Game Theory Through The Computational Lens

Some Points of Contact Between Theoretical Computer Science and Economics

Tim Roughgarden
(Stanford)

First Point of Contact

von Neumann

First Point of Contact

Origins of game theory:

- "Zur Theorie der Gesellschaftsspiele" (1928)
- *Theory of Games and Economic Behavior* (1944, with Morgenstern)

von Neumann

First Point of Contact

von Neumann

Origins of game theory:

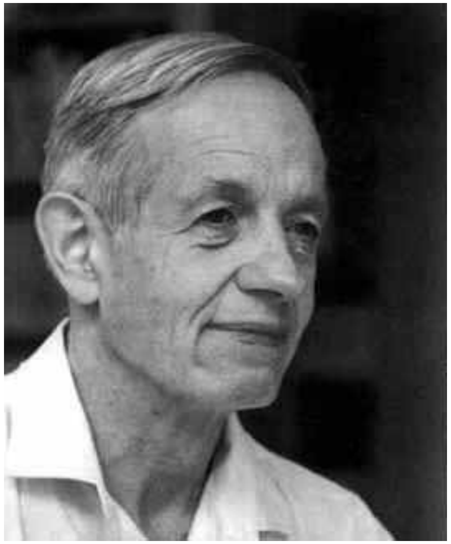
- "Zur Theorie der Gesellschaftsspiele” (1928)
- *Theory of Games and Economic Behavior* (1944, with Morgenstern)

Early contributions to computing:

- ENIAC (UPenn, 1945)
- IAS machine (1945-1951)

Games and Nash Equilibria

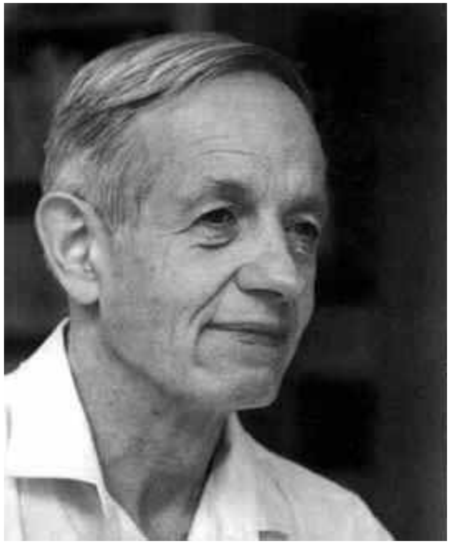
Nash's Theorem (1950): every finite noncooperative game has at least one (Nash) equilibrium.



Nash

Games and Nash Equilibria

Nash's Theorem (1950): every finite noncooperative game has at least one (Nash) equilibrium.

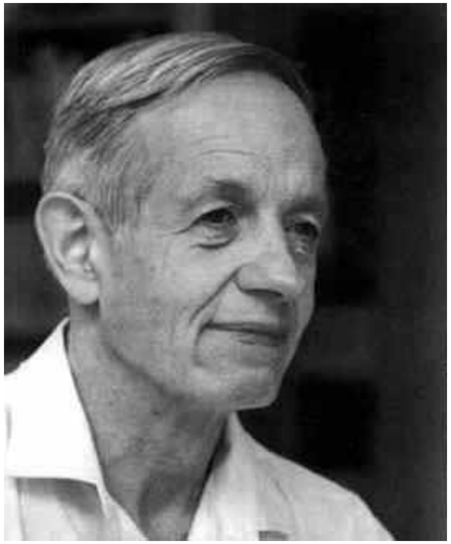


Nash

An equilibrium

Games and Nash Equilibria

Nash's Theorem (1950): every finite noncooperative game has at least one (Nash) equilibrium.



Nash

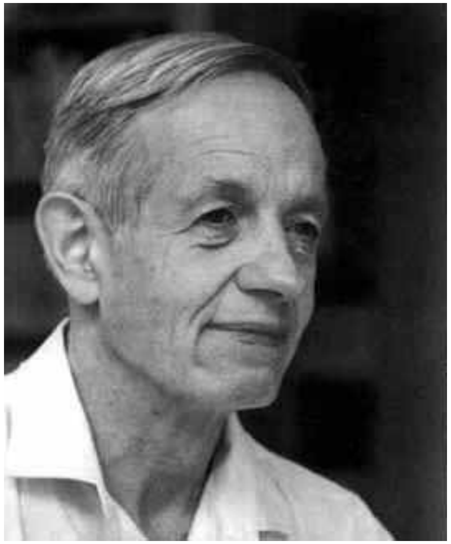
Not an equilibrium

Example: Chicken

Nash's Theorem (1950): every finite noncooperative game has at least one (Nash) equilibrium.

	Go straight	Swerve
Go straight	-1, -1	1, 0
Swerve	0, 1	0, 0

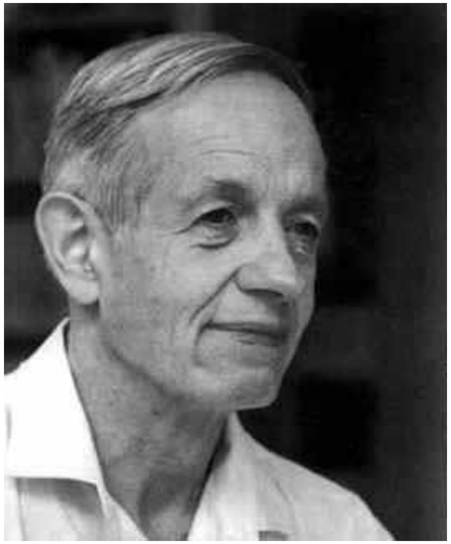
Chicken



Nash

Example: Chicken

Nash's Theorem (1950): every finite noncooperative game has at least one (Nash) equilibrium.



Nash

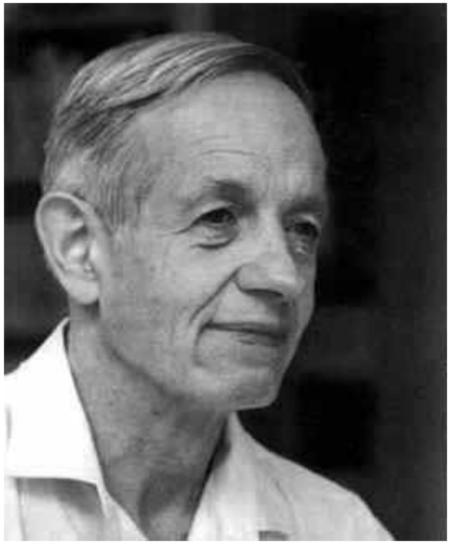
	Go straight	Swerve
Go straight	-1, -1	1, 0
Swerve	0, 1	0, 0

Chicken

Equilibrium #1

Example: Chicken

Nash's Theorem (1950): every finite noncooperative game has at least one (Nash) equilibrium.



Nash

	Go straight	Swerve
Go straight	-1, -1	1, 0
Swerve	0, 1	0, 0

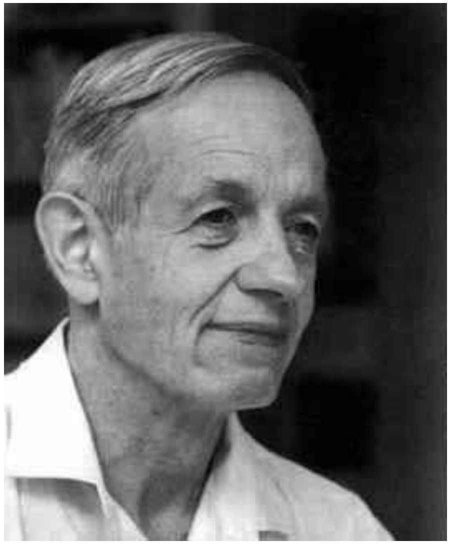
Chicken

Equilibrium #1

Equilibrium #2

Example: Rock-Paper-Scissors

Nash's Theorem (1950): every finite noncooperative game has at least one (Nash) equilibrium.



Nash

	Rock	Paper	Scissors
Rock	0, 0	-1, 1	1, -1
Paper	1, -1	0, 0	-1, 1
Scissors	-1, 1	1, -1	0, 0

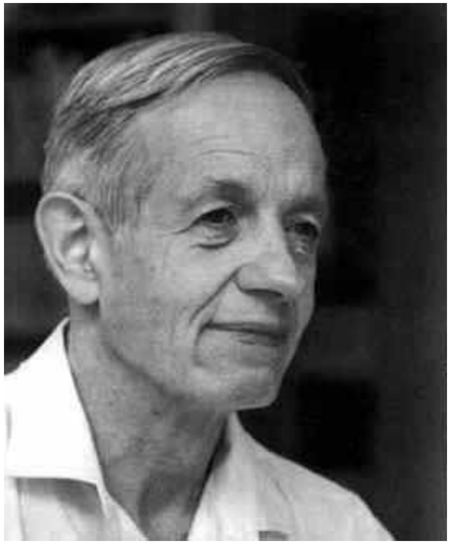
Example: Rock-Paper-Scissors

Nash's Theorem (1950): every finite noncooperative game has at least one (Nash) equilibrium.

Randomize: ⅓, ⅓, ⅓

	Rock	Paper	Scissors
Rock	0, 0	-1, 1	1, -1
Paper	1, -1	0, 0	-1, 1
Scissors	-1, 1	1, -1	0, 0

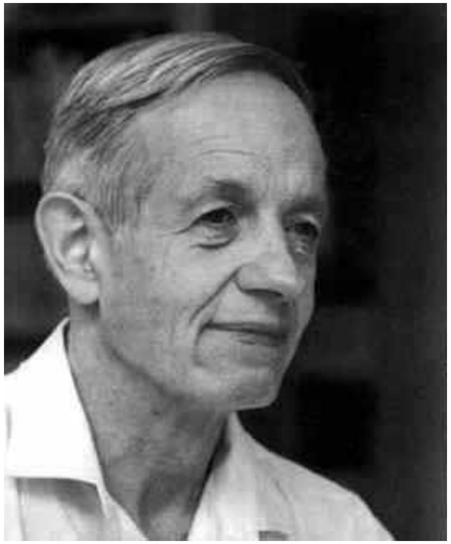
Randomize: ⅓, ⅓, ⅓



Nash

Example: Rock-Paper-Scissors

Nash's Theorem (1950): every finite noncooperative game has at least one (Nash) equilibrium.

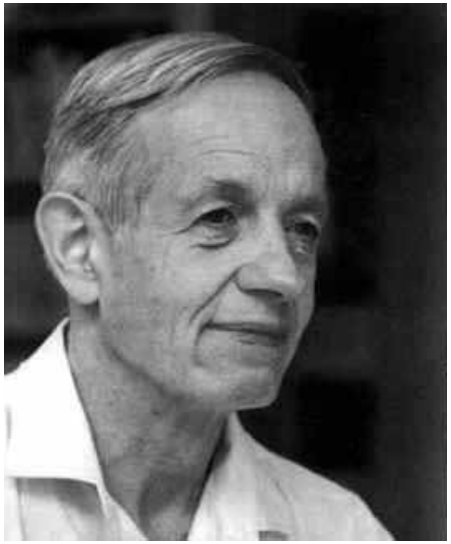


Nash

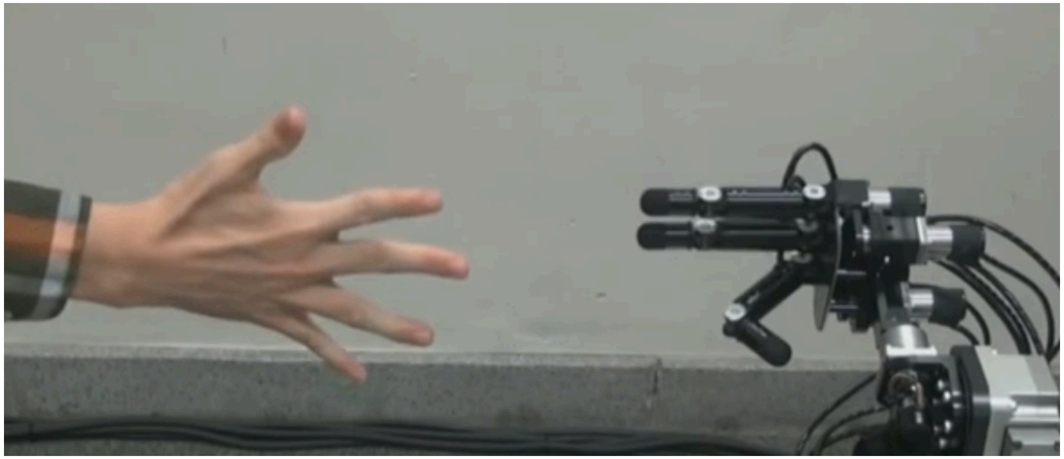
Rock-Paper-Scissors Championship

Example: Rock-Paper-Scissors

Nash's Theorem (1950): every finite noncooperative game has at least one (Nash) equilibrium.



Nash



Janken robot (University of Tokyo)

Turing and Unsolvable Problems

Origins of computer science:

ON COMPUTABLE NUMBERS, WITH AN APPLICATION TO THE ENTSCHEIDUNGSPROBLEM

By A. M. TURING.

[Received 28 May, 1936.—Read 12 November, 1936.]

Alan Turing

Turing and Unsolvable Problems

Origins of computer science:

ON COMPUTABLE NUMBERS, WITH AN APPLICATION TO THE ENTSCHEIDUNGSPROBLEM

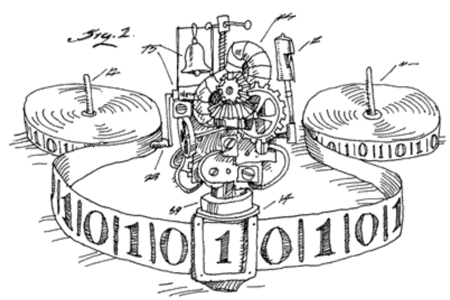
By A. M. TURING.

[Received 28 May, 1936.—Read 12 November, 1936.]

Alan Turing

Importance:

- formal model of computation ("Turing machine")



Turing and Unsolvable Problems

Origins of computer science:

ON COMPUTABLE NUMBERS, WITH AN APPLICATION TO THE ENTSCHEIDUNGSPROBLEM

By A. M. Turing.

[Received 28 May, 1936.—Read 12 November, 1936.]

Alan Turing

Importance:

- formal model of computation
- existence of unsolvable problems
  - example: the "halting problem"

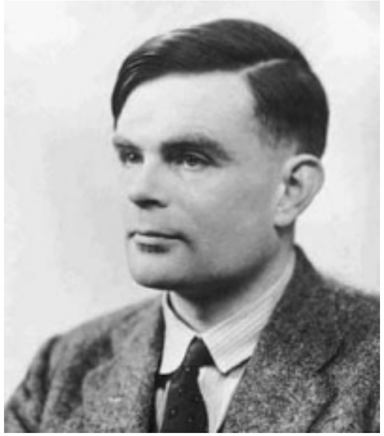
Turing and Unsolvable Problems

Origins of computer science:

ON COMPUTABLE NUMBERS, WITH AN APPLICATION TO THE ENTSCHEIDUNGSPROBLEM

By A. M. Turing.

[Received 28 May, 1936.—Read 12 November, 1936.]



Alan Turing

Importance:

- formal model of computation
- existence of unsolvable problems
  - example: the "halting problem"

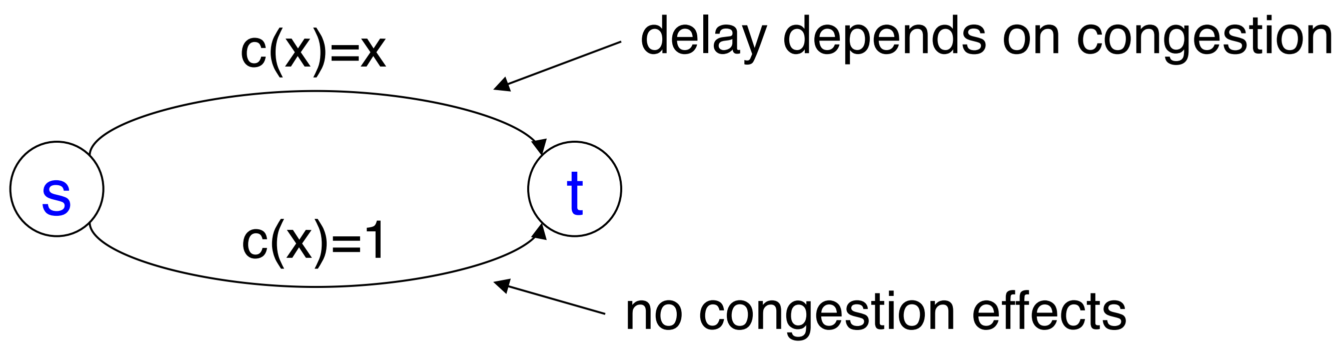
Outline

1. Introduction: von Neumann, Nash, and Turing
2. *Approximation: A Constructive Compromise*
3. Auction Design: The Rubber Meets the Road
4. Complexity: Critiquing the Nash Equilibrium
5. Conclusions


THE PRICE OF ANARCHY
IS IT OKAY TO BE SELFISH?
FIRE DEPT.
39
FIRE DEPT.

Pigou's Example

Example: one unit of traffic travelling from s to t

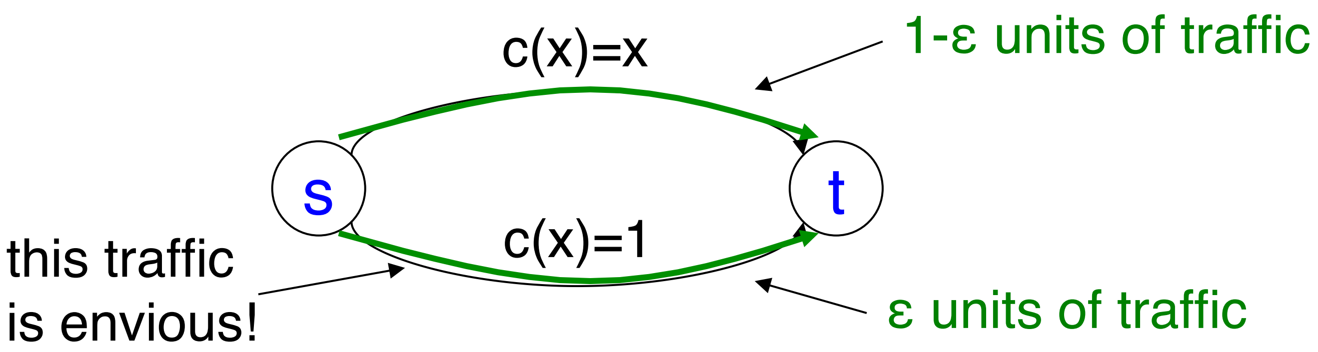


Question: what will selfish drivers do?

- assume everyone wants smallest-possible delay
- [Pigou 1920]

Equilibrium in Pigou's Example

Claim: all traffic will take the top route.

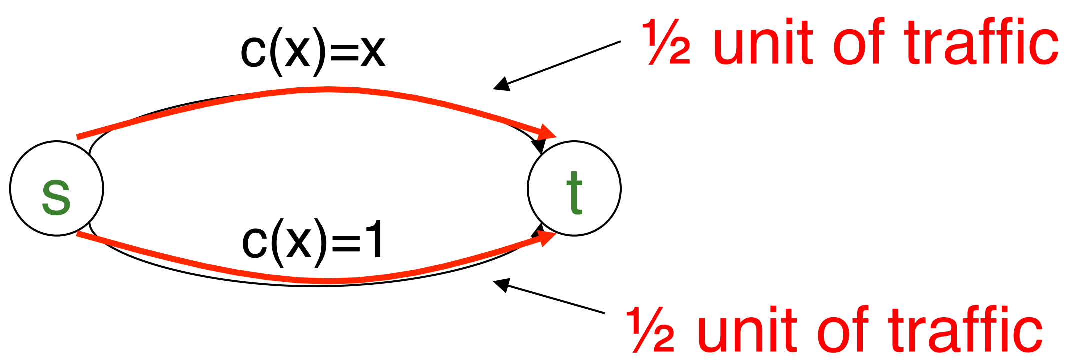


Reason:

- $\varepsilon > 0 \rightarrow$ traffic on bottom is envious
- $\varepsilon = 0 \rightarrow$ equilibrium
  - all traffic incurs one unit of delay

Can We Do Better?

Consider instead: traffic split equally

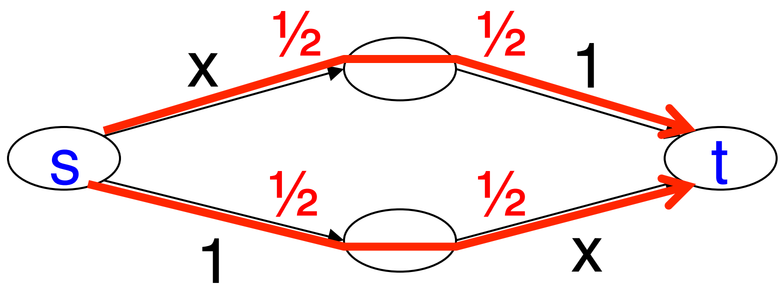


Improvement:

- half of traffic has delay 1 (same as before)
- half of traffic has delay ½ (much improved!)
- "price of anarchy" [Kousoupias/Papadimitriou 99] = 4/3

Braess's Paradox (1968)

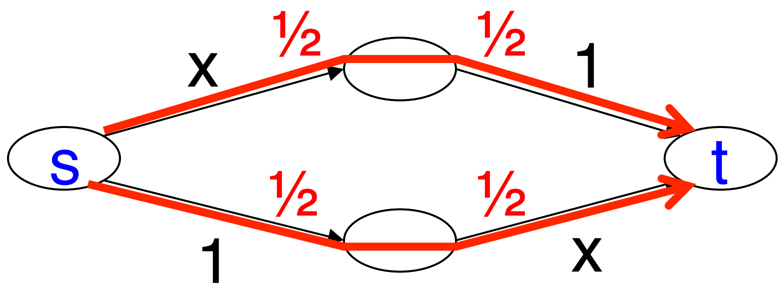
Initial Network:



Commute time = 1.5

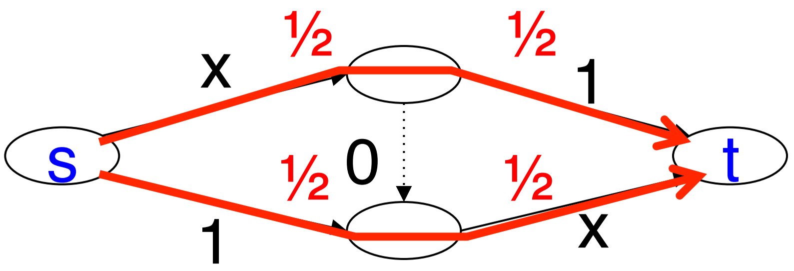
Braess's Paradox (1968)

Initial Network:



Commute time = 1.5

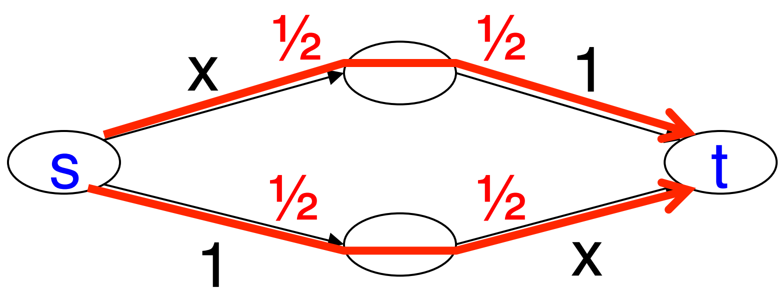
Augmented Network:



Now what?

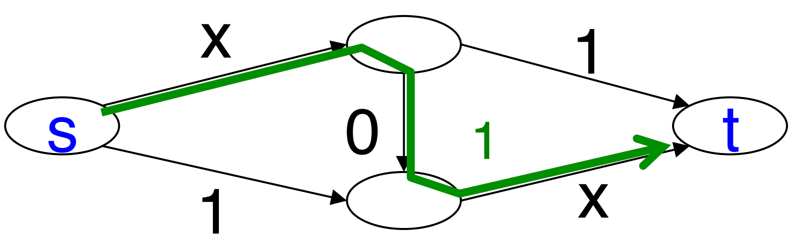
Braess's Paradox (1968)

Initial Network:



Commute time = 1.5

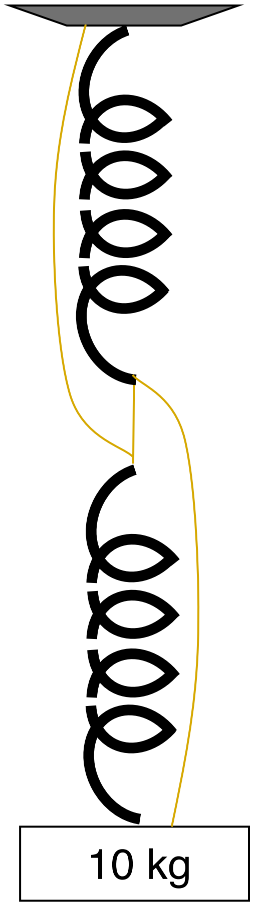
Augmented Network:



Commute time = 2

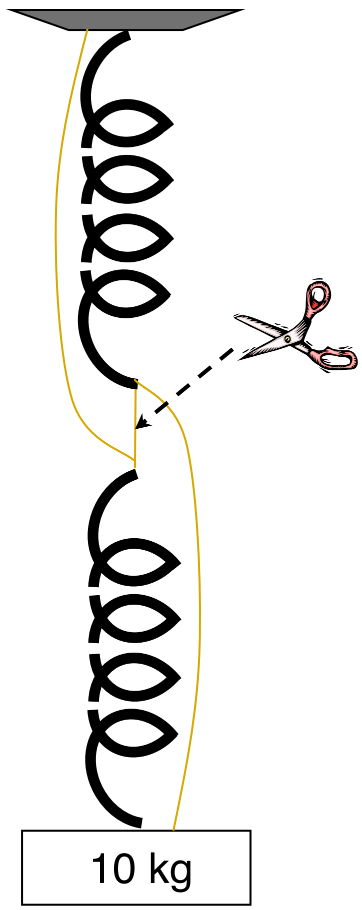
Price of anarchy = 4/3 in augmented network (again!)

Strings and Springs



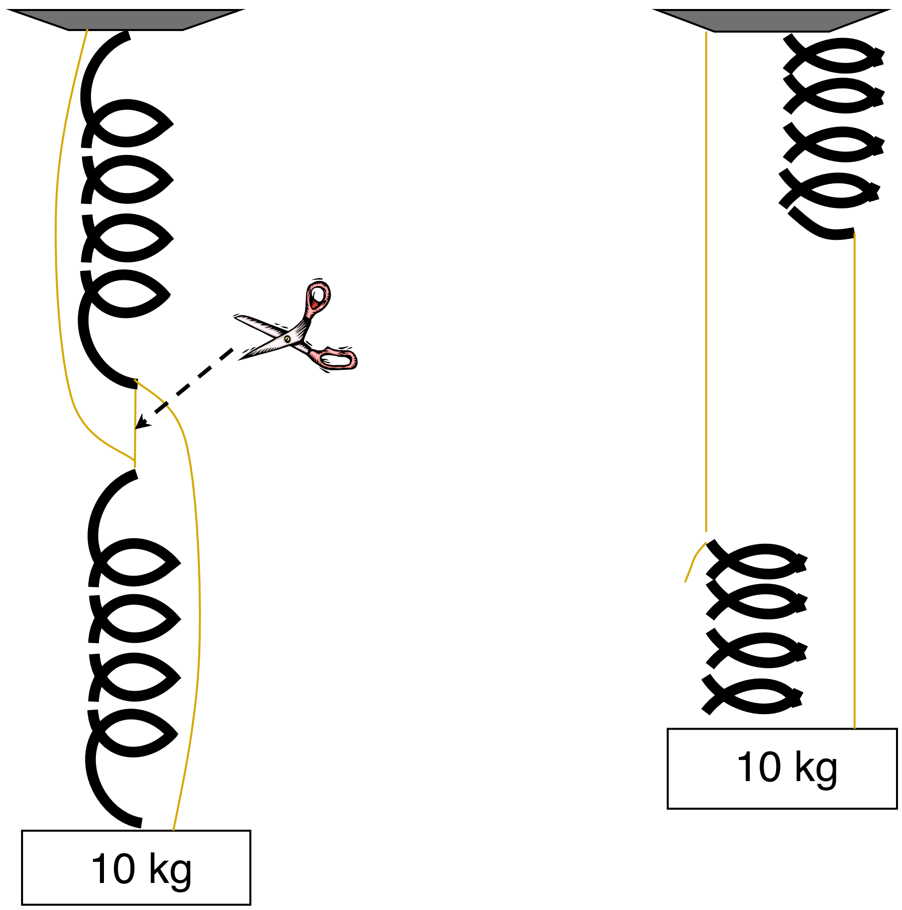
Cohen and Horowitz (Nature, 1991)

Strings and Springs



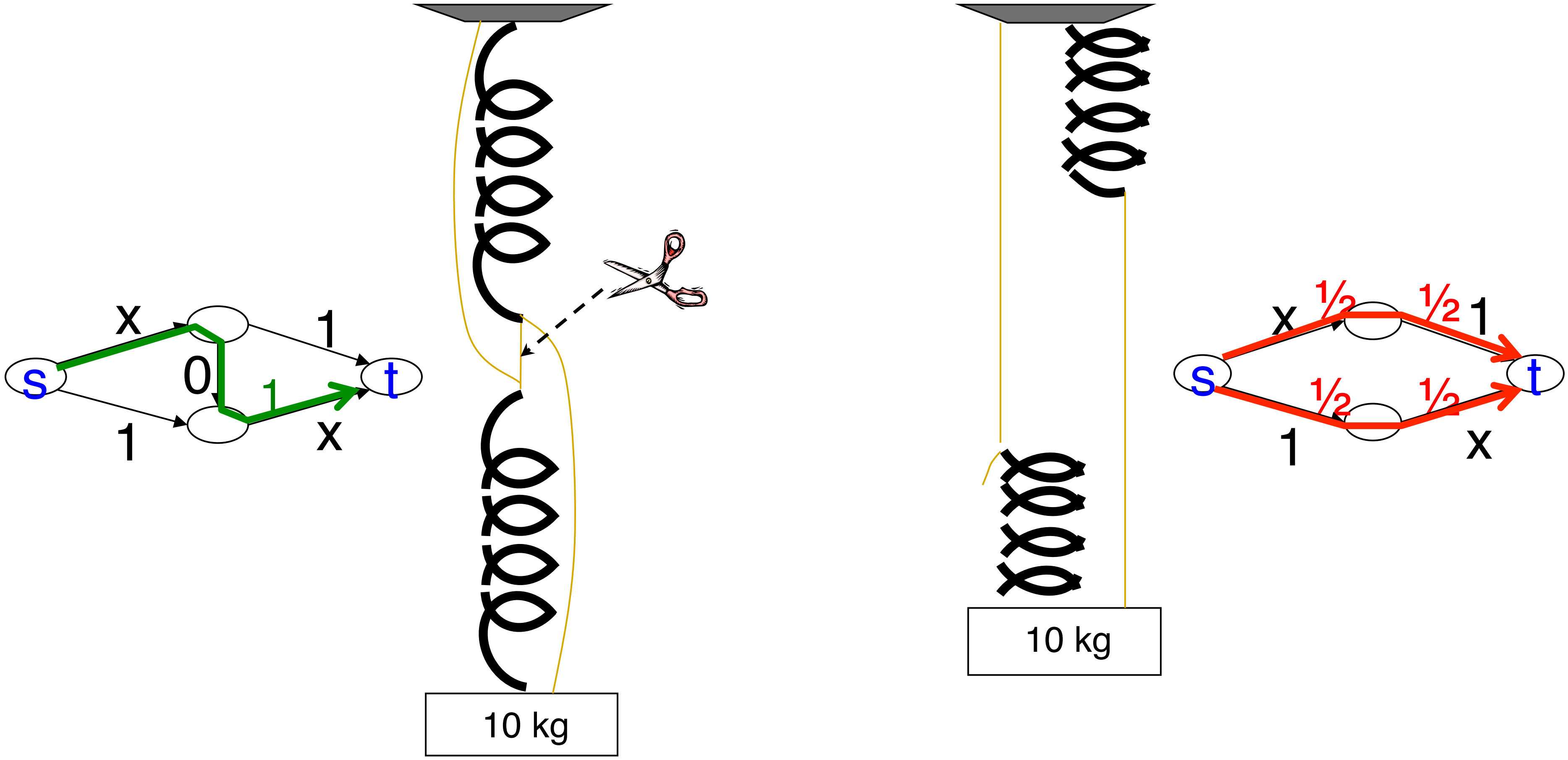
Cohen and Horowitz (Nature, 1991)

Strings and Springs



Cohen and Horowitz (Nature, 1991)

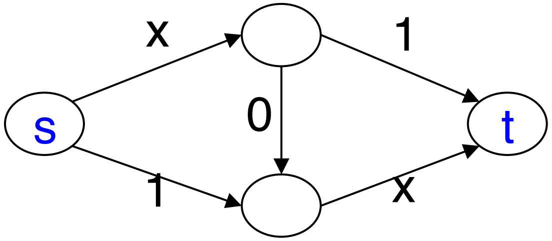
Strings and Springs

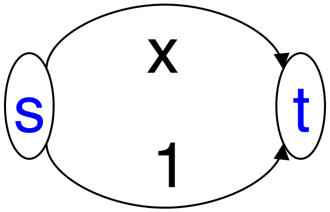


Cohen and Horowitz (Nature, 1991)

When Is the Price of Anarchy Bounded?

Examples so far:





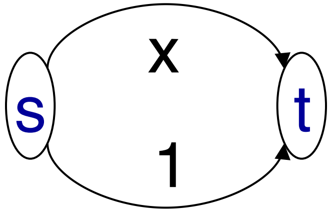
Question: does the price of anarchy stay small in bigger networks?

Affine Cost Functions

Defn: *affine cost function* has the form
$c_e(x)=a_ex+b_e$ [for $a_e, b_e \geq 0$].

Theorem: [Roughgarden/Tardos 02] for every network with affine cost functions:

average delay of equilibrium $\leq$ 4/3 $\times$ optimal average delay



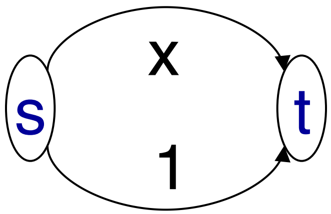
Affine Cost Functions

Defn: *affine cost function* has the form

$c_e(x)=a_e x+b_e$ [for $a_e, b_e \geq 0$].

Theorem: [Roughgarden/Tardos 02] for every network with affine cost functions:

average delay of equilibrium $\leq$ 4/3 $\times$ optimal average delay



Metaphor: electrical current in networks of resistors.

Question: why wasn't this proved in the 20th century by economists, or transportation scientists, or...?

Digression: Map Coloring

Rule: adjacent countries get distinct colors.

Digression: Map Coloring

need 3 colors (one per country)

Rule: adjacent countries get distinct colors.

Digression: Map Coloring

Rule: adjacent countries get distinct colors.

Digression: Map Coloring

needs 3 colors
(odd cycle)

Rule: adjacent countries get distinct colors.

Digression: Map Coloring

needs a 4^{th} color
(touches entire odd cycle)

needs 3 colors
(odd cycle)

Rule: adjacent countries get distinct colors.

Digression: Map Coloring

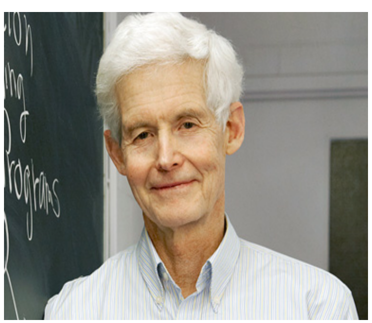
Rule: adjacent countries get distinct colors.

NP-Completeness

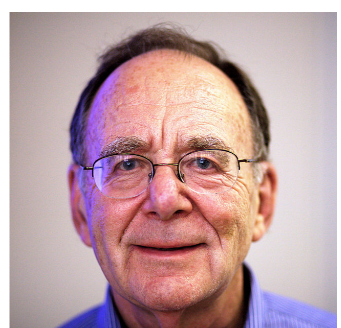
Cook-Karp-Levin Theorem (1971-1973):

Many fundamental problems are “NP-complete.”

- solvable in principle via exhaustive search
- no significantly better algorithms exist (if P≠NP)
- compromises required (use heuristics, tackle special/small cases, buy lots of hardware, etc.)



Stephen Cook



Richard Karp

Leonid Levin

Outline

1. Introduction: von Neumann, Nash, and Turing
2. Approximation: A Constructive Compromise
3. *Auction Design: The Rubber Meets the Road*
4. Complexity: Critiquing the Nash Equilibrium
5. Conclusions

F.C.C. Backs Proposal to Realign Airwaves

The New York Times

September 28, 2012 **By EDWARD WYATT**

WASHINGTON — The government took a big step on Friday to aid the creation of new high-speed wireless Internet networks that could fuel the development of the next generation of smartphones and tablets, and devices that haven't even been thought of yet.

The five-member Federal Communications Commission unanimously approved a sweeping, though preliminary, proposal to reclaim public airwaves now used for broadcast television and auction them off for use in wireless broadband networks, with a portion of the proceeds paid to the broadcasters.

The initiative, which the F.C.C. said would be the first in which any government would pay to reclaim public airwaves with the intention of selling them, would help satisfy what many industry experts say is booming demand for wireless Internet capacity.

Mobile broadband traffic will increase more than thirtyfold by 2015, the commission estimates. Without additional airwaves to handle the traffic, officials say, consumers will face more dropped calls, connection delays and slower downloads of data.

FCC Incentive Auction

Broadcast Television Incentive Auction (3/16-3/17):

- Reverse Auction: buy TV broadcast licenses
  - Final tally: ≈$10 billion cost
- Forward Auction: sell wireless broadband licenses
  - Final tally: ≈$20 billion revenue
- Revenue to cover auction costs, fund a new first responder network, reduce the deficit (!)
  - “Middle Class Tax Relief and Job Creation Act”

Bad Designs Cost Billions

New Zealand, 1990:

- simultaneous sealed-bid 2^{nd}-price auctions for 10 interchangeable TV broadcasting licenses
  - creates tricky coordination problem
- projected revenue: 250M

Bad Designs Cost Billions

New Zealand, 1990:

- simultaneous sealed-bid 2nd-price auctions for 10 interchangeable TV broadcasting licenses
  - creates tricky coordination problem
- projected revenue: 250M; actual = 36M
- often huge difference between top two bids

US, 2016: $10s of billions at stake.

Reverse Auction Format

"Descending Clock Auction":

[Milgrom/Segal 14] (extending [Moulin/Shenker 01],[Mehta/Roughgarden/Sundararajan 09])

- each round, each broadcaster offered a buyout price (decreases over time)
  - declined ➔ exits, retains license
    accepted ➔ moves to next round

Milgrom

Segal

The Stopping Rule

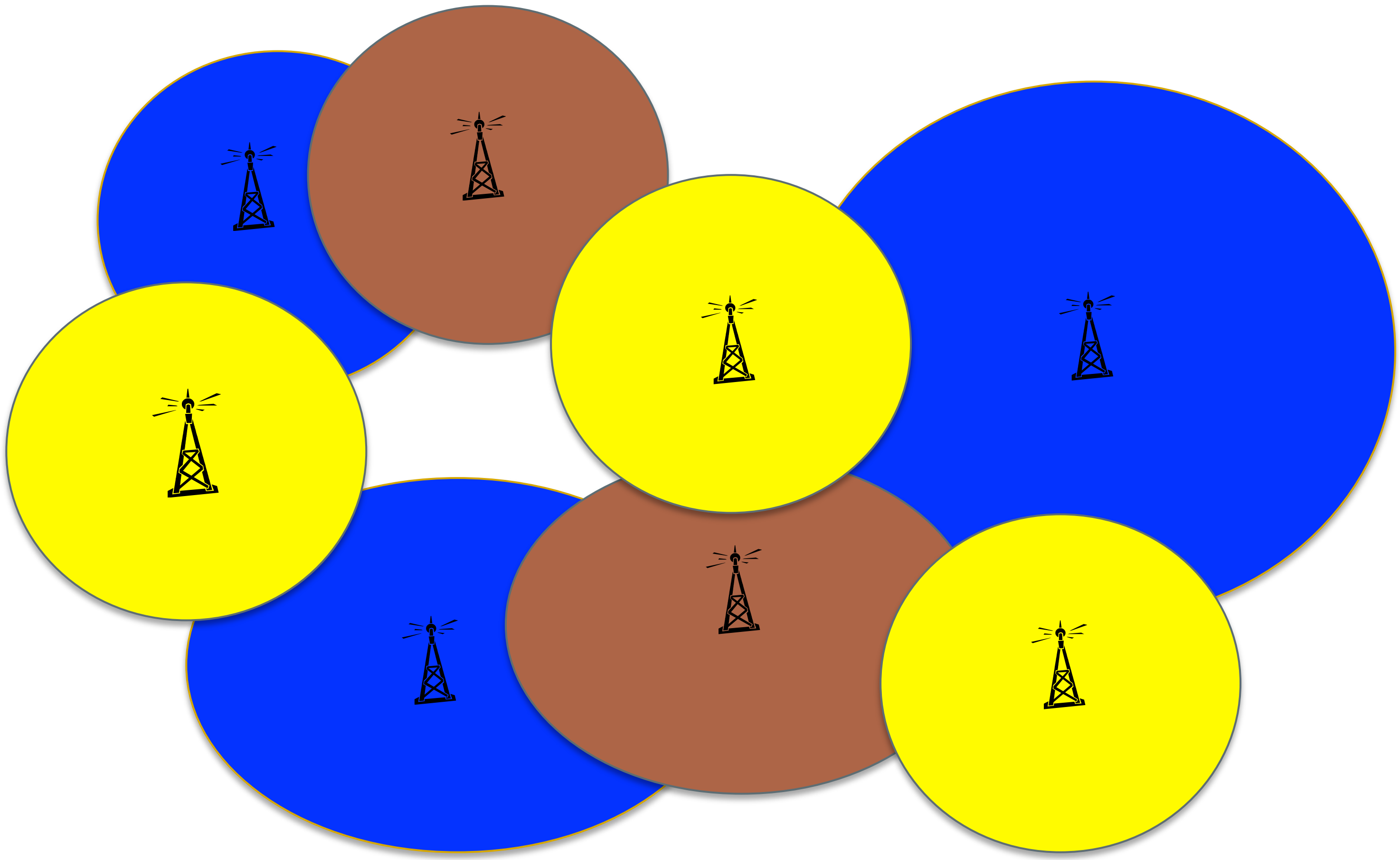
Intuition: stop auction when prices are as low as possible, subject to clearing enough spectrum.

Example goal: from channels 38-51, clear 10 of them nationwide.

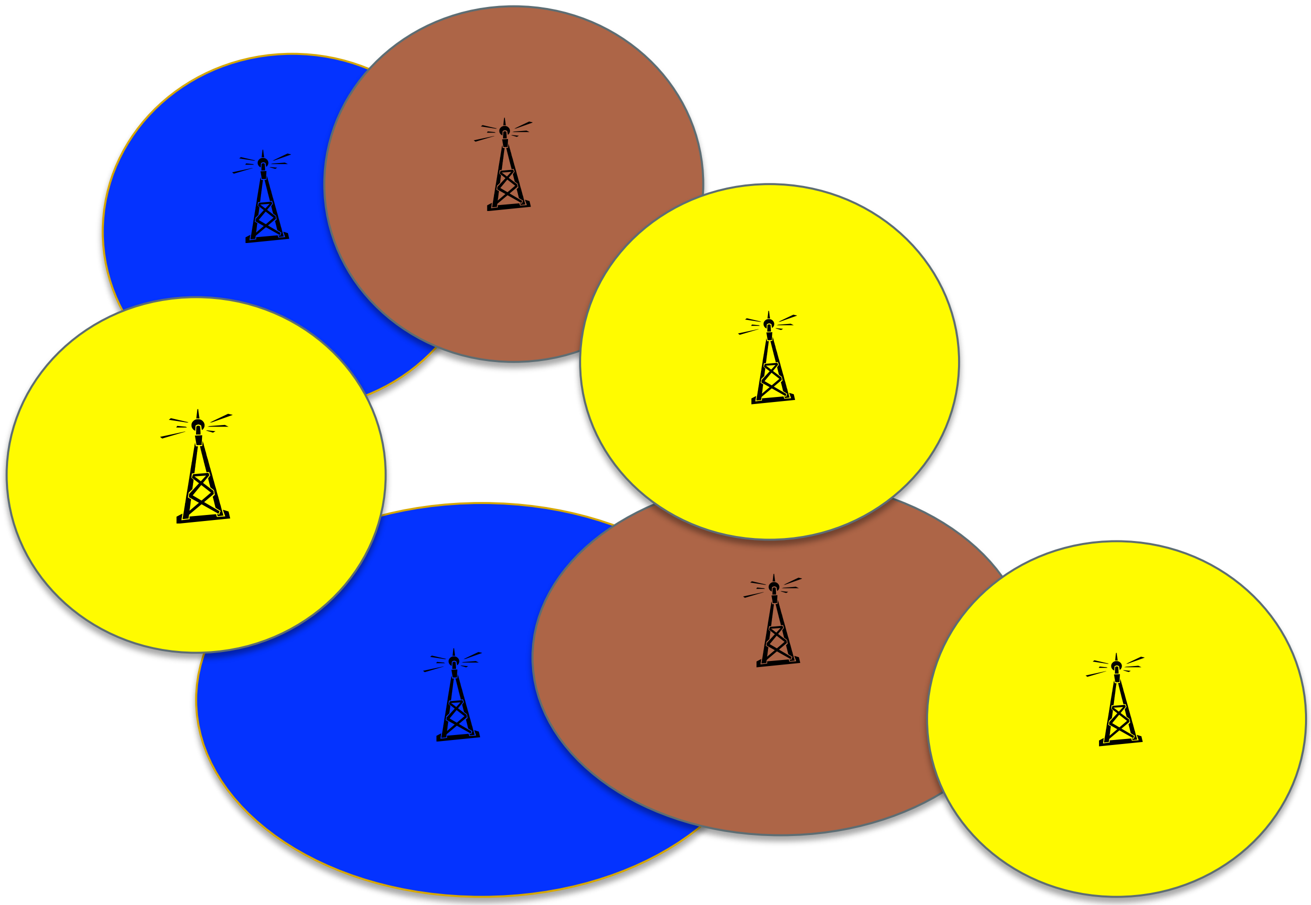
Issue: buyouts scattered across channels.

Solution: repack remaining TV stations into a smaller subset of channels (e.g., 38-41).

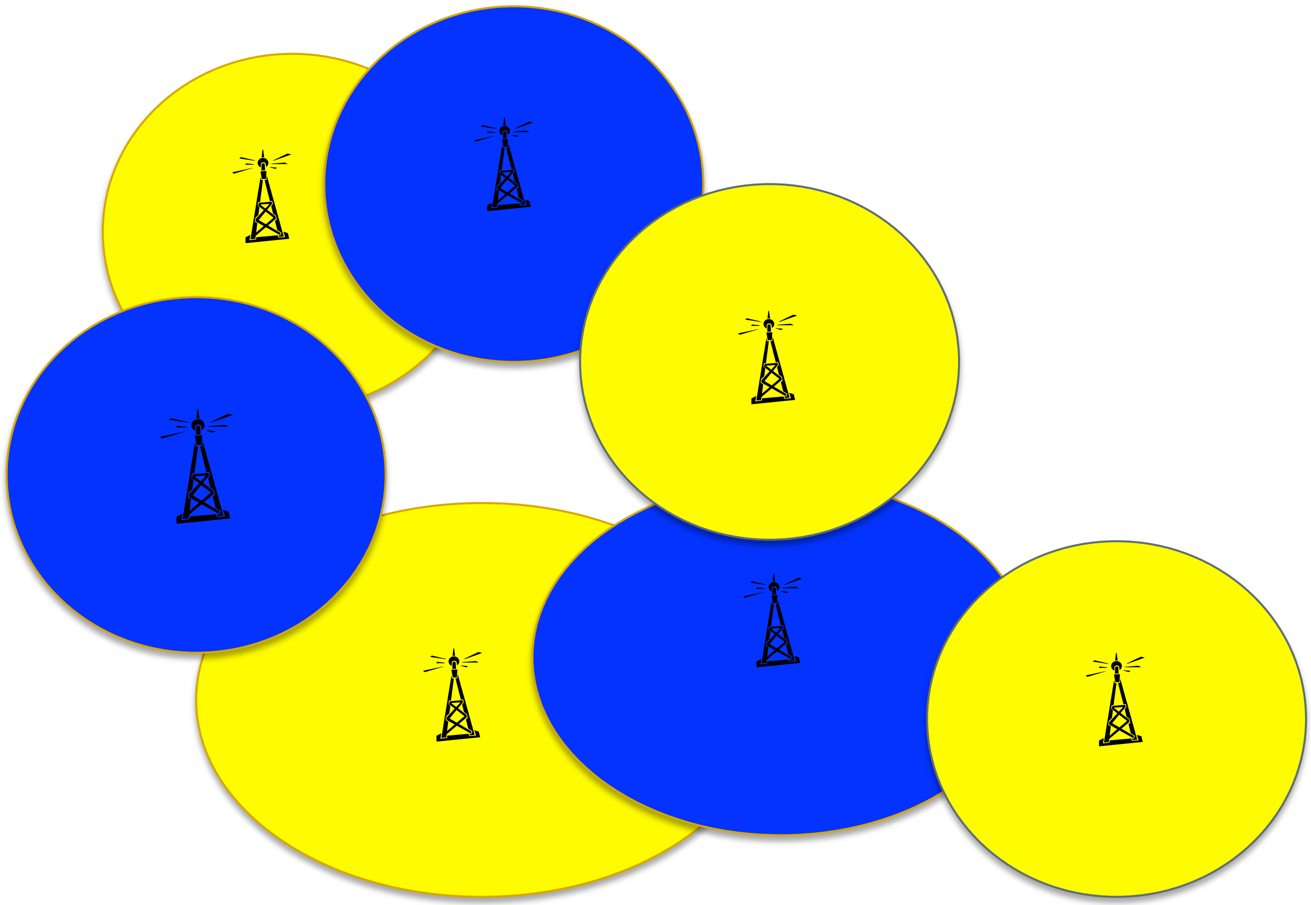
The Repacking Problem



The Repacking Problem



The Repacking Problem



The Need for Algorithms

Cool fact: state-of-the-art algorithms for solving NP-complete problems both necessary and sufficient to solve repacking problem quickly.

Leyton-Brown

[Leyton-Brown et al. 13, 14, 17]

- encode as satisfiability (SAT)
- use presolvers, solver configuations tuned to interference constraints, caching tricks

Outline

1. Introduction: von Neumann, Nash, and Turing
2. Approximation: A Constructive Compromise
3. Auction Design: The Rubber Meets the Road
4. *Complexity: Critiquing the Nash Equilibrium*
5. Conclusions

Constructive Nash's Theorem?

Nash's Theorem (1950): every finite game has at least one Nash equilibrium.

- many interpretations require the determination of an equilibrium
  - by the players, designer, etc.
- fixed-point proof offers no help

An equilibrium

Challenge: "more constructive" version.

- cf., "bounded rationality" [Simon]

Classifying the complexity of computing a Nash equilibrium

Idea: is computing a Nash equilibrium NP-complete?

Classifying the complexity of computing a Nash equilibrium

Idea: is computing a Nash equilibrium NP-complete?

Answer: [Megiddo 88] probably not.

- reason: guaranteed existence

Classifying the complexity of computing a Nash equilibrium

Idea: is computing a Nash equilibrium NP-complete?

Answer: [Megiddo 88] probably not.

- reason: guaranteed existence

Upshot: Need to refine NP to obtain the right complexity class.

Proposal: [Papadimitriou 94] PPAD.

Papadimitriou

Nash Meets Turing

Theorem: [Daskalakis/Goldberg/Papadimitriou 06, Chen/Deng/Teng 06] Computing a Nash equilibrium is PPAD-complete.

- also intractable in other senses [Etessami/Yannakakis 07, Hart/Mansour 09]
- even approximate Nash equilibria [Rubinstein 15,16], [Roughgarden/Weinstein 16], [Babichenko/Rubinstein 17]

Interpretation: no general constructive version of Nash's theorem. Compromises required.

Conclusions

- many points of contact between theory CS and game theory/econ over past 15 years
  - including many not mentioned today

Conclusions

- many points of contact between theory CS and game theory/econ over past 15 years
- many 21st-century computer science applications require economic reasoning
  - routing in communication networks
  - auctions for online advertising
  - cryptocurrencies
  - etc.

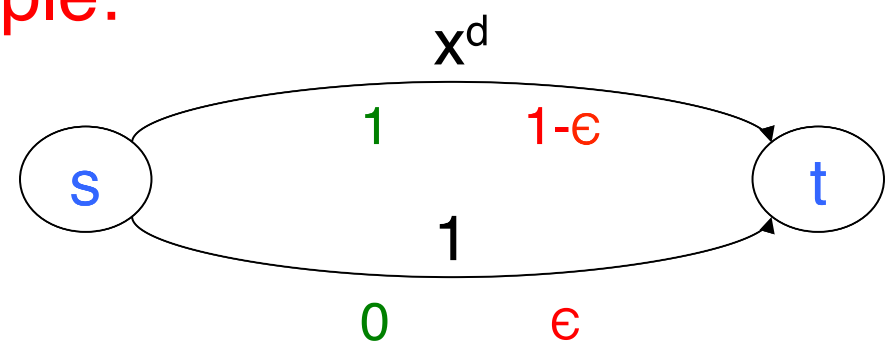
Conclusions

- many points of contact between theory CS and game theory/econ over past 15 years
- many 21st-century computer science applications require economic reasoning
- theory CS can articulate computational barriers and offers constructive compromises
  - approximation
  - workarounds for NP-complete problems
  - intractability of Nash equilibria

FIN

A Nonlinear Pigou Network

Bad Example: (d large)



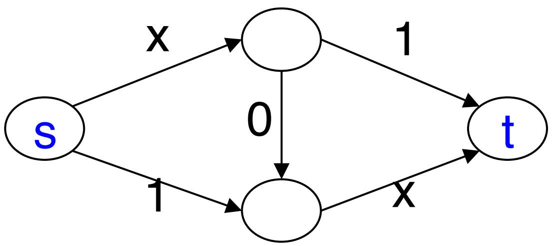
equilibrium has average delay 1, optimal → 0

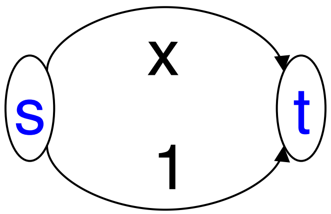
Upshot: price of anarchy unbounded as d tends to infinity.

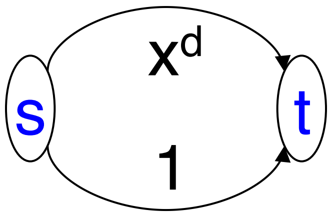
Goal: weakest-possible conditions under which the price of anarchy is small.

When Is the Price of Anarchy Bounded?

Examples so far:





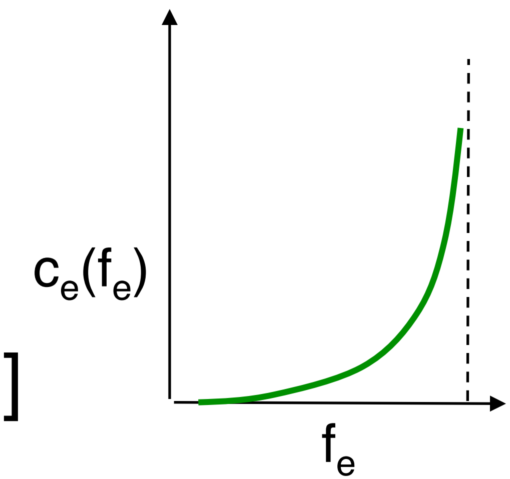


Hope (revised): imposing additional structure on the cost functions helps.

- worry: bad things happen in larger networks

Benefit of Overprovisioning

M/M/1 Cost Functions: $c(x) = 1/(u_e - x)$



Suppose: β fraction of each edge unused [i.e., max utilization (1-β)%]

Then: Price of anarchy is at most $\frac{1}{2}(1+1/\sqrt{\beta})$

- arbitrary network size/topology, traffic matrix

Moral: Even modest (10%) over-provisioning sufficient for near-optimal routing.

The Evidence Against

Rabin

- Rabin (1957): “It is quite obvious that not all games considered in the theory can actually be played by human beings.”
- Gilboa/Zemel (1989): Many problems about Nash equilibria are NP-complete.
- Dutta (2003): algebraic universality of Nash equilibria
- Hart/Mas-Colell (2003): no natural learning algorithms
- Savani/von Stengel (2004): Lemke-Howson can require exponential time to compute an equilibrium