_E^{\lambda-1}\kappa_g^{\frac{\varepsilon}{\varepsilon-1}(\lambda-1)}C_{gt}^{\lambda-1}+(1-\nu)^{\lambda}A_{Pt}^{\lambda-1}}L.$$

Using $d\ln A_{ct} = d\ln A_{st}$, this implies that

$$
\begin{aligned}
d\ln P_t \quad \to \quad & -\left(\varepsilon-1+\frac{(1-\lambda)(1-\nu)^{\lambda}A_P^{\lambda-1}}{\nu^{\lambda}\widetilde{A}_E^{\lambda-1}\kappa_g^{\frac{\varepsilon(\lambda-1)}{\varepsilon-1}}C_{gt}^{\lambda-1}+(1-\nu)^{\lambda}A_P^{\lambda-1}}\right)d\ln C_{gt} \\
& +\varepsilon\frac{\xi_c\kappa_c^{\varepsilon}C_c^{\varepsilon}\frac{C_c}{A_c}+\xi_s\kappa_s^{\varepsilon}C_s^{\varepsilon}\frac{C_s}{A_s}}{\xi_c\kappa_c^{\varepsilon}C_c^{\varepsilon}+\xi_s\kappa_s^{\varepsilon}C_s^{\varepsilon}}d\ln A_{ct}+\varepsilon\frac{\xi_s\kappa_s^{\varepsilon}C_s^{\varepsilon}}{\xi_c\kappa_c^{\varepsilon}C_c^{\varepsilon}+\xi_s\kappa_s^{\varepsilon}C_s^{\varepsilon}}\frac{C_s}{B_s}d\ln B_s.
\end{aligned}
$$

Therefore emissions will increase asymptotically following the shale gas boom provided that C_{gt} decreases and A_{ct} and A_{st} increase. We prove that this is the case by contradiction.

Assume that C_{gt} does not decrease for all t. Denote by t_A the first time that $d\ln C_{gt} > 0$, then if $\ln\gamma$ is small enough, it must be that $d\ln C_{gt_A} \approx d\ln C_{gt_A-1} \approx 0$, so that $d\ln A_{ct_A} < 0$ according to Lemma 3 and A_{ct} must decline at some point.

Assume now that A_{ct} does not increase for all t. Denote by t_A the first time that $d\ln A_{ct_A} < 0$, as argued before it must be that $ds_{ft_A} < 0$. Log differentiate f_{t_A} to obtain:

$$\begin{aligned} d\ln f_{t_A} &= -(\varepsilon-1)\, d\ln C_{g(t_A-1)} + \frac{\frac{1}{A_{st_A}}\kappa_s^{\varepsilon} C_{st_A}^{\varepsilon}}{\frac{1}{A_{ct_A}}\kappa_c^{\varepsilon} C_{ct_A}^{\varepsilon} + \frac{1}{A_{st_A}}\kappa_s^{\varepsilon} C_{st_A}^{\varepsilon}} \frac{C_{st_A}}{B_s}\varepsilon d\ln B_s \\ &+ \frac{\frac{1}{A_{ct_A}}\kappa_c^{\varepsilon} C_{ct_A}^{\varepsilon}\left(\varepsilon\frac{C_{ct_A}}{A_{ct_A}}-1\right) + \frac{1}{A_{st_A}}\kappa_s^{\varepsilon} C_{st_A}^{\varepsilon}\left(\varepsilon\frac{C_{st_A}}{A_{st_A}}-1\right)}{\frac{1}{A_{ct_A}}\kappa_c^{\varepsilon} C_{ct_A}^{\varepsilon} + \frac{1}{A_{st_A}}\kappa_s^{\varepsilon} C_{st_A}^{\varepsilon}} d\ln A_{c(t-1)}. \end{aligned}$$

Following a shale gas boom $d\ln B_s > 0$. Since $d\ln A_{ct_{A-1}} > 0 > d\ln A_{ct_A}$, then for $\ln\gamma$ small, we have $d\ln A_{ct_{A-1}} \approx d\ln A_{ct_A} \approx 0$, using lemma 2 we have that $d\ln C_{g(t_A-1)} < 0$, so that we must have $d\ln f_{t_A} > 0$ but this implies that $ds_{ft_A} > 0$ which is a contradiction. Therefore A_{ct} must increase for all t's.

This establishes that emissions must increase asymptotically.

8.4 Extending the theoretical results to the calibrated model

8.4.1 Equilibrium

Following similar steps as those used in the baseline model to derive (10) and using the definition of E_{ft}, we get that given technologies and the level of overall demand for energy E_t, the demand for the different type of electricities are given by:

$$E_{c,t} = \kappa_c^{\sigma}\left(\frac{C_{ct}}{C_{ft}}\right)^{\sigma} E_{ft} \text{ and } E_{s,t} = \kappa_s^{\sigma}\left(\frac{C_{st}}{C_{ft}}\right)^{\sigma} E_{ft}, \tag{38}$$

within fossil fuels and

$$E_{f,t} = \left(\frac{C_{ft}}{C_{Et}}\right)^{\varepsilon} E_t \text{ and } E_{g,t} = \kappa_g^{\varepsilon}\left(\frac{C_{gt}}{C_{Et}}\right)^{\varepsilon} E_t, \tag{39}$$

for fossil fuel and clean energy. The quantity of energy is itself given by (28).

To determine the level of E_t (that is to solve for the input allocation), note that cost minimization in energy production and the production of good Y_{Pt} leads directly:

$$\frac{K_{Et}}{L_{Et}} = \frac{1-\phi}{\phi}\frac{w_t}{\rho_t}, \tag{40}$$

$$\frac{K_{Pt}}{L_{Pt}} = \frac{1-\varphi}{\varphi}\frac{w_t}{\rho_t}. \tag{41}$$

Profit maximization in the final good sector leads to the relative demand:

$$\frac{\left(\frac{w_t}{\phi}\right)^{\phi}\left(\frac{\rho_t}{1-\phi}\right)^{1-\phi}}{\left(\frac{w_t}{\varphi}\right)^{\varphi}\left(\frac{\rho_t}{1-\varphi}\right)^{1-\varphi}} = \frac{\nu \widetilde{A}_{Et}^{\frac{\lambda-1}{\lambda}} C_{Et}^{\frac{\lambda-1}{\lambda}} \left(L_{Et}^{\phi} K_{Et}^{1-\phi}\right)_t^{\frac{-1}{\lambda}}}{(1-\nu) A_{Pt}^{\frac{\lambda-1}{\lambda}} \left(L_{Pt}^{\varphi} K_{Pt}^{1-\varphi}\right)^{\frac{-1}{\lambda}}}. \tag{42}$$

Normalizing the price of the final good to 1, we obtain:

$$1 = (1-\nu)^{\lambda}\left(\frac{\gamma}{A_{Pt}}\left(\frac{w_t}{\varphi}\right)^{\varphi}\left(\frac{\rho_t}{1-\varphi}\right)^{1-\varphi}\right)^{1-\lambda} + \nu^{\lambda}\left(\frac{\gamma}{\widetilde{A}_{Et} C_{Et}}\left(\frac{w_t}{\phi}\right)^{\phi}\left(\frac{\rho_t}{1-\phi}\right)^{1-\phi}\right)^{1-\lambda}. \tag{43}$$

Together with the two factor market clearing equations, (40), (41), (42) and (43) determine the equilibrium value of w_t, ρ_t, L_{Pt}, K_{Pt}, L_{Et}, K_{Et}. In particular, the calibration results are based on a re-computation of the macroeconomic equilibrium using these conditions.

From this, we obtain that $E_t = g\left(C_{Et}\right)$ with g increasing. This is intuitive but to derive it formally, note that the system simplifies in two equations which determine $\frac{w_t}{\rho_t}$ and E_t:

$$\left(\frac{1-\varphi}{\varphi}L\frac{w_t}{\rho_t}+\left(\frac{1}{\phi}-\frac{1}{\varphi}\right)\left(\frac{\phi}{1-\phi}\right)^{1-\phi}\frac{E_t}{C_{Et}}\left(\frac{w_t}{\rho_t}\right)^{\phi}\right)=K, \tag{44}$$

$$\frac{\varphi^{\varphi+(1-\varphi)\frac{1}{\lambda}}\left(1-\varphi\right)^{(1-\varphi)\left(1-\frac{1}{\lambda}\right)}}{\phi^{\phi}\left(1-\phi\right)^{1-\phi}}\left(\frac{w_t}{\rho_t}\right)^{(\phi-\varphi)-\frac{1}{\lambda}(1-\varphi)}=\frac{\nu\widetilde{A}_{Et}^{\frac{\lambda-1}{\lambda}}C_{Et}}{\left(1-\nu\right)A_{Pt}^{\frac{\lambda-1}{\lambda}}}\left(\frac{L}{E_t}-\frac{1}{C_{Et}\left(\frac{1-\phi}{\phi}\frac{w_t}{\rho_t}\right)^{1-\phi}}\right)^{\frac{1}{\lambda}}. \tag{45}$$

Assuming that $\varphi > \phi$, the first equation traces a negative relationship between $\frac{w_t}{\rho_t}$ and E_t while an increase in C_{Et} moves the relationship to the right in the $(E_t, w_t/\rho_t)$ space. The second equation leads to a positive relationship which also moves to the right as C_{Et} increases. Therefore E_t increases in C_{Et}. By symmetry this also holds when $\varphi \leq \phi$.

We can then write the equilibrium level of pollution as $P_t = \xi_{Et}E_t$, where the average effective emission rate per unit of electricity is now given by

$$\xi_{E,t}=\left(\xi_{c,t}\kappa_c^{\sigma}\left(\frac{C_{ct}}{C_{ft}}\right)^{\sigma}+\xi_{s,t}\kappa_s^{\sigma}\left(\frac{C_{st}}{C_{ft}}\right)^{\sigma}\right)\left(\frac{C_{ft}}{C_{Et}}\right)^{\varepsilon}. \tag{46}$$

8.4.2 Comparative statics

We now derive the comparative statics results. We get that

$$\frac{\partial\ln\xi_E}{\partial\ln B_{st}}=\underbrace{\varepsilon\frac{\partial\ln\left(C_{ft}/C_{Et}\right)}{\partial\ln B_{st}}}_{Sub_g\text{: substitution effect away from green}}+\underbrace{\frac{\partial\ln\left(\xi_c\kappa_c^{\sigma}\left(\frac{C_{ct}}{C_{ft}}\right)^{\sigma}+\xi_s\kappa_s^{\sigma}\left(\frac{C_{st}}{C_{ft}}\right)^{\sigma}\right)}{\partial\ln\left(B_{st}\right)}}_{Sub_f\text{: substitution within fossil fuels}}$$

The substitution effect away from green electricity is naturally positive:

$$Sub_g=\varepsilon\frac{\kappa_g^{\varepsilon}A_{gt}^{\varepsilon-1}}{C_{Et}^{\varepsilon-1}}\frac{\kappa_s^{\sigma}C_{st}^{\sigma-1}}{C_{ft}^{\sigma-1}}\frac{B_{st}}{C_{st}}. \tag{47}$$

We use (25), (38) and the fact that the price of the fossil fuel aggregate is given by

$$p_{ft}=\left(\kappa_c^{\sigma}p_{ct}^{1-\sigma}+\kappa_s^{\sigma}p_{st}^{1-\sigma}\right)^{\frac{1}{1-\sigma}}=\frac{\gamma c_{Et}}{C_{ft}},$$

to get that the expenditure share of gas electricity in fossil fuel electricity obeys:

$$\theta_{sft}=\frac{p_{st}E_{st}}{p_{ft}E_{ft}}=\frac{\kappa_s^{\sigma}C_{st}^{\sigma-1}}{C_{ft}^{\sigma-1}}.$$

The expenditure share on clean energy, using (25), (27) and (39), is given by:

$$\Theta_{gt} = \frac{p_{gt}E_{gt}}{p_{Et}E_t} = \frac{\kappa_g^{\varepsilon} A_{gt}^{\varepsilon-1}}{C_{Et}^{\varepsilon-1}}.$$

We then can rewrite (47) as

$$Sub_g = \varepsilon \Theta_{gt} \theta_{sft} \frac{B_{st}}{C_{st}}.$$

Further, we have

$$\begin{aligned} Sub_f &= -\sigma \frac{\kappa_c^{\sigma} C_{ct}^{\sigma-1} \kappa_s^{\sigma} C_{st}^{\sigma-1}}{\left(\xi_{c,t}\kappa_c^{\sigma} C_{ct}^{\sigma} + \xi_{s,t}\kappa_s^{\sigma} C_{st}^{\sigma}\right) C_{ft}^{\sigma-1}} \left(\xi_c C_{ct} - \xi_s C_{st}\right) \frac{B_{st}}{C_{st}} \\ &= -\sigma \theta_{sft} \frac{P_{c,t}}{P_t} \left(1 - \frac{\xi_s C_{st}}{\xi_c C_{ct}}\right) \frac{B_{st}}{C_{st}}, \end{aligned}$$

where

$$\frac{P_{ct}}{P_t} = \frac{\xi_c \kappa_c^{\sigma} C_{ct}^{\sigma}}{\xi_{c,t}\kappa_c^{\sigma} C_{ct}^{\sigma} + \xi_{s,t}\kappa_s^{\sigma} C_{st}^{\sigma}}$$

is the pollution share of coal based electricity. Therefore the substitution effect within fossil fuel is negative as long as $\xi_c C_{ct} > \xi_s C_{st}$ holds. Overall, we obtain equation (29).

To obtain $\frac{\partial \ln E_t}{\partial \ln C_{Et}}$, we log differentiate (44) and (45), from which we get:

$$\frac{1-\varphi}{\varphi} L \frac{\partial \ln\left(\frac{w_t}{\rho_t}\right)}{\partial \ln C_{Et}} + \left(\frac{1}{\phi} - \frac{1}{\varphi}\right) L_E \left(\frac{\partial \ln E_t}{\partial \ln C_{Et}} - 1 + \phi \frac{\partial \ln\left(\frac{w_t}{\rho_t}\right)}{\partial \ln C_{Et}}\right) = 0, \tag{48}$$

$$\left((\phi-\varphi) - \frac{1}{\lambda}(1-\varphi)\right) \frac{\partial \ln\left(\frac{w_t}{\rho_t}\right)}{\partial \ln C_{Et}} = 1 + \frac{1}{\lambda}\left(-\frac{\partial \ln E_t}{\partial \ln C_{Et}} \frac{L}{L_P} + \frac{L_E}{L_P}\left(1 + (1-\phi)\frac{\partial \ln\left(\frac{w_t}{\rho_t}\right)}{\partial \ln C_{Et}}\right)\right), \tag{49}$$

where we used that $L_E = \left(\frac{\phi}{1-\phi}\frac{\rho_t}{w_t}\right)^{1-\phi} \frac{E}{C_E}$. Re-arranging terms we then get that:

$$\frac{\partial \ln E_t}{\partial \ln C_{Et}} = 1 - \frac{(1-\lambda)\phi\left((1-\varphi)L_P + (1-\phi)L_E\right)L_P}{\lambda(\varphi-\phi)^2 L_P L_E + (\varphi L_E + \phi L_P)\left(((1-\varphi)L_P + (1-\phi)L_E)\right)},$$

so that $\frac{\partial \ln E_t}{\partial \ln C_{Et}} \in (0,1)$: since energy and the production inputs are complement, $\lambda < 1$, resources move toward the production input when the productivity of the energy sector goes up). The scale effect is given by

$$\frac{\partial \ln E_t}{\partial \ln B_{st}} = \frac{\partial \ln E_t}{\partial \ln C_{Et}} \frac{\partial \ln C_{Et}}{\partial \ln B_{st}} = \Theta_{st} \frac{C_{st}}{B_{st}} \frac{\partial \ln E_t}{\partial \ln C_{Et}}$$

We then obtain

$$\frac{\partial \ln P_t}{\partial \ln B_{st}} = \frac{\Theta_{st} C_{st}}{B_{st}} \left[\underbrace{\varepsilon \frac{\Theta_{gt}}{\Theta_{ft}}}_{\text{substitution away from green}} - \underbrace{\frac{1}{\Theta_{ft}} \sigma \frac{P_{ct}}{P_t}\left(1 - \frac{\xi_s C_{st}}{\xi_c C_{ct}}\right)}_{\text{substitution within fossil fuels}} + \underbrace{\frac{\partial \ln E_t}{\partial \ln C_{Et}}}_{\text{scale effect}} \right].$$

For $\xi_c >> \xi_s$, we get that

$$\varepsilon \frac{\Theta_{gt}}{\Theta_{ft}} - \frac{1}{\Theta_{ft}} \sigma \frac{P_{ct}}{P_t} \left(1 - \frac{\xi_s C_{st}}{\xi_c C_{ct}}\right) |_{\xi_c >> \xi_s} \approx -\varepsilon - \frac{1}{\Theta_{ft}} (\sigma - \varepsilon) .$$

Therefore since $\sigma \geq \varepsilon > 1$ and since $\frac{\partial \ln E_t}{\partial \ln C_{Et}} < 1$, we get that $\frac{\partial \ln P_t}{\partial \ln B_{st}} < 0$ for $\xi_c >> \xi_s$.

Furthermore, if C_{gt} grows while C_{st} and C_{ct} stay constant, then $\Theta_{gt} \to 1$, so that

$$\varepsilon \frac{\Theta_{gt}}{\Theta_{ft}} - \frac{1}{\Theta_{ft}} \sigma \frac{P_{ct}}{P_t} \left(1 - \frac{\xi_s C_{st}}{\xi_c C_{ct}}\right) |_{\Theta_{gt} \to 1} \approx \frac{1}{\Theta_{ft}} \left(\varepsilon - \sigma \frac{P_{ct}}{P_t} \left(1 - \frac{\xi_s C_{st}}{\xi_c C_{ct}}\right)\right).$$

As $\Theta_{ft} \to 0$, the substitution effect dominates the scale effect for t large enough, so that a shale gas boom at $t = 0$ will eventually lead to an increase in emissions only if

$$\varepsilon > \frac{P_{c0}}{P_0} \left(1 - \frac{\xi_s C_{s0}}{\xi_c C_{c0}}\right) \sigma.$$

We then obtain the modified Proposition 1:

Proposition 6 *i) A shale gas boom (that is a one time increase in B_s at time $t = 0$) leads to a decrease in emissions in the short-run provided that the natural gas is sufficiently clean compared to coal (for ξ_s/ξ_c small enough).*
ii) If all future innovations in the energy sector occur in clean technologies, then for t large enough, the shale gas boom will increase emissions if $\varepsilon > \frac{P_{c0}}{P_0} \left(1 - \frac{\xi_s C_{s0}}{\xi_c C_{c0}}\right) \sigma$.

8.4.3 Innovation allocation

Assume that innovation occurs as in Section 3.3. Then the expected profits of an innovator in clean technologies are still given by (21) and those of an innovator in fossil fuel power plant technologies by (22). Therefore (23) is replaced by:

$$\frac{\Pi_{gt}}{\Pi_{ft}} = \frac{\eta_g s_{gt}^{-\psi} \kappa_g^{\varepsilon} A_{gt}^{\varepsilon-1}}{\eta_f s_{ft}^{-\psi} \left(\frac{\kappa_c^{\sigma} C_{ct}^{\sigma}}{A_c} + \frac{\kappa_s^{\sigma} C_{st}^{\sigma}}{A_s}\right) C_{ft}^{\varepsilon-\sigma}} = 1.$$

We can rewrite this equation as:

$$f\left(s_{gt}, A_{ct-1}, B_{ct}, A_{st-1}, B_{st}, C_{gt-1}\right) = 1$$

where the function f is now defined as

$$f \equiv \frac{\eta_f s_{gt}^{\psi} \left(\kappa_c^{\sigma} \frac{\gamma^{-\eta_f s_{ft}^{1-\psi}}}{A_{ct-1}} \left(\frac{\gamma^{-\eta_f s_{ft}^{1-\psi}}}{A_{ct-1}} + \frac{1}{B_{ct}}\right)^{-\sigma} + \kappa_s^{\sigma} \frac{\gamma^{-\eta_f s_{ft}^{1-\psi}}}{A_{st-1}} \left(\frac{\gamma^{-\eta_f s_{ft}^{1-\psi}}}{A_{st-1}} + \frac{1}{B_{st}}\right)^{-\sigma}\right)}{\eta_g \kappa_g^{\varepsilon} s_{ft}^{\psi} C_{gt-1}^{\varepsilon-1} \gamma^{\eta_g s_{gt}^{1-\psi} (\varepsilon-1)}}$$
$$\times \left(\kappa_c^{\sigma} \left(\frac{\gamma^{-\eta_f s_{ft}^{1-\psi}}}{A_{ct-1}} + \frac{1}{B_{ct}}\right)^{1-\sigma} + \kappa_s^{\sigma} \left(\frac{\gamma^{-\eta_f s_{ft}^{1-\psi}}}{A_{st-1}} + \frac{1}{B_{st}}\right)^{1-\sigma}\right)^{\frac{\varepsilon-\sigma}{\sigma-1}}$$

We then get that

$$
\begin{aligned}
& \frac{\partial \ln f}{\partial \ln s_{gt}} \\
= \; & \psi - \eta_g (\varepsilon - 1)(1-\psi)(\ln \gamma) s_{gt}^{1-\psi} + \psi \frac{s_{gt}}{s_{ft}} \\
& + \frac{\eta_f (1-\psi) \ln(\gamma) s_{ft}^{1-\psi} \frac{s_{gt}}{s_{ft}} \left(\kappa_c^\sigma \frac{C_{ct}^\sigma}{A_{ct}} \left(1 - \sigma \frac{B_{ct}}{B_{ct}+A_{ct}}\right) + \kappa_s^\sigma \frac{C_{st}^\sigma}{A_{st}} \left(1 - \sigma \frac{B_{st}}{B_{st}+A_{st}}\right)\right)}{\kappa_c^\sigma \frac{C_{ct}^\sigma}{A_{ct}} + \kappa_s^\sigma \frac{C_{st}^\sigma}{A_{st}}} \\
& + \frac{\varepsilon - \sigma}{\sigma - 1} \frac{\partial \ln \left(\kappa_c^\sigma \left(\frac{\gamma^{-\eta_f s_{ft}^{1-\psi}}}{A_{ct-1}} + \frac{1}{B_{ct}} \right)^{1-\sigma} + \kappa_s^\sigma \left(\frac{\gamma^{-\eta_f s_{ft}^{1-\psi}}}{A_{st-1}} + \frac{1}{B_{st}} \right)^{1-\sigma} \right)^{\frac{\varepsilon-\sigma}{\sigma-1}}}{\partial \ln s_{gt}} \\
\geq \; & \psi - \eta_g (\varepsilon - 1)(1-\psi)(\ln \gamma) s_{gt}^{1-\psi} + \left(\psi - \eta_f (\sigma - 1)(1-\psi)(\ln \gamma) s_{ft}^{1-\psi} \right) \frac{s_{gt}}{s_{ft}},
\end{aligned}
$$

since $\sigma \geq \varepsilon$ and $\left(\kappa_c^\sigma \left(\frac{\gamma^{-\eta_f s_{ft}^{1-\psi}}}{A_{ct-1}} + \frac{1}{B_{ct}} \right)^{1-\sigma} + \kappa_s^\sigma \left(\frac{\gamma^{-\eta_f s_{ft}^{1-\psi}}}{A_{st-1}} + \frac{1}{B_{st}} \right)^{1-\sigma} \right)^{\frac{\varepsilon-\sigma}{\sigma-1}}$ is decreasing in s_{gt}. Therefore f increases in s_g provided that $(\ln \gamma) \max\left(\eta_g (\varepsilon - 1), \eta_f (\sigma - 1)\right) < \psi/(1-\psi)$ in which case the equilibrium is uniquely defined.

We get that

$$
\frac{\partial \ln f}{\partial \ln B_{st}} = \frac{\kappa_c^\sigma C_{ct}^{\sigma-1} \left(\sigma \frac{C_{st}}{A_{st}} - (\sigma - \varepsilon) \frac{C_{ct}}{A_{ct}} \right) + \varepsilon \kappa_s^\sigma \frac{C_{st}^\sigma}{A_{st}}}{\left(\kappa_c^\sigma \frac{C_{ct}^\sigma}{A_{ct}} + \kappa_s^\sigma \frac{C_{st}^\sigma}{A_{st}} \right) \left(\kappa_c^\sigma C_{ct}^{\sigma-1} + \kappa_s^\sigma C_{st}^{\sigma-1} \right)} \kappa_s^\sigma \frac{C_{st}^\sigma}{B_{st}},
$$

so that $\frac{\partial \ln f}{\partial \ln B_{st}} > 0$ if and only if

$$
\varepsilon \left(1 + \frac{\kappa_s^\sigma C_{st}^{\sigma-1}}{\kappa_c^\sigma C_{ct}^{\sigma-1}} \right) > (\sigma - \varepsilon) \left(\frac{1 + \frac{A_{st}}{B_{st}}}{1 + \frac{A_{ct}}{B_{ct}}} - 1 \right),
$$

that is provided that either σ is close enough to ε or $\frac{A_{st}}{B_{st}}$ is not too large relative to $\frac{A_{ct}}{B_{ct}}$—in fact, if $\sigma > \varepsilon$, the above inequality will be violated for B_{st} low enough. Intuitively, innovation toward the fossil fuel sector is higher when there is a large gap between productivity in coal and natural gas technology; an increase in B_{st} may not lead to more innovation in fossil fuel technologies when it is not very useful, that is when B_{st} is low.

Further,

$$
\begin{aligned}
\frac{\partial \ln f}{\partial \ln A_{c(t-1)}} &= \left(\frac{\sigma \frac{C_{ct}}{A_{ct}} - 1}{\kappa_c^\sigma \frac{C_{ct}^\sigma}{A_c} + \kappa_s^\sigma \frac{C_{st}^\sigma}{A_s}} + \frac{\varepsilon - \sigma}{\kappa_c^\sigma C_{ct}^{\sigma-1} + \kappa_s^\sigma C_{st}^{\sigma-1}} \right) \kappa_c^\sigma \frac{C_{ct}^\sigma}{A_c} \\
&= \left(\frac{\left(\varepsilon \frac{C_{ct}}{A_{ct}} - 1 \right) \left(\kappa_c^\sigma C_{ct}^{\sigma-1} + \kappa_s^\sigma C_{st}^{\sigma-1} \right) - (\sigma - \varepsilon) \left(\frac{C_{st}}{A_s} - \frac{C_{ct}}{A_{ct}} \right) \kappa_s^\sigma C_{st}^{\sigma-1}}{\left(\kappa_c^\sigma \frac{C_{ct}^\sigma}{A_c} + \kappa_s^\sigma \frac{C_{st}^\sigma}{A_s} \right) \left(\kappa_c^\sigma C_{ct}^{\sigma-1} + \kappa_s^\sigma C_{st}^{\sigma-1} \right)} \right) \kappa_c^\sigma \frac{C_{ct}^\sigma}{A_c}
\end{aligned}
$$

so that $\frac{\partial \ln f}{\partial \ln A_{c(t-1)}} > 0$ if and only if

$$\left(\varepsilon - \frac{A_{ct}}{C_{ct}}\right)\left(1 + \frac{\kappa_c^\sigma C_{ct}^{\sigma-1}}{\kappa_s^\sigma C_{st}^{\sigma-1}}\right) > (\sigma - \varepsilon)\left(\frac{1+\frac{A_{ct}}{B_{ct}}}{1+\frac{A_{st}}{B_{st}}} - 1\right).$$

By symmetry we get that $\frac{\partial \ln f}{\partial \ln A_{c(t-1)}} > 0$ if and only if

$$\left(\varepsilon - \frac{A_{st}}{C_{st}}\right)\left(1 + \frac{\kappa_s^\sigma C_{st}^{\sigma-1}}{\kappa_c^\sigma C_{ct}^{\sigma-1}}\right) > (\sigma - \varepsilon)\left(\frac{1+\frac{A_{st}}{B_{st}}}{1+\frac{A_{ct}}{B_{ct}}} - 1\right).$$

Therefore we obtain similar comparatie statics as in the baseline model provided that either $\frac{A_{st}}{B_{st}}$ and $\frac{A_{ct}}{B_{ct}}$ are not too far away or σ is not too large relative to ε. Therefore Proposition 3 is modified and becomes:

Proposition 7 *Assume that* $(\ln \gamma) \max\left(\eta_g(\varepsilon - 1), \eta_f(\sigma - 1)\right) < \frac{\psi}{1-\psi}$. *Then a shale gas boom at* $t = 0$ *(an increase in* B_{s0}*) leads to a decrease in innovation in green technology at* $t = 0$ *(a decrease in* s_{g0}*) provided that* σ *is not too large relative to* ε *and* $\frac{A_{s0}}{B_{s0}}$ *is not too large relative to* $\frac{A_{c0}}{B_{c0}}$ *Furthermore if (i)* $\min\left(\frac{B_{ct}}{A_{ct-1}}, \frac{B_{st}}{A_{st-1}}\right) > \frac{\gamma^{\eta_f}}{\varepsilon-1}$ *and (ii) either* σ *is close to* ε *or* $\frac{A_{st-1}}{B_{st}}$ *is close to* $\frac{A_{ct-1}}{B_{ct}}$ *for all* $t > 0$*, then green innovation declines for all* $t > 0$*.*

9 Appendix C: Calibration details

9.1 Calibration of electricity substitution parameter λ

The elasticity of substitution λ is calibrated based on the literature with appropriate modification since we are focused only on electricity. The literature, when differentiating electricity, typically estimates or calibrates parameters for nested final goods production functions with electricity and non-electric energy. In the 'background' of our framework we might thus imagine a production function:

$$F_t = \{\gamma_Y (A_{Yt} Y_t)^{\frac{\sigma_1 - 1}{\sigma_1}} + (1-\gamma_Y)\left[\gamma_{Elec}(E_{Elec})^{\frac{v_1-1}{v_1}} + (1-\gamma_{Elec})(E_{NonElec})^{\frac{v_1-1}{v_1}}\right]^{\left(\frac{v_1-1}{v_1}\right)\left(\frac{\sigma_1-1}{\sigma_1}\right)}\}^{\frac{\sigma_1-1}{\sigma_1}} \tag{50}$$

For our calibration, we need $\sigma_{Y_P,Elec}$ and/or σ_{Elec,Y_P}. The literature provides some examples or estimates of the parameters in (50). The Morishima elasticities are then (Anderson and Moroney, 1993):[14]

$$\begin{aligned} \sigma_{Elec,Y_P} &= \gamma_{Elec} \cdot \sigma_1 + (1-\gamma_{Elec}) \cdot v_1 \\ \sigma_{Y_P,Elec} &= \sigma_1 \end{aligned}$$

Standard values for $\sigma_1 \sim \sigma_{KL,E}$ from the literature are $0.4 - 0.5$ (e.g., Chen et al., 2017; Van der Werf, 2008; Böringer and Rutherford, 2008; Bosetti et al., 2007). As several major models

[14] Intuitively, they are not symmetric (whereas the Allen-Uzawa elasticities would be) because a change in the price of electricity also changes the relative prices of electric and non-electric energy, whereas a change in the price of Y_P does not (Frieling and Madner, 2016).

moreover assume $v_1 = 0.5$ (e.g., Chen et al., 2017; Bosetti et al., 2007), for our purposes, we would have $\sigma_{Elec,Y_P} = \sigma_{Y_P,Elec} = 0.5$ for any value of γ_{Elec}. We thus keep $\lambda = 0.5$ as a benchmark, but consider considerably lower values given the empirical evidence of very low capital-labor and energy substitution elasticities presented by Hassler, Krusell, and Olovsson (2012).

9.2 Benchmark employment in electricity (and resource) sectors

NIPA Table 6.5D: Full-Time Equivalent Employees by Industry		
	2008 ('000s)	Share
Mining - Oil and gas extraction	160	
Mining - Support activities for mining	325	
Utilities	554	
Manufacturing - Petroleum and coal products	115	
Transportation - Pipeline transportation	39	
Total Energy:	1193	
Total FTE	127478	0.9358%
Total Private Industries FTE	108027	1.1044%

9.3 Profit Margins and γ Calibration

The following tables present after-tax profits per dollar of sales for corporations in three relevant industries ("Petroleum and coal products," "All Durable Manufacturing," and "All Wholesale Trade") for 2004-2014 (Source: U.S. Census Bureau Quarterly Financial Report for Manufacturing, Mining, and Trade Corporations, 2004-2014).

Petroleum and Coal Products: Profits per Dollar of Sales (cents)					
After-tax (cents/dollar)	Q1	Q2	Q3	Q4	Avg.
2004	8.2	9.5	8.9	10.6	9.3
2005	10.1	9.2	8.4	9.6	9.325
2006	9.8	11.5	11.3	9.6	10.55
2007	10.9	10.6	8.7	8.2	9.6
2008	8.4	8.0	10.2	-8.5	4.525
2009	6.5	4.8	5.9	3.9	5.275
2010	6.8	0.7	6.4	6.2	5.025
2011	8.4	7.9	7.8	6.7	7.700
2012	6.8	8.6	6.8	6.6	7.200
2013	7.5	3.8	4.8	5.7	5.450
2014	6.3	5.7	6.3	4.8	5.775
Average					**7.25**

All Durable Manufacturing: Profits per Dollar of Sales (cents)					
After-tax (cents/dollar)	Q1	Q2	Q3	Q4	Avg.
2004	5.7	7.0	5.8	5.7	6.05
2005	10.1	9.2	8.4	9.6	9.325
2006	7.4	6.8	6.9	6.2	6.825
2007	6.6	7.9	2.4	5.8	5.675
2008	6.0	4.2	5.1	-7.1	2.05
2009	-1.9	0.6	4.6	5.0	2.075
2010	7.2	9.6	8.7	8.6	8.525
2011	9.5	10.3	9.6	9.3	9.675
2012	9.1	9.5	8.0	7.3	8.475
2013	9.0	9.2	9.5	9.2	9.225
2014	8.4	10.0	10.1	9.2	9.425
Average					**7.03**

All Wholesale Trade: Profits per Dollar of Sales (cents)					
After-tax (cents/dollar)	Q1	Q2	Q3	Q4	Avg.
2004	2.0	2.3	2.4	2.0	2.175
2005	1.9	2.3	2.1	2.4	2.175
2006	2.1	2.1	2.4	1.8	2.1
2007	2.1	2.3	2.0	1.7	2.025
2008	1.3	1.8	1.8	0.1	1.25
2009	-0.1	0.9	1.1	1.4	0.825
2010	1.2	1.7	1.7	1.4	1.5
2011	1.8	1.8	1.7	1.1	1.6
2012	1.3	2.0	1.8	1.3	1.6
2013	2.0	1.6	1.8	1.5	1.725
2014	1.7	1.8	2.2	1.2	1.725
Average					**1.7**

Weighted Average After-Tax Profits per Dollar of Sales (cents)		
Industry	Avg.	Income Share (2011)
Petroleum and coal products	7.25	.23
All durable manufacturing	7.03	.70
All wholesale trade	1.7	.06
Weighted Average:		6.69
⇒ Implied γ :		1.07

9.4 Equilibrium conditions matched by calibration

After the calibration of the parameters as described in sub-section 4.2, we solve for the remaining unknowns to satisfy the following set of equations at the initial observed labor shares L_{E0}, L_{P0}, GDP Y_0, aggregate capital K_0, energy prices, etc.:

$$
\begin{aligned}
\text{13 Unknowns} \quad &: \quad A_{g,0}, A_{c,0}, A_{s,0}, B_{c,0}, B_{s,0}, C_{f,0}, C_{E,0}, A_{P,0}, K_{E,0}, K_{P,0}, c_{E,0}, w_0, \rho_0 \quad (51)\\
Y_0 \quad &= \quad \left((1-\nu)\left(A_{P,0} L_{P,0}^{\varphi} K_{P,0}^{1-\varphi}\right)^{\frac{\lambda-1}{\lambda}} + \nu \left(\widetilde{A}_{E,0} E_0\right)^{\frac{\lambda-1}{\lambda}} \right)^{\frac{\lambda}{\lambda-1}}\\
K_0 \quad &= \quad K_{E,0} + K_{P,0}\\
c_{E,0} \quad &= \quad \left(\frac{w_0}{\phi}\right)^{\phi} \left(\frac{\rho_0}{1-\phi}\right)^{1-\phi}\\
E_0 \quad &= \quad C_{E,0} L_{E,0}^{\phi} K_{E,0}^{1-\phi}\\
\frac{K_{E,0}}{L_{E,0}} \quad &= \quad \frac{1-\phi}{\phi}\frac{w_0}{\rho_0} \text{ and } \frac{K_{P,0}}{L_{P,0}} = \frac{1-\varphi}{\varphi}\frac{w_0}{\rho_0} \text{ from (40) and (41).}\\
A_{g,0} \quad &= \quad \frac{\gamma c_{E,0}}{p_{g,0}^{y}},\ A_{c,0} = \frac{\gamma c_{E,0}}{p_{c,0}^{y}} \text{ and } A_{s,0} = \frac{\gamma c_{E,0}}{p_{s,0}^{y}}\\
B_{c,0} \quad &= \quad \frac{\gamma c_{E,0}}{p_{c,0}^{r}} \text{ and } B_{s,0} = \frac{\gamma c_{E,0}}{p_{s,0}^{r}}\\
C_{f,0} \quad &= \quad \left(\kappa_c^{\sigma} \left(\frac{1}{A_{c,0}} + \frac{1}{B_{c,0}}\right)^{1-\sigma} + \kappa_s^{\sigma} \left(\frac{1}{A_{s,0}} + \frac{1}{B_{s,0}}\right)^{1-\sigma} \right)^{\frac{1}{\sigma-1}}\\
C_{E,0} \quad &= \quad \left(\kappa_g^{\varepsilon} A_{g,0}^{\varepsilon-1} + C_{f,0}^{\varepsilon-1}\right)^{\frac{1}{\varepsilon-1}}
\end{aligned}
$$

9.4.1 Extended parameter value table

Finally, the parameters and initial endogenous unknowns whose values are not listed in Table (3) already are as follows in the benchmark:

Parameter	Value
$A_{g,0}$	0.0950
$A_{c,0}$	0.1794
$A_{s,0}$	0.3361
$B_{c,0}$	0.3817
$B_{s,0}$	0.1607
$C_{f,0}$	0.0262
$C_{E,0}$	0.0389
$A_{P,0}$	46.0079
$K_{E,0}$	1.6432e+03
$K_{P,0}$	4.8941e+04
$c_{E,0}$	6.5456
w_0	817.2825
ρ_0	0.0813